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26 May 2023

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May 26, 2023

Abstract

Knowledge creation either in isolation or joint with another person, using either face to face or internet contact, incorporating internet search ability is analyzed. In addition to formal knowledge, tacit knowledge plays an essential role in the knowledge production process. Due to a myopic decision rule used by knowledge workers, the sink point of the dynamic system is found not be the most effective long run profile for knowledge creation. The framework is applied to pandemic restrictions on face to face communication; workers with longer commutes experience less of a relative productivity loss from restrictions than workers with shorter commutes.

JEL codes: D83, L86 Keywords: Knowledge creation; Tacit knowledge; Multimodal communication; Pandemic restrictions

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1 Introduction

How does knowledge creation function when there are multiple ways for people to communicate, for example face to face or using the internet? Relative to the one communication channel case, what different patterns of joint research among knowledge workers emerge, and how is the productivity of research work affected?

Through a sequence of four papers, Berliant and Fujita (2008, 2009, 2011, 2012), we have developed a micromodel of knowledge creation based on the interactions among a group of heterogeneous people. These papers form the basis for the analysis here. In Berliant and Fujita (2008), we develop the basic model and analyze interactions among a group of knowledge workers under the assumption of symmetry of the state of knowledge. Berliant and Fujita (2009) investigates further the case of two knowledge workers, relaxing the symmetry assumption, whereas Berliant and Fujita (2011) embeds the basic model in a growth framework to analyze dynamics and the efficiency properties of equilibrium. Finally, Berliant and Fujita (2012) constructs a two region version of the model to examine the emergence of cultures in the knowledge production community.

To address our motivating questions, as depicted in Figure 1, a person, say i, with knowledge K_i can develop new ideas in isolation while interacting with *the Server*. Or person i may create new ideas jointly with another person, say j, by interacting through *the Net* or by working *F2F (Face to Face)*. Using this extended model, we can examine the impact of rapidly developing ICT (including AI) on knowledge creation activities.

Figure 1

This paper also aims to provide a theoretical framework for the study of specific recent issues such as the impact of the Covid-19 pandemic on knowledge creation, or the effect of urban structure on the productivity of knowledge workers as well as on the pattern of knowledge work in large cities.

Our main findings are as follows. In contrast with our previous work, where tacit knowledge was implicit, the incorporation of tacit knowledge explicitly into the framework changes the results. In particular, knowledge is generated not only by patent output but also through search and independent thinking in preparation for joint or independent research. This represents tacit knowledge retained by a specific worker for future work. It slows the accumulation of knowledge in common. Our conclusions are threefold. First, the dynamics of the system with two workers and tacit knowledge are, in the end, very different from the dynamics without tacit knowledge. Second, the steady state will not, in general, be the state with highest productivity. The net effect is that achieving and maintaining the highest productivity profile of knowledge in common and differential knowledge requires more heterogeneity or larger research groups than we found in our previous work. Third, the effect of tacit knowledge on knowledge productivity is not internalized by the knowledge workers. We shall provide more discussion of these ideas in the conclusions.

Applying this framework to pandemic restrictions, we show, for example, how the productivity of knowledge workers with longer commutes to work is affected less than those with shorter commutes when pandemic restrictions on face to face work are implemented.

Our analysis proceeds as follows. In section 2, we develop our model of two knowledge workers using multiple modes of communication and generating tacit knowledge in addition to patents, and analyze the steady state. In section 3, we apply this analysis to consider pandemic effects. Section 4 considers knowledge growth under symmetry, whereas section 5 analyzes dynamics in the two person case. Section 6 presents our conclusions and suggestions for future research. An appendix contains a discussion of the multiple roles played by knowledge in common in joint knowledge creation.

2 The model with two persons in the stationary state

In this section, we consider two persons/researchers, i and j, and extend the model of Berliant and Fujita (2008, 2009) by incorporating multiple modes of interaction. Wherever possible, we use the same notation as in Berliant and Fujita (2008, 2009).

Following on our earlier work, we model knowledge creation as a process of opening up boxes containing ideas. The labels on the boxes, that describe their contents, are known to all, but it takes time to understand the *contents* of the boxes. An example of such a box of knowledge is the creation of this paper by its authors. The title is its label. Another example is a new recipe for curry rice. The rate at which the boxes can be opened depends on the stock already opened by a particular person, either alone or with someone else. When working in isolation, the total stock of knowledge or boxes already opened by that person affects the rate at which new boxes are opened. When working jointly, both the total number of boxes opened previously and their profile matter. Specifically, whether they were opened together, and thus become mutual knowledge, or independently, and thus become exclusive knowledge, matters.

As there is an infinite number of boxes or potential ideas, we assume that the probability that knowledge workers who are not working together open the same box is zero.

In what follows, in contrast with our earlier work, we allow more channels of communication between knowledge workers, namely face to face and internet communication. Moreover, we decompose knowledge creation into more elementary units or phases, to be described formally in this section. These phases involve internet search and thinking on one's own, both when creating knowledge alone and when preparing to work with someone else. When working with someone else, there will also be time spent communicating either face to face or over the internet. We allow each person to optimize over the allocation of time or frequency to these various activities. For example, when creating a new recipe for curry rice, a chef might search the internet for recipes good and bad (respecting reviews on line), and then might engage in trial and error. The allocation of time is chosen by the chef. Much of what the chef learns becomes tacit knowledge *beyond* the final recipe; this recipe might be secret and used by his restaurant, or sold. The rate at which new recipes are created will depend on the chef's experience with this type of knowledge work, but also on *the tacit knowledge accumulated through the creation process.*

To elaborate our model, let us consider a specific time, $t \in [0, \infty)$, and let the following variables represent the state of each person's knowledge at time t (whenever clear, dropping t for simplicity):

n_k: the size of person k's knowledge (or number of ideas known by person k at time t); $k = i, j$

$n_{ij}^c \equiv n_{ji}^c$: the size of knowledge that i and j both know, or the *common knowledge* for i and j

n_{ij}^d: the size of knowledge known by i but not known by j, or the *differential knowledge* of i from j,

n_{ji}^d: the size of knowledge known by j but not known by i, or the *differential knowledge* of j from i.

By definition,

$$n_i = n_{ij}^c + n_{ij}^d, \;\; n_j = n_{ij}^c + n_{ji}^d.$$

Let

$$n^{ij} \equiv n_{ij}^c + n_{ij}^d + n_{ji}^d = n_i + n_j - n_{ij}^c$$

be the size of total knowledge that is known either by i or j.

Next, we define the proportion of each type of knowledge in the total size of knowledge n^{ij}:

$$\begin{aligned} m_{ij}^c &= \frac{n_{ij}^c}{n^{ij}}, \\ m_{ij}^d &= \frac{n_{ij}^d}{n^{ij}}, \;\; m_{ji}^d = \frac{n_{ji}^d}{n^{ij}}, \end{aligned}$$

implying that

$$m_{ij}^c + m_{ij}^d + m_{ji}^d = 1, \tag{1}$$

and hence

$$n_i = n^{ij} \cdot (1 - m_{ji}^d), \;\; n_j = n^{ij} \cdot (1 - m_{ij}^d), \tag{2}$$

or

$$\frac{n_i}{n^{ij}} = 1 - m_{ji}^d, \;\; \frac{n_j}{n^{ij}} = 1 - m_{ij}^d.$$

Using this notation, we describe next the two alternative ways of creating knowledge (at time t).

In what follows, consistent with the notation introduced above, lower case letters such as i and j represent persons, whereas upper case letters represent activities. Examples of the latter include I, representing *isolated* or *independent* activity, and J, representing *joint* activity or activity for the purpose of *joint* knowledge creation. Likewise, S represents *search* activity, to be explained next.

What we mean by *search* is to search the web by oneself to prepare for knowledge creation activity that will occur either in isolation or jointly with another. Examples are using Google or ChatGPT. The activity is a form of directed search, in contrast with undirected search. That is, the search terms or questions guide the use of the web in an important and nonrandom way, and are progressively refined over the time used for search. Just a few years ago, when beginning a project, an economics researcher would search the Econlit database, for example, using search terms that define the new project. The purpose would be to find related papers at the frontier of knowledge and to see similar work in terms of assumptions, implications, models, and empirics. For example, aside from the references we knew about from previous joint work, to compose this paper we searched on key phrases such as "R&D during the Covid 19 pandemic." Refinement of search terms, as well as digesting material, takes time and effort. Nowadays, with Google Scholar and ChatGPT, the effectiveness of search has improved dramatically over primitive times. We will parameterize this effectiveness of directed search in our model.

(i) Knowledge creation *in Isolation* by each person consists of two distinct activities: thinking in isolation and searching the web for help. We assume that knowledge creation in isolation by person i is governed by the following equation:

$$A_{ii}^{I} = \alpha_I \cdot [\omega_{IS} \cdot (\alpha_{IS} \cdot n_i)]^{\rho_{IS}} \cdot [\omega_{IT} \cdot (\alpha_{IT} \cdot n_i)]^{\rho_{IT}} \quad (\alpha_I, \alpha_{IS}, \alpha_{IT} > 0) \tag{3}$$

where A_{ii}^{I} is the number of ideas produced. On the right hand side, $\rho_{IS}, \rho_{IT} \geq 0$ are fixed parameters that weight the search and thought activities with $\rho_{IS} + \rho_{IT} = 1$. We divide knowledge creation activity in a time period into two parts: search in isolation with frequency $\omega_{IS} \geq 0$ and thinking in isolation $\omega_{IT} \geq 0$, where $\omega_{IS} + \omega_{IT} = 1$. In both cases, productivity depends on what a person already knows.[1] The expression $[\omega_{IS} \cdot (\alpha_{IS} \cdot n_i)]^{\rho_{IS}}$ gives the total productivity of the search frequency, where $\alpha_{IS} \cdot n_i$ is the output of the search activity. Here $\alpha_{IS} > 0$ represents the effectiveness of search, whereas $\alpha_{IT} > 0$ represents the effectiveness of thinking alone. The expression $[\omega_{IT} \cdot (\alpha_{IT} \cdot n_i)]^{\rho_{IT}}$ gives the total productivity of the thought frequency, where $\alpha_{IT} \cdot n_i$ is the output of the thought activity.

This can be rewritten as:

$$A_{ii}^{I} = \alpha_I \cdot \alpha_{IS}^{\rho_{IS}} \cdot \alpha_{IT}^{\rho_{IT}} \cdot \omega_{IS}^{\rho_{IS}} \cdot \omega_{IT}^{\rho_{IT}} \cdot n_i$$

[1] "What you can learn from Wikipedia depends on what you already know."

Knowledge worker i optimizes the choice of frequency over the two activities:

$$Max\left\{A_{ii}^{I}\middle|\,\omega_{IS}+\omega_{IT}=1,\omega_{IS}\geq 0,\omega_{IT}\geq 0\right\}$$

yielding the optimal choices of frequencies ω_{IS}^{*} and ω_{IT}^{*}:

$$\omega_{IS}^{*}=\rho_{IS},\omega_{IT}^{*}=\rho_{IT}$$

Thus, the optimized value of knowledge output is:

$$\begin{aligned} A_{ii}^{I*} &= \Phi_I\cdot n_i \\ \text{where }\Phi_I &\equiv \alpha_I\cdot(\alpha_{IS}\cdot\rho_{IS})^{\rho_{IS}}\cdot(\alpha_{IT}\cdot\rho_{IT})^{\rho_{IT}} \end{aligned} \quad (4)$$

Likewise, for person j, we have

$$A_{jj}^{I*}=\Phi_I\cdot n_j \quad (5)$$

where Φ_I represents the same function of parameters as above. We may note that the search productivity parameter α_{IS} depends on several factors. First, it depends on the search technology at the time the work is done. For example, Google (1998) preceded Google Scholar (2004) and more recently ChatGPT. Second, it depends on the knowledge stock at the time, for example in Wikipedia. Third, it depends on the learning capacity of person i. Similarly for the parameter ρ_{IS}. In the long run, parameters might change, but here, for simplicity, we take the parameters as fixed.

(ii) *Joint* knowledge creation by two persons is conducted through the combination of the following three *basic activities* with appropriate frequencies.
• *Working/thinking in Isolation* for the common purpose of joint knowledge creation, which is governed by the equation:

$$a_{ii}^{I}=\alpha_{JI}\cdot[\omega_{JIS}\cdot(\alpha_{JIS}\cdot n_i)]^{\rho_{JIS}}\cdot[\omega_{JIT}\cdot(\alpha_{JIT}\cdot n_i)]^{\rho_{JIT}} \quad (\alpha_{JI},\alpha_{JIS},\alpha_{JIT}>0) \quad (6)$$

where ρ_{JIS} and ρ_{JIT} are positive parameters such that

$$\rho_{JIS}+\rho_{JIT}=1,$$

whereas ω_{JIS} and ω_{JIT} are endogenous frequencies such that

$$\omega_{JIS}+\omega_{JIT}=1,\ \omega_{JIS}\geq 0,\ \omega_{JIT}\geq 0.$$

The left hand side of the equation (6), a_{ii}^{I}, represents the number of *intermediate ideas* created specifically for the purpose of *joint* knowledge creation. This is analogous to equation (3), but in the context of preparing for joint knowledge creation. The endogenous frequencies ω_{JIS} and ω_{JIT} again represent the time devoted to *search* and *thinking* activities, respectively. Solving the optimization problem similarly to the case of knowledge creation in isolation, the optimal choices of frequencies ω^*_{JIS} and ω^*_{JIT} are given by:

$$\omega^*_{JIS} = \rho_{JIS}, \omega^*_{JIT} = \rho_{JIT}$$

The optimized value of knowledge production is given by:

$$\begin{aligned} a^{I*}_{ii} &= \Phi_{JI} \cdot n_i \\ \text{where } \Phi_{JI} &\equiv \alpha_{JI} \cdot (\alpha_{JIS} \cdot \rho_{JIS})^{\rho_{JIS}} \cdot (\alpha_{JIT} \cdot \rho_{JIT})^{\rho_{JIT}} \end{aligned} \tag{7}$$

Likewise, for person j, we have:

$$a^{I*}_{jj} = \Phi_{JI} \cdot n_j \tag{8}$$

where Φ_{JI} represents the same function of parameters as above.

• Jointly working *F2F*, conducted through the combination of two activities, searching the web independently and thinking together, is governed by the following equation:

$$\begin{aligned} a^F_{ij} &= a^F_{ji} = \alpha_F \cdot [\omega_{ISF} \cdot (\alpha_{ISF} \cdot n_i + \alpha_{ISF} \cdot n_j)]^{\rho_{ISF}} \\ &\times \left\{ \left[\omega_{JTF} \cdot \left(\alpha_{JTF} \cdot (\beta_F(\theta_F) \cdot n^c_{ij})^{1-\theta_F} \cdot (n^d_{ij} \cdot n^d_{ji})^{\frac{\theta_F}{2}} \right) \right] \right\}^{\rho_{JTF}} \end{aligned} \tag{9}$$

where ρ_{ISF} and ρ_{JIT} are positive parameters such that

$$\rho_{ISF} + \rho_{JTF} = 1,$$

whereas ω_{ISF} and ω_{JTF} are endogenous frequencies such that

$$\omega_{ISF} + \omega_{JTF} = 1, \ \omega_{ISF} \geq 0, \ \omega_{JTF} \geq 0.$$

The left hand side of this equation, a^F_{ij}, is the quantity of new intermediate ideas generated by F2F work for the purpose of joint creation. The right hand side of the first line of the equation describes independent search when working face to face. Each of the workers has their own web access, and searches independently at endogenous frequency ω_{ISF}. What person learns from the web is proportional to their knowledge stock n_i or n_j. The weight of this search activity in knowledge production is parameterized by ρ_{ISF}, whereas its effectiveness is parameterized by α_{ISF}.

The second line of the equation represents how the frequency ω_{JTF} of face to face joint thinking translates into new intermediate ideas. When two persons are thinking jointly, new intermediate ideas are generated at a rate proportional to the normalized product of their knowledge in common, n_{ij}^c, the differential knowledge of i from j, n_{ij}^d, and the differential knowledge of j from i, n_{ji}^d. The rate of creation of new intermediate ideas is high when the proportions of knowledge in common, knowledge exclusive to person i, and knowledge exclusive to person j are in balance. The parameter θ_F represents the weight on differential knowledge as opposed to knowledge in common in the production of new ideas. Knowledge in common is necessary for communication between the two persons, whereas knowledge exclusive to one person or the other implies more heterogeneity or originality in the collaboration. Finally, $\beta_F(\theta_F) \geq 1$ is used to represent an implicit weight on knowledge in common as follows. If the two researchers work separately, the stock of knowledge in common is counted twice ($\beta_F(\theta_F) = 2$), since it is a part of the stock of each of the people working independently. Furthermore, knowledge in common plays an important role in facilitating communication between the two workers when they work together. So relative to working independently, the size of the stock of knowledge in common while working jointly should be counted more than once. However, the degree of double counting can be limited. We will explain the nature of the function $\beta_F(\theta_F)$ in more detail in the Appendix.

Knowledge workers i and j jointly optimize the choice of frequencies ω_{IS} and ω_{IT} over the two activities:

$$Max \left\{ a_{ij}^F \middle| \, \omega_{ISF} + \omega_{JTF} = 1, \; \omega_{ISF} \geq 0, \; \omega_{JTF} \geq 0. \right\}$$

yielding the optimal choices of frequencies ω_{ISF}^* and ω_{JIT}^*:

$$\omega_{ISF}^* = \rho_{ISF}, \omega_{JTF}^* = \rho_{JTF}$$

Thus, the optimized value of knowledge output created face to face is:

$$\begin{aligned} a_{ij}^{F*} &= \Phi_{JF} \cdot (n_i + n_j)^{\rho_{ISF}} \cdot \left[\left(n_{ij}^c \right)^{1-\theta_F} \cdot (n_{ij}^d \cdot n_{ji}^d)^{\frac{\theta_F}{2}} \right]^{\rho_{JTF}} \qquad (10) \\ \text{where } \Phi_{JF} &\equiv \alpha_F \cdot (\alpha_{ISF} \cdot \rho_{ISF})^{\rho_{ISF}} \cdot (\alpha_{JTF} \cdot \rho_{JTF})^{\rho_{JTF}} \cdot \beta_F(\theta_F)^{(1-\theta_F) \cdot \rho_{JTF}}. \end{aligned}$$

- Similarly, jointly working through the *Net* is governed by the equation:

$$\begin{aligned} a_{ij}^N &= a_{ji}^N = \alpha_N \cdot [\omega_{ISN} \cdot (\alpha_{ISN} \cdot n_i + \alpha_{ISN} \cdot n_j)]^{\rho_{ISN}} \\ &\times \left\{ \left[\omega_{JTN} \cdot \left(\alpha_{JTN} \cdot (\beta_N(\theta_N) \cdot n_{ij}^c)^{1-\theta_N} \cdot (n_{ij}^d \cdot n_{ji}^d)^{\frac{\theta_N}{2}} \right) \right] \right\}^{\rho_{JTN}}. \end{aligned}$$

where ρ_{ISN} and ρ_{JTN} are positive parameters such that

$$\rho_{ISN} + \rho_{JTN} = 1,$$

whereas ω_{ISN} and ω_{JTN} are endogenous frequencies such that

$$\omega_{ISN} + \omega_{JIT} = 1,\ \omega_{ISN} \geq 0,\ \omega_{JTN} \geq 0.$$

The two types of activities in new intermediate idea production are again independent search on the Net and joint thinking through the Net, where the frequency, ω_{ISN} and ω_{JTN}, of each is determined endogenously. We define $\beta_N(\theta_N)$ analogously to $\beta_F(\theta_F)$.

