Residential Land Use and Utilities of Multiple Generations with Lifespan Perspectives and Demographic Dynamics

Kono, Tatsuhito and Tsutaki, Keisuke

Tohoku University, Tohoku University

June 2023

Residential Land Use and Utilities of Multiple Generations with Lifespan Perspectives and Demographic Dynamics

Tatsuhito Kono [a*] and Keisuke Tsutaki [a]

[a] Graduate School of Information Sciences, Tohoku University

Aoba 6-6-06, Aramaki, Aobaku, Sendai, Miyagi 980-8579, Japan

[*] Corresponding author

E-mail addresses: kono@tohoku.ac.jp, keisuke.tsutaki.q7@dc.tohoku.ac.jp

Abstract. Demographic dynamics and spatial distribution of urban amenities bring about spatially different benefits to young, middle-aged, and elderly people, thereby affecting residential location patterns. Using an overlapping generations model in a closed city with two zones with different amenity levels, we demonstrate how young, middle-aged, and elderly generations with lifespan perspectives reside in the two zones with their interplay across periods and locations and analyze the residents' welfare levels. We find that, unlike a static situation, there is no steady residential pattern in which middle-aged or elderly households live in both of the two zones when they optimize their residential locations throughout their life. Our numerical simulation reveals two findings useful for policy making: first, urban amenities should be unevenly distributed across the city from a perspective of lifetime utility; second, different demographic changes lead to different desirable residential patterns in terms of utility. Finally, we check the robustness of these findings for the case of the expansion of remote work.

Keywords: Urban land use, Overlapping generations model, Urban amenities, Demographic dynamics

1. Introduction

The spatial residential pattern differs greatly across generations. For example, the tendency of young and elderly people to reside downtown has been found in many Japanese cities in recent years. According to Okada (2014), in Tokyo, this tendency started in the late 1990s. With a crowding-out effect caused by this residential tendency of elderly and young generations, the middle-aged generation tends to reside in suburbs. In the USA also, the difference in residential patterns among generations has been reported (Dutzik et al. (2014), Circella et al. (2017), and Lee et. al (2019)).

Such recent residential patterns have been caused by the spatial distribution of amenities, and differences in preference for amenities among generations. In the system of cities, Albouy and Stuart (2020) empirically show that local amenities play an important role in determining residential locations compared to the productivities of traded goods. Similarly, within a city, Circella et al. (2017) show that young people not living with their parents are more likely to reside in areas with good access to urban amenities than middle-aged people and young people living with their parents. Okada (2014) points out that urban amenities, such as the accessibility of educational facilities and medical facilities, become more varied and richer than those in the suburbs, and the urban amenities have caused the recent migration to downtown districts in Japan[1].

In such a recent trend, there is a circular causality between the 'amenity changes' and 'change in spatial residential pattern'. That is, if amenities preferred by a certain generation are provided in the center, the generation is attracted to the center. When some specific

[1] Okada (2014) points out that, in Tokyo, this tendency started in the late 1990s. Okada (2014) refers to several factors related to the current migration within a city. First, the population of major cities in Japan is decreasing, which causes the suburbs to become inconvenient in terms of amenities such as shopping and parks. Secondly, modern urban life has become increasingly attractive for residents. Thirdly, due to the decline in the birthrate, small houses have become popular, and accordingly the need for large houses in the suburbs has diminished. Finally, the urban amenities have become more varied and richer than those in the suburbs.

generation agglomerates at some locations, providing amenities for this generation there is efficient. In this causality, heterogeneity in preference for amenities among generations plays an important role. Likewise, the spatial distribution of urban amenities is a key factor to determine residential land use with multiple generations.

The current study examines how urban amenities affect the residential spatial distribution and the spatial distribution of housing. Elucidating this mechanism is important for policy makers designing urban amenity improvements and other urban policies including land use regulation.

The spatial distribution of residents has been an important topic in urban economics for a long time. So, there is a vast relevant literature. Among them, Tabuchi (2019) shows a novel mechanism in which households with different incomes are spatially sorted or collocated according to the distance from the central business district (CBD). In addition, he empirically shows that such a collocation of heterogenous people is actually observed in the Tokyo metropolitan area. However, in the literature, including Tabuchi (2019), there are no papers focusing on the spatial sorting among generations. Shimizu et al. (2014) classify the amenities into twenty-four categories and examine how a concentration of these amenities affects housing rents and population concentration in Tokyo by empirical analysis. But Shimizu et al. (2014) have not shown the mechanism in the background. Gaigné el al. (2022) show income sorting according to commuting distance and amenities with a Dutch dataset.

Urban land use is determined by the demand for residential use and the supply of housing. To explore residential land use with multiple generations, the following two points are important: (i) an intertemporal perspective and (ii) the durability of housing. Point (i) is necessary because residents have a long-term viewpoint to optimize their residential location and consumption. Point (ii), the durability of housing is necessary because the future rents affect the current supply of buildings, which are constructed from a long-term viewpoint. If the future rents are high, because the lifespan profit increases, the current supply of buildings increases. Static urban land use models developed by Alonso (1964), Muth (1969), and Mills (1972) cannot take account of such long-term perspective mechanisms because housing is malleable in the static models.

Dynamic urban models, developed by Evans (1975) and Muth (1975), have considered dynamic changes in zonal population and the durability of housing. Many papers (e.g., Anas, 1978; Fujita, 1982; Turnbull, 1988; Wheaton, 1982a; Braid, 1991)[2] assume durable buildings, which are not torn down and rebuilt. Brueckner (1980, 1981), Wheaton (1982b) and Braid (2001) consider the deterioration of buildings in terms of quality, which can be a main cause of redevelopment. The current paper does not take account of the deterioration of buildings but supposes a lifespan of buildings for simplicity.

The adoption of an overlapping generations (OLG) model is the most natural way to express the dynamics of young, middle-aged, and elderly generations. However, the combination of an OLG model and a multiple zonal model would be complex. Only a few papers explore urban land use with multiple generations. For example, Yonemoto (2007) constructs two-period models with multiple zones. Englund (1986), Hardman and Ioannides (1995) and Brueckner and Pereira (1997) construct OLG models to explore housing markets, but they do not consider multiple zones. Duranton (2000) constructs an OLG model with two zone types (urban and suburban zones) to investigate characteristics of the market equilibrium and the first best economy. However, although the Duranton (2000) model includes young and elderly generations, young people live in urban zones and elderly people live in the suburbs. This perfect segregation is derived from the assumption that the workplaces exist only in urban areas and that no amenities exist other than the workplaces. Moreover, Duranton (2000) does not consider durable housing stock. Althaus (2004)

[2] Dynamic models can be classified from the perspective of their future expectations: myopic or perfect expectation. These papers show that the results can differ between these two expectations. The current paper adopts perfect expectation.

constructs a three-period life cycle model with two regions. To maximize lifetime utilities, households choose optimal dynamical choices of residential locations throughout their lifetimes. However, Althaus (2004) does not consider the case in which multiple dynamical residential choices are in equilibrium and assume that all households make the same dynamical choices throughout their lifetimes.

An OLG model with two zones having different amenity types is constructed by Kono et al. (2012) to explore how young and elderly generations reside in the zones and to demonstrate the equilibrium utility paths of young and elderly people. Kono et al. (2012) consider two zones and durable housing stock. They also take account of lifespan perspectives of agents. However, there are three limitations when applying the model to produce some policy implications for the current topic. First, in this model, people do not make trips to the business areas. Setting the business areas is necessary to characterize the center of the city. Second, there are only two generations. Considering at least three generations is necessary to express the recent trend of residential patterns discussed in the first paragraph of the Introduction. So, we take account of three generations: young, middle-aged, and elderly. Third, the savings and floor area for a household are fixed in Kono et al. (2012). These assumptions are too simplistic for investigating the lifetime utility of households living in urban or suburban areas. Therefore, the current paper takes account of endogenous changes in floor areas, setting a Cobb-Douglas utility function.

In summary, we explore how young, middle-aged, and elderly generations reside in the two zones and classify the residential patterns according to the demographic dynamics (decrease, increase, and constant). As a model structure, we adopt an OLG model with the three generations; we use a closed city model with two zones, each having different amenities, to express urban areas. Young and middle-aged people work, while elderly people are retired. The agents are residents and developers, who supply residential buildings that are durable but not permanent.

Section 2 develops the model. Section 3 presents the respective market equilibria of each residential pattern. Section 4 uses numerical simulations to investigate which residential patterns emerge with demographic changes and which amenity allocations are desirable. Section 5 concludes the paper.

2. Model

The model is an OLG model in a closed city composed of the CBD and two residential zones labeled 1 and 2. The two zones are characterized by their different amenities. The CBD is in Zone 1. The two zones are connected by a bridge. The travel cost from Zone 1 to the CBD is assumed to be zero. Figure 1 shows the shape of the city.

Fig. 1 The city

The model is based on several simplifying assumptions. First, all the residents rent housing from absentee owners. However, even if they own housing, when the housing can be sold at the present value, the results do not change. Second, our model includes the assumption of free migration. The cost of migration can be negligible if the cost is spread over a long time. Finally, all agents, who are residents, developers, and absentee landowners, behave with perfect foresight. In real situations, these suppositions may not necessarily apply perfectly. We analyze our model to establish an ideal benchmark for analyzing real situations.

2.1 Three generations

This paper explores how the distribution of amenities changes the utility of each generation and changes the residential pattern. We assume three generations: young people living alone and independently; middle-aged married people living with children; elderly married people with no children at home. The utility functions and the budget constraints are different among the three generations.

We assume the following point about the budget constraints. Young people spend all of their income consuming goods. In other words, young people have no savings. Indeed, the median value of the financial assets held by single people in their twenties and those in their thirties in Japan are only 50,000 yen and 400,000 yen, respectively. In addition, 45.4% of single people in their twenties have no financial assets (Survey of Household Finances (2018))[3].

Elderly people are retired, so their working time is zero, and they make fewer trips to the CBD than the other generations. Each person optimizes the allocation of the composite good and their residential zones throughout the middle-aged and the elderly periods to maximize their utilities, subject to the combined income constraint and the separate time constraints. As we already explained, in the young period, there are no variables connected to the decisions of the middle-aged or the elderly. In contrast, the choices of the middle-aged are connected to those of the elderly via savings.

