 Preferential Trade Agreements as Insurance

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14 November 2022
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November 14, 2022

Abstract

We investigate preferential trade agreement (PTA) formation when risk averse countries face demand uncertainty and, hence, have an insurance motive for pursuing trade integration. In this environment, when deciding which type of PTA - if any - they wish to form, countries seek to maximise their net welfare; that is, their expected utility less a risk premium. The desire for insurance influences, not just whether a particular PTA forms, but also the preferred depth of integration. We analyze the insurance implications of free trade agreements (FTAs), customs unions (CUs), and countries choosing to stand alone. We further distinguish between shallow CUs and deep CUs; in the former, members maximise the sum of their individual net welfares, while in the latter they maximise the net value of the sum of their individual expected welfares. We show that differences in country risk attitudes and the levels of risk they face, as well as the degree to which these risks are correlated with each other, each, and together, influence the formation and design of TAs. When countries’ demands are uncorrelated, they form a deep CU if their levels of risk aversion are sufficiently different. If, however, their risk attitudes are similar, countries opt for shallower trade integration - either a shallow CU or a FTA - if they face low levels of uncertainty, and choose to stand alone if one country faces a sufficiently high level of uncertainty. When countries’ demands are correlated, they tend to form a deep CU if their demands are strongly negatively correlated, a FTA if their demands are strongly positively correlated and a shallow CU when their demands are weakly correlated. Intuitively, differences in country risk attitudes (i.e., their degree of risk aversion) act as an additional source of comparative advantage. Deeper integration - particularly via a CU - permits less risk averse members to essentially export their relative partiality for risk to more risk averse partners, thereby effectively providing the latter with insurance.

Keywords: Trade Agreement, Free Trade Area, Customs Union, Insurance, Uncertainty, Risk Premium.

JEL Classification: F12, F13, F15, D81.

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1 Introduction

For decades, globalization increased the exposure of firms and consumers to political and economic uncertainty originating from abroad. Recently, however, there has been an apparent retreat from international trade integration exemplified by the United Kingdom’s exit from the European Union, the US-initiated renegotiation of NAFTA, the withdrawal of the US from the Trans-Pacific Partnership (TPP) and frequent trade skirmishes - if not outright trade wars - involving the European Union, the US, China, Australia and other nations. Moreover, this apparent rollback of globalization - at least as far as regionalism is concerned - has occurred against a backdrop of seemingly heightened geopolitical and other risks; a febrile atmosphere characterized, among other things, by increased division and instability in once equable political environments, the apparent waning of the global hegemony of the United States, an unfamiliar global macro-economy characterized by unconventional monetary policy and ubiquitous and disruptive technological change.

It is unclear whether the recent retreat from regionalism marks the beginning of a long-term trend, nor to what extent and how it is related to greater political and economic uncertainty. In any case, there has been a groundswell of political support for countries to retreat from a wholesale embrace of global integration that has characterized much of the last half-century. Brexit, for example, has been justified based on the United Kingdom “taking back control of its destiny.”\footnote{1UK Prime Minister Boris Johnson, December 24 2020.} One interpretation of this is that the UK has regained the flexibility of unilateral policy-setting allowing it to better respond to challenges in a world of heightened uncertainty. An alternative interpretation is that, by extricating itself from the European Union’s embrace, the UK is depriving itself of protections that might arise from policy coordination, leaving itself more exposed to global risks. Likewise, the US demand to renegotiate NAFTA and walk-away from the TPP reflects broad dissatisfaction in the US with the impact of trade integration and an acceptance that the cost-benefit calculus of preferential trading agreements (PTAs) has changed.

This paper re-examines the role of uncertainty in the formation of PTAs. In particular, we seek to understand how the presence of demand uncertainty and countries’ attitudes to risk influence their decision whether or not to join (or leave) a PTA and the kind of PTA they join. The role of uncertainty in PTA formation and design has received surprisingly little attention in the literature. Even less attention has been paid to examining how countries’ attitudes to risk influence PTA formation.

This paper discusses two inter-related motives for PTA formation which have yet to be adequately addressed in the literature: insurance and diversification. We demonstrate that in a world characterized by demand uncertainty, countries may decide whether or not to form a PTA based on the degree to which it will mitigate...
the costs of economic uncertainty for members. Moreover, we show that this consideration influences the type of PTA that members decide to form, particularly the depth of integration they prefer.

We develop a partial equilibrium model of a trading world characterized by (linear) demand uncertainty and imperfect competition between firms. There are three countries, each of which can be risk-averse. One of them (Country 3) is assumed to eschew PTA membership. We focus on understanding the motivations for the other two countries (1 and 2) to join a PTA, if any. While important, deriving equilibrium trade coalitions is beyond the scope of the current paper and worthy of separate study.2

We define a country’s welfare as the sum of its consumer and producer surplus. We assume that the expected utility of a country’s welfare can be expressed by the standard approximation of expected utility as the "mean-variance" of its welfare, namely, expected welfare minus a risk premium. We define this (mean-variance) as the country’s net value of welfare (NVW).

Countries play a three-stage game. In the first stage, countries 1 and 2 choose whether to form a PTA and how best to design it. In the second stage, optimal tariffs are chosen by all countries (including the excluded Country 3). In the third stage, firms choose their profit maximizing outputs. Demand uncertainty is assumed to be resolved after the second stage but before the third stage. Hence, while PTA formation and optimal tariff choice occur under uncertainty, firms choose outputs under certainty.

In the first stage, countries 1 and 2 can choose between forming a free trade area (FTA), a "shallow" customs union (SCU), a "deep" customs union (DCU), or to stand alone (SA). In a SCU, countries 1 and 2 choose a common external tariff to maximize the sum of their NVWs. A DCU differs from a SCU in two respects. First, members of a DCU choose the common external tariff to maximize the net value of the sum of their welfares. In other words, in a SCU, they maximize the sum of their expected utilities of welfare, whereas, in a DCU, they maximize the expected utility of the sums of their welfare. Clearly, with nonlinear utility functions, the former and the latter will yield different outcomes. Second, the maximization of the net value of the sum of the DCU members’ welfares requires the adoption of a “common” or “joint” measure of risk-aversion.

The distinction between a SCU and a DCU is crucial in this paper because it raises particular issues with respect to how one should model the depth of trade integration in the presence of uncertainty. We demonstrate that a DCU is a deeper form of integration than a SCU because members of the former - in contrast to the latter - are required to calculate a ‘joint risk premium’ that accounts for any correlation between their individual welfares. Moreover, in contrast to a SCU, in calculating a joint risk premium, members must also agree on a joint measure of risk aversion.

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2For a state-of-the-art analysis of equilibrium PTA formation, see Saggi and Yildiz (2010).
The introduction of risk aversion has several important implications (which are absent when countries are risk-neutral) for analyzing PTA formation. First, compared to the benchmark case in which putative member countries are risk-neutral (or, equivalently, there is no uncertainty), the introduction of risk-averse members implies that depth in customs union (CU) formation is now a relevant consideration for them. While a SCU and DCU are identical if PTA members are risk-neutral, this is no longer the case if at least one member is risk-averse. Countries 1 and 2 must now incorporate the difference between shallow and deep CUs into their deliberations. One particularly difficult problem they now face is the choice of the “joint” measure of risk-aversion.

Second, a (shallow or deep) CU, characterized by a common external tariff, involves greater risk-sharing among members than a FTA. This conclusion follows directly from the fact that, given agents’ concave utility functions, bargaining or collusive equilibria always result in Pareto efficient risk-sharing, i.e., efficient insurance. This is not the case with non-cooperative interactions as in the case of a FTA.\(^3\)

Moreover, while both a SCU and a DCU (being cooperative) confer insurance benefits on members, with a DCU, the impact of uncertainty and risk aversion are fully internalized. In other words, all possible externalities arising from uncertainty, risk aversion, and demand correlations are fully taken into account (and, hence, risk-sharing is Pareto efficient).

A third implication arising from the consideration of risk aversion is the introduction of additional difference between countries, namely, differences in risk and risk aversion. These difference are important because, in a world in which uncertainty matters, one can usefully think of country risk and risk attitudes as additional sources of comparative advantage.\(^4\) The fact that uncertainty exists and that one country is sufficiently risk-averse relative to its partner confers a comparative advantage on the relatively risk-sanguine nation. By agreeing to form a CU - and especially a DCU - rather than a FTA, the less risk-averse member is, essentially, exporting lower risk-aversion, thus, effectively providing insurance to its more risk-averse partner. In short, through judicious choice of PTA type, member countries can ‘trade’ risk attitudes or, equivalently, insurance to mutual advantage and, by doing so, capture and maximize the gains from risk-sharing inherent in a CU. The same argument applies to differences in risk. By agreeing to form a CU - and especially a DCU - rather than a FTA, the less risky member is, essentially, exporting lower risk to its more risky partner.

A fourth implication arising from the consideration of risk aversion is that it introduces diversification considerations and benefits. Again, while all PTAs involve some diversification benefits, such benefits increase with the degree of cooperation and will, therefore, be most valuable in a DCU. Moreover, diversification benefits become

\(^3\)However, although there is no direct risk-sharing in non-cooperative interactions, some implicit risk-sharing, albeit not Pareto efficient, may still occur (depending on the parameters of the model).

\(^4\)We thank Eyal Winter for alerting us to this interpretation. See Appelbaum (2020) for a discussion of the effects of differences in risk and risk aversion on trade patterns and the gains from trade.
even more important when markets are correlated.5

A final implication of risk aversion is that the level of uncertainty facing countries also matters when they decide whether or not to form a PTA and, if so, what type. In particular, for a given degree of risk aversion, a country’s desire for insurance-via-trade-policy-coordination will be greater (less) the higher (lower) the level of uncertainty faced. A SCU is less likely to be observed when either or both members face significant levels of uncertainty.

Our results show that when countries’ demands are uncorrelated, they form a DCU if their levels of risk aversion are sufficiently different. If, however, their risk attitudes are similar, members opt for shallower trade integration - either a SCU or a FTA - if they face low levels of uncertainty and choose to stand alone if one country faces a sufficiently high level of uncertainty. When countries’ demands are correlated, they tend to form a DCU if their demands are strongly negatively correlated, a FTA if their demands are strongly positively correlated, and a SCU when their demands are weakly correlated. As already explained, differences in country risk attitudes (i.e., their degree of absolute risk aversion) act as an additional source of comparative advantage, permitting a less risk-averse member to export their relative risk neutrality to a more risk-averse partner, thereby providing them with insurance.

The role of member country risk aversion and insurance has received surprisingly little attention in the PTA literature. A notable exception includes Perroni and Whalley (2000). They employ a calibrated static general equilibrium trade model to argue that in PTAs, small countries with little negotiating power may make policy concessions (i.e., pay a risk premium) to secure “safe-haven trade arrangements” and their access to a large market.6 Wu (2005) distinguishes between the self-insurance and self-protection motives for trade bloc membership where the former reduces the size of any loss from a trade war and the latter reduces the probability of a trade war occurring. A country decides the amount of integration it wishes to pursue to minimize its exposure to the costs of a trade war. Wu (2005) finds that an increase in the cost of self-protection dampens a country’s interest in purchasing integration, while a moderate increase in the threat of a trade war will encourage a country to purchase greater integration.