Knowledge workers i and j jointly optimize the choice of frequencies ω_{IS} and ω_{IT} over the two activities:

$$Max \left\{ a_{ij}^N \middle| \omega_{ISN} + \omega_{JTN} = 1,\ \omega_{ISN} \geq 0,\ \omega_{JTN} \geq 0. \right\}$$

yielding the optimal choices of frequencies ω_{ISN}^* and ω_{JTN}^*:

$$\omega_{ISN}^* = \rho_{ISN}, \omega_{JTN}^* = \rho_{JTN}$$

Thus, the optimized value of knowledge output created using the Net is:

$$a_{ij}^{N*} = \Phi_{JN} \cdot (n_i + n_j)^{\rho_{ISN}} \cdot \left[\left(n_{ij}^c \right)^{1-\theta_N} \cdot (n_{ij}^d \cdot n_{ji}^d)^{\frac{\theta_N}{2}} \right]^{\rho_{JTN}} \quad (11)$$

$$\text{where } \Phi_{JN} \equiv \alpha_N \cdot (\alpha_{ISN} \cdot \rho_{ISN})^{\rho_{ISN}} \cdot (\alpha_{JTN} \cdot \rho_{JTN})^{\rho_{JTN}} \cdot \beta_N(\theta_N)^{(1-\theta_N)\cdot\rho_{JTN}}.$$

The final output of the joint work for knowledge creation is generated by combining the outputs of three basic activities, (7), (10) and (11), as follows:

$$A_{ij}^J = \left[\lambda_I \cdot (a_{ii}^{I*} + a_{jj}^{I*}) \right]^{\rho_I} \cdot \left[\lambda_F \cdot a_{ij}^{F*} \right]^{\rho_F} \cdot \left[\lambda_N \cdot a_{ij}^{N*} \right]^{\rho_N}, \quad (12)$$

where each of ρ_I, ρ_F and ρ_N represents a given positive weight on each type of basic activity such that

$$\rho_I + \rho_F + \rho_N = 1,$$

whereas each of λ_I, λ_F and λ_N denotes the frequency of each basic activity over a unit of time. The two persons can jointly choose λ_I, λ_F and λ_N freely, subject to the following constraint:

$$(1 + \varepsilon_I) \cdot \lambda_I + (1 + \varepsilon_F) \cdot \lambda_F + (1 + \varepsilon_N) \cdot \lambda_N = 1,\ \lambda_I \geq 0,\ \lambda_F \geq 0,\ \lambda_N \geq 0, \quad (13)$$

where $\varepsilon_k > 0$ represents the *lead time* which reflects *the proportion of time-loss* for each person in preparing for various types of meetings $k = I, F, N$. For example, ε_F reflects the time cost of preparing for F2F meetings, such as commuting time to the common CBD office, or to the common university office.[2]

(iii) In joint K-creation, under the given state of knowledge $\{n_i, n_j, n_{ij}^c, n_{ij}^d, n_{ji}^d\}$ at time t, the two persons choose jointly the optimal combination of $\{\lambda_I, \lambda_F, \lambda_N\}$ by solving the next problem:

$$\max\{A_{ij}^J \mid (1+\varepsilon_I)\cdot\lambda_I+(1+\varepsilon_F)\cdot\lambda_F+(1+\varepsilon_N)\cdot\lambda_N = 1,\ \lambda_I \geq 0,\ \lambda_F \geq 0,\ \lambda_N \geq 0\}, \tag{14}$$

where, rewriting equation (12), we have

$$A_{ij}^J = {\lambda_I}^{\rho_I} \cdot {\lambda_F}^{\rho_F} \cdot {\lambda_N}^{\rho_N} \cdot \left[a_{ii}^{I*} + a_{jj}^{I*}\right]^{\rho_I} \cdot \left[a_{ij}^{F*}\right]^{\rho_F} \cdot \left[a_{ij}^{N*}\right]^{\rho_N}. \tag{15}$$

In the right side of this equation, terms involving a's are independent of λ's. Hence, the problem (14) amounts to the next simple problem:

$$\max\{{\lambda_I}^{\rho_I}\cdot{\lambda_F}^{\rho_F}\cdot{\lambda_N}^{\rho_N} \mid (1+\varepsilon_I)\cdot\lambda_I+(1+\varepsilon_F)\cdot\lambda_F+(1+\varepsilon_N)\cdot\lambda_N = 1,\ \lambda_I \geq 0,\ \lambda_F \geq 0,\ \lambda_N \geq 0\}, \tag{16}$$

which yields the following solution:

$$\lambda_I^* = \frac{\rho_I}{1+\varepsilon_I},\ \lambda_F^* = \frac{\rho_F}{1+\varepsilon_F},\ \lambda_N^* = \frac{\rho_N}{1+\varepsilon_N}. \tag{17}$$

Not surprisingly, frequency of the activity, λ_k^*, decreases as lead time ε_k increases.

Substituting (17) into (15), the maximum value of A_{ij}^J is given by

$$\begin{aligned} A_{ij}^{J*} \equiv\ & (1+\varepsilon_I)^{-\rho_I} \cdot (1+\varepsilon_F)^{-\rho_F} \cdot (1+\varepsilon_N)^{-\rho_N} \cdot {\rho_I}^{\rho_I} \cdot {\rho_F}^{\rho_F} \cdot {\rho_N}^{\rho_N} \\ & \times \left[a_{ii}^{I*} + a_{jj}^{I*}\right]^{\rho_I} \cdot \left[a_{ij}^{F*}\right]^{\rho_F} \cdot \left[a_{ij}^{N*}\right]^{\rho_N}. \end{aligned} \tag{18}$$

Finally, substitution of (7), (10) and (11) into this equation leads to

$$\begin{aligned} A_{ij}^{J*} =\ & \Phi_J \cdot [n_i + n_j]^{\rho_I+\rho_{ISF}\cdot\rho_F+\rho_{ISN}\cdot\rho_N} \cdot \left[(n_{ij}^c)^{1-\theta_F} \cdot (n_{ij}^d \cdot n_{ji}^d)^{\frac{\theta_F}{2}}\right]^{\rho_{JTF}\cdot\rho_F} \\ & \times \left[(n_{ij}^c)^{1-\theta_N} \cdot (n_{ij}^d \cdot n_{ji}^d)^{\frac{\theta_N}{2}}\right]^{\rho_{JTN}\cdot\rho_N}, \end{aligned} \tag{19}$$

[2] In Battiston, Vidal and Kirchmair (2021), the core role of communication in organizations is assumed to be to transmit information that helps co-workers do their job better. In this context, the time cost incurred by the sender is emphasized. In the present context of joint *knowledge creation*, in contrast, the time involved in F2F communication among researchers itself is not a cost but an essential input for producing new ideas jointly.

where

$$
\begin{aligned}
\Phi_J \equiv\ & (1+\varepsilon_I)^{-\rho_I}\cdot(1+\varepsilon_F)^{-\rho_F}\cdot(1+\varepsilon_N)^{-\rho_N}\cdot(\alpha_{JI}\cdot\rho_I)^{\rho_I}\cdot(\alpha_F\cdot\rho_F)^{\rho_F}\cdot(\alpha_N\cdot\rho_N)^{\rho_N} \\
& \times(\alpha_{JIS}\cdot\rho_{JIS})^{\rho_{JIS}\cdot\rho_I}\cdot(\alpha_{JIT}\cdot\rho_{JIT})^{\rho_{JIT}\cdot\rho_I} \\
& \times(\alpha_{ISF}\cdot\rho_{ISF})^{\rho_{ISF}\cdot\rho_F}\cdot(\alpha_{JTF}\cdot\rho_{JTF})^{\rho_{JTF}\cdot\rho_F} \\
& \times(\alpha_{ISN}\cdot\rho_{ISN})^{\rho_{ISN}\cdot\rho_N}\cdot(\alpha_{JTN}\cdot\rho_{JTN})^{\rho_{JTN}\cdot\rho_N} \\
& \times\beta_F(\theta_F)^{(1-\theta_F)\cdot\rho_{JTF}\cdot\rho_F}\cdot\beta_N(\theta_N)^{(1-\theta_N)\cdot\rho_{JTN}\cdot\rho_N} \qquad (20)
\end{aligned}
$$

and

$$
\rho_I+\rho_F+\rho_N=1,\ \rho_{JIS}+\rho_{JIT}=1,\ \rho_{ISF}+\rho_{JTF}=1,\ \rho_{ISN}+\rho_{JTN}=1 \quad (21)
$$

(iv) Next, recalling the definitions in the first part of this section, we can rewrite (19) in terms of m^c_{ij}, m^d_{ij} and m^d_{ji} as follows:

$$
\begin{aligned}
A^{J*}_{ij} =\ & n^{ij}\cdot\Phi_J\cdot\left[(1-m^d_{ji})+(1-m^d_{ij})\right]^{\rho_I+\rho_{ISF}\cdot\rho_F+\rho_{ISN}\cdot\rho_N}\cdot\left[(m^c_{ij})^{1-\theta_F}\cdot(m^d_{ij}\cdot m^d_{ji})^{\frac{\theta_F}{2}}\right]^{\rho_{JTF}\cdot\rho_F} \\
& \times\left[(m^c_{ij})^{1-\theta_N}\cdot(m^d_{ij}\cdot m^d_{ji})^{\frac{\theta_N}{2}}\right]^{\rho_{JTN}\cdot\rho_N} \qquad (22)
\end{aligned}
$$

where Φ_J is a function of parameters given by (20).

3 The impact of regulating communication modes on joint knowledge productivity

> "The World Health Organization officially declared Covid-19 a pandemic on March 11. Within a few weeks, an estimated 16 million U.S. knowledge workers had switched to working remotely to flatten the curve of the health crisis, according to a new survey by Slack.
>
> This amounts to nearly one-quarter of all knowledge workers in the U.S., and that proportion has climbed even higher as more states have urged citizens to stay home."
>
> Hanson (2020)

During the Covid-19 pandemic, the intensity of use of communication channels between researchers changed, as many switched to work from home, for instance. How does the availability and use of both electronic and face to face communication change from an exogenous event, and how does it affect the patterns and volume of knowledge creation? How do pandemic restrictions change the use of various modes of communication as well as the characteristics of the knowledge creation process?

In this section, using our model developed in the previous section, we analyze the effect of restrictions on face to face communication in our model.

There is a large and rapidly expanding literature on the economic effects of the Covid19 pandemic. Here we focus on *the effects of the pandemic on the knowledge creation activity when multiple channels of communication are present*, and in particular on the productivity of researchers under pandemic restrictions. How do they change their choice of joint or individual work, and how do they change their modality of joint work (face to face or internet communication) with the imposition of pandemic restrictions? And how does this interact with the differing commuting cost faced by workers living at various distances from work?

There is some interesting empirical work associated with these questions. Morikawa (2020) finds significant effects, as well as significant heterogeneity in effects, of pandemic restrictions on workers in Japan. For example, worker productivity was reduced by 30-40% when working from home as opposed to commuting to work. We shall discuss in more detail below how further empirical results from this paper support our theory. Inoue et al (2022) examine the effect of the Spanish flu pandemic from the early 20th century on patent productivity in industries where face to face communication was important, and find a huge effect. Finally, Yamauchi et al (2022) find a significant negative shock to patent applications as a result of the Covid-19 pandemic. Interestingly, they find that shocks are fine tuned even to the timing of the waves of the pandemic in Japan,[3] suggesting upheavals in the modes of collaborations used in R&D.

In the formulation (14), the combination of communication modes, $\{\lambda_I, \lambda_F, \lambda_N\}$, has been freely chosen. In reality, however, there exist numerous restrictions on the usage of communication modes; some are explicit whereas others are implicit. For example, old metropolises, such as London, New York and Tokyo, have big CBDs that formed a long time ago when commuting railways were predominant. In these metropolises, before the Covid-19 pandemic a large proportion of workers were forced to commute to CBD offices to ease F2F

[3] See their Figure 3.

communications. Likewise, professors and students at traditional universities have been forced to commute to their main campuses for ease of F2F communications. For another example, conversely, during the recent Covid-19 pandemic, many governments introduced regulations that forced a large proportion of workers and students in large cities to work at their homes through the net while discouraging F2F meetings. Specifically, in the early period of the Covid-19 pandemic, the Japanese government asked business offices in large cities to reduce their commuting workers by 80% by switching their working status to WFH (working from home) to avoid the transmission of corona virus through F2F communications.

Given such examples of restrictions on communication modes in the real world, let us try to apply our model in the previous section to study the impact of such restrictions on joint knowledge productivity. Here, we focus on the regulation of F2F communications, so we set $\varepsilon_I = \varepsilon_N = 0$ and slightly modify the maximization problem of (14) as follows:

$$\max\{A_{ij}^J \mid \lambda_I + (1+\varepsilon_F)\bar{\lambda}_F + \lambda_N = 1,\ \lambda_I \geq 0,\, \lambda_N \geq 0\}, \tag{23}$$

where $\bar{\lambda}_F$ is a parameter exogenously specified such that

$$0 < \bar{\lambda}_F < \frac{1}{1+\varepsilon_F}. \tag{24}$$

That is, we fix the frequency of F2F communications at $\bar{\lambda}_F$, and then let the two persons choose λ_I and λ_N optimally. In this context, similar to the case of (16), the problem (23) amounts to the next problem:

$$\max\{{\lambda_I}^{\rho_I} \cdot {\bar{\lambda}_F}^{\rho_F} \cdot {\lambda_N}^{\rho_N} \mid \lambda_I + (1+\varepsilon_F)\bar{\lambda}_F + \lambda_N = 1,\ \lambda_I \geq 0,\, \lambda_N \geq 0\} \tag{25}$$

which yields the following solution:

$$\hat{\lambda}_I = \rho_I \cdot \left(\frac{1-(1+\varepsilon_F)\bar{\lambda}_F}{1-\rho_F}\right) \equiv \rho_I \cdot \left[1 + \frac{(1+\varepsilon_F)\cdot(\lambda_F^* - \bar{\lambda}_F)}{1-\rho_F}\right], \tag{26}$$

$$\hat{\lambda}_N = \rho_N \cdot \left(\frac{1-(1+\varepsilon_F)\bar{\lambda}_F}{1-\rho_F}\right) \equiv \rho_N \cdot \left[1 + \frac{(1+\varepsilon_F)\cdot(\lambda_F^* - \bar{\lambda}_F)}{1-\rho_F}\right], \tag{27}$$

where $\lambda_F^* \equiv \rho_F/(1+\varepsilon_F)$. We can see that when $\bar{\lambda}_F = \lambda_F^*$, it holds that $\hat{\lambda}_I = \rho_I \equiv \lambda_I^*$ and $\hat{\lambda}_N = \rho_N \equiv \lambda_N^*$, not surprisingly; whereas when $\bar{\lambda}_F \neq \lambda_F^*$, $\hat{\lambda}_I$ and $\hat{\lambda}_N$ are adjusted so as to accommodate the difference between $\bar{\lambda}_F$ and λ_F^*.

Substituting $\hat{\lambda}_I$ and $\hat{\lambda}_N$ in (26) and (27) into equation (15), we get

$$\begin{aligned}
\hat{A}^J_{ij}(\bar{\lambda}_F) &\equiv {\rho_I}^{\rho_I} \cdot {\rho_N}^{\rho_N} \cdot \left[\frac{1-(1+\varepsilon_F)\bar{\lambda}_F}{1-\rho_F}\right]^{\rho_I+\rho_N} \cdot \left(\bar{\lambda}_F\right)^{\rho_F} \\
&\quad \times \left[a^I_{ii}+a^I_{jj}\right]^{\rho_I} \cdot \left[a^F_{ij}\right]^{\rho_F} \cdot \left[a^N_{ij}\right]^{\rho_N} \\
&= (1+\varepsilon_F)^{-\rho_F} \cdot {\rho_I}^{\rho_I} \cdot {\rho_F}^{\rho_F} \cdot {\rho_N}^{\rho_N} \cdot \left[1+\frac{(1+\varepsilon_F)\cdot(\lambda^*_F-\bar{\lambda}_F)}{1-\rho_F}\right]^{\rho_I+\rho_N} \cdot \left(\frac{\bar{\lambda}_F}{\lambda^*_F}\right)^{\rho_F} \\
&\quad \times \left[a^I_{ii}+a^I_{jj}\right]^{\rho_I} \cdot \left[a^F_{ij}\right]^{\rho_F} \cdot \left[a^N_{ij}\right]^{\rho_N}, \qquad (28)
\end{aligned}$$

where the last equality follows from $\lambda^*_F \equiv \rho_F/(1+\varepsilon_F)$. This equation gives the maximum value of A^J_{ij} when λ_F is fixed at $\bar{\lambda}_F \in (0, 1/(1+\varepsilon_F))$.

Using (18) and (28), we divide $\hat{A}^J_{ij}(\bar{\lambda}_F)$ by A^{J*}_{ij}, and obtain

$$h(\bar{\lambda}_F) \equiv \frac{\hat{A}^J_{ij}(\bar{\lambda}_F)}{A^{J*}_{ij}} = \left[1+\frac{(1+\varepsilon_F)\cdot(\lambda^*_F-\bar{\lambda}_F)}{1-\rho_F}\right]^{\rho_I+\rho_N} \cdot \left(\frac{\bar{\lambda}_F}{\lambda^*_F}\right)^{\rho_F}, \qquad (29)$$

since $\varepsilon_I = \varepsilon_N = 0$. We call $h(\bar{\lambda}_F)$ the *relative productivity function*, which measures the effect of the restriction $\lambda_F = \bar{\lambda}_F$ on joint knowledge productivity. We can readily see from (29) that when $\bar{\lambda}_F = \lambda^*_F$, $h(\lambda^*_F) = 1$ as to be expected.

