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Abstract

The existing literature on optimal specific import tariff only compares the pre and the post-tariff market equilibriums in order to account for the efficiency losses or gains. However, when the government imposes a tariff, the foreign producer's cost (to the extent of his/ her exports) jumps to the pre-tariff cost plus the amount of specific tariff affecting the import quantity and hence pushing the market out of equilibrium. The supply and the demand of the commodity on which a tariff is imposed then adjust over time to bring the new post-tariff market equilibrium. **The price adjustment mechanism is based on the fact that when the imposition of a tariff leads the market out of equilibrium, the buyers' and sellers' decisions are not coordinated at the current prices.** The existing literature does not take into account the efficiency losses during the adjustment process while computing the optimal specific import tariffs. This paper develops a dynamic model and derives an optimal specific import tariff path minimizing the efficiency losses (output and/ or consumption lost) during the dynamic adjustment process as well as the post-tariff market equilibrium subject to a tariff revenue constraint. (JEL F10, F11, H20, H21, H30)

Keywords: Specific Import Tariff Path, Tariff Revenue, Dynamic Efficiency, Price Adjustment Path, Equilibrium

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1 Introduction

The theory of optimal tariff has a long history. The possibility that a tariff could improve national welfare for a large country in international markets was first noted by Robert Torrens as depicted in Torrens (1844), Robbins (1959), O'Brien (1975) and Viner (2016). Mill (1844) showed how different elasticities affect the degree of terms-of trade improvement. Sidgwick (1887) constructed an optimum tariff model. In particular, Sidgwick stressed three points. First is the importance of monopsony power in achieving terms-of-trade improvement. No country, he said, could expect to improve its terms of trade by means of tariff unless it supplied a considerable part of the whole demand for the foreign products. Second, a tariff affects the terms of trade through its impact on reciprocal demands. Specifically, A's tariff reduces her demand for B's good, thus producing an excess world supply of that good. This excess supply is only eliminated by a deterioration in B's terms of trade. Third, the effectiveness of A's tariff depends upon the elasticity of B's offer curve. If that curve is almost totally inelastic, as when B urgently requires A's good at any price, the terms-of-trade gain realized by A comes at the cost of little or no shrinkage in her export volume. But if B's offer curve is perfectly elastic, as when she can readily substitute third-country goods for A's good in her consumption mix, A's tariff will have no effect other than diminishing her (A's) real trade volume. Edgeworth (1894) provided the key insight regarding when a country should impose a tariff. It was shown that as long as a foreign country's offer curve was not perfectly elastic, a country could gain from a tariff. In this case, the decrease in import demand on account of a tariff leads to a reduction in the price of all units imported and this first order gain offsets the distortion losses from lower imports.

Bickerdike (1906) and Bickerdike (1907) contributed some innovations to optimum tariff theory. C. F. Bickerdike specified a new welfare gain from trade restriction. As an alternative to Edgeworth's indifference curve measure, he defined the net benefit of an import duty as the sum of the tax revenue collected from foreigners through lower import prices less the deadweight loss in consumers' surplus caused by the shrinkage in trade volume. Kaldor (1940) asserted that a tariff always benefits the levying country provided that the duty is not too large, that the country has monopoly power in world markets, and that other countries do not retaliate with tariffs of their own. The classic formula for the optimum tariff was later made famous by Kahn (1947), Graaff (1949) and Johnson (2013). Grossman and Helpman (1995) extend their endogenous trade policy model to the case where a country is "large", i.e. it faces finite export supply elasticities. Bond (1990) generalizes to higher dimensions the two good result that the optimal trade policy for a large country is an import tariff. Ogawa (2007) theoretically analyzes the optimal tariff problem that arises in a large country with market power. It is not hard to find examples of first class theorists arguing that it provides the underlying motive for the world trading system (Bagwell and Staiger (1999)) while others argue that it is little more than an intellectual curiosity with no practical value in all but the largest countries

(Krugman and Obstfeld (1997)). Felbermayr, Jung and Larch (2013) characterize analytically the optimal tariff of a large one-sector economy with monopolistic competition and firm heterogeneity in general equilibrium. Cato (2017) examines the relationship between the optimal tariff structure and the degree of penetration.

According to the World Bank, "Regardless of whether a tariff is bound or applied on preferential versus non-discriminatory basis, the tariff can take several forms. The most common is an ad valorem tariff, which means that the customs duty is calculated as a percentage of the value of the product. Many countries' tariff schedules also include a variety of non ad valorem tariffs.

- Specific tariffs are computed on the physical quantity of the good being imported, e.g., Australia's 2005 schedule includes a tariff of $1.22/ kg on certain types of cheeses and the United States charges $0.68 per live goat. The physical quantity may be expressed in ways that are difficult to determine without laboratory equipment. The European Union charges duties on certain dairy products based on the weight of lactic matter in the product, and the United States charges a tariff on raw cane sugar that varies with the sucrose content of sugar: 1.4606 cents/ kg less 0.020668 cents/ kg for each degree under 100 degrees (and fractions of a degree in proportion) but not less than 0.943854 cents/ kg.

- Mixed tariffs are expressed as either a specific or an ad valorem rate, depending on which generates the most (or sometimes least) revenue. For example, Indian duties on certain rayon fabrics are either 15 percent ad valorem or Rs. 87 per square meter, whichever is higher.

- Compound tariffs include both ad valorem and a specific component. For example, Pakistan charges Rs. 0.88 per liter of some petroleum products plus 25 percent ad valorem.

- Tariff rate quotas are made up of a low tariff rate on an initial increment of imports (the within-quota quantity) and a very high tariff rate on imports entering above that initial amount.

Trade economists typically argue that these non ad valorem tariffs are less transparent and more distorting, i.e., that they drive a bigger wedge between domestic and international prices. In addition, their economic impact changes as world prices change. The share of tariff lines with non ad valorem rates varies across countries. WITS Advanced Query can compute the share of non-ad valorem tariff lines when it profiles a county's tariff schedule."

The existing literature on optimal specific import tariff only compares the pre and the post-tariff market equilibriums in order to account for the efficiency losses or gains. However, when the government imposes a tariff, the foreign producer's cost (to the extent of his/ her exports) jumps to the pre-tariff cost plus the amount of specific tariff affecting the import quantity and hence pushing the market out of equilibrium. The supply and the demand of the commodity on which a tariff is imposed then adjust over time to bring the new post-tariff market equilibrium. **The price adjustment mechanism is based on the fact that when the imposition of a tariff leads the market out of equilibrium, the buyers' and sellers' decisions are not coordinated at the current prices.** The existing literature does not take into account the efficiency losses during the adjustment process while computing the optimal specific import tariffs. This paper develops a dynamic model and derives an optimal specific import tariff path minimizing the efficiency losses (output and/ or consumption lost) during the dynamic adjustment process as well as the post-tariff market equilibrium subject to a tariff revenue constraint.

The remainder of this paper is organized as follows: Section 2 explains how the individual components of the market system are joined together to form a dynamic market model. Section 3 provides the solution of the model with a specific import tariff imposed. Section 4 derives an optimal specific import tariff path minimizing the efficiency losses subject to a tariff revenue constraint in a specific time period. **Section 5 presents a comparison of the static and the dynamic efficiency loss as a result of a specific import tariff.** Section 6 summarizes the findings and concludes. Section 7 explains the future research prospects.

2 The Model

Let's assume that there is a perfectly competitive market of a single homogeneous commodity in equilibrium (so our starting point is when the market is already in equilibrium). There are four market agents, i.e. domestic producer, foreign producer, consumer and a middleman. The role of middleman is motivated by the real world scenario where the producer and the consumer seldom directly meet for a transaction to take place. The existence of retailers, wholesalers, financial institutions, educational institutions and the hospitals reflect the presence of middlemen between producers and consumers in most of the economic activity going on. **Furthermore, assuming the role of the middleman as an intermediary between the foreign producer and the local consumer is also realistic as it is quite common for the foreign producers to sell the products to an intermediary importer in the domestic country (who subsquently sells the products to consumers) rather than the local consumers directly. Also, as the producer is a price taker in a perfectly competitive market, who would change the price in case a shock happens to the market? The price cannot automatically jump to bring the new equilibrium, rather realistically speaking, some agents have to change the price in their own benefit which leads to the new equilibrium, where they find it optimal not to change the price further and the market stays in equilibrium until another shock hits the market. It does not limit the generality of the analysis in any case as the initial and the final equilibrium values are identical to those in a static model. It only adds to the static model in terms of dynamics of the price in**

a perfectly competitive market as a result of a shock. The producers produce the goods and supply those to the middleman, who keeps an inventory of the goods and sells those to the consumer at the market price. The producers and the middleman have an objective of maximizing their profits and the consumer's objective is to maximize utility.

The price adjustment mechanism is based on the fact that when a shock leads the market out of equilibrium, the buyers' and sellers' decisions are not coordinated at the current prices. An example can illustrate the working of this market. Consider that the market is initially in equilibrium. The middleman has an equilibrium stock of inventory. Then, an exogenous supply contraction will decrease the stock of inventory, due to consumers' demand could not match with the –now lower– units supplied by the producers at the current price. This lower supply is reflected in the depleted inventory held by the middleman. The middleman will increase the price so that the producers will find optimal to produce a higher level of output. A new equilibrium with a higher price and a lower level of output is then reached. The equilibrium is defined as follows:

(i) The producers and the middleman maximize their profits and the consumer maximizes her utility subject to the constraints they face (mentioned in their individual dynamic optimization problems in Section 2).

