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Abstract: The present paper discusses the possibility of free licensing in a model of differentiated duopoly. We have shown that given the market size, the degree of product differentiation and the unit cost of input production, free licensing will occur if the transferred technology is not much superior and the market price of input is sufficiently large. If, however, any of market size, input cost or product substitution goes up, the possibility of free licensing will fall. Our result has an important implication in the context of transboundary pollution. The overall welfare under free licensing will be higher unambiguously.

Keywords: Transferred technology, free licensing, product differentiation, input price, cross-border pollution.

JEL Classification: D43; D45; L13; L24.

1. Introduction

Technology licensing is a common phenomenon in industries. A firm owning a superior production technology licenses its technology to another firm initially holding relatively backward technologies. Technology transfer reduces the licensee's cost of production or improves the quality of the product. If the transferor is a non-producing firm or it does not compete in the transferee's market after technology transfer takes place, then such a transfer is always feasible to the extent that it enhances the transferee's profit. In such a situation the problem of the transferor is to design a contract so as to extract maximum possible surplus. On the other hand, if the licensor and the licensee compete in the same market place, technology licensing reduces the operational profit of the licensor. Then, technology licensing may take place provided that the licensee can fully compensate for the loss of payoff of the licensor. The literature on technology licensing discusses, among other things, the optimal licensing contracts.[1] Typically, licensing takes place against a payment by the licensee in the form of a fixed fee and/or royalty. There is also a literature that shows that sometimes technology licensing results in an upward shift of the market demand. This further counters the negative effect of technology transfer on the transferor's profit. For instance, in Shepard (1987), licensing induces quality competition, and this acts as quality commitment that

[1] On optimal licensing contracts one may look at some selected works like Kabiraj (2004), Sen and Tauman (2007), Erutku and Richelle (2007), Lee and Kabiraj (2011), Sinha (2016), Kabiraj and Kabiraj (2017), Liu and Tauman (2019), and Mukherjee (2020).

increases industry demand. Similarly, in Stamatopoulos and Tauman (2008), licensing of a quality-improving innovation directly affects consumers' preferences and their willingness to pay for the product. They have studied the problem in a logit demand framework and have shown that the optimal licensing contract depends on whether the market is fully covered or not, and whether the consumers' heterogeneity is sufficiently large.

Then a natural question that comes up is: Can free licensing of technology to a rival be profitable? This question is not without any context; it has important implication to some real-life problems. For example, consider the problem of cross-border pollution. In the literature of environmental economics, one important debated issue relates to the cross-border pollution generated from production of final goods. It is often complained by the developed countries that developing countries, which generally produce goods and services using backward technologies, generate pollution to such an extent that not only the country of origin is adversely affected but, in fact, all other countries are similarly affected by the cross-border movement of pollutants. After all, production by backward technologies damage the atmosphere, air space and waterbody by means of emitting carbon, gaseous pollutants and other obnoxious chemicals and particles.[2] The developing countries, in turn, ask the developed countries to transfer their superior technologies freely and the associated inputs at a subsidized price so that the overall pollution generation becomes least, and this in turn benefits the developed countries as well. There are several studies that show that pollution in one country can have serious environmental consequences in others. Reducing global pollution and warming-up requires increased cooperation among the nations of both developed and developing countries. This cooperation should include, to the extent possible, the right of access to environment friendly technologies by all others (e.g., see Jeffery (1992) and West (2020)).

Similarly, one may find implications of free licensing in the context of music industry. It is shown by Peitz and Waelbroeck (2006) that the music industry may benefit by allowing free downloading to music consumers. This is indeed the case when consumers' taste heterogeneity and product diversity are sufficiently large. Here the industry gains from file-sharing networks. Note that free licensing of innovations in the computer software industry is also common. However, free and

[2] See Keresztesi et al. (2020) and the references therein.

open source software appears to be close to sharing of knowledge and cooperation in research. A very comprehensive and insightful study on this issue is found in Lerner and Tirole (2002).

In the present paper we construct a differentiated duopoly model with quantity competition when initially one firm holds a superior production technology which, if transferred, will save resources of the transferee, and hence reduce unit production cost. Further, the licensor sells inputs to the licensee compatible with the production technology. Although by sharing its production knowledge the licensor creates competition from the licensee, but the former's loss of profit due to competition may be outweighed by the revenue it earns from the sale of inputs to the competitor, hence free licensing of a superior technology may be mutually beneficial. Before we go to the model and results, we first briefly discuss the existing literature on free licensing. This will help the readers place our paper properly in the literature.

