The Optimal Antitrust Policies for Vertical Price Restraints in a Non-Green Supply Chain

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Abstract. This paper studies the optimal antitrust policies for vertical price restraints in an infinitely-lived non-green supply chain channel that emits air pollution during production. The channel involves a supplier and a retailer that can either engage in sequential (Stackelberg) price competition where the supplier moves first or engage in vertical price coordination where they choose the retail price to maximize their joint profits and choose the wholesale price using the generalized Nash bargaining. We first consider the absence of an antitrust authority and characterize a necessary and sufficient condition for the stability of coordination, which we call internal stability. Then, we characterize the socially optimal antitrust policies. The policies we consider involve the costly auditing of the channel to detect coordination at a fixed probability in each period and a penalty fee charged to the channel members in case coordination is detected. When coordination is internally unstable, it is socially optimal to prevent its formation if the relative abatement cost of collusive emissions is sufficiently large or if the minimum cost of auditing is sufficiently small. In the case where coordination is internally stable, destabilization is also an option for the antitrust authority. In this case, our necessary and sufficient conditions characterizing the optimal antitrust decisions imply that it is socially optimal to destabilize (allow) the vertical price coordination of the channel if both the minimum cost of auditing and the relative abatement cost of collusive emissions are sufficiently small (large) and to prevent it otherwise.

Keywords: Supply chain; vertical price coordination; vertical price restraints; antitrust policy.

JEL codes: D43; L11, L22, L42, Q52.

1 Introduction

This paper deals with the characterization of optimal antitrust policies for vertical price restraints in a supply chain channel involving one supplier and one retailer. For this simple channel, it is well known by the work of Spengler (1950) that the sequential competition in prices would lead to a problem known as double marginalization (or a chain of monopolization) where the downstream (retail) price of the product would exceed the monopoly price due to the double markups charged by the supplier and the retailer consecutively. However, this problem is eliminated, as also shown by Spengler (1950), if the channel members become vertically integrated (or coordinated) since the retail price that maximizes their joint profits would be equal to the monopoly price in the downstream market. Hence, the collusive output and the induced consumer surplus become higher and exceed their monopolistic levels due to the elimination of double marginalization unlike in the case of a horizontal industry. Consequently, the vertical integration or coordination of a simple channel involving successive monopolies would not call for antitrust enforcement on the grounds of increasing consumer surplus.

But, things may change when the supply chain channel has more than two echelons. For example, if the channel involves more than one retailer, vertical price restraints known as resale-price maintenance (a price floor or ceiling put by a supplier on the prices of some retailers) can facilitate inter-brand collusion between suppliers and reduce or eliminate intra-brand competition between retailers. In fact, in many jurisdictions, involving Sections 1-2 of the Sherman Act in the United States and Articles 81-82 of the European Communities Treaty, vertical restraints such as resale-price maintenance are considered to be illegal per se, calling for antitrust action, as they restrict competition or usually lead to an abuse of dominant position.

In this paper, we focus on another rationale, namely the adverse environmental effects of production, for the use of antitrust policies for vertical price restraints even when the supply chain channel involves two members only. To model this rationale, we consider a non-green channel that emits a certain amount of air pollution for each unit of its production and assume that the abatement cost of this pollution is borne by consumers. Following Nordhaus (2008), we assume that the abatement cost is (non-linearly) increasing with the quantity of the channel's output. Therefore, this cost becomes higher when the channel members coordinate, than when they compete, countering the positive effect of coordination on social welfare. This means that the solution to the problem of whether the antitrust authority should allow the channel to coordinate using vertical price restraints depends on whether the positive effect of coordination via an increase in the output is large enough to offset the negative effect via an increase in the abatement cost. If the above-mentioned negative effect is arbitrarily small, then the channel should trivially be allowed to coordinate. But, if this is not the case, then the problem of the antitrust authority becomes complicated as it should also take into consideration the social costs of the antitrust policies that it may use.

What is key for our model is the assumption that consumers bear the abatement cost of pollution. In the polar case where the abatement cost of pollution is borne by the industry instead of consumers, the negative effect of price coordination would be (partially) internalized by the industry, alleviating or eliminating the need for antitrust action against the supply chain. However, this case is not observed in reality. Even though the public awareness on cleaner production and the role of carbon taxes as a major tool to mitigate carbon emissions and facilitate energy transition to low carbon sources has undoubtedly grown in the last decades especially in developed countries, carbon taxes on corporate sector are still very low and ineffective worldwide (Ryan et al, 2023; Zheng et al., 2023). Not surprisingly, the air quality is very poor across the world, causing severe health damages. A data report by World Health Organization (WHO, 2022) reveals that "In 2019, 99% of the world's population was living in places where the WHO air quality guidelines levels were not met. ... Ambient (outdoor) air pollution is estimated to have caused 4.2 million premature deaths worldwide in 2019." Thus, our assumption that the abatement cost is borne by consumers takes into account both the insufficient adoption of carbon taxes to mitigate carbon emissions and its adverse health consequences on the world population.

In our paper, we use the antitrust policy measures earlier employed by Harrington Jr. (2014) in the context of deterring collusion in a horizontal oligopoly. These measures, when modified for our purpose, involve (i) auditing the supply chain to detect any price coordination at a fixed probability and (ii) a fixed penalty fee charged to both members of the channel in case their coordination is detected. Following Friedman (1971) that dealt with collusion in a horizontal industry, we model vertical price coordination as the equilibrium of an infinitely repeated game where the members of the channel seek to sustain coordination non-cooperatively, using the grim punishment under which deviation by any member from the coordination outcome triggers the permanent reversion to the non-cooperative (Stackelberg) equilibrium outcome. In this setup, we say that the vertical price coordination is *internally stable* if it can be sustained in the absence of any antitrust policy and *internally unstable* otherwise.

We characterize the socially-optimal antitrust policies separately in the case where the vertical price coordination of a given supply chain is internally unstable and the case where it is stable. In the former case, we show that it is socially optimal to allow the vertical price coordination if the relative abatement cost of induced emissions is sufficiently small (or if the minimum cost of auditing is sufficiently large) and to prevent it otherwise. On the other hand, in the case where the vertical price coordination is stable, the antitrust authority is no longer restricted to either allowing or preventing it. Now, it may also consider the option of destabilizing the vertical price coordination externally. After characterizing necessary and sufficient conditions for optimal antitrust decisions, we show that it is socially optimal to destabilize (allow) the vertical price coordination of a given channel if both the minimum cost of auditing and the relative abatement cost of emissions generated by coordination are sufficiently small (large) and to prevent it otherwise.

The remainder of the paper is organized as follows. Section 2 presents a brief literature review, Section 3 presents the basic structures, and Section 4 contains our results. Finally, Section 5 concludes.

2 Literature Review

This paper studies the optimal antitrust policies for vertical price restraints in a non-green supply chain involving a single supplier and a single retailer; it can be thus related to the theoretical studies in the following three streams of literature: (i) vertical price restraints in supply chains, (ii) deterrence of coordination or collusion, and (iii) sustainable supply chain management.

2.1 Vertical price restraints in supply chains

The literature on supply chains, and in particular vertical price restraints/coordination, is pioneered by the seminal work of Spengler (1950), who showed that the sequential competition in a supply chain involving two echelons leads to a vertical externality known as double marginalization (or markups), which is eliminated if the channel members become vertically integrated or coordinated in prices. This work showed that in simple supply chains, sequential competition may be less desirable than vertical coordination in terms of consumer welfare. Following Spengler (1950), the literature on supply chains expanded rapidly with an initial focus on vertical integration. Bork (1954) and Burstein (1960) showed that a firm that is the sole producer of a good that is used as an input by a downstream industry with fixed proportions of production (or, in general, any economic process) does not have an incentive to vertically integrate with that industry, whereas Vernon and Graham (1971) argued, and Schmalensee (73) proved, that this result is reversed when the monopolized input is employed in variable proportions by a downstream industry. Blair and Kaserman (1978) examined the incentive for vertical integration in the presence of uncertainty and showed that vertical integration can reduce uncertainty about the availability of the product. Dixit (1983) and Gallini and Winter (1983) extended the analysis of Spengler (1950) to a supply chain model involving a monopolistically competitive retail market. Dixit (1983) showed that the equilibrium under vertical full integration yields higher social welfare than the unintegrated outcome of monopolistic competition, whereas Gallini and Winter (1983) derived conditions under which the upstream monopolist can implement the equilibrium under vertical full integration by imposing resale-price maintenance. Rey and Tirole (1986) showed that when retailers face demand and cost uncertainty, resale-price maintenance cannot eliminate the vertical price distortion. Ordover et al. (1990) studied vertical integration between Bertrand duopolists in the upstream industry and differentiated Bertrand duopolists in the downstream industry and showed that the integration between an upstream and a downstream firm could raise final good prices. O'Brien and Shaffer (1992) showed that if the contracts between the upstream firm and downstream retailers are secret, then non-linear pricing can no longer yield the outcome of vertical integration. Several studies showed that a two-part tariff contract can be more efficient than a wholesale price contract (Cachon and Kök, 2010, Corbett et al., 2004, Feng and Lu, 2013). Huang et al. (2016) examined the effect of various pricing strategies and channel power structures on channel members' pricing competition and cooperation in a two-tier supply chain. More recently, Li et al. (2019) studied contract design in a cross-sales supply chain where retailers have private information about demand, and Liu et al. (2020) investigated the impact of different consumer network acceptances on the optimal pricing, demand and profit of single-channel and dual-channel supply chains.

2.2 Deterrence of coordination or collusion

Several studies dealt with antitrust policies in industries with vertical or horizontal integration. For vertical industries, a belief that was formed as early as the 1950s by a group of economists and lawyers at the University of Chicago dominated academic circles for many decades –even up to now to some extent– as to the social desirability of vertical integration or coordination. This belief was that the anticompetitive effects of vertical restraints or vertical integration would only arise from regulatory evasion or horizontal effects such as collusion among the suppliers or retailers. Therefore, competitive policies or antitrust action were only needed to alleviate these 'side issues' related to vertical restraints. (The validity of this belief was empirically tested in many studies. A review by Cooper et al. (2005) revealed that empirical papers published between 1984 and 2004 on the effects of vertical integration provide little evidence for the claim that vertical integration or restraints may be harmful to consumers. Also, see O'Brien (2008) for a comprehensive survey, classifying theoretical models in supply chains, where the welfare effects of vertical restraints become either positive, negative, or ambiguous.) However, even these side issues do not appear in a simple supply chain with successive monopolies as considered in our model. Absent any environmental effect of production, there is no reason for taking any antitrust action in successive monopolies; which is why we cannot refer to any relevant theoretical study for deterring vertical coordination in prices. Nonetheless, our paper greatly merits from sister literature studying collusion in horizontal industries, since vertical coordination in our model might be simply interpreted as collusion in a horizontal duopoly, once we should incline to ignore whether the firms in the industry make their strategic moves sequentially or simultaneously. In fact, as we mentioned earlier, we will model vertical price coordination in our model following the approach developed by Friedman (1971) to deal with collusion in a horizontal industry. Thus, we believe it will be helpful to summarise below some of the major works dealing with the deterrence of collusion in horizontal industries.

Among some early works, Block et al. (1981) and Besanko and Spulber (1989, 1990) showed in oligopolistic setups that the cartel's equilibrium price is increasing in antitrust penalties. Souam (2001) compared the effect of two regimes of antitrust penalties, one based on revenues of the industry and the other on the damages caused to consumers, on the deterrence levels of collusion in a Cournot oligopoly. Frezal (2006) showed that a certain but non-stationary audit strategy is more effective to deter collusion than an uncertain and stationary audit strategy. Martin (2006) compared the welfare effects of a deterrence-based competition policy on the cooperative and non-cooperative collusive equilibria of a Cournot oligopoly under demand uncertainty. Bartolini and Zazzaro (2011) showed that the antitrust fines, whenever sufficiently low, may have anti-competitive effects as they may lead firms in a Cournot oligopoly to form coarser cartel coalitions. Finally, Harrington Jr. (2014), from which we borrow the antitrust measures used in our model, studied the size of penalty fees required to destabilize the collusion in an infinitely repeated oligopoly.