In this context, therefore, a trade war acts as a threat-point in PTA negotiations. Hence, countries are willing to pay a premium to join a PTA directly via side-payments, as in Perroni and Whalley (2000), or indirectly via an investment in integration, as in Wu (2005). In contrast, we model the source of the underlying uncertainty - demand uncertainty in our case. This approach allows us not only to capture the welfare implications of

5The role of the diversification effects in determining trade patterns and the gains from trade are discussed in Appelbaum (2020).

6Harris and Robertson (2009) use a dynamic general equilibrium framework to estimate the potential benefits to Australia of the Australia-US FTA in the case of a global trade war occurring. While they term this the “insurance benefit” of the agreement, they do not use the term "insurance" in the formal way intended in our paper. Harris and Robertson (2009) do not explicitly model uncertainty or country attitudes to risk.
uncertainty and risk aversion more broadly, but it also permits us to characterize the nature of risk premia in terms of both the nature of country risk attitudes and the level of risk they face. Crucially, as our results below reveal, even when insurance considerations are taken into account, it is not always the case that standing alone (i.e. a trade war) is the worst outcome for individual countries. Indeed, if one country faces sufficiently high uncertainty, then standing alone may be preferable to any PTA since doing so provides them with maximum flexibility to unilaterally choose their trade policy vis-à-vis the rest of the world.

While little attention has been paid to insurance in PTA formation, extensive literature exists dealing with the role of trade agreements (TAs) in dealing with trade policy uncertainty. Handley (2014) and Handley and Limão (2015, 2017) show that a country may be motivated to join an agreement if, by doing so, it can reduce trade policy uncertainty faced by its domestic producers. To the extent that membership implies a credible reduction in tariffs levied against its domestic producers, they will be encouraged to make sunk cost investments, translating into increased exports. The current paper differs from these analyses in several ways. First, we consider uncertainty in underlying economic parameters, not trade policies per se. Of course, even in our framework, a change in demand uncertainty will influence countries’ optimal tariff choices, so, indirectly at least, trade policy uncertainty plays a role in our analysis. However, our main focus is to investigate the insurance implications of PTAs and understand how PTAs should be designed when uncertainty and risk aversion are present, issues on which the above papers do not seek to engage. Moreover, these papers do not seek to distinguish between different types of TAs and so do not address the issue of how uncertainty impacts on the choice of depth of integration or the choice between FTAs and CUs more generally.

In a paper that is closest in spirit to ours, Limão and Maggi (2015) investigate the conditions under which TA formation is motivated by a country’s desire to either reduce or enhance its exposure to the negative and positive externalities from trade policies implemented by its trading partners. Contrary to the TA-as-insurance literature previously discussed, Limão and Maggi (2015) note that TA formation may be motivated by a desire to either reduce or increase exposure to uncertainty, not just the former. They differentiate between two sources of uncertainty a country may face: either that which arises from a “political economy” (i.e. pure policy) shock instigated by a trade partner or an “economic shock” that affects the country directly as well as via its trade partner’s policy response. In the former case, the country’s preference for policy risk determines whether it views noncooperation as “too risky”; if it does, it will pursue a TA; otherwise, it will not. In the latter case, a country’s PTA decision depends not only on its own policy risk preference but also on the direct welfare impact of the economic shock on the country (i.e., holding the trade partner’s policy unchanged).

Attention has also recently turned to political uncertainty over TA ratification; see Cole, Lake and Zissimos (2021). Typically, all TA members must individually ratify an agreement before it can come into force. However, lobbying between anti- and pro-trade liberalizing forces may endanger ratification by one or more members.
Limão and Maggi (2015) demonstrate that if individuals are income-risk neutral, a country may decide to form a TA to increase its risk exposure. If, instead, individuals are sufficiently income-risk averse, a country will have a risk-reducing motive to form a TA, analogous to that discussed in the TA-as-insurance literature above. Moreover, a country’s risk-reducing motive increases as its economy becomes more specialized and open and when its export supply elasticity is low.

Our current paper differs from Limão and Maggi’s analysis beyond the modelling framework employed. First, we explicitly address the issue of the optimal design of PTAs. This allows us to gain several important insights into the implications of risk aversion and uncertainty for optimal trade agreement formation and design. Second, in our modelling framework, all countries (including the excluded Country 3) are asymmetric and set their trade policies optimally. In contrast, Limão and Maggi concentrate first on only one policy-active country and then, when they introduce a second policy-active country, they assume that both are symmetric. In the PTA literature, strategic interaction among asymmetric countries has significant implications for PTA formation. As indicated above, the introduction of risk aversion accentuates this by introducing another source of asymmetry between members.

Finally, Appelbaum and Melatos (2016) explicitly model PTA formation in the presence of demand and cost uncertainty when countries are risk-neutral and levy optimal tariffs on imports. They find that every PTA has an option value which is the expected value of (perfect) information. As long as member countries’ welfare functions are convex in the random variables (which happens when at least some decisions are made after uncertainty is resolved), then the option value of any PTA will increase with greater demand or cost uncertainty (modelled as a mean-preserving spread). This convexity will be greater for a PTA that provides members with greater degrees of freedom in tariff choice and greater degrees of tariff policy coordination. As such, however, as uncertainty increases in a given random variable, the option value of a PTA (and the Nash equilibrium member welfare associated with it) rises by more the more convex the PTA’s member welfare function is with respect to that random variable. In this way, as the level of uncertainty changes, the relative attractiveness of PTAs can also change. Since the role of option values has already been discussed in Appelbaum and Melatos (2016), we will not focus on that aspect in this paper unless it is necessary.

For the case of linear demand and cost functions, Appelbaum and Melatos (2016) show that as member demand uncertainty increases, members increasingly prefer a FTA to a CU. Members value the greater degrees of freedom in tariff choice implied by the former; policy coordination benefits are muted since markets are

\(^8\) See, for example, Riezman (1985) and Melatos and Woodland (2007).

\(^9\) As implied by Blackwell’s Theorem.

\(^10\) See Lake (2019) for a demonstration of the role of PTA flexibility in a dynamic setting without uncertainty. In this case, flexibility refers to the ability of PTA members to form overlapping agreements subsequently.
segmented and member welfare functions are additive. On the other hand, an increase in cost uncertainty makes a FTA relatively less attractive relative to a CU; members value policy coordination over freedom in tariff choice in order to internalize cost externalities between them. The current paper extends our previous analysis by relaxing the assumption of risk neutrality so as to investigate the role of insurance as a motive for PTA formation.

2 The General Framework

Consider a world comprising three countries, each populated by one representative firm, in which PTAs can form. Assume that one country, Country 3 here, is “passive” in the sense that it does not sign PTAs. Countries 1 and 2, on the other hand, are “active”; they may negotiate a bilateral PTA if they wish. It is further assumed that countries 1 and 2 can choose between two alternative types of bilateral trade blocs - a FTA or a CU - where a CU can either be “shallow” (SCU) or “deep” (DCU). A DCU differs from a SCU in two respects. First, DCU members optimally (and simultaneously) choose a “joint” measure of absolute risk aversion. Second, they use a “joint risk premium”. The joint risk premium will be discussed in detail below. Alternatively, countries 1 and 2 may prefer not to form a bilateral trade bloc and, instead, stand alone (SA).

Define the set of four possible coalition structures as \( Y = \{sa, fta, scu, dcu\} \). In what follows, all elements in \( Y \), including the \( sa \) case, are referred to as “types” of PTAs. Consistent with WTO rules, this paper assumes that, regardless of which coalition structure eventuates, the most-favoured-nation (MFN) principle holds. That is, unless they are a member of a formal discriminatory trading arrangement - either a FTA or a CU in this paper - countries must implement identical trade policies (e.g. identical tariff rates) on all their trade partners.

In our framework, the three countries engage in a multi-stage trade policy game. In stage one, countries 1 and 2 choose a PTA, \( y \in Y \) and associated lump-sum transfers, \( K^y_i, i = 1, 2 \). A choice of a DCU subsumes the simultaneous choice of an optimal “blended” measure of absolute risk aversion. In stage two, given the PTA that has formed, all three countries choose their tariffs, \( t_{ij}, i, j = 1, 3 \), where \( t_{ij} \) denotes the tariff Country \( i \) pays Country \( j \), and where \( t_{ii} = 0 \). In stage three, given the previously chosen PTA and tariffs, firms in the three countries choose their outputs in each market. These outputs are denoted by \( q_{ij} \), the quantity that firm \( i \) sells in Country \( j \). For simplicity, we assume that there is one firm domiciled in each country. Country \( i \)'s firm is referred to as firm \( i \).

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11 The partial equilibrium model adopted in this section is based on a model used by Ornelas (2007) and others to analyze PTA formation in a certain world.
12 The assumption that Country 3 is passive means that we do not have to consider the case of global free trade. This simplifies the analysis significantly as, otherwise, we would have to consider all possible coalition structures among the three countries, not just those involving countries 1 and 2. It turns out that in this model, without uncertainty, global free trade dominates all other types of trade agreements. On the other hand, once uncertainty is introduced, the primacy of global free trade can no longer be guaranteed. Detailed analysis of the preference for global free trade under uncertainty is left for future research.
13 Assuming multiple firms yields little additional insight for our purposes while making the analysis more cumbersome.
We assume that markets are segmented. This assumption significantly simplifies the analysis without compromising our desire to focus on the role of uncertainty in PTA formation. Country $j$’s demand function is given by:

$$p_j = a_j - Q_j, \ j = 1..3,$$

where $a_j > 0$ is a demand parameter, and $Q_j = \sum_{i=1}^{3} q_{ij}$ is the aggregate output sold in Country $j$. We define the demand vector as $a = (a_1, a_2, a_3)$. The firms’ technology in the three countries is captured by their marginal (and average) costs $c_i, \ i = 1..3$. The vector of marginal costs is given by $c = (c_1, c_2, c_3)$.

2.1 The Source and Resolution of Uncertainty

We assume that countries face uncertain demands.\(^{14}\) Specifically, demand parameters $(a_j, j = 1..3)$ are random variables with a vector of means $\mu \equiv (\mu_1, \mu_2, \mu_3)$, where $\mu_i = E(a_i)$, and a 3x3 covariance matrix $\Sigma$, with $\sigma_{ij} = Cov(a_i, a_j)$ and associated $\rho_{ij}$. We assume that the properties of the distribution of the vector $a$, specifically its moments, are common knowledge to all players in the game.

