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EXPLOITATION OF COLLECTIVE BARGAINING IN THE LABOR MARKET

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ABSTRACT. We consider a collective bargaining model for the determination of labor hours and wages in the presence of a government that can tax corporate income. The original form of this model was developed by Del Rey et al. (2022) in the absence of taxes. Using our modified model with taxes, we investigate whether workers or the firm can exploit the bargaining equilibrium, with the help of Sertel's (1992a) pre-donation idea, by committing to transfer a part of their would-be payoffs to the other party. We show that pre-donation by any party cannot affect the equilibrium hours or the social welfare while it may affect the welfare distribution in the economy if and only if the corporate tax rate is positive. In case this tax rate is positive, we find that making pre-donation to the firm is beneficial for workers. Moreover, the optimal pre-donation of workers enables them to fully extract the tax revenue that the government could obtain in the absence of pre-donation while keeping the welfare of the firm unchanged. On the other hand, making pre-donation to workers is harmful to the firm and beneficial for the government while having no effect on the welfare of workers.

KEYWORDS: Wage determination; Nash bargaining; pre-donation; exploitation.

JEL CODES: C78; E24; J31.

1 Introduction

Labor markets in many countries heavily use collective bargaining protocols for the determination of wages.¹ Accordingly, following the seminal works of McDonald and Solow (1981), Grout (1984), and Ellis and Fender (1985), researchers in labor economics have extensively studied the equilibrium predictions of firm or industry- level collective bargaining models. Such models typically involve a two-player single-shot cooperative bargaining game, where a labor union (as one of the players) bargains on behalf of a set of workers collectively with a single firm or industry (the other player) over the hourly wage rates and, sometimes, the total work hours. The bargaining in these models is typically resolved according to a generalized Nash rule (Nash, 1950; Roth, 1979) that maximizes the product of the two players' net agreement payoffs raised to some fixed and asymmetric bargaining powers. This bargaining rule selects a Pareto optimal solution in the bargaining set of payoffs facing the players; i.e., it would not be possible to make one of the bargaining parties better off without making the other party worse off if the players mutually agreed to switch to any other payoff allocation (implied by any other bargaining rule).

Accordingly, it is generally believed that as long as the players' bargaining set of payoffs is correctly specified within the model so that each payoff allocation on the frontier of this set is consistent with efficient utilization of labor resources, no Pareto improvement (even in the weak sense) should be possible on the bargaining solution implied by the generalized Nash rule (or any other bargaining rule satisfying Pareto optimality). However, this belief is true only in cases where the bargaining set facing the players is the same before and during the bargaining process. An interesting work of Sertel (1992a) showed that alterations of bargaining sets via pre-donations by some players before bargaining takes place may yield weak Pareto gains; i.e., the pre-donor may strictly gain from his/her pre-donation while the recipient's payoff would remain the same. In this paper, we investigate whether pre-

¹According to the 2021 database of OECD on Institutional Characteristics of Trade Unions, Wage Setting, State Intervention and Social Pacts (ICTWSS), the collective bargaining coverage rate between 2009-2018 scattered around an average annual value of 46.5 (with a yearly standard variation of 33.9) for 39 OECD countries.

donation may yield weak or strict Pareto gains in the labor market as well.

Before proceeding any further, it will be useful to give some background on the idea of pre-donation. In a two-player bargaining game, unilateral pre-donation is defined as a pre-bargaining commitment for any player to give, to the other player, a fixed share of her would-be payoffs in the bargaining set. Sertel's (1992a) work showed that in two-player bargaining problems where the bargaining set has linear and asymmetric frontiers, the Nash bargaining rule can be manipulated by the player whose ideal payoff in the bargaining set is higher; moreover, the bargaining outcome resulting under the optimal pre-donation of this strategic player would coincide with the outcome of the Talmudic division rule in the absence of any pre-donation. Several studies including Sertel (1992b), Orbay (2003) and Akin et al. (2011) showed that the manipulability result of Sertel (1992a) can be extended to domains under other bargaining rules, involving the Kalai and Smorodinsky rule (1975), the Maschler and Perles rule (1981), and various utilitarian rules (Kalai, 1985). Akyol (2008) further showed that incentives for pre-donation also exist in two-player non-cooperative games where players adopt either Stackelberg or Nash strategies in bilaterally choosing their pre-donation shares. Recently, Saglam (2023a) characterized conditions on two-player bargaining problems and rules under which unilateral pre-donation leads to Pareto welfare gains.

In addition to the above works that restricted their attention to abstract game settings, there are also works that studied the effect of pre-donation in microeconomic problems or environments like we currently do in our paper. As such, Orbay (2003) studied a cartel's problem of dividing a market between two firms that are asymmetric concerning their marginal costs of production and showed that the efficient firm has the incentive to pre-donate under both the Kalai and Smorodinsky rule (1975) and the Maschler and Perles rule (1981). More recently, Saglam (2022) incorporated pre-donation into the optimal regulation of a natural monopoly under asymmetric cost information and showed that the regulated monopoly has the incentive to pre-donate while no such incentive exists for consumers. He also showed that the optimal pre-donation of the monopoly may lead to ex-ante (and possibly ex-post) Pareto gains in welfare. In a similar vein, Saglam (2023b) studied whether similar Pareto gains from pre-donation may arise in a vertically related industry where two downstream producers of a homogenous good

jointly bargain, before they compete in quantities, with an upstream producer over a linear input price using the generalized Nash rule. Like Saglam (2022), Saglam (2023b) finds that disposition of players towards pre-donation is asymmetric. He finds that the downstream industry has no incentive to make pre-donation whereas the upstream producer has such an incentive provided that its bargaining power is sufficiently small. Moreover, the optimal pre-donation of the upstream producer yields strict Pareto gains.