2.3 Sustainable (green) supply chain management

This strand of literature emerged mostly in the last two decades. Several studies in this strand combined the profit performance and the environmental performance of supply chains to highlight the trade-off between them (Nagurney and Toyasaki, 2003; Sheu et al., 2005; Lu et al., 2007; Neto et al., 2008, among others). Pistikopoulos and Hugo (2005) integrated environmental concerns with long-range planning and design of supply chain networks. Nagurney et al. (2006) developed a supply chain model in which the suppliers can produce a single product in different plants with distinct environmental emissions. Guillen-Gosalbez and Grossmann

(2009) dealt with the optimal design and planning of sustainable chemical supply chains under uncertainty. Ramudhin et al. (2010) studied the design of carbon market sensitive green supply chain networks. Chaabane et al. (2012) studied the problem of designing sustainable supply chains under the emission trading scheme. Das and Posinasetti (2015) studied a closed loop supply chain model under environmental concerns to improve overall supply chain performance. Wang et al. (2017) studied the (non)alignment between profitability and environmental targets in a reverse supply chain, while Wang et al. (2020) studied the desirability of the collusive behavior of retailers in a closed-loop supply chain both from the aspect of environmental benefits and social welfare. Recently, a strand of literature (Hafezalkotob, 2015, 2018; Madani and Rasti-Barzoki, 2017; Yang et al., 2019, Yu et al., 2012, among others) dealt with the role of government intervention (subsidies and taxes) in competitive and cooperative games between manufacturers in two-channel industries involving green and/or non-green supply chains.

2.4 Location of our work

To the best of our knowledge, a game-theoretical analysis of optimal antitrust decisions and policies to deal with vertical price restraints in supply chains with successive monopolies is novel to our work. However, our work can be partially related to several articles in the supply chain or oligopoly literature, involving Spengler (1950), Friedman (1971), Harrington, Jr. (2014), and Bolatto and Lambertini (2017), among possibly others. In Table 1, we briefly summarize these articles to highlight the location of our work.

Table 1
Summary of Related Articles and Location of Our Work
(✓ = existent)

	Spengler (1950)	Friedman (1971)	Harrington, Jr. (2014)	Bolatto and Lambertini (2017)	Our Work
Environmental Concern					✓
Pollution Emission					✓
Cost of Abatement					✓
Horizontal Industry		✓	✓		
Vertical Industry	✓			✓	✓
Single Channel	✓			✓	✓
Successive Monopolies	✓			✓	✓
Single Period	✓				
Infinite Horizon		✓	✓	✓	✓
Simultaneous Competition		✓	✓		
Sequential Competition	✓			✓	✓
Horizontal Collusion		✓	✓		
Vertical Coordination	✓			✓	✓
Linear Prices	✓	✓	✓		✓
Two-Part Tariffs				✓	
Nash Bargaining					✓
Antitrust Policies			✓		✓
Auditing with Uncertainty			✓		✓
Dynamic Penalty Fees			✓		
Constant Penalty Fees					✓
Destabilization Policies			✓		✓
Prevention Policies					✓

Briefly, we borrow the basic structures of our supply chain model from Spengler (1950). Both his work and ours are dealing with linear prices, whereas a related paper by Bolatto and Lambertini (2017) analyzed the same model under two-part tariffs. However, neither Spengler (1950) nor Bolatto and Lambertini (2017) included any antitrust treatment. We borrow the infinitely-repeated game structure which we use to model non-cooperative coordination from Friedman (1971), who developed this structure to model non-cooperative collusion in oligopolies. The same structure was also used by Bolatto and Lambertini (2017). On the other hand, we borrow our antitrust measures that deal with vertical price coordination from Harrington, Jr. (2014), who used these measures to deter collusion in horizontal industries. However, unlike our work, neither of the above-mentioned four articles contains any environmental concerns.

3 Basic Structures

We consider a supply chain involving one supplier and one retailer that live for infinite periods. The retailer is the downstream member of the chain (echelon 1) while the supplier is the upstream member (echelon 2). Periods are indexed by $t = 0, 1, 2, \dots, \infty$. We assume that the retailer, the supplier, and consumers discount each period by a common factor $\delta \in (0, 1)$.

In each period, the supplier produces a good at the constant marginal cost c normalized to zero. We assume that the fixed cost of production is zero, too. For each period, we let p_2 denote the price charged by the supplier to the retailer for each unit of its product and let p_1 denote the price charged by the retailer in the downstream market. (Since our model does not involve any time-varying parameter, prices and other variables will be time-invariant.) In each period, the retailer faces a linear demand function given by

$$D(p_1) = a - p_1, \tag{1}$$

where $a > 0$. Once the retailer chooses its price p_1, the quantity it orders from the supplier is determined as $q_1 = D(p_1)$. The cost information of the supplier and the demand information of the retailer are common knowledge.

Given the above structures, the one-period profits of the retailer and the supplier are

$$\pi_1(p_1, p_2) = (p_1 - p_2)D(p_1) \tag{2}$$

and

$$\pi_2(p_1, p_2) = p_2 D(p_1) \tag{3}$$

respectively. Consequently, the lifetime profits of the retailer and the supplier (discounted to any given period) are

$$V_1 = \frac{\pi_1(p_1, p_2)}{1 - \delta} \tag{4}$$

and

$$V_2 = \frac{\pi_2(p_1, p_2)}{1 - \delta} \tag{5}$$

respectively.

Given the retail price p_1, the gross benefit of consumers in the downstream market is equal to $\int_0^{D(p_1)} D^{-1}(q)dq$ in each period. Accordingly, their net surplus is

$$CS(p_1) = \int_0^{D(p_1)} D^{-1}(q)dq - p_1 D(p_1) = \frac{[D(p_1)]^2}{2} = \frac{(a-p_1)^2}{2}. \quad (6)$$

Consumers' net gain is less than their net surplus in each period since we assume that there exists an abatement cost of pollution generated by the production of the channel, and this cost is borne by consumers. Here, we let γ denote the amount of air pollution (CO_2) emitted by the channel for each unit of its production. Following Nordhaus (2008), we assume that the abatement cost in each period is given by

$$k = \varphi\mu^{\epsilon} \quad (7)$$

where $\varphi > 0$ is a scale parameter, μ is the reduction in emissions from the baseline to the policy target level (which we assume to be zero), and ϵ is the exponent reflecting the nonlinearity in costs for larger reductions. Nordhaus (2008) sets ϵ at 2.8, nearly cubic for all countries and periods in his study. Following him (with a slight simplification for tractability and clarity), we set $\epsilon = 3$, implying $k \equiv \varphi\mu^3$. Thus, in each period the abatement cost $k \equiv k(p_1)$ borne by consumers becomes

$$k(p_1) = \varphi\left[\gamma D(p_1)\right]^3 = \varphi\gamma^3\left[a - p_1\right]^3. \quad (8)$$

With the abatement cost being taken into account, the net gain of consumers in each period becomes

$$NCG(p_1) = CS(p_1) - k(p_1) = \frac{(a-p_1)^2}{2} - \varphi\gamma^3\left[a-p_1\right]^3. \quad (9)$$

Consequently, the lifetime welfare of consumers (discounted to any given period) becomes

$$V_{cons} = \frac{NCG(p_1)}{1-\delta}. \quad (10)$$

Finally, we can define the social welfare $SW(p_1, p_2)$ in each period as the sum of the consumer net gain and the total profit of the channel. That is,

$$SW(p_1, p_2) = NCG(p_1) + \pi_1(p_1, p_2) + \pi_2(p_1, p_2). \quad (11)$$

Accordingly, we can calculate the lifetime welfare of the society V_{soc} (discounted to any given period) as

$$V_{soc} = V_1 + V_2 + V_{cons}. \quad (12)$$

We present the summary of notations used in our model and the next sections in Table 2 below.

Table 2
Summary of Notations

Notation	Description
t	The index of time; $t = 0, 1, 2, \ldots,$
i	The index of echelons; $i = 1 \implies$ the retailer, $i = 2 \implies$ the supplier
δ	The discount factor of the retailer, the supplier, and consumers
p_1	The price charged by the retailer to consumers
p_2	The price charged by the supplier to the retailer
$D(p_1)$	The demand function in the downstream market
a	The size of demand in the downstream market
γ	The pollution emitted by the channel for each unit of the product
φ	The abatement cost of reducing one unit of emission to zero
$k(p_1)$	The one-period abatement cost
λ_1	The bargaining power of the retailer
θ	The probability of detecting coordination
$A(\theta)$	The auditing cost of detecting coordination at probability θ
f	The penalty fee charged to each echelon in case coordination is detected
f^m	The upper threshold for the penalty multiple
$\pi_1(p_1, p_2)$	The one-period profit of the retailer
$\pi_2(p_1, p_2)$	The one-period profit of the supplier
$CS(p_1)$	The one-period consumer surplus
$NCG(p_1)$	The one-period net gain of consumers
$SW(p_1, p_2)$	The one-period social welfare
V_1	The lifetime profit of the retailer
V_2	The lifetime profit of the supplier
V_{cons}	The lifetime welfare of consumers
V_{soc}	The lifetime welfare of the society
$\mathcal{VPC}$	Vertical price coordination
$\mathcal{SC}$	Supply chain

4 Results

We will first calculate the non-cooperative (vertical competition) outcomes and the cooperative (vertical coordination) outcomes for a given supply chain in the absence of an antitrust authority.

4.1 Vertical Price Competition

Here, we assume that the supplier and the retailer engage in a two-stage non-cooperative game, called Stackelberg price competition, in each period. In the first stage of this sequential game, the supplier, acting as the leader, determines its price p_2 charged to the retailer for each unit of its product, and in the second stage the retailer, acting as the follower, determines the retail price p_1 and the order quantity $q_1 = D(p_1)$. Solving this game backward, the supplier first solves the retailer's optimization problem in the second stage for each p_2. That is, the supplier (as well as the retailer) solves for each period the problem given by

$$\max_{p_1 \geq 0} \pi_1(p_1, p_2) \tag{13}$$

using (2). The first-order condition yields the retailer's reaction function

$$p_1(p_2) = \frac{a}{2} + \frac{p_2}{2}. \tag{14}$$

Inserting (14) into (3), we obtain the problem faced by the supplier in the first stage of each period:

$$\max_{p_2 \geq 0} \pi_2(p_1(p_2), p_2) = p_2 D(p_1(p_2)) = \frac{p_2}{2}(a - p_2) \tag{15}$$

The first-order condition for (15) yields the optimal (equilibrium) wholesale price for each period, given by

$$p_2^S = \frac{a}{2}, \tag{16}$$

where the superscript S stands for the Stackelberg equilibrium. Using (16), we can calculate the retailer's equilibrium price $p_1^S = p_1(p_2^S) = 3a/4$ and equilibrium order $q_1^S = D(p_1^S) = a/4$ for each period. In result, the supplier and the retailer can enjoy in each period the equilibrium profits

$$\pi_1^S = \pi_1(p_1^S, p_2^S) = (p_1^S - p_2^S)D(p_1^S) = \frac{a^2}{16} \tag{17}$$

and

$$\pi_2^S = \pi_2(p_1^S, p_2^S) = p_2^S D(p_1^S) = \frac{a^2}{8} \tag{18}$$

respectively. Thus, their lifetime equilibrium profits can be calculated as

$$V_1^S = \frac{1}{1-\delta}\frac{a^2}{16} \tag{19}$$

and

$$V_2^S = \frac{1}{1-\delta}\frac{a^2}{8} \tag{20}$$

respectively. On the other hand, the one-period net gain of consumers can be calculated as

$$\begin{aligned} NCG^S \equiv NCG(p_1^S) &= \frac{(a - p_1^S)^2}{2} - \varphi\gamma^3 \left[a - p_1^S\right]^3 \\ &= \frac{a^2}{32} - \frac{\varphi\gamma^3 a^3}{64}. \end{aligned} \tag{21}$$

Consequently, the lifetime welfare of consumers and society become

$$V_{cons}^S = \frac{1}{1-\delta}\left(\frac{a^2}{32} - \frac{\varphi\gamma^3 a^3}{64}\right) \tag{22}$$

and

$$V_{soc}^S = \frac{1}{1-\delta}\left(\frac{7a^2}{32} - \frac{\varphi\gamma^3 a^3}{64}\right) \tag{23}$$

respectively.