There are only a few works, in the literature, that discuss the possibility of free licensing, that is, the case when technology is transferred free of cost; still it is a rational decision on the part of the transferor which wants to maximize its overall payoff. The existing literature focuses mostly on network externalities and the shift of market demand for the product. We have already mentioned the work of Peitz and Waelbroeck (2006). Among others, Conner (1995) analyzes the benefits of market expansion from licensing and derives conditions for which it is profitable for an incumbent to license its technology for free to an entrant who uses the licensed technology to introduce its own product and compete in the incumbent's market. The paper suggests that innovator's best strategy may be to encourage clones of its products when a network externality is present. Then Boivin and Langinier (2005) extend the analysis to examine whether free licensing can be profitable even in a homogeneous good duopoly setting. The paper explicitly assumes that the structure of the market influences the market demand. In particular, consumers' willingness to pay for a product is larger when it is duopoly than when it is monopoly. Thus, the paper assumes that licensing results in an upward shift of the market demand function. The increase in demand resulting from licensing can be large enough to induce the incumbent to share its technology.

In contrast, in our paper there is neither network externality nor market demand shift effect. Moreover, pre-licensing situation is also a duopoly. So, licensing of a superior production technology to a rival will intensify competition in the product market and reduce the licensor's payoff from market competition depending on the degree of substitution between the products sold by them. Since we assume that the licensor can supply the relevant input at a cheaper rate than the market price, therefore the rival will buy the input from the licensor. So, when the superior production technology is transferred, revenue from input sale must have to outweigh the loss from product market competition. The interesting feature of our model is that although the licensor sells inputs at a price higher than the input cost of production, but the input demand of the licensee may increase or decrease in the post-licensing situation. The reason is that under the transferred technology input requirement per unit of output is reduced. Therefore, the revenue from input sale may go up or fall, and the loss of profit from the product market competition will depend on the degree of product differentiability. First of all, we have shown that there cannot be any free licensing technology if the products they produce are homogeneous. Then given that the products are not perfect substitutes to each other, free licensing of technology can occur only if the transferred technology is not much superior and the licensor can sell inputs at a sufficiently high price, that is, the input market is critically imperfect. The reason is that a much superior transferred technology will reduce the licensor's operating profit to a significant amount, and unless input price is high enough, additional revenue from input sale will not be sufficient to compensate for the loss of profit. If the input production cost increases or the degree of product substitution rises, it is natural that free licensing is more difficult to occur. But interestingly, we see that an increase in market size also reduces the possibility of free licensing.

There are a number of papers dealing with licensing in a differentiated goods model (e.g., see Wang and Yang (1999), Mukherjee and Balasubramanian (2001), Wang (2002), and Bagchi and Mukherjee (2014)). Our paper, however, differs from those in various respects. First, those papers consider technology transfer under a fee or royalty contract (or under two-part tariff contracts), whereas we consider licensing completely free of cost. Second, in our paper the licensor has two sources of profits, viz., profits from product market operation and also from input sale. Third, in those papers in the post-licensing situation the licensor and the licensee have generally symmetric costs of production. In our paper superior technology reduces input requirements per unit of final good production, but even after licensing the cost asymmetry between the firms prevails, although the gap is reduced. Generally, a larger market size increases the possibility of licensing. In our paper, possibility of free licensing gets reduced.

There are some works (e.g., Mukherjee (2019) and Mukherjee et al. (2009)) which although do not discuss free licensing but can be implicated. These papers show that there are situations when entry of a new firm in the industry can benefit the incumbent(s). The papers show that entry in the product market has its effect in the input market, that results in lowering input price for the incumbents. Contrarily, in our paper licening occurs in between the product market competitors, and it has no effect on the input price. However, licensing increases input sale of the licensor, hence its revenue from input sale goes up.

Finally, we have derived welfare implications of our results. We have shown that under free licensing, not only consumers' surplus and industry profit go up, but also under some conditions the overall pollution level goes down.

The layout of the paper is the following. In Section 2, we present the model and results of the paper, then Section 3 concludes the paper. Some proofs are relegated in the appendix.