We can now turn back to our problem of interest in this paper. In the next section, we will present the basic structures of a collective bargaining model for the determination of wages and hours in the labor market and we will later investigate using this model whether and how workers and/or the firm may use pre-donation transfers that are unobservable to the government to exploit the bargaining equilibrium. To strengthen our results under pre-donation, we desire the equilibrium predictions of our bargaining model in the absence of pre-donation to be in line with empirical evidence. Thus, we choose to borrow the main structures of our model from Del Rey et al. (2022) that can successfully explain the presence of hump-shaped relationship between hours and hourly wages, empirically observed for several countries by a number of authors involving Yurdagul (2017) and Bick et al. (2022). Briefly, the model of Del Rey et al. (2022) includes a firm and a set of workers that act as a union. The payoffs of both workers and the firm are linear in the wage rate ω and nonlinear in the number of working hours h supplied by workers, while both ω and h are determined through a collective bargaining process between workers and the firm according to the generalized Nash rule. We extend this bargaining model of Del Rey et al. (2022) by adding a government sector that may charge corporate taxes on the firm's operating profit. Indeed, the presence or absence of these taxes will uniquely identify whether a non-zero amount of pre-donation can arise in the equilibrium of our bargaining model.

Our results show that pre-donation by either workers or the firm can never affect the equilibrium levels of the working hours, output, and social welfare in the economy. However, pre-donation may affect the welfare distribution in the economy only in the case where the corporate tax rate is positive. In such a case, any amount of pre-donation by the firm would reduce its net bargaining payoff and increase the tax revenue of the government while leading to no change in the net bargaining payoff of workers. Therefore, the

optimal pre-donation of the firm is zero. On the other hand, workers have the incentive for making pre-donation. By increasing their pre-donation to the firm as large as possible, they can increase their equilibrium payoff and reduce the government's equilibrium tax revenue while yielding no change in the firm's equilibrium payoff. Interestingly, the optimal pre-donation of workers, under some mild participation constraints, enables them to fully extract the tax revenue the government could obtain in the absence of pre-donation.

The remainder of the paper is organized as follows. Section 2 presents the basic structures and Section 3 contains our results. Finally, Section 4 concludes.

2 Basic Structures

We will incorporate pre-donation into a modified version of the bargaining model introduced by Del Rey et al. (2022) for studying wage and hour relationship in the labor market. We will first define the payoffs in the absence of pre-donation. In case the bargaining ends with an agreement, the payoff of workers is

$$W(h, \omega) = h\omega - xh^\mu \quad (1)$$

where h denotes the hours of work, ω denotes the hourly wage, and xh^μ stands for the cost of working h hours, with $x > 0$ and $\mu > 1$ being fixed parameters. In case bargaining ends with disagreement, workers incur a fixed cost of job search, denoted by $F > 0$. Hence, the net bargaining payoff of workers is $W + F$ in case bargaining ends with an agreement.

On the other hand, the agreement payoff of the firm is given by

$$J(h, \omega) = (1 - \tau)(ah^\lambda - h\omega - T) \quad (2)$$

where ah^λ is the output produced using h hours of work, with $a > 0$ and $\lambda < 1$ being fixed parameters. Above, T denotes a fixed cost of hiring workers and $\tau \in [0, 1]$ stands for the corporate (income) tax rate. Thus, J is the after-tax bargaining payoff of the firm if an agreement is reached. In case a disagreement occurs, the firm does not produce and its income becomes

zero. Hence, the net bargaining payoff of the firm is J in case bargaining ends with an agreement.

Workers and the firm determine the hourly wage ω and the hours of work h through collectively bargaining using the generalized Nash rule (Nash, 1950; Roth, 1979). In the absence of pre-donation, workers and the firm face the following problem:

$$\max_{h, \omega} [W(h, \omega) + F]^\beta [J(h, \omega)]^{1-\beta} \quad (3)$$

where $\beta \in [0, 1]$ and $1 - \beta$ denote the bargaining powers of workers and the firm, respectively.

Whenever the corporate tax rate, τ , is zero in the above problem (in the definition of J), we obtain the bargaining problem presented by Del Rey et al. (2022). In the next section, we will further extend the above bargaining problem (defined for any value of $\tau \in [0, 1]$) by allowing either workers or the firm to pre-donate a pre-determined fraction, $\theta \in [0, 1)$, of their net bargaining payoff, at all possible values h and ω , to the other party before the bargaining occurs. We will assume that pre-donation transfers are not observable to the government and therefore cannot be taxed.

3 Results

We will consider separately two cases in which either workers or the firm will pre-donate unilaterally a fraction of their net agreement payoffs to the other party before the bargaining takes place.

3.1 Workers Pre-donate Unilaterally

Here, we assume that workers pre-donate unilaterally a constant fraction, $\theta \in [0, 1)$, of their net agreement payoff, at all possible values h and ω , to the firm before the bargaining occurs. After pre-donation, the net agreement payoffs of workers and the firm will be

$$V^W(\theta, h, \omega) = (1 - \theta)[W(h, \omega) + F] \quad (4)$$

and

$$V^F(\theta, h, \omega) = J(h, \omega) + \theta[W(h, \omega) + F] \quad (5)$$

respectively. In equation (5), we observe that the pre-donation transfer $\theta[W(h, \omega) + F]$ received by the firm from workers is not taxed. This is because we assumed that pre-donation transfers are not observed by the government. Given equations (4)-(5), the firm and workers will face the bargaining problem

$$\max_{h, \omega} [V^W(\theta, h, \omega)]^\beta [V^F(\theta, h, \omega)]^{1-\beta}, \quad (6)$$

where $\beta \in [0, 1]$ is pre-determined and fixed. The solution to this problem will be $h^*(\theta)$ and $\omega^*(\theta)$, implying the equilibrium payoffs $V^{W^*}(\theta) \equiv V^W(\theta, h^*(\theta), \omega^*(\theta))$ and $V^{F^*}(\theta) \equiv V^F(\theta, h^*(\theta), \omega^*(\theta))$ for workers and the firm, respectively.