In general, uncertainty can be resolved at different points of the multi-stage trade policy game. It may be resolved before all decisions (choice of PTA, tariffs and outputs) are made (“early resolution”); after some, but not all, decisions are made (“intermediate resolution”), or after all decisions are made (“late resolution”). As demonstrated by Appelbaum and Melatos (2016), the ability to make at least some decisions after uncertainty is resolved introduces value-of-information considerations (i.e., option values) into the analysis. In the case of “early resolution”, however, such considerations do not arise.

In this paper, we follow Appelbaum and Melatos (2016) and assume that uncertainty is resolved after some, but not all, decisions are made. Specifically, the PTA and tariffs are chosen under uncertainty, but outputs are chosen after uncertainty is resolved. Unlike Appelbaum and Melatos (2016), however, we now add insurance and diversification (which can be viewed as risk management) considerations by introducing risk aversion.\(^{15}\) In this model, therefore, both value-of-information and insurance considerations are present.

We use the certainty case, where decisions in all stages of the game (PTA formation, tariffs and firm outputs) are made with full knowledge of demand conditions, as the reference case for comparisons.

3 Stage 3: Output Choice

In stage 3, when outputs are chosen, demand conditions are already known. Thus, the three firms choose their outputs simultaneously in a Cournot-Nash game, given the tariffs chosen by all three countries and the PTA

\(^{14}\) Alternatively, we can assume that costs, or both demands and costs, are uncertain. Since our objective is simply to demonstrate the insurance aspects of PTAs, we do not consider these alternatives in the current paper.

\(^{15}\) Since their model was not meant to address the role of insurance in PTA formation, Appelbaum and Melatos (2016) assumed that all parties were risk-neutral.
chosen by countries 1 and 2. Using the demand functions defined above and the tariff rates chosen by each country, the profit firm \( i \) makes from selling in Country \( j \) is given by:

\[
\pi_{ij} = a_j - \sum_{k=1}^{3} q_{kij} - c_i - t_{ij}q_{ij} = \pi_{ij}(q^j, t_{ij}; a_j, c_i), \quad i, j = 1..3,
\]

(2)

where \( q^j = (q_{1ij}, q_{2ij}, q_{3ij}) \) is the vector of quantities sold in Country \( j \).

Since markets are segmented, the Nash equilibrium quantities in Country \( j \) are obtained by the simultaneous solution to the three countries’ profit maximization problems given by:

\[
\max_{q_{ij}} \pi_{ij}(q^j, t_{ij}; a_j, c_i), \quad i = 1..3.
\]

(3)

Let the Nash Equilibrium quantities in Country \( j \), be denoted as \( q^*_ij \). It is straightforward to show that:

\[
q^*_ij = q^*ij(t^j; a_j, c) \equiv \frac{1}{4} [a_j + \sum_{k \neq i} (c_k + t_{kj}) - 3(c_i + t_{ij})], \quad i = 1..3,
\]

(4)

where \( t^j = (t_{1ij}, t_{2ij}, t_{3ij}) \) is the vector of tariffs levied by Country \( j \). Note that while \( q^*_ij \) depends on the vectors \( c \) and \( t^j \), it only depends on Country \( j \)'s demand parameter \( a_j \) (and not on \( a_{h \neq j} \)). The Nash Equilibrium quantities in Country \( j \) can be written alternatively as the vector, \( q^*j = q^{*j}(t^j; a_j, c) \).

Using equations (2) and (4), the corresponding Nash equilibrium profits, denoted as \( \pi^*ij[t^j; a_j, c] \), can be calculated as:

\[
\pi^*ij[t^j; a_j, c] \equiv \pi_{ij}[q^{*j}(t^j; a_j, c), t_{ij}; a_j, c_i] = \frac{1}{16} [a_j + \sum_{h \neq i} (c_h + t_{hj}) - 3(c_i + t_{ij})]^2, \quad i = 1..3.
\]

(5)

Whereas (not surprisingly) \( \pi^*ij[t^j; a_j, c] \) is decreasing in \( c_i \) and \( t_{ij} \), it is increasing in \( c_{h \neq i} \) and \( t_{h \neq i,j} \). Moreover, \( \pi^*ij[t^j; a_j, c] \) is convex in \( a_j, c \).\(^{16}\)

4 Stage 2: Tariff Choice

In stage 2, facing uncertain demand conditions, the countries choose their tariffs given the PTA (if any) formed in stage 1. We define the welfare of Country \( i \) (welfare minus lump sum transfers) as the sum of consumer surplus, producer surplus and tariff revenue. Using the Nash equilibrium quantities derived above, we can explicitly write Country \( i \)'s welfare in stage 3 as:

\[
w_i(t; a, c) \equiv \frac{1}{2} Q_i^2 + \sum_{j=1}^{3} \pi^*ij[t^j; a_j, c] + \sum_{j \neq i} q^*_ij(t^j; a_i, c)t_{ji},
\]

(6)

where \( t \) is the set (matrix) of all tariffs and \( Q_i = \sum_{j=1}^{3} q^*_ij \). Since demand conditions are uncertain, each \( w_i(t; a, c) \) is also a random variable. In fact, it is straightforward to show that the countries’ welfare functions in equations

\(^{16}\) As Appelbaum and Melatos (2016) demonstrate, it is this convexity that gives rise to value-of-information effects.
(6) are quadratic functions of the random variables $a$ and the choice variables $t$. For the rest of the paper, we will not be concerned with (nor examine the role of) the cost parameters. Thus, for notational convenience, we write country $i$’s welfare as $w_i(t; a)$ instead of $w_i(t; a, c)$.

In general, when analyzing uncertainty models, we would adopt the expected utility approach by assuming that the three countries make their decisions (in this case, the choice of tariffs) by maximizing the expected utility of their welfare. In other words, if we define the utility of a random variable (i.e., lottery) $w_i(t; a)$ as $U_i[w_i(t; a)]$, and if the utility function has the expected utility form, then the countries choose their tariffs by maximizing $U_i[w_i(t; a)] = E\{u_i[w_i(t; a)]\}$, where $u_i[w_i(t; a)]$ is the utility of the realizations of welfare in country $i$ (corresponding to the realization of the vector $a$).

Unfortunately, using general (increasing and concave) utility functions substantially complicates the analysis. For example, often (even for a single decision-maker), knowledge of properties of high order derivatives of the utility function is required (e.g., properties of measures of risk aversion depend on third-order derivatives). In our case, this is even more complicated because we have to obtain the Nash Equilibrium of a three-party multi-stage game under uncertainty. In addition, in general, the nonlinearity of the utility functions requires knowledge of more than just the first two moments of the underlying distribution functions. Since our main objective is to demonstrate the role of insurance considerations in PTA formation, we prefer to be parsimonious and use the simplest framework possible to achieve this. Thus, we use the standard approximation of the expected utility, given by:

$$E\{u_i[w_i(t; a)]\} \approx u_i\{E[w_i(t; a)] - \frac{1}{2}R_i Var[w_i(t; a)]\},$$

where the function $u_i$ is strictly monotonically increasing and concave in $w_i$, $Var(w_i)$ is the variance of $w_i$ and $R_i$ is the measure of absolute risk aversion in Country $i$, which we assume is constant. Country $i$’s “risk premium” is, therefore, given by

$$\theta_i \equiv \frac{1}{2}R_i Var(w_i).$$

Since (given our linear demand and cost functions) $w_i$ is quadratic in $a \equiv (a_1, a_2, a_3)$, it follows that the mean of a country’s welfare, $E(w_i(t; a))$, depends only on the first two moments (means, $\mu$, and covariance matrix,
it is straightforward to show that, although \( w_i(t; a) \) is quadratic, it does not have “cross-terms” involving products of the \( a_i \)'s, namely, we have \( \frac{\partial^2 w_i(t; a)}{\partial a_i \partial a_j} = 0 \), for all \( i, j = 1..3 \). As a result, \( E[w_i(t; a)] \) does not contain the covariances of the \( a_i \)'s.

Unfortunately, given that \( w_i(t; a) \) is a quadratic function of the random variables, the calculation of its variance is difficult because it involves moments of order three and four. To overcome this difficulty, we follow the standard practice in statistics and use a linear approximation of the variance of \( w_i(t; a) \), given by:

\[
Var[w_i(t; a)] \approx \sum_{j=1}^{3} \sum_{k=1}^{3} \frac{\partial w_i(t; \mu)}{\partial \mu_j} \frac{\partial w_i(t; \mu)}{\partial \mu_k} \sigma_{jk}
\] (8)

With this approximation, the variance of \( w_i(t; a) \) depends only on the means, \( \mu \), and the (full) covariance matrix, \( \Sigma \) (that, unlike \( E[w_i(t; a)] \), does involve covariances), but not on higher moments of the distribution.\(^{20}\)

The approximation of the expected utility in equation (7) can, therefore, be written as:

\[
E\{u_i[w_i(t; a)]\} \approx u_i\{M_i(t; \mu, \Sigma, R_i)\},
\] (9)

where \( M_i(t; \mu, \Sigma, R_i) \) is the “risk-adjusted” expected value of welfare (expected value of welfare, \( E(w_i(t; a)) \), adjusted by the the risk premium, \( \frac{1}{2} R_i Var(w_i) \)). In the following we refer to \( M_i(t; \mu, \Sigma, R_i) \) as the net value of welfare (NVW).

Clearly, this objective function is rather simple: only the first two moments are required. But, as was stated above, it is sufficient for our purposes; it allows for risk aversion and gives rise to a simple and relatively simple-to-calculate risk premium.

4.1 Standing Alone and FTA

To examine the choice of tariffs, we must consider the tariff restrictions implied by the four possible coalition structures that countries 1 and 2 can choose to form. In this section, we concentrate on the cases in which countries 1 and 2 choose to stand alone or form a FTA.

First, note that in all cases, we have: \( t_{11} = t_{22} = t_{33} = 0 \). When the countries stand alone in the first stage, the tariff restrictions are given by the MFN rules, given by:

\[
t_{21} = t_{31} \equiv t_{1}^{sa}, \quad t_{12} = t_{32} \equiv t_{2}^{sa}, \quad t_{13} = t_{23} \equiv t_{3}^{sa}, \quad t_{ii} = 0.
\] (10)

Thus, each country chooses a single tariff. These tariffs are given by: \( t_{1}^{sa}, t_{2}^{sa}, t_{3}^{sa} \).

If, instead, countries 1 and 2 form a FTA in the first stage, then \( t_{12} = t_{21} = 0 \); each member grants their partner discriminatory duty-free access to the domestic market. Moreover, the MFN rule requires that \( t_{13} = t_{23} \).