(ii) The quantity supplied by the producers equals the quantity consumed by the consumer (and hence the inventory does not change when the market is in equilibrium). The conditions for the existence of the equilibrium (Routh–Hurwitz stability criterion, which provides a necessary and sufficient condition for the stability of a linear dynamical system) have been mentioned in Section 3.

As the set up is for a perfectly competitive market, therefore, the middleman who sells the goods to the consumer at the market price is a price taker when the market is in equilibrium. When the market is out of equilibrium, the middleman can change the price along the dynamic adjustment path until the new equilibrium arrives, where again the middleman becomes a price taker. The government announces and imposes a specific import tariff at the same time (the expectations of the agents will be taken into account in a future research project when the dates of announcement and implementation of the import tariff are different). When a specific tariff is imposed, the market does not suddenly jump to the post-tariff market equilibrium, rather the price adjusts over time to bring the new equilibrium. This adjustment process involves endogenous decision making (in their own interest) by all the agents in the market, i.e. consumer, producers and the middleman **as follows: Suppose there are two producers in a market who produce a perishable good and sell it to a middleman who further sells it to a consumer living in a community. The consumer and the middleman buy a quantity exactly equal to the quantity the producers produce in each time period, and the market stays in equilibrium. If the government announces and imposes a tariff on one of the producers, which decreases**

the supply of this product, the demand of the consumer will remain partially unmet by the end of the time period in which the tariff was imposed. Assuming that the producers and the middleman can change the production and the price respectively, immediately, had the middleman known the exact pattern of new supply and demand (after she would change the price), she would immediately pick the price to maximize her profits and clear the market without wasting her profits through unmet demand. However, she lacks this information, so the middleman increases the price based on her best guess about the new supply at the new price (based on the quantity of the reduced production), driving the market close to the new equilibrium. At the higher price, the producers produce a higher quantity than before. If in the following time period, their production sold to the middleman fully meets the demand of the consumer, the middleman will know that the new equilibrium has arrived, however, if there is still some unmet demand, the middleman will increase the price further (and the producers, the production accordingly) to bring the market closer to the new equilibrium. The market will eventually settle at the new equilibrium after some efficiency loss. The efficiency loss by the imposition of the tariff is the output that could not be produced in each time period during the adjustment process. A new equilibrium with a deadweight loss due to tariff is finally arrived at. The total efficiency loss as a result of tariff is the loss during the adjustment process plus the loss in the final equilibrium.

For the mathematical treatment, the objective of each of the four market agents is maximized through the first order conditions of their objective functions and to capture the collective result of their individual actions, the equations representing their individual actions are solved simultaneously. For simplification, we assume that after the imposition of the import tariff, the new equilibrium is not too far from the initial equilibrium. This assumption makes the linearization of supply and demand curves quite reasonable. Please look at figure 1 (the time axis is not shown). Linearization seems to be a good approximation when we move from point a to b, whereas it is not a good approximation when we move from point a to c. For modeling the movement of the market from point a to c, we need to model a non-linear dynamical system (which is not covered under the scope of this paper).

2.1 Middleman

The middleman purchases goods from the producers and sells those to the consumer for profit. As happens in the real world, the middleman does not buy and sell exactly the same quantity at all points in time, thus he holds an inventory of the goods purchased to be sold subsequently. Inventory is an intermediary stage between supply and demand which reflects the quantum of difference between supply and demand of the goods in the market. If the inventory remains the same, it implies that demand and supply rates are the same. An increase or decrease in inventory implies a change in supply, demand or both at different rates.

Please look at figure 2 to understand the link between inventory, supply, demand and prices. When the supply curve shifts to the right (while demand remains the same), the inventory in the market increases at the initial price, and the new equilibrium brings the price down. Similarly, when the demand curve shifts to the right (while supply remains constant), the inventory depletes from the market at the previous price and the new equilibrium brings the price up. This shows that there is an inverse relationship between an inventory change and a price change (all else the same). If both the supply and demand curves shift by the same magnitude such that the inventory does not change, then price will also remain the same. Inventory unifies the supply and demand shocks in the sense that they are both affecting the same factor, i.e. inventory and are basically the faces of the same coin. Therefore each kind of shock is in fact just an inventory shock. From the above mentioned discussion, we have seen that there is an inverse relationship between an inventory change and a price change. Now let's discuss the mechanism which brings about such a change. Consider a market of homogeneous goods where the middlemen, such as whole salers, retailers, etc. hold inventories, incur some cost for holding those, and sell products to the consumers to make profits. The cost is a positive function of the size of an inventory, i.e. a larger inventory costs more to hold as compared to a smaller inventory. In the absence of an exogenous shock, if the supply and demand rates are equal then the system is in equilibrium and the price does not vary with time.

Suppose that a technological advancement decreases the marginal cost of production and increases the supply rate, whereas the demand rate remains the same. As the demand and supply rates are no longer equal, therefore the difference will appear somewhere in the economy in the form of piled up inventories. As the production flows from the producers to the consumer through the middleman, therefore it is reasonable to assume that the middleman will be holding the net difference (Explanation: The piled up inventories can also be in the form of producers' inventories of finished goods, which does not change the key point that a difference of supply and demand rates directly affect the inventories). The economy will not be able to sustain this situation indefinitely, and the middlemen will have to think of some means of getting rid of piled up inventories. The only resort they have is to decrease the price which brings the demand up along the demand curve. In a perfectly competitive market, the price will eventually come down to equalize the new marginal cost, however the adjustment path depends on how the middlemen react to the change in their inventories. Notice that the marginal cost of production has decreased but the marginal cost of holding an extra unit of inventory for the middleman has increased. This is an intuitive explanation which is theoretically consistent with the demand, supply, utility and profit maximization by a consumer and a producer respectively. In the real world, we see examples of this behavior of middlemen, e.g. as consumers, we enjoy the *end of year sales*, offers such as *buy one get one free*, *gift offers* if you buy above a certain quantity threshold, etc. For a mathematical treatment, we need to consider the profit maximization problem of the *middleman* as follows:

2.1.1 Static Problem

Let's first consider the static problem (Explanation: The static problem means that the middleman's objective is myopic rather than doing dynamic optimization) of the middleman as follows:

$$\Pi = pq(p) - \varsigma(m(p,e)), \tag{1}$$

where
Π = profit,
p = market price,
$q(p)$ = quantity sold at price p,
m = inventory (total number of goods held by the *middleman*),
e = other factors which influence inventory other than the market price including the middleman's purchase price from the producer,
$\varsigma(m(p,e))$ = cost as a function of inventory (increasing in inventory).
The first order condition (with respect to price) is as follows:

$$pq'(p) + q(p) - \varsigma'(m(p,e))m_1'(p,e) = 0, \tag{2}$$

The middleman has an incentive to change the price only during the adjustment process and will incur losses by deviating from the price (equal to the marginal cost) when the market is in equilibrium. During the adjustment process, the demand does not equal the supply and the market drifts toward the new equilibrium (however, the price cannot move automatically and it is reasonable to assume that some economic agent moves the price in his own benefit), therefore a price change by the middleman in the direction of bringing the new equilibrium is not against the market forces, so he does not lose business by changing price on the adjustment path unlike when the market is in equilibrium and where the middleman faces an infinitely elastic demand as follows:

$$\begin{aligned} pq'(p) + q(p) &= \varsigma'(m(p,e))m_1'(p,e), \\ p\left[1 + \frac{1}{\textit{demand elasticity}}\right] &= \varsigma'(m(p,e))\frac{m_1'(p,e)}{q'(p)}. \end{aligned}$$

The right hand side of the above expression is the marginal cost which equals the price when the middleman faces an infinitely elastic demand. Suppose that as a result of a supply shock, the marginal cost of production decreases, and the supply curve shifts downwards (if the marginal cost of production decreases either for domestic producer, foreign producer or both, the total domestic supply curve will shift downwards in all the three cases however by different magnitudes). Now the competitive market is out of equilibrium as the demand does not equal the supply at the previous equilibrium price. The price must eventually decrease to bring the new equilibrium, however, the price will not jump to equalize the demand and supply, and rather the middleman will continue charging a price higher than the new marginal cost until the market forces make him realize that the supply has increased and he needs to lower the price to satisfy the profit maximizing condition. The similar is the case of a reverse supply shock, where the price must eventually increase to bring the new equilibrium. In this case, the middleman will continue charging a price lower than the marginal cost until the market forces make him increase the price, in which case it is the consumer who is the short term beneficiary. Again, the consumer will be paying a price less than the marginal cost only during the adjustment process and only until the middleman increases the price. The equilibrium price is equal to the marginal cost of production plus the marginal cost of storage (i.e. the total marginal cost) in the absence of any kind of a tax, tariff, etc., so neither does the middleman earn any economic rent, nor does the consumer benefit by paying a price less than the marginal cost when the competitive market is in equilibrium.