4.2 Vertical Price Coordination

Here, we consider a vertical industry where in each period the supplier and retailer coordinate in prices to maximize their joint profits denoted by $\pi(p_1, p_2)$. Thus, the channel solves the following problem:

$$\max_{p_1,p_2\geq 0} \pi(p_1, p_2) \equiv \pi_1(p_1, p_2) + \pi_2(p_1, p_2) = p_1 D(p_1) = p_1(a - p_1) \tag{24}$$

After the retailer and the supplier find the coordination prices (p_1^C, p_2^C) that solve (24), they start moving sequentially in each period. The retailer orders quantity $q_1^C = D(p_1^C)$ from the supplier, the supplier charges price p_2^C to the retailer for each unit of its ordered product, and the retailer charges the price p_1^C to customers in the downstream market.

To solve the coordination problem in (24), we can easily check that the associated first-order condition implies the optimal retail price

$$p_1^C = \frac{a}{2}. \tag{25}$$

Using this price, we can calculate the per-period joint profit of the channel as

$$\pi^C = p_1^C D(p_1^C) = \frac{a^2}{4}. \tag{26}$$

Notice from above that vertical coordination enables the channel to extract, in each period, the monopoly profit in the downstream market. We assume that the supplier and the retailer share this monopoly profit using the generalized Nash bargaining rule (Nash, 1950; Roth, 1979) that maximizes the product of the channel members' net agreement payoffs raised to some fixed and asymmetric bargaining powers.

That is, the supplier and the retailer cooperatively solve, for each period, the problem given by

$$\max_{\pi_1,\pi_2\geq 0} (\pi_1 - d_1)^{\lambda_1} (\pi_2 - d_2)^{1-\lambda_1} \tag{27}$$

subject to

$$\pi_1 + \pi_2 = \pi_t^C. \tag{28}$$

Above, d_1 and d_2 are disagreement payoffs of the retailer and the supplier in case the bargaining fails, and λ_1 and $1-\lambda_1$ are their bargaining powers respectively. Although the supplier has the advantage of being the first mover in the sequential (Stackelberg) competition and thus gets the lion's share of the channel profits in equilibrium, we need not assume this advantageous position (i.e., the case of $\lambda_1 = 0$) for the supplier under vertical coordination as well since the formation of such coordination (to share any excess gain over the competitive profits) requires the mutual and symmetric cooperation of both channel members. Hence, we assume $\lambda \in [0, 1]$ to cover all possible distributions of coordination gains.

To determine the disagreement payoffs of the retailer and the supplier endogenously within our model, we assume that the two parties engage in the Stackelberg competition if their bargaining fails. Thus, we set $d_1 = \pi_1^S = a^2/16$ and $d_2 = \pi_2^S = a^2/8$.

Now, we turn back to the bargaining problem (27)-(28). The profit distribution (π_1^C, π_2^C) solves this problem only if it satisfies the first-order condition

$$\frac{\pi_1^C - d_1}{\lambda_1} = \frac{\pi_2^C - d_2}{1 - \lambda_1}. \tag{29}$$

Using (28) to insert $\pi_2^C = \pi_t^C - \pi_1^C$ into the above equation, we obtain

$$\pi_1^C = (1 + \lambda_1)\frac{a^2}{16} \tag{30}$$

and

$$\pi_2^C = (3 - \lambda_1)\frac{a^2}{16}. \tag{31}$$

Equations (2)-(3) and (30)-(31) together imply

$$\frac{p_1^C - p_2^C}{p_2^C} = \frac{1 + \lambda_1}{3 - \lambda_1} \tag{32}$$

or

$$p_2^C = \frac{(3 - \lambda_1)}{4} p_1^C. \tag{33}$$

Also, using (25) and (34) we obtain

$$p_2^C = \frac{(3 - \lambda_1)a}{8}. \tag{34}$$

Notice from the above results that the additional total profit of the channel, when the two members are coordinating (instead of competing) with each other, is $\pi_t^C - \pi_1^S - \pi_2^S = a^2/16$. Under coordination, the retailer obtains the fraction λ_1 of this additional sum and its profit becomes $\pi_1^C = \pi_1^S + \lambda_1 a^2/16$, whereas the supplier gets what is left and its profit becomes $\pi_2^C = \pi_2^S + (1 - \lambda_1)a^2/16$. Apparently, the channel member whose bargaining power is zero cannot obtain any gain from coordination. That is, each member benefits from coordination if and only if $\lambda_1 \in (0, 1)$.

We can calculate the lifetime profits of the retailer and the supplier under coordination as

$$V_1^C = \frac{1 + \lambda_1}{1 - \delta}\frac{a^2}{16} \tag{35}$$

and

$$V_2^C = \frac{3 - \lambda_1}{1 - \delta}\frac{a^2}{16} \tag{36}$$

respectively. Also, we can calculate the one-period net gain of consumers under coordination as

$$\begin{aligned} NCG^C \equiv NCG(p_1^C) &= \frac{(a - p_1^C)^2}{2} - \varphi\gamma^3 \left[a - p_1^C\right]^3 \\ &= \frac{a^2}{8} - \frac{\varphi\gamma^3 a^3}{8}. \end{aligned} \tag{37}$$

Accordingly, the lifetime welfare of consumers and society become

$$V_{cons}^{C} = \frac{1}{1-\delta}\left(\frac{a^2}{8} - \frac{\varphi\gamma^3 a^3}{8}\right) \tag{38}$$

and

$$V_{soc}^{C} = \frac{1}{1-\delta}\left(\frac{3a^2}{8} - \frac{\varphi\gamma^3 a^3}{8}\right) \tag{39}$$

respectively. Notice that when the channel engages in vertical price coordination, the per-period abatement cost of emissions is $\varphi\gamma^3 a^3/8$ whereas the per-period consumer surplus is $a^2/8$. The ratio of these two magnitudes is $\varphi\gamma^3 a$, which we will call the relative abatement cost of emissions generated by vertical price coordination. Also, we will hereafter represent a supply chain by the set of parameters $\mathcal{SC} = \langle a, \gamma, \varphi, \delta, \lambda_1 \rangle$ and abbreviate vertical price coordination by $\mathcal{VPC}$, whenever convenient.

4.3 Stability of Vertical Price Coordination in the Absence of Antitrust Policies

Here, we consider an industry where the antitrust authority does not make any attempt to prevent or destabilize $\mathcal{VPC}$ of the supply chain. So, any stable (sustainable) coordination in this industry must be self-enforcing for the members in the channel. Accordingly, we will call $\mathcal{VPC}$ *internally unstable* if and only if it is not self-enforcing. Below, we will first investigate when $\mathcal{VPC}$ can be internally unstable, borrowing from the repeated game approach developed by Friedman (1971) to analyze the stability of collusion in horizontal industries.

We assume a setup where all decisions are made in the first period, for simplicity. The members of the channel seek to sustain $\mathcal{VPC}$ using the grim punishment under which the deviation by any member triggers the permanent reversion to the non-cooperative equilibrium outcome. Below, we will first explore whether any member can increase its one-period profit by deviating from $\mathcal{VPC}$. If the answer is yes, then we will study whether such a one-period deviation is worthwhile for any member of the channel against the threat of being permanently punished by the other member.

Recall that the one-period profit of echelon i from $\mathcal{VPC}$ is equal to $\pi_i^C = \pi_i^S + (1-\lambda_i)a^2/16$. The supplier ($i = 2$) as the first mover in $\mathcal{VPC}$ has no incentive to deviate in any period from the $\mathcal{VPC}$ price p_2^C if its bargaining power is positive, i.e., $1 - \lambda_1 > 0$ and has no strict incentive to deviate if $\lambda_1 = 1$. This is because the supplier can simply anticipate that if it deviates from p_2^C, the retailer will use its best response as a follower, and they will end up playing their equilibrium strategies in the Stackelberg game, which will yield a lower (the same) profit to the supplier if $\lambda_1 < 1$ ($\lambda_1 = 1$).

However, the situation is different for the retailer as it possesses the advantage of being the second mover. Whenever the retailer should play its move to decide whether to deviate from the $\mathcal{VPC}$ price p_1^C or not, it knows the fact that the supplier has already charged its $\mathcal{VPC}$ price p_2^C to the retailer and delivered units $q_1^C = D(p_1^C)$ ordered by the retailer. If the retailer chooses to cheat and charge a deviation price p_1^{dev-1} to customers, it can earn the profit

$$\pi_1^{dev-1}(p_1^{dev-1}, p_1^C, p_2^C) = (p_1^{dev-1} - p_2^C)\min\{D(p_1^{dev-1}), D(p_1^C)\}. \tag{40}$$

The retailer's problem is then to solve

$$\max_{p_1^{dev-1} \geq 0} \pi_1^{dev-1}(p_1^{dev-1}, p_1^C, p_2^C). \tag{41}$$

We will solve the problem in (41) by supposing first $\min\{D(p_1^{dev-1}), D(p_1^C)\} = D(p_1^{dev-1})$ and verifying later whether this supposition is correct. The first-order condition for (41) would imply

$$p_1^{dev-1} = \frac{a}{2} + \frac{p_2^C}{2} = \frac{(11-\lambda_1)a}{16} \tag{42}$$

and

$$q_1^{dev-1} = D(p_1^{dev-1}) = a - bp_1^{dev-1} = \frac{(5+\lambda_1)a}{16}. \tag{43}$$

Recall that $p_1^C = a/(2b)$ and $q_1^C = D(p_1^C) = a/2$. Therefore, $q_1^{dev-1} < q_1^C$, implying that the supposition $\min\{D(p_1^{dev-1}), D(p_1^C)\} = D(p_1^{dev-1})$ was correct. If the retailer deviates from the $\mathcal{VPC}$ price p_1^C, it has to leave a total of $q_1^C - q_1^{dev-1} = \frac{(3-\lambda_1)a}{16}$ units of ordered goods unsold. The amount of unsold units is decreasing in λ_1; but it is always positive.

Using (42) and (43), we can calculate the one-period profit of the retailer in the period it deviates from the $\mathcal{VPC}$ price as follows:

$$\pi_1^{dev-1}(p_1^{dev-1}, p_1^C, p_2^C) = \frac{(5+\lambda_1)(11-\lambda_1)a^2}{256}. \tag{44}$$

Using $\pi_1^C = (1+\lambda_1)a^2/16$, we can then calculate the retailer's one-period profit gain from deviation as

$$\pi_1^{dev-1} - \pi_1^C = \frac{(39 - 10\lambda_1 - \lambda_1^2)a^2}{256}. \tag{45}$$

Apparently, $\pi_1^{dev-1} - \pi_1^C$ is always positive. (It monotonically increases from its lowest value $28a^2/(256)$ to its highest value $39a^2/(256)$ while λ_1 is varied from 1 to 0.) Thus, we have established so far that by deviating from the $\mathcal{VPC}$ price p_1^C to the price p_1^{dev-1} and leaving the amount of units $q_1^C - q_1^{dev-1}$ unsold (even destroying it wastefully), the retailer can increase its one-period profit.

Now, we will explore whether/when the deviation of the retailer is worthwhile in the face of the grim punishment it will trigger. The retailer's lifetime profit when it always complies with $\mathcal{VPC}$ is V_1^C. On the other hand, when the retailer deviates from $\mathcal{VPC}$ in any period and gets punished thereafter to enjoy the competitive (Stackelberg) profit stream, its lifetime profit becomes

$$\begin{aligned} V_1^{dev-1} &= \pi_1^{dev-1} + \delta V_1^S \\ &= \frac{(5+\lambda_1)(11-\lambda_1)a^2}{256} + \frac{\delta}{1-\delta}\frac{a^2}{16}. \end{aligned} \tag{46}$$

We should remember that the supplier has no incentive to deviate from $\mathcal{VPC}$. This is because $V_2^{dev-2} = V_2^S$ and therefore $V_2^C \geq V_2^{dev-2}$ always holds since we already know from (20) and (36) that

$$V_2^C = V_2^S + \frac{(1-\lambda_1)}{(1-\delta)}\frac{a^2}{16}. \tag{47}$$

So, $\mathcal{VPC}$ is internally stable if and only if $V_1^C \geq V_1^{dev-1}$. These observations lead us to the following result.