Notice that the (exogenously-determined and fixed) fraction τ of the firm's gross profit is accrued to the government as an income tax. Thus, the government's equilibrium payoff is

$$V^{G^*}(\theta) = \tau [a(h^*(\theta))^\lambda - h^*(\theta)\omega^*(\theta) - T] = \frac{\tau}{1-\tau} J(h^*(\theta), \omega^*(\theta)). \quad (7)$$

Having defined all equilibrium payoffs, we can now write the equilibrium social welfare as

$$SW^*(\theta) = V^{G^*}(\theta) + V^{F^*}(\theta) + V^{W^*}(\theta). \quad (8)$$

Using (4), (5), and (7), we can rewrite (8) as

$$SW^*(\theta) = a(h^*(\theta))^\lambda - T + F - x(h^*(\theta))^\mu. \quad (9)$$

Now, we turn to the optimization problem of workers. Before bargaining takes place, workers should optimally choose the value of the pre-donation parameter $\theta \in [0, 1]$. However, their choice should be acceptable to both the firm and the government: the firm should not reject the pre-donation of workers on the ground that accepting it would make them worse off whereas the government should not prohibit bargaining between the firm and workers on the ground that its tax income becomes negative. Given these two constraints, the optimization problem of workers, before bargaining takes place, is given by

$$\max_{\theta \in [0, 1]} V^{W^*}(\theta) \quad \text{s.t.} \quad V^{F^*}(\theta) \geq V^{F^*}(0) \quad \text{and} \quad V^{G^*}(\theta) \geq 0. \quad (10)$$

(Since the government is not among the bargaining parties, workers do not need to set the government's reservation income to the tax income $V^{G^*}(0)$ the government would enjoy in a bargaining situation without pre-donations.) The solution θ^* to the above problem will yield the equilibrium payoffs $V^{G^*}(\theta^*)$, $V^{F^*}(\theta^*)$, $V^{W^*}(\theta^*)$, and $SW^*(\theta^*)$ for the government, the firm, workers, and society, respectively. Using backward induction, workers will first calculate the solution $(h^*(\theta), \omega^*(\theta))$ to the bargaining problem in (6) for each value of $\theta \in [0, 1)$ and thereby construct the equilibrium payoffs in their optimization problem (10).

For the bargaining problem (6), the first-order condition that determines the wage rate is

$$\beta(J + \theta(W + F)) \frac{\partial W}{\partial \omega} = -(1 - \beta)(W + F) \frac{\partial J + \theta W}{\partial \omega}. \quad (11)$$

Inserting $\partial W / \partial \omega = h$ and $\partial J / \partial \omega = -(1 - \tau)h$ into above, we obtain

$$\beta(J + \theta(W + F)) = (1 - \beta)(1 - \tau - \theta)(W + F). \quad (12)$$

Substituting (1) and (2) into (12) yields the wage equation

$$\omega(\theta, h) = \frac{\beta(1 - \tau)}{1 - \tau - \theta} \left(ah^{\lambda-1} - \frac{T}{h} \right) + \left(1 - \frac{\beta(1 - \tau)}{1 - \tau - \theta} \right) \left(xh^{\mu-1} - \frac{F}{h} \right). \quad (13)$$

Likewise, the first-order condition that determines the hours of work is

$$\beta(J + \theta(W + F)) \frac{\partial W}{\partial h} = -(1 - \beta)(W + F) \frac{\partial J + \theta W}{\partial h}. \quad (14)$$

Substituting (12) into above, we obtain

$$(1 - \tau - \theta) \frac{\partial W}{\partial h} = - \frac{\partial J + \theta W}{\partial h} \quad (15)$$

or

$$(1 - \tau) \frac{\partial W}{\partial h} = - \frac{\partial J}{\partial h}. \quad (16)$$

Using (1) and (2), we obtain

$$\frac{\partial W}{\partial h} = \omega - \mu x h^{\mu-1} \quad (17)$$

and

$$\frac{\partial J}{\partial h} = (1 - \tau)(\lambda ah^{\lambda-1} - \omega). \quad (18)$$

Substituting (17) and (18) in (16), we obtain

$$\mu x h^{\mu-1} = \lambda ah^{\lambda-1}, \quad (19)$$

implying

$$h^*(\theta) = \left(\frac{a\lambda}{\mu x} \right)^{\frac{1}{\mu-\lambda}}. \quad (20)$$

Notice that the equilibrium level of working hours $h^*(\theta)$ in (20) is independent of both θ and τ , i.e., insensitive to whether or how much workers pre-donate before bargaining takes place and at what rate the payoff of the firm is taxed by the government. Thus, we have obtained the same equilibrium level of working hours as in Del Rey et al. (2022), where $\theta = 0$ and $\tau = 0$.

Let $h^* \equiv h^*(\theta)$. Substituting this into equation (13) yields the equilibrium wage rate $\omega^*(\theta) \equiv \omega(\theta, h^*(\theta))$, where

$$\begin{aligned} \omega^*(\theta) &= \frac{\beta(1-\tau)}{1-\tau-\theta} \left(a(h^*)^{\lambda-1} - \frac{T}{h^*} \right) + \\ &\quad \left(1 - \frac{\beta(1-\tau)}{1-\tau-\theta} \right) \left(x(h^*)^{\mu-1} - \frac{F}{h^*} \right). \end{aligned} \quad (21)$$

Apparently, the equilibrium wage rate $\omega^*(\theta)$ depends on θ . On the other hand, using $h^* \equiv h^*(\theta)$ and equation (9), we can show that the equilibrium social welfare $SW^* \equiv SW^*(\theta)$ is independent of θ , as we have

$$SW^* = a(h^*)^\lambda - T + F - x(h^*)^\mu. \quad (22)$$

Using (21) and (22), we can then calculate the equilibrium payoffs of workers, the firm, and the government as

$$\begin{aligned} V^{W^*}(\theta) &= (1-\theta)[W(h^*, \omega^*(\theta)) + F] \\ &= (1-\theta)[h^*\omega^*(\theta) - x(h^*)^\mu + F] \\ &= (1-\theta)\frac{\beta(1-\tau)}{1-\tau-\theta} SW^*, \end{aligned} \quad (23)$$

$$\begin{aligned}
V^{F^*}(\theta) &= J(h^*, \omega^*(\theta)) + \theta[W(h^*, \omega^*(\theta)) + F] \\
&= (1 - \tau) [a(h^*)^\lambda - h^* \omega^*(\theta) - T] + \frac{\theta}{1 - \theta} V^{W^*}(\theta) \\
&= (1 - \tau) \left[1 - \frac{\beta(1 - \tau)}{1 - \tau - \theta} \right] SW^* + \frac{\theta \beta(1 - \tau)}{1 - \tau - \theta} SW^* \\
&= (1 - \tau)(1 - \beta) SW^*, \tag{24}
\end{aligned}$$

and

$$\begin{aligned}
V^{G^*}(\theta) &= \frac{\tau}{1 - \tau} J(h^*, \omega^*(\theta)) \\
&= \tau [a(h^*)^\lambda - h^* \omega^*(\theta) - T] \\
&= \tau \left[1 - \frac{\beta(1 - \tau)}{1 - \tau - \theta} \right] SW^* \tag{25}
\end{aligned}$$

respectively.