2. Model

Consider a differentiated duopoly. Two firms produce differentiated products and compete in quantities. Let q_1 and q_2 be the quantities produced by firm 1 and firm 2, respectively. The market demand as faced by firm i is given by

$$P_i = a - q_i - bq_j, \quad i \neq j; i, j = 1,2$$

where P_i is the price of product i, and $a > 0$ is the common demand parameter, representing market size. Finally, b measures the degree of substitution between the products such that $b = 0$ means two products are independent (so zero substitution) and $b = 1$ means two products are perfectly homogeneous; hence we assume $b \in (0, 1)$.

Initially, the firms have the same production technology, that is, each product requires one unit of one common input. Firm 1 can produce this input at a cost of $c > 0$ per unit, whereas the input is also available in the market at a price r; $c < r < a$. Hence firm 1 can sell the input to firm 2 at a limit price, r. For simplicity, we assume that no other inputs are required for production. Hence initially, firm 1 has a unit cost c per unit of its output and firm 2's unit cost of production is r.

Now let us assume that before the game starts, firm 1 comes up with an innovation that reduces its input requirement to m per unit of output; $0 < m < 1$. Then firm 1 has an option to license its superior technology to firm 2. In this paper we consider free licensing if licensing is to occur, and free licensing will occur if it is profitable. Then under no licensing situation firm 1's unit cost of production will be mc and that of firm 2 will be r. However, if licensing occurs, their unit costs of production will be, respectively, mc and mr. Below we first consider no-licensing equilibrium, and then examine the possibility of free licensing under the initial assumptions:

(A1) $a > r > c > 0$, $0 < b < 1$ and $0 < m < 1$

2.1 No licensing

Under no licensing situation, the profit functions of firm 1 and firm 2 are respectively,

$$\Pi_1 = [(a - mc) - q_1 - bq_2]q_1 + (r - c)q_2 \tag{1a}$$

$$\Pi_2 = [(a - r) - q_2 - bq_1]q_2 \tag{1b}$$

Then their profit maximizing outputs can be solved from the two first order conditions, $\frac{\partial \Pi_i}{\partial q_i} = 0, i = 1, 2$. The unique equilibrium outputs of the firms will be,[3]

$$q_{1n} = \frac{(2-b)a-2mc+br}{4-b^2} \tag{2a}$$

$$q_{2n} = \frac{(2-b)a-2r+bmc}{4-b^2} \tag{2b}$$

where the subscript n denotes no-licensing. Since we assume initial duopoly, we must need to satisfy that $q_{2n} > 0$, i.e.,

(A2) $2r < (2 - b)a + bmc$, i.e., $r < \frac{(2-b)a + bmc}{2} \equiv \bar{r}(m)$.[4]

Therefore, in equilibrium the no-licensing payoffs of the firms are

$$\Pi_{1n} = q_{1n}^2 + (r - c)q_{2n} \tag{3a}$$

[3] Note that the second order and uniqueness conditions are satisfied.

[4] Note that $\bar{r}(m) < a$ true for all $m \in [0, 1]$.

$$\Pi_{2n} = q_{2n}^2 \tag{3b}$$

2.2 Free Licensing

We first derive the quantities and payoffs of the firms if free licensing occurs. Then we find conditions under which free licensing will occur. Free licensing will be profitable if under free licensing the licensor's profit is strictly higher than the no-licensing payoff and the licensee is not worse off.

Under free licensing firm 1 and firm 2 will maximize respectively the following profit expressions:

$$\Pi_1 = [(a - mc) - q_1 - bq_2]q_1 + m(r - c)q_2 \tag{4a}$$

$$\Pi_2 = [(a - mr) - q_2 - bq_1]q_2 \tag{4b}$$

Solving the above maximization problems, we derive the equilibrium quantities to be produced under free licensing by firm 1 and firm 2, given by