Differentiating $h(\bar{\lambda}_F)$ yields

$$h'(\bar{\lambda}_F) = h(\bar{\lambda}_F) \cdot \frac{(1+\varepsilon_F)\cdot(\lambda^*_F-\bar{\lambda}_F)}{[1-(1+\varepsilon_F)\bar{\lambda}_F]\cdot\bar{\lambda}_F}, \qquad (30)$$

implying that

$$h'(\bar{\lambda}_F) \gtreqless 0 \text{ as } \bar{\lambda}_F \lesseqgtr \lambda^*_F \text{ for } \bar{\lambda}_F \in \left(0, \frac{1}{1+\varepsilon_F}\right) \qquad (31)$$

Thus, $h(\bar{\lambda}_F)$ is strictly quasi-concave on $[0, 1/(1+\varepsilon_F)]$, achieving the maximum value, 1, at $\bar{\lambda}_F = \lambda^*_F$. We can also readily see from (29) and (30) that

$$\begin{aligned}
\lim_{\bar{\lambda}_F\to 0} h(\bar{\lambda}_F) &= 0, \ \lim_{\bar{\lambda}_F\to 0} h'(\bar{\lambda}_F) = \infty, \\
\lim_{\bar{\lambda}_F\to 1/(1+\varepsilon_F)} h(\bar{\lambda}_F) &= 0, \ \lim_{\bar{\lambda}_F\to 1/(1+\varepsilon_F)} h'(\bar{\lambda}_F) = -\infty,
\end{aligned}$$

implying that $h(\bar{\lambda}_F)$ decreases sharply towards 0 as either $\bar{\lambda}_F$ decreases towards zero, or $\bar{\lambda}_F$ increases towards $1/(1+\varepsilon_F)$.

For numerical illustration, Figure 2 depicts the shape of function $h(\bar{\lambda}_F)$ for parameters,

$$\rho_F = \frac{1}{2}, \ \rho_I = \rho_N = \frac{1}{4}, \ \varepsilon_F = 0.2,$$

implying that

$$\lambda^*_F = \frac{\rho_F}{1+\varepsilon_F} = \frac{1}{2.4}.$$

Figure 2

In this figure, when parameter $\bar{\lambda}_F$ is set either at $\bar{\lambda}_{F1} = 0.2\lambda_F^*$ or at $\bar{\lambda}_{F2} = 1.8\lambda_F^*$, the value of $h(\bar{\lambda}_F)$ is 0.6, implying 40% reduction of the joint knowledge productivity by either restriction. The case of $\bar{\lambda}_{F1} = 0.2\lambda_F^*$ may correspond to the instance when the Japanese government asked 80% of office workers in big cities to work at their homes in the early period of the Covid-19 pandemic.[4] The case of $\bar{\lambda}_{F2} = 1.8\lambda_F^*$ might suggest the possible reduction of joint knowledge productivity by the de facto enforcement of commuting to CBD offices on their knowledge workers.

Notice also from the shape of function $h(\bar{\lambda}_F)$ that when the restriction on λ_F is such that

$$0 \leq \lambda_F \leq \bar{\lambda}_F,$$

the optimal joint choice of λ_F is given at $\bar{\lambda}_F$ if $\bar{\lambda}_F \leq \lambda_F^*$; in contrast, if $\bar{\lambda}_F > \lambda_F^*$, this restriction is not effective. Likewise, when the restriction on λ_F is such that

$$\bar{\lambda}_F \leq \lambda_F \leq \frac{1}{1+\varepsilon_F},$$

then the optimal joint choice of λ_F is given at $\bar{\lambda}_F$ if $\bar{\lambda}_F \geq \lambda_F^*$; in contrast, if $\bar{\lambda}_F < \lambda_F^*$, this restriction is not effective.

In order to study more closely the nature of relative productivity function, substituting $\lambda_F^* = \rho_F/(1+\varepsilon_F)$ into equation (29), we rewrite (29) as follows:

$$h(\bar{\lambda}_F; \rho_F, \varepsilon_F) \equiv \frac{A_{ij}^J(\bar{\lambda}_F)}{A_{ij}^{J*}} = \left[\frac{1-(1+\varepsilon_F)\bar{\lambda}_F}{1-\rho_F}\right]^{1-\rho_F} \cdot \left(\frac{(1+\varepsilon_F)\bar{\lambda}_F}{\rho_F}\right)^{\rho_F}, \tag{32}$$

which is a function of three parameters: the regulation parameter $\bar{\lambda}_F$, and original parameters ρ_F and ε_F. First, to examine how the change in F2F intensity parameter ρ_F affects the value of relative productivity at given $\bar{\lambda}_F$ and ε_F, we differentiate (32) with ρ_F, and obtain

$$\frac{\partial h(\bar{\lambda}_F; \rho_F, \varepsilon_F)}{\partial \rho_F} = h(\bar{\lambda}_F; \rho_F, \varepsilon_F) \cdot \log\left[1 + \frac{(1+\varepsilon_F)\cdot(\bar{\lambda}_F - \frac{\rho_F}{1+\varepsilon_F})}{\rho_F \cdot [1-(1+\varepsilon_F)\bar{\lambda}_F]}\right] \tag{33}$$

[4]The empirical productivity of working from home during the Covid-19 pandemic in Japan was examined by Morikawa (2020). It is reported there that in the early period of the Covid-19 pandemic in Japan, the mean productivity of workers *WFH (working from home)* relative to *WWC (working with commuting)* at their usual workplace was about 60-70%. Thus, the case $\bar{\lambda}_{F1} = 0.2\lambda_F^*$ in Figure 2 happens to correspond rather well to the Japanese experience. Needless to say, empirical applications of the results in this section to the real world requires careful examination of the situations before and after the imposition of such regulations.

implying that

$$\frac{\partial h(\bar{\lambda}_F;\rho_F,\varepsilon_F)}{\partial \rho_F} \lesseqgtr 0 \text{ as } \bar{\lambda}_F \lesseqgtr \frac{\rho_F}{1+\varepsilon_F} \equiv \lambda_F^*. \tag{34}$$

Using this result, we can obtain Figure 3.

Figure 3

In drawing this figure, we choose two different values of F2F-intensity parameter ρ_F such that

$$\rho_{F1} < \rho_{F2}. \tag{35}$$

Then, given each chosen value of ρ_F, we can draw the corresponding relative productivity curve. Since the curve $h(\bar{\lambda}_F;\rho_{F1},\varepsilon_F)$ achieves the maximum value of 1 at $\lambda_{F1}^* = \rho_{F1}/(1+\varepsilon_F)$ and the curve $h(\bar{\lambda}_F;\rho_{F2},\varepsilon_F)$ at $\lambda_{F2}^* = \rho_{F2}/(1+\varepsilon_F)$, and since $\rho_{F1} < \rho_{F2}$, the left bold-curve corresponds to function $h(\bar{\lambda}_F;\rho_{F1},\varepsilon_F)$ whereas the right broken-curve corresponds to $h(\bar{\lambda}_F;\rho_{F2},\varepsilon_F)$.

In Figure 3, two fixed values, $\bar{\lambda}_{F1}$ and $\bar{\lambda}_{F2}$, of F2F-regulation parameter are chosen such that

$$\bar{\lambda}_{F1} < \lambda_{F1}^* = \frac{\rho_{F1}}{1+\varepsilon_F} < \lambda_{F2}^* = \frac{\rho_{F2}}{1+\varepsilon_F} < \bar{\lambda}_{F2}. \tag{36}$$

Then, as predicted by (33), as the curve $h(\bar{\lambda}_F;\rho_{F1},\varepsilon_F)$ shifts to the curve $h(\bar{\lambda}_F;\rho_{F2},\varepsilon_F)$, at $\bar{\lambda}_{F1}$ the relative productivity decreases from a to a', whereas at $\bar{\lambda}_{F2}$ the relative productivity increases b to b'.

For concreteness, given ρ_{F1} and ρ_{F2} as in (35), we may say that the system of joint knowledge creation associated with ρ_{F2} is *more F2F intensive* than the system associated with ρ_{F1}. Then, we can summarize the observations above as follows:

Proposition 1: Let us consider two systems of joint knowledge creation associated with different levels of F2F-intensity, ρ_{F1} and ρ_{F2}. Set two different levels of F2F regulation, $\bar{\lambda}_{F1}$ and $\bar{\lambda}_{F2}$, such that relation (35) holds. Then,
(i) Under the regulation $\bar{\lambda}_{F1} < \lambda_{F1}^*$, relative productivity in the joint system of knowledge creation with the higher F2F-intensity decreases more than in the system with the lower F2F intensity;
(ii) Under the regulation $\bar{\lambda}_{F2} > \lambda_{F2}^*$, relative productivity in the joint system of knowledge creation with the lower F2F-intensity decreases more than in the system with the higher F2F intensity.

In the proposition above, case (i) may correspond to the situation with strong request for WFH (working from home) introduced in the early period of the Covid-19 pandemic, whereas case (ii) may reflect the situation of forced WWC (working with commuting) in large old cities.

Regarding empirical evidence, Inoue et al (2022) look at the effect of the Spanish flu epidemic in the early 1900's on patent applications using a difference in differences approach, where the control group is industries where face to face communication is not intensive, and the treatment group is contact intensive industries. In accordance with Proposition 1(i), they find that the pandemic caused patent applications in contact intensive industries to decline significantly relative to the control group.

Likewise, let us next examine how the change in *time-loss parameter* ε_F affects the value of relative productivity at given $\bar{\lambda}_F$ and ρ_F. Differentiating (32) with respect to ε_F yields

$$\frac{\partial h(\bar{\lambda}_F;\rho_F,\varepsilon_F)}{\partial \varepsilon_F} = h(\bar{\lambda}_F;\rho_F,\varepsilon_F)\cdot \frac{\frac{\rho_F}{1+\varepsilon_F}-\bar{\lambda}_F}{1-(1+\varepsilon_F)\bar{\lambda}_F}, \tag{37}$$

implying that

$$\begin{aligned}\frac{\partial h(\bar{\lambda}_F;\rho_F,\varepsilon_F)}{\partial \varepsilon_F} &\gtreqless 0 \text{ as } \bar{\lambda}_F \lesseqgtr \frac{\rho_F}{1+\varepsilon_F} \equiv \lambda_F^*(\rho_F,\varepsilon_F) \\ &\Leftrightarrow \varepsilon_F \lesseqgtr \frac{\rho_F-\bar{\lambda}_F}{\bar{\lambda}_F}\end{aligned} \tag{38}$$

where $\lambda_F^*(\rho_F,\varepsilon_F)$ represents the optimal frequency of λ_F under the set of parameters, ρ_F and ε_F. Based on (38), we can draw Figure 4.

Figure 4

In drawing this figure, we choose two different values of time-loss parameter ε such that

$$\varepsilon_{F1} < \varepsilon_{F2}. \tag{39}$$

Then, for each chosen value of ε_F, we can draw the corresponding relative productivity curve as in Figure 4. Given (39), it holds that $1/(1+\varepsilon_{F1}) > 1/(1+\varepsilon_{F2})$ and $\lambda_{F1}^* \equiv \lambda_F^*(\rho_F,\varepsilon_{F1}) = \rho_F/(1+\varepsilon_{F1}) > \lambda_{F2}^* \equiv \lambda_F^*(\rho_F,\varepsilon_{F2}) = \rho_F/(1+\varepsilon_{F2})$. Hence, the right bold-curve corresponds to function $h(\bar{\lambda}_F;\rho_F,\varepsilon_{F1})$ and the left broken-curve corresponds to $h(\bar{\lambda}_F;\rho_F,\varepsilon_{F2})$.

In Figure 4, two fixed values of F2F regulation parameter, $\bar{\lambda}_{Fa}$ and $\bar{\lambda}_{Fb}$, are chosen such that

$$\bar{\lambda}_{Fa} < \lambda^*_{F2} \equiv \frac{\rho_F}{1+\varepsilon_{F2}} < \lambda^*_{F1} \equiv \frac{\rho_F}{1+\varepsilon_{F1}} < \bar{\lambda}_{Fb} \tag{40}$$

Then, as predicted by (38), when the bold-curve $h(\bar{\lambda}_F;\rho_F,\varepsilon_{F1})$ shifts to the broken-curve $h(\bar{\lambda}_F;\rho_F,\varepsilon_{F2})$, at $\bar{\lambda}_{Fa}$ the relative productivity *increases* from a_1 to a_2, whereas at $\bar{\lambda}_{Fb}$ the relative productivity *decreases* from b_1 to b_2.

For an intuitive understanding of the result above, let us consider *two pairs* of joint-knowledge workers, (i_1, j_1) and (i_2, j_2). And, suppose that ε_{F1} represents the *commuting time* of each i_1 and j_1 to the common CBD office, whereas ε_{F2} the commuting time of each i_2 and j_2 to the common CBD office. Here, as indicated by (39), each of (i_2, j_2) resides farther from the CBD than each of (i_1, j_1). Thus, the bold-curve in Figure 4 represents the relative productivity curve of pair (i_1, j_1) with *shorter commuting time* ε_{F1}, whereas the broken-curve represents the relative productivity curve of pair (i_2, j_2) with *longer commuting time* ε_{F2}.

In this context, given the regulation $\bar{\lambda}_{Fa}$ in Figure 4, the *deviation* of $\bar{\lambda}_{Fa}$ from the optimal F2F frequency for pair (i_1, j_1) with shorter commuting time ε_{F1} equals $(\lambda^*_{F1} - \bar{\lambda}_{Fa})$, whereas the *deviation* for pair (i_2, j_2) with longer commuting time ε_{F2} equals $(\lambda^*_{F2} - \bar{\lambda}_{Fa})$, where

$$\lambda^*_{F2} - \bar{\lambda}_{Fa} < \lambda^*_{F1} - \bar{\lambda}_{Fa}$$

Thus, the relative productivity loss $(1 - a_2)$ for the pair with the longer commuting time is smaller than the relative productivity loss $(1 - a_1)$ for the pair with the shorter commuting time.

Conversely, given another regulation $\bar{\lambda}_{Fb}$ in Figure 4, we can see that

$$\bar{\lambda}_{Fb} - \lambda^*_{F1} < \bar{\lambda}_{Fb} - \lambda^*_{F2}, \tag{41}$$

implying that the relative productivity loss $(1-b_1)$ for the pair with the shorter commuting time is smaller than the relative productivity loss $(1 - b_2)$ for the pair with longer commuting time.

We may summarize the observation above as follows:

Proposition 2: Let us consider two pairs of joint knowledge workers, (i_1, j_1) and (i_2, j_2), such that each of (i_1, j_1) has commuting time ε_{F1} to the common CBD office whereas each of (i_2, j_2) has commuting time ε_{F2} to the common CBD office, where $\varepsilon_{F1} < \varepsilon_{F2}$. In this context, let us consider two different levels of F2F regulation, $\bar{\lambda}_{Fa}$ and $\bar{\lambda}_{Fb}$, such that relation (41) holds as depicted in Figure 4. Then,

(i) Under the regulation $\bar{\lambda}_{Fa} < \lambda^*_{F2}$, the relative productivity decrease for the pair with longer commuting ε_{F2} is smaller than that for the pair with shorter commuting time ε_{F1}.[5]

(ii) Under the regulation $\bar{\lambda}_{Fb} > \lambda^*_{F1}$, the relative productivity decrease for the pair with shorter commuting time ε_{F1} is smaller than that for the pair with longer commuting time ε_{F2}.

In Morikawa (2020), it is reported that in the early period of the Covid-19 pandemic in Japan, "long-distance commuters tended to exhibit a relatively small reduction in productivity when participating in the WFH arrangement," which happens to be consistent with Proposition 2(i).

4 The knowledge growth rate in the pairwise symmetric case

Before going further, we must specify what is the objective of knowledge creation by each person. Specifically, we need to clarify how to output A^{J*}_{ij} is split / not split between i and j. In this connection, we introduced in our early papers two different specifications. In Berliant and Fujita (2008, 2009), assuming that A^{J*}_{ij} contributes directly to increasing each person's felicity (or instantaneous utility) at that time, A^{J*}_{ij} is not split between the two persons. In contrast, in Berliant and Fujita (2011, 2012), a fixed proportion of every collection of ideas created are assumed to become new patents, which are sold at a given market price at that time. The revenue from new patents is split evenly if persons i and j produce new ideas together. Although either specification would lead to similar results, in this paper we adopt the latter specification of Berliant and Fujita (2011, 2012).

To go further, we must define the rule used by each person to decide whether they create new ideas jointly or in isolation. For this purpose, we assume that income for each person derives from selling new ideas created as patents. The revenue from new patents is split evenly if persons i and j produce new ideas together.

[5] In Morikawa (2020), it is reported that in the early period of the Covid-19 pandemic in Japan, "long-distance commuters tended to exhibit a relatively small reduction in productivity when participating in the WFH arrangement," which happens to be consistent with Proposition 2(i).

Let $\delta_{ij}(t) \in \{0, 1\}$ be a function such that if

$$\delta_{ij}(t) = \delta_{ji}(t) = 1 \text{ for } j \neq i, \tag{42}$$

then joint knowledge creation occurs at time t. In contrast, if

$$\delta_{ii}(t) = \delta_{jj}(t) = 1, \tag{43}$$

then each person creates new knowledge in isolation. Notice that since the two persons must agree either to work jointly or to work in isolation each, either (42) or (43) can occur exclusively at time t.