\(^{20}\)Note that \( \frac{\partial^2 w_i(t; a)}{\partial a_i \partial a_j} = 0 \). But, for the “cross terms” in equation (8), we have: \( \frac{\partial w_i(t; a)}{\partial \mu_j} \frac{\partial w_i(t; a)}{\partial \mu_k} \neq 0 \). Hence the whole covariance matrix, \( \Sigma \) appears in \( Var[w_i(t; a)] \).
Thus, under a FTA, we have:

\[ t_{12} = t_{21} = 0, \quad t_{31} \equiv t_{1}^{hta}, \quad t_{32} = t_{2}^{hta}, \quad t_{13} = t_{23}^{hta}, \quad t_{ii} = 0. \]  (11)

Again, each country chooses a single tariff. These tariffs are given by: \( t_{1}^{hta}, t_{2}^{hta}, t_{3}^{hta} \).

Implicit in our definition of a FTA is the assumption that the rules of origin required to support the different external tariff rates levied by countries 1 and 2 on the excluded Country 3 are completely effectively enforced and that, consequently, there is no trade deflection between the FTA members.

We can satisfy the restrictions in (10) and (11) by substituting them directly into each country’s objective function. Define the resulting utility functions for \( sa \) and \( fta \) as:

\[ u_{i}(M_{i}(t^{y}; \mu, \Sigma, R_{i})) = u_{i}\left( E(w_{i}(t^{y}; a)) - \frac{1}{2} R_{i} \text{var}[w_{i}(t^{y}; a)] \right) : \; y = sa, fta, \; i = 1..3 \]  \hspace{1cm} (12)

where \( t^{y} = (t_{1}^{y}, t_{2}^{y}, t_{3}^{y}) \).

First, note that since the utility functions \( u_{i}(M_{i}(t^{y}; \mu, \Sigma, R_{i})) \), \( i = 1..3 \), are strictly monotonically increasing in the NVW, \( M_{i} \), the maximization of \( u_{i}(M_{i}(t^{y}; \mu, \Sigma, R_{i})) \) is equivalent to the maximization of \( M_{i}^{y}(t^{y}; \mu, \Sigma, R_{i}) \), in the sense that they yield the same solutions for the optimal tariffs. Thus, the three countries’ maximization problems can, therefore, be written as:

\[ \max_{t^{y}_{i}}\{ M_{i}(t^{y}; \mu, \Sigma, R_{i}) \}, \; i = 1..3, \; y = sa, fta \} .\]

It is quite straightforward to verify that each country’s NVW function, \( M_{i}(t^{y}; \mu, \Sigma, R_{i}) \), is strictly concave in its own tariff. In addition, it is additively separable in all tariffs, if and only if all covariances are zero. This, in turn, implies that tariffs are strategically neutral if and only if all covariances are zero. Therefore, in general, even with segmented markets, tariffs are not strategically neutral (unlike the standard strategic neutrality in this literature). Consequently, we cannot solve for each \( t^{y}_{i} \) separately. Furthermore, the NVW functions are (i) functions of the first and second moments and (ii) quadratic in the first moments but linear in the second moments.

Let the Nash equilibrium tariff in Country \( i = 1..3 \), for coalition structures \( y = sa, fta \), be denoted as \( t^{y}_{i} \) and let the vector of measures of absolute risk aversion be denoted as: \( R \equiv (R_{1}, R_{2}, R_{3}) \). Then, we have:

\[ t_{i}^{ssa} = t_{i}^{ssa}(\mu, \Sigma, R), \; i = 1..3 \]  \hspace{1cm} (13)

\[ t_{i}^{sfta} = t_{i}^{sfta}(\mu, \Sigma, R), \; i = 1, 2, \]

\[ t_{3}^{sfta} = t_{3}^{sfta}(\mu, \Sigma, R) \]
where Nash equilibrium tariffs are not strategically neutral if covariances are non-zero.\footnote{The Nash equilibrium tariff functions in the presence of country risk aversion are very complicated. However, to give a general flavour of their appearance, we provide the Nash equilibrium tariff solutions in Appendix A.1 for the case (analysed in detail when we present our results later) in which \( R_1 = 1, R_2 = R_3 = 0 \) and all covariances are non-zero.}

Now, define the corresponding Nash equilibrium NVW in each country as \( M^*_i(t^y; \mu, \Sigma, R_i) \). Evaluated at the Nash equilibrium tariffs, this yields:

\[
M^*_i(\mu, \Sigma, R) \equiv M_i[t^y(\mu, \Sigma, R); \mu, \Sigma, R_i, c], \quad i = 1, 3, \quad y = sa, fta,
\]

(14)

where \( t^y(\mu, \Sigma, R) = [t^y_1, t^y_2, t^y_3] \) is the vector of equilibrium tariffs.

### 4.2 Customs Union

Before we can proceed with the choice of tariffs in a CU, we first must specify the CU’s objective function. Unfortunately, this question has not been adequately addressed in the literature. As should be clear, the choice of a CU objective function introduces all the usual difficulties that we inevitably face when we choose a social welfare function (e.g., the Arrow Impossibility Theorem). Since the choice of a social welfare function is normally not the main focus of the PTA literature, the CU’s objective function is often taken simply as a sum (or weighted sum) of members’ welfare, \( \sum_i w_i \).\footnote{One exception is Melatos and Woodland (2007), who investigate Pareto optimal delegation in customs unions. However, even there, analysis of the choice of CU social welfare function does not go beyond consideration of the weighted sum of member welfare; the focus is merely on identifying the optimal weights to choose.}

In this paper, with uncertainty and risk-averse countries, choosing a social welfare function becomes much more important and difficult. For example, even with a “simple”, say linear, social welfare function, it is not clear whether we should consider the utility of the sum of the members’ welfares or the sum of utilities of members’ welfares (and, of course, there is no reason to restrict ourselves to simple summations). Similarly, if we apply the linear approximation as in equation (7) above, do we take the sum of members’ net values of welfare or the net value of the sum of their welfares?

In what follows, we examine two distinct CU types: “deep” CU integration and “shallow” CU integration. In a SCU, putative member countries choose to maximize the sum of their (individual) NVWs. In a DCU, member countries choose to maximize the net value of the sum of their welfares (the mean of the sum of their welfares minus a joint risk premium). Under this interpretation, as we demonstrate below, a DCU is a deeper form of PTA in two ways. First, when DCU members (jointly) calculate their “joint risk premium”, they take into account the correlation between their (individual) welfare functions \( (w_i) \).

In contrast, in a SCU, member countries do not consider the correlation between their \( w_i \) functions, so their overall risk premium is simply the sum of the individual risk premia. This is one sense in which integration can be interpreted as deeper: it allows for greater internalization of welfare-correlation externalities. The second sense in which a DCU is a deeper form of trade integration is that members must agree on an optimal joint measure of
absolute risk aversion. In general, deeper integration involves a greater blending of all other individual country characteristics, not just attitudes toward risk. But, since we focus on the risk and insurance aspects of PTAs, we restrict ourselves to the blending of member risk attitudes.

As should be clear, the ex-ante choice of an ex-post joint welfare function is a complex problem. Specifically, the parties need to choose the welfare function of an entity - a CU - that is “yet to be born” (and may never be born); and they must make this choice before the entity comes into being and based on that same social welfare function that does not yet exist. An in-depth analysis of this more general problem is beyond the scope of this paper, so we do not pursue it further here. An example of such a problem is discussed in a general context in Appelbaum (2021).

4.3 Shallow CU Integration (SCU)

If countries 1 and 2 form a SCU in the first stage, they levy a common external tariff on Country 3 so, in addition to the restrictions in (11), we also have \( t_{31} = t_{32} \). Thus with a SCU we have:

\[
\begin{align*}
  t_{12} &= t_{21} = 0, \\
  t_{13} &= t_{23} = t_{3}^{scu}, \\
  t_{31} &= t_{32} = t_{3}^{scu}, \\
  t_{ii} &= 0.
\end{align*}
\]  

(15)

Note that now we have seven restrictions, which means that we only have two tariffs to solve: \( t_{3}^{scu} \) and \( t_{3}^{scu} \). In this case, the two SCU members choose only one tariff. Therefore, as pointed out in Appelbaum and Melatos (2016), relative to both fta and sa, SCU members have fewer (trade policy) degrees of freedom to respond to changes in the trading environment.

Substituting the restrictions listed in equation (15) directly into each \( w_i(t; a) \) yields the net value of welfare for Country \( i \) as:

\[
M_i(t^{scu}; \mu, \Sigma, R_i) = E(w_i(t^{scu}; a)) - \frac{1}{2} R_i \text{var}[w_i(t^{scu}; a)], \quad i = 1..3
\]

where \( t^{scu} = (t_{scu}^{scu}, t_{3}^{scu}) \).

Now, remember that in the sa and fta cases above, countries maximized their net values of welfare \( (M_i(t^{y}; \mu, \Sigma, R_i)) \), rather than the utilities of the net values of welfare \( (u_i\{M_i(t^{y}; \mu, \Sigma, R_i)\}) \). To simplify matters and to be consistent with the sa and fta cases, we do the same here. Namely, we assume that countries 1 and 2 jointly choose their common external tariff by maximizing the sum of their net values of welfare\(^{23}\):

\[
M_{12}^{scu} (t^{cu}; \mu, \Sigma, R_1, R_2) \equiv M_1(t^{scu}; \mu, \Sigma, R_1) + M_2(t^{scu}; \mu, \Sigma, R_2).
\]

(17)

\(^{23}\)In principle, in a more general case, the two countries would maximize some “social welfare” function of \( u_1\{M_i(t^{y}; \mu, \Sigma, R_i)\} \) and \( u_2\{M_2(t^{y}; \mu, \Sigma, R_i)\} \). We do not pursue this case here, but we will address a welfare function’s choice when we consider the "depth" of a CU.
Note that from equation (17) it follows that the “joint” risk premium in the SCU case, denoted as $\theta^{scu}$, is given by:

$$
\theta^{scu} = \frac{1}{2} R_1 \sum_{j=1}^{3} \sum_{k=1}^{3} \frac{\partial w_1(t; \mu)}{\partial \mu_j} \frac{\partial w_1(t; \mu)}{\partial \mu_k} \sigma_{jk} + \frac{1}{2} R_2 \sum_{j=1}^{3} \sum_{k=1}^{3} \frac{\partial w_2(t; \mu)}{\partial \mu_j} \frac{\partial w_2(t; \mu)}{\partial \mu_k} \sigma_{jk}
$$

(18)

This joint premium is a weighted average of the terms $\sum_{j=1}^{3} \sum_{k=1}^{3} \frac{\partial w_1(t; \mu)}{\partial \mu_j} \frac{\partial w_1(t; \mu)}{\partial \mu_k} \sigma_{jk}$ and $\sum_{j=1}^{3} \sum_{k=1}^{3} \frac{\partial w_2(t; \mu)}{\partial \mu_j} \frac{\partial w_2(t; \mu)}{\partial \mu_k} \sigma_{jk}$, with the weights taken as $R_1$ and $R_2$. 