If $\theta = 0$, then the above equilibrium payoffs become $V^{W^*}(0) = \beta SW^*$, $V^{F^*}(0) = (1 - \tau)(1 - \beta) SW^*$, and $V^{G^*}(0) = \tau(1 - \beta) SW^*$. So, whenever workers make no pre-donation, they obtain the fraction β of the total social welfare and the remaining fraction, $1 - \beta$, is shared by the firm and the government with their shares being $1 - \tau$ and τ respectively.

We should here remember that the equilibrium payoff of the firm is independent of pre-donation as is the equilibrium social welfare, since we have $V^{F^*}(\theta) = (1 - \tau)(1 - \beta) SW^*$. Moreover, if the corporate tax rate becomes zero, i.e. $\tau = 0$, the equilibrium payoffs of workers and the government also become independent of θ , as we will have $V^{W^*}(\theta) = \beta SW^*$, $V^{F^*}(\theta) = (1 - \beta) SW^*$, and $V^{G^*}(\theta) = 0$. Thus, whenever our model reduces to the model of Del Rey et al. (2022), where the firm does not pay any income taxes ($\tau = 0$), the pre-donation of workers does not affect the equilibrium payoff distribution under the generalized Nash bargaining. On the other hand, whenever $\tau > 0$, equations (23) and (25) show that the equilibrium payoffs of workers and the government are affected by the fraction θ of payoffs pre-donated by workers. We state these results below.

Proposition 1. *The pre-donation of workers (their choice of θ) can never affect the equilibrium social welfare, SW^* , and the equilibrium payoff of the firm, $V^{F^*}(\theta)$. However, it affects the equilibrium payoffs of workers and the government, $V^{W^*}(\theta)$ and $V^{G^*}(\theta)$, if and only if $\tau > 0$.*

Proposition 1 implies that whenever $\tau > 0$, the pre-donation of workers has a non-zero effect on their equilibrium payoff coupled with an opposite effect of equal size on the equilibrium payoff of the government; i.e. for any non-negative amount of change in θ , denoted by $\Delta\theta \geq 0$, the changes in the equilibrium payoffs satisfy $\Delta V^{W^*}(\theta) = -\Delta V^{G^*}(\theta) \neq 0$ and $\Delta V^{F^*}(\theta) = \Delta SW^* = 0$.

To explore how the equilibrium payoffs of workers and the firm, $V^{W^*}(\theta)$ and $V^{G^*}(\theta)$, are actually affected by a change in θ whenever $\tau > 0$, we need to make the following assumption.

Assumption 1. $SW^* > 0$.

The above assumption says that the equilibrium social welfare is positive. Equations (20) and (22) imply that this assumption is independent of θ . Since, $SW^* = a(h^*)^\lambda - T + F - xa(h^*)^\mu$ and $h^* = (a\lambda/(\mu x))^{1/(\mu-\lambda)}$, the assumption $SW^* > 0$ restricts only the parameters a, x, λ, μ, T , and F . Clearly, this assumption holds if, for example, F is sufficiently large.

Proposition 2. *If Assumption 1 holds, then for all $\theta \in [0, 1)$ it is true that*

- (i) *the equilibrium wage rate, $\omega^*(\theta)$, is increasing in θ ,*
- (ii) *the equilibrium payoff of workers, $V^{W^*}(\theta)$, is increasing in θ , and*
- (iii) *the equilibrium payoff of the government $V^{G^*}(\theta)$ is decreasing in θ .*

Proof. Let Assumption 1 hold.

(i) Using (22), one can rewrite (21) as

$$\omega^*(\theta) = \frac{\beta(1-\tau)}{1-\tau-\theta} \left(\frac{SW^*}{h^*} \right) + x(h^*)^{\mu-1} - \frac{F}{h^*}. \quad (26)$$

Differentiating $\omega^*(\theta)$ with respect to θ , we obtain

$$\frac{\partial \omega^*(\theta)}{\partial \theta} = \frac{\beta(1-\tau)}{(1-\tau-\theta)^2} \left(\frac{SW^*}{h^*} \right). \quad (27)$$

We know from (20) that $h^* > 0$ since $a > 0$, $\lambda \in (0,1)$, $\mu > 1$ and $x > 0$ by assumptions in Section 2. Moreover, $SW^* > 0$ since Assumption 1 holds. Therefore, $\partial \omega^*(\theta)/\partial \theta > 0$, i.e., $\omega^*(\theta)$ is increasing in $\theta \in [0, 1)$.

(ii) Differentiating $V^{W^*}(\theta)$ in (23) with respect to θ , we obtain

$$\frac{\partial V^{W^*}(\theta)}{\partial \theta} = \tau \frac{\beta(1-\tau)}{(1-\tau-\theta)^2} SW^* > 0, \quad (28)$$

implying that $V^{W^*}(\theta)$ is increasing in $\theta \in [0, 1)$.