$$q_{1f} = \frac{(2-b)a - 2mc + bmr}{4-b^2} \tag{5a}$$

$$q_{2f} = \frac{(2-b)a - 2mr + bmc}{4-b^2} \tag{5b}$$

and the payoffs under equilibrium are

$$\Pi_{1f} = q_{1f}^2 + m(r - c)q_{2f} \tag{6a}$$

$$\Pi_{2f} = q_{2f}^2 \tag{6b}$$

Then comparing (2) and (5), we can immediately see that

$$q_{1f} < q_{1n}, \ \ q_{2f} > q_{2n} \tag{7}$$

that is, under free licensing firm 1's output falls and firm 2's increases. The simple reason is that under licensing production knowledge is shared; as a result, firm 2 uses the same superior technology, hence its cost of production falls compared to no-licensing situation. Then given (7), comparing (3) and (6) we have $\Pi_{2f} > \Pi_{2n}$, that is, firm 2's profit under licensing goes up. However, firm 1's overall profit may or may not increase; therefore, free licensing may or may not be profitable. The reason is that firm 1's operational profit falls certainly, i.e., $q_{1f}^2 < q_{1n}^2$, but its volume of input sales to firm 2 is ambiguous, because $q_{2f} > q_{2n}$, but $m < 1$ so that $mq_{2f} \gtrless q_{2n}$.[5] Even when the volume of input sales goes up, its increased revenue from input sale may not be large enough to overcompensate the loss of operational profit.

Therefore, given assumptions (A1) and (A2), free licensing will occur if and only if $(\Pi_{1f} - \Pi_{1n}) > 0$. In the next section we shall derive the conditions under which free licensing will be profitable. In our analysis we focus on the role of two parameters, m and r, that is, the parameters for production technology and the input market competition, respectively. Hence, we take the parameters a, b and c as fixed, and then examine for which combination of m and r free licensing will be profitable. Finally, we study the effect of a change of a, b or c on the critical values of m and r.

2.3 Conditions for Free Licensing

We have already shown that given assumptions (A1) and (A2), free licensing will be profitable if and only if $\left(\Pi_{1f} - \Pi_{1n}\right) > 0$. Now,

$$\Pi_{1f} - \Pi_{1n} = (r-c)\left(mq_{2f} - q_{2n}\right) - (q_{1n}^2 - q_{1f}^2)$$

Hence,

$$\Pi_{1f} - \Pi_{1n} = \frac{(1-m)[\left(4-b^2\right)(r-c)\{2r(1+m)-(2-b)a-bmc\} - br\{2(2-b)a-4mc+br(1+m)\}]}{(4-b^2)^2}$$

$$= \frac{(1-m)}{(4-b^2)^2} H(a,b,c,m,r) \tag{8}$$

where

$$H(a,b,c,m,r) = [(4-b^2)(r-c)\{2r(1+m)-(2-b)a-bmc\}$$
$$- br\{2(2-b)a - 4mc + br(1+m)\}]$$

[5] $mq_{2f} - q_{2n} = \frac{(1-m)[2r\,(1+m)-(2-b)a-bmc]}{4-b^2}$

Then,

$$\Pi_{1f} - \Pi_{1n} > 0 \iff H(a,b,c,m,r) > 0 \text{ for all } m \in [0,1) \tag{9}$$

Immediately, as a special case, we have the following two results.

Proposition 1: Free licensing is never profitable if the products are perfect substitute to each other (i.e., $b = 1$).

Proof: See *Appendix 1*.

In *Appendix 1* we have shown that $H(a,b,c,m,r) < 0$ when $b = 1$, hence free licensing is not profitable if the products are homogeneous. The reason is that if the products are homogeneous, the fierce competition will reduce firm 1's operational profit substantially, making the overall profit to fall.

Proposition 2: If the products are independent (i.e., $b = 0$), free licensing is profitable if and only if $a < r(1+m)$.

Proof: See *Appendix 2*.

The proposition states that when the degree of substitution between the products is zero, free licensing will not be profitable unless the market size is below a critical level, or the transferred technology must not be very much efficient. The intuition is the following. Since products are independent, so there is no competition, hence firm 1's output and operational profit will remain unchanged. But total input sale to firm 2 may or may not go up. If technology is very superior (that is, m is small enough), effectively input sale to firm 2 will fall, hence revenue from input sale will also fall.

Before we derive the main results of the paper, the following results are helpful regarding the behavior of $H(.)$ function.

Proposition 3: Given (A1), for any (a,b,c) and $r = \bar{r}(m)$ there exists a unique $m = m^*$ for which (i) $H(a,b,c,m^*,\bar{r}(m^*)) = 0$ holds, and (ii) for all $m \in [m^*, 1]$, we have $\frac{\partial H(a,b,c,m,\bar{r}(m))}{\partial m} > 0$.