Country 3 maximizes its objective function as before. Hence, the problems of the SCU and Country 3 are, respectively, given by:

$$
\max_{t^{cu}} \{ M_1^{scu}(t^{cu}; \mu, \Sigma, R_1, R_2) \}, \quad \text{and} \quad \max_{t^{cu}} \{ M_3(t^{cu}; \mu, \Sigma, R_3) \}.
$$

Once again, the objective functions of the SCU and Country 3 are strictly concave in their own tariff, and tariffs are strategically neutral if and only if all covariances are zero. Moreover, as before, the NVW functions only depend on the first and second moments, respectively, quadratically and linearly.

Let the Nash equilibrium tariffs in the SCU case be denoted as $t^{scu} = [t^{scu}_1(\mu, \Sigma, R), t^{scu}_3(\mu, \Sigma, R)].$ The corresponding Nash equilibrium NVW functions (for the CU and Country 3) can be written as:

$$
M_1^{scu}(\mu, \Sigma, R) = M_1^{scu}[t^{scu}(\mu, \Sigma, R); \mu, \Sigma, R]
$$

$$
M_3^{scu}(\mu, \Sigma, R) = M_3^{scu}[t^{scu}(\mu, \Sigma, R); \mu, \Sigma, R]
$$

(19)

4.4 Deep CU Integration (DCU)

If countries 1 and 2 form a DCU in the first stage, their objective function is different, but their tariff constraints are the same as in the SCU case. Thus, first, we can define:

$$
t_{12} = t_{21} = 0, \quad t_{13} = t_{23} = t^{dcu}_3, \quad t_{31} = t_{32} = t^{dcu}_2, \quad t_{ii} = 0
$$

(20)

and

$$
w_{12}(t^{dcu}; a) = w_1(t^{dcu}; a) + w_2(t^{dcu}; a).
$$

Second, we define the “average” measure of absolute risk aversion as:

$$
R_1^\gamma = \gamma R_1 + (1 - \gamma) R_2, \quad 0 \leq \gamma \leq 1
$$

where $\gamma$ is the optimal “blending” parameter, chosen in stage 1. We refer to this case as the $dcu^\gamma$ case. First, we examine the choice of TAs, for a given value of $\gamma$. 

---

24 Appendix A.1 provides the Nash equilibrium tariff solutions in the SCU case where $R_1 = 1$, $R_2 = R_3 = 0$ and all covariances are non-zero.
We take the expected utility of joint welfare as:

\[ E\{u_{12}[w_{12}(t^{dcu}; a)]\} \approx u_{12}\{E[w_{12}(t^{dcu}; a)] - \frac{1}{2} R_{12}^\gamma Var([w_{12}(t^{dcu}; a)])\} = u_{12}\{M_{12}^{dcu}(t^{dcu}; \mu, R_1, R_2, \gamma)\} \]

where \( u_{12} \) is some monotonically increasing (in \( w_{12} \)) social welfare function and,

\[ M_{12}^{dcu}(t^{dcu}; \mu, R_1, R_2, \gamma) \equiv E[w_{12}(t^{dcu}; a)] - \frac{1}{2} R_{12}^\gamma Var([w_{12}(t^{dcu}; a)]) \] (21)

is the net value of joint welfare. As was the case above, we approximate the variance of \( w_{12}(t; a) \) as:

\[ Var([w_{12}(t; a)]) \approx \sum_{j=1}^{3} \sum_{k=1}^{3} \frac{\partial w_{12}(t; \mu)}{\partial \mu_j} \frac{\partial w_{12}(t; \mu)}{\partial \mu_k} \sigma_{jk}. \]

Since \( u_{12}\{\cdot\} \) is strictly, monotonically, increasing in the net value of joint welfare, the maximization of \( u_{12}\{E[w_{12}(t^{dcu}; a)] - \frac{1}{2} R_{12}^\gamma Var([w_{12}(t^{dcu}; a)])\} \) is equivalent to the maximization of \( M_{12}^{dcu}(t^{dcu}; \mu, R_1, R_2, \gamma) \), in the sense that they yield the same solutions for the optimal tariffs. The objective function in the \( dcu^\gamma \) case is, therefore, given in equation (21).

Consider the risk premium in the \( dcu^\gamma \) case, defined as \( \theta^{dcu^\gamma} \). To simplify the expression for the risk premium, and to be able to better appreciate the effects of deeper integration, it is useful to re-write the risk premium as follows. First, define \( \beta^h \) and \( \lambda^{hn} \) as:

\[ \beta^h_j = \frac{\partial w_h(t; \mu)}{\partial \mu_j}, \quad h = 1, 2, \quad j = 1, 2, 3 \]

\[ \lambda^{hn} = \sum_{j=1}^{3} \sum_{k=1}^{3} \beta^h_j \beta^k_n \sigma_{jk} \]

The risk premium in the \( dcu^\gamma \) case can, then, be written as:

\[ \theta^{dcu^\gamma} = \frac{R_{12}^\gamma}{2} \{ \lambda^{11} + \lambda^{22} + 2\lambda^{12} \} \] (22)

Using the same notation, we can re-write the risk premium in the SCU case as:

\[ \theta^{scu} = \frac{1}{2} R_1 \lambda^{11} + \frac{1}{2} R_2 \lambda^{22} \]

As we can see, in addition to the measures of risk aversion being different, \( \theta^{dcu^\gamma} \) contains an extra term involving \( \lambda^{12} \). This term depends not only on the correlation of demand parameters (the \( a_i \)'s) but also on the correlation between the two members' welfare functions, \( w_1(t; a) \) and \( w_2(t; a) \). To see this, remember that, since \( w_1(t; a) \) and \( w_2(t; a) \) are quadratic functions of the random variables, they can be correlated even if the underlying demand parameters are uncorrelated; that is, \( \lambda^{12} \) does not vanish even if \( \sigma_{ij} = 0, \; i \neq j \) because it still includes the
variances, $\sigma_{jj}$, which are not zero (as long as there is uncertainty). This can be viewed as an additional source of correlation.

The difference between the two risk premia can, therefore, be written as:

$$\theta^{dcu} - \theta^{scu} = \frac{1}{2} [R_1^\gamma - R_1] \lambda^{11} + \frac{1}{2} [R_2^\gamma - R_2] \lambda^{22} + R_{12}^\gamma \lambda^{12}$$

This shows that (among other things) deeper integration depends on (i) the correlation between the countries’ welfare functions and (ii) the deviations of individual countries’ measures of risk aversion from their blended measure. These two elements are sources of insurance and diversification benefits in the DCU case.

The tariff choice problems of the DCU and Country 3 are, therefore, respectively, given by:

$$\max_{t^{dcu}} M_{dcu}(t^{dcu}; \mu, \Sigma, R_1, R_2, \gamma), \quad \text{and} \quad \max_{t_3^{dcu}} M_3(t_3^{dcu}; \mu, \Sigma, R_3).$$

The objective functions of the DCU and excluded Country 3 have the same characteristics as previously noted.

Let the Nash equilibrium tariffs in the $dcu^\gamma$ case be denoted as $t^{*dcu^\gamma} = (t_1^{dcu}, t_3^{dcu}).$ These can be written as:

$$t_1^{*dcu^\gamma} = t_1^{*dcu^\gamma}(\mu, \Sigma, R, \gamma),$$

$$t_3^{*dcu^\gamma} = t_3^{*dcu^\gamma}(\mu, \Sigma, R, \gamma)$$

The corresponding Nash equilibrium net value of welfare functions in the $dcu^\gamma$ case (in the DCU and Country 3) can be written as:

$$M_1^{*dcu^\gamma}(\mu, \Sigma, R, \gamma) \equiv M_{dcu}^\gamma[t^{*dcu^\gamma}(\mu, \Sigma, R, \gamma); \mu, \Sigma, R, \gamma]$$

$$M_3^{*dcu^\gamma}(\mu, \Sigma, R, \gamma) \equiv M_3[t^{*dcu^\gamma}(\mu, \Sigma, R, \gamma); \mu, \Sigma, R, \gamma]$$

4.5 Stage 1: The Choice of PTA

In stage 1, countries 1 and 2 choose a PTA and the “optimal depth” ($\gamma$) simultaneously. To describe the solution, first, we extend the definition of total country 1 and 2 Nash equilibrium net values of welfare to the $sa$ and $fta$ cases. Second, we redefine the set of all possible PTAs by $Y = \{sa, fta, scu, dcu\}$.

The total Nash equilibrium net values of the welfare of countries 1 and 2, for any PTA, $y \in Y \equiv \{sa, fta, scu, dcu\}$ can be written as:

---

25 The Nash equilibrium tariff solutions in the DCU case are given in Appendix A.1
26 First note that, in principle, for all TAs, we should add side-payments, where $K_{12}^{y} + K_{2}^{y} = 0$. But, in the CU cases, these would cancel each other. Note also that since, by construction, the $K_{12}^{y}$ are fixed, the transfer payments themselves cannot be an insurance tool. Alternatively, it is possible to consider a scenario where the side payments are contingent on the state of the word. For example, we may take the linear transfer scheme $K_{12}^{y} = a_1 y_1 + a_2 y_2$, where $a_1 + a_2 = 0$. In the “usual risk-sharing” framework with symmetric (verifiable) information, it is the case that the risk-neutral party provides full insurance to the risk-averse one. Here, this is not always the case.
\[ M_{12}^{y}(\mu, \Sigma, R), \quad \text{if } y = sa, fta, scu \]
\[ M_{12}^{dcu}(\mu, \Sigma, R, \gamma), \quad \text{if } y = dcu^{\gamma} \]

Thus, the optimal PTA is obtained from the solution of the following problem:27

\[ \max \{ \max_y [M_{12}^{y}(\mu, \Sigma, R) : y = sa, fta, scu], \; \max_{\gamma} [M_{12}^{dcu}(\mu, \Sigma, R, \gamma)] \} \] (25)

We can view stage 1 as the choice among the following PTAs: SA, FTA, SCU and a continuum of types of dcu's, where a different value of \( \gamma \) characterizes each possible type of DCU.

Consider the choice of \( \gamma \). First, as shown in the previous section, one aspect of deeper integration is that the correlation between the countries’ welfare functions is taken into account. Problem (25) addresses the other aspect of deeper integration: the optimal blend of the measures of absolute risk aversion.

The optimal value of the “blending parameter”, \( \gamma \), is obtained by solving the problem:

\[ \max \{ M_{12}^{dcu}(t^{dcu}, \mu, \Sigma, R, \gamma); \mu, \Sigma, R, \gamma \}, \; 0 \leq \gamma \leq 1 \} \] (26)

where \( M_{12}^{dcu}(\mu, \Sigma, R, \gamma) \) is the Nash equilibrium net value of welfare in the dcu case, as given by equation (24).