Proposition 1. *$\mathcal{VPC}$ of a supply chain $\mathcal{SC} = \langle a, \gamma, \varphi, \delta, \lambda_1 \rangle$ is internally stable if and only if $\delta \in [\bar{\delta}(\lambda_1), 1]$, where*

$$\bar{\delta}(\lambda_1) = 1 - \frac{16\lambda_1}{39 + 6\lambda_1 - \lambda_1^2}. \tag{48}$$

Proof. We have already established that the supplier has no incentive to deviate from the $\mathcal{VPC}$ plan where it moves first in each period since its deviation would trigger Stackelberg equilibrium play that yields a lower profit even in the period of deviation. Thus, $\mathcal{VPC}$ of a supply chain is internally stable if and only if $V_1^C \geq V_1^{dev-1}$ holds. Using (35) and (46), we can rewrite this inequality as

$$\frac{(1+\lambda_1)}{1-\delta}\frac{a^2}{16} \geq \frac{(5+\lambda_1)(11-\lambda_1)a^2}{256} + \frac{\delta}{1-\delta}\frac{a^2}{16}, \tag{49}$$

which holds if and only if

$$\Gamma(\delta, \lambda_1) \equiv \lambda_1 \left[10 + \lambda_1(1-\delta) + 6\delta\right] - 39(1-\delta) \geq 0. \tag{50}$$

Notice that for any $\lambda_1 \in (0, 1)$, we have $\Gamma(\delta, \lambda_1) > 0$ if δ is sufficiently close to 1 and $\Gamma(\delta, \lambda_1) < 0$ if δ is sufficiently close to 0. Since $\Gamma(\delta, \lambda_1)$ is continuous in δ, there exists $\bar{\delta}(\lambda_1)$ such that the equation $\Gamma(\bar{\delta}(\lambda_1), \lambda_1) = 0$ holds. One can easily solve this equation to find $\bar{\delta}(\lambda_1)$ as in (48). Clearly, $\bar{\delta}(\lambda_1) \in (0, 1)$ for any $\lambda_1 \in (0, 1]$ and $\bar{\delta}(\lambda_1) = 1$ if $\lambda_1 = 0$. Moreover, since $\Gamma(\delta, \lambda_1)$ is increasing in δ, the threshold $\bar{\delta}(\lambda_1)$ is unique, and therefore $\Gamma(\delta, \lambda_1) \geq 0$ if and only if $\delta \in [\bar{\delta}(\lambda_1), 1]$. ■

Since $\bar{\delta}(\lambda_1)$ is positive for each $\lambda_1 \in [0, 1]$, we have $(\bar{\delta}(\lambda_1), 1] \subsetneq [0, 1]$. Given this observation, Proposition 1 implies the following.

Corollary 1. *$\mathcal{VPC}$ of a supply chain $\mathcal{SC} = \langle a, \gamma, \varphi,\ \delta, \lambda_1 \rangle$ is internally unstable if and only if $\delta \in [0, \bar{\delta}(\lambda_1))$.*

The above result states that $\mathcal{VPC}$ is internally unstable if and only if the retailer is sufficiently impatient (myopic). In such a case, the retailer's one-period gain from cheating would exceed its discounted sum of future losses caused by the permanent punishment it would have triggered. Notice that $\mathcal{VPC}$ of the supply chain is weakly unstable (as well as weakly stable) if $\delta = \bar{\delta}$, in which case $V_1^C = V_1^{dev-1}$. We should also notice that the threshold value of the retailer's time discount, $\bar{\delta}(\lambda_1)$, is decreasing in λ_1, the bargaining power of the retailer. As λ_1 decreases from 1 to 0, the threshold $\bar{\delta}$ rises from 7/11 to 1 monotonically. Thus, the lower the value of λ_1, the wider the interval $[0, \bar{\delta}(\lambda_1))$, and the higher the likelihood that $\mathcal{VPC}$ of a supply chain (with randomly generated parameters from a known distribution) would be unstable.

Whenever the strict inequality $V_i^C > V_i^{dev-1}$ can hold for each i, we say that $\mathcal{VPC}$ is *strongly* internally stable.

Corollary 2. *$\mathcal{VPC}$ of a supply chain $\mathcal{SC} = \langle a, \gamma, \varphi, \delta, \lambda_1 \rangle$ is strongly internally stable only if $\lambda_1 \in (0, 1)$.*

Proof. We should notice from (47) that if $\lambda_1 \neq 1$, then $V_2^C \geq V_2^S = V_2^{dev-2}$ holds with strict inequality; i.e. $V_2^C > V_2^{dev-2}$. Also, Proposition 1 and equation (50) imply that the strict inequality $V_1^C > V_1^{dev-1}$ holds if $\delta \in (\bar{\delta}(\lambda_1), 1]$. However, such δ can exist only if $\lambda_1 \neq 0$ and $\bar{\delta}(\lambda_1) \neq 1$. Thus, $V_i^C > V_i^{dev-1}$ can hold for each i only if $\lambda \in (0, 1)$. ■

Proposition 1 and Corollary 2 together imply that $\mathcal{VPC}$ of a supply chain $\mathcal{SC}$ is strongly internally-stable if $\lambda_1 \in (0, 1)$ and $\delta \in (\bar{\delta}(\lambda_1), 1]$ are both satisfied.

4.4 Vertical Price Coordination in the Presence of an Antitrust Authority

Here, we consider a supply chain that engages in $\mathcal{VPC}$ in the presence of a socially-benevolent antitrust authority. This authority has three policy options against $\mathcal{VPC}$: (i) to take no measures and allow $\mathcal{VPC}$, (ii) to take measures that will prevent the formation of $\mathcal{VPC}$, (iii) to take measures that will destabilize $\mathcal{VPC}$ whenever it is internally stable. For our investigations, we will assume that the antitrust authority will be using the same kind of policy measures irrespective of whether it targets prevention or destabilization of $\mathcal{VPC}$. These policy measures, which we borrow from Harrington Jr. (2014) and modify for our purpose, are auditing the supply chain with a fixed probability of detection and penalizing it in case $\mathcal{VPC}$ is detected.

In more detail, we assume that in each period where the supply chain is using $\mathcal{VPC}$, there is an exogenous probability $\theta \in (0, 1)$ that the antitrust authority will discover it and will penalize each echelon to pay a fee $f \geq 0$ in the period of discovery. In line with antitrust practices, we assume that f cannot exceed a certain multiple (penalty multiple) of the total gain of the channel from $\mathcal{VPC}$, i.e., $f \leq f^m(\pi^C - \pi_1^S - \pi_2^S) = f^m a^2/16$ where f^m is an exogenously determined parameter. In addition, we let $f^m > 1/2$ to ensure that the total penalty fee paid by the channel $(2f^m a^2/16)$ in case it is convicted for $\mathcal{VPC}$ is higher than the one-period gain of the channel from $\mathcal{VPC}$. We also assume that any penalty fee paid by the channel to the antitrust authority is distributed to consumers at the end of the period the fee is collected. Given these descriptions, we will denote an antitrust policy throughout this section by a pair of parameters (θ, f) restricted to their assumed domains.

We assume that auditing the supply chain and detecting potential $\mathcal{VPC}$ is costly for society, in particular for consumers. To maintain a detection probability of $\theta \in (0, 1)$, consumers should incur, in each period, a monetary cost of $A(\theta) = m/(1 - \theta)$, where $m > 0$ is exogenously given. However, consumers should incur no such cost if $\theta = 0$; i.e., $A(0) = 0$. Notice here that the cost function $A(\theta)$ is increasing over $(0, 1)$. Moreover, perfect auditing is infinitely costly; i.e., $\lim_{\theta \to 1} A(\theta) = \infty$. Also, note that $\lim_{\theta \to 0} A(\theta) = m$. Thus, we can call the scale parameter m as the minimum cost of auditing. Even if the probability of detection is arbitrarily small, this cost exists, i.e., $\lim_{\theta \to 0} A(\theta) > 0$. This assumption is reasonable since the antitrust authority has to bear some fixed costs of auditing once it decides to audit the activities of the supply chain.

For consumers, the lifetime cost of detecting $\mathcal{VPC}$ at a likelihood θ is equal to $A(\theta)/(1 - \delta)$. We will consider this cost in all social welfare calculations whenever the antitrust authority audits the activities of the supply chain.

4.4.1 The Optimal Antitrust Policy to Prevent Vertical Price Coordination

Here, we will characterize the optimal antitrust policy under which each echelon in the supply chain will prefer Stackelberg competition to $\mathcal{VPC}$. Consider any admissible antitrust policy (θ, f). Let $\tilde{V}_i^C(\theta, f)$ and $\tilde{V}_i^S(\theta, f)$ denote for each $i \in \{1, 2\}$ the lifetime profit of echelon i under $\mathcal{VPC}$ and under the Stackelberg competition respectively. Likewise, let $\tilde{V}_{cons}^C(\theta, f)$, $\tilde{V}_{cons}^S(\theta, f), \tilde{V}_{soc}^C(\theta, f)$, and $\tilde{V}_{soc}^S(\theta, f)$ denote the lifetime welfares of the consumers and society under $\mathcal{VPC}$ and under the Stackelberg competition respectively. Then, we can formally define the problem of the antitrust authority as follows:

$$\max_{\theta, f} \ \tilde{V}_{soc}^S(\theta, f) \quad \text{subject to:} \tag{51}$$

$$\tilde{V}_i^S(\theta, f) \geq \tilde{V}_i^C(\theta, f) \ \text{ for some } i \in \{1, 2\} \tag{52}$$

$$\tilde{V}_i^S(\theta, f) \geq 0 \tag{53}$$

$$\theta \in [0, 1) \ \text{ and } f \in [0, f^m a^2/16]. \tag{54}$$

To solve the above problem, we have to first calculate $\tilde{V}_{soc}^S(\theta, f)$, $\tilde{V}_1^S(\theta, f)$, $\tilde{V}_1^C(\theta, f)$, $\tilde{V}_2^S(\theta, f)$, and $\tilde{V}_2^C(\theta, f)$ for each (θ, f) that satisfies (54). Clearly, $\tilde{V}_i^S(\theta, f) = V_i^S$ for each $i \in \{1, 2\}$; the antitrust policy does not affect the lifetime profit of any member of the channel, when the members engage in the Stackelberg competition in each period. (If the members do not ever coordinate in prices, they are never convicted for using a vertical price restraint and never have to pay any penalty fees.) However, the lifetime welfare of consumers is affected by the antitrust policy, since the cost of auditing the channel is borne by consumers, implying that

$$\tilde{V}_{cons}^S(\theta, f) = V_{cons}^S - \frac{A(\theta)}{1 - \delta}. \tag{55}$$

Therefore, when the channel that is audited always engages in Stackelberg competition, the lifetime welfare of the society becomes

$$\begin{aligned}
\tilde{V}_{soc}^S(\theta, f) &= \tilde{V}_{cons}^S(\theta, f) + \tilde{V}_1^S(\theta, f) + \tilde{V}_2^S(\theta, f) \\
&= V_{cons}^S - \frac{A(\theta)}{1 - \delta} + V_1^S + V_2^S \\
&= V_{soc}^S - \frac{A(\theta)}{1 - \delta} \\
&= \frac{1}{1 - \delta}\left(\frac{7a^2}{32} - \frac{\varphi \gamma^3 a^3}{64}\right) - \frac{m}{(1 - \delta)(1 - \theta)}.
\end{aligned} \tag{56}$$

Now, we will calculate $\tilde{V}_i^C(\theta, f)$ for each $i \in \{1,2\}$. If the channel plans to coordinate in each period until the coordination is detected, then the lifetime profit of member i, $\tilde{V}_i^C(\theta, f)$, must satisfy the following recursive equation:

$$\tilde{V}_i^C(\theta, f) = \pi_i^C + \theta\left(\delta\tilde{V}_i^S(\theta, f) - f\right) + (1-\theta)\delta\tilde{V}_i^C(\theta, f). \tag{57}$$