Proof. Define

$$I(a,b,c,m) = H(a,b,c,m,\bar{r}(m))$$

Then it can be shown that

$$I(a,b,c,m) = \frac{(2-b)a + bmc}{4} J(a,b,c,m),$$

where

$$J(a,b,c,m) = Z_1(a,b,c)\, m^2 + Z_2(a,b,c)\, m + Z_3(a,b,c)$$

and

$$Z_1(a,b,c) = 8bc - 3b^3c$$

$$Z_2(a,b,c) = 16a - 16c - 8ab + 8bc - 6ab^2 + 4b^2c + 3ab^3 - b^3c$$

$$Z_3(a,b,c) = ab^3 + 2ab^2 - 8ab.$$

Note that $Z_1 > 0$ for all $b \in (0,1)$. Further,

$$I(a,b,c,m) = 0 \text{ iff } J(a,b,c,m) = 0$$

Then we have,

(i) $J(a,b,c,0) = -ab(2-b)(4+b) < 0$

(ii) $J(a,b,c,1) = 4(a-c)(4-b^2)(1-b) > 0$, and

(iii) J is strictly convex and quadratic function of m with at most two real roots.

Hence, by intermediate value theorem (IVT), there exists $m = m^*$ such that $J(a,b,c,m^*) = 0$ and $J(a,b,c,m) > 0$ for all $m \in (m^*, 1]$. Further, $\frac{\partial J}{\partial m} > 0$ in some neighborhood of m^*, hence the proposition.[6] □

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[6] Further note the following. Define $Y(b) := \frac{b(4+b)}{(8-3b^2)}$. Then (i) $J(a,b,c,Y(b)) = \frac{-8(1-b)b(2+b)(4+b)}{(8-3b^2)}c < 0$ and (ii) $\frac{\partial J}{\partial m}\big(a,b,c,Y(b)\big) = [16a - 8ab - 6ab^2 + 3ab^3 - 16c + 8bc + 12b^2c + b^3c] > 0$. Hence J is increasing in m at least over the interval $[Y(b),1]$. This also means that $Y(b) < m^* < 1$. This result will be useful later.

Therefore, this proposition proves that in the interval of $[0, 1]$, $H(a, b, c, m, \bar{r}(m)) \gtreqless 0$ according as $m \gtreqless m^*$. Next consider the following proposition.

Proposition 4: Given any (a, b, c), for all $m \in (m^*, 1)$ we have, a unique $\underline{r}\ (m) \in (\text{c}, \bar{r}(m))$ that solves $H(a, b, c, m, r) = 0$.

Proof. Let $m \in (m^*, 1)$. First, note that

$$H(a, b, c, m, c) = -bc[2(2 - b)a - 4mc + bc(1 + m)]$$

$$= -bc[2(2 - b)(a - mc) + bc(1 - m)] < 0$$

And in Proposition 3 it is shown that for all $(m^*, 1)$, $H(a, b, c, m, \bar{r}(m)) > 0$. So, by IVT, there exists at least one $\underline{r}\ (m) \in (\text{c}, \bar{r}(m))$ that *solves* $H(a, b, c, m, r) = 0$.

Further note that $H(a, b, c, m, 0) > 0$. Hence, again by IVT, there exists another $\underline{r}\ (m) \in (0, \text{c})$ for which $H(a, b, c, m, r) = 0$.

Since $H(a, b, c, m, r)$ is a quadratic polynomial of r, so, there are exactly these two $\underline{r}$ for which $H(a, b, c, m, r) = 0$ can hold, but one $\underline{r} < c$ and the other $\underline{r} > c$. Hence, given assumption (A1), from now on, by $\underline{r}$ we will always mean $\underline{r} > c$. □

We are now in a position to state the basic result of the paper stated in the following proposition.

Proposition 5: Given assumption (A1) and (A2), for all $m \in (m^*, 1)$, there exist $\underline{r}(m) \in (\text{c}, \bar{r}(m))$ such that for all $r \in (\underline{r}(m), \bar{r}(m))$, we must have $\Pi_{1f} - \Pi_{1n} > 0$.