Is there an interior solution for \( \gamma \)? As equations (24) (21) and (26) show, although \( M_{12}^{dcu}(t^{dcu}, \mu, \Sigma, R, R_1, R_2; \gamma) \equiv E[w_{12}(t^{dcu}; a)] - \frac{1}{2} R_{12}^2 \text{Var}([w_{12}(t^{dcu}; a)]) \) is linear in \( \gamma \), the Nash equilibrium net value, \( M_{12}^{dcu}(t^{dcu}, \mu, \Sigma, R, \gamma); \mu, \Sigma, R, \gamma \), is not. The reason for this is clear: the Nash equilibrium tariffs, \( t^{dcu}_{12}(\mu, \Sigma, R, \gamma) \) and \( t^{dcu}_{3}(\mu, \Sigma, R, \gamma) \) are functions of \( \gamma \). Nevertheless, although it appears that, in general, an interior solution for \( \gamma \) is possible, this is not the case. Since risk neutrality always confers an advantage in the face of risk, it is jointly optimal for the DCU members to choose a corner solution for \( \gamma \). Namely, DCU members will always choose the lower of the two measures of absolute risk aversion to represent the union. Thus, we have the following proposition:28

**Proposition 1** Let the solution to problem (26) be denoted as: \( \gamma^* = \gamma^*(\mu, \Sigma, R) \). Then \( \gamma^* \) satisfies:

\[ R_{12}^{\gamma^*} = \gamma^* R_1 + (1 - \gamma^*) R_2 = \min \{ R_1, R_2 \}, \quad \text{for all } (\mu, \Sigma, R). \]

Proposition 1 says that, for all \( (\mu, \Sigma, R) \), the DCU’s optimal measure of absolute risk aversion is always \( \min \{ R_1, R_2 \} \). Now, given \( \gamma^* \), we denote the corresponding maximum of the net value of welfare as \( M_{12}^{dcu^*}(\mu, \Sigma, R) \).

We refer to this as the dcu* case (the optimally blended DCU).

27A proof of this statement is available on request. It is useful to note that while the agreement’s choice is always unique, the transfers are not uniquely determined. Since our objective is to identify the optimal PTA, this is not a major problem here.

28A similar result in the context of marriage partnerships is shown in Appelbaum (2021).
The stage 1 problem can, therefore, now be re-written as:

\[
\max\{ \max_y [M_{y1}^y(\mu, \Sigma, R) : y = sa, fta, scu, decu^*], \ M_{12}^{decu^*}(\mu, \Sigma, R) \}
\]  

(27)

In other words, countries 1 and 2 compare the total Nash equilibrium net values of welfare of the four possible coalition structures, \{sa, fta, scu, decu\}, given by \(M_{12}^{sa} \mu, \Sigma, R\), \(M_{12}^{fta} \mu, \Sigma, R\), \(M_{12}^{scu} \mu, \Sigma, R\) and \(M_{12}^{decu}\), and choose the one with the highest corresponding value.

5 Preferred PTAs

In this section, we investigate the effects of demand uncertainty on the choice of PTA. Specifically, we examine the role of risk aversion and the consequent insurance and diversification considerations in PTA formation. To clarify the issues, we first examine the benchmark case in which all countries are risk-neutral; that is when insurance and diversification do not play a roll in the choice of PTA.

Before proceeding, it is important to note that the results discussed henceforth are presented in terms of the characteristics of the putative member countries 1 and 2. That is, the results presented below hold for fixed demand characteristics of Country 3; in particular, the value of Country 3’s mean demand (\(\mu_3\)), Country 3’s risk attitude (\(R_3\)) and \(\sigma_3\), the level of demand uncertainty (if any) faced by Country 3. As already argued, when country demands are uncorrelated, Country 3’s characteristics do not influence the PTA formation decisions of countries 1 and 2. This is for two reasons. First, because of our segmented markets assumption. Second, because Country 3, by assumption, is a passive player that does not make PTA formation decisions. Remember, however, that when country demands are correlated, the excluded Country 3’s characteristics can influence the PTA formation decision of countries 1 and 2.

5.1 Benchmark Case: All PTA Members Are risk-neutral

If both putative PTA member countries (1 and 2) are risk-neutral, \(R_1 = R_2 = 0\) and, consequently, member risk premia are also zero.\(^{29}\) To facilitate the comparison of the risk neutrality and risk aversion cases, we need to assign values to the variables \(a_i\) in the risk-neutral case. We follow the standard practice and assume that the certain values are taken as the means of the random variables (i.e., by the \(\mu\) vector).

Since we derive closed-form solutions for member country preferences over different PTAs (and standing alone) in a certain world, by fixing \(\mu_3\) we can illustrate these preferences in a simple diagram in \((\mu_1, \mu_2)\) space. Figure 1 represents the benchmark case in which both countries 1 and 2 are risk-neutral; equivalently, the case in which they face no demand uncertainty.\(^{30}\)

\(^{29}\) Equally, if there is no uncertainty, all variances are zero, and consequently, all risk premia are again zero.

\(^{30}\) All the figures presented in this section are based on the (complicated) closed-form solution to the model derived earlier.
Figure 1: PTA preferences for countries 1 and 2 when both are risk neutral, all countries face an identical level of uncertainty $\sigma_1 = \sigma_2 = \sigma_3 = 0.5$, and all country demands are uncorrelated.

Note that, in this situation, joint member welfare from a SCU and a DCU is identical because the mean of the sum of individual member welfares is exactly equal to the sum of the means of individual member welfares. In Figure 1, it is also assumed that all countries (putative PTA members in particular) face an identical level of uncertainty, $\sigma_1 = \sigma_2 = \sigma_3 = 0.5$, and all country demands are uncorrelated.

The “V” in Figure 1 traces out the locus of country 1 and 2 mean demands for which they are jointly indifferent between forming a (shallow or deep) CU and a FTA. Within the arms of this “V” locus, $cu \succ fta$; that is, countries 1 and 2 jointly prefer a (shallow or deep) CU to a FTA when they have sufficiently similar mean demands. Outside the arms of each “V”, towards the top-left and bottom-right of Figure 1, $fta \succ cu$; countries 1 and 2 jointly prefer a FTA to a (shallow or deep) CU when their mean demands are sufficiently different. Standing alone ($sa$) is always the least preferred option for countries 1 and 2 when insurance and diversification considerations are absent either because there is uncertainty, but members are risk-neutral or because there is no uncertainty. Since all country demands are uncorrelated, this pattern of member (joint) PTA preference holds regardless of the characteristics of the excluded Country 3.
The intuition for the pattern of member PTA preferences under risk neutrality (or, equivalently, no uncertainty) was first explained in Appelbaum and Melatos (2016). When member countries are sufficiently similar, the benefits of policy coordination in a CU - that is, the additional rent extracted from the excluded country by members jointly choosing the optimal common external tariff - outweigh the costs of members committing to set a common external tariff rate that differs from their unilaterally preferred rate. However, when members are sufficiently different, the loss-of-autonomy costs implied by policy coordination in a CU will tend to outweigh any monopoly power benefits accruing from it.

5.2 Introducing Risk Aversion when Country Demands are Uncorrelated

In the previous section, we assumed that all countries were risk-neutral and, hence, uncertainty was not a consideration in TA formation decisions. To understand the role of uncertainty - and the desire for ID - in PTA formation, we now assume that countries 1 and 2 - the putative PTA members - are risk-averse. Note that, for country $i$'s risk premium to be strictly positive, we require both that $\sigma_i^2 > 0$ and $R_i > 0$.

Figure 2 shows what kind of trade agreement (if any) is likely to form when countries 1 and 2 are identically risk-averse ($R_1 = R_2 = 1$) but can face different levels of demand uncertainty ($\sigma_1 \neq \sigma_2$). Since throughout Figure

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Figure 2: PTA outcomes for countries 1 and 2 as the levels of uncertainty they face, $(\sigma_1, \sigma_2)$, vary. Countries 1 and 2 are characterised by identical levels of risk aversion ($R_1 = R_2 = 1$) and mean demand ($\mu_1 = \mu_2 = 1$). The excluded country characteristics do not influence the outcome but are assumed to be $R_3 = 0$, $\mu_3 = 1$ and $\sigma_3 = 0.5$.

2 countries 1 and 2 are also characterized by identical levels of mean demand ($\mu_1 = \mu_2 = 1$), the only potential source of difference between them is the level of uncertainty each faces. As such, cells along the main diagonal
of Figure 2 where $\sigma_1 = \sigma_2$ represent situations in which countries 1 and 2 are identical. The shading in Figure 2 denotes the type of PTA that countries 1 and 2 will prefer to form; respectively, yellow, green and red for FTA, SCU and standing alone. The number within each cell identifies the joint preference ranking across all possible PTAs considered here. Hence, for example, while a SCU is jointly most preferred by countries 1 and 2 in each green cell in Figure 2, the cells labelled “8” and “11” have FTA ranked second and DCU third in the former, while DCU is ranked second and FTA third in the latter.\(^{31}\) Finally, note that in Figure 2 country demands are uncorrelated ($\rho_{ij} = 0$) and so the characteristics of the excluded Country 3 do not influence the outcome; in any case, they are set to be $R_3 = 0$, $\mu_3 = 1$ and $\sigma_3 = 0.5$.

Figure 2 emphasizes that the absolute level of uncertainty faced by potential PTA members matters when they jointly decide on the type of PTA (if any) that they wish to form. Countries will prefer to stand alone if either one or both face excessive levels of demand uncertainty; that is, in the red-shaded region in which either $\sigma_1$ or $\sigma_2$ or both are sufficiently large. On the other hand, if countries 1 and 2 face sufficiently similar and low levels of uncertainty - the green-shaded region - they will most likely form a SCU. Finally, countries 1 and 2 choose to form a FTA if, despite both facing relatively low levels of risk, these levels are sufficiently different - the yellow-shaded regions. Note that in Figure 2 countries 1 and 2 never choose to form a DCU regardless of the levels of uncertainty either faces.

The insights provided by Figure 2 can be summarized in Proposition 2:

**Proposition 2** Consider a three-country world in which all country demands are uncorrelated and one country eschews PTA membership. If the remaining two countries are both identically sized and characterized by an identical degree of risk aversion, they will

(i) **Stand alone if either faces a sufficiently high level of uncertainty.**

(ii) **Form a SCU (FTA) if the levels of uncertainty they face are sufficiently low and similar (different).**

The intuition behind Figure 2 and Proposition 2 can be explained as follows. First, since in Figure 2 the putative PTA members are identical and, in particular, identically risk-averse, then the insurance benefits associated with either a DCU or SCU, but especially the former, are muted since neither member has a risk-aversion comparative advantage. As such, a DCU is never observed in Figure 2 even when one putative PTA member faces significant levels of uncertainty. At the same time, the high risk facing both countries implies that the option value of SA is high.\(^{32}\)

\(^{31}\)Note that there are 24 possible rankings of the four TA types considered in this paper. These are listed in Appendix A.2. In each figure that follows, the associated key only lists those rankings that are observed in that figure.