In the right-hand side of the above equation, the first term, π_i^C, is the profit of member i in the first period of coordination. After each period is over, detection and conviction of coordination occur with probability θ, thus the second term is the present value of the continuation profit obtained from the outcome of the (audited) Stackelberg competition, which member i is forced to enjoy with probability θ. (Notice that the penalty fee is paid at the end of the period where the conviction of coordination occurs.) The last term is the present value of its lifetime profit in case the coordination of the channel is not detected in the first period and the channel can coordinate with probability $1-\theta$ in the next period. Using (57) and the fact that $\tilde{V}_i^S(\theta, f) = V_i^S$, we can calculate

$$\tilde{V}_i^C(\theta, f) = \frac{\pi_i^C + \theta\delta V_i^S - \theta f}{1-(1-\theta)\delta}. \tag{58}$$

Now, we can turn back to our optimization problem in (51)-(54). The feasibility constraint in (53) will always hold since $\tilde{V}_i^S(\theta, f) = V_i^S$ for each $i \in \{1,2\}$ and $V_1^S, V_2^S > 0$ by equations (19)-(20). On the other hand, the feasibility constraint in (52) implies that the inequality $V_i^S \geq \tilde{V}_i^C(\theta, f)$ holds for some $i \in \{1,2\}$, further implying $\theta f \geq \min\left\{V_1^C - V_1^S, V_2^C - V_2^S\right\}$ or equivalently $\theta f \geq \min\{\lambda_1, 1-\lambda_1\}\, a^2/16$. The optimization problem in (51)-(54) then reduces to

$$\max_{\theta, f} \quad \frac{1}{1-\delta}\left(\frac{7a^2}{32} - \frac{\varphi\gamma^3 a^3}{64}\right) - \frac{m}{(1-\delta)(1-\theta)} \qquad \text{subject to:} \tag{59}$$

$$\theta f \geq \min\{\lambda_1, 1-\lambda_1\}\frac{a^2}{16} \tag{60}$$

$$\theta \in [0,1) \;\text{ and }\; f \in [0, f^m a^2/16]. \tag{61}$$

We can then state the following.

Proposition 2. *For any supply chain $\mathcal{SC} = \langle a, \gamma, \varphi, \delta, \lambda_1\rangle$, the optimal antitrust policy that prevents (P) the formation of $\mathcal{VPC}$ is a pair (θ^P, f^P) such that*

$$\theta^P = \frac{1}{f^m}\min\{\lambda_1, 1-\lambda_1\} \tag{62}$$

and

$$f^P = f^m a^2/16. \tag{63}$$

Proof. Consider any supply chain $\mathcal{SC} = \langle a, \gamma, \varphi, \delta, \lambda_1 \rangle$. Notice that the objective $\tilde{V}^S_{soc}(\theta, f)$ in (59) is decreasing in θ while it is independent of f. Therefore, to maximize $\tilde{V}^S_{soc}(\theta, f)$, we have to choose θ as small as possible. Then, condition (60) would imply that we have to increase f as high as possible. Thus, the optimal antitrust policy that prevents the formation of $\mathcal{VPC}$ is a pair (θ^P, f^P) where $f^P = f^m a^2/16$ and $\theta^P = \min\{\lambda_1, 1-\lambda_1\}/f^m$. ■

Since $\lambda_1 \in [0,1]$, the detection probability θ^P that prevents the formation of $\mathcal{VPC}$ attains its minimum value at zero if the channel members have asymmetric bargaining powers, i.e., λ_1 is equal to 0 or 1. On the other hand, θ^P attains its maximum value at $1/(2f^m)$ if the channel members have symmetric bargaining powers, i.e., $\lambda_1 = 1/2$. Notice that our earlier assumption that $f^m > 1/2$ ensures that $\theta^P \in [0,1)$.

Using the above proposition, we can calculate the lifetime welfare obtained by the society when the formation of $\mathcal{VPC}$ is (optimally) prevented.

$$\tilde{V}^S_{soc}(\theta^P, f^P) = \frac{1}{1-\delta}\left(\frac{7a^2}{32} - \frac{\varphi\gamma^3 a^3}{64} - \frac{m}{1-\min\{\lambda_1, 1-\lambda_1\}/f^m}\right). \tag{64}$$

Notice that $\tilde{V}^S_i(\theta^P, f^P) \geq 0$ if and only if

$$\frac{a^2}{64}\left(14 - \varphi\gamma^3 a\right) \geq \frac{m}{1-\min\{\lambda_1, 1-\lambda_1\}/f^m}. \tag{65}$$

Also, notice that $\tilde{V}^S_i(\theta^P, f^P)$ is increasing in f^m. This is because $\tilde{V}^S_i(\theta^P, f^P)$ is negatively affected by the cost of auditing, $A(\theta^P)/(1-\delta)$, whereas θ^P is decreasing in f^m.

4.4.2 The Optimal Antitrust Policy to Destabilize Vertical Price Coordination

Here, we will assume that the supply chain channel is already under $\mathcal{VPC}$ and calculate the optimal antitrust policy that will induce at least one of the echelons to deviate. So, consider any admissible antitrust policy (θ, f). Let $\tilde{V}^{dev-i}_j(\theta, f)$ denote for each $j \in \{1,2\}$ the lifetime profit of member j when member $i \in \{1,2\}$ unilaterally deviates from $\mathcal{VPC}$ in any period, say the first period. Likewise, let $\tilde{V}^{dev-i}_{cons}(\theta, f)$, and $\tilde{V}^{dev-i}_{soc}(\theta, f)$ denote the corresponding lifetime welfare of the consumers and society when member $i \in \{1,2\}$ unilaterally deviates from $\mathcal{VPC}$ in the first period.

If member i of the channel decides to deviate and thus end $\mathcal{VPC}$, then its lifetime profit becomes

$$\tilde{V}^{dev-i}_i(\theta, f) = \pi^{dev-i}_i + \delta\tilde{V}^S_i(\theta, f) - \theta f. \tag{66}$$

Recall from Section 4.3 that deviations are unilateral. If one of the channel members deviates from $\mathcal{VPC}$ in the first period, the other member remains to stick to its price in that period and starts the grim punishment only in the next period. So, if $\mathcal{VPC}$ of the channel is broken up, none of the members will be charging a competitive price in the period of deviation. Hence, both members are likely, when the period of deviation is over, to be convicted for $\mathcal{VPC}$ with probability θ and to pay the expected penalty fee θf as shown in equation (66).

We say that $\mathcal{VPC}$ can be destabilized by the antitrust policy (θ, f) if and only if there exists some $i \in \{1,2\}$ such that $\tilde{V}^{dev-i}_i(\theta, f) \geq \tilde{V}^C_i(\theta, f)$. We should also recall from Section 4.3 that when the channel is under $\mathcal{VPC}$, the immediate deviation profit of the supplier, π_2^{dev-2}, is equal to π_2^S since it has the disadvantage of being the first mover under vertical coordination. So, the condition $\tilde{V}_i^{dev-i}(\theta, f) \geq \tilde{V}_i^C(\theta, f)$ cannot be true if $i = 2$. Therefore, $\mathcal{VPC}$ can be destabilized by the antitrust policy (θ, f) if and only if $\tilde{V}_1^{dev-1}(\theta, f) \geq \tilde{V}_1^C(\theta, f)$, i.e., the retailer finds it gainful to unilaterally deviate from $\mathcal{VPC}$.

Now, we are ready to write the problem of the antitrust authority that aims to destabilize $\mathcal{VPC}$ of a given supply chain $\mathcal{SC}$:

$$\max_{\theta, f} \tilde{V}_{soc}^{dev-1}(\theta, f) \quad \text{subject to:} \tag{67}$$

$$\tilde{V}_1^{dev-1}(\theta, f) \geq \tilde{V}_1^C(\theta, f) \tag{68}$$

$$\theta \in [0, 1) \text{ and } f \in [0, f^m a^2/16]. \tag{69}$$

Using $\tilde{V}_1^S(\theta, f) = V_1^S$ and (58), the feasibility constraint in (68) can be rewritten as

$$\pi_1^{dev-1} + \delta V_1^S - \theta f \geq \frac{\pi_1^C + \theta\delta V_1^S - \theta f}{1 - (1 - \theta)\delta}, \tag{70}$$

which is true if and only if

$$(1 - \theta)\delta \left[\pi_1^{dev-1} - \pi_1^C + \left(\pi_1^C - \pi_1^S - \theta f\right)\right] \leq \pi_1^{dev-1} - \pi_1^C. \tag{71}$$

Substituting for π_1^{dev-1}, π_1^C, and π_1^S in the above inequality using (17), (30), and (44) respectively, we observe that the condition in (71) can be true if and only if

$$\begin{aligned} &\left(\frac{(5 + \lambda_1)(11 - \lambda_1)}{16} - 1 - \lambda_1\right) \\ &-(1 - \theta)\delta \left(\frac{(5 + \lambda_1)(11 - \lambda_1)}{16} - 1 - \frac{\theta f}{a^2/16}\right) \geq 0. \end{aligned} \tag{72}$$

On the other hand, the lifetime social welfare $\tilde{V}_{soc}^{dev-1}(\theta, f)$ obtained when the retailer (echelon 1) deviates from $\mathcal{VPC}$ can be calculated as follows:

$$\begin{aligned} \tilde{V}_{soc}^{dev-1}(\theta, f) &= \left(\pi_1^{dev-1} - \theta f\right) + \left(\pi_2^C - \theta f\right) + \left(NCG^C + 2\theta f\right) \\ &\quad + \delta \tilde{V}_{soc}^S(\theta, f) - \frac{A(\theta)}{1 - \delta} \\ &= \frac{(5 + \lambda_1)(11 - \lambda_1)a^2}{256} + (3 - \lambda_1)\frac{a^2}{16} + \frac{a^2}{8} - \frac{\varphi\gamma^3 a^3}{8} \\ &\quad + \frac{\delta}{1 - \delta}\left(\frac{7a^2}{32} - \frac{\varphi\gamma^3 a^3}{64}\right) - \frac{m}{(1 - \delta)(1 - \theta)}. \end{aligned} \tag{73}$$

Whenever $\theta = 0$, the last term in equation (73) drops since $A(0) = 0$ by assumption. Thus, we have

$$\begin{aligned}\tilde{V}_{soc}^{dev-1}(0,f) &= \frac{(5+\lambda_1)(11-\lambda_1)a^2}{256} + (3-\lambda_1)\frac{a^2}{16} + \frac{a^2}{8} - \frac{\varphi\gamma^3 a^3}{8} \\ &\quad + \frac{\delta}{1-\delta}\left(\frac{7a^2}{32} - \frac{\varphi\gamma^3 a^3}{64}\right). \end{aligned} \tag{74}$$

for any $f \in [0, f^m a^2/16]$. The problem of the antitrust authority in (67)-(69) then reduces to maximize equation (73), subject to the inequality in (72), over the set of admissible policies (θ, f) such that $\theta \in [0,1)$ and $f \in [0, f^m a^2/16]$.

Notice here that if $\mathcal{VPC}$ of the channel is internally unstable, which is the case if $\delta \in [0, \bar{\delta}(\lambda_1)$, then the optimal antitrust policy becomes $(\theta, f) = (0,0)$. The reason is that auditing $\mathcal{VPC}$ (at any $\theta > 0$) is socially costly (yielding a monetary cost $A(\theta) > 0$) while the penalty fee does not directly affect social welfare. Thus, we are interested in the destabilization of $\mathcal{VPC}$ only if the supply chain is internally stable.