2.2 Residents

Residents live through three periods. During the first period, they are young and working; in the second, they are middle-aged and working; in the third, they are elderly and retired. Figure 2 shows the overlapping generations (OLG) structure. The number of people who are young in period t is expressed as $\bar{N}^{y,t}$. Everyone lives through three periods, so the number who are middle-aged in period t+1, expressed as $\bar{N}^{m,t+1}$, is equal to $\bar{N}^{y,t}$ and the number who are elderly in period t+1, expressed as $\bar{N}^{o,t+2}$, is equal to $\bar{N}^{m,t+1}$ The population dynamics, i.e. the path of $\bar{N}^{y,t}(t=0,1,...)$, is given exogenously.

[3] We can factor in the savings of the young by combining their utility maximization behaviors with the middle-aged and elderly utility maximization behaviors. However, as noted in our main text, their savings are very low. So, factoring in the savings of the young does not affect the results much.

Every resident in Zone $i \in \{1,2\}$ consumes some quantity of a composite good c_i, leisure time l_i, and floor area h_i throughout their lifetime. The utility of a young person in period *t* is

$$\max_{i,c,l,h} u^{y,t} = V^y\left(c_i^{y,t}, l_i^{y,t}, h_i^{y,t}, e_i^{y}\right). \tag{1}$$

The utility of a middle-aged to elderly person who is middle-aged in period *t* is

$$\begin{aligned}\max_{i,j,c,l,h} u^{mo,t} &= V^m\left(c_i^{m,t}, l_i^{m,t}, h_i^{m,t}, e_i^{m}\right) \\ &\quad + \frac{1}{1+\rho} V^o\left(c_j^{o,t+1}, l_j^{o,t+1}, h_j^{o,t+1}, e_j^{o}\right),\end{aligned} \tag{2}$$

where ρ is the discount rate, and where e_i^y, e_i^m, and e_i^o represent the amount or level of amenities for young, middle-aged, and elderly people, respectively. The utility arising from these amenities differs because young, middle-aged, and elderly people normally enjoy different amenities. For instance, young people highly value the accessibility of restaurants, amusement facilities, and so on; middle-aged people prefer libraries, public parks, hospitals, and facilities that are useful for raising children; elderly people often use public parks and hospitals.

The budget constraint of a young person at period *t* is expressed as

$$w^{y,t} L_i^{y,t} = c_i^{y,t} + r_i^{y,t} h_i^{y,t} + \sum_{k=1}^{i} \Psi_{k-1,k}\ , \tag{3}$$

where $w^{y,t}$ is the exogenous labor income in young age, $L_i^{y,t}$ is working time, $r_i^{y,t}$ is residential floor rent per unit area and $\sum_{k=1}^{i} \Psi_{k-1,k}$ is the total travel expenses from Zone *i* to the CBD. The price of the composite good is assumed to be constant at 1 as numeraire. The budget constraint of a middle-aged person is expressed as

$$w^{m,t} L_i^{m,t} = c_i^{m,t} + r_i^{m,t} h_i^{m,t} + s^{m,t} + \sum_{k=1}^{i} \Psi_{k-1,k}\ , \tag{4}$$

and that of an elderly person is expressed as

$$\left(1+\pi\right) s^{m,t} = c_i^{o,t+1} + r_i^{o,t+1} h_i^{o,t+1} + \theta \sum_{k=1}^{i} \Psi_{k-1,k}\ . \tag{5}$$

Fig. 2 The three-generation overlapping model

The middle-aged person in period t becomes an elderly person in period t+1. The income of an elderly person is equal to the savings accumulated in middle age, i.e. $(1+\pi)s^{m,t}$, where π is the exogenously given interest rate. We assume that $\pi = \rho$ at market equilibrium. $w^{m,t}$ is the exogenous labor income in middle age.

Following Tabuchi (2019), each resident's total time $\overline{T}$ is fixed and divided among working time L_i^t, leisure time l_i^t, and commuting time $\sum_{k=1}^{i} M_{k-1,k}$.

$$\overline{T} = l_i^{y,t} + L_i^{y,t} + \sum_{k=1}^{i} M_{k-1,k} \quad \text{for the young generation,} \tag{6}$$

$$\overline{T} = l_i^{m,t} + L_i^{m,t} + \sum_{k=1}^{i} M_{k-1,k} \quad \text{for the middle-aged generation,} \tag{7}$$

$$\overline{T} = l_i^{o,t+1} + \theta \sum_{k=1}^{i} M_{k-1,k} \quad \text{for the elderly generation.} \tag{8}$$

To concretely solve the equilibria, we use a Cobb-Douglas utility function.

$$\begin{aligned} V^{y,t} &= \delta_1\left(\alpha^y \ln c_1^{y,t} + \beta^y \ln l_1^{y,t} + \gamma^y \ln h_1^{y,t} + \ln e_1^y\right) \\ &\quad + \delta_2\left(\alpha^y \ln c_2^{y,t} + \beta^y \ln l_2^{y,t} + \gamma^y \ln h_2^{y,t} + \ln e_2^y\right), \end{aligned} \tag{9}$$

$$\begin{aligned} V^{m,t} &= \delta_1\left(\alpha^m \ln c_1^{m,t} + \beta^m \ln l_1^{m,t} + \gamma^m \ln h_1^{m,t} + \ln e_1^m\right) \\ &\quad + \delta_2\left(\alpha^m \ln c_2^{m,t} + \beta^y \ln l_2^{m,t} + \gamma^m \ln h_2^{m,t} + \ln e_2^m\right), \end{aligned} \tag{10}$$

$$\begin{aligned} V^{o,t+1} &= \delta_1\left(\alpha^o \ln c_1^{o,t+1} + \beta^o \ln l_1^{o,t+1} + \gamma^o \ln h_1^{o,t+1} + \ln e_1^o\right) \\ &\quad + \delta_2\left(\alpha^o \ln c_2^{o,t+1} + \beta^o \ln l_2^{o,t+1} + \gamma^o \ln h_2^{o,t+1} + \ln e_2^o\right), \end{aligned} \tag{11}$$

$$\alpha^y + \beta^y + \gamma^y = 1,$$

$$\frac{1}{1+\rho}\left(\alpha^o + \gamma^o\right) + \alpha^m + \beta^m + \gamma^m = 1,$$

$$\delta_1 + \delta_2 = 1 \text{ and } \delta_1\delta_2 = 0,$$

where δ_i indicates that the person lives in Zone *i*. α^k, β^k, and γ^k *for* $k \in \{y, m, o\}$ are preference parameters for people in life stage *k*. α^k, β^k, and γ^k are elasticities of consumption, leisure time, and floor area, respectively. People have different preferences during the three periods, having different parameters.

2.3 Developers

Developers rent land from absentee landowners and construct housing, which can be multi-story buildings in which many residents dwell. The floor space of the residential buildings is used by residents. Developers receive the floor rents from residents. The floor rent of the residential houses is determined by the highest bid-rent of residents.

Residential buildings are durable, but not permanent. We assume that the duration of use is two periods. Our model divides an individual's lifetime, excluding childhood, into three periods. The period can be any time period. But we can regard one period as about 20 years. In our numerical simulations, we use this period. Accordingly, the building duration period is presumed to be 40-50 years. This duration is realistic for solid homes such as concrete, brick, two-by-four wooden buildings[4]. Assuming two periods of building duration in an OLG model, the future rent affects the present supply[5].

Developers construct housing every period. Rent-taking competitive developers maximize the profit of each building constructed every period. Perfectly competitive developers can be treated as an aggregate developer. Therefore, we treat the developers as one developer maximizing the profit of buildings constructed every period.

---

[4] The average lifespans of houses in the U.S.A. and England are 55 years and 77 years, respectively, according to an estimation by the Japanese Ministry of Land, Infrastructure, Transport and Tourism based on data from the American Housing Survey (2001, 2005) and Housing and Construction Statistics (1996, 2001).

[5] Developers are assumed to have perfect foresight related to the next-period rents. This can be justified to some degree as follows. In the model, life is divided into four periods: childhood, young, middle-aged, and elderly. We model only the latter three periods because children cannot choose their housing independently. Agents can find the number of young people in the next period from the number of children in the current period. The same can be said for the numbers of middle-aged and elderly people. In this way, the next period's population is known to developers who need this information for housing construction.

The buildings' profit is defined as the floor rent revenue minus the building cost. The building is durable: it is useful for two periods. Therefore, the revenue from the building constructed in period t is the built floor space Q_i^t in period t times the present value of the rents over the two periods t and t+1, i.e. $r_i^t + r_i^{t+1}/(1+\pi)$. The building cost comprises the costs of building materials y_i^t and land H_i^t. Consequently, the profit from a building constructed in period t is expressed as

$$max \Pi^t = \sum_i^2 \left[\left(r_i^t + \frac{r_i^{t+1}}{1+\pi} \right) Q_i^t - y_i^t - R_i^t H_i^t \right], \tag{12}$$

where the price of the building material is assumed to be constant at 1 for simplicity, and where the land rent is R_i^t. Furthermore, r_i^{t+1} in (12) expresses the future rent that developers suppose when supplying buildings. For simplicity, the building quality is assumed not to deteriorate over the two periods.

Assuming that the inputs of the production function are housing materials (e.g., cement, steel, and/or wood) y_i^t and land H_i^t, the floor production function in Zone i is represented as

$$Q_i^t = \left(y_i^t\right)^{\eta} \left(H_i^t\right)^{1-\eta} \quad \eta \in (0,1). \tag{13}$$

Eq. (13) implies that, as the built floor space Q_i^t increases subject to land space H_i^t, more building material y_i^t is needed for one acre of floor space. In other words, the marginal cost of the floor space increases concomitantly with the height of the multi-story building.

The developer behavior is expressed by maximizing Eq. (12) subject to Eq. (13) with respect to building material y_i^t and land H_i^t. For simplicity, the total available area of each Zone $i \in \{1,2\}$ is assumed to be constant: 2H. Buildings are built every period in half the zone area, i.e. H. The building duration time is two periods, so that one-half of the zone area, i.e. H, is occupied by first-period buildings and the other half, H, is occupied by second-period buildings every period[6]. Consequently, the total existing floor area in Zone $i \in \{1,2\}$ at time *t* is $Q_i^{t-1} + Q_i^t$ where Q_i^t represents first-period buildings and Q_i^{t-1} signifies second-period buildings at time *t*. To obtain closed-form solutions, we set η of the floor production function as $\eta = 1/2$. For simple notation, we use $\bar{H}$, defined as $\bar{H} \equiv H^{1/2}$. Each utility maximization with income constraints and time constraints is described in Appendix A.