This result simply follows from the results of underlying Propositions 3 and 4. Proposition 5 states that, given assumptions (A1) and (A2), in our structure free licensing will occur when r belongs to an interval and m is above a critical level. The reason is that under licensing, firm 1's operational profit always falls. Now, if m is very small (that is, technology is much superior), then under licensing loss of firm 1's operational profit will be to the extent that it cannot be outweighed by the gain from input sale, if any. On the other hand, if r is not sufficiently high, increase in revenue from input sale will not be large enough for free licensing to be profitable.

To facilitate the diagrammatic presentation of the result and to explore the result further, consider the following two loci of (m, r),

$$\phi(m,r) \equiv (\,\overline{r}(m) - r) \; = \; 0 \tag{10}$$

$$\psi(m,r) \equiv (\Pi_{1f} \; - \Pi_{1n}) \; = \; 0 \tag{11}$$

Note that (10) and (11) correspond to the two inequalities (A2) and (9), respectively. Then it can be shown (see *Appendix 3*) that:

$$\left[\frac{\partial \overline{r}(m)}{\partial m}\right]_{\phi(m,r)} > \; 0, \text{and} \; \left[\frac{\partial \underline{r}(m)}{\partial m}\right]_{\psi(m,r)} \; < \; 0 \tag{12}$$

Therefore, (10) generates an upward sloping locus such that for all (m, r) below this locus, assumption (A2) is satisfied. Similarly, (11) gives a falling locus of (m, r) such that for all (m, r), condition (9) is satisfied. Hence, the intersection point of these two loci corresponds to the critical m^*. Then for all values of (m, r) in the area bounded by these two loci for $m \; > m^*$, free licensing is profitable. Given the nature of these two loci, it is also clear that as m goes up, the interval [$\underline{r}$, $\overline{r}$] becomes more widened for $m \in (m^* \, , 1)$. We write the result more formally in the following proposition.

Proposition 6: $\frac{\partial(\overline{r} - \underline{r})}{\partial m} > 0$ for all $m \in (m^*, 1)$.

This states that as technological superiority falls, the gap between $\underline{r}$ and $\overline{r}$ increases; this means, the possibility of free licensing also increases. The results underlying Propositions 5 and 6 are depicted in *Figure 1*.

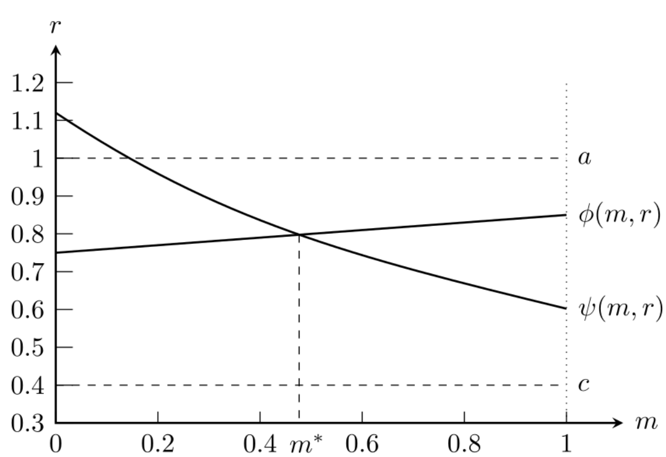

Figure 1: Possibility of free licensing, given any (a, b, c)

We may, as well, check the results for the following example.

Example 1: Let $a = 1$, $b = 0.5$ and $c = 0.4$. Then $m^* \approx 0.4745$. Hence, we have the following results given in Table 1.

Table 1: Free licensing interval of r for any given m

m	$\underline{r}$	$\overline{r}$
0.5	0.7858	0.80
0.6	0.7438	0.81
0.7	0.7066	0.82
0.8	0.6733	0.83
0.9	0.6435	0.84

2.4 Comparative Static Analysis

We now study the effect of the change of a, b or c on m^* which is solved from $I(a, b, c, m) = 0$, i.e. $J(a, b, c, m) = 0$. We have the following results.