Let $y_i(t)$ be the income of each person at time t, and let $\Pi(t)$ be the price of patents at time t. Then suppressing t for notational simplicity:

$$y_i = \Pi \cdot \left[\delta_{ii} \cdot A_{ii}^{I*} + \delta_{ij} \cdot A_{ij}^{J*}/2\right] \quad \text{for } j \neq i. \tag{44}$$

Since person i chooses δ_{ii} and δ_{ij} so as to maximize y_i and similarly for person j, it follows that[6]

$$\delta_{ij} = \delta_{ji} = 1 \Longleftrightarrow A_{ij}^{J*}/2 > A_{ii}^{I*} \text{ and } A_{ij}^{J*}/2 > A_{jj}^{I*}, \tag{45}$$

$$\delta_{ii} = \delta_{jj} = 1 \Longleftrightarrow A_{ij}^{J*}/2 \leq A_{ii}^{I*} \text{ or } A_{ij}^{J*}/2 \leq A_{jj}^{I*}. \tag{46}$$

To study this system in greater detail, in the following we focus on the special case where persons i and j are *pairwise symmetric* in terms of knowledge heterogeneity. Specifically, suppose at time t that (suppressing t for notational simplicity):

$$n_i = n_j \equiv n. \tag{47}$$

By definition,

$$n^c \equiv n_{ij}^c = n_{ji}^c.$$

Then since $n_{ij}^d = n - n^c$, it follows that

$$n_{ij}^d = n_{ji}^d \equiv n^d, \tag{48}$$

and

$$n = n^c + n^d, \ n^{ij} = n^c + 2n^d = n + n^d, \tag{49}$$

[6]Recall from equation (12) that $A_{ij}^{J*} = A_{ji}^{J*}$ by definition. In equations (45) and (46), we can use either strict inequality or weak inequality. However, since the case of ties is not important in the following analysis, for convenience we use strict inequality in (45) whereas we use weak inequality in (46).

implying that

$$\begin{aligned} m^d &\equiv m^d_{ij} = m^d_{ji} = \frac{n^d}{n^{ij}},\ m^c \equiv m^c_{ij} = m^c_{ji} = \frac{m^c}{n^{ij}}, \\ m^c &\equiv m^c_{ij} = m^c_{ji} = \frac{n^c}{n^{ij}} = 1 - 2m^d. \end{aligned} \tag{50}$$

It also follows from (2) that

$$n = n^{ij} \cdot (1 - m^d), \tag{51}$$

Furthermore, from (4) and (5),

$$A^{I*}_{ii} = A^{I*}_{jj} = \Phi_I \cdot n. \tag{52}$$

Hence, relations (45) and (46) can be restated as

$$\delta_{ij} = \delta_{ji} = 1 \Longleftrightarrow A^{J*}_{ij}/2 > \Phi_I \cdot n, \tag{53}$$

$$\delta_{ii} = \delta_{jj} = 1 \Longleftrightarrow A^{J*}_{ij}/2 \leq \Phi_I \cdot n, \tag{54}$$

where equation (22) for A^{J*}_{ij} is now given by

$$\begin{aligned} A^{J*}_{ij} = {} & n^{ij} \cdot \Phi_J \cdot \left[2 \cdot (1 - m^d)\right]^{\rho_I + \rho_{ISF} \cdot \rho_F + \rho_{ISN} \cdot \rho_N} \cdot \left[(1 - 2m^d)^{1-\theta_F} \cdot (m^d)^{\theta_F}\right]^{\rho_{JTF} \cdot \rho_F} \\ & \times \left[(1 - 2m^d)^{1-\theta_N} \cdot (m^d)^{\theta_N}\right]^{\rho_{JTN} \cdot \rho_N} \end{aligned} \tag{55}$$

Using (51) and rearranging terms, let us define *the knowledge growth rate for joint creation per person* by

$$\begin{aligned} g_J(m^d) \equiv \frac{A^{J*}_{ij}/2}{n} & = \Omega \cdot (1 - m^d)^{\rho_I + \rho_{ISF} \cdot \rho_F + \rho_{ISN} \cdot \rho_N - 1} \cdot (1 - 2m^d)^{(1-\theta_F)\rho_{JTF} \cdot \rho_F + (1-\theta_N)\rho_{JTN} \cdot \rho_N} \\ & \quad \times (m^d)^{\theta_F \rho_{JTF} \cdot \rho_F + \theta_N \rho_{JTN} \cdot \rho_N}, \\ & = \Omega \cdot (1 - m^d)^a \cdot (1 - 2m^d)^b \cdot (m^d)^c \end{aligned} \tag{56}$$

where

$$\begin{aligned} \Omega &\equiv \Phi_J \cdot 2^a \\ a &\equiv \rho_I + \rho_{ISF} \cdot \rho_F + \rho_{ISN} \cdot \rho_N - 1 \\ b &\equiv (1 - \theta_F)\rho_{JTF} \cdot \rho_F + (1 - \theta_N)\rho_{JTN} \cdot \rho_N \\ c &\equiv \theta_F \rho_{JTF} \cdot \rho_F + \theta_N \rho_{JTN} \cdot \rho_N \end{aligned} \tag{57}$$

We may note that the knowledge growth rate defined by equation (56) represents the "public aspect" of the knowledge creation and accumulation process. That is, A^{J*}_{ij} in equation (56) represents the newly created "formal knowledge" to be accumulated in the "Server" in Figure 1 as public documents, which are accessible by any person in the future. In contrast, as discussed in Section 5, "inside the brain" of each person i and j, the tacit knowledge generated in the process of creating A_{ij}^{J*} is also accumulated for the future activity of knowledge creation. Furthermore, in equation (56), A_{ij}^{J*} is divided by 2 to account for the number of "patents" created per person.

By definition,

$$a + b + c = 0.$$

Since the function $g_J(m^d)$ contains only m^d as a variable, differentiating $g_J(m^d)$ yields

$$g_J'(m^d) = g_J(m^d) \cdot \frac{c - (b + 2c) \cdot m^d}{m^d \cdot (1 - m^d) \cdot (2 - m^d)}. \tag{58}$$

Let us define

$$\begin{aligned} m^B &\equiv \frac{1}{2 + \frac{b}{c}} \\ &= \frac{1}{2 + \frac{(1-\theta_F)\rho_{JTF} \cdot \rho_F + (1-\theta_N)\rho_{JTN} \cdot \rho_N}{\theta_F \rho_{JTF} \cdot \rho_F + \theta_N \rho_{JTN} \cdot \rho_N}} \end{aligned} \tag{59}$$

which is smaller than 1/2 since $\theta_F < 1$ and $\theta_N < 1$. Thus, (58) implies that

$$g_J'(m^d) \gtreqless 0 \text{ as } m^d \lesseqgtr m^B \text{ for } m^d \in \left(0, \frac{1}{2}\right) \tag{60}$$

Hence, $g_J(m^d)$ is strictly quasi-concave on $[0, 1/2]$, achieving its maximal value at m^B, which we call the "Bliss Point."

Next, for the case of knowledge creation *in isolation*, from equations (4) and (5), *the knowledge growth rate for each person* is obtained as follows:

$$\frac{A_{ii}^{I*}}{n_i} = \frac{A_{jj}^{I*}}{n_j} = \Phi_I, \tag{61}$$

which is constant for the two persons.

Overall, the knowledge growth rate per person is given by:

$$g(m^d) = \max\left\{\Phi_I, g_J(m^d)\right\} \tag{62}$$

Using function $g_J(m^d)$, the selection rule (45) and (46) can be restated as

$$\delta_{ij} = \delta_{ji} = 1 \Longleftrightarrow g_J(m^d) > \Phi_I \tag{63}$$

$$\delta_{ii} = \delta_{jj} = 1 \Longleftrightarrow g_J(m^d) \leq \Phi_I \tag{64}$$

Figure 5 depicts the shape of the function $g_J(m^d)$ for parameters:

$$\begin{aligned}
\rho_F &= \frac{1}{2},\ \rho_I = \rho_N = \frac{1}{4},\ \rho_{JIS} = \rho_{JIT} = \frac{1}{2}, \rho_{ISF} = \rho_{ISN} = \frac{1}{4}, \rho_{JTF} = \rho_{JTN} = \frac{3}{4}, \\
\theta_F &= \frac{1}{4}, \theta_N = \frac{1}{2}, \varepsilon_F = 0.2,, \varepsilon_I = \varepsilon_N = 0, \beta_F(\theta_F) = \beta_N(\theta_N) = 4, \\
\alpha_F &= 2, \alpha_{JI} = \alpha_N = 4,\ \alpha_{JIS} = \alpha_{JIT} = \alpha_{JSF} = \alpha_{JTF} = \alpha_{ISF} = \alpha_{ISN} = \alpha_{JTN} = 3
\end{aligned}$$

together yielding

$$m_B = 0.4,\ g_J(m^B) = 1.0.$$

Figure 5

In Figure 5, the horizontal line with height

$$\Phi_I = 0.5$$

represents the knowledge growth rate when i and j are *working separately in isolation*, which is obtained from the set of parameters:

$$\alpha_I = \alpha_{IS} = \alpha_{IT} = 1, \rho_{IS} = \rho_{IT} = \frac{1}{2}.$$

This horizontal line intersects the $g_J(m^d)$ curve at points E and H. Hence,

$$\delta_{ij} = \delta_{ji} = 1 \text{ for } m^d \in (m^E, m^H), \tag{65}$$

$$\delta_{ii} = \delta_{jj} = 1 \text{ for } m^d \in [0, m^E] \cup [m^H, \frac{1}{2}]. \tag{66}$$

(Some comparative statics of $g(m^d)$ might be conducted.)

5 Dynamics of the two-person system

"Tacit knowledge consists partly of technical skills—the kind of informal, hard-to-pin-down skills captured in the term 'know-how.' A master craftsman after years of experience develops a wealth of expertise 'at his fingertips.' But he is often unable to articulate the scientific or technical principles behind what he knows."

Nonaka (2007)

Thus far, through Sections 2 to 4, the size of each person's knowledge as well as the relative composition of two persons' knowledge have been treated parametrically. In this section, we examine how the size of each person's knowledge as well as the relative composition of two persons' knowledge change endogenously over time when they continue interacting by sequentially choosing either to work alone or to work together for knowledge creation.

To study such dynamics of the two-person system, in the following we focus on the special case where persons i and j have the same size of knowledge at the initial time, $t = 0$; namely

$$n_i(0) = n_j(0) \equiv n(0). \tag{67}$$

It holds by definition that

$$n^c_{ij}(0) = n^c_{ji}(0) \equiv n^c(0),$$

thus as shown in equations (47), (48) and (49), condition (67) means

$$n^d_{ij}(0) = n^d_{ji}(0) \equiv n^d(0), \tag{68}$$

and

$$m^d_{ij}(0) = m^d_{ji}(0) \equiv m^d(0). \tag{69}$$

Given that the initial state of knowledge is symmetric for persons i and j as above, as seen below, it turns out that the equilibrium configuration at any time also maintains pairwise symmetry between the two persons; at any time $t \in [0, \infty)$,[7]

$$n_i(t) = n_j(t) \equiv n(t), \tag{70}$$

and hence,

$$n^d_{ij}(t) = n^d_{ji}(t) \equiv n^d(t),$$
$$m^d_{ij}(t) = m^d_{ji}(t) \equiv m^d(t),$$

whereas, by definition,

$$n^c_{ij}(t) = n^c_{ji}(t) \equiv n^c(t).$$

In this context of a pairwise-symmetric equilibrium path, our main goal is to examine how *the knowledge growth rate per person* changes in the long-run.

[7] This can be seen from the fact that when condition (70) is met at any time t (say, $t = 0$), the right hand side of the differential equation specifying $\dot{n}(t)$ (also $\dot{n}^d(t)$ and $\dot{m}^d(t)$) does not involve any variable that is specific to i or j. For a more precise explanation of this point, please see Berliant and Fujita (2011).

As defined in equation (62), given $m^d \in [0, \frac{1}{2}]$, the knowledge growth rate per person, $g(m^d)$, is given by

$$g(m^d) = \max\left\{\Phi_I, g_J(m^d)\right\}.$$

To make the analysis interesting, let us consider the case

$$g_J(m^B) > \Phi_I. \tag{71}$$

Then, as depicted in Figure 5, on the horizontal axis m^d, there exist m^E and m^H such that $0 < m^E < m^B < m^H < \frac{1}{2}$ and

$$g_J(m^d) > \Phi_I \text{ for } m^d \in (m^E, m^H),$$

$$\Phi_I > g_J(m^d) \text{ for } m^d \in \left[0, m^E\right) \cup (m^H, \frac{1}{2}]$$

Let $m^d(0)$ be the initial value of m^d at time 0. First, let us assume that

$$g_J(m^d(0)) > \Phi_I. \tag{72}$$

That is, $m^d(0)$ locates between m^E and m^H in Figure 5. Then, the two persons will continue to work jointly (i.e., $\delta_{ij} = \delta_{ji} = 1$) as long as

$$g_J(m^d(t)) > \Phi_I. \tag{73}$$

In order to know which direction $m^d(t)$ moves and how long the two persons will continue working jointly, let us examine the dynamics of m^d when condition (73) holds. To do so, recalling the definition $m^d \equiv n^d/n^{ij}$ and dropping t for simplicity, we have that:

$$\begin{aligned} \dot{m}^d &= \frac{\dot{n}^d}{n^{ij}} - \frac{n^d \cdot \dot{n}^{ij}}{(n^{ij})^2} \\ &= (n^{ij})^{-1} \cdot [\dot{n}^d - \dot{n}^{ij} \cdot m^d], \end{aligned}$$

Since $n^{ij} \equiv n^c + 2n^d$, it follows that

$$\dot{m}^d = (n^{ij})^{-1} \cdot [\dot{n}^d - (\dot{n}^c + 2\dot{n}^d) \cdot m^d] \tag{74}$$

implying that

$$\dot{m}^d \gtreqless 0 \Leftrightarrow \dot{n}^d - (\dot{n}^c + 2\dot{n}^d) \cdot m^d \gtreqless 0. \tag{75}$$

Hence, to identify the sign of $\dot{m}^d$, we must know the values of $\dot{n}^d$ and $\dot{n}^c$. That is, when $m^d(t)$ locates in between m^E and m^H in Figure 5, the knowledge growth rate $g_J(m^d(t))$ is realized for each person, and therefore we must calculate how much *differential knowledge* ($\dot{n}^d(t)$) for each person and how much *common knowledge* ($\dot{n}^c(t)$) for the two persons has been generated at that moment, t.

However, knowing the values of $\dot{n}^d(t)$ and $\dot{n}^c(t)$ is not a simple task because joint knowledge creation is composed, as explained in Section 2 (ii), of three layers of activities: the production of the final output A_{ij}^{J*}, which is generated by combining the outputs of three types of basic activities, whereas each basic activity consists of multiple sub-activities. Since the final output A_{ij}^{J*} is produced jointly, it will naturally become a part of *new common knowledge* for the two persons. The output of a basic activity or a sub-activity is not only used as the input for an activity at the next higher level, but it is also kept as new *tacit knowledge* for knowledge creation in the future; when the output is produced jointly, that tacit knowledge becomes a part of new *common knowledge* for the two persons. In contrast, when the output is independently created by a person, say i, it becomes a part of new *differential knowledge* of i from j. Thus, calculating the values of $\dot{n}^d(t)$ and $\dot{n}^c(t)$ is a complex task, which is attained through a sequence of steps as follows.

In the first step, we clarify the structural relationship among all basic activities and sub-activities. As depicted in Figure 6, the relationship among all of the activities used to generate the final output A_{ij}^{J*} is represented as a *tree*. Table 1 provides further description of this *activity tree*.

Figure 6

Table 1

The activity tree depicted in Figure 6 consists of *three tiers* of knowledge creation activities. We may note that in describing a tree in mathematics, the term “level” is used instead of “tier.” In the present context, however, the diagram shown in Figure 6 resembles the structure of a *supply chain* that produces a final industrial product (e.g., automobiles) by sequentially assembling various parts produced through *multiple tiers* of many factories. Thus, here we use the term “tier.” The major difference between “automobile production” and the present “knowledge production” must be noted. That is, all parts used in the production process of a car “disappear” into the car at the end of the assembly process. In contrast, the tacit knowledge created and used in the knowledge production process is accumulated for knowledge creation in the future.

In Figure 6, each node denoted by a black square ■ or a black dot • represents a *joint activity* by persons i and j, whereas each node denoted by a white circle ∘ represents an *independent activity* conducted by either i or j. Each node is labelled by the output of the corresponding activity, where the sign $*$ attached to an output means that the frequency variables (ω's and / or λ's) have been chosen optimally. The precise description of each output is provided in the fourth column of Table 1.

The top node ■ in Figure 6 represents the final output A_{ij}^{J*}. The second tier consists of four basic activities: a_{ii}^{I*} (the output of searching and thinking by i), a_{jj}^{I*} (the output of searching and thinking by j), a_{ij}^{F*} (the output of independent searching by i and j, and joint thinking by i and j while jointly working F2F), and a_{ij}^{N*} (the output of independent searching by i and j, and joint thinking by i and j while jointly working through the Net).

The third tier consist of four groups of sub-activities: $\overset{*JIS}{a_i}$ and $\overset{*JIT}{a_i}$ (sub-activities of a_{ii}^{I*}), $\overset{*JIS}{a_j}$ and $\overset{*JIT}{a_j}$ (sub-activities of a_{jj}^{I*}), $\overset{*ISF}{a_i}$, $\overset{*ISF}{a_j}$ and $\overset{*JTF}{a_{ij}}$ (sub-activities of a_{ij}^{F*}), $\overset{*ISN}{a_i}$, $\overset{*ISN}{a_j}$ and $\overset{*JTN}{a_{ij}}$ (sub-activities of a_{ij}^{N*}). Further description of each sub-activity is provided in the third column (*type*) of Table 1.

In the second step, the output of each activity / sub-activity is represented by an equation, which is listed in the fourth column of Table 1. (The output description of each sub-activity in the fourth column comes from the equation of the corresponding second-tier activity in the text.)

Although each equation in the fourth column of Table 1 represents new tacit knowledge generated by each activity / sub-activity, different pieces of new tacit knowledge cannot simply be added up because each one has different importance for the creation of new knowledge in the future. Thus, in the third step, we attach a "weight" to each specific output, say x. For the weight on x, following the tradition of microeconomics, we use the value, $\partial A_{ij}^{J*}/\partial x$, which represents marginal contribution of x to the final output A_{ij}^{J*}. Then, we calculate the (total) contribution of x to the final output A_{ij}^{J*} by

$$\frac{\partial A_{ij}^{J*}}{\partial x} \cdot x, \tag{76}$$

which we call the *imputed value* of x, using the terminology of microeconomics. Here, it must be noted that in each calculation of (76), the *pairwise symmetry* stated in equation (70), namely $n_i(t) = n_j(t) \equiv n(t)$, is taken into account.