For the mathematical treatment, suppose that as a result of a supply shock (while demand remains the same) such as a technological advancement which reduces the marginal cost of production and increases the supply by the producers (either by the domestic producer, foreign producer or both), if the middleman wants to hold an extra unit of inventory, his marginal cost of holding an extra unit i.e. $\varsigma'(m(p,e))\frac{m_1'(p,e)}{q'(p)}$ is higher at the previous price, because the term $\varsigma'(m(p,e))$ is higher at the previous price. This might be on account of higher storage charges because of increased demand of warehouses, godowns, etc. after increased supply in the market. The second term, i.e. $\frac{m_1'(p,e)}{q'(p)}$ is a function of price, and is the same as before as the price has not changed yet (we are assuming that the middleman's purchase price is the same as before as the producer is a price taker during the adjustment process as well and always charges a fixed fraction of the market price to the middleman). A discrete analog of this scenario is that the middleman maximizes profits in each time period without considering the future time periods, and in each time period he takes the purchase price from the producer as given and only chooses the sale price. This implies that on the previous price, now the middleman faces

$$\frac{\partial \Pi}{\partial p} = pq'(p) + q(p) - \varsigma'(m(p,e))m_1'(p,e) < 0, \tag{3}$$

which means that the middleman must decrease the price to hold an extra unit of inventory to satisfy the profit maximizing condition after the supply shock. Please notice that in this static scenario, the short term gains accrued from the decreased marginal cost of production will be reaped by the producer, as his marginal cost has decreased but he charges the same price to the middleman until the middleman changes the price. If we plot together various profit maximizing combinations of inventories and the respective prices chosen by a middleman, we will get a downward sloping *inventory curve* with the price on the y-axis and the inventory on the x-axis. This is analogous to the concept of *supply* and *demand curves* for the profit maximizing producers and the utility maximizing consumers respectively.

2.1.2 Dynamic Problem

Now let's consider the dynamic problem of the middleman. In a dynamic setting, the middleman maximizes the present discounted value of the future stream of profits, and his present value at time zero is as follows:

$$V(0) = \int_0^\infty [pq(p) - \varsigma(m(p,e))]\, e^{-rt} dt, \tag{4}$$

r denotes the discount rate. $p(t)$ is the *control variable* and $m(t)$ the *state variable*. The maximization problem can be written as

$$\underset{\{p(t)\}}{Max} V(0) = \int_0^\infty [pq(p) - \varsigma(m(p,e))]\, e^{-rt} dt,$$

subject to the constraints that
$\dot{m}(t) = m_1'(p(t), e(p(t), z))\dot{p}(t) + m_2'(p(t), e(p(t), z))e_1'(p(t), z)\dot{p}(t)$ (state equation, describing how the state variable changes with time; z are exogenous factors),
$m(0) = m_s$ (initial condition),
$m(t) \geq 0$ (non-negativity constraint on state variable),
$m(\infty)$ free (terminal condition).
The current-value Hamiltonian for this case is

$$\widetilde{H} = p(t)q(p(t)) - \varsigma(m(p(t), e(p(t), z))) + \mu(t)\dot{p}(t) \left[\begin{array}{c} m_1'(p(t), e(p(t), z)) + m_2'(p(t), e(p(t), z)) * \\ e_1'(p(t), z) \end{array} \right]. \tag{5}$$

Now the maximizing conditions are as follows:
(i) $p^*(t)$ maximizes $\widetilde{H}$ for all t: $\frac{\partial \widetilde{H}}{\partial p} = 0$,
(ii) $\dot{\mu} - r\mu = -\frac{\partial \widetilde{H}}{\partial m}$,
(iii) $\dot{m}^* = \frac{\partial \widetilde{H}}{\partial \mu}$ (this just gives back the state equation),
(iv) $\lim_{t \to \infty} \mu(t)m(t)e^{-rt} = 0$ (the transversality condition).
The first two conditions are as follows:

$$\frac{\partial \widetilde{H}}{\partial p} = 0, \tag{6}$$

and

$$\dot{\mu} - r\mu = -\frac{\partial \widetilde{H}}{\partial m} = \varsigma'(m(p(t), e(p(t), z))). \tag{7}$$

When the market is in equilibrium, $\dot{p}(t) = 0$, and the expression $\frac{\partial \widetilde{H}}{\partial p}$ boils down to the following (see appendix):

$$p(t)\left[1+\frac{1}{demand\ elasticity}\right]=\varsigma'(m(p(t),e(p(t),z)))\left\{\frac{m_1'(p(t),e(p(t),z))}{q'(p(t))}+\frac{m_2'(p(t),e(p(t),z))e_1'(p(t),z)}{q'(p(t))}\right\},$$

suggesting that the price equals the marginal cost (the right hand side of the above expression is the marginal cost in a dynamic setting, which is different from that in a static problem on account of the fact that in a dynamic setting the middleman also takes into account the impact of price chosen on his purchase price from the producer) when the demand is infinitely elastic. Now suppose that as a result of a supply shock, if the middleman wants to hold an extra unit of inventory, then the marginal cost of holding an extra unit is higher because the term $\varsigma'(m(p(t),e(p(t),z)))$ is higher at the previous price at that point in time. The term in parentheses in the expression of the marginal cost, i.e. $\frac{m_1'(p(t),e(p(t),z))}{q'(p(t))}+\frac{m_2'(p(t),e(p(t),z))e_1'(p(t),z)}{q'(p(t))}$ is a function of price and is the same at the previous price. This implies that on the previous price, now the middleman faces

$$\frac{\partial\widetilde{H}}{\partial p}<0.$$

Therefore in order to satisfy the condition of dynamic optimization, the middleman must decrease the price for an increase in inventory. This implies a negative relationship between price and inventory. The concept of inventory unifies the market supply and demand. If the supply and demand rates are equal, the market is in a steady state equilibrium. If a difference of finite magnitude is created between the supply and demand rates and the consumer and the producer do not react to a price change induced by a difference in the supply and demand rates, the price will continue changing until the system saturates. This behavior can be depicted by the following formulation:

$$\textit{Price change} \propto \textit{change in market inventory.}$$

$$P = \textit{price change.}$$

$$M = m - m_s = \textit{change in inventory in the market,}$$

$$m = \textit{inventory at time } t,$$

$$m_s = \textit{inventory in steady state equilibrium.}$$

$$\textit{Input} - \textit{output} = \frac{dm}{dt} = \frac{d(m - m_s)}{dt} = \frac{dM}{dt},$$

$$\text{or } M = \int (\textit{input} - \textit{output})\, dt.$$

$$\textit{Price change} \propto \int (\textit{supply rate} - \textit{demand rate})\, dt, \text{ or}$$

$$P = -K_m \int (\textit{supply rate} - \textit{demand rate})\, dt,$$

where K_m is the proportionality constant. A negative sign indicates that when (*supply rate* $-$ *demand rate*) is positive, then P is negative (i.e. price decreases). The above equation can be re-arranged as follows:

$$\int (\textit{supply rate} - \textit{demand rate})\, dt = -\frac{P}{K_m}, \text{ or}$$

$$\int (w_i - w_0)\, dt = -\frac{P}{K_m}, \tag{8}$$

$$w_i = \textit{supply rate},$$

$$w_0 = \textit{demand rate},$$

$$K_m = \textit{dimensional constant.}$$

Let at time $t = 0$, *supply rate* $=$ *demand rate* (market is in a steady state equilibrium), then eq. (8) can be written as

$$\int (w_{is} - w_{0s})\, dt = 0. \tag{9}$$

The subscript s indicates the steady state equilibrium and $P = 0$ in steady state. Subtracting eq. (9) from eq. (8), we get:

$$\int (w_i - w_{is})\, dt - \int (w_0 - w_{0s})\, dt = -\frac{P}{K_m}, \text{ or}$$

$$\int (W_i - W_0)\, dt = -\frac{P}{K_m}, \tag{10}$$

where $w_i - w_{is} = W_i = \textit{change in supply rate},$

$w_0 - w_{0s} = W_0 = \textit{change in demand rate.}$

P, W_i and W_0 are deviation variables, which indicate deviation from the steady state equilibrium. The initial values of the deviation variables are zero. Eq. (10) may also be written as follows:

$$P = -K_m \int W dt = -K_m M, \tag{11}$$

where $W = W_i - W_0$. If P gets a jump as a result of some factor other than an inventory change, that is considered as a separate input and can be added to eq. (11) as follows:

$$P = -K_m \int W dt + J = -K_m M + J. \tag{11a}$$

Similarly, there can be an exogenous shock in inventory other than the price feedback.