Proposition 7: We have

$$\frac{\partial m^*}{\partial a} < 0, \frac{\partial m^*}{\partial b} > 0 \text{ and } \frac{\partial m^*}{\partial c} > 0$$

Proof is given in *Appendix 4*. We have the following implications of the above comparative static results. Given the parameter vector (a, b, c, m, r), let us assume that initially free licensing is profitable. This means, given (a, b, c), m and r are such that they belong to the relevant interval. Now suppose that a goes up, keeping all the remaining four parameters unchanged. Then the locus $\phi(m, r) = 0$ shifts up, that is, for every m, $\overline{r}$ will be higher. Again, when a goes up, the locus $\psi(m, r) = 0$ shifts to the right, implying that for every m, $\underline{r}$ will be higher.[7] But given $\frac{\partial m^*}{\partial a} < 0$, the cut-off m^* will fall, hence the interval of m for feasible free licensing goes up. But since $\underline{r}$ goes up as a consequence of an increase in a, the initial (m, r) may or may not be within the new feasible free licensing zone. Therefore, the implication of an increase in market size is that if increase in a is small, free licensing will continue to be profitable, but if it is large, in the new

[7] This follows from the fact that $\frac{\partial H}{\partial a} = -[(4 - b^2)(r - c) + 2br](2 - b) < 0$ and $\frac{\partial H}{\partial r} > 0$ at some neighborhood of $\underline{r}$.

parametric situation, free licensing may not be profitable. The result in *Figure 2* shows that given an initial parameter vector $(a, b, c, m, r) = (1.0, 0.5, 0.4, 0.8, 0.75)$, if a goes up to 1.1, free licensing continues to be profitable, but for $a = 1.25$, free licensing conditions fail to be satisfied.

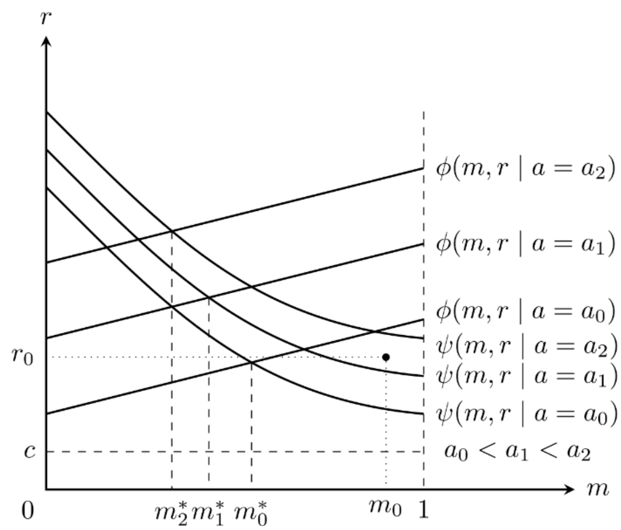

Figure 2: Effect of the change of market size on free licensing

Finally, since $\frac{\partial m^*}{\partial b} > 0$ and $\frac{\partial m^*}{\partial c} > 0$, therefore when either *b* or *c* alone increases, m^*goes up; hence the possibility of free licensing goes down when either the degree of product differentiation falls or the cost of (input) production increases. The reason is that operational profit in either case will fall much under licensing, but the corresponding revenue from input sale is unlikely to increase sufficiently.

2.5 Implication of Free Licensing to Environmental Problem

We are now in a position to derive the implication of free licensing in the context of global pollution. Let s_0 be the pollution per unit of output generated for using old (backward) technology, and s_1 be the same for new or superior technology. It is then reasonable to assume $s_0 > s_1 > 0$. Then total pollution generated from production in the pre-licensing and post- free licensing situations will be respectively,

$$L_0 = s_1 q_{1n} + s_0 q_{2n}$$

$$L_1 = s_1 q_{1f} + s_1 q_{2f}$$

Then free licensing will reduce global pollution level if and only if $L_1 < L_0$, that is,

$$[s_1 q_{2f} - s_0 q_{2n}] + s_1[q_{1f} - q_{1n}] < 0 \tag{13}$$

Since $q_{2f} > q_{2n}$, $q_{1f} < q_{1n}$ and $s_1 < s_0$, therefore the sufficient condition that (13) will be satisfied is:

$$\frac{s_0}{s_1} > \frac{q_{2f}}{q_{2n}} \tag{14}$$

Therefore, (14) requires that s_0 will be sufficiently larger than s_1. Note that $q_{2f} > q_{2n}$, but under free licensing m has to be greater than m^*; hence q_{2f} cannot be much larger than q_{2n}.