(iii) Differentiating $V^{G^*}(\theta)$ in (25) with respect to θ , we obtain

$$\frac{\partial V^{G^*}(\theta)}{\partial \theta} = -\tau \frac{\beta(1-\tau)}{(1-\tau-\theta)^2} SW^* < 0, \quad (29)$$

implying that $V^{G^*}(\theta)$ is decreasing in $\theta \in [0, 1)$. ■

Having the insights in Proposition 2, we can now turn to workers' pre-bargaining problem (10), where they should optimally choose the value of the pre-donation parameter θ in $[0, 1)$. Here, workers must satisfy two constraints. One of them is $V^{F^*}(\theta) \geq V^{F^*}(0)$; the firm should not prefer rejecting the pre-donation from workers (implied by θ) to accepting it. We know from (24) that $V^{F^*}(\theta) = (1-\tau)(1-\beta)SW^*$ is independent of θ . Thus, the above constraint is satisfied at all values of θ . The other constraint that workers must satisfy is $V^{G^*}(\theta) \geq 0$, i.e., the equilibrium payoff (tax revenue) of the government must not be negative after any amount of pre-donation. We recall from (25) that $V^{G^*}(\theta) \geq 0$ if and only if $1-\tau-\theta \geq \beta(1-\tau)$ or equivalently $\theta \leq (1-\tau)(1-\beta)$. Thus, workers can only consider values of $\theta \in [0, (1-\tau)(1-\beta))$. Then, their pre-bargaining problem (10) can be simplified as

$$\max_{\theta \in [0, (1-\tau)(1-\beta)]} V^{W^*}(\theta). \quad (30)$$

Proposition 3. *Let $\tau > 0$ and Assumption 1 hold. Then,*

(i) *the pre-donation of workers is optimal for them if and only if they offer the fraction $\theta^* = (1 - \tau)(1 - \beta)$ of their net bargaining payoff to the firm,*

(ii) *whenever the pre-donation of workers is optimal for them, i.e. $\theta = \theta^*$, the equilibrium payoffs of workers, the firm, and the government, are respectively given by*

$$V^{W^*}(\theta^*) = [\beta + \tau(1 - \beta)]SW^* \quad (31)$$

$$V^{F^*}(\theta^*) = (1 - \tau)(1 - \beta)SW^* \quad (32)$$

$$V^{G^*}(\theta^*) = 0. \quad (33)$$

Proof. Let $\tau > 0$ and Assumption 1 hold.

(i) The optimal pre-donation of workers, θ^* , must solve the problem in (30), where θ^* must lie in the interval $[0, (1 - \tau)(1 - \beta)] \subset [0, 1)$. Under Assumption 1, Proposition 2-(ii) shows that the equilibrium payoff of workers, $V^{W^*}(\theta)$, is increasing in $\theta \in [0, 1)$. Therefore, we must have $\theta^* = (1 - \tau)(1 - \beta)$.

(ii) Inserting $\theta^* = (1 - \tau)(1 - \beta)$ to equations (23), (24), and (25) yields the equilibrium payoffs (31), (32), and (33), respectively. ■

Note that the pre-donation $\theta^* = (1 - \tau)(1 - \beta)$ is optimal for workers if and only if $\tau > 0$. When τ is arbitrarily close to zero, θ^* becomes arbitrarily close $(1 - \beta)$. Interestingly, the equilibrium payoffs in (31), (32), and (33) would then arbitrarily close to

$$V^{W^*}(1 - \beta) = \beta SW^* \quad (34)$$

$$V^{F^*}(1 - \beta) = (1 - \beta)SW^* \quad (35)$$

$$V^{G^*}(1 - \beta) = 0. \quad (36)$$

We can state this observation in the following Corollary.

Corollary 1. *Let Assumption 1 hold. If τ is positive but arbitrarily small, then the bargaining equilibrium payoffs obtained when workers optimally pre-donate are arbitrarily close to the bargaining equilibrium payoffs in Del Rey et al. (2022), where there is no pre-donation ($\theta = 0$) or corporate taxes ($\tau = 0$).*

3.2 The Firm Pre-donates Unilaterally

Here, we assume that the firm pre-donates unilaterally a constant fraction, $\theta \in [0, 1)$, of its net bargaining payoff, at all possible values h and ω , to workers before the bargaining occurs. After pre-donation, the net bargaining payoffs of workers and the firm will be

$$\tilde{V}^W(\theta, h, \omega) = W(h, \omega) + F + \theta J(h, \omega) \quad (37)$$

and

$$\tilde{V}^F(\theta, h, \omega) = (1 - \theta)J(h, \omega) \quad (38)$$

respectively. (We will be using the tilde (\sim) sign to distinguish some variables from those in Section 3.1.) In consequence, the bargaining problem will be

$$\max_{h, \omega} \left[\tilde{V}^W(\theta, h, \omega) \right]^\beta \left[\tilde{V}^F(\theta, h, \omega) \right]^{1-\beta}. \quad (39)$$

The solution to this problem will be $\tilde{h}^*(\theta)$ and $\tilde{\omega}^*(\theta)$, implying the equilibrium payoffs $\tilde{V}^{W^*}(\theta) \equiv \tilde{V}^W(\theta, \tilde{h}^*(\theta), \tilde{\omega}^*(\theta))$ and $\tilde{V}^{F^*}(\theta) \equiv \tilde{V}^F(\theta, \tilde{h}^*(\theta), \tilde{\omega}^*(\theta))$ for workers and the firm, respectively.