From utility maximization, developer profit maximization, and housing supply-demand equilibrium $Q_i^{t-1} + Q_i^t = \sum_{i,g} h_i^{g,t} N_i^{g,t}$, a second-order difference equation for market land prices (14) can be derived as,

$$\begin{aligned}&\frac{\left(\bar{H}\right)^2}{2}\left\{r_i^t\left(r_i^{t-1}+\frac{r_i^t}{1+\pi}\right)+r_i^t\left(r_i^t+\frac{r_i^{t+1}}{1+\pi}\right)\right\}\\&=\gamma^y B_i N_i^{y,t}+\gamma^m D_{j,k} N_{j,k}{}^{m,t}+\gamma^o D_{j,k} N_{j,k}{}^{o,t} \quad for\ i,j,k \in \{1,2\},\end{aligned} \tag{14}$$

where $B_i = w^{y,t}\left(\bar{T} - \sum_{l=1}^{i} M_{l-1,l}\right) - \sum_{l=1}^{i} \Psi_{l-1,l}$ and $D_{j,k} = w^{m,t}\left(\bar{T} - \sum_{l=1}^{j} M_{l-1,l}\right) - \sum_{l=1}^{j} \Psi_{l-1,l}$ $- \frac{\theta}{(1+\pi)} \sum_{l=1}^{k} \Psi_{l-1,l}$, and *i, j,* and *k* denote the zones chosen by young, middle, and elderly households, respectively. The variables in Eq. (14) are the market land rents at *t-1*, *t*, and *t+1*, and population *N* is given exogenously. In other words, given the residential patterns of people, we obtain the market land rent in every period.

3. Residential patterns

This section explores the residential patterns by theoretical analysis. For population dynamics, we analyze population decline, population growth, and the constant population. The population dynamics are given exogenously.

---

[6] The land within one zone is homogeneous. Therefore, this setting may be natural.

Developers rent the floor area of the buildings to the highest bidder. The bidders in each period are people of three generations: young, middle-aged, and elderly. Having three generations and two zones, this model produces many residential patterns. We define the notation of residential patterns as follows: “Y”,”M”, and “O” mean young, middle-aged, and elderly: “/” is the border of the two zones; “Y/” implies that young people live in Zone 1, and “/O” implies that elderly people live in Zone 2.

We follow Lopez (2019)’s classification of residential patterns.

Segregation: All the people in each generation live in one of the two zones. e.g., Y/MO.

Partial Segregation: People in one or two of the three generations live in both zones. The other generations live in one of the two zones. e.g., YM/YO, YM/MO, and YM/YMO.

Integration: Both zones contain all three generations. That is, YMO/YMO.

There are locational choices at the beginning of each age period. At stable equilibrium, there is no incentive for anyone in any generation to change their residential location. We examine whether a household can increase its utility when it moves from their current zone to the other in each stage. We assume that because one person has very little effect on the whole economy, the migration of one person does not change the land rents and other market prices. Young people maximize their utility in their stage only. A middle-aged household maximizes the lifetime utility throughout their middle-aged and elderly stages. A middle-aged household can choose the residential location choice path $\{m, o\}$ and their savings.

The processes for deriving the equilibria of each residential pattern are in Appendix B. We can prove Proposition 1. The proof is described in Appendix C.

Proposition 1. *There is no steady residential location pattern in which middle-aged or elderly households live in both zones, where a steady residential pattern implies that the same residential pattern continues over time.*

Proposition 1 means that the following residential patterns are not in equilibrium when the same pattern continues: YM/MO, YMO/M, MO/YM, M/YMO, YO/MO, YMO/O, MO/YO, and O/YMO. Proposition 1 is an intriguing feature in a dynamic model in the sense that this does not hold if the model is a static model, which maximizes utilities in only one period.

Proposition 1 is derived by choosing the optimal lifetime path of location choices in the middle and elderly stages. We intuitively explain how Proposition 1 holds. First, we explain that the case where the middle-aged generation resides in both zones does not hold, and next, the case where the elderly generation resides in both zones does not hold.

For the middle-aged people, we compare the case in which middle-aged people live in both zones does not hold with the case in which they live in only one zone. When they live in both zones (e.g., YM/MO), middle-aged people live together with elderly people in one zone. This raises the rent in the zone because the zone is more crowded, compared to the case in which middle-aged people live separately from elderly people. Middle-aged people would like to avoid this location pattern because their future rents are higher than in the other location pattern when they become elderly.

Next, we explain why the elderly generation does not reside in both zones. Note that the savings in middle age depend on the choices of residential locations in elderly age as well as middle age through changes in the transportation costs to the central zone. For example, in YO/MO, there are two types of middle-aged households: first, middle-aged households who plan to live in Zone 1 in the elderly stage ($\{m,o\}=\{2,1\}$); second, the ones who plan to live in Zone 2 in the elderly stage ($\{m,o\}=\{2,2\}$). The savings for the two types are different. The savings for the path $\{m,o\}=\{2,2\}$ are larger than one for the path $\{m,o\}=\{2,1\}$ if $1+\pi>\alpha^o+\gamma^o$. This inequality, $1+\pi>\alpha^o+\gamma^o$, is derived in Appendix A. Larger savings lead to a smaller consumption of the floor area in middle age. Compared to Y/MO (i.e., residential path $\{m,o\}=\{2,2\}$), the middle-aged generation whose path is $\{m,o\}=\{2,1\}$ raises the rent in Zone 2. Middle-aged people would like to avoid this location pattern because their future rents are higher than in the other location pattern when they become elderly.

In contrast to the middle-aged and elderly people, young people can live in both zones in a steady location pattern, as many static urban models have shown. The different condition from the middle-aged and elderly people is that young people do not have savings, so they do not have any variable connected to their future choices. Accordingly, young people determine their location based on the utilities only in the current period. As a result, if the utility level decreases due to the increase in population in both zones, young people live in both zones.

In real cities, middle-aged and elderly households live in both urban and suburban areas. Unlike the current setting, in reality, there are various things affecting the location choices. For example, people must take care of their parents, or vice versa. In addition, people of the same age can have different preferences. Our study ignores these differences and idiosyncratic factors and focuses on the differences among age groups. Nevertheless, the mechanisms we clarify here exist in real cities as well.

Next, we use numerical simulations to discuss the effect of demographic dynamics and amenities on residential patterns and lifetime utility.

4. Numerical simulations

As shown in Section 3, our model has multiple equilibria. To compare the utilities in the multiple equilibria specifically, we need to calibrate parameters. Calibration is shown in Section 4.1. Section 4.2 shows numerical simulations of utilities and residential patterns, using several sets of amenities and demographic dynamics.

4.1 Calibration

We calibrate the parameters, using the data in the city of Sendai and in Japan. One period is set at 20 years. The CBD is Sendai City Hall, and Zone 1 is set within a range of 2km from the CBD. Zone 2 is set within a range of 2km from the center of Izumi Park Town, which is a suburban town in Sendai[7]. We suppose that the center of Izumi Park Town is Sendai-Izumi Premium Outlets.

The one-way travel cost $\Psi_{1,2}$ and time $M_{1,2}$ (from Zone 2 to Zone 1) are set as 640 yen and 30 minutes, which are shown in Miyagi Transportation (2019), assuming that people travel by bus. The travel cost per year is set at 404,907 yen/year, and time is set at 316.3 hours/year. Both are estimated with the data of the number of trips for an employed worker per year, which is shown in the Sendai Person-Trip Survey (2019). The trip ratio of the elderly to the young and middle-aged θ is set at 0.462 as in the Sendai Person-Trip Survey.

The interest rate per year is set at 2%. The total available time $\overline{T}$ is set as 13.465 hours/day derived from MIC's Survey on Time Use and Leisure Activities (2016). We assume the following two points: first, the wage rate of the middle-aged is the average wage rate of Miyagi Prefecture residents; second, the young wage rate w^y is lower than that of the middle-aged. Based on the above assumptions, the middle-aged wage rate w^m is set at 1,800 yen/hour. This is derived from data from the Miyagi Prefectural Government and the Ministry of Health, Labour and Welfare. The young wage rate w^y is set at 1,200 yen/hour.

Parameters $\alpha^y, \alpha^m, \alpha^o$ in the Cobb-Douglas utility function by age are 0.398, 0.287, and 0.149. Parameters $\beta^y, \beta^m, \beta^o$ for leisure time are 0.493, 0.551, and 0.493. Parameters $\gamma^y, \gamma^m, \gamma^o$ for floor area are 0.109, 0.070, and 0.049.

To analyze dynamical changes, we need to set the endogenous variables in the first and last periods. For that purpose, we assume that the system is in steady states in both the first and last periods. In other words, the population has been static before the dynamic changes in

[7] The total population of zone 1 is 125,883. The number of people 20-39 years old is 39,639, constituting 31.5% of the total population. The numbers of people 40-59 and over 60 years old are 33,529 (26.6%) and 28,598 (22.7%), respectively. The total population of zone 2 is 21,983. The numbers for 20-39, 40-59, and over 60 years old are 3,886 (17.7%), 5,973 (27.2%), and 7,327 (33.3%), respectively. This data is provided by the *2015 Population Census of Japan*.

population, and will be static again after the change. Let $t \in \{0,1,...,6\}$. The economies before t=0 and after t=6 are assumed to be both in steady states. Accordingly, the endogenous variables at t=0 and t=6 can be solved when the population is constant.

In the case of population decline, the population of young people declines by 20% of the initial population at t=2 and 3, and after t=3, the population of young people becomes 60% of the initial population. We assume that the populations of the middle-aged and elderly generations do not decrease due to death before the end of their natural lifespan. The middle-aged population declines one period later than the young one, and the elderly population declines one period later than the middle-aged one. In the population growth, in contrast to decline, the population increases by 20% of its initial value. In the steady state, the population is assumed to be constant in each period.