Proposition 3. *For any supply chain $\mathcal{SC} = \langle a, \gamma, \varphi, \delta, \lambda_1 \rangle$, the optimal antitrust policy that destabilizes (D) $\mathcal{VPC}$ whenever it is internally stable is a pair (θ^D, f^D) such that*

$$\begin{aligned}\theta^D &= \frac{1}{2} + \frac{39 + 6\lambda_1 - \lambda_1^2}{32 f^m} \\ &\quad - \left[\left(\frac{1}{2} + \frac{39 + 6\lambda_1 - \lambda_1^2}{32 f^m}\right)^2 + \frac{(1-\delta)(39 + 6\lambda_1 - \lambda_1^2) - 16\lambda_1}{16\delta f^m}\right]^{1/2} \end{aligned} \tag{75}$$

and

$$f^D = f^m a^2/16. \tag{76}$$

Proof. Consider any supply chain $\mathcal{SC} = \langle a, \gamma, \varphi, \delta, \lambda_1 \rangle$. Define

$$\Gamma(\theta) = (1-\delta)(A-1) - \lambda_1 + \delta\left(A - 1 + \frac{16f}{a^2}\right)\theta - \delta\left(\frac{16f}{a^2}\right)\theta^2 \tag{77}$$

where

$$A = \frac{(5+\lambda_1)(11-\lambda_1)}{16}. \tag{78}$$

Notice that the destabilization condition in (72) is equivalent to $\Gamma(\theta) \geq 0$. If $\mathcal{VPC}$ is internally stable, then Proposition 1 implies that $\delta \in [\bar{\delta}(\lambda_1), 1]$ where $\bar{\delta}(\lambda_1)$ satisfies (48). This implies that $(1-\delta)(A-1) - \lambda_1 < 0$. Then, $\Gamma(\theta) \geq 0$ cannot hold when $\theta = 0$. In fact, we have $\Gamma(0) = (1-\delta)(A-1) - \lambda_1 < 0$ and $\Gamma(1) = A - 1 - \lambda_1 = (39 - 10\lambda_1 - \lambda_1^2)/16 > 0$ since $\lambda_1 \in [0,1]$. Thus, the continuity of $\Gamma(\theta)$ with respect to θ ensures that there exists θ such that

$\Gamma(\theta) = 0$. This value of θ, which we call $\theta^D(f)$, is calculated as

$$\theta^D(f) = \frac{1}{2} + \frac{39 + 6\lambda_1 - \lambda_1^2}{512f/a^2} - \left[\left(\frac{1}{2} + \frac{39 + 6\lambda_1 - \lambda_1^2}{512f/a^2}\right)^2 + \frac{(1-\delta)(39 + 6\lambda_1 - \lambda_1^2) - 16\lambda_1}{256\delta f/a^2}\right]^{1/2}. \tag{79}$$

One can easily check that the second solution to $\Gamma(\theta) = 0$ is greater than 1. Notice that $\theta^D(f)$ is decreasing in f and attains its minimum when $f = f^m a^2/16$. Let us define $\theta^D \equiv \theta^D(f^m a^2/16)$. Clearly, the lifetime social welfare $\tilde{V}_{soc}^{dev-1}(\theta, f)$ in (73) is maximized at $\theta = \theta^D$ and $f = f^D = f^m a^2/16$. Thus, the optimal antitrust policy that destabilizes $\mathcal{VPC}$ is (θ^D, f^D), which completes the proof. ■

We should notice from (63) and (76) the optimal penalty fee is the same ($f^P = f^D$) irrespective of whether the antitrust policy targets to prevent or destabilize $\mathcal{VPC}$. On the other hand, comparing (62) and (75) immediately shows that similar equality is not true for the optimal detection probabilities (θ^P and θ^D). We should also notice that the detection probability θ^D is increasing in λ_1. This is because the function $\Gamma(\theta)$ in equation (77) is a parabola (with arms opening downwards) that shifts downwards when λ_1 rises. Since θ^D is the unique root of $\Gamma(\theta)$ on the interval $[0, 1]$ with the other root being on the right of it, it follows that θ^D becomes higher if λ_1 rises. So, the higher the bargaining power of the retailer (hence its collusive profit) is, the tighter the antitrust auditing should be since the channel member that may have the incentive to deviate from $\mathcal{VPC}$ is only the retailer.

Given Proposition 3, the optimal antitrust policy that destabilizes $\mathcal{VPC}$ induces the lifetime social welfare $\tilde{V}_{soc}^{dev-1}(\theta^D, f^D)$.

4.5 The Optimal Antitrust Decision

Here, we will finally solve the problem of the antitrust authority which has to decide whether it should allow, prevent, or destabilize $\mathcal{VPC}$ of a given supply chain. The rankings of these options from the viewpoint of the society will depend on whether the anticipated coordination is internally stable or unstable since the antitrust authority requires the use of costly destabilizing measures only in the former case. Here, recall from Proposition 1 that if δ is lower than a threshold $\bar{\delta}(\lambda_1)$, then the retailer finds it optimal to immediately deviate from $\mathcal{VPC}$ once it is formed. In such a case, $\mathcal{VPC}$ becomes internally destabilized without requiring any antitrust policy. However, if δ is at least as high as $\bar{\delta}(\lambda_1)$, $\mathcal{VPC}$ becomes internally stable and it can be externally (and optimally) destabilized within our model using the antitrust policy (θ^D, f^D) characterized in Proposition 3. So, we will solve the decision problem of the antitrust authority in two separate cases depending upon whether $\mathcal{VPC}$ is internally unstable or not.

4.5.1 The Optimal Antitrust Decision When Vertical Price Coordination Is Internally Unstable

Here, we consider the case where $\mathcal{VPC}$ of a supply chain $\mathcal{SC} = \langle a, \gamma, \varphi, \delta, \lambda_1\rangle$ is internally unstable, which arises if and only if $\delta \in [0, \bar{\delta}(\lambda_1))$. In this case, the antitrust authority can reason as follows: If it allows the channel to engage in $\mathcal{VPC}$ freely, it will not need any measure to destabilize $\mathcal{VPC}$ that is anticipated to be formed since it will be internally unstable; hence, the optimal antitrust policy will be $(\theta, f) = (0,0)$ and the lifetime social welfare will be $\tilde{V}_{soc}^{dev-1}(0,0)$. On the other hand, if the antitrust authority uses the policy that prevents the formation of $\mathcal{VPC}$, the lifetime social welfare will be $\tilde{V}_{soc}^{S}(\theta^P, f^P)$. So, the antitrust authority should allow the channel to engage in $\mathcal{VPC}$ (in anticipation that it will dissolve by the immediate deviation of the retailer) if and only if $\tilde{V}_{soc}^{dev-1}(0,0) \geq \tilde{V}_{soc}^{S}(\theta^P, f^P)$, and conversely it should prevent the supply chain from engaging in $\mathcal{VPC}$ if $\tilde{V}_{soc}^{dev-1}(0,0) < \tilde{V}_{soc}^{S}(\theta^P, f^P)$. (For convenience, we assume that the antitrust authority should allow $\mathcal{VPC}$ when $\tilde{V}_{soc}^{dev-1}(0,0) = \tilde{V}_{soc}^{S}(\theta^P, f^P)$.)

Substituting for the lifetime social welfares in the first inequality using equation (64) evaluated at the policy (θ^P, f^P) and equation (74) evaluated at the policy $(\theta, f) = (0,0)$, we can easily show that $\tilde{V}_{soc}^{dev-1}(0,0) \geq \tilde{V}_{soc}^{S}(\theta^P, f^P)$ holds if and only if the following inequality holds:

$$(1-\delta)\left(28\varphi\gamma^3 a - 79 + 10\lambda_1 + \lambda_1^2\right) \leq \frac{256m}{a^2}\left(\frac{1}{1-\theta^P}\right). \tag{80}$$

Theorem 1. *Let $\mathcal{SC} = \langle a, \gamma, \varphi, \delta, \lambda_1 \rangle$ be a supply chain that finds $\mathcal{VPC}$ internally unstable; i.e., $\delta \in [0, \bar{\delta}(\lambda_1))$. Then, it is socially optimal to*

(i) allow $\mathcal{VPC}$ if condition (80) holds,

(ii) prevent $\mathcal{VPC}$ if condition (80) does not hold.

We should notice that the higher the relative abatement cost of the emissions ($\varphi\gamma^3 a$) generated by $\mathcal{VPC}$, the tighter the inequality condition (80). Since $\mathcal{VPC}$ of the channel is internally unstable, the term $1-\delta$ is confined to the interval $(1-\bar{\delta}(\lambda_1), 1]$. Moreover, λ_1 is bounded within the interval $[0,1]$. However, our model does not limit the parameters φ, γ, a, and m from above. Thus, using the inequality condition (80) we can calculate for any given supply chain $\mathcal{SC}$ a critical value $\bar{x}(\mathcal{SC})$ corresponding to each parameter $x \in \{\varphi, \gamma, a, m\}$ such that (80) holds with equality. Using these critical values we can state the following result.

Corollary 2. *Let $\mathcal{SC} = \langle a, \gamma, \varphi, \delta, \lambda_1 \rangle$ be a supply chain that finds $\mathcal{VPC}$ internally unstable. Then, it is socially optimal to allow $\mathcal{SC}$ to engage in $\mathcal{VPC}$ if and only if $x \leq \bar{x}(\mathcal{SC})$ for any $x \in \{\varphi, \gamma, a\}$ or $m \geq \bar{m}(\mathcal{SC})$.*

Corollary 2 implies that it is socially optimal to prevent a channel from engaging in $\mathcal{VPC}$, whenever it is internally unstable, if the relative abatement cost of emissions generated by $\mathcal{VPC}$ is sufficiently large or if the minimum cost of auditing is sufficiently small.

4.5.2 The Optimal Antitrust Decision When Vertical Price Coordination Is Internally Stable

Here, we consider the decision problem of the antitrust authority under the knowledge that $\mathcal{VPC}$ of the supply chain will be internally stable; i.e., it will not be broken up without a destabilizing antitrust policy. So, unlike in the previous subsection, the antitrust authority has now three policy options to deal with $\mathcal{VPC}$: (i) to follow the (null) antitrust policy $(\theta, f) = (0, 0)$, which will allow the channel to engage in $\mathcal{VPC}$ under no intervention, (ii) to follow the antitrust policy (θ^P, f^P), characterized in Proposition 2, which will prevent the channel from engaging in $\mathcal{VPC}$, or (iii) to follow the antitrust policy (θ^D, f^D), characterized in Proposition 3, which will destabilize $\mathcal{VPC}$ of the channel. Of these three options, the antitrust authority will choose the one(s) that would lead to the highest lifetime social welfare.

To solve the described choice problem of the antitrust authority, we will first make pairwise welfare comparisons corresponding to the three options. Thus, we will first compare options (i) and (ii). Formally, we will characterize a necessary and sufficient condition under which the antitrust authority should prefer allowing $\mathcal{VPC}$ in favor of preventing it. This condition is simply the inequality condition $V^C \geq \tilde{V}^S_{soc}(\theta^P, f^P)$, meaning that the lifetime social welfare obtained when the channel engages in $\mathcal{VPC}$ without facing any antitrust policy is not less than the lifetime social welfare obtained under the antitrust policy (θ^P, f^P) that ensures that the channel engages in Stackelberg competition in each period. (For convenience, we have assumed that the antitrust authority should allow $\mathcal{VPC}$ when the aforementioned condition holds with equality.)

Proposition 4. *Let $\mathcal{SC} = \langle a, \gamma, \varphi, \delta, \lambda_1 \rangle$ be a supply chain that finds $\mathcal{VPC}$ internally stable; i.e., $\delta \in [\bar{\delta}(\lambda_1), 1]$. Then, it is socially more desirable to allow $\mathcal{VPC}$ than to prevent it if and only if the following condition holds:*

$$\frac{a^2}{64}\left(7\varphi\gamma^3 a - 10\right) \leq \frac{m}{1-\theta^P} \tag{81}$$

Proof. Rewriting the inequality $V^C_{soc} \geq \tilde{V}^S_{soc}(\theta^P, f^P)$ using (39) and (56) yields the inequality in (81) after some simple arrangements. ■

The left-hand side of (81) is the social gain from preventing the formation of $\mathcal{VPC}$ if it is costless to achieve this. It is socially less desirable to prevent $\mathcal{VPC}$ than to allow it if and only if the induced social gain from preventing $\mathcal{VPC}$ is offset by the cost of auditing that appears on the right-hand side of (81). Proposition 4 immediately suggests that preventing $\mathcal{VPC}$ is socially more desirable than allowing it if and only if condition (81) does not hold. We should also notice that (81) holds if φ, γ, or a is sufficiently small. Moreover, the lower the value of f^m, the easier to satisfy this inequality condition. More interestingly, for a given m and f^m, this condition becomes most rigid when the supplier and the retailer have asymmetric bargaining powers ($\lambda_1 = 0$ or $\lambda_1 = 1$) and becomes most relaxed when they have symmetric bargaining powers ($\lambda_1 = 1/2$).