We can calculate the government's equilibrium payoff as

$$\tilde{V}^{G^*}(\theta) = \tau \left[a(\tilde{h}^*(\theta))^\lambda - \tilde{h}^*(\theta)\tilde{\omega}^*(\theta) - T \right] = \frac{\tau}{1 - \tau} J(\tilde{h}^*(\theta), \tilde{\omega}^*(\theta)). \quad (40)$$

We can now define the equilibrium social welfare as

$$\widetilde{SW}^*(\theta) = \tilde{V}^{G^*}(\theta) + \tilde{V}^{F^*}(\theta) + \tilde{V}^{W^*}(\theta). \quad (41)$$

Using (37), (38), and (40), we can rewrite (41) as

$$\widetilde{SW}^*(\theta) = a(\tilde{h}^*(\theta))^\lambda - T + F - x(\tilde{h}^*(\theta))^\mu. \quad (42)$$

Now, we turn to the optimization problem of workers. Before bargaining occurs, the firm should optimally choose the value of the pre-donation parameter $\theta \in [0, 1)$. However, its choice should be acceptable to both workers and the government: workers should not reject the pre-donation of the firm on the ground that accepting it would make them worse off and the government should not prohibit pre-donation of the firm on the ground that its tax

income becomes negative. Given these two constraints, the pre-bargaining problem of the firm is given by

$$\max_{\theta \in [0,1]} \tilde{V}^{F^*}(\theta) \quad \text{s.t.} \quad \tilde{V}^{W^*}(\theta) \geq \tilde{V}^{W^*}(0) \quad \text{and} \quad \tilde{V}^{G^*}(\theta) \geq 0. \quad (43)$$

The solution $\tilde{\theta}^*$ to this problem will yield the equilibrium payoffs $\tilde{V}^{G^*}(\tilde{\theta}^*)$, $\tilde{V}^{F^*}(\tilde{\theta}^*)$, $\tilde{V}^{W^*}(\tilde{\theta}^*)$, and $\tilde{S}W^*(\tilde{\theta}^*)$ respectively for the government, the firm, workers, and society. Using backward induction, the firm will first calculate the solution $(\tilde{h}^*(\theta), \tilde{\omega}^*(\theta))$ to the bargaining problem in (39) for each value of $\theta \in [0, 1)$ and thereby construct the equilibrium payoffs in its optimization problem (43).

For the bargaining problem (39), the first-order condition that determines the wage rate is

$$\beta J \frac{\partial W}{\partial \omega} = -[(1 - \beta)(W + F) + \theta J] \frac{\partial J}{\partial \omega}. \quad (44)$$

Inserting $\partial W/\partial \omega = h$ and $\partial J/\partial \omega = -(1 - \tau)h$ into above, we obtain

$$(\beta - \theta(1 - \tau))J = (1 - \beta)(1 - \tau)(W + F). \quad (45)$$

Substituting (1) and (2) into (45) yields the wage equation

$$\tilde{\omega}(\theta, h) = \frac{\beta - \theta(1 - \tau)}{1 - \theta(1 - \tau)} \left(ah^{\lambda-1} - \frac{T}{h} \right) + \frac{1 - \beta}{1 - \theta(1 - \tau)} \left(xh^{\mu-1} - \frac{F}{h} \right). \quad (46)$$

Likewise, the first-order condition that determines the hours of work is

$$\beta J \frac{\partial W}{\partial h} = -[(1 - \beta)(W + F) + \theta J] \frac{\partial J}{\partial h}. \quad (47)$$

Substituting (45) into above, we obtain

$$(1 - \tau) \frac{\partial W}{\partial h} = -\frac{\partial J}{\partial h}. \quad (48)$$

Substituting (17) and (18) in (48), we obtain

$$\mu x h^{\mu-1} = \lambda a h^{\lambda-1}, \quad (49)$$

implying

$$\tilde{h}^*(\theta) = \left(\frac{a\lambda}{\mu x} \right)^{\frac{1}{\mu-\lambda}}. \quad (50)$$

The equilibrium level of working hours $\tilde{h}^*(\theta)$ in (50) is independent of θ as in both Section 3.1 and Del Rey et al. (2022). Using $\tilde{h}^* \equiv \tilde{h}^*(\theta)$ and equation (42), we can show that the equilibrium social welfare $\widetilde{SW}^* \equiv \widetilde{SW}^*(\theta)$ is independent of θ , as we have

$$\widetilde{SW}^* = a(\tilde{h}^*)^\lambda - T + F - x(\tilde{h}^*)^\mu. \quad (51)$$

Noticing that the pair of equilibrium working hours and social welfare, $(\tilde{h}^*, \widetilde{SW}^*)$, we have calculated above is the same as the equilibrium pair (h^*, SW^*) we found in Section 3.1, where pre-donation is made by workers, we can state the following remark.

Remark 1. *The equilibrium working hours and the equilibrium social welfare are both independent of which bargaining party makes the pre-donation and how much it does.*

However, we will see that the equilibrium wage rate $\tilde{\omega}^*(\theta)$ depends on θ as well as the identity of who makes pre-donation. Substituting h^* into equation (46) yields the equilibrium wage rate $\tilde{\omega}^*(\theta) \equiv \tilde{\omega}(\theta, \tilde{h}^*)$, where

$$\begin{aligned} \tilde{\omega}^*(\theta) &= \left[1 - \frac{1-\beta}{1-\theta(1-\tau)} \right] \left(a(\tilde{h}^*)^{\lambda-1} - \frac{T}{\tilde{h}^*} \right) + \\ &\quad \left[\frac{1-\beta}{1-\theta(1-\tau)} \right] \left(x(\tilde{h}^*)^{\mu-1} - \frac{F}{\tilde{h}^*} \right). \end{aligned} \quad (52)$$

Using (51) and (52), we can now rewrite the equilibrium payoffs of workers, the firm, and the government as

$$\begin{aligned} \tilde{V}^{W^*}(\theta) &= W(\tilde{h}^*, \tilde{\omega}^*(\theta)) + F + \theta J(\tilde{h}^*, \tilde{\omega}^*(\theta)) \\ &= \tilde{h}^* \tilde{\omega}^*(\theta) - x(\tilde{h}^*)^\mu + F + \theta(1-\tau) \left[a(\tilde{h}^*)^\lambda - \tilde{h}^* \tilde{\omega}^*(\theta) - T \right] \\ &= \beta \widetilde{SW}^*, \end{aligned} \quad (53)$$

$$\begin{aligned}
\tilde{V}^{F^*}(\theta) &= (1 - \theta)J(h^*, \omega^*(\theta)) \\
&= (1 - \theta)(1 - \tau) [a(h^*)^\lambda - h^* \omega^*(\theta) - T] \\
&= \left[1 - \frac{\tau}{1 - \theta(1 - \tau)}\right] (1 - \beta)\widetilde{SW}^*, \tag{54}
\end{aligned}$$

and

$$\begin{aligned}
\tilde{V}^{G^*}(\theta) &= \frac{\tau}{1 - \tau} J(h^*, \omega^*(\theta)) = \left[\frac{\tau}{(1 - \tau)(1 - \theta)}\right] \tilde{V}^{F^*}(h^*, \omega^*(\theta)) \\
&= \left[\frac{\tau}{1 - \theta(1 - \tau)}\right] (1 - \beta)\widetilde{SW}^* \tag{55}
\end{aligned}$$

respectively.