For example, let us focus on the top row in the third tier of Table 1. On this row, the last term, $A_{ij}^{J*} \cdot (\rho_I \cdot \rho_{JIS}/2)$, is obtained as follows. First, substituting

ρ_{JIS} (the optimal value of ω_{JIS}) for ω_{JIS} and ρ_{JIT} (the optimal value of ω_{JIT}) for ω_{JIT} in equation (6), the optimized value of a^I_{ii} is given by

$$a^{I*}_{ii} = \alpha_{JI} \cdot [\overset{*}{a}{}^{JIS}_i]^{\rho_{JIS}} \cdot [\overset{*}{a}{}^{JIT}_i]^{\rho_{JIT}}, \tag{77}$$

where

$$\overset{*}{a}{}^{JIS}_i = \rho_{JIS} \cdot \alpha_{JIS} \cdot n_i, \; \overset{*}{a}{}^{JIT}_i = \rho_{JIT} \cdot \alpha_{JIT} \cdot n_i. \tag{78}$$

Similarly, we can obtain a^{I*}_{jj} (detail about which is not important here). Next, using (77) and (78), from (23) we can obtain that

$$\frac{\partial A^{J*}_{ij}}{\partial \overset{*}{a}{}^{JIS}_i} \cdot \overset{*}{a}{}^{JIS}_i = A^{J*}_{ij} \cdot \frac{\rho_I \cdot \rho_{JIS} \cdot a^{I*}_{ii}}{a^{I*}_{ii} + a^{I*}_{jj}}.$$

Since $n_i(t) = n_j(t) = n(t)$ at any t, it follows that $a^{I*}_{ii} = a^{I*}_{jj}$. Hence,

$$\frac{\partial A^{J*}_{ij}}{\partial \overset{*}{a}{}^{JIS}_i} \cdot \overset{*}{a}{}^{JIS}_i = A^{J*}_{ij} \cdot \frac{\rho_I \cdot \rho_{JIS}}{2}, \tag{79}$$

which gives the last term on the top row of the third tier associated with the output, $\overset{*}{a}{}^{JIS}_i$, in Table 1.

Likewise, for each output x, we can calculate the imputed value, $(\partial A^{J*}_{ij}/\partial x) \cdot x$, of each x as listed in the last column of Table 1.

Next, using the terms listed in the last column of Table 1, we calculate the values of $\dot{n}^d(t)$ and $\dot{n}^c(t)$. Before doing so, however, let us observe from the last column of Table 1 that

$$\begin{aligned} & \text{the sum of imputed values in the second tier} \\ = \; & A^{J*}_{ij} \cdot \frac{\rho_I}{2} + A^{J*}_{ij} \cdot \frac{\rho_I}{2} + A^{J*}_{ij} \cdot \rho_F + A^{J*}_{ij} \cdot \rho_N \\ = \; & A^{J*}_{ij} \cdot (\rho_I + \rho_F + \rho_N) \\ = \; & A^{J*}_{ij}, \end{aligned} \tag{80}$$

since $\rho_I + \rho_F + \rho_N = 1$. Likewise, using the relations among ρ's noted in (21), it can be readily confirmed that

$$\begin{aligned} & \text{the sum of imputed values in the third tier} \\ = \; & A^{J*}_{ij}. \end{aligned} \tag{81}$$

Therefore, when the final output A^{J*}_{ij} is created, the same size of tacit knowledge (in terms of imputed values) is generated in tier 2 as well as in tier 3.[8]

[8] If we recall the microeconomics of production, this result is not surprising. That is, "all production functions / sub-production functions" in each tier are composed of Cobb-

In calculating the values of $\dot{n}^d(t)$ and $\dot{n}(t)$, we may take into account all imputed values in tier 2 and tier 3. However, the contents of tacit knowledge generated in tier 2 seem rather close to those in tier 3. Thus, in order to avoid double counting of similar pieces of tacit knowledge, we consider only the tacit knowledge generated in tier 3.[9] In this context, the value of $\dot{n}^d$ for each person, say i, is obtained by summing up the imputed values of all third-tier activities by person i that are identified by $\circ$ circles in the second column of Table 1 as follows:

$$\begin{aligned}\dot{n}^d &= A_{ij}^{J*} \cdot \frac{\rho_I \cdot \rho_{JIS}}{2} + A_{ij}^{J*} \cdot \frac{\rho_I \cdot \rho_{JIT}}{2} + A_{ij}^{J*} \cdot \frac{\rho_F \cdot \rho_{ISF}}{2} + A_{ij}^{J*} \cdot \frac{\rho_N \cdot \rho_{ISN}}{2} \\ &= \frac{A_{ij}^{J*}}{2} \cdot (\rho_I + \rho_F \cdot \rho_{ISF} + \rho_N \cdot \rho_{ISN}), \qquad (82)\end{aligned}$$

using the identity $\rho_{JIS} + \rho_{JIT} = 1$. This equation represents the size of the *differential knowledge* generated by each person at the time. Next, the value of $\dot{n}^c$ is obtained as the sum of final output A_{ij}^{J*} and the imputed values of third-tier joint activities by i and j as follows:

$$\begin{aligned}\dot{n}^c &= A_{ij}^{J*} + A_{ij}^{J*} \cdot \rho_F \cdot \rho_{JTF} + A_{ij}^{J*} \cdot \rho_N \cdot \rho_{JTN} \\ &= A_{ij}^{J*} \cdot (1 + \rho_F \cdot \rho_{JTF} + \rho_N \cdot \rho_{JTN}). \qquad (83)\end{aligned}$$

This equation represents the size of *common knowledge* generated by the joint work of i and j.

Finally, substituting (82) and (83) into equation (74) and noting that $\dot{n}^c + 2\dot{n}^d = 2A_{ij}^{J*}$, we have that

$$\dot{m}^d = (n^{ij})^{-1} \cdot \frac{A_{ij}^{J*}}{2} \cdot [(\rho_I + \rho_F \cdot \rho_{ISF} + \rho_N \cdot \rho_{ISN}) - 4m^d] \quad \text{for } m^d \in (m^E, m^H). \qquad (84)$$

Let us define

$$\tilde{m} \equiv \frac{\rho_I + \rho_F \cdot \rho_{ISF} + \rho_N \cdot \rho_{ISN}}{4}, \qquad (85)$$

which represents the *stationary point* of $\dot{m}^d$. Then, it follows from (84) that

$$\dot{m}^d \gtreqless 0 \text{ as } m^d \lesseqgtr \tilde{m} \quad \text{for } m^d \in (m^E, m^H). \qquad (86)$$

Douglas functions, and each function displays constant returns to scale as can be seen from (21). Hence, the sum of all imputed values in each tier equals the value of the final output, the price of which is normalized to be 1.

[9] Even if we take into account all pieces of tacit knowledge in tier 2 and tier 3, the main results below would not change qualitatively.

Although we have identified the sign of $\dot{m}^d$ when the two persons work jointly, we must also consider the case when two persons working independently. Fortunately, this case is rather simple. First, by definition of $m^d \equiv n^d/n^{ij}$, the dynamics of m^d are given by equation (74) also in the present context of each person working in isolation. Hence, to identify the sign of $\dot{m}^d$, we need to know the values of $\dot{n}^d$ and $\dot{n}^c$ in the present context. To do so, we focus on person i and rewrite equation (4) as follows:

$$A_{ii}^{I*} = \alpha_I \cdot [\overset{*}{a}_i^{IS}]^{\rho_{IS}} \cdot [\overset{*}{a}_i^{IT}]^{\rho_{IT}}, \tag{87}$$

where

$$\overset{*}{a}_i^{IS} = \rho_{IS} \cdot \alpha_{IS} \cdot n_i, \; \overset{*}{a}_i^{IT} = \rho_{IT} \cdot \alpha_{IT} \cdot n_i. \tag{88}$$

Thus, the activity tree for the knowledge creation by person i in isolation can be represented simply as in Figure 7.

Figure 7

The top node $\square$ in Figure 7 represents the final output created by person i in isolation. The second tier consists of two basic activities: $\overset{*}{a}_i^{IS}$ (the intermediate output of searching by i) and $\overset{*}{a}_i^{IT}$ (the intermediate output of thinking by i). From (87), the imputed value of each intermediate output can be obtained as follows:

$$\frac{\partial A_{ii}^{I*}}{\partial \overset{*}{a}_i^{IS}} \cdot \overset{*}{a}_i^{IS} = A_{ii}^{I*} \cdot \rho_{IS}, \; \frac{\partial A_{ii}^{I*}}{\partial \overset{*}{a}_i^{IT}} \cdot \overset{*}{a}_i^{IT} = A_{ii}^{I*} \cdot \rho_{IT}. \tag{89}$$

implying that:

$$\begin{aligned} & \text{the sum of imputed value in the second tier} \\ = \; & A_{ii}^{I*} \cdot \rho_{IS} + A_{ii}^{I*} \cdot \rho_{IT} \\ = \; & A_{ii}^{I*}, \end{aligned} \tag{90}$$

since $\rho_{IS} + \rho_{IT} = 1$.

By assumption, since person i is working in isolation (from j), the formal output A_{ii}^{I*} and imputed values of second-tier activities by person i become *differential knowledge* of person i (from j).[10] Hence, the value of $\dot{n}^d$ for

[10]The formal output A_{ii}^{I*} by person i will be registered in the knowledge stock in the Server. Given that the size of the total stock of knowledge in the Server is almost infinite, it can be safely assumed that the probability of person j finding A_{ii}^{I*} in the Server is nearly zero (and vice versa for i). Even if we assume that a part of A_{ii}^{I*} and A_{ij}^{I*} is transferred to each other by some mechanism, it will not change the sign of $\dot{m}^d$ below.

person i is obtained as follows:

$$\begin{aligned} \dot{n}^d &= A_{ii}^{I*} + A_{ii}^{I*} \cdot \rho_{IS} + A_{ii}^{I*} \cdot \rho_{IT} \\ &= 2A_{ii}^{I*}, \end{aligned} \tag{91}$$

whereas no common knowledge is generated for the two persons:

$$\dot{n}^c = 0. \tag{92}$$

Substituting (90) and (91) into (74) yields that

$$\dot{m}^d = (n^{ij})^{-1} \cdot 2A_{ii}^{I*} \cdot (1 - 2m^d) > 0 \text{ for } m^d \in [0, \frac{1}{2}). \tag{93}$$

Therefore, whenever each person is working in isolation, the proportion of differential knowledge, m^d, will keep increasing.

Now, combining the two situations of working jointly and working in isolation by two persons, we can derive the overall dynamics of m^d. Again, we assume that condition (70) holds. Then, as shown in Figure 8, depending on the relative position of $\tilde{m}$ and m^B on the m^d-axis, we have three different diagrams for the dynamics of two-person system.

Figure 8

In Figure 8, (a), (b) and (c) respectively show one of three possible dynamics of m^d, whereas diagram (d) explains the determination of the relationship between $\tilde{m}$ and m^B.[11]

First, diagram (a) shows the dynamics of m^d for the case, $m^E < \tilde{m} < m^B$. As depicted in this diagram, if the initial position, $m^d(0)$, at time 0 is to the left of $\tilde{m}$, then $m^d(t)$ gradually increases towards $\tilde{m}$. If $\tilde{m} < m(0) < m^H$, then $m^d(t)$ gradually decreases towards $\tilde{m}$. Only when $m^d(0) > m^H$ does $m^d(t)$ move away from $\tilde{m}$ toward 1/2. Hence, whenever $m^d(0) < m^H$, $m^d(t)$ approaches *the sink point*, $\tilde{m}$, which is to the left of the bliss point m^B.

Next, diagram (b) depicts the dynamics of m^d for the case $m^B < \tilde{m} < m^H$. In this case, except when $m^d(0) > m^H$, $m^d(t)$ approaches the sink point $\tilde{m}$ to the right of the bliss point m^B.

[11] Theoretically speaking, there exists the possibility of a fourth case where $m^B < m^H < \tilde{m} < 1/2$. This can happen only in the extreme situation where both θ_F and θ_N are close zero, and Φ_I and $g_J(m^B)$ are nearly equal. Hence, in the following discussion, we neglect this fourth case. Actually, it is possible that $\tilde{m} = m^B$. But this case happens on a set of measure zero, so we neglect it as well.

Third, diagram (c) is the case where $\tilde{m} < m^E$, which happens when $g_J(\tilde{m}) < \Phi_I$. In this case, except when $m^d(0) > m^H$, $m^d(t)$ approaches the sink point m^E, where the K-growth rate is much lower than at the bliss point m^B.

Diagram (d) in Figure 8 explains when each of the three possible cases of dynamics happens. For the purpose of intuitive understanding of when each case happens, we here focus on the special situation where

$$\theta_F = \theta_N \equiv \theta. \tag{94}$$

That is, both in equation (10) (the K-production function for joint thinking F2F) and equation (11) (the K-production function for joint thinking through the Net), the weight on differential knowledge has the same value. In this case, from (59), we have that

$$m^B = \frac{1}{1+\frac{1}{\theta}} \equiv m^B(\theta), \tag{95}$$

meaning that the value of m^B is uniquely determined by the value of parameter θ. In diagram (d) of Figure 8, function $m^B(\theta)$ is depicted by the bold curve for $\theta \in [0, 1]$.

In contrast, the value of $\tilde{m}$ is determined, as in (85), by the values of ρ's, independently of θ. Hence, here we treat $\tilde{m}$ as a single parameter. By definition, the maximum value of $\tilde{m}$ equals 1/4:

$$\max \tilde{m} = \frac{1}{4}.$$

Given each $\tilde{m} \in (0, 1/4]$, as shown in diagram (d), by setting

$$\tilde{m} = m^B(\theta), \tag{96}$$

the value of $\theta(\tilde{m})$ is uniquely determined by $\tilde{m} = m^B$.

Now, given $\tilde{m} \in (0, 1/4]$, we can see from diagram (d) that

$$\tilde{m} < m^B(\theta) \text{ for } \theta \in (\theta(\tilde{m}), 1/2). \tag{97}$$

That is, given $\tilde{m} \in (0, 1/4]$, if we take the value of θ sufficiently large so that $\theta > \theta(\tilde{m})$, then the corresponding bliss point, $m^B(\theta)$, locates to the right of $\tilde{m}$, which corresponds to diagrams (a) and (c) in Figure 8. By definition, $\theta > \theta(\tilde{m})$ means that *the weight on differential knowledge in knowledge sub-production functions (10) and (11) is sufficiently large*. Furthermore, when θ is close $\theta(\tilde{m})$, then we have diagram (a). On the other hand, when θ is much larger than $\theta(\tilde{m})$, then the corresponding bliss point m^B is far to the right of $\tilde{m}$, yielding diagram (c).

In contrast to (97), we can see from diagram (d) that for each $\tilde{m} \in (0, 1/4)$,

$$\tilde{m} > m^B(\theta) \text{ for } \theta \in (0, \theta(\tilde{m})). \tag{98}$$

That is, given $\tilde{m} \in (0, 1/4)$, if the value of θ is sufficiently small so that $\theta < \theta(\tilde{m})$, then the corresponding bliss point, $m^B(\theta)$, locates in the left of $\tilde{m}$, which corresponds to diagram (b) in Figure 8. By definition, $\theta < \theta(\tilde{m})$ means that *the weight on common knowledge* in sub-production functions (10) and (11) *is sufficiently large.*

We have seen through Figure 8 that except when $\tilde{m}$ happens to coincide with the bliss point m^B, K-growth rate at the sink point is lower than at the bliss point. In particular, when the bliss point m^B is close 1/2 (which happens when the weight on differential knowledge in K-subproduction functions (10) and (11) is sufficiently large), we have diagram (c), where the K-growth rate at the sink point m^E is much lower than that at the bliss point. Thus, it is important to ask: When the K-growth rate at the sink point is lower than that at the bliss point, what possible mechanism could make the new sink point coincide with the bliss point? Before examining this question further in the last section, however, we need to ask another fundamental question here.

Recall the discussion near the end of Section 4 stating that A_{ij}^{J*} in equation (56) represents newly created *formal knowledge* to be accumulated in the Server as public documents. However, we have just seen that the knowledge production process also yields a large amount of *tacit knowledge* which will be accumulated *inside the brain* of each person for future knowledge creation activity. Thus, we need to know the growth rate of *total knowledge* per person, including both formal knowledge and tacit knowledge. Furthermore, we must examine the relationship between the knowledge growth rate defined by equation (56) and the growth rate of total knowledge for each person.

To answer these questions, let us first calculate the growth rate of total knowledge, $\dot{n}_i$, for person i *when working in isolation.* From (91) and (92), we have that

$$\begin{aligned} \dot{n}_i &= \dot{n}^d + \dot{n}^c \\ &= 2A_{ii}^{I*}. \end{aligned} \tag{99}$$

Thus, recalling (61), the growth rate of total knowledge for person i when working in isolation is given by

$$G_I \equiv \frac{\dot{n}_i}{n_i} = \frac{2A_{ii}^{I*}}{n_i} = 2\Phi_I. \tag{100}$$

Next, when person i is working jointly with j, from (82) and (83) we have that

$$\begin{aligned}
\dot{n}_i &= \dot{n}_i^c + \dot{n}_i^d = \dot{n}^c + \dot{n}^d \\
&= A_{ij}^{J*} \cdot (1 + \rho_F \cdot \rho_{JTF} + \rho_N \cdot \rho_{JTN}) + \frac{A_{ij}^{J*}}{2} \cdot (\rho_I + \rho_F \cdot \rho_{ISF} + \rho_N \cdot \rho_{ISN}) \\
&= A_{ij}^{J*} \cdot (1 + \frac{1 + \rho_F \cdot \rho_{JTF} + \rho_N \cdot \rho_{JTN}}{2}), \qquad (101)
\end{aligned}$$

by using a relation in (21). Hence, the growth rate of total knowledge for person i when working jointly with j is given by

$$\begin{aligned}
G_J(m^d) &\equiv \frac{\dot{n}_i}{n_i} = \frac{A_{ij}^{J*}}{n_i} \cdot (1 + \frac{1 + \rho_F \cdot \rho_{JTF} + \rho_N \cdot \rho_{JTN}}{2}) \\
&= \frac{A_{ij}^{J*}/2}{n_i} \cdot (3 + \rho_F \cdot \rho_{JTF} + \rho_N \cdot \rho_{JTN}) \\
&= g_J(m^d) \cdot (3 + \rho_F \cdot \rho_{JTF} + \rho_N \cdot \rho_{JTN}), \qquad (102)
\end{aligned}$$

recalling definition (56) and setting $n_i = n$.