\(^{32}\)As is shown in Appelbaum and Melatos (2016).
In the green-shaded region of Figure 2 in which countries 1 and 2 are not just identical but also face sufficiently similar and low levels of uncertainty, they prefer a SCU to either a FTA or a DCU. The similarity of the PTA members ensures that their joint benefit from policy coordination outweighs the costs to them of committing to an external tariff rate other than their unilaterally preferred rate. Moreover, the low and similar levels of uncertainty they face and their identical risk aversion act to curtail any insurance benefits from deeper integration via a DCU (or, for that matter, a SCU). In addition, although diversification benefits exist even with no correlation, often, the absence of correlation also curtail the diversification benefits of deep integration. At the same time, the low risk also reduces their option value benefits.

On the other hand, in the yellow-shaded regions in Figure 2 when countries 1 and 2 face sufficiently different - but still low - levels of uncertainty, they choose to form a FTA in preference to a SCU since this cost-benefit calculus of policy coordination is reversed. In addition, the fact that uncertainty is relatively low helps to diminish the attraction of a DCU (or a SCU) as an insurance mechanism.

In the red-shaded region of Figure 2 countries 1 and 2 choose to stand alone. Appelbaum and Melatos (2016) showed that when countries face elevated levels of demand uncertainty, they want to have maximum flexibility to unilaterally choose their trade policy vis-a-vis the rest of the world. Similarly, in our framework, the joint welfare countries 1 and 2 derive from standing alone compared to being members of a CU or a FTA, increases with the level of uncertainty either one or both of them faces. Remember that the results in Figure 2 are based on the assumption that countries 1 and 2 have identical levels of risk aversion. This assumption - which is relaxed later in the paper - greatly reduces the efficacy of CUs as instruments of insurance.

5.3 Degrees of Risk Aversion and PTA Formation with Uncorrelated Demands

We demonstrated that in the presence of identically risk-averse members, insurance considerations acquire importance that they did not have when all members were risk-neutral. We now examine how differences in degrees of risk aversion between putative PTA members can affect PTA formation.

Figures 3 and 4 show what happens to trade agreement outcomes when the degree of risk aversion can differ between PTA members. For the moment, we continue to assume that all country demands are uncorrelated. We relax this assumption later in the paper. As before, the shading in the figures denotes the type of PTA that countries 1 and 2 will prefer to form; respectively, yellow, green, blue and red for FTA, SCU, DCU and standing alone. The number within each cell identifies the joint preference ranking across all possible PTAs considered.

Figures 3 and 4 show trade agreement outcomes for countries 1 and 2 as Country 1’s degree of risk aversion.

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33Appelbaum and Melatos (2016) also show that countries that face increased uncertainty in costs seek greater trade policy coordination. As such, if countries 1 and 2 faced greater production cost uncertainty, as opposed to demand uncertainty, a SCU or even a DCU could be observed in Figure 2.
Figure 3: PTA outcomes for countries 1 and 2 as Country 1’s risk aversion ($R_1$) and mean demand ($\mu_2$) vary relative to those of Country 2. In each case, $R_2 = 1$, $R_3 = 0$, $\mu_2 = \mu_3 = 1$, all countries (PTA members in particular) face an identical level of uncertainty, $\sigma_1 = \sigma_2 = \sigma_3 = 0.5$, and all country demands are uncorrelated. The bolded cell at $(R_1, \mu_1) = (1, 1)$ represents the case in which countries 1 and 2 are identically symmetric.

($R_1$) varies relative to that of Country 2. In Figure 3, Country 1’s mean demand ($\mu_1$) also varies relative to Country 2 while they face an identical level of demand uncertainty ($\sigma_1 = \sigma_2 = 0.5$). In Figure 4 the level of demand uncertainty faced by country 1 ($\sigma_1$) varies relative to Country 2 while they are endowed with identical mean demands ($\mu_1 = \mu_2$).

Throughout Figure 3, $R_2 = 1$, $R_3 = 0$, $\mu_2 = \mu_3 = 1$ and $\sigma_3 = 0.5$. As such, the bolded cell at $(R_1, \mu_1) = (1, 1)$ represents the case in which the putative PTA members are identical. Note that since Country 2’s risk aversion and demand parameters have been normalized to unity, the horizontal and vertical axes in Figure 3 can be interpreted, respectively, as the ratios $\frac{R_1}{R_2}$ and $\frac{\mu_1}{\mu_2}$. Hence, moving from left to right in Figure 3 results in Country 1 becoming more risk-averse relative to Country 2. Moving vertically upwards in Figure 3 is akin to making Country 1 larger relative to Country 2.

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34 Throughout this paper, the values of country absolute risk aversion parameters are limited to the range (1,3). Estimates of country absolute risk aversion in the literature tend to vary widely from less than one as in Saha, Shumway and Talpaz (1994) to four (Wolf and Pohlman, 1983) and even greater than 14 (Chavas and Holt, 1996). Meanwhile, Barseghyan, Molinari, O’Donoghue and Teitelbaum (2018), in a wide-ranging survey of the literature, report that most estimates of absolute risk aversion tend to be extremely low, less than 0.1. In this paper, we are mainly interested in understanding how changes in a country’s degree of absolute risk aversion and how differences in the levels between countries influence their PTA decisions.
Figure 4: PTA outcomes for countries 1 and 2 as Country 1’s risk aversion ($R_1$) and the level of risk it faces ($\sigma_1$) vary relative to those of Country 2; the latter are normalised to: ($R_2, \sigma_2$) = (1, 1). Countries 1 and 2 are characterised by identical levels of mean demand ($\mu_1 = \mu_2 = 1$). The excluded country characteristics do not influence the outcome - as country demands are uncorrelated - but are assumed to be $R_3 = 0$, $\mu_3 = 1$ and $\sigma_3 = 0.5$.

In Figure 4, the level of uncertainty faced by Country 2, as well as Country 2’s degree of risk aversion, have both been normalized to unity. Hence, the vertical axis measures the level of uncertainty Country 1 faces relative to that faced by its PTA partner, $\frac{\sigma_1}{\sigma_2}$. The horizontal axis, meanwhile, measures Country 1’s degree of risk aversion relative to that of its partner, $\frac{R_1}{R_2}$. In other words, as we move vertically upwards in Figure 4, Country 1 faces increasing levels of uncertainty relative to Country 2. As we move horizontally to the right, Country 1 becomes increasingly risk-averse compared to Country 2. Once again, the bolded-border cell, at ($R_1, \sigma_1$) = (1, 1) this time, represents the situation in which countries 1 and 2 are identical.

Inspection of Figures 3 and 4 reveals that if one member country is sufficiently risk-averse relative to the other - that is, if $R_1$ is sufficiently high - then countries 1 and 2 choose to form a DCU. Note the blue-shaded region towards the right of Figure 3 and the bottom-right of Figure 4. This result is summarized in Proposition

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7 SCU, FTA, SA, DCU
15 DCU, SA, FTA, SCU
19 SA, FTA, DCU, SCU
23 SA, DCU, FTA, SCU

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Proposition 3  Consider a three-country world in which all country demands are uncorrelated and one country eschews PTA membership. If the remaining two countries face sufficiently similar levels of demand uncertainty, they will form a DCU if either is sufficiently risk-averse relative to the other.

Figures 3 and 4 also confirm that, just as in the benchmark (risk-neutral) case, putative members will prefer to form a CU - in this case, a SCU - when they are sufficiently similar; note, the green-shaded regions situated in the neighborhood of the bolded-border cell in each figure. Also analogously to the benchmark case, countries will choose to form a FTA if they are sufficiently different in size, provided that they are not too different in terms of their degree of absolute risk aversion or face significantly different levels of uncertainty. Note the yellow-shaded region towards the top-left of Figure 3 where Country 2 is relatively large but is not too risk-averse compared to Country 1. Note also that countries 1 and 2 never choose to join a FTA in Figure 4. Finally, consistent with Figure 2, countries 1 and 2 prefer to stand alone when at least one of them faces a sufficiently high level of uncertainty; note the red-shaded area towards the top of Figure 4.

The intuition supporting Proposition 3 is similar to that presented earlier. Countries are more likely to value insurance - and, hence, prefer a DCU - when their degrees of risk aversion are sufficiently different. This is because deeper integration makes efficient risk-sharing more attractive. Effectively, it allows the less risk-averse member to export its relatively low risk aversion to its more risk-averse partner. However, as argued before, if the level of uncertainty faced by a country becomes too great, then their desire for flexibility (that is, to unilaterally choose their trade policy vis-a-vis the rest of the world), and hence its the greater option-value, will outweigh their desire for insurance. In that case, countries will choose to stand alone rather than form a PTA.

When countries 1 and 2 face low uncertainty levels, their desire for insurance and trade policy flexibility is diminished. As such, they will favour intermediate forms of integration - a SCU (or FTA) when they face sufficiently similar (or different) levels of uncertainty - to the more extreme forms of shallow or deep integration, respectively, standing alone or a DCU. As already explained, a SCU is preferred when member similarity ensures that their beggar-thy-(excluded)-neighbor joint benefits from trade policy coordination outweigh the costs associated with members committing to an external tariff rate other than their unilaterally preferred rate. On the other hand, a FTA is preferred when member dissimilarity reverses this cost-benefit calculus of policy coordination.

Finally, note that the introduction of risk aversion yields more nuanced PTA preferences among dissimilar countries than was the case in the benchmark (risk-neutral) case. Remember that in Figure 1, the putative
members preferred to form a CU the greater their (demand or size) similarity, and a FTA, the greater their (demand or size) dissimilarity. However, in the presence of risk aversion, Figure 3 shows that significant size differences between countries may not necessarily lead them to choose to form a FTA; they may, instead, choose to form a DCU. As explained above, this difference in PTA choice rests on the extent to which member degrees of absolute risk aversion differ. If different-sized members are characterized by sufficiently similar degrees of risk aversion, they will form a FTA - just as in the risk-neutral case. On the other hand, if different sized members also display significant differences in absolute risk aversion, they will prefer to form a DCU.

5.4 Introducing Risk Aversion when Member Demands are Correlated

This section investigates the implications for PTA formation when country demands are correlated. Figures 5 and 6 illustrate how the nature of member country demand correlation (its strength, as well as whether it is positive or negative) impacts their PTA formation decisions.

In Figure 5, countries 1 and 2 face identical levels of demand uncertainty ($\sigma_1 = \sigma_2 = 1$) and are endowed with identical degrees of risk aversion ($R_1 = R_2 = 1$). However, members can differ in relative size ($\mu_2 = 1$ while $\mu_1$ can vary). So Figure 5 demonstrates how variations in country size and demand correlation interact to influence PTA formation decisions.

In Figure 6, countries 1 and 2 face identical levels of demand uncertainty and are identically sized ($\mu_1 = \mu_2 = 1$). Now, however, members can differ in their relative attitude to risk ($R_2 = 1$ while $R_1$ can vary). So Figure 6 demonstrates how variations in country risk attitudes and demand correlation interact to influence PTA formation. Note that in both Figures 5 and 6, the putative PTA members, countries 1 and 2, are identical in the left-most column along which, respectively, $\mu_1 = 1$ and $R_1 = 1$.