If $\theta = 0$, then the above equilibrium payoffs become $\tilde{V}^{W^*}(0) = \beta SW^*$, $\tilde{V}^{F^*}(0) = (1 - \tau)(1 - \beta)SW^*$, and $\tilde{V}^{G^*}(0) = \tau(1 - \beta)SW^*$, as in Section 3.1. We should also notice from (53) that the equilibrium payoff of workers, $\tilde{V}^{W^*}(\theta)$, is independent of the pre-donation parameter θ . Moreover, if the corporate tax rate becomes zero, i.e. $\tau = 0$, then the equilibrium payoffs of the firm and the government also become independent of θ . Thus, whenever our model reduces to the model of Del Rey et al. (2022), where the firm does not pay any income taxes ($\tau = 0$), the pre-donation of the firm has no effect on the equilibrium payoff distribution under Nash bargaining, as we also found in Section 3.1 where pre-donation is made by workers. On the other hand, whenever $\tau > 0$, equations (54) and (55) show that the equilibrium payoffs of the firm and the government are affected by the fraction θ of payoffs pre-donated by the firm. We state these results below.

Proposition 4. *Pre-donation of the firm (its choice of θ) can never affect the equilibrium social welfare, \widetilde{SW}^* , and the equilibrium payoff of workers, $\tilde{V}^{W^*}(\theta)$. However, it affects the equilibrium payoffs of the firm and the government, $\tilde{V}^{F^*}(\theta)$ and $\tilde{V}^{G^*}(\theta)$, if and only if $\tau > 0$.*

Proposition 4 implies that whenever $\tau > 0$, the pre-donation of the firm has a non-zero effect on its equilibrium payoff along with an opposite effect of equal size on the equilibrium payoff of the government; i.e. for any

non-negative amount of change in θ , denoted by $\Delta\theta \geq 0$, the changes in the equilibrium payoffs satisfy $\Delta\tilde{V}^{F^*}(\theta) = -\Delta\tilde{V}^{G^*}(\theta) \neq 0$ and $\Delta\tilde{V}^{W^*}(\theta) = \Delta\widetilde{SW}^* = 0$.

Proposition 5. *If Assumption 1 holds, then for all $\theta \in [0, 1)$ it is true that*

- (i) *the equilibrium wage rate, $\tilde{\omega}^*(\theta)$, is decreasing in θ ,*
- (ii) *the equilibrium payoff of the firm, $\tilde{V}^{F^*}(\theta)$, is decreasing in θ , and*
- (iii) *the equilibrium payoff of the government $\tilde{V}^{G^*}(\theta)$ is increasing in θ .*

Proof. Let Assumption 1 hold.

(i) Using (51), one can rewrite (52) as

$$\tilde{\omega}^*(\theta) = \frac{\beta - \theta(1 - \tau)}{1 - \theta(1 - \tau)} \left(\frac{\widetilde{SW}^*}{\tilde{h}^*} \right) + x(\tilde{h}^*)^{\mu-1} - \frac{F}{\tilde{h}^*}. \quad (56)$$

Differentiating $\tilde{\omega}^*(\theta)$ with respect to θ , we obtain

$$\frac{\partial \tilde{\omega}^*(\theta)}{\partial \theta} = -\frac{(1 - \beta)(1 - \tau)}{(1 - \theta(1 - \tau))^2} \left(\frac{\widetilde{SW}^*}{\tilde{h}^*} \right) < 0. \quad (57)$$

Thus, $\tilde{\omega}^*(\theta)$ is decreasing in $\theta \in [0, 1)$.

(ii) Differentiating $\tilde{V}^{F^*}(\theta)$ in (54) with respect to θ , we obtain

$$\frac{\partial \tilde{V}^{F^*}(\theta)}{\partial \theta} = -\frac{\tau(1 - \tau)}{(1 - \theta(1 - \tau))^2} (1 - \beta)\widetilde{SW}^* < 0, \quad (58)$$

implying that $\tilde{V}^{F^*}(\theta)$ is decreasing in $\theta \in [0, 1)$.

(iii) Differentiating $\tilde{V}^{G^*}(\theta)$ in (55) with respect to θ , we obtain

$$\frac{\partial \tilde{V}^{G^*}(\theta)}{\partial \theta} = \frac{\tau(1 - \tau)}{(1 - \theta(1 - \tau))^2} (1 - \beta)\widetilde{SW}^* > 0, \quad (59)$$

implying that $\tilde{V}^{G^*}(\theta)$ is increasing in $\theta \in [0, 1)$. ■

We can now consider the firm's pre-bargaining problem (43), where it should optimally choose the value of its pre-donation parameter θ in $[0, 1)$. Here, the firm must satisfy two constraints. One of them is $\tilde{V}^{W^*}(\theta) \geq \tilde{V}^{W^*}(0)$; workers should not prefer rejecting the pre-donation from the firm (implied by θ) to accepting it. We know from (53) that $\tilde{V}^{W^*}(\theta) = \beta \widetilde{SW}^*$, which is independent of θ . Thus, the above constraint is satisfied at all values of θ . The other constraint that the firm must satisfy is $\tilde{V}^{G^*}(\theta) \geq 0$, i.e., the equilibrium payoff (tax revenue) of the government must not be negative after any amount of pre-donation. We should recall from (55) that $\tilde{V}^{G^*}(\theta) \geq 0$ if and only if $\tau[1 - \tau(1 - \theta)](1 - \beta) \geq 0$ or equivalently $\theta(1 - \tau) \leq 1$. This last inequality always holds since $\tau \in [0, 1]$ and $\theta \in [0, 1)$ by assumption. Therefore, $\tilde{V}^{G^*}(\theta) \geq 0$ for all $\theta \in [0, 1)$. Then, the firm's pre-bargaining problem (43) can be simplified as

$$\max_{\theta \in [0, 1)} \tilde{V}^{F^*}(\theta). \quad (60)$$

Proposition 6. *Let $\tau > 0$ and Assumption 1 hold. Then, $\tilde{\theta}^* = 0$ is the unique solution to the problem (60); i.e., the firm finds it optimal not to pre-donate.*