4.2 Result

In Section 4.2, we numerically demonstrate how the results differ between population constant and population dynamics. The results of numerical simulations are shown as three-dimensional figures. Our study focuses on the effect of amenities. Amenity ratios between zones $\ln e_1^k / e_2^k \;\; for \; k \in \{y,m,o\}$ are set in the range of -0.5 to 0.5. The X-axis shows the difference in the zone amenity for middle-aged people between zones. The Y-axis shows the difference in the zone amenity for old people between zones. The Z-axis shows the sum of the discounted lifetime utilities for a household. In a demographic dynamic setting, we calculate the lifetime utility for a household by birth generation. In order to determine the solution, it is necessary to set the difference in zone amenities for young people. However, it is impossible to represent a four-dimensional diagram. Therefore, we show a three-dimensional figure by fixing amenities for young people at certain levels.

In the following, some important observations in the numerical results are presented as main findings. First, to clarify the relationship between amenities and lifetime utility, we show two figures of lifetime utility, where the X- and Y-axes are measured in the logarithmic scale, so that the distribution of amenities is evenly distributed between the two zones as the value is zero. Fig. 3 is a figure of lifetime utility for the case of $\ln e_1^y / e_2^y = 0$ with a population decline. Fig. 4 is that for the case of $\ln e_1^y / e_2^y = 0$ with a population growth. Fig. 5 is that for the case of $\ln e_1^y / e_2^y = 0$ with a constant population.

Fig. 3 Discounted lifetime utility ($\ln \frac{e_1{}^y}{e_2{}^y} = 0$, population decline, t=1)

Fig. 4 Discounted lifetime utility ($\ln \frac{e_1{}^y}{e_2{}^y} = 0$, population growth, t=1)

Fig. 5 Discounted lifetime utility ($\ln \frac{e_1{}^y}{e_2{}^y} = 0$, constant population)

Figs. 3, 4, and 5 show the lifetime utilities are negatively sloped toward the center of the figure. In other words, placing more amenities in one of the two zones results in an equilibrium with a higher lifetime utility. We confirm that this tendency holds even regardless of the setting of amenity distribution, population dynamics, and period t. These results can be summarized as Main Finding 1.

Main Finding 1. *The equilibrium lifetime utility is higher when amenities are very unequally distributed between zones than when they are equally distributed.*

The reason for Main Finding 1 to hold can be explained by location pattern as follows. For location patterns of segregation (e.g., Y/MO), the utility increases as more amenities are allocated to the zone where people with high preference for the amenities live. For location patterns of partial segregation (e.g., YMO/Y), the increase in utility by allocating more preferred amenities to one zone overcomes the decrease in utility due to congestion caused by the unevenness of the amenities.

Main Finding 1 provides the following implications for urban amenity policies. Amenities for households of a certain age group should be concentrated in certain areas, rather than being located evenly throughout the city. In other words, planning areas that are attractive to a certain age group is a good policy from the perspective of utility. For example, entertainment facilities that appeal to the young residents should be located mainly in the city center, while parks and nursery schools that appeal to the child-rearing generation should be located mainly in the suburbs.

Next, we explore how the results change according to the demographic changes. For instance, we show the results from t=1 to 6 in the case of a population decline and the constant population, where $\ln e_1^y / e_2^y = -0.1$ in Figs. 6 and 7, respectively. The yellow stars in Fig. 6 and 7 illustrate the residential pattern with the highest lifetime utility.

MO/Y has the highest utility among all the patterns at t=2 and 3 in the case of population decline in Fig. 6, while in the constant population in Fig. 7, YMO/Y has the highest lifetime utility. From this, we obtain Main Finding 2.

Main Finding 2. *The residential pattern with the highest lifetime utility may differ between the constant population case and an unsteady demographic case.*

The mechanism of Main Finding 2 is that population decline permits the young generation to live in one zone. We explain this using Figs. 6 and 7. YMO/Y in the constant population indicates that the population of the young generation is too large to live in one zone for young generation. The population of the young generation declines at t=2 and 3. At t=2, the populations of the young, middle-aged, and elderly generations are 8,000, 10,000, and 10,000, respectively. This population and age structure change permits the young generation to live in one zone. In other words, the pattern of partial segregation changes to segregation.

Next, we explain the area having the rich amenities for the middle-aged and elderly in Zone 1. After *t*=2, there are two equilibria: YM/YO and YM/O. The reason why YM/O occurs is the same as above: population decline. At t=3, the populations of the young, middle-aged, and elderly generations are 6,000, 8,000, and 10,000, respectively. The decrease in the young and middle-aged populations reduces congestion in Zone 1, and this permits the young generation to live in one zone.

Note that the previous residential pattern with the highest lifetime utility becomes unstable during t=4 and 5. This result indicates that the residential pattern with the highest lifetime utility may change.

Fig. 6 Discount lifetime utility ($\ln \frac{e_1^y}{e_2^y} = -0.1$, t=1 to 6, population decline)

Fig. 7 Discounted lifetime utility ($\ln \frac{e_1{}^y}{e_2{}^y} = -0.1$, constant population)

4.3 Effects of Remote Work

This subsection discusses the robustness when the parameters are changed by exogenous shocks. The impact of COVID-19 has significantly changed how people work. Globally, the number of commuter trips has decreased, and the number of days people work from home has increased. As a result, more and more people highly value the comfort and space of their homes rather than the short commute to the workplace. We discuss the case in which such a change in preference and decrease in the number of commuting trips will continue in the future. The purpose of this analysis is to demonstrate whether Main Findings 1 and 2 hold when parameters are changed.

We set parameters as follows. The housing preference shares γ^y and γ^m of the young and middle-aged groups are set at 110% of the current values, the consumption preference shares α^y and α^m at 90%, and the transportation costs of the young and middle-aged groups at 50%.

First, we check Main Finding 1. Figs. 8, 9, and 10 show the results for the same settings except parameters related to remote work as in Figs. 3, 4, and 5. Figs. 8, 9, and 10 show the lifetime utilities are negatively sloped toward the center of the figure. Therefore, Main Finding 1 holds in the case of remote work, too.

Second, we check Main Finding 2. Figs. 11 and 12 show the results. Fig. 11 shows the case of population decline, $\ln e_1^y / e_2^y = 0.1$, and remote work. Fig. 12 shows the case of constant population, $\ln e_1^y / e_2^y = 0.1$, and remote work. The residential pattern with the highest lifetime utility in Fig. 11 during t=2 to 4 is in Y/MO, and that in Fig. 12 is in YO/M. Therefore, Main Finding 2 holds in remote work. However, the mechanism of Main Finding 2 in remote work is different from that in the baseline parameter settings. The mechanism in the baseline settings involves a change in the pattern with an amenity allocation fixed because of population decline. Unlike this mechanism, as shown in Fig. 11, the pattern YO/M with $(\ln e_1^m / e_2^m, \ln e_1^o / e_2^o) = (-0.5, 0.5)$ holds during t=1 to 6 in a remote work setting. As the population declines, the desirable amenity distribution changes from $(\ln e_1^m / e_2^m, \ln e_1^o / e_2^o) = (-0.5, 0.5)$ to $(\ln e_1^m / e_2^m, \ln e_1^o / e_2^o) = (-0.5, -0.5)$ as shown in Fig. 11. That is, there are multiple ways to generate Main Finding 2.

Fig. 8 Discounted lifetime utility ($\ln \frac{e_1^y}{e_2^y} = 0$, population decline, t=1, remote work)

Fig. 9 Discounted lifetime utility ($\ln \frac{e_1^y}{e_2^y} = 0$, population growth, t=1, remote work)

Fig. 10 Discounted lifetime utility ($\ln \frac{e_1^y}{e_2^y} = 0$, constant population, remote work)

Fig. 11 Discounted lifetime utility ($\ln \frac{e_1^y}{e_2^y} = 0.1$, t=1 to 6, population decline, remote work)

Fig. 12 Discounted lifetime utility ($\ln \frac{e_1^y}{e_2^y} = 0.1$, constant population, remote work)

5. Conclusion

Using an OLG model in a closed city with two zones, we clarify how the distribution of amenities for young, middle-aged, and elderly generations affects the residential location patterns and their utilities. In particular, we take account of their dynamic optimal location choices throughout their lives.

Our model has many residential patterns for young, middle-aged, and elderly people. As a theoretical result, we reveal that there is no residential pattern in which middle-aged or elderly households live in both zones when the pattern remains the same because people choose their current location expecting to obtain a high utility in the future. Next, the numerical simulation provides some policy implications. First, amenities for households of a certain age group should be concentrated in certain areas, rather than being located evenly throughout the city. This is because the utility increases as more amenities are allocated to the zone where people with high preference for the amenities live. Second, the desirable residential pattern from the viewpoint of lifetime utility changes depending on the demographics and the distribution of amenities. This is because population and age structure change leads to a change in the desirable residential pattern. As additional research, we check the robustness when the parameters are changed by an increase in remote work.

Future studies on the topic of this study can be pursued along various avenues[8]. Here, we will explain two of them. First, the current model sets only two zones. This setting can yield closed-form results, which are useful for capturing mechanisms. But if we extend our model to a multiple zone model, then we obtain richer results associated with the relation between urban land use and demographic dynamics. Second, the model does not deal with vacant houses. In addition, buildings are durable for two periods, while the floor size per household is not preserved and changes in each period. There is a problem of an increase in the number of vacant houses due to the aging of the population in Japan, which cannot be

[8] Besides the topics shown here, urban externalities can be introduced. See Kono and Joshi (2019) for traffic congestion, Yoshida and Kono (2020, 2022) for biological externalities, and Domon et al. (2022) for carbon dioxide externalities.

addressed in this model. By developing a more realistic model of the real housing market, we can obtain suggestions on the vacant house problem associated with changes in life stages.