2.2 Producers

The foreign and the domestic producers' objective is identical, therefore the problem of only one of the producers has been considered and the results extended to both of them. The producer maximizes the present discounted value of the future stream of profits, and his present value at time zero is as follows:

$$V(0) = \int_0^\infty \left[\alpha p(t) F\left(K\left(t\right), L\left(t\right)\right) - w(t) L\left(t\right) - \Re(t) I(t)\right] e^{-rt} dt, \tag{12}$$

α is the fraction of the market price the producer charges to the middleman. r denotes the discount rate. $L(t)$ (labor) and $I(t)$ (level of investment) are the *control variables* and $K(t)$ the *state variable.* The maximization problem can be written as

$$\underset{\{L(t),I(t)\}}{Max} V(0) = \int_0^\infty \left[\alpha p(t) F\left(K\left(t\right), L\left(t\right)\right) - w(t) L\left(t\right) - \Re(t) I(t)\right] e^{-rt} dt,$$

subject to the constraints that
$\dot{K}(t) = I(t) - \delta K(t)$ (state equation, describing how the state variable changes with time),

$K(0) = K_0$ (initial condition),
$K(t) \geq 0$ (non-negativity constraint on state variable),
$K(\infty)$ free (terminal condition).
The current-value Hamiltonian for this case is

$$\widetilde{H} = \alpha p(t)F\left(K\left(t\right), L\left(t\right)\right) - w(t)L\left(t\right) - \Re(t)I(t) + \mu(t)\left[I(t) - \delta K(t)\right]. \tag{13}$$

Now the maximizing conditions are as follows:
(i) $L^*(t)$ and $I^*(t)$ maximize $\widetilde{H}$ for all t: $\frac{\partial \widetilde{H}}{\partial L} = 0$ and $\frac{\partial \widetilde{H}}{\partial I} = 0$,
(ii) $\dot{\mu} - r\mu = -\frac{\partial \widetilde{H}}{\partial K}$,
(iii) $\dot{K}^* = \frac{\partial \widetilde{H}}{\partial \mu}$ (this just gives back the state equation),
(iv) $\lim_{t\to\infty} \mu(t)K(t)e^{-rt} = 0$ (the transversality condition).
The first two conditions are as follows:

$$\frac{\partial \widetilde{H}}{\partial L} = \alpha p(t)F_2'\left(K\left(t\right), L\left(t\right)\right) - w(t) = 0, \tag{14}$$

$$\frac{\partial \widetilde{H}}{\partial I} = -\Re(t) + \mu(t) = 0, \tag{15}$$

and

$$\dot{\mu} - r\mu = -\frac{\partial \widetilde{H}}{\partial K} = -\left[\alpha p(t)F_1'\left(K\left(t\right), L\left(t\right)\right) - \delta\mu(t)\right]. \tag{16}$$

Substituting the value of $\dot{\mu}$ and μ from eq. (15) into eq. (16) yields

$$\alpha p(t)F_1'\left(K\left(t\right), L\left(t\right)\right) - (r + \delta)\Re(t) + \dot{\Re}(t) = 0.$$

If the price, i.e. $p(t)$ goes up, (at the previous level of investment and labor) the producer faces

$$\alpha p(t)F_2'\left(K\left(t\right), L\left(t\right)\right) - w(t) > 0,$$

$$\alpha p(t)F_1'\left(K\left(t\right), L\left(t\right)\right) - (r + \delta)\Re(t) + \dot{\Re}(t) > 0.$$

Therefore in order to satisfy the condition of dynamic optimization after the price increase, the producer must increase the production level. Let p = market price, c = a reference price (such as the retail price which includes the production cost, profit of producer and profit of the middleman). c is a parameter which may vary with time or be kept fixed for a limited time period, e.g. the cost of a product may vary over time or can also remain constant for a while. It is the reference point with respect to which the variation in p is considered by the producer for decision making.

$$W_m = \textit{Change in production due to change in price},$$

$(p - c)$ acts as an incentive for the producer to produce more. We can write:

$$W_m \propto \alpha(p - c), \text{ or}$$

$$W_m = K_s(p - c). \tag{17}$$

When the market is in equilibrium, then $W_m = 0$, or

$$0 = K_s(p_s - c_s). \tag{18}$$

K_s is the proportionality constant. p_s and c_s are the steady state equilibrium values. Subtracting eq. (18) from eq. (17), we get:

$$W_m = K_s\left[(p - p_s) - (c - c_s)\right] = -K_s\left(C - P\right) = -K_s\varepsilon, \tag{19}$$

where W_m, C and P are deviation variables.

2.3 Consumer

The consumer maximizes the present discounted value of the future stream of utilities, and his present value at time zero is as follows:

$$V(0) = \int_0^\infty U(x(t))e^{-\rho t}dt, \tag{20}$$

ρ denotes the discount rate and $x(t)$ is the *control variable*. The maximization problem can be written as

$$\underset{\{x(t)\}}{Max} V(0) = \int_0^\infty U(x(t))e^{-\rho t}dt,$$

subject to the constraints that
$\dot{a}(t) = R(t)a(t) + w(t) - p(t)x(t)$ (state equation, describing how the state variable changes with time). $a(t)$ is asset holdings (a *state variable*) and $w(t)$ and $R(t)$ are exogenous time path of wages and return on assets.
$a(0) = a_s$ (initial condition),
$a(t) \geq 0$ (non-negativity constraint on state variable),
$a(\infty)$ free (terminal condition).
The current-value Hamiltonian for this case is

$$\widetilde{H} = U(x(t)) + \mu(t)\left[R\left(t\right)a\left(t\right) + w\left(t\right) - p\left(t\right)x\left(t\right)\right]. \tag{21}$$

Now the maximizing conditions are as follows:
(i) $x^*(t)$ maximizes $\widetilde{H}$ for all t: $\frac{\partial \widetilde{H}}{\partial x} = 0$,
(ii) $\dot{\mu} - \rho\mu = -\frac{\partial \widetilde{H}}{\partial a}$,
(iii) $\dot{a}^* = \frac{\partial \widetilde{H}}{\partial \mu}$ (this just gives back the state equation),
(iv) $\lim\limits_{t\to\infty} \mu(t)a(t)e^{-\rho t} = 0$ (the transversality condition).
The first two conditions are as follows:

$$\frac{\partial \widetilde{H}}{\partial x} = U'\left(x\left(t\right)\right) - \mu(t)p(t) = 0, \tag{22}$$

and

$$\dot{\mu} - \rho\mu = -\frac{\partial \widetilde{H}}{\partial a} = -\mu(t)R(t). \tag{23}$$

If the price of good x goes up, the consumer faces (at the previous level of consumption)

$$\frac{\partial \widetilde{H}}{\partial x} = U'\left(x\left(t\right)\right) - \mu(t)p(t) < 0.$$

Therefore in order to satisfy the condition of dynamic optimization after the price increase, the consumer must decrease the consumption of good x. Let the change in demand be proportional to the change in price, i.e. P. Then we can write:

$$\textit{Change in demand} \propto P, \text{ or}$$

$$W_d = -K_d P. \tag{24}$$

W_d is the change in demand due to P; when P is positive W_d is negative.

3 Solution of the Model with a Specific Import Tariff

As $W_m(t)$ is the total supply in the domestic market, it includes both the domestic supply as well the imports, and thus can be bifurcated as follows:

$$W_m(t) = -K_{sd}\left[C_d(t) - P(t)\right] - K_{se}\left[C_e(t) - P(t)\right], \tag{25}$$

where the subscripts d and e denote the domestic producer and the exporter (foreign producer) in the foreign country respectively. The solution of the model can be written as follows:

$$\frac{dP(t)}{dt} + K_m(K_{sd} + K_{se} + K_d)P(t) = K_m\left[K_{sd}C_d(t) + K_{se}C_e(t)\right]. \quad (26)$$

If $C_e(t) = T$, and $C_d(t) = 0$, i.e. the government imposes a per unit tariff on the imports at $t = 0$, then the above differential equation becomes as follows:

$$\frac{dP(t)}{dt} + K_m(K_{sd} + K_{se} + K_d)P(t) = K_m K_{se} T. \quad (27)$$

The Routh–Hurwitz stability criterion (which provides a necessary and sufficient condition for stability of a linear dynamical system) for the stability of the above differential equation is $K_m(K_{sd} + K_{se} + K_d) > 0$, which holds as K_m, K_{sd}, K_{se} and K_d are all defined to be positive. This ensures that, away from a given initial equilibrium, every adjustment mechanism will lead to another equilibrium. The solution has the form

$$P(t) = C_1 + C_2 e^{-[K_m(K_{sd}+K_{se}+K_d)]t}. \quad (28)$$

Substituting the values of C_1 and C_2 in eq. (28), we get:

$$P(t) = \frac{K_{se}T}{(K_{sd} + K_{se} + K_d)} - \frac{K_{se}T}{(K_{sd} + K_{se} + K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}. \quad (29)$$

When $t = 0$, $P(0) = 0$ (the initial condition), and when $t = \infty$, $P(\infty) = \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}$ (the final steady state equilibrium value). The price dynamics as a result of a tariff for a small or a large country depends on the parameters K_{sd}, K_{se}, K_d, K_m and T. **In the final equilibrium, the quantity demanded must equal the quantity supplied, which holds.**

4 An Optimal Import Tariff Path

The efficiency loss as a result of an import tariff, generally mentioned in the economics literature is the dead weight loss as a result of comparisons of the pre and post tariff market equilibriums. However, the dynamic picture shows that there is some efficiency loss on the dynamic adjustment path to the new equilibrium as well after the tariff. After the imposition of the tariff, the import cost jumps to the previous cost plus the import tariff affecting the import quantity. The demand does not equal the supply any longer and the price adjusts over time to bring the new equilibrium price which is higher than the previous equilibrium price by a magnitude depending on the elasticity of demand and supply schedules. A pile up of inventory indicates a higher supply than demand, and a depletion of inventory occurs when demand is higher than the supply in a given time period. When the demand and supply again become equal, the new equilibrium has arrived. If the demand and supply are different, the output and/ or consumption is being lost at that point in time. Furthermore, some production is being wasted in comparison with that in the previous equilibrium. Therefore if we sum up either the consumption lost or the production lost (which are both equivalent), we get the total efficiency loss in terms of quantity, which is as follows:

$$EL = \int_0^\infty W_m(t)dt = M(t) + \int_0^\infty W_d(t)dt. \tag{30}$$

By eq. (25), the change in total domestic supply as a result of a tariff is as follows:

$$W_m(0) = -K_{sd}\left[C_d(0) - P(0)\right] - K_{se}\left[C_e(0) - P(0)\right] = -K_{se}T, \tag{31}$$

as $P(0) = 0$.