Putting together (100) and (102), *the growth rate of total knowledge per person* is given by:

$$G(m^d) = \begin{cases} 2\Phi_I \quad \text{when } \Phi_I \geq g_J(m^d), \\ g_J(m^d) \cdot (3 + \rho_F \cdot \rho_{JTF} + \rho_N \cdot \rho_{JTN}) \quad \text{when } g_J(m^d) > \Phi_I. \end{cases} \tag{103}$$

Let us recall equation (62) which defines *the growth rate of public knowledge* per person. By comparing equations (62) and (103), we can see that the two functions are significantly different, but they also have a close relationship. First, we can observe that each term inside the parentheses of equation (103) is a product of the corresponding term in equation (62) with a constant. Specifically, the first term in equation (103) is twice the corresponding term in equation (62), reflecting the fact that the total knowledge newly created in isolation includes the same amount of tacit knowledge (given by (90)) as formal knowledge. The second term in equation (103) is more than the triple of the corresponding term in equation (62), reflecting the fact that the total knowledge newly acquired by person i through joint work with person j includes the *full amount* of newly created formal knowledge, A_{ij}^{J*}, and the tacit knowledge represented by equations (82) and (83).[12]

Next, observe from equation (102) that since function $G_J(m^d)$ is the product of $g_J(m^d)$ and a constant, it follows that

$$\begin{aligned} m^B &\equiv \text{ the bliss point of function } g_J(m^d) \\ &= \text{ the bliss point of function } G_J(m^d), \end{aligned} \tag{104}$$

where m^B is defined by (59), meaning that the maximum values of two functions, $g_J(m^d)$ and $G_J(m^d)$, are attained at the same point, $m^B \in (0, 1/2)$.

We are now ready to summarize the dual dynamics of formal-K and tacit-K in the two-person system. Notice that along the symmetric equilibrium path, at any $t \in [0, \infty)$, if values of $n_i(t) = n_j(t) \equiv n(t) > 0$ and $m^d(t) \in (0, 1/2)$ are known, then values of all the rest of structural variables, $n^c(t) \equiv n_{ij}^c(t) = n_{ji}^c(t)$, $n^d(t) \equiv n_{ij}^d(t) = n_{ji}^d(t)$, $n^{ij}(t) \equiv n(t) + n^d(t)$, $m^c(t) = 1 - 2m^d(t)$, $A_{ij}^{J*}(t)$, $A_{ii}^{I*}(t)$, $g_J(m^d(t))$, $g_I(m^d(t))$ are uniquely determined, together with the values of choice rules,

$$\delta_{ii}(t) = \delta_{jj}(t) = 1 \text{ if } \Phi_I \geq g_J(m^d(t)), \;\; \delta_{ij}(t) = \delta_{ji}(t) = 1 \text{ if } \Phi_I < g_J(m^d(t)). \tag{105}$$

Hence, given the initial values, $n(0) > 0$ and $m^d(0) \in (0, 1/2)$, solving the system of differential equations, (84) and (93) for $\dot{m}^d$, and (99) and (101) for $\dot{n} = \dot{n}_i = \dot{n}_j$, together with the choice rule (105), the equilibrium path of $m^d(t)$ and $n(t)$, together with all the rest of the structural variables, is uniquely determined for $t \in [0, \infty)$.

For example, let us focus on the case of diagram (a) depicted in Figure 8. In the context of diagram (a), Figure 9 synthesizes the dual dynamics of formal-K and tacit-K in the two-person system.

Figure 9

In the bottom part of Figure 9, diagram (a) in Figure 8 is duplicated, representing the dynamics of formal-K for the case of $m^E < \tilde{m} < m^B$. The

[12] Recall that in the definition of the function $g_J(m^d)$ in (56), the formal K-output A_{ij}^{J*} created by joint work is *divided by* 2 to account for the number of "patents" created *per person*. In contrast, in deriving equation (100), the full amount of knowledge embodied in these "patents," A_{ij}^{J*}, is considered a part of *new knowledge acquired by each person.*

top part of Figure 9 depicts the dynamics of total-K. Comparing the bottom part and the top part of Figure 9, we can recognize similarities and differences between the two dynamics. Since the upper curve $G_J(m^d)$ is the product of the lower curve $g_J(m^d)$ with the constant $(3 + \rho_F \cdot \rho_{JTF} + \rho_N \cdot \rho_{JTN})$, the two curves share the same bliss point, m^B, and the same sink point, $\tilde{m}$, on the m^d-axis. Hence, in the phase of joint work by i and j, and hence when $m^E < m^d(t) < m^H$, the two dynamics are essentially parallel, leading respectively to the sink points S and S^* at the same $\tilde{m}$. However, since the constant of multiplication, $(3 + \rho_F \cdot \rho_{JTF} + \rho_N \cdot \rho_{JTN})$, is more than 3, the upper curve $G_J(m^d)$ is much higher than the lower curve $g_J(m^d)$. This indicates that by working jointly for knowledge creation, the accumulation of total knowledge by each person is much higher than that of formal knowledge created jointly.

In contrast, the phase of knowledge creation in isolation by each person independently, the two dynamics are significantly different. In the bottom part of Figure 9, formal-K dynamics, when $m^d(0) < m^E$, the switch from working in isolation to joint work occurs at the point E, where the horizontal line with of height Φ_I crosses the $g_J(m^d)$ curve. Since m^E on the m^d-axis represents the real switching point based on the switching rule noted in (105), in the top part of Figure 9 the switching is realized at point E' when the growth rate of total knowledge, $2\Phi_I$, from working in isolation is much lower than that from working together. The opposite situation happens at point m^H on the m^d-axis. Hence, from the view-point of total knowledge accumulation, the *myopic switching rule* (105) is not optimal. This is in contrast with our previous work.

6 Conclusions

Building on our earlier work, we have developed a model of knowledge creation in the context of two persons when multiple modes of communication are available, and knowledge workers can independently use the internet for the purpose of search. Tacit knowledge plays a huge role in our analysis. We apply our model to analyze consequences of Covid policies and firm requirements for office presence. When considering long run dynamics, the sink point of the system generally yields a suboptimal configuration of knowledge differentiation. Finally, we find that if knowledge workers use a myopic rule to determine whether they work jointly or independently, they will not internalize tacit knowledge creation in their decisions, leading to a second source of inefficiency. In summary, we have seen the importance of tacit knowledge when considering the process of knowledge creation in the long run.

How can these two inefficiencies be addressed? For the first type of inefficiency, illustrated in Figure 8(a,c), the solution would be to have larger research group sizes as pointed out in our earlier work. This prevents the dynamic buildup of knowledge in common, so the bliss point can be decentralized using the myopic core. Our restriction to two persons here does not allow that, so the extension of the model to more than two persons would have to be considered.

For the situation illustrated in Figure 8(b), again we should consider an extension of the model to more knowledge workers. If the workers have a diversity of backgrounds in terms of differential knowledge relative to each other at time 0, initial partners for knowledge creation could be chosen so that this case is excluded. In other words, it would be best if a person's initial partner were chosen so that knowledge in common is large. Both partners would find this optimal in the long run, and thus could be optimal for forward looking agents. Another way to justify this is to notice that in Figure 8(b), every knowledge state to the right of the bliss point can be matched in terms of knowledge growth by a state to the left of the bliss point, but the (absolute value of the) slope of the knowledge creation function is *higher to the left of the bliss point* for each level of growth rate or point on the vertical axis. So even if agents are myopic, the *derivative* of the rate of growth to the left of the bliss point is higher at the same level of knowledge growth as the corresponding point to the right of the bliss point. Given a choice of backgrounds for their initial partner, myopic agents would choose an initial partner with more knowledge in common. That is to say, it is a weakly dominant myopic strategy. The assumption here is that if there is an initial partner with a profile to the right of the bliss point available, then there is also one to the left of the bliss point with the same or higher initial K-growth rate available.

Regarding the second inefficiency, a result of the myopic rule and tacit knowledge, the use of forward looking agents should be investigated.

To sum up, there is much further work to be done to analyze the microeconomic dynamics of knowledge creation in settings with tacit knowledge.

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7 Appendix: The many roles played by common knowledge in joint knowledge creation

7.1 Preliminaries

In equation (9), we introduced the function $\beta_F(\theta_F)$ in order to represent an implicit weight on knowledge in common during F2F creation. As explained next, when persons i and j work jointly for knowledge creation, the stock of knowledge in common, n_{ij}^c, should play multiple roles.

For example, when two economists are working together, basic mathematics and microeconomics constitute a part of their knowledge in common; they are used by each person while collaborating, and the two persons are able to communicate effectively based on such common knowledge. So, relative to working independently, the size of the stock of knowledge in common while working jointly should be counted more than once. However, the degree of double counting should be limited.

Likewise, when two persons are jointly working through the Net, we introduced the function $\beta_N(\theta_N)$ in order to represent an implicit weight on knowledge in common. In this Appendix, we examine the question: What are appropriate functional forms for $\beta_F(\theta_F)$ and $\beta_N(\theta_N)$ to represent the multiple roles of common knowledge in joint creation?

Here, our major concern is the influence of the functions $\beta_F(\theta_F)$ and $\beta_N(\theta_N)$ on the per person knowledge growth rate $g_J(m^d)$, defined by (56). To examine this effect, we decompose the function Ω given by (57) as follows:

$$\Omega = \hat{\Omega} \cdot P, \tag{A.1}$$

where

$$P \equiv \beta_F(\theta_F)^{(1-\theta_F)\cdot\rho_{JTF}\cdot\rho_F} \cdot \beta_N(\theta_N)^{(1-\theta_N)\cdot\rho_{JTN}\cdot\rho_N}, \tag{A.2}$$

$$\hat{\Omega} \equiv \frac{\Omega}{P}. \tag{A.3}$$

By definition, the new function $\hat{\Omega}$ does not contain θ_F nor θ_N. Using these functions, the knowledge growth rate function (56) can be rewritten as follows:

$$g_J(m^d) = \hat{\Omega} \cdot P \cdot Q, \tag{A.4}$$

where

$$Q \equiv (1-m^d)^a \cdot (1-2m^d)^b \cdot (m^d)^c, \tag{A.5}$$

and where a, b and c are functions of parameters defined in (57).

Now, substituting the bliss point functions (59) into (A.4) and (A.5), we have

$$g_J(m^B) = \hat{\Omega} \cdot P \cdot Q, \tag{A.6}$$

where

$$Q = \frac{c^c \cdot (\delta - c)^{\delta - c}}{\delta^\delta}, \tag{A.7}$$

and

$$\begin{aligned} c &\equiv \theta_F \cdot \rho_{JTF} \cdot \rho_F + \theta_N \cdot \rho_{JTN} \cdot \rho_N, \\ \delta &\equiv \rho_{JTF} \cdot \rho_F + \rho_{JTN} \cdot \rho_N. \end{aligned} \tag{A.8}$$

By definition, δ is independent of θ_F and θ_N.

7.2 The Symmetric Case

First, let us consider the special case where

$$\theta_F = \theta_N \equiv \theta, \tag{A.9}$$

and the functions β_F and β_N are assumed to be the same:

$$\beta_F(\theta_F) = \beta_N(\theta_N) \equiv \beta(\theta) \text{ for } \theta \in [0, 1]. \tag{A.10}$$

Then, substituting (A.9) into (59) yields

$$m^B = \frac{1}{1 + \frac{1}{\theta}} \equiv m^B(\theta), \tag{A.11}$$

whereas P and Q respectively become

$$P = \beta(\theta)^{(1-\theta)\delta} \equiv P(\theta), \tag{A.12}$$

$$Q = \theta^{\theta\delta} \cdot (1 - \theta)^{(1-\theta)\delta} \equiv Q(\theta). \tag{A.13}$$

Substituting (A.11), (A.12) and (A.13) into (A.6) yields

$$g_J(m^B(\theta)) = \hat{\Omega} \cdot P(\theta) \cdot Q(\theta).$$

or

$$g_J(m^B(\theta))/\hat{\Omega} = P(\theta) \cdot Q(\theta), \tag{A.14}$$

which represents the (normalized) *bliss point growth rate of knowledge* per person as a function of θ.

In order to capture the essential nature of the issue at hand, first let us assume that

$$\beta(\theta) = 1 \text{ and hence } P(\theta) = 1 \text{ for all } \theta \in [0,1], \tag{A.15}$$

implying that

$$g_J(m^B(\theta))/\hat{\Omega} = Q(\theta). \tag{A.16}$$

In diagram (a) of Figure A, the function (A.16) is represented by a curve, which we call *the (normalized) bliss curve.*

Figure A

From (A.13), it can be readily shown that the bliss curve, $Q(\theta)$, is strictly convex on $[0, 1]$ whereas

$$Q(0) = Q(1) = 1, \tag{A.17}$$

and

$$\min\{Q(\theta) \mid \theta \in [0,1]\} = Q(\frac{1}{2}) = 2^{-\delta}, \tag{A.18}$$

That is, the bliss curve achieves the minimum at $\theta = 1/2$. The corresponding proportion of differential knowledge of each person is obtained from (A.11) as

$$m^B(\frac{1}{2}) = \frac{1}{3}, \tag{A.19}$$

and hence the corresponding proportion of common knowledge is

$$m^c = 1 - 2m^B(\frac{1}{2}) = \frac{1}{3}. \tag{A.20}$$

Unfortunately, however, the implications of (A.18), (A.19) and (A.20) are inappropriate. That is, when we introduced the production function (9) for joint knowledge creation F2F, we said, "The rate of creation of new intermediate ideas is high when the proportion of knowledge in common, knowledge exclusive to person i, and knowledge exclusive to person j are in balance." However, (A.17), (A.18) and (A.19) together mean almost the opposite. That is, the rate of creation of new intermediate ideas at the bliss point is *lowest* when the weight on differential knowledge and the weight on common knowledge are in balance (i.e., $\theta = 1 - \theta = 1/2$) and when the proportion of knowledge in common and knowledge exclusive to each person are in balance (i.e., $m_{ij}^d = m_{ji}^d = m_{ij}^c = 1/3$).

In order to remedy this undesirable result, we now abandon the assumption (A.15), and introduce a more appropriate functional form for $\beta(\theta)$, whereas the simplifying assumptions (A.9) and (A.10) are maintained for the remainder of this subsection. It is evident from diagram (b) of Figure A that when the curve $Q(\theta)$ is *strictly convex* on $[0,1]$, in order to make the new bliss curve $P(\theta)\cdot Q(\theta)$ be *strictly concave*, function $P(\theta)$ should be *strictly concave* on $[0,1]$. To obtain such a strictly concave function $P(\theta)$, let us consider the following functional form for β:

$$\beta(\theta) = \theta^{-\frac{\theta\phi}{1-\theta}} \cdot (1-\theta)^{-\phi} \quad \text{for } \theta \in [0,1], \tag{A.21}$$

where ϕ is a positive constant. Then, substituting (A.21) into (A.12) leads to

$$\begin{aligned} P(\theta) &= \theta^{-\theta\phi\delta} \cdot (1-\theta)^{-(1-\theta)\phi\delta} \\ &= Q(\theta)^{-\phi}, \end{aligned} \tag{A.22}$$

using (A.13). As depicted in diagram (b) of Figure A, since $Q(\theta)$ is a strictly convex curve, the $P(\theta)$ *curve defined by (A.22) with* $\phi > 0$ *is strictly concave.* Furthermore, substituting (A.22) into (A.14) yields

$$P(\theta) \cdot Q(\theta) = Q(\theta)^{-(\phi-1)}. \tag{A.23}$$

Hence, *when* $\phi > 1$*, the new bliss curve,* $P(\theta)\cdot Q(\theta)$*, is strictly concave* on $[0,1]$, whereas

$$P(0)\cdot Q(0) = P(1)\cdot Q(1) = 1 \tag{A.24}$$

and

$$\max\{P(\theta)\cdot Q(\theta) \mid \theta \in [0,1]\} = Q(\frac{1}{2})^{-(\phi-1)} = 2^{\delta(\phi-1)} > 1. \tag{A.25}$$

using (A.18). Furthermore, setting $\theta = \frac{1}{2}$ in (A.11), we have (A.19) and (A.20) as before.

Summarizing the analysis above, we can conclude as follows:

Proposition A.1: Consider the special case where $\theta_F = \theta_N$. If we specify the weight function $\beta(\theta)$ on knowledge in common as (A.21) with $\phi > 1$, then the bliss point growth rate of knowledge defined by (A.14) attains its highest value at $\theta = 1/2$ with $m^B(\frac{1}{2}) = m^c = 1/3$. That is, the knowledge growth rate at the bliss point is highest when the weight on differential knowledge and weight on common knowledge are in balance (i.e., $\theta = 1-\theta = \frac{1}{2}$) and when the proportion of knowledge in common and proportion of differential knowledge of each person are in balance (i.e., $m^c_{ij} = m^d_{ij} = m^d_{ji} = 1/3$).

7.3 The General Case

In this subsection, we show that the desirable result above holds in the general case without assuming $\theta_F = \theta_N$.