Appendices

Appendix A: Derivation process of maximization of utility

First, we derive the demands of the young from their behavior, as follows.

$$c^y = \alpha^y \left\{ w^{y,t} \left(\bar{T} - \sum_{k=1}^{i} M_{k-1,k} \right) - \sum_{k=1}^{i} \Psi_{k-1,k} \right\}, \tag{15}$$

$$l^y = \frac{\beta^y}{w^y} \left\{ w^{y,t} \left(\bar{T} - \sum_{k=1}^{i} M_{k-1,k} \right) - \sum_{k=1}^{i} \Psi_{k-1,k} \right\}, \tag{16}$$

$$h^y = \frac{\gamma^y}{r^y} \left\{ w^{y,t} \left(\bar{T} - \sum_{k=1}^{i} M_{k-1,k} \right) - \sum_{k=1}^{i} \Psi_{k-1,k} \right\}. \tag{17}$$

Second, we derive the demands in middle age and elderly age, as follows.

$$c^m = \alpha^m \left\{ w^{m,t} \left(\bar{T} - \sum_{k=1}^{i^m} M_{k-1,k} \right) - \sum_{k=1}^{i^m} \Psi_{k-1,k} - \frac{\theta}{(1+\pi)} \sum_{j=1}^{i^o} \Psi_{j-1,j} \right\}, \tag{18}$$

$$l^m = \frac{\beta^m}{w^m} \left\{ w^{m,t} \left(\bar{T} - \sum_{k=1}^{i^m} M_{k-1,k} \right) - \sum_{k=1}^{i^m} \Psi_{k-1,k} - \frac{\theta}{(1+\pi)} \sum_{j=1}^{i^o} \Psi_{j-1,j} \right\}, \tag{19}$$

$$h^m = \frac{\gamma^m}{r^m} \left\{ w^{m,t} \left(\bar{T} - \sum_{k=1}^{i^m} M_{k-1,k} \right) - \sum_{k=1}^{i^m} \Psi_{k-1,k} - \frac{\theta}{(1+\pi)} \sum_{j=1}^{i^o} \Psi_{j-1,j} \right\}, \tag{20}$$

$$c^o = \alpha^o \left\{ w^{m,t} \left(\bar{T} - \sum_{k=1}^{i^m} M_{k-1,k} \right) - \sum_{k=1}^{i^m} \Psi_{k-1,k} - \frac{\theta}{(1+\pi)} \sum_{j=1}^{i^o} \Psi_{j-1,j} \right\}, \tag{21}$$

$$l^o = \bar{T} - \theta \sum_{j=1}^{i^o} M_{j-1,j}, \tag{22}$$

$$h^o = \frac{\gamma^o}{r^o} \left\{ w^{m,t} \left(\bar{T} - \sum_{k=1}^{i^m} M_{k-1,k} \right) - \sum_{k=1}^{i^m} \Psi_{k-1,k} - \frac{\theta}{(1+\pi)} \sum_{j=1}^{i^o} \Psi_{j-1,j} \right\}. \tag{23}$$

where i^y, i^m, i^o express zones in which each generation lives.

The savings are derived as follows,

$$s^{m,t} = \frac{1}{(1+\pi)}\left((\alpha^o+\gamma^o)w^{m,t}\left(\bar{T}-\sum_{k=1}^{i^m} M_{k-1,k}\right)-(\alpha^o+\gamma^o)\sum_{k=1}^{i^m}\Psi_{k-1,k}+\left(1-\frac{(\alpha^o+\gamma^o)}{(1+\pi)}\right)\theta\sum_{j=1}^{i^o}\Psi_{j-1,j}\right). \tag{24}$$

Appendix B: The processes of deriving the equilibria of each residential pattern

Segregation

The simultaneous equations are as follows.

$$\begin{cases} r_i^{t+1} = f_i(r_i^{t-1}, r_i^t) & \\ r_i^0 = \overline{r_i^0} & i \in \{1,2\} \\ r_i^T = \overline{r_i^T} & t \in \{0,1,\ldots,T\} \end{cases} \tag{25}$$

Note that, as the main text explains, we assume that the system is in steady states in both the first and last periods. The first equation in Eq. (25) represents Eq. (14). $\overline{r_i^0}$ and $\overline{r_i^T}$ mean the land rent in Zone *i* at the initial period (0) and at the last period (*T*) in a steady state.

Partial Segregation

To solve Eq. (14) in the case that people in the young generation live in both zones, we set the utility of young people to be constant regardless of which zone they live in, $V_1^y = V_2^y$. As in the case of segregation, we assume that the initial and terminal phases are steady states. The simultaneous equations are as follows.

$$\begin{cases} r_i^{t+1} = f_i(r_i^{t-1}, r_i^t, N_i^{y,t}) & \\ r_i^0 = \overline{r_i^0} & i \in \{1,2\} \\ r_i^T = \overline{r_i^T} & t \in \{0,1,\ldots,T\} \\ V_1^{y,t} = V_2^{y,t} & \end{cases} \tag{26}$$

The first equation in Eq. (26) has $N_i^{y,t}$ as a variable. $N_i^{y,t}$ represents the population of young households living in Zone *i*. In the case where young households reside in both zones, they are stable when $\partial V_1^y / \partial N_1^y < 0$ and $\partial V_2^y / \partial N_2^y < 0$. For the middle-aged and the elderly, the pattern is stable when the lifetime utility (discounted utility sum) cannot be increased by changing the path of location choices.

Appendix C: Proof of Proposition 1

To prove Proposition 1, we set patterns in which middle-aged or elderly people reside in both zones and show that these patterns do not hold in equilibrium. These patterns are in interior equilibria in the sense that middle-aged or elderly populations, $N_{i,j}^{m,t}$ or $N_{i,j}^{o,t}$, where *i* is the zone of their residence in the middle stage, and j is the zone of their planned residence in the elderly stage, are determined as an interior solution of the equation system. To give a formal proof, we have to show the results of all the locational patterns (ten patterns). However, that would be long and tedious. So, we give a proof sketch which can show the proof step and the important points.

The following conditions are necessary for the equilibrium. First, the utility of a person from middle age to elderly age is constant regardless of which residential path they choose, i.e., $V_{i,j}^{m,t} + V_{i,j}^{o,t+1} / \left(1+\rho\right) = V_{i',j'}^{m,t} + V_{i',j'}^{o,t+1} / \left(1+\rho\right)$ for $\{i, j\} \neq \{i', j'\}$, where *i* is the zone of their residence in the middle stage and j is the zone of their planned residence in the elderly stage. For greater readability, we define $\tilde{V}_{i,j}^{t} \equiv V_{i,j}^{m,t} + V_{i,j}^{o,t+1} / \left(1+\rho\right)$ hereafter. This represents the utility of a person from middle age to elderly age. Second, the middle-aged households maximize the lifetime utility with regard to the number of residents in the two zones, i.e., $\max_{N_{i,j}^{m,t}, N_{i',j'}^{m,t}} \tilde{V}_{i,j}^{t}$. This is because any household chooses its residential location choices to maximize its lifetime utility. When $\max_{N_{i,j}^{m,t}, N_{i',j'}^{m,t}} \tilde{V}_{i,j}^{t}$ has interior solutions, the derivative of $\tilde{V}_{i,j}^{t} \equiv V_{i,j}^{m,t} + V_{i,j}^{o,t+1} \big/ \left(1+\rho\right)$ with respect to $N_{i,j}^{m,t}$ and $N_{i',j'}^{m,t}$ must be 0 at the solutions. We prove that this does not hold as follows.

First, we explain the cases of interior equilibria where the middle-aged reside in both zones (i.e., YM/MO, YMO/M, MO/YM, and M/YMO). Take YM/MO as an example. In this pattern, there are two residential paths: first, living in Zone 1 in middle age and in Zone 2 in elderly age, and second, living in Zone 2 in middle age and in Zone 2 in elderly age. These location path choices, $N_{1,2}^{m,t}$ and $N_{2,2}^{m,t}$, should maximize the equilibrium lifetime utility.

$$\max_{N_{1,2}^{m,t},N_{2,2}^{m,t}} \tilde{V}_{1,2}^{t} = \tilde{V}_{2,2}^{t}. \tag{27}$$

Eq. (27) implies that the utilities of the two residential paths are common, and the equilibrium allocation of the middle-aged people should maximize the equilibrium utility because if some deviation from the allocation increases the utility, people in middle age will change their residential location.

Because the total number of people in middle age is fixed, considering the derivative with respect to either $N_{1,2}^{m,t}$ or $N_{2,2}^{m,t}$ is sufficient. Here, we calculate the derivative of $\tilde{V}_{1,2}{}^{t}$ and $\tilde{V}_{2,2}{}^{t}$ with respect to $N_{2,2}^{m,t}$ to obtain

$$\begin{aligned}
&\frac{\partial \tilde{V}_{1,2}^{t}}{\partial N_{2,2}^{m,t}} = \frac{\partial \tilde{V}_{2,2}^{t}}{\partial N_{2,2}^{m,t}} \\
&= -\frac{\gamma^{m}\gamma^{o}D_{2,2}}{(1+\rho)(\gamma^{o}D_{1,2}N_{1,2}^{m,t-1}+\gamma^{o}D_{2,2}N_{2,2}^{m,t-1}+\gamma^{m}D_{2,2}N_{2,2}^{m,t}-\frac{1}{2}\overline{H}^{2}r_{2}{}^{t}(r_{2}{}^{t-1}+\frac{2+\rho}{1+\rho}r_{2}{}^{t}))} \\
&= -\frac{\gamma^{m}\gamma^{o}\underline{D_{2,2}}}{\frac{1}{2}\overline{H}^{2}\underline{r_{2}{}^{t}r_{2}{}^{t+1}}} < 0.
\end{aligned} \tag{28}$$

The sign of Eq. (28) is negative. γ^{m} and γ^{o} in the numerator of Eq. (28) are preference parameters for housing in the middle-aged and elderly and are positive. $D_{2,2}$, which is the net income of the middle-aged and the elderly after subtracting the transportation costs for the two periods from their income when people live in Zone 2 from middle age to elderly age, is positive, too. Hence, the numerator is positive. Since $\overline{H}$ in the denominator is the square root of the area H of the zone and r_{i}^{t} is the land rent, the denominator is also positive. Therefore, Eq. (28) is negative.

As a result, the path in which people live in Zone 2 from middle age to elderly age is not valid, and only the path to move from Zone 1 to 2 remains, resulting in a YM/O residential pattern. The intuitive reason for this is to avoid having the middle-aged live in Zone 2, which would make the zone crowded and raise the rent of the elderly.

In the other cases of interior equilibria where the middle-aged reside in both zones (i.e., YMO/M, MO/YM, and M/YMO), the underlined terms in Eq. (28) change (e.g., in the case of YMO/M, $D_{2,2} \rightarrow D_{1,1}$ and $r_2^{\,t} \rightarrow r_1^{\,t}$). These changes are caused by the difference in changes in r_j^{t+1}. r_j^{t+1} depends on whether the young or the elderly live in j. However, these changes in any location pattern do not affect the sign of the equation corresponding to Eq. (28). Hence, there are no interior equilibria where the middle-aged reside in both zones.