Therefore, at time zero, the inventory goes down by $K_{se}T$ (as the demand remains the same). Please look at figure 3, where the supply difference jumps to $K_{se}T$, i.e. the decrease in supply because of tariff at $t = 0$. The demand does not equal the supply any longer, and the market forces come into play. The supply along with the price adjusts over time and the new equilibrium arrives, i.e. $W_m(\infty)$. The shaded area is the efficiency loss (the amount of supply lost) during the adjustment process. The area between the lines $W_m(t) = 0$, and $W_m(t) = W_m(\infty)$ is the efficiency loss resulting from a difference in pre and post tariff market equilibriums.
Eq. $(11a)$ can also be written as (see appendix)

$$P(t) = -K_m M(t) - K_m K_{se}T, \text{ or}$$
$$M(t) = -\frac{1}{K_m}\left[P(t) + K_m K_{se}T\right]. \tag{32}$$

With Tariff Revenue Constraint:
The expression for the tariff revenue is as follows:

$$TR = T\left[w_{ime}(0) - K_{se}\left\{T - P(t)\right\}\right]. \tag{33}$$

If we want to minimize the efficiency loss subject to the constraint that tariff revenue generated is greater than or equal to G in a given time period, our problem is as follows:

$$\min_T EL \quad \text{s.t.} \quad TR \geq G.$$

The choice variable is the tariff rate, and the constraint is binding. The Lagrangian for the above problem is as follows:

$$\mathcal{L} = -\int_0^\infty \left[\mathbf{K}_{sd}\left[C_d(t) - P(t)\right] + \mathbf{K}_{se}\left[C_e(t) - P(t)\right]\right] dt + \lambda\left[G - T\left[w_{ime}(0) - K_{se}\left\{T - P(t)\right\}\right]\right]$$
$$= \int_0^\infty \left[-K_{se}T + \frac{K_{se}(K_{sd}+K_{se})T}{(K_{sd}+K_{se}+K_d)} - \frac{K_{se}(K_{sd}+K_{se})T}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right] dt$$
$$+ \lambda\left[G - T\left[w_{ime}(0) - K_{se}\left\{T - \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}\right]\right].$$

Taking the first order condition with respect to T, we get:

$$T = \frac{\lambda w_{ime}(0) - \int_0^\infty \left[-K_{se} + \frac{K_{se}(K_{sd}+K_{se})}{(K_{sd}+K_{se}+K_d)} - \frac{K_{se}(K_{sd}+K_{se})}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right] dt}{2\lambda K_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right]}. \tag{34}$$

Taking the first order condition with respect to λ, we get:

$$G - T\left[w_{ime}(0) - K_{se}\left\{T - \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}\right] = 0. \tag{35}$$

Substituting the value of T from eq. (34) into (35), we get:

$$\lambda = \frac{J}{\sqrt{w_{ime}^2(0) - 4QG}}.$$

$$Q = K_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right],$$
$$J = J = \int_0^\infty \left[-K_{se} + \frac{K_{se}(K_{sd}+K_{se})}{(K_{sd}+K_{se}+K_d)} - \frac{K_{se}(K_{sd}+K_{se})}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right] dt.$$

Eq. (34) can also be written as

$$T = \frac{\lambda w_{ime}(0) - J}{2\lambda Q}. \tag{36}$$

Substituting the value of λ into eq. (36), we get:

$$T(t) = \frac{w_{ime}(0) - \sqrt{w_{ime}^2(0) - 4QG}}{2Q}. \tag{37}$$

The second order condition for minimization has been checked (see appendix). Suppose that the government wants to generate a revenue of $1000 by imposing a tariff on a certain good. The initial import quantity of that good is 100, and the value of each one of K_m, K_{sd}, K_{se} and K_d is equal to one. Substituting these values in eq. (37) yields

$$T(0) = \frac{100 - \sqrt{10000 - 4000}}{2} = 11.27,$$

where $Q = 1 - 0.333 + 0.333e^{-3t}$, and at $t = 0$, $Q = 1$. The tariff revenue generated is $TR = T\left[w_{ime}(0) - QT\right] = 1000$. Now when $t = \infty$, $Q = 0.666$. This implies that

$$T(\infty) = \frac{100 - \sqrt{10000 - 2664}}{1.332} = 10.77.$$

The tariff revenue is again 1000 as desired. Therefore the optimal tariff is that the government should impose a tariff rate of $11.27 per unit quanititity initially and then gradually decrease the tariff over time up to a final rate of $10.77 per unit quantity of the same good.

5 Static Versus Dynamic Efficiency Loss

From eq. (24), the change in the demand as a result of a price change is as follows:

$$W_d(t) = -K_d P(t).$$

In the final equilibrium, we have

$$\begin{aligned} W_d(\infty) &= -K_d P(\infty), \text{ or} \\ W_d(\infty) &= \frac{-K_d K_{se} T}{(K_{sd} + K_{se} + K_d)}. \end{aligned} \tag{38}$$

The above expression gives the quantity of total consumption lost as a result of the imposition of an import tariff T in the final equilibrium as compared to that in the initial equilibrium or the static efficiency loss. The dynamic efficiency loss is given by eq. (30) as follows:

$$\begin{aligned} EL &= \int_0^{t_e} \left[-K_{se}T + \frac{K_{se}(K_{sd} + K_{se})T}{(K_{sd} + K_{se} + K_d)} - \frac{K_{se}(K_{sd} + K_{se})T}{(K_{sd} + K_{se} + K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t} \right] dt, \\ &= -K_{se}Tt_e + \frac{K_{se}(K_{sd} + K_{se})Tt_e}{(K_{sd} + K_{se} + K_d)} + \frac{K_{se}(K_{sd} + K_{se})Te^{-[K_m(K_{sd}+K_{se}+K_d)]t_e}}{K_m(K_{sd} + K_{se} + K_d)^2} - \frac{K_{se}(K_{sd} + K_{se})T}{K_m(K_{sd} + K_{se} + K_d)^2}, \end{aligned} \tag{39}$$

where the new equilibrium arrives in t_e time. For a specific import tariff of \$10 and the value of each one of K_m, K_{sd}, K_{se} and K_d equal to one, the static efficiency loss is 13.32, whereas the dynamic efficiency loss is 15.62 units of consumption (with a value of t_e equal to 4).

6 Conclusion

When a government imposes an import tariff, the foreign producer's cost (to the extent of his/her exports) jumps to the pre-tariff cost plus the amount of the specific tariff affecting the import quantity and hence pushing the domestic market out of equilibrium. The supply and the demand of the commodity on which a tariff is imposed then adjust over time to bring the new post-tariff market equilibrium. The existing literature does not take into account the efficiency losses during the adjustment process while computing the optimal specific import tariffs. It is important to take into consideration the efficiency losses during the adjustment process while deriving an optimal tariff schedule. Eq. (37) gives an optimal specific import tariff path over time which generates the same desired revenue at any given point in time considering the adjustment of demand and supply over time. The expression is a function of the slopes of the demand, supply (domestic as well as foreign) and the inventory curves, and the initial pre-tariff equilibrium quantity. The expression is much more complex as compared to the optimal tariff expression in the existing literature, which only compares the pre and the post tariff equilibriums.

7 Future Research Prospects

Potential future research areas are as follows:

Dynamic welfare analysis: A complete dynamic welfare analysis against various governmental policies, such as ad valorem tariff, compound tariff, export tariff, non-tariff barriers, such as import licenses, export licenses, quotas, subsidies, voluntary export restraints, local content requirements, embargo, currency devaluation, trade restriction, etc. can be carried out and the optimal governmental policy instruments can be derived following the methodology developed in this paper.

8 Appendix:

8.1 Dynamic Problem of the Middleman

In a dynamic setting, the middleman maximizes the present discounted value of the future stream of profits, and his present value at time zero is as follows:

$$\mathbf{V(0)} = \int_0^{\infty} \left[pq(p) - \varsigma(m(p,e))\right] \mathbf{e}^{-rt}\mathbf{dt}, \tag{40}$$

r **denotes the discount rate.** $p(t)$ **is the control variable and** $m(t)$ **the state variable. The maximization problem can be written as**

$$\underset{\{p(t)\}}{Max}\mathbf{V(0)} = \int_0^\infty [pq(p) - \varsigma(m(p,e))]\,\mathbf{e}^{-rt}\mathbf{dt},$$

subject to the constraints that
$\dot{m}(t) = m_1'(p(t), e(p(t), z))\dot{p}(t) + m_2'(p(t), e(p(t), z))e_1'(p(t), z)\dot{p}(t)$ **(state equation, describing how the state variable changes with time;** z **are exogenous factors),**
$m(0) = m_s$ **(initial condition),**
$m(t) \geq 0$ **(non-negativity constraint on state variable),**
$m(\infty)$ **free (terminal condition).**
The current-value Hamiltonian for this case is

$$\widetilde{H} = \mathbf{p(t)q(p(t))} - \varsigma(\mathbf{m(p(t), e(p(t), z))}) + \mu(\mathbf{t})\dot{p}(\mathbf{t}) \begin{bmatrix} m_1'(p(t), e(p(t), z)) + m_2'(p(t), e(p(t), z)) * \\ e_1'(p(t), z) \end{bmatrix}. \tag{41}$$