Figure 5 shows that, provided that members are sufficiently different in size (where $\frac{\mu_1}{\mu_2} \geq 1.2$ in Figure 5), the more strongly and positively correlated are member demands, the more likely they are to form a FTA (the yellow-shaded region). The more strongly and negatively correlated are member demands, the more likely they are to form a DCU (the blue-shaded region). On the other hand, if member demands are only weakly correlated, whether positively or negatively, then members are most likely to opt for a SCU (the green-shaded region).

In Figure 6, given each member’s absolute risk aversion, they will form a DCU if their demands are not too positively correlated. If member risk attitudes are sufficiently similar and their demands are not too negatively correlated, they will prefer to form a SCU. Finally, if member demands are not too negatively correlated and member risk attitudes are sufficiently different, they will choose to stand alone.

Figures 5 and 6 suggest the following proposition:
Figure 5: PTA outcomes for countries 1 and 2 as their mean demand levels, as well as the correlation between their demands ($\rho_{12}$), vary. Note that Country 2’s mean demand is normalised throughout ($\mu_2 = 1$). Countries 1 and 2 face identical levels of uncertainty ($\sigma_1 = \sigma_2 = 1$) and identical degrees of risk aversion ($R_1 = R_2 = 1$). The excluded country characteristics are assumed to be $R_3 = 1$, $\mu_3 = 1$ and $\sigma_3 = 0.5$.

**Proposition 4** Consider a three-country world in which one country eschews PTA membership. Everything else being equal, the remaining two countries are more likely to:

(i) Form a DCU the more negatively (or less positively) correlated their demands.

(ii) Form a FTA if their demands are strongly positively correlated, and they differ sufficiently in size.

(iii) Form a SCU if their demands are not too negatively correlated, and their risk-aversion is sufficiently similar.

(iv) Choose to stand alone if their demands are sufficiently positively correlated and if either is sufficiently risk-averse relative to the other.

The intuition underlying Proposition 4 and Figures 5 and 6 is closely related to that provided for the case of uncorrelated demands. If members differ significantly in their degrees of risk aversion, or if their demands are

| $\rho_{12}$ | 1  | 0.9 | 0.8 | 0.7 | 0.6 | 0.5 | 0.4 | 0.3 | 0.2 | 0.1 | 0  | -0.1 | -0.2 | -0.3 | -0.4 | -0.5 | -0.6 | -0.7 | -0.8 | -0.9 | -1 |
|-------------|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|------|------|------|------|------|------|------|------|-----|
| 1           | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
| 0.9         | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
| 0.8         | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
| 0.7         | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
| 0.6         | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
| 0.5         | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
| 0.4         | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
| 0.3         | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
| 0.2         | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
| 0.1         | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
| 0           | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
| -0.1        | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
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| -0.3        | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
| -0.4        | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
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| -0.7        | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
| -0.8        | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
| -0.9        | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |
| -1          | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 1    |

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Figure 6: PTA outcomes for countries 1 and 2 as the correlation between member demands ($\rho_{12}$), as well as their levels of risk aversion, vary. Note that Country 2’s risk aversion parameter is normalised throughout ($R_2 = 1$). Countries 1 and 2 face identical levels of uncertainty ($\sigma_1 = \sigma_2 = 1$) and identical mean demands ($\mu_1 = \mu_2 = 1$). The excluded country characteristics are assumed to be $R_3 = 1$, $\mu_3 = 1$ and $\sigma_3 = 0.5$.

Negatively correlated, then it is mutually beneficial for them to seek to insure against any demand uncertainty they face by forming a DCU and exploiting the relatively risk-sanguine member’s comparative advantage in low risk-aversion. In addition, in this case, the deeper the integration (i.e., in the DCU case), the greater the diversification benefits.

If member demands are strongly positively correlated and differ (significantly) in size (but not in measures of risk aversion), the benefits of risk-sharing and diversification are low, making a SCU or a DCU less attractive. Moreover, their size difference makes policy coordination via a CU relatively costly. As a result, countries 1 and 2 will tend to prefer a FTA in such situations. As noted previously, FTAs tend to arise in situations where the member countries differ substantially in their size or the levels of uncertainty they face. Consistent with this,
note that a FTA is not observed in Figure 6 since both countries are identically sized (and face identical levels of uncertainty).

The more strongly negatively correlated are member demands, the greater is their demand for ID, and, hence, the motivation to form a CU. This is the case regardless of any differences in their size or levels of uncertainty they face. Note how in Figure 5 PTA preference graduate from a FTA to a SCU and, finally, to a DCU as we move vertically downwards from strong positive to strong negative demand correlation. This is also the case in Figure 5, at least where countries 1 and 2 have sufficiently similar degrees of risk aversion.

When member demands are highly correlated, risk-sharing and diversification are less attractive risk-management tools. Hence, when their demands are sufficiently strongly positively correlated, members prefer to stand alone rather than form a DCU even when one of them is significantly risk-averse relative to the other. Note how in Figure 6 member preference for standing alone becomes more pronounced the more strongly positively correlated their demands, even as the ratio \( \frac{R_1}{R_2} \) becomes large.

6 Conclusion

In this paper, we have investigated the formation of PTAs when countries that are characterized by risk aversion face demand uncertainty. In such an environment, countries have an additional welfare consideration - a desire for insurance - when determining whether or not to join a PTA and the type of PTA they wish to form. This desire to insure exists side-by-side with the need to consider the option value inherent in any PTA; the latter depending on the twin forces of the degrees of freedom in tariff choice and the degree of tariff policy coordination. We find that the insurance motivation for PTA formation influences not just whether a PTA forms, but also the depth of integration. As such, we distinguish, not just between FTAs and CUs, but also between shallow and deep CUs. When country demands are uncorrelated, but their attitudes to risk differ significantly, they will choose to pursue deep integration - a deep CU. However, when their levels of risk aversion are sufficiently similar, they will choose shallower integration - either a shallow CU or a FTA. When country demands are strongly negatively correlated, they will tend to pursue deeper integration while if their demands are weakly or positively correlated, they will tend to pursue shallower integration. We have argued that, intuitively, it is useful to think of differences in country risk attitudes as an additional source of comparative advantage. Deeper integration permits more risk averse members to insure themselves against demand risk by importing relative risk neutrality from their partners. Finally, we also observe countries deciding to stand alone and eschew PTA membership. This tends to occur when at least one country faces sufficiently high absolute levels of risk; the benefits of flexibility in tariff choice tend to outweigh any insurance or policy coordination benefits that might
otherwise accrue.

A Appendix

A.1 Solutions for Tariffs

In all cases below, we provide Nash equilibrium tariff solutions when $R_1 = 1$, $R_2 = R_3 = 0$ and all covariances are non-zero.

1. SA and FTA Cases:

When all countries stand alone:

$$t_{i}^{\text{sa}} = -\frac{1}{105(123+3\sigma_{i}^2)} \{14c_1 (40 - 20\sigma_{12} - 21\sigma_{13} - 20\sigma_{21} - 21\sigma_{31}) - 2c_2 (1960 - 50\sigma_{12} - 63\sigma_{13} - 50\sigma_{21} - 63\sigma_{31}) + 6c_3 (840 + 10\sigma_{12} + 21\sigma_{13} + 10\sigma_{21} + 21\sigma_{31}) - 280\mu_1 + 20\mu_2 (\sigma_{12} + \sigma_{21}) + 7\mu_3 (\sigma_{13} + \sigma_{31}) \} - \frac{105\sigma_{i}^2}{9(c_1+c_2+c_3-11\mu_1)} \};$$

$$t_{2}^{\text{fta}} = \frac{1}{21} (7c_1 - c_2 - 9c_3 + 3\mu_1);$$

$$t_{3}^{\text{fta}} = -\frac{1}{10} (c_1 + c_2 + c_3 - 3\mu_3).$$

2. The SCU Case:

Under SCU(1, 2):

$$t_{i}^{\text{scu}} = -\frac{1}{5(608+6(\sigma_{12}+\sigma_{21})+9\sigma_{i}^2+4\sigma_{i}^2)} \{14c_1 (40 - 20\sigma_{12} - 21\sigma_{13} - 20\sigma_{21} - 21\sigma_{31}) + c_2 (18\sigma_{13} + 10\sigma_{21} + 12\sigma_{23} + 18\sigma_{31} + 12\sigma_{32}) + c_3 (18\sigma_{13} + 10\sigma_{21} + 12\sigma_{23} + 18\sigma_{31} + 12\sigma_{32}) + \sigma_{21} (55\mu_1 + 15\mu_2) + 5\sigma_{12} (2c_2 + 2c_3 + 11\mu_1 + 3\mu_2) - 80[2c_2 - 14c_3 + 5(\mu_1 + \mu_2)] + \mu_3 (6\sigma_{13} + 4\sigma_{23} + 6\sigma_{31} + 4\sigma_{32}) - 15\sigma_{i}^2 (c_2 + c_3 - 11\mu_1) + 20\sigma_{i}^2 (c_2 + c_3 + \mu_2) - c_1 (160 + 90\sigma_{12} + 42\sigma_{13} + 90\sigma_{21} + 28\sigma_{23} + 42\sigma_{31} + 28\sigma_{32} + 135\sigma_{i}^2 + 60\sigma_{i}^2) \}$$

$$t_{3}^{\text{scu}} = -\frac{1}{10} (c_1 + c_2 + c_3 - 3\mu_3).$$

3. The DCU Case:
Under DCU(1, 2):

\[ t_{d1}^* = \frac{1}{1216+50(\sigma_{12}+\sigma_{21})^2} \left\{ 2c_1 \left( 32 + 35\sigma_{12} + 8\sigma_{13} + 35\sigma_{21} + 8\sigma_{23} + 8\sigma_{31} + 8\sigma_{32} + 35\sigma_1^2 + 35\sigma_2^2 \right) + \\
2c_2 \left( 32 + 35\sigma_{12} + 8\sigma_{13} + 35\sigma_{21} + 8\sigma_{23} + 8\sigma_{31} + 8\sigma_{32} + 35\sigma_1^2 + 35\sigma_2^2 \right) - \\
c_3 \left( 448 + 10\sigma_{12} + 24\sigma_{13} + 10\sigma_{21} + 24\sigma_{23} + 24\sigma_{31} + 24\sigma_{32} \right) - 65\mu_1 (\sigma_{12} + \sigma_{21}) - 65\mu_2 (\sigma_{12} + \sigma_{21} + 160 (\mu_1 + \mu_2) \\
-8\mu_3 (\sigma_{13} + \sigma_{23} + \sigma_{31} + \sigma_{32}) - 10\sigma_1^2 (c_3 + 13\mu_1) - 10\sigma_2^2 (c_3 + 13\mu_2) \right\}; \]

\[ t_{d2}^* = -\frac{1}{16} (c_1 + c_2 + c_3 - 3\mu_3). \]

### A.2 Possible TA joint preference orderings for countries 1 and 2

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