Next, we explain the cases of interior equilibria where elderly people reside in both zones (i.e., YO/MO, MO/YO, YMO/O, and O/YMO). The derivation process is similar to the above. Take YMO/O as an example to explain the patterns where elderly people reside in both zones. In equilibrium, YMO/O has $\max_{N_{1,1}{}^{m,t}, N_{1,2}{}^{m,t}} \tilde{V}_{1,1}{}^{t} = \tilde{V}_{1,2}{}^{t}$. $\tilde{V}_{1,1}{}^{t}$ and $\tilde{V}_{1,2}{}^{t}$ represent the utility of a person from middle age to elderly age. The former is the utility of a person living in Zone 1 in both middle age and elderly age. The latter is the utility of a person who lives in Zone 1 in middle age and Zone 2 in elderly age. The signs of the derivative of $\tilde{V}_{1,1}{}^{t}$ and $\tilde{V}_{1,2}{}^{t}$ with respect to $N_{1,1}^{m,t}$ and $N_{1,2}^{m,t}$ are obtained as follows.

$$\frac{\partial \tilde{V}_{1,1}^{t}}{\partial N_{1,1}^{m,t}} \neq 0, \text{ and } \frac{\partial \tilde{V}_{1,2}^{t}}{\partial N_{1,1}^{m,t}} = 0 \,. \tag{29}$$

We describe the detailed solution of Eq. (29) in Eq. (30).

Defining $A^{t} = \gamma^{o} D_{1,1} N_{1,1}^{m,t-1} + \gamma^{m} D_{1,1} N_{1,1}^{m,t} + \gamma^{m} D_{1,2} N_{1,2}^{m,t} + \gamma^{y} B_{1} N^{y,t}$, we obtain Eq. (30)

$$\partial \tilde{V}^{t}_{1,1} \big/ \partial N_{1,1}^{m,t}$$
$$= -\frac{\gamma^{m}\gamma^{o} D_{1,1}}{(1+\rho)\left[A^{t} - \frac{1}{2\overline{H}^{2}\left(r_1^{t-1}\right)^{2}}\left(-2(1+\rho)\left(A^{t-1}+\overline{H}^{2}\left(\left(r_1^{t-1}\right)^{2}+r_1^{t-2}r_1^{t-1}\right)\right)\right)\left(-2(1+\rho)(2+\rho)(A^{t-1})+\overline{H}^{2}\left((1+\rho)^{2}+(2+\rho)\right)\left(r_1^{t-1}\right)^{2}+\overline{H}^{2}(1+\rho)(2+\rho)r_1^{t-2}r_1^{t-1}\right)\right]}. \tag{30}$$

The above equation has only positive variables, so the denominator does not equal 0. In other words, YMO/O cannot hold as an interior equilibrium, implying that YMO/O should change into other patterns, such as YMO/none and YM/O.

Similar to cases of interior equilibria where elderly people reside in both zones, Eq. (30) changes in the other cases (i.e., YO/MO, MO/YO, O/YMO). Unlike the case in which the middle-aged live in both zones, the equation corresponding to Eq. (30) changes to a somewhat large extent. However, this change cannot affect the sign of the equation. Hence, there are no interior equilibria where the elderly people reside in both zones.

Finally, in the case where the middle-aged and elderly generations live in both zones (i.e., YMO/MO, and MO/YMO) also, as a result of a similar discussion, interior equilibria cannot be established.

The above discussions in the case of Partial segregation prove Proposition 1.

References

Albouy, D., & Stuart, B. A., 2020. Urban population and amenities: The neoclassical model of location. International economic review, 61(1), 127-158.

Alonso, W., 1964. Location and land use. Harvard University Press, Cambridge, MA.

Althaus, P. G. 2004. A Life-cycle Model of Locational Choice in a Two-region Economy. Journal of Regional Science, 44, 351–366.

Anas, A., 1978. “Dynamics of urban residential growth,” Journal of Urban Economics, 5, 66–87.

Arnott, R. J., 1980. “A Simple Urban Growth Model with Durable Housing,” Regional Science and Urban Economics, 10, 53–76.

Barro, R. J., 1974. “Are government bonds net wealth?” Journal of Political Economy, 82, 1095-117.

Braid, R. M., 1991. “Residential Spatial Growth with Perfect Foresight and Multiple Income Groups,” Journal of Urban Economics, 30, 385-407.

Braid, R. M., 2001. "Spatial Growth and Redevelopment with Perfect Foresight and Durable Housing," Journal of Urban Economics, 49, 425–452.

Brueckner, J. K., 1980. "A Vintage Model of Urban Growth," Journal of Urban Economics, 8, 389–402.

Brueckner, J. K., 1981. "A Dynamic Model of Housing Production, Journal of Urban Economics, 10, 1–14.

Brueckner, J. K., Pereira, A. M., 1997. "Housing wealth and the economy's adjustment to unanticipated shocks," Regional Science and Urban Economics, 27, 497–513.

Brueckner, J. K., 2000. "Urban Growth Models with Durable Housing: An Overview," in J.-M. Huriot and J.-F. Thisse (eds.), Economics of Cities. Cambridge University Press, pp. 263–289.

Circella, G., Fulton, L., Handy, S. L., & Mokhtarian, P. L. 2017. The Multimodal Behavior of Millennials : Exploring Differences in Travel Choices between Young Adults and Gen Xers in California. (January).

Domon, S., Hirota, M., Kono, T., Managi, S., & Matsuki, Y. (2022). The long-run effects of congestion tolls, carbon tax, and land use regulations on urban CO2 emissions. Regional Science and Urban Economics, 92, 103750.

Dutzik, T., Inglis, J., & Baxandall, P., 2014. Millennials in Motion. U.S. PIRG Education Fund Frontier Group.

Duranton, G., 2000. "Urbanization, Urban Structure, and Growth," in J.-M. Huriot and J.-F. Thisse (eds.), Economics of Cities. Cambridge University Press, pp. 290–317.

Evans, A. W., 1975. "Rent and Housing in the Theory of Urban Growth," Journal of Regional Science, 15, 113–125.

Englund, P., 1986. "Transaction Costs, Capital-Gains Taxes, and Housing Demand," Journal of Urban Economics, 20, 274–290.

Fujita, M., 1982. "Spatial patterns of residential development," Journal of Urban Economics, 12, 22–52.

Gaigné, C., Koster, H. R. A., Moizeau, F., & Thisse, J.-F., 2022. Who lives where in the city? Amenities, commuting and income sorting. Journal of Urban Economics, 128, 103394.

Hardman, A. M. and Ioannides, Y. M. 1995. “Moving behavior and the housing market,” Regional Science and Urban Economics, 25, 21–39.

Kono, T., & Joshi, K. K. (2019). Traffic congestion and land use regulations: Theory and policy analysis. Elsevier.

Kono, T., Kotoku, T., Otazawa, T., 2012. Residential land use with demographic dynamics of young and old generations. Journal of Housing Economics, 21, 283-295.

Lee, Y., Lee, B., & Shubho, M. T. H. 2019. Urban revival by Millennials? Intraurban net migration patterns of young adults, 1980–2010. Journal of Regional Science, 59(3), 538–566.

Mills, E. S., 1967. “An aggregative model of resource allocation in a metropolitan area,” American Economic Review, 57, 197–210.

Ministry of Health, Labour and Welfare, 2020. Labour Survey. Retrieved January, 2022. from https://www.mhlw.go.jp/toukei/youran/index-roudou.html

Ministry of Internal Affairs and Communications, 2014. National Survey of Family Income and Expenditure. Retrieved January, 2020, from https://www.stat.go.jp/data/zensho/2014/index.html

Ministry of Internal Affairs and Communications, 2019. National Survey of Family Income, Consumption and Wealth. Retrieved January, 2022, from https://www.stat.go.jp/data/zenkokukakei/2019/index.html

Ministry of Internal Affairs and Communications, 2016. Survey on Time Use and Leisure Activities. Retrieved January, 2020, from https://www.stat.go.jp/data/shakai/2016/index.html

Ministry of Land, Infrastructure, Transport and Tourism, Japan, “Average duration period” of houses in the U.S.A. and England. Retrieved May 2012 from http://www.mlit.go.jp/jutakukentiku/house/singi/syakaishihon/bunkakai/14bunkakai/14bunka_sankou04.pdf

Miyagi Prefectural Government. Miyagi Prefecture shichousonmin keizai keisan. Retrieved January, 2022. from https://www.pref.miyagi.jp/soshiki/toukei/sityousonmin.html

Miyagi Transportation official web site. Retrieved January, 2020, from http://www.miyakou.co.jp/top.php

Muth, R. F., 1969. Cities and Housing. University of Chicago Press.

Muth, R. F., 1975. “A Vintage Model of the Housing Stock,” Papers and Proceedings of the Regional Science Association, 30, 141–156.

Okada, Y., 2014. Young and elderly people return to urban centers Products and services for urban residents become a new growth industry (this title is translated by us). (in Japanese) Retrieved December, 2019, from https://www.mizuho-ri.co.jp/publication/research/pdf/insight/pl140331.pdf

Sendai., 2019. Sendai toshiken Person-Trip Survey. Retrieved December, 2019, from https://www.city.sendai.jp/kotsu-kekaku/kurashi/machi/kotsu/data/pt.html

Shimizu, C., Yasumoto, S., Asami, Y., & Clark, T, N., 2014. Do Urban Amenities Drive Housing Rent?. HIT-REFINED Working Paper Series 9, Institute of Economic Research, Hitotsubashi University.

Tabuchi, T., 2019. Do the rich and poor collocate in large cities?. Journal of Urban Economics, 113, 103186.

Turnbull, G. K., 1988, “Residential development in an open city,” Regional Science and Urban Economics, 18, 307–320.

Wheaton, W. C., 1982a. “Urban Residential Growth under Perfect Foresight.” Journal of Urban Economics, 12, 1–12.

Wheaton, W. C., 1982b. “Urban Spatial Development with Durable but Replaceable Capital,” Journal of Urban Economics, 12, 53–67.

Yonemoto, K., 2007. “Endogenous determination of historical amenities and the residential location choice,” Annals of Regional Science, 41, 967–993.

Yoshida, J., & Kono, T. (2020). Land use policies considering a natural ecosystem. Regional Science and Urban Economics, 83, 103552.