Now the maximizing conditions are as follows:
(i) $p^*(t)$ **maximizes** $\widetilde{H}$ **for all** t**:** $\frac{\partial \widetilde{H}}{\partial p} = 0$,
(ii) $\dot{\mu} - r\mu = -\frac{\partial \widetilde{H}}{\partial m}$,
(iii) $\dot{m}^* = \frac{\partial \widetilde{H}}{\partial \mu}$ **(this just gives back the state equation),**
(iv) $\lim\limits_{t \to \infty} \mu(t)m(t)e^{-rt} = 0$ **(the transversality condition).**
The first two conditions are as follows:

$$\begin{aligned} \frac{\partial \widetilde{H}}{\partial p} &= q(p(t)) + p(t)q'(p(t)) - \varsigma'(m(p(t), e(p(t), z))) \begin{Bmatrix} m_1'(p(t), e(p(t), z)) + m_2'(p(t), e(p(t), z)) * \\ e_1'(p(t), z) \end{Bmatrix} \\ &+ \mu(t)\dot{p}(t) * \begin{bmatrix} m_{11}''(p(t), e(p(t), z)) + m_{12}''(p(t), e(p(t), z))e_1'(p(t), z) + \\ m_{21}''(p(t), e(p(t), z))e_1'(p(t), z) + m_{22}''(p(t), e(p(t), z))e_1'^2(p(t), z) + \\ m_2'(p(t), e(p(t), z))e_{11}''(p(t), z) \end{bmatrix} \\ &= 0, \end{aligned} \tag{42}$$

and

$$\dot{\mu} - \mathbf{r}\mu = -\frac{\partial \widetilde{H}}{\partial m} = \varsigma'(\mathbf{m(p(t), e(p(t), z))}). \tag{43}$$

When the market is in equilibrium, $\dot{p}(t) = 0$**, and the expression** $\frac{\partial \widetilde{H}}{\partial p}$ **boils down to the following:**

$$q(p(t)) + p(t)q'(p(t)) - \varsigma'(m(p(t), e(p(t), z))) \left\{ \begin{array}{c} m_1'(p(t), e(p(t), z)) + m_2'(p(t), e(p(t), z))* \\ e_1'(p(t), z) \end{array} \right\}$$
$$= 0,$$

$$p(t)q'(p(t)) + q(p(t)) = \varsigma'(m(p(t), e(p(t), z))) \left\{ \begin{array}{c} m_1'(p(t), e(p(t), z)) + m_2'(p(t), e(p(t), z))* \\ e_1'(p(t), z) \end{array} \right\},$$
$$p(t)\left[1 + \frac{1}{demand\ elasticity}\right] = \varsigma'(m(p(t), e(p(t), z))) \left\{ \frac{m_1'(p(t), e(p(t), z))}{q'(p(t))} + \frac{m_2'(p(t), e(p(t), z))e_1'(p(t), z)}{q'(p(t))} \right\},$$

suggesting that the price equals the marginal cost (the right hand side of the above expression is the marginal cost in a dynamic setting, which is different from that in a static problem on account of the fact that in a dynamic setting the middleman also takes into account the impact of price chosen on his purchase price from the producer) when the demand is infinitely elastic. Now suppose that as a result of a supply shock, if the middleman wants to hold an extra unit of inventory, then the marginal cost of holding an extra unit is higher because the term $\varsigma'(m(p(t), e(p(t), z)))$ is higher at the previous price at that point in time. The term in parentheses in the expression of the marginal cost, i.e. $\frac{m_1'(p(t), e(p(t), z))}{q'(p(t))} + \frac{m_2'(p(t), e(p(t), z))e_1'(p(t), z)}{q'(p(t))}$ is a function of price and is the same at the previous price. This implies that on the previous price, now the middleman faces

$$\frac{\partial \widetilde{H}}{\partial p} = q(p(t)) + p(t)q'(p(t)) - \varsigma'(m(p(t), e(p(t), z))) \left\{ \begin{array}{c} m_1'(p(t), e(p(t), z)) + m_2'(p(t), e(p(t), z))* \\ e_1'(p(t), z) \end{array} \right\}$$
$$+ \mu(t)\dot{p}(t) * \left[\begin{array}{c} m_{11}''(p(t), e(p(t), z)) + m_{12}''(p(t), e(p(t), z))e_1'(p(t), z) + \\ m_{21}''(p(t), e(p(t), z))e_1'(p(t), z) + m_{22}''(p(t), e(p(t), z))e_1'^2(p(t), z) + \\ m_2'(p(t), e(p(t), z))e_{11}''(p(t), z) \end{array} \right]$$
$$< 0.$$

Therefore in order to satisfy the condition of dynamic optimization, the middleman must decrease the price for an increase in inventory. This implies a negative relationship between price and inventory. The concept of inventory unifies the market supply and demand. If the supply and demand rates are equal, the market is in a steady state equilibrium. If a difference of finite magnitude is created between the

supply and demand rates and the consumer and the producer do not react to a price change induced by a difference in the supply and demand rates, the price will continue changing until the system saturates. This behavior can be depicted by the following formulation:

$$\begin{aligned}
\mathit{Price\ change} &\propto \mathit{change\ in\ market\ inventory.}\\
P &= \mathit{price\ change.}\\
M &= m - m_s = \mathit{change\ in\ inventory\ in\ the\ market,}\\
m &= \mathit{inventory\ at\ time\ t,}\\
m_s &= \mathit{inventory\ in\ steady\ state\ equilibrium.}\\
\mathit{Input} - \mathit{output} &= \frac{dm}{dt} = \frac{d(m-m_s)}{dt} = \frac{dM}{dt},\\
\text{or } M &= \textstyle\int (\mathit{input} - \mathit{output})\, dt.\\
\mathit{Price\ change} &\propto \textstyle\int (\mathit{supply\ rate} - \mathit{demand\ rate})\, dt, \text{ or}\\
P &= -K_m \textstyle\int (\mathit{supply\ rate} - \mathit{demand\ rate})\, dt,
\end{aligned}$$

where K_m is the proportionality constant. A negative sign indicates that when $(\mathit{supply\ rate} - \mathit{demand\ ra}$ is positive, then P is negative (i.e. price decreases). The above equation can be re-arranged as follows:

$$\int (\mathit{supply\ rate} - \mathit{demand\ rate})\, \mathbf{dt} = -\frac{P}{K_m}, \text{ or}$$

$$\int (w_i - w_0)\, \mathbf{dt} = -\frac{P}{K_m}, \tag{44}$$

$$\begin{aligned}
w_i &= \mathit{supply\ rate,}\\
w_0 &= \mathit{demand\ rate,}\\
K_m &= \mathit{dimensional\ constant.}
\end{aligned}$$

Let at time $t = 0$, supply rate = demand rate (market is in a steady state equilibrium), then eq. (44) **can be written as**

$$\int (w_{is} - w_{0s})\, \mathbf{dt} = \mathbf{0}. \tag{45}$$

The subscript s indicates the steady state equilibrium and $P = 0$ in steady state. Subtracting eq. (45) from eq. (44), we get:

$$\int (w_i - w_{is})\, \mathbf{dt} - \int (w_0 - w_{0s})\, \mathbf{dt} = -\frac{P}{K_m}, \text{ or}$$

$$\int (W_i - W_0)\, \mathbf{dt} = -\frac{P}{K_m}, \tag{46}$$

$$\text{where } w_i - w_{is} = W_i = \textit{change in supply rate,}$$

$$w_0 - w_{0s} = W_0 = \textit{change in demand rate.}$$

P, W_i **and** W_0 **are deviation variables, which indicate deviation from the steady state equilibrium. The initial values of the deviation variables are zero. Eq.** (46) **may also be written as follows:**

$$\mathbf{P} = -\mathbf{K}_m \int \mathbf{W}\mathbf{dt} = -\mathbf{K}_m \mathbf{M}, \tag{47}$$

where $W = W_i - W_0$. **If** P **gets a jump as a result of some factor other than an inventory change, that is considered as a separate input and can be added to eq.** (47) **as follows:**

$$\mathbf{P} = -\mathbf{K}_m \int \mathbf{W}\mathbf{dt} + \mathbf{J} = -\mathbf{K}_m \mathbf{M} + \mathbf{J}. \tag{47a}$$

Similarly, there can be an exogenous shock in inventory other than the price feedback.