Proof. Let $\tau > 0$ and Assumption 1 hold. The optimal pre-donation by the firm, $\tilde{\theta}^*$, must solve the problem (60) since $\tau > 0$. According to this problem, $\tilde{\theta}^*$ must lie in $[0, 1)$. Under Assumption 1, Proposition 5-(ii) shows that the equilibrium payoff of the firm, $V^{F^*}(\theta)$, is decreasing in $\theta \in [0, 1)$. Therefore, we must have $\tilde{\theta}^* = 0$. ■

Inserting $\tilde{\theta}^* = 0$ to equations (53), (54), and (55) yields the following equilibrium payoffs of workers, the firm, and the government when there is no pre-donation:

$$\tilde{V}^{W^*}(\theta^*) = \beta \widetilde{SW}^* \quad (61)$$

$$\tilde{V}^{F^*}(\theta^*) = (1 - \tau)(1 - \beta) \widetilde{SW}^* \quad (62)$$

$$\tilde{V}^{G^*}(\theta^*) = \tau(1 - \beta) \widetilde{SW}^* \quad (63)$$

Whenever $\tau = 0$, the above payoffs coincide with the equilibrium payoffs in Del Rey et al. (2022). Remembering that the equilibrium social welfare is independent of who makes pre-donation and how much it does, i.e.,

$\widetilde{SW}^* = SW^*$, we can compare the payoff equations (61)-(63) to the equations (31)-(33) that we found in Section 3.1 to observe the following remark.

Remark 2. *The equilibrium payoff of the firm is the same when either workers or the firm can optimally choose how much to pre-donate to the other party. On the other hand, whenever workers are allowed to make pre-donation optimally, they can fully extract the tax revenue that the government could obtain in the absence of pre-donation.*

Comparing (61)-(63) to (31)-(33), we also observe that if τ is arbitrarily close to zero, then the equilibrium payoffs obtained whenever the firm optimally chooses the pre-donation parameter as $\theta^* = 0$ and the equilibrium payoffs obtained whenever workers optimally choose the pre-donation parameter as $\theta^* = (1 - \tau)(1 - \beta)$ are arbitrarily close to each other.

4 Conclusion

In this paper, we extended the model of Del Rey et al. (2022) –which describes a generalized Nash bargaining process in the labor market– by adding a government that can tax the firm’s operating profit. Using this extended model, we studied whether and how pre-donation made by either workers or the firm to the other party before bargaining takes place could affect the equilibrium outcomes in the labor market.

Our results showed that pre-donation by any party can never affect the equilibrium levels of the working hours, output, and social welfare in the economy; thus it would never lead to any productive inefficiency. However, pre-donation may change the welfare distribution in society if and only if the (corporate) tax rate applied to the firm’s operating profit is positive. In the case where the corporate tax rate is positive, we showed that the equilibrium wage rate, hence the implied welfare distribution in society, is sensitive to pre-donation by both workers and the firm. In detail, the equilibrium wage rate is increasing in the fraction of payoffs pre-donated when the pre-donor is workers and decreasing when the pre-donor is the firm. However, the firm has never any incentive to make pre-donation to workers as it would reduce

its net bargaining payoff and increase the tax revenue of the government while leading to no change in the net bargaining payoff of workers. Therefore, the optimal pre-donation of the firm turns out to be zero.

Workers, on the other hand, find making any amount of pre-donation more beneficial than making none. If they increase their pre-donation, workers' equilibrium payoff becomes higher and the government's equilibrium tax revenue becomes lower while the firm's equilibrium payoff is never changed. The optimal pre-donation of workers enables them, in addition to enjoying their equilibrium wage earning that is a constant fraction of the equilibrium social welfare, to fully extract the tax revenue the government could obtain in the absence of pre-donation. The optimal pre-donation of workers can lead to this consequential welfare transfer from the government to themselves by raising the wage rate to such a high level that the operating profit of the firm, hence the tax revenue of the government, becomes zero. The loss of the firm from its foregone operating profit is compensated by an equal amount of pre-donation by workers at the equilibrium of the bargaining process so that the net welfare of the firm is not affected by the existence or the amount of pre-donation it receives.

Overall, our results showed that workers can exploit the collective bargaining equilibrium in the labor market when the corporate tax rate is not equal to zero. Moreover, the larger this tax rate, the larger the incentive of workers for exploitation; i.e., the larger the gains of workers from pre-donation. On the other hand, the firm becomes neither better nor worse off because of any amount of pre-donation it receives.

At this point, we should notice that in this paper we only dealt with the possible equilibrium predictions of pre-donation when it is practically or legally allowed. In reality, it may not be befitting for a government to allow workers to make pre-donation, or to make any incentivizing lump-sum transfer for that matter, to the corporate sector after it becomes aware that it would reduce, and even eliminate when it's set optimally for workers, its corporate tax revenues. Certainly, a theoretically well-informed government could prevent pre-donation if it can detect or verify it. This argument might be sensible, for instance, if the government could attend the bargaining sessions between workers and the firm. In cases such a detection or verification is not possible, the government could be advised to institute some red-flag

rules on equilibrium wage rates or corporate profits since too high values of the former or too low values of the latter may indicate a strong likelihood of pre-donation. Such red-flag rules accompanied by some discouraging penalty schemes may reduce pre-donation, but cannot eliminate it (without harming productive efficiency) at least in cases where workers and the firm have private information about their bargaining payoffs. Future research may profitably study the incentives of workers and the firm for making pre-donation under situations of asymmetric information as well as possible measures of the government to prevent or fight it.

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