As before, let us assume that $\beta_F(\cdot)$ and $\beta_N(\cdot)$ have the same functional form given by (A.21) with $\phi > 1$. Then, (A.2) becomes

$$P = P_F(\theta_F) \cdot P_N(\theta_N) \equiv P(\theta_F, \theta_N), \tag{A.26}$$

where

$$\begin{aligned} P_F(\theta_F) &\equiv \beta(\theta_F)^{(1-\theta_F)\cdot\rho_{JTF}\cdot\rho_F} \\ &= \theta_F{}^{-\theta_F\cdot\rho_{JTF}\cdot\rho_F\cdot\phi} \cdot (1-\theta_F)^{-(1-\theta_F)\cdot\rho_{JTF}\cdot\rho_F\cdot\phi}, \end{aligned} \tag{A.27}$$

$$\begin{aligned} P_N(\theta_N) &\equiv \beta(\theta_N)^{(1-\theta_N)\cdot\rho_{JTN}\cdot\rho_N} \\ &= \theta_N{}^{-\theta_N\cdot\rho_{JTN}\cdot\rho_N\cdot\phi} \cdot (1-\theta_N)^{-(1-\theta_N)\cdot\rho_{JTN}\cdot\rho_N\cdot\phi}. \end{aligned} \tag{A.28}$$

Substituting (A.26) into (A.6) yields

$$g_J(m^B) = \hat{\Omega} \cdot P(\theta_F, \theta_N) \cdot Q, \tag{A.29}$$

where Q is given by (A.7) together with (A.8). Noting that parameter c defined in (A.8) contains θ_F and θ_N as arguments, differentiation of (A.29) with θ_F and θ_N yields respectively that

$$\begin{aligned} \frac{\partial g_J(m^B)}{\partial \theta_F} &= 0 \Rightarrow \left(\frac{\theta_F}{1-\theta_F}\right)^{\phi} = \frac{c}{\delta - c}, \\ \frac{\partial g_J(m^B)}{\partial \theta_N} &= 0 \Rightarrow \left(\frac{\theta_N}{1-\theta_N}\right)^{\phi} = \frac{c}{\delta - c}, \end{aligned}$$

meaning that

$$\frac{\theta_F}{1-\theta_F} = \frac{\theta_N}{1-\theta_N} = \left(\frac{c}{\delta - c}\right)^{1/\phi}. \tag{A.30}$$

The first equality above implies that

$$\theta_F = \theta_N \equiv \theta, \tag{A.31}$$

which in turns implies from (A.8) that

$$\frac{c}{\delta - c} = \frac{\theta}{1-\theta}. \tag{A.32}$$

Since it is assumed that $\phi > 1$, conditions (A.30), (A.31) and (A.32) together imply that

$$\theta_F^* = \theta_N^* = \frac{1}{2}. \tag{A.33}$$

where $*$ means the values of θ_F and θ_N that satisfy the first-order conditions for optimization. Substituting (A.33) into (59) yields

$$m^B = \frac{1}{3} \equiv m^{B*} \text{ at } \theta_F^* = \theta_N^* = \frac{1}{2}, \tag{A.34}$$

which in turn, from (A.26) and (A.7), yields

$$P(\theta_F^*, \theta_N^*) = 2^{\delta\phi} \text{ and } Q = 2^{-\delta}.$$

Hence, from (A.29),

$$g_J(m^{B*}) = \hat{\Omega} \cdot 2^{\delta(\phi-1)} > \hat{\Omega} \tag{A.35}$$

since $\phi > 1$. On the other hand, from definition (A.6), it follows that

$$g_J(0) = g_J\left(\frac{1}{2}\right) = \hat{\Omega}. \tag{A.36}$$

From (A.35) and (A.16), we can conclude that $g_J(m^{B*})$ represents the *maximum value* of $g_J(m^d)$ on the parameter space $\{(\theta_F, \theta_N) \mid 0 \leq \theta_F \leq 1, 0 \leq \theta_N \leq 1\}$.

Summarizing the results above, we can generalize Proposition A.1 as follows:

Proposition A.2: Let us specify the weight functions, $\beta_F(\cdot)$ and $\beta_N(\cdot)$, on knowledge in common by the same functional form given by (A.21) with $\phi > 1$. Then, on the parameter space $\{(\theta_F, \theta_N) \mid 0 \leq \theta_F \leq 1, 0 \leq \theta_N \leq 1\}$, the bliss point growth function given by (A.29) achieves its maximum value when

$$\theta_F^* = \theta_N^* = \frac{1}{2},$$

and hence

$$m^{B*} = \frac{1}{3}, m^{c*} = \frac{1}{3}.$$

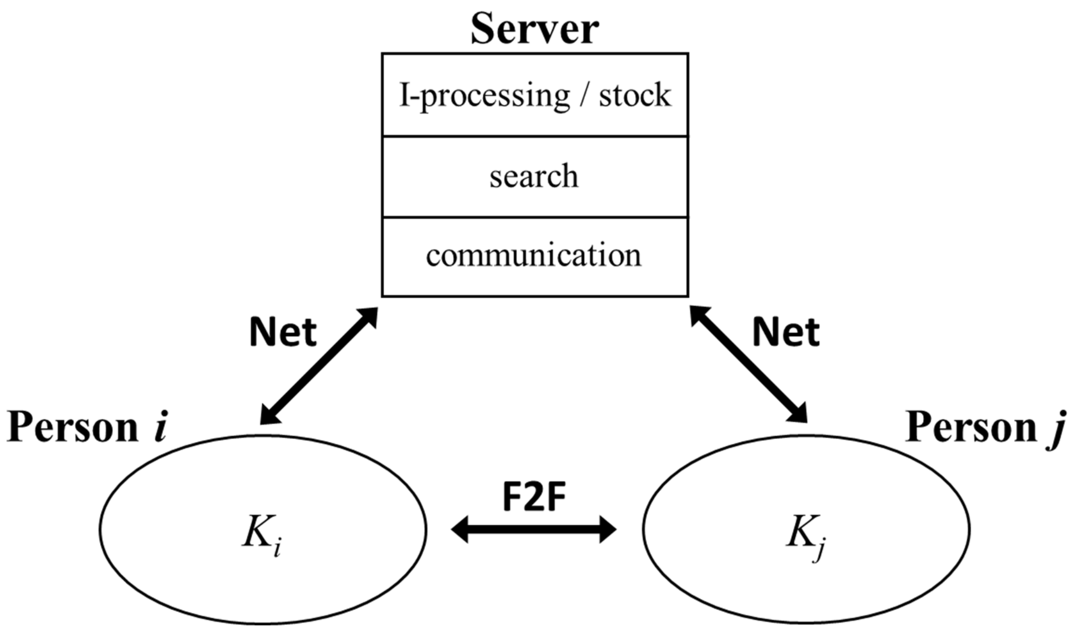

Figure 1. Knowledge creation through multiple modes of communication

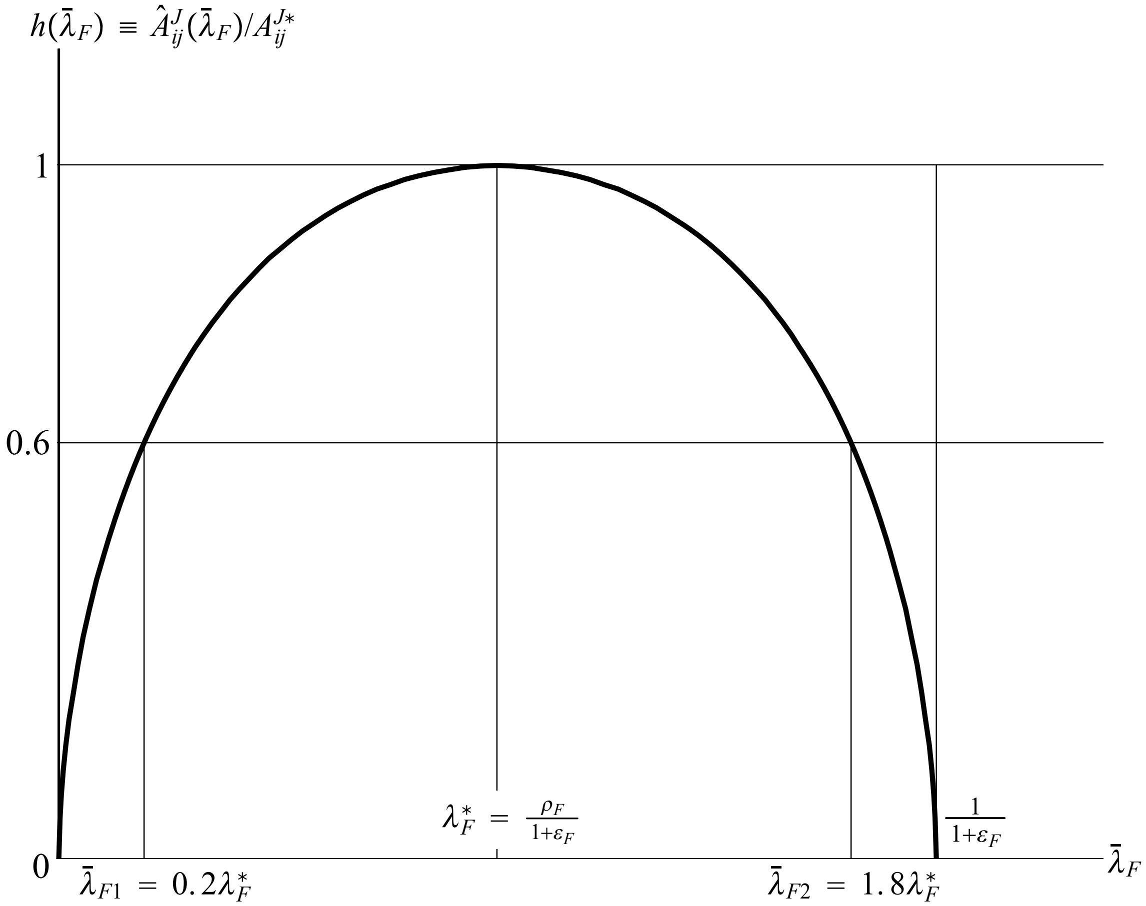

Figure 2. The relative productivity curve $h(\bar{\lambda}_F)$ when $\rho_F = \frac{1}{2}$, $\rho_I = \rho_N = \frac{1}{4}$, $\varepsilon_F = 0.2$.

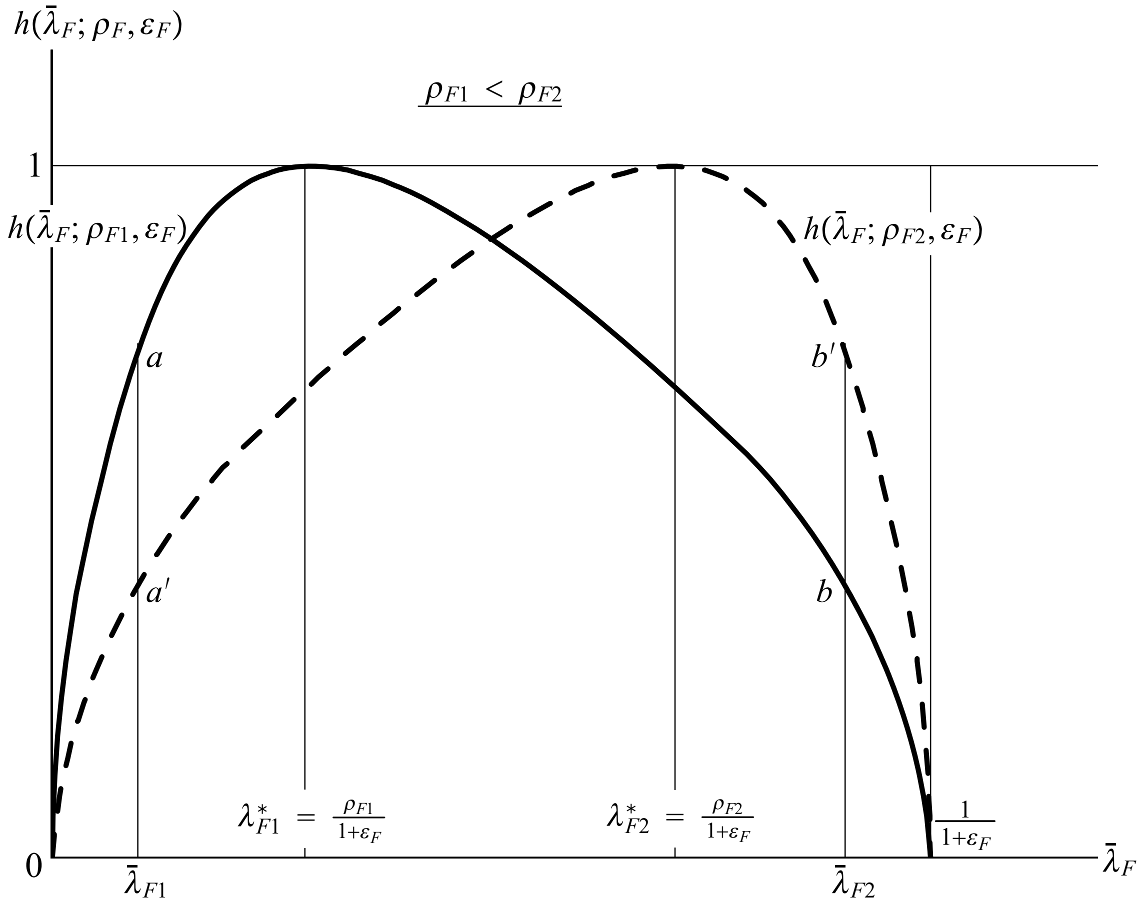

Figure 3. The effect of different ρ_F on the shape of relative productivity curve $h(\bar{\lambda}_F; \rho_F, \varepsilon_F)$, where $\rho_{F1} < \rho_{F2}$.

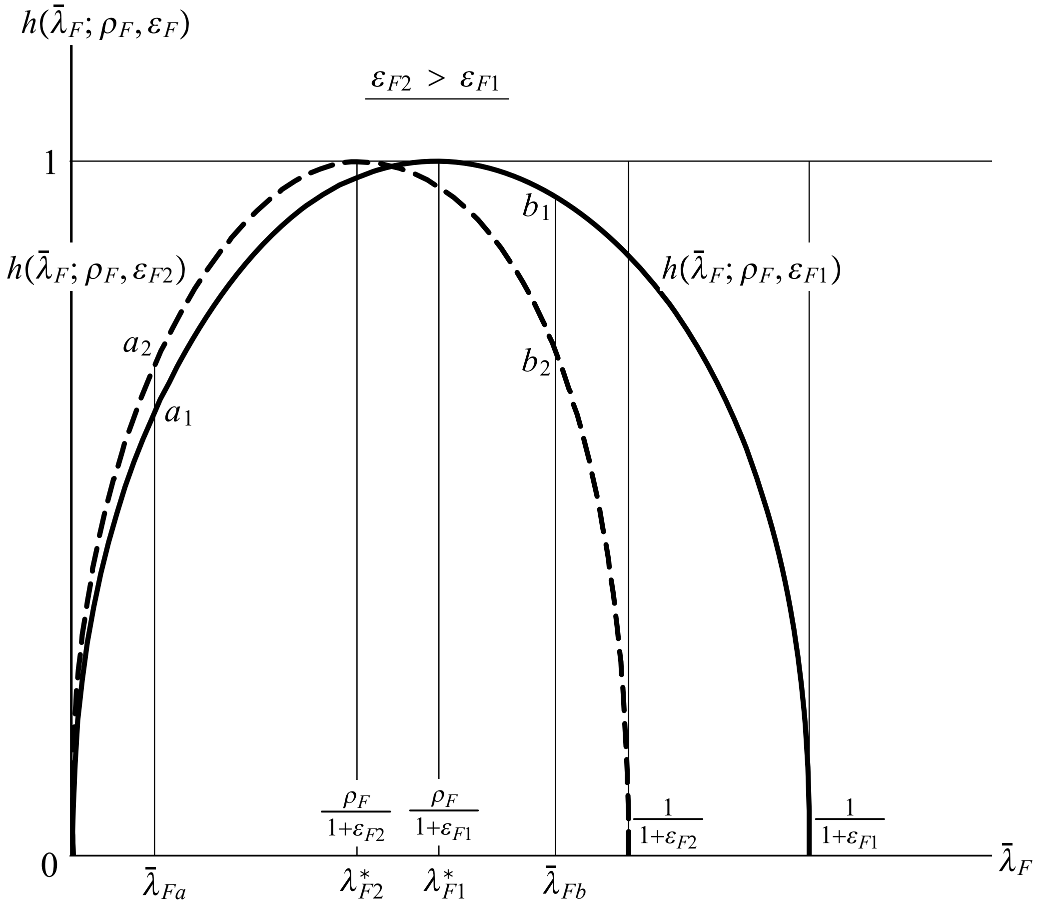

Figure 4. The effect of different ε_F on the shape of relative productivity curve $h(\bar{\lambda}_F; \rho_F, \varepsilon_F)$, where $\varepsilon_{F2} > \varepsilon_{F1}$.

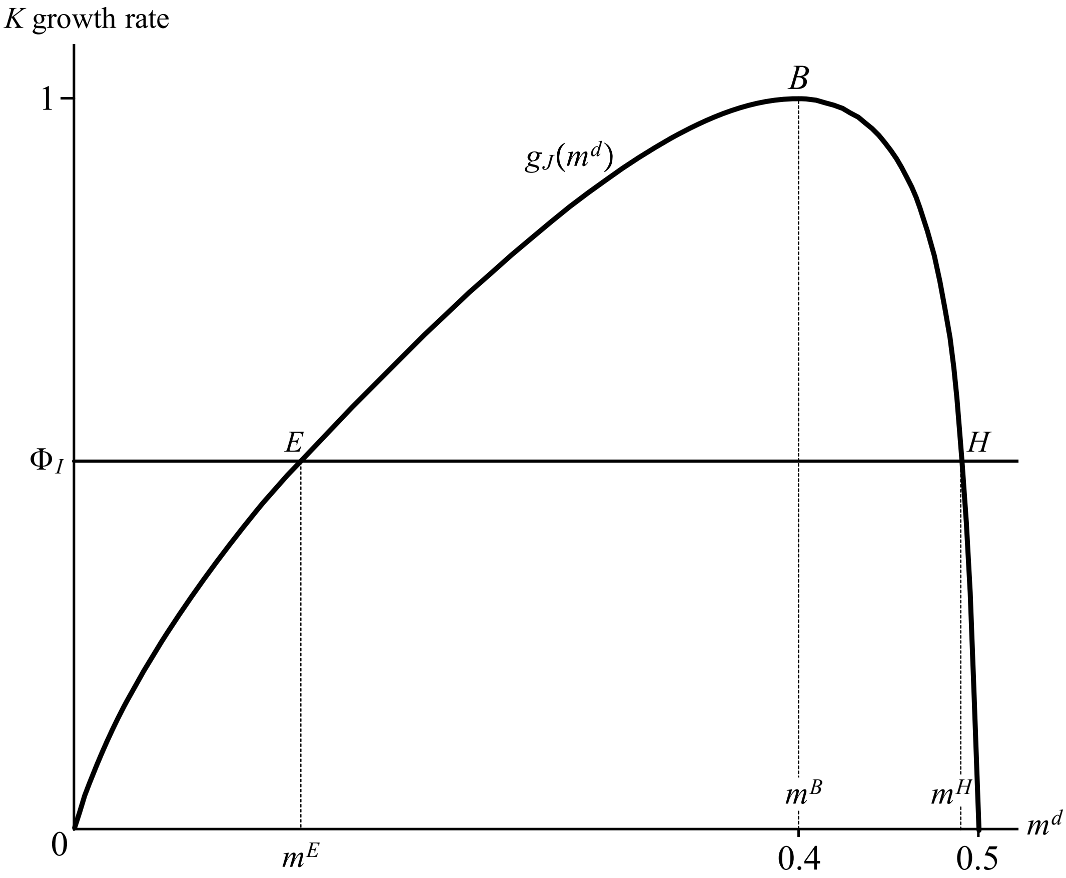

Figure 5. The knowledge growth rate curve $g_J(m^d)$ and the Bliss Point m^B.