8.2 Solution of the Model with a Specific Import Tariff

From eqs. $(11a)$, (19) **and** (24) **we have the following expressions:**

$$\begin{aligned} \frac{dP(t)}{dt} &= -K_m W(t), \\ W_m(t) &= -K_s \varepsilon(t), \\ \varepsilon(t) &= C(t) - P(t), \\ W_d(t) &= -K_d P(t), \end{aligned}$$

and

$$\mathbf{W(t)} = \mathbf{W}_m(\mathbf{t}) - \mathbf{W}_d(\mathbf{t}),$$

if there is no exogenous change in supply and demand. As $W_m(t)$ **is the total supply in the domestic market, it includes both the domestic supply as well the imports, and**

thus can be bifurcated as follows:

$$\mathbf{W}_m(\mathbf{t}) = -\mathbf{K}_{sd}\left[C_d(t) - P(t)\right] - \mathbf{K}_{se}\left[C_e(t) - P(t)\right], \tag{48}$$

where the subscripts d and e denote the domestic producer and the exporter (foreign producer) in the foreign country respectively. From the above equations, we can write

$$\begin{aligned}\frac{dP(t)}{dt} &= -K_m\left[W_m(t) - W_d(t)\right] \\ &= -K_m\left[-K_{sd}\left\{C_d(t) - P(t)\right\} - K_{se}\left\{C_e(t) - P(t)\right\} + K_d P(t)\right] \\ &= -K_m\left[-K_{sd}C_d(t) - K_{se}C_e(t) + (K_{sd} + K_{se} + K_d)P(t)\right].\end{aligned}$$

The above expression can be rearranged as follows:

$$\frac{dP(t)}{dt} + \mathbf{K}_m(\mathbf{K}_{sd} + \mathbf{K}_{se} + \mathbf{K}_d)\mathbf{P}(\mathbf{t}) = \mathbf{K}_m\left[K_{sd}C_d(t) + K_{se}C_e(t)\right]. \tag{49}$$

If $C_e(t) = T$, and $C_d(t) = 0$, i.e. the government imposes a per unit tariff on the imports at $t = 0$, then we can solve the above differential equation as follows:

$$\frac{dP(t)}{dt} + \mathbf{K}_m(\mathbf{K}_{sd} + \mathbf{K}_{se} + \mathbf{K}_d)\mathbf{P}(\mathbf{t}) = \mathbf{K}_m\mathbf{K}_{se}\mathbf{T}. \tag{50}$$

The characteristic function of the differential equation is as follows:

$$\mathbf{x} + \mathbf{K}_m(\mathbf{K}_{sd} + \mathbf{K}_{se} + \mathbf{K}_d) = \mathbf{0}.$$

The characteristic function has a single root given by:

$$x = -K_m(K_{sd} + K_{se} + K_d).$$

Thus the complementary solution is

$$\mathbf{P}_c(\mathbf{t}) = \mathbf{C}_2\mathbf{e}^{-[K_m(K_{sd} + K_{se} + K_d)]t}.$$

The particular solution has the form

$$\mathbf{P}_p(\mathbf{t}) = \mathbf{C}_1.$$

Thus the solution has the form

$$\mathbf{P}(\mathbf{t}) = \mathbf{C}_1 + \mathbf{C}_2\mathbf{e}^{-[K_m(K_{sd} + K_{se} + K_d)]t}. \tag{51}$$

The constant C_1 is determined by substitution into the differential equation as follows:

$$-\mathbf{K}_m(\mathbf{K}_{sd} + \mathbf{K}_{se} + \mathbf{K}_d)\mathbf{C}_2\mathbf{e}^{-[K_m(K_{sd} + K_{se} + K_d)]t} + \mathbf{K}_m(\mathbf{K}_{sd} + \mathbf{K}_{se} + \mathbf{K}_d)\mathbf{C}_1$$

$$+\mathbf{K}_m(\mathbf{K}_{sd}+\mathbf{K}_{se}+\mathbf{K}_d)\mathbf{C}_2\mathbf{e}^{-[K_m(K_{sd}+K_{se}+K_d)]t}=\mathbf{K}_m\mathbf{K}_{se}\mathbf{T},$$

$$\mathbf{C}_1=\frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}.$$

C_2 is determined by the initial condition as follows:

$$P(0)=\frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}+C_2=0,$$
$$C_2=-\frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}.$$

Substituting the values of C_1 and C_2 in eq. (51), **we get:**

$$\mathbf{P}(\mathbf{t})=\frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}-\frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}\mathbf{e}^{-[K_m(K_{sd}+K_{se}+K_d)]t}. \tag{52}$$

When $t=0$, $P(0)=0$ (the initial condition), and when $t=\infty$, $P(\infty)=\frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}$ (the final steady state equilibrium value). The price dynamics as a result of a tariff for a small or a large country depends on the parameters K_{sd}, K_{se}, K_d, K_m and T.

8.3 An Optimal Import Tariff Path

The efficiency loss as a result of an import tariff, generally mentioned in the economics literature is the dead weight loss as a result of comparisons of the pre and post tariff market equilibriums. However, the dynamic picture shows that there is some efficiency loss on the dynamic adjustment path to the new equilibrium as well after the tariff. After the imposition of the tariff, the import cost jumps to the previous cost plus the import tariff affecting the import quantity. The demand does not equal the supply any longer and the price adjusts over time to bring the new equilibrium price which is higher than the previous equilibrium price by a magnitude depending on the elasticity of demand and supply schedules**.** A pile up of inventory indicates a higher supply than demand, and a depletion of inventory occurs when demand is higher than the supply in a given time period. When the demand and supply again become equal, the new equilibrium has arrived. If the demand and supply are different, the output and/ or consumption is being lost at that point in time. Furthermore, some production is being wasted in comparison with that in the previous equilibrium. Therefore if we sum up either the consumption lost or the production lost (which are both equivalent), we get the total efficiency loss in terms of quantity, which is as follows:

$$EL=\int_0^\infty W_m(t)dt=M(t)+\int_0^\infty W_d(t)dt. \tag{53}$$

By eq. (48), the change in total domestic supply as a result of a tariff is as follows:

$$W_m(0) = -K_{sd}\left[C_d(0) - P(0)\right] - K_{se}\left[C_e(0) - P(0)\right] = -K_{se}T,$$

as $P(0) = 0$.

Therefore, at time zero, the inventory goes down by $K_{se}T$ (as the demand remains the same). Please look at figure 3, where the supply difference jumps to $K_{se}T$, i.e. the decrease in supply because of tariff at $t = 0$. The demand does not equal the supply any longer, and the market forces come into play. The supply along with the price adjusts over time and the new equilibrium arrives, i.e. $W_m(\infty)$. The shaded area is the efficiency loss (the amount of supply lost) during the adjustment process. The area between the lines $W_m(t) = 0$, and $W_m(t) = W_m(\infty)$ is the efficiency loss resulting from a difference in pre and post tariff market equilibriums. From eq. $(47a)$, we have

$$P(t) = -K_m M(t) + J.$$

The value of J can be found through the initial conditions as follows:

$$P(0) = -K_m M(0) + J,$$

$$0 = K_m K_{se} T + J,$$

$$J = -K_m K_{se} T.$$

Substituting the value of J in eq. $(47a)$, we get

$$P(t) = -K_m M(t) - K_m K_{se} T, \text{ or}$$

$$M(t) = -\frac{1}{K_m}\left[P(t) + K_m K_{se} T\right]. \tag{54}$$

With Tariff Revenue Constraint:
From eq. (48), the change in supply due to a change in price is as follows:

$$W_m(t) = -K_{sd}\left[C_d(t) - P(t)\right] - K_{se}\left[C_e(t) - P(t)\right].$$

The component of supply from the exporter in the foreign country on which tariff is levied is $-K_{se}\left[C_e(t) - P(t)\right]$, i.e.

$$W_{me}(t) = -K_{se}\left[C_e(t) - P(t)\right],$$

$$w_{nme}(t) - w_{ime}(0) = -K_{se}\left[C_e(t) - P(t)\right], \tag{55}$$

where $w_{ime}(0)$ is the initial import quantity and $w_{nme}(t)$ is the new import quantity after tariff, because $W_{me}(t)$ is a deviation variable, i.e. deviation from the initial equilibrium value. Therefore the expression for the tariff revenue is as follows:

$$TR = T\left[w_{ime}(0) - K_{se}\left\{T - P(t)\right\}\right]. \tag{56}$$

If we want to minimize the efficiency loss subject to the constraint that tariff revenue generated is greater than or equal to G in a given time period, our problem is as follows:

$$\min_T EL \quad \text{s.t.} \quad TR \geq G.$$

The choice variable is the tariff rate, and the constraint is binding. The Lagrangian for the above problem is as follows:

$$\begin{aligned}
\mathcal{L} &= -\int_0^\infty \left[\mathbf{K}_{sd}\left[C_d(t) - P(t)\right] + \mathbf{K}_{se}\left[C_e(t) - P(t)\right]\right] dt + \lambda\left[G - T\left[w_{ime}(0) - K_{se}\left\{T - P(t)\right\}\right]\right] \\
&= \int_0^\infty \left[-K_{se}T + \frac{K_{se}(K_{sd}+K_{se})T}{(K_{sd}+K_{se}+K_d)} - \frac{K_{se}(K_{sd}+K_{se})T}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right] dt \\
&+ \lambda\left[G - T\left[w_{ime}(0) - K_{se}\left\{T - \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}\right]\right].
\end{aligned}$$

Taking the first order condition with respect to T, we get:

$$\begin{aligned}
&\int_0^\infty \left[-K_{se} + \frac{K_{se}(K_{sd}+K_{se})}{(K_{sd}+K_{se}+K_d)} - \frac{K_{se}(K_{sd}+K_{se})}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right] dt \\
&- \lambda\left[w_{ime}(0) - K_{se}\left\{T - \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}\right] \\
&\qquad +\lambda T K_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right] = 0.
\end{aligned}$$