Yoshida, J., & Kono, T. (2022). Cities and biodiversity: Spatial efficiency of land use. Journal of Economic Behavior & Organization, 197, 685-705.

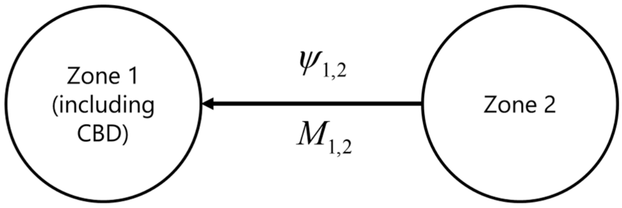

Fig. 3 The city

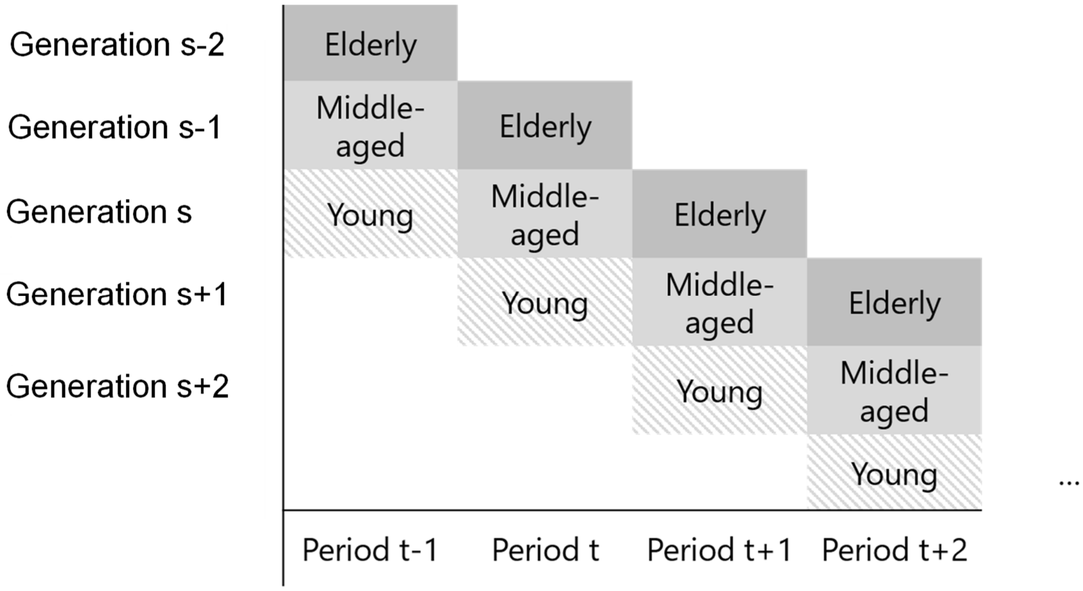

Fig. 4 The three-generation overlapping model

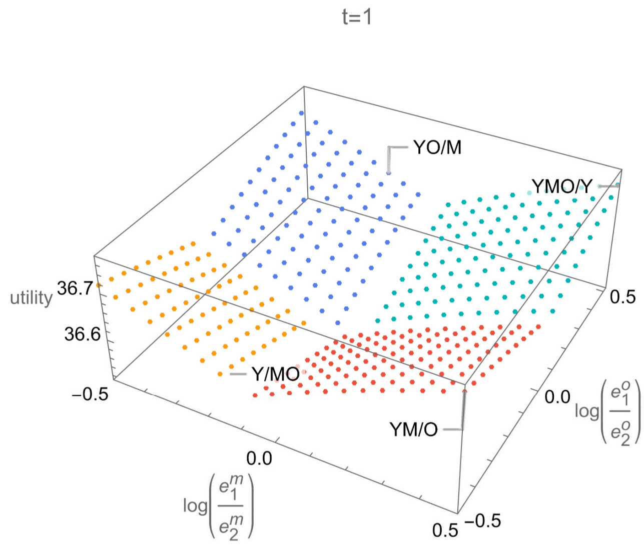

Fig. 3 Discounted lifetime utility ($\ln \frac{e_1{}^y}{e_2{}^y} = 0$, population decline, t=1)

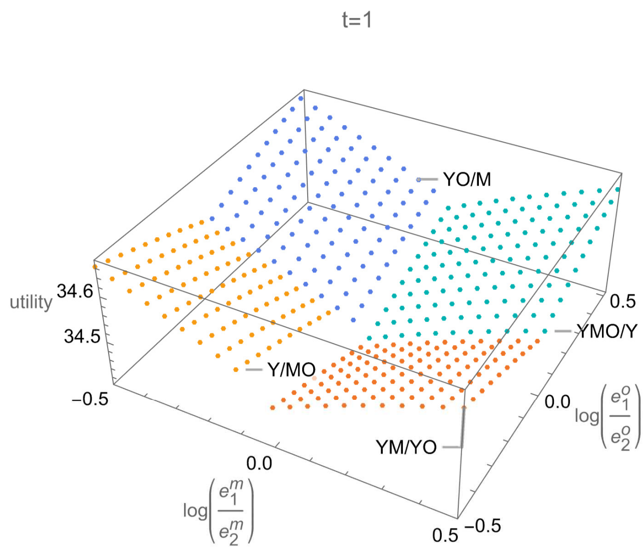

Fig. 4 Discounted lifetime utility ($\ln \frac{e_1{}^y}{e_2{}^y} = 0$, population growth, t=1)

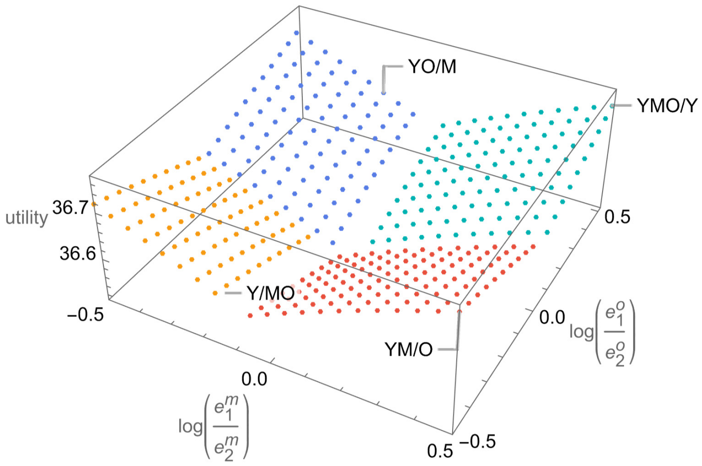

Fig. 5 Discounted lifetime utility ($\ln\frac{e_1{}^y}{e_2{}^y} = 0$, constant population)

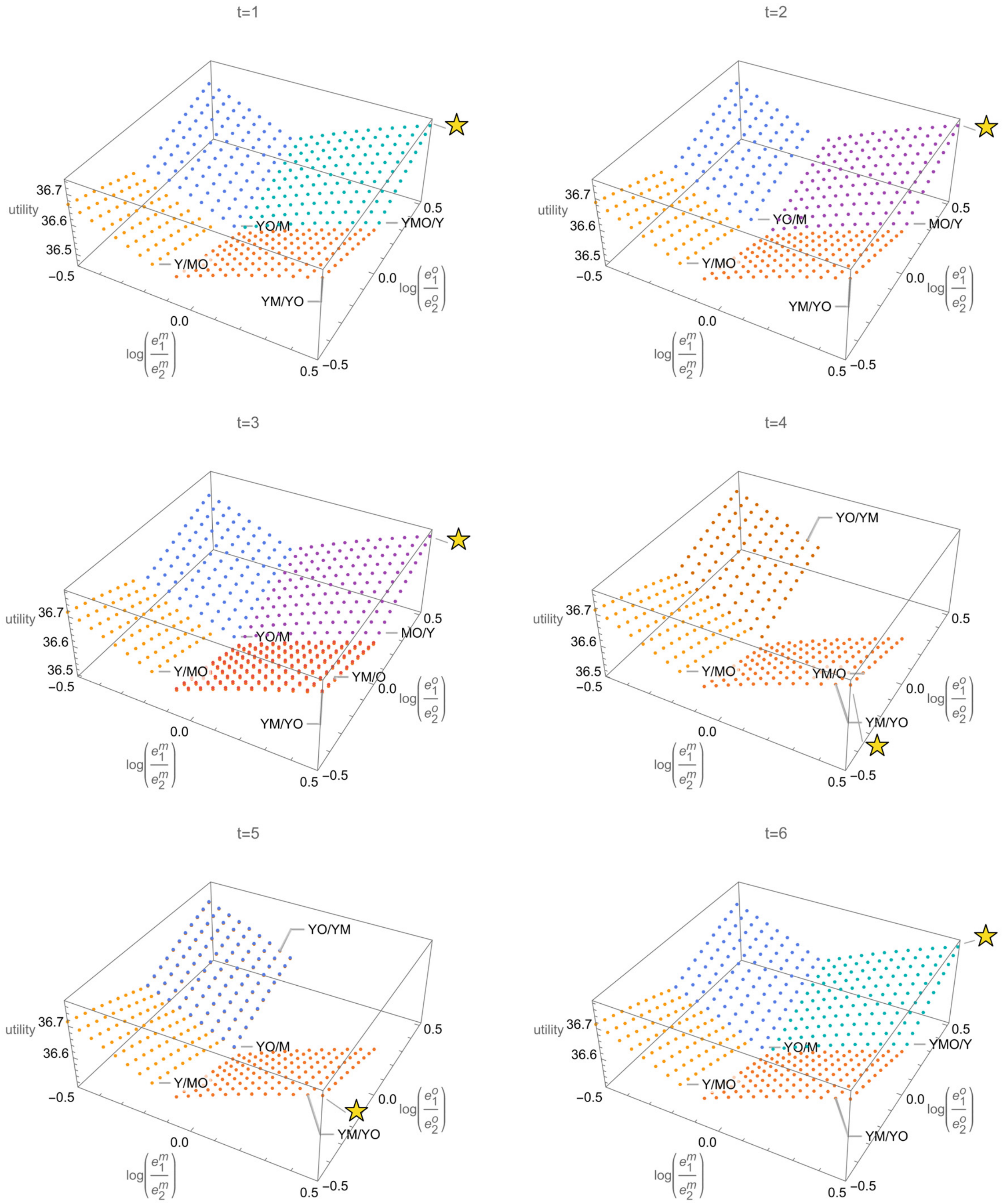

Fig. 6 Discounted lifetime utility ($\ln \frac{e_1^y}{e_2^y} = -0.1$, t=1 to 6, population decline)