Now, we will compare options (ii) and (iii) of the antitrust authority, namely preventing and destabilizing $\mathcal{VPC}$ of the channel, respectively. Option (ii) will be socially more desirable than option (iii) if and only if the inequality condition $\tilde{V}^S_{soc}(\theta^P, f^P) \geq \tilde{V}^{dev-1}_{soc}(\theta^D, f^D)$ holds. (For convenience, we have assumed that the antitrust authority should prevent $\mathcal{VPC}$ when the above condition holds with equality.)

Proposition 5. *Let $\mathcal{SC} = \langle a, \gamma, \varphi, \delta, \lambda_1 \rangle$ be a supply chain that finds $\mathcal{VPC}$ internally stable;*

i.e., $\delta \in [\bar{\delta}(\lambda_1), 1]$. *Then, it is socially more desirable to prevent* $\mathcal{VPC}$ *than to destabilize it if and only if the following condition holds:*

$$(1-\delta)\frac{a^2}{256}\left(28\varphi\gamma^3 a - 79 + 10\lambda_1 + \lambda_1^2\right) \geq \frac{m}{1-\theta^P} - \frac{m}{1-\theta^D}. \tag{82}$$

Proof. Given any supply chain $\mathcal{SC} = \langle a, \gamma, \varphi, \delta, \lambda_1 \rangle$, preventing the formation of $\mathcal{VPC}$ that is internally stable is socially more desirable than to destabilize $\mathcal{VPC}$ that is formed if and only if $\tilde{V}^S_{soc}(\theta^P, f^P) > \tilde{V}^{dev-1}_{soc}(\theta^D, f^D)$. Substituting for the lifetime social welfares in the above inequality using equation (64) evaluated at the policy (θ^P, f^P) and equation (73) evaluated at the policy $(\theta^D, f^D) = (\theta^D, f^D)$, we can easily show that the aforementioned inequality holds if and only if (82) is satisfied. ■

Recall from Theorem 1-ii that whenever a channel finds $\mathcal{VPC}$ internally unstable, it is socially optimal to prevent $\mathcal{VPC}$ if condition (80) does not hold, implying

$$(1-\delta)\left(28\varphi\gamma^3 a - 79 + 10\lambda_1 + \lambda_1^2\right) > \frac{256m}{a^2}\left(\frac{1}{1-\theta^P}\right). \tag{$\star$}$$

Comparing condition (82) with condition ($\star$), we should notice that since $\theta^D \in (0,1)$, condition (82) holds if condition ($\star$) holds. Therefore, the set of discount factors δ at which the antitrust authority should prefer preventing $\mathcal{VPC}$ of a channel in favor of destabilizing it is not wider (and possibly narrower) when $\mathcal{VPC}$ is internally stable than when it is not. This is because destabilizing $\mathcal{VPC}$ does not require any auditing cost if (and only if) $\mathcal{VPC}$ is internally unstable. Therefore, the social welfare gain from preventing $\mathcal{VPC}$, $\tilde{V}^S_{soc} - \tilde{V}^{dev-1}_{soc}$, instead of destabilizing it attains its highest (lowest) value if $\mathcal{VPC}$ is internally unstable (stable).

Finally, we will compare options (i) and (iii) of the antitrust authority, namely allowing and destabilizing $\mathcal{VPC}$ of the channel, respectively. Option (i) will be socially more desirable than option (iii) if and only if the inequality condition $V^C_{soc} \geq \tilde{V}^{dev-1}_{soc}(\theta^D, f^D)$ holds. (For convenience, we have assumed that the antitrust authority should choose option (i) and allow $\mathcal{VPC}$ when the above condition holds with equality.)

Proposition 6. *Let* $\mathcal{SC} = \langle a, \gamma, \varphi, \delta, \lambda_1 \rangle$ *be a supply chain that finds* $\mathcal{VPC}$ *internally stable; i.e.,* $\delta \in [\bar{\delta}(\lambda_1), 1]$. *Then, it is socially more desirable to allow* $\mathcal{VPC}$ *than to destabilize it if and only if the following condition holds:*

$$(1-\delta)\left(28\varphi\gamma^3 a - 79 + 10\lambda_1 + \lambda_1^2\right) \geq \left(28\varphi\gamma^3 a - 40\right) - \frac{256\,m}{a^2(1-\theta^D)}. \tag{83}$$

Proof. Given any supply chain $\mathcal{SC} = \langle a, \gamma, \varphi, \delta, \lambda_1 \rangle$, allowing the formation of $\mathcal{VPC}$ that is internally stable is socially more desirable than destabilizing it if and only if $V^C_{soc} \geq \tilde{V}^{dev-1}_{soc}(\theta^D, f^D)$. Substituting for the lifetime social welfares in the above inequality using equation (64) and equation (73) evaluated at the policy (θ^D, f^D), we can easily show that the aforementioned inequality holds if and only if (83) is satisfied. ■

We should notice that if m becomes extremely large or a becomes extremely small while all other parameters are kept constant, condition (83) can be satisfied for any value of δ at which $\mathcal{VPC}$ becomes internally stable. On the other hand, if m is arbitrarily small, then (83) may or may not hold. If the parameters φ, γ, and/or a are high enough so that the right-hand side of (83) is positive while $m \simeq 0$, then the left-hand side of this condition becomes always smaller than the right-hand side and destabilization of $\mathcal{VPC}$ becomes a better alternative than allowing $\mathcal{VPC}$ irrespective of how far δ is close to 1 in its admissible domain. However, if the right-hand side of (83) is negative while $m \simeq 0$, then the left-hand side can be higher than the right-hand side of this condition and allowing $\mathcal{VPC}$ becomes socially more desirable provided that δ is sufficiently close to 1.

Using our results in Propositions 4-6, we are ready to state the second of our main results.

Theorem 2. *Let $\mathcal{SC} = \langle a, \gamma, \varphi, \delta, \lambda_1 \rangle$ be a supply chain that finds $\mathcal{VPC}$ internally stable; i.e., $\delta \in [\bar{\delta}(\lambda_1), 1]$. Then, it is socially optimal to*

(i) allow $\mathcal{VPC}$ if and only if conditions (81) and (83) hold.

(ii) prevent $\mathcal{VPC}$ if and only if condition (81) does not hold and (82) holds.

(iii) destabilize $\mathcal{VPC}$ if and only if neither condition (82) nor (83) holds.

Proof. Part (i) follows from Propositions 4 and 6, part (ii) follows from Propositions 4 and 5, and part (iii) follows from Propositions 5 and 6. ■

Considering Theorem 1 and Theorem 2 together, we should notice that the optimal decision of the antitrust authority to deal with $\mathcal{VPC}$ of a channel can be obtained by satisfying or unsatisfying at least two and at most three of five conditions that involve inequalities (80), (81), (82), (83), and an interval condition on δ ensuring internal stability of $\mathcal{VPC}$, i.e., $\delta \in [\bar{\delta}(\lambda_1), 1]$. In Table 3, we summarize the results in Theorem 1 and Theorem 2 in relation to these five conditions.

Table 3
Summary of Our Main Results
(✓ = satisfied; X = not satisfied)

$\delta \in [\bar{\delta}(\lambda_1), 1]$	Ineq. (80)	Ineq. (81)	Ineq. (82)	Ineq. (83)	Optimal Decision	Theorem
X	✓				Allow $\mathcal{VPC}$	1-i
X	X				Prevent $\mathcal{VPC}$	1-ii
✓		✓		✓	Allow $\mathcal{VPC}$	2-i
✓		X	✓		Prevent $\mathcal{VPC}$	2-ii
✓			X	X	Destabilize $\mathcal{VPC}$	2-iii

Inspecting how the inequalities (81), (82), and (83) are affected by the minimum cost of auditing (m) and the relative abatement cost of emissions ($\varphi\gamma^3 a$) generated by $\mathcal{VPC}$, we obtain the following corollary to Theorem 2.

Corollary 3. *Let $\mathcal{SC} = \langle a, \gamma, \varphi, \delta, \lambda_1 \rangle$ be a supply chain that finds$\mathcal{VPC}$ internally stable. Then, it is socially optimal to*

(i) allow $\mathcal{VPC}$ if the minimum cost of auditing (m) is sufficiently large.

(ii) prevent $\mathcal{VPC}$ if the minimum cost of auditing (m) is sufficiently small but the relative abatement cost of collusive emissions ($\varphi\gamma^3 a$) generated by $\mathcal{VPC}$ is sufficiently large.

(iii) destabilize $\mathcal{VPC}$ if both the minimum cost of auditing (m) and the relative abatement cost of emissions ($\varphi\gamma^3 a$) generated by $\mathcal{VPC}$ are sufficiently small.

Corollary 3 shows that the antitrust authority should always allow $\mathcal{VPC}$ if the minimum cost of auditing is sufficiently large. On the other hand, if this cost is sufficiently small, then its optimal decision will depend on the relative abatement cost of collusive emissions generated by $\mathcal{VPC}$. Recall that the output produced, and the pollution emitted, are higher in any period when the channel is under $\mathcal{VPC}$ than when it is under the Stackelberg competition. Also recall that if the channel is prevented from engaging in $\mathcal{VPC}$, then in each period the members produce the Stackelberg output whereas if they are allowed to engage in $\mathcal{VPC}$ to be destabilized by an antitrust policy, then they produce the $\mathcal{VPC}$ output in the first period and the Stackelberg output thereafter. Thus, the lifetime output, hence the lifetime pollution emitted, is lower when $\mathcal{VPC}$ is prevented than when it is destabilized. But, preventing $\mathcal{VPC}$ that is internally stable may be more costly than destabilizing it since in the former case there is no incentive for the retailer to deviate in any period and therefore the cost of auditing at a positive detection probability would add up to a social welfare loss in each period $\mathcal{VPC}$ is not detected. Certainly, if the relative abatement cost of emissions generated by $\mathcal{VPC}$ is sufficiently large, then the option of preventing $\mathcal{VPC}$ becomes socially more desirable than destabilizing it. Of course, the social ranking between these two options is reversed if the relative abatement cost of collusive emissions generated by $\mathcal{VPC}$ is sufficiently small like the minimal cost of auditing.

5 Conclusion

This paper studied the optimal antitrust policies dealing with vertical price coordination in an infinitely-lived non-green supply chain channel. Taking the abatement cost of the air pollution emitted by the channel during production into account when we calculate the social welfare function, we characterized the socially optimal antitrust policies for two distinct cases depending on whether $\mathcal{VPC}$ of the channel is internally stable or not. For the case where $\mathcal{VPC}$ is internally unstable, we found that it is socially optimal to allow $\mathcal{VPC}$ (in anticipation that it will be dissolved by the deviation of the retailer) if the relative abatement cost of $\mathcal{VPC}$ emissions is sufficiently small or if the minimum cost of auditing is sufficiently large. On the other hand, in the case where the $\mathcal{VPC}$ is stable, the antitrust authority has an additional option to consider, which is destabilizing $\mathcal{VPC}$ externally. To obtain the pairwise social rankings between the three options of the antitrust authority, which are allowing, preventing, and destabilizing $\mathcal{VPC}$, we calculated the optimal antitrust policies that either prevent or destabilize $\mathcal{VPC}$, and using the aforementioned social rankings we obtained necessary and sufficient conditions under which either allowing, preventing, or destabilizing $\mathcal{VPC}$ can become socially optimal. These conditions suggested that it is socially optimal to destabilize (allow) $\mathcal{VPC}$ if both the minimum cost of auditing and the relative abatement cost of $\mathcal{VPC}$ emissions are sufficiently small (large) and to prevent $\mathcal{VPC}$ otherwise.