This implies that

$$\begin{aligned}
&\int_0^\infty \left[-K_{se} + \frac{K_{se}(K_{sd}+K_{se})}{(K_{sd}+K_{se}+K_d)} - \frac{K_{se}(K_{sd}+K_{se})}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right] dt \\
&+ 2\lambda T K_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right]
\end{aligned}$$

$$= \lambda w_{ime}(0),$$

or

$$T = \frac{\lambda w_{ime}(0) - \int\limits_0^\infty \left[-K_{se} + \frac{K_{se}(K_{sd}+K_{se})}{(K_{sd}+K_{se}+K_d)} - \frac{K_{se}(K_{sd}+K_{se})}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right] dt}{2\lambda K_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right]}. \quad (57)$$

Taking the first order condition with respect to λ, we get:

$$G - T\left[w_{ime}(0) - K_{se}\left\{T - \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}\right] = 0. \quad (58)$$

Substituting the value of T from eq. (57) into (58), we get:

$$\begin{aligned} G = w_{ime}(0). &\frac{\lambda w_{ime}(0) - \int\limits_0^\infty \left[-K_{se} + \frac{K_{se}(K_{sd}+K_{se})}{(K_{sd}+K_{se}+K_d)} - \frac{K_{se}(K_{sd}+K_{se})}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right] dt}{2\lambda K_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right]} \\ - K_{se}&\left\{1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\} \\ * &\left[\frac{\lambda w_{ime}(0) - \int\limits_0^\infty \left[-K_{se} + \frac{K_{se}(K_{sd}+K_{se})}{(K_{sd}+K_{se}+K_d)} - \frac{K_{se}(K_{sd}+K_{se})}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right] dt}{2\lambda K_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right]}\right]^2, \end{aligned}$$

or $4\lambda^2 QG = 2\lambda^2 w_{ime}^2(0) - 2\lambda w_{ime}(0)J - \lambda^2 w_{ime}^2(0) - J^2 + 2\lambda w_{ime}(0)J$,

$$\text{where } Q = K_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right],$$

$$J = \int\limits_0^\infty \left[-K_{se} + \frac{K_{se}(K_{sd}+K_{se})}{(K_{sd}+K_{se}+K_d)} - \frac{K_{se}(K_{sd}+K_{se})}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right] dt.$$

This implies that

$$\left\{w_{ime}^2(0) - 4QG\right\}\lambda^2 - J^2 = 0.$$

$$\lambda = \frac{J}{\sqrt{w_{ime}^2(0) - 4QG}}.$$

Eq. (57) can also be written as

$$T = \frac{\lambda w_{ime}(0) - J}{2\lambda Q}. \tag{59}$$

Substituting the value of λ into eq. (59), we get:

$$T(t) = \frac{\frac{w_{ime}(0)J}{\sqrt{w_{ime}^2(0) - 4QG}} - J}{\frac{2QJ}{\sqrt{w_{ime}^2(0) - 4QG}}},$$

$$T(t) = \frac{w_{ime}(0) - \sqrt{w_{ime}^2(0) - 4QG}}{2Q}. \tag{60}$$

In order to check the second order condition for minimization, we proceed as follows: The Lagrangian can be written as

$$\mathcal{L} = JT + \lambda \left[G - T \left(w_{ime}(0) - QT \right) \right].$$

The Bordered Hessian matrix of the Lagrange function is as follows:

$$BH = \begin{bmatrix} 0 & w_{ime}(0) - 2QT \\ w_{ime}(0) - 2QT & \frac{2QJ}{\sqrt{w_{ime}^2(0) - 4QG}} \end{bmatrix},$$

the determinant of which is negative as $-\left(w_{ime}(0) - 2QT \right)^2 < 0$, which implies that the efficiency loss is minimized.

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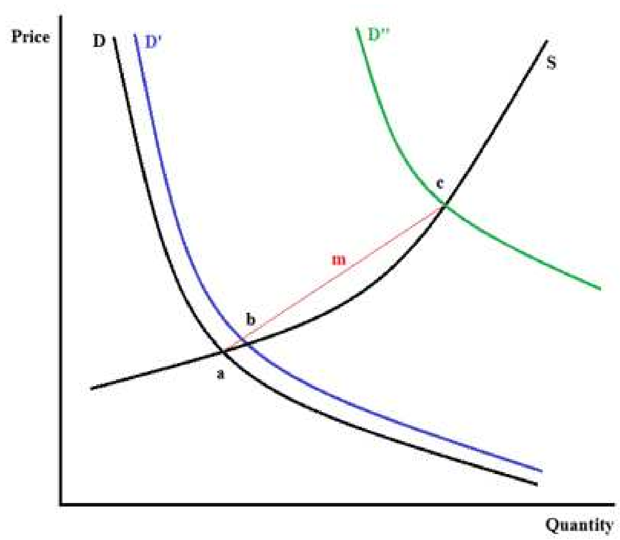

Figure 1: When is Linearity a Reasonable Assumption?

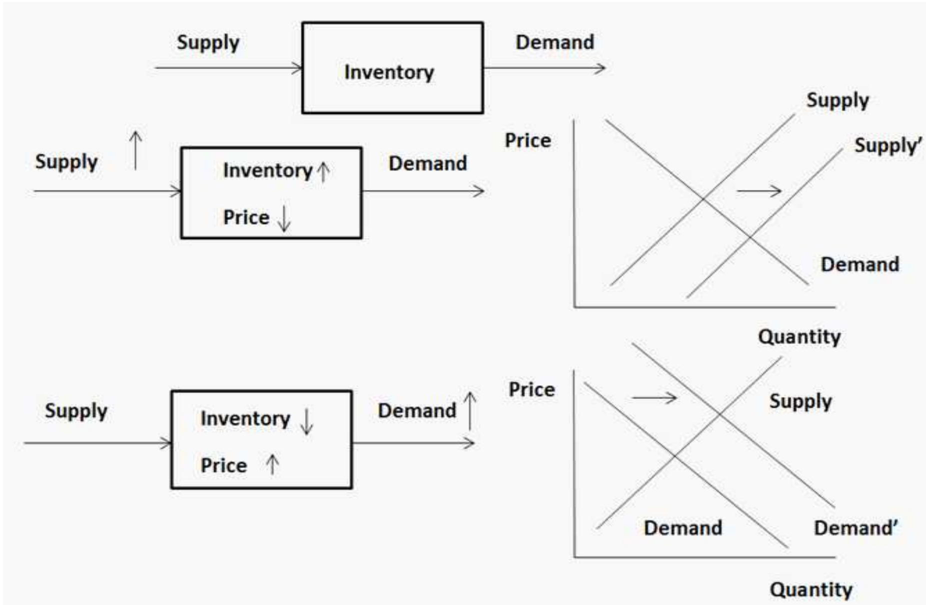

Figure 2: Movement of Price with Inventory.

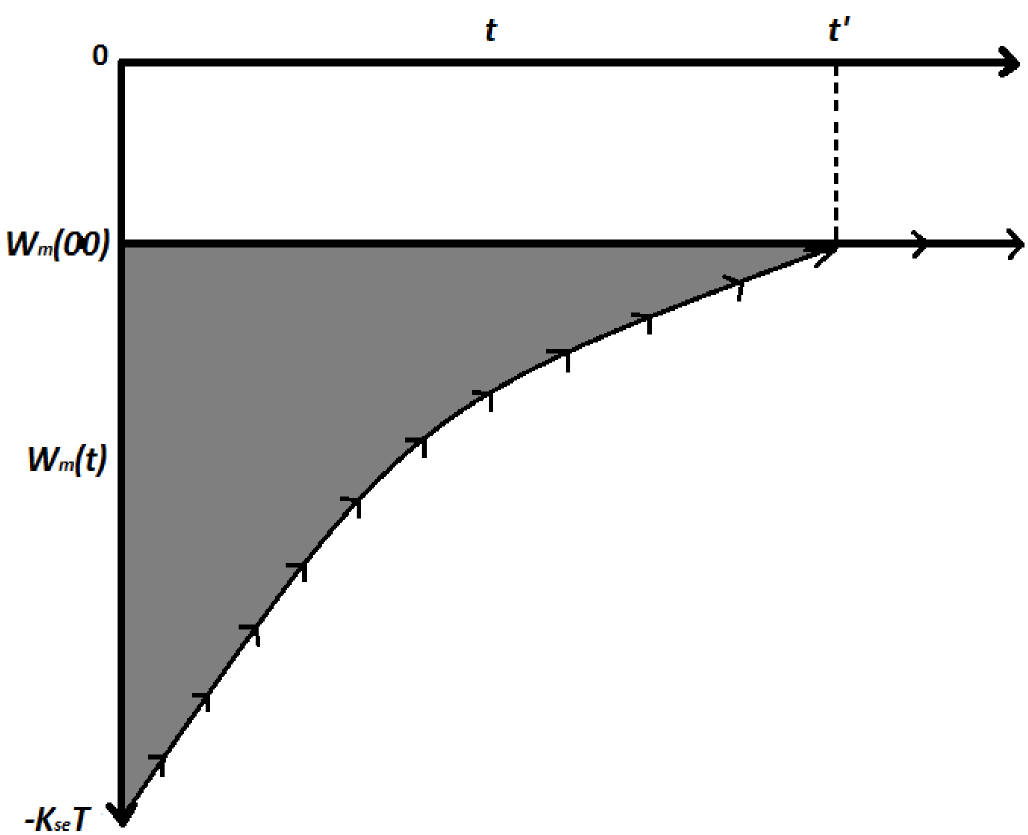

Figure 3: Dynamic Efficiency Loss because of an Import Tariff.