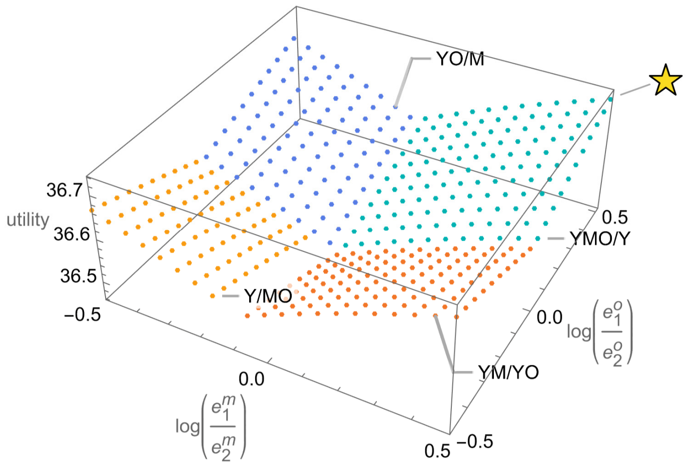

Fig. 7 Discounted lifetime utility ($\ln \frac{e_1{}^y}{e_2{}^y} = -0.1$, constant population)

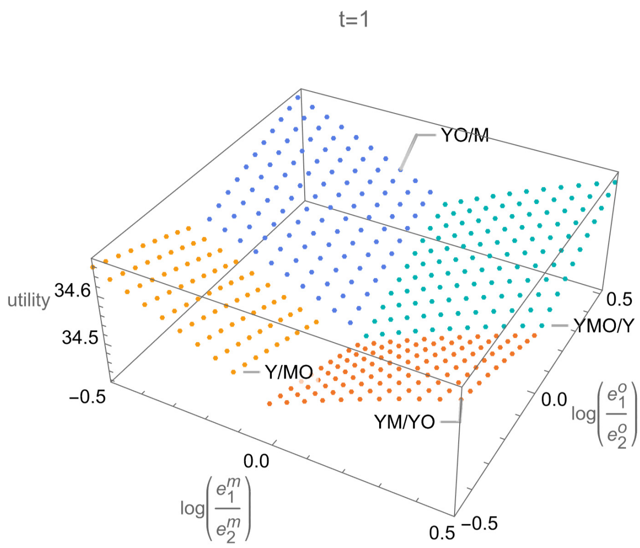

Fig. 8 Discounted lifetime utility ($\ln \frac{e_1{}^y}{e_2{}^y} = 0$, population decline, t=1, remote work)

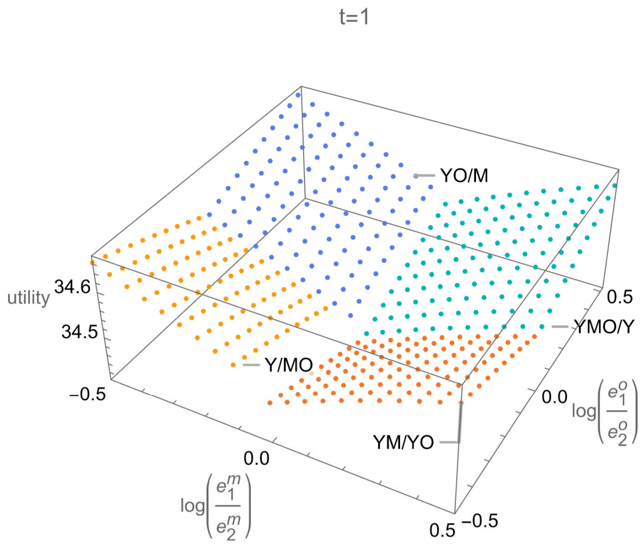

Fig. 9 Discounted lifetime utility ($\ln \frac{e_1{}^y}{e_2{}^y} = 0$, population growth, t=1, remote work)

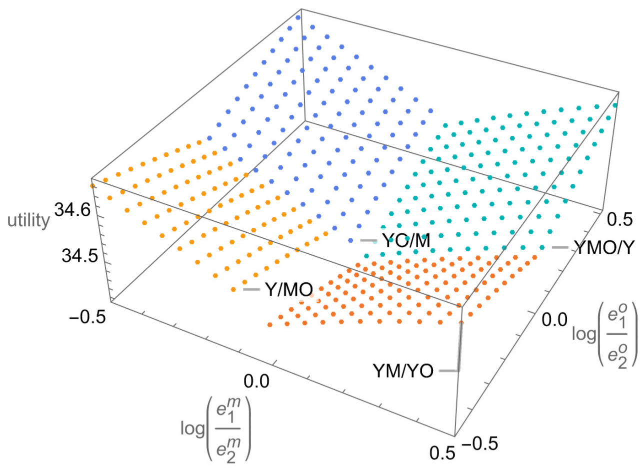

Fig. 10 Discounted d lifetime utility ($\ln \frac{e_1{}^y}{e_2{}^y} = 0$, constant population, remote work)

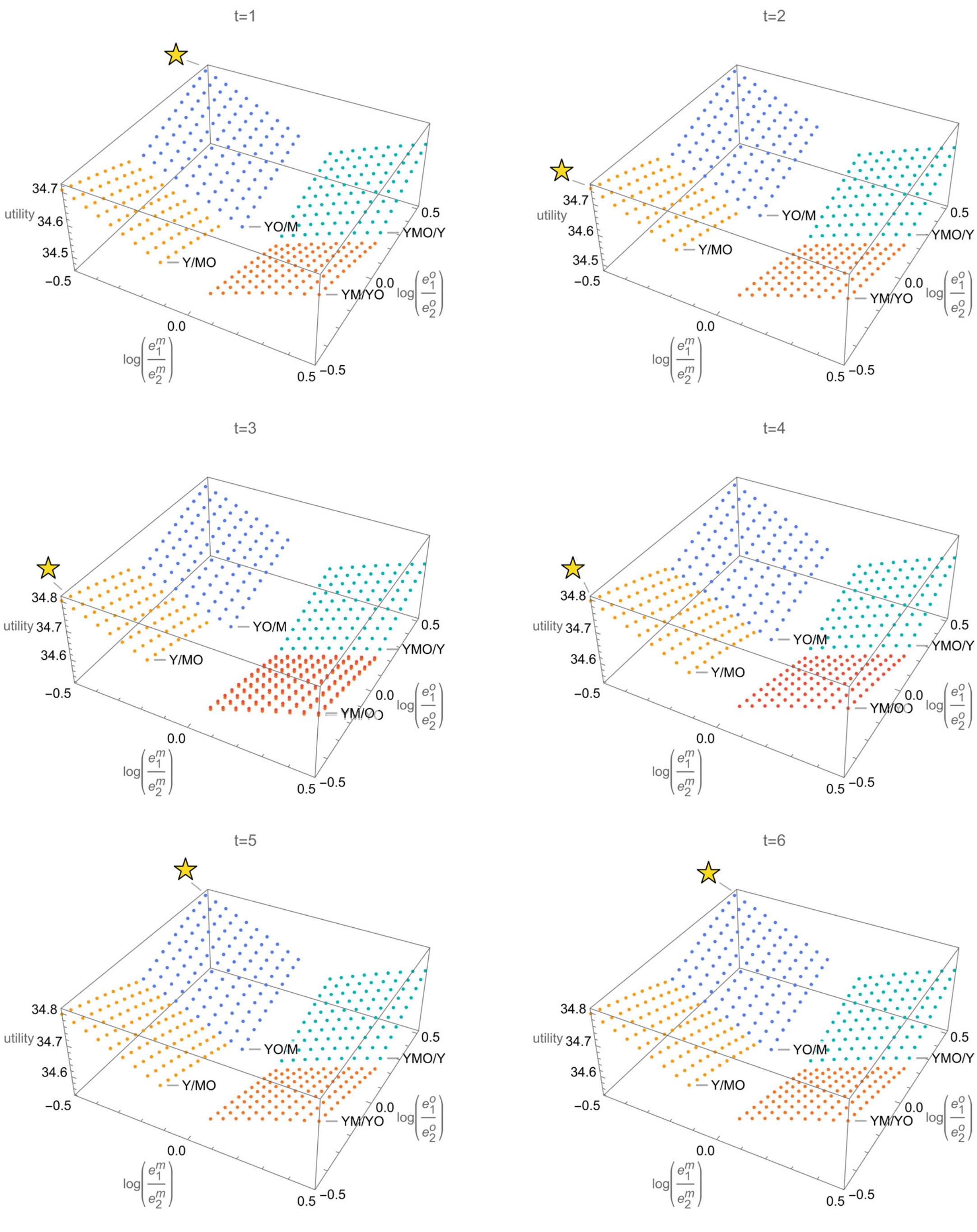

Fig. 11 Discounted lifetime utility ($\ln\frac{e_1{}^y}{e_2{}^y} = 0.1$, t=1 to 6, population decline, remote work)

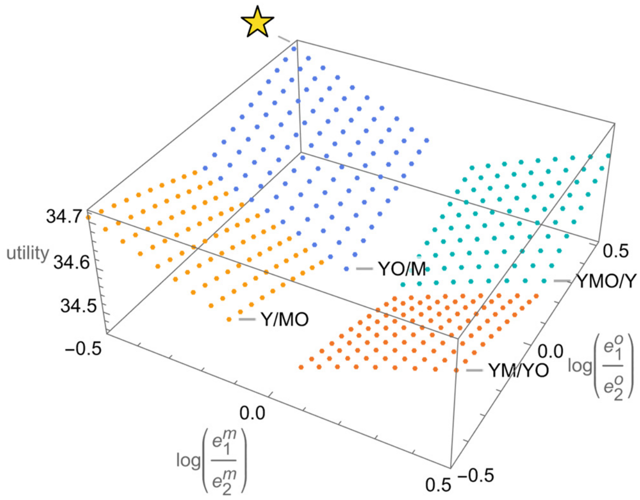

Fig. 12 Discounted lifetime utility ($\ln \frac{e_1{}^y}{e_2{}^y} = 0.1$, constant population, remote work)