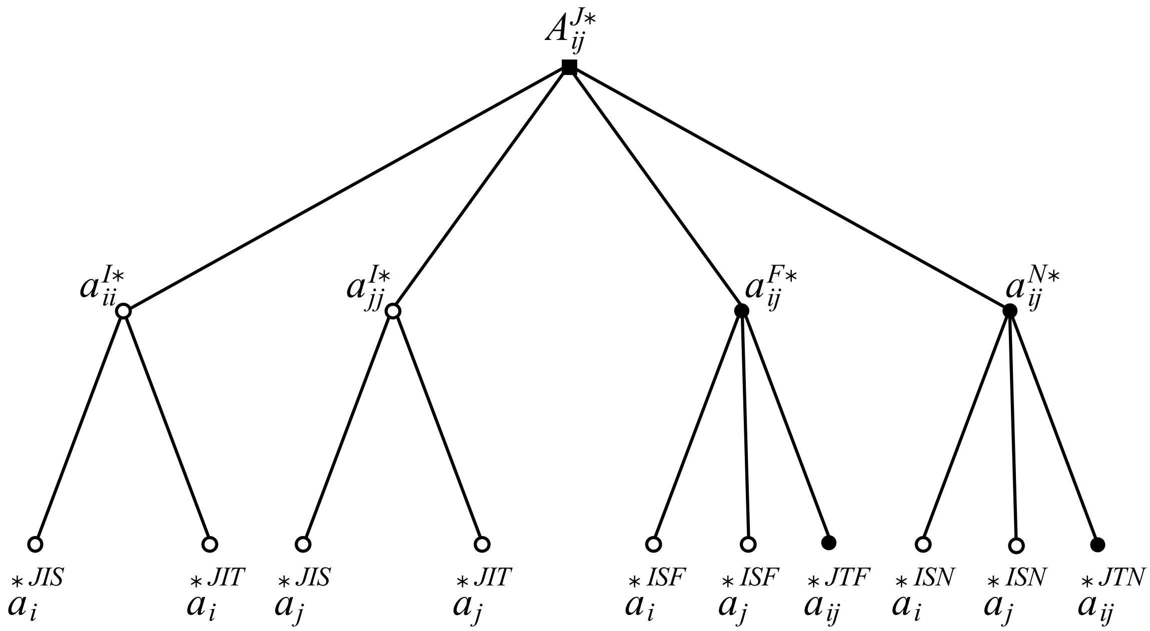

Figure 6. The activity tree for joint knowledge creation:

■ the final output of the joint work

● representing a joint activity with subscript *ij*,

○ representing an independent activity for the purpose of joint creation with subscript *i*, *ii*, *j* or *jj*.

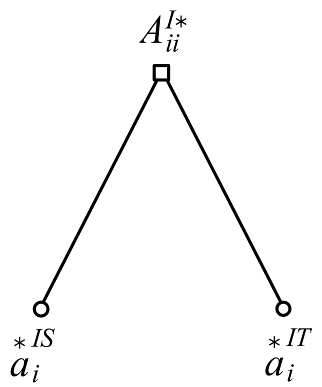

Figure 7. The activity tree for the knowledge creation by person *i* in isolation.

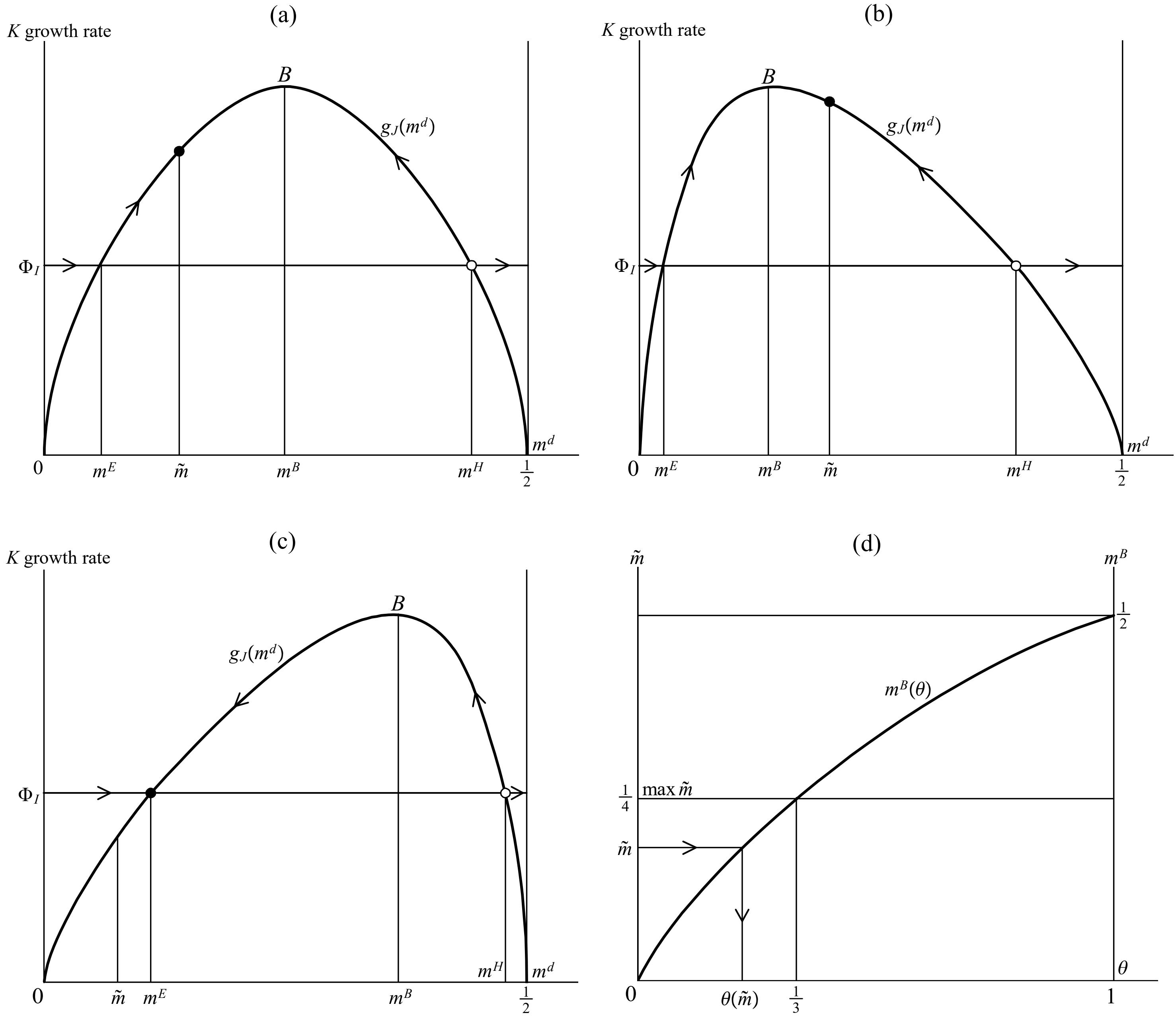

Figure 8. The three possible cases, (a), (b) and (c), for the dynamics of two-person system; and diagram (d) for explaining the relationship between $\tilde{m}$ and m^B.

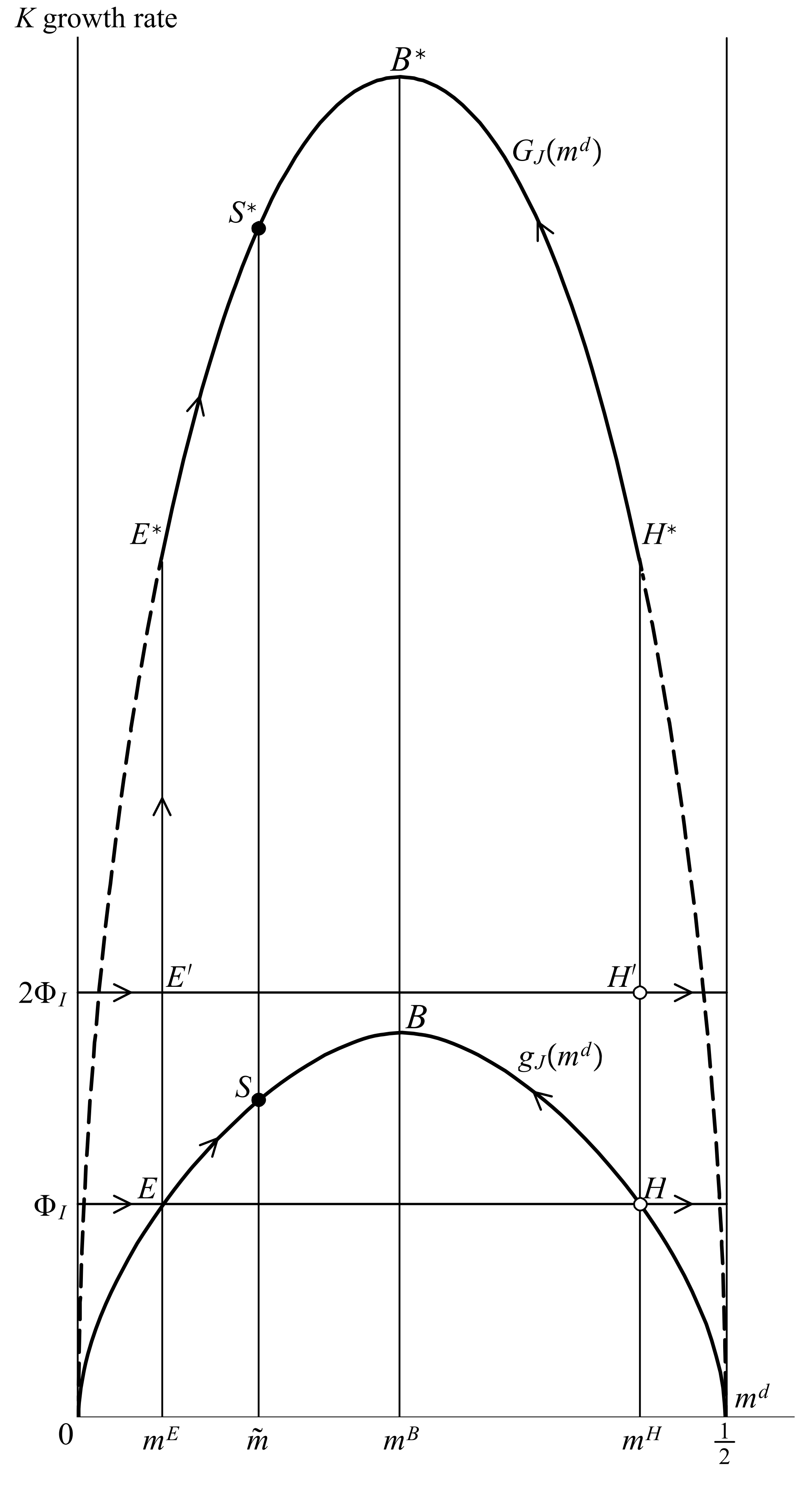

Figure 9. Dual dynamics of formal-K and total-K for the two-person system.

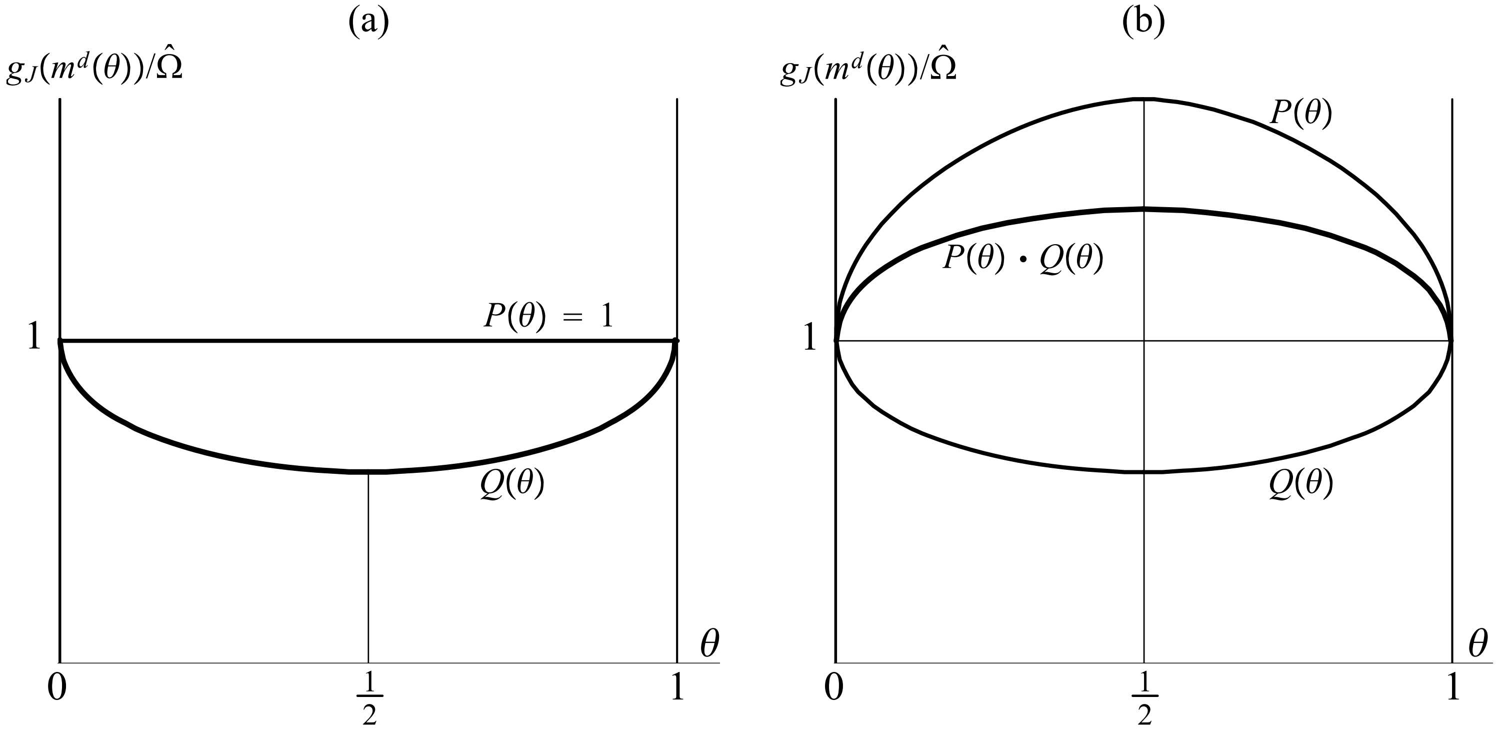

Figure A. (a) Bliss curve $Q(\theta)$ when $P(\theta) = 1$, (b) Bliss curve $P(\theta) \cdot Q(\theta)$ when $P(\theta) = Q(\theta)^{-\phi}$ and $\phi > 1$.

tier	activity output	type	description of output x	imputed value of x: $(\partial A_{ij}^{J*}/\partial x)\cdot x$
top	■ A_{ij}^{J*}	final output	equation (18)	A_{ij}^{J*}
second tier	○ a_{ii}^{I*}	creation by i	equation (7)	$A_{ij}^{J*}\cdot\frac{\rho_I}{2}$
	○ a_{jj}^{I*}	creation by j	equation (8)	$A_{ij}^{J*}\cdot\frac{\rho_I}{2}$
	● a_{ij}^{F*}	joint in F2F	equation (10)	$A_{ij}^{J*}\cdot\rho_F$
	● a_{ij}^{N*}	joint in Net	equation (11)	$A_{ij}^{J*}\cdot\rho_N$
third tier	○ a_i^{*JIS}	search by i	$\rho_{JIS}\cdot\alpha_{JIS}\cdot n_i$	$A_{ij}^{J*}\cdot\frac{\rho_I\cdot\rho_{JIS}}{2}$
	○ a_i^{*JIT}	thinking by i	$\rho_{JIT}\cdot\alpha_{JIT}\cdot n_i$	$A_{ij}^{J*}\cdot\frac{\rho_I\cdot\rho_{JIT}}{2}$
	○ a_j^{*JIS}	search by j	$\rho_{JIS}\cdot\alpha_{JIS}\cdot n_j$	$A_{ij}^{J*}\cdot\frac{\rho_I\cdot\rho_{JIS}}{2}$
	○ a_j^{*JIT}	thinking by j	$\rho_{JIT}\cdot\alpha_{JIT}\cdot n_j$	$A_{ij}^{J*}\cdot\frac{\rho_I\cdot\rho_{JIT}}{2}$
	○ a_i^{*ISF}	search by i while F2F	$\rho_{ISF}\cdot\alpha_{ISF}\cdot n_i$	$A_{ij}^{J*}\cdot\frac{\rho_F\cdot\rho_{ISF}}{2}$
	○ a_j^{*ISF}	search by j while F2F	$\rho_{ISF}\cdot\alpha_{ISF}\cdot n_j$	$A_{ij}^{J*}\cdot\frac{\rho_F\cdot\rho_{ISF}}{2}$
	● a_{ij}^{*JTF}	joint thinking while F2F	$\rho_{JTF}\cdot\alpha_{JTF}\cdot(\beta_F(\theta_F)\cdot n_{ij}^c)^{1-\theta_F}\times(n_{ij}^d\cdot n_{ji}^d)^{\frac{\theta_F}{2}}$	$A_{ij}^{J*}\cdot\rho_F\cdot\rho_{JTF}$
	○ a_i^{*ISN}	search by i while Net	$\rho_{ISN}\cdot\alpha_{ISN}\cdot n_i$	$A_{ij}^{J*}\cdot\frac{\rho_N\cdot\rho_{ISN}}{2}$
	○ a_j^{*ISN}	search by j while Net	$\rho_{ISN}\cdot\alpha_{ISN}\cdot n_j$	$A_{ij}^{J*}\cdot\frac{\rho_N\cdot\rho_{ISN}}{2}$
	● a_{ij}^{*JTN}	joint thinking while Net	$\rho_{JTN}\cdot\alpha_{JTN}\cdot(\beta_N(\theta_N)\cdot n_{ij}^c)^{1-\theta_N}\times(n_{ij}^d\cdot n_{ji}^d)^{\frac{\theta_N}{2}}$	$A_{ij}^{J*}\cdot\rho_N\cdot\rho_{JTN}$

Table 1. Further description of the activity tree for joint knowledge creation in Figure 6