The model we constructed in this paper can be extended in several directions. First, one can change the assumption about the abatement cost burden in the society. We assumed that the abatement cost of pollution emitted by the channel is fully borne by consumers. This assumption may be argued to be plausible in countries where the legal enforcement of green technologies and abatement of emissions is weak. However, one can also study the case where the cost of abatement is accrued partially or completely to the polluting member of the channel, the supplier. Clearly, whether the supplier or consumers pay the bill of the abatement costs does not directly affect the social welfare function, since we conventionally modeled this function to be the linear sum of the channel profits and consumers surplus net of the abatement costs and auditing costs associated with the antitrust policy in effect. However, the distribution of the abatement costs among the members of the society would indirectly affect social welfare through its effect on the output supplied, and profit earned, by the channel both under the case of competition and under the case of vertical price coordination (with or without the deviation of the retailer). Consequently, a modeling change in the cost of abatement would affect all social welfare comparisons conducted by the antitrust authority in evaluating the desirability of its policy alternatives.

A second direction to extend our model is to endogenize the bargaining powers of the channel members. In our model, we assumed that these powers were exogenously given. Given this assumption, we found that the relative bargaining power of the retailer (λ_1) affects the inequality conditions (80), (81), (82), and (83) as well as the interval condition on the common discount factor δ related to the internal stability of vertical price coordination, which are used by the antitrust authority to calculate the socially-optimal antitrust decision. Once the channel members realize that their bargaining powers may be pivotal in affecting the antitrust decision and in consequence their equilibrium welfares, there is no reason why they should stick to a pair of exogenously-ascribed bargaining power coefficients in situations where they could mutually gain from using another pair of coefficients and thus directing the antitrust decision to their best interests. Future studies may consider such situations and the question of whether/how the channel members would coordinate between themselves to optimally choose their bargaining powers in response to the optimal antitrust decisions.

A third direction for extension is to investigate the effect of market coordination devices other than linear prices. Our model assumed that the channel members use linear prices, which require that each ordered unit is sold at the same price to the buyer. Linear prices make the most sense when the supplier faces many (risk-averse) retailers in the presence of an uncertain demand function since the cooperation between the retailers to alleviate demand shocks may prevent total price discrimination by the supplier. However, in our model involving a single retailer, the supplier may profitably consider other allocation mechanisms as well. Linear prices not only limit the maximal welfare of the channel against the welfare of consumers, but also limit the distribution of welfare among the channel members. For example, under the Stackelberg competition with linear prices, the supplier cannot earn more than two-thirds of the channel's profit despite its mover advantage. (In fact, the supplier could only get half of the channel's profit if the demand curve were of a (non-linear) form with constant price elasticity.) In contrast, under a two-part tariff that involves a wholesale price equal to the marginal cost and a fixed fee paid (by the retailer to the supplier), the supplier can extract all channel profits through the fixed fee. (The vertical externality, or double marginalization, can also be eliminated if the supplier uses resale-price maintenance to constrain the retailer's margin to zero or if the supplier and retailer agree on a revenue or profit sharing contract.) The resulting change in the competitive profits of the channel members due to their switching from a linear price to a two-part-tariff would also affect their profits from vertical price coordination as long as the bargaining process takes the competitive profits as their disagreement payoffs and maximizes the Nash product of their profit gains from agreement. The change in competition and coordination profits would affect all social welfare calculations, and consequently the optimal antitrust policies and decisions, too.

A fourth direction for future research is to extend our work by considering a type of coordination distinct from non-cooperative (tacit) coordination. Following the treatment of Friedman (1971) on collusion, we modeled coordination in our model as the non-cooperative equilibrium of an infinitely repeated game where the coordinating members of the channel need not make an explicit agreement. Another alternative is to consider the cooperative coordination of the supply chain, a possibility first modeled by Martin (2006) in the context of collusion for a horizontal industry where the oligopolistic firms, to make unilateral defections unprofitable, expand total output above the level that would be chosen by a monopolistic supplier. While such an expansion would increase consumer surplus in our model at the expense of a reduction in the channel members' profits from coordination, it would also increase the pollution generated by the supply chain and the corresponding abatement cost accrued to society, leading to a welfare tradeoff. One can investigate how this tradeoff might affect the social desirability of allowing, preventing, or destabilizing the cooperative price coordination of the supply chain in comparison to the case where the coordination is non-cooperative as assumed in our model.

References

Bartolini D, Zazzaro A (2011) The impact of antitrust fines on the formation of collusive cartels. The B.E. Journal of Economic Analysis & Policy, 11:1, Article 58.

Besanko D, Spulber DF (1989) Antitrust enforcement under asymmetric information. Economic Journal, 99, 408–425.

Besanko D, Spulber DF (1990) Are treble damages neutral? Sequential equilibrium and private antitrust enforcement. American Economic Review, 80, 870–887.

Blair RD, Kaserman DL (1978) Uncertainty and the incentive for vertical integration. Southern Economic Journal, 45, 266–272.

Block MK, Nold FC, Sidak JG (1981) The deterrent effect of antitrust enforcement. Journal of Political Economy, 89, 429–445.

Bolatto S, Lambertini L (2017) Collusive vertical relations. Working Papers wp1103, Dipartimento Scienze Economiche, Universita' di Bologna.

Bork RH (1954) Vertical Integration and the Sherman Act: The Legal History of an Economic Misconception, University of Chicago Law Review, 22:1, 157–201.

Burstein ML (1960) A theory of full-time forcing. Northwestern University Law Review, 55, 62–95.

Cachon GP, Kök AG (2010) Competing manufacturers in a retail supply chain: On contractual form and coordination. Management Science, 56:3, 571–589.

Cooper JC, Froeb LM, O'Brien D, Vita MG (2005) Vertical antitrust policy as a problem of inference. International Journal of Industrial Organization, 23:7-8, 639–664.

Corbett CJ, Zhou D, Tang CS (2004) Designing supply contracts: Contract type and information asymmetry. Management Science, 50:4, 550–559.

Das K, Posinasetti NR (2015) Addressing environmental concerns in closed loop supply chains design and planning. International Journal of Production Economics, 163, 34–47.

Dixit A (1983) Vertical integration in a monopolistically competitive industry. International Journal of Industrial Organization, 1, 63–78.

Feng Q, Lu LX (2013) Supply chain contracting under competition: Bilateral bargaining vs. Stackelberg. Production and Operations Management, 22:3, 661–675.

Frezal S (2006) On optimal cartel deterrence policies. International Journal of Industrial Organization, 24, 1231–1240.

Friedman JW (1971) A noncooperative equilibrium for supergames. Review of Economic Studies, 38, 1–12.

Gallini NT, Winter RA (1983) On vertical control in monopolistic competition. International Journal of Industrial Organization, 1, 275–286.

Guillen-Gosalbez G, Grossmann IE (2009) Optimal design and planning of sustainable chemical supply chains under uncertainty. AIChE Journal, 55:1, 99–121.

Hafezalkotob A (2015) Competition of two green and regular supply chains under environmental protection and revenue seeking policies of government. Computers & Industrial Engineering, 82, 103–114.

Hafezalkotob A (2018) Direct and indirect intervention schemas of government in the competition between green and non-green supply chains. Journal of Cleaner Production, 170, 753–772.

Harrington Jr. JE (2014) Penalties and the deterrence of unlawful collusion, Economics Letters, 124, 33–36.

Huang H, Ke H, Wang L (2016) Equilibrium analysis of pricing competition and cooperation in supply chain with one common manufacturer and duopoly retailers. International Journal of Production Economics, 178, 12–21.

Li X, Chen J, Ai X (2019) Contract design in a cross-sales supply chain with demand information asymmetry. European Journal of Operational Research, 275:3, 939–956.

Liu Y, Li J, Ren W, Forrest JYL (2020) Differentiated products pricing with consumer network acceptance in a dual-channel supply chain. Electronic Commerce Research and Applications, 39, 100915.

Lu LYY, Wu CH, Kuo TC (2007) Environmental principles applicable to green supplier evaluation by using multi-objective decision analysis. Sustainable Design and Manufacture, International Journal of Production Research, 45:18-19, 4317–4331.

Madani SR, Rasti-Barzoki M (2017) Sustainable supply chain management with pricing, greening and governmental tariffs determining strategies: A game-theoretic approach. Computers & Industrial Engineering, 105, 287–298.

Martin S (2006) Competition policy, collusion, and tacit collusion. International Journal of Industrial Organization, 24, 1299–1332.

Nagurney A, Liu Z, Woolley T (2006) Optimal endogenous carbon taxes for electric power supply chains with power plants. Mathematical and Computer Modelling, 44:9–10, 899–916.

Nagurney A, Toyasaki F (2003) Supply chain supernetworks and environmental criteria. Transportation Research Part D: Transport and Environment, 8:3, 185–213.

Nash JF (1950) The bargaining problem. Econometrica, 18, 155–162.

Neto JQF, Bloemhof-Ruwaard JM, van Nunen JAEE, van Heck E (2008) Designing and evaluating sustainable logistics networks. International Journal of Production Economics, 111:2, 195–208.

Nordhaus WD (2008) A Question of Balance: Weighing the Options on Global Warming Policies. Yale University Press, New Haven.

O'Brien DP (2008) The antitrust treatment of vertical restraints: Beyond the possibility theorems. In: Ljusberg J-E (Ed.), The Pros and Cons of Vertical Restraints. The Pros and Cons Series of the Swedish Competition Authority, Volume 7, pp. 40–101.

O'Brien D, Shaffer G (1992) Vertical control with bilateral contracts. Rand Journal of Economics, 23, 299–308.

Ordover J, Saloner G, Salop S. (1990) Equilibrium vertical foreclosure. American Economic Review, 80, 127–42.

O'Ryan R, Nasirov S, Osorio H (2023) Assessment of the potential impacts of a carbon tax in Chile using dynamic CGE model. Journal of Cleaner Production, 403, 136694.

Pistikopoulos EN, Hugo A (2005) Environmentally conscious long-range planning and design of supply chain networks. Journal of Cleaner Production, 13:15, 1428–1448.

Ramudhin A, Chaabane A, Paquet M (2010) Carbon market sensitive sustainable supply chain network design. International Journal of Management Science and Engineering Management, 5:1, 30–38.

Rey P, Tirole J (1986) The logic of vertical restraints, American Economic Review, 76, 921–939.

Roth A (1979) *Axiomatic Models of Bargaining*. Berlin: Springer-Verlag.

Schmalensee R (1973) A note on the theory of vertical integration. Journal of Political Economy, 81:2, 442–449.

Sheu J-B, Chou Y-H, Hu C-C, 2005. An integrated logistics operational model for green-supply chain management. Transportation Research Part E: Logistics and Transportation Review, 41:4, 287–313.

Souam S (2001) Optimal antitrust policy under different regimes of fines. International Journal of Industrial Organization, 19, 1–26.

Spengler J (1950) Vertical integration and anti-trust policy. Journal of Political Economy, 58, 347–352.

Vernon JM, Graham DA (1971) Profitability of monopolization by vertical integration. Journal of Political Economy, 79:4, 924-925.

Wang L, Cai G, Tsay AA, Vakharia AJ (2017) Design of the reverse channel for remanufacturing: Must profit-maximization harm the environment? Production and Operations Management, 26:8, 1585–1603.

Wang Q, Hong X, Gong YY, Chen WA (2020) Collusion or not: The optimal choice of competing retailers in a closed-loop supply chain. International Journal of Production Economics, 225, 107580.

WHO (2022) World Health Organization Fact Sheet. https://www.who.int/news-room/fact-sheets/detail/ambient-(outdoor)-air-quality-and-health?

Yang D, Wang J, Song D (2019) Channel structure strategies of supply chains with varying green cost and governmental interventions. Sustainability, 12:1, 113.

Yu W, Wang Y, Feng W, Bao L, Han R (2022) Low carbon strategy analysis with two competing supply chain considering carbon taxation. Computers & Industrial Engineering, 169, 108203.

Zheng P, Pei W, Pan W (2023) Impact of different carbon tax conditions on the behavioral strategies of new energy vehicle manufacturers and governments - A dynamic analysis and simulation based on prospect theory. Journal of Cleaner Production, 407, 137132.