Finally, we conclude this section by noting the welfare implication of our result. Under free licensing, each firm's profit is going up, so industry profit increases. Industry output also increases, because $q_{1f} + q_{2f} > q_{1n} + q_{2n}$. This means consumers' surplus also increases unambiguously. Moreover, when (14) holds, overall environment becomes less polluted. This, further increases welfare. Hence under this situation, the overall welfare under free licensing must increase unambiguously.

3. Conclusion

In the present paper we have studied the possibility of free licensing, and that too without any network externality or demand shift effect. We have constructed a differentiated duopoly model where the transferred technology reduces the input requirements of the licensee. The licensor also sells inputs to the licensee. We have derived conditions under which the revenue from input sales outweighs the licensor's loss of profit from competition. We show that given the market size, the degree of product differentiability and the cost of input production, free licensing is profitable provided that the transferred technology is not too superior and at the same time the input price at which the transferor sells inputs to the transferee is not very low. We have further shown that as any of market size, degree of product substitutability and input production cost increases, the possibility of free licensing decreases.

Finally, we have derived implications of our results to the environmental problem. To the extent superior technology generates relatively less pollution, free licensing is likely to reduce global pollution. In our paper the overall welfare unambiguously goes up.

Appendix

Appendix 1: *Proof of Proposition 1*

To show that when $b = 1$, there does not exist any tuple (a, c, m, r) for which $\Pi_{1f} > \Pi_{1n}$ holds, first note that the assumption (A2) is reduced to $a + mc > 2r$, and

$$H(a, 1, c, m, r) = 3(r - c)(2r(1 + m) - a - mc) - r(2a - 4mc + r(1 + m)).$$

Now,

$$\begin{aligned} & H(a, 1, c, m, r) \\ & = 3(r - c)(2r(1 + m) - a - mc) - r(2a - 4mc + r(1 + m)) \\ & < 3(r - c)(2rm) - r(2a - 4mc + r(1 + m)) \ [\text{since } a + mc > 2r] \\ & = r[6(r - c)m - (2a - 4mc + r(1 + m))] \\ & = r[6rm - 6mc - 2a + 4mc - r - rm] \\ & = r[5rm - 2mc - 2a - r] \\ & < r[5rm - 4r - r] \qquad [\text{as } a + mc > 2r] \\ & = -5r^2(1 - m) < 0 . \qquad \square \end{aligned}$$

Appendix 2: *Proof of Proposition 2*

When $b = 0$, the assumption (A2) becomes $a > r$. Then

$$H(a, 0, c, m, r) = 4(r - c)(2r(1 + m) - 2a).$$

So,

$$H(a, 0, c, m, r) > 0 \Leftrightarrow r(1 + m) > a. \qquad \square$$

Appendix 3: *Proof of* $\left[\frac{\partial \overline{r}(m)}{\partial m}\right]_{\phi(m,r)} > 0$, and $\left[\frac{\partial \underline{r}(m)}{\partial m}\right]_{\psi(m,r)} < 0$.

Note that Proof of $\left[\frac{\partial \overline{r}(m)}{\partial m}\right]_{\phi(m,r)} > 0$ follows straight from (10).

To prove the second part, first note that for any m (and given a, b and c), $\underline{r}$ is the solution of the equation $\Pi_{1f} - \Pi_{1n} = 0$ or, $H(a,b,c,m,r) = 0$. Then,

$$\left[\frac{\partial \underline{r}(m)}{\partial m}\right]_{\psi(m,r)} = -\left[\frac{\partial H/\partial m}{\partial H/\partial r}\right]_{r=\underline{r}}$$

Now, to see the sign of $(\partial H/\partial m)$, note that

$$\partial H/\partial m = [2(4-b^2)(r-c) - b^2 r]r + [4r - (4-b^2)(r-c)]bc$$

Now, when $r = c$, we have $(\partial H/\partial m) = 4bc^2 - b^2c^2 > 0$. Also, $\frac{\partial^2 H}{\partial m \partial r} > 0$.

So, for all $r \geq c$, we have $(\partial H/\partial m) > 0$.

To see the sign of $(\partial H/\partial r) > 0$, first note that we can write

$$H(a,b,c,m,r) = S_1(a,b,c,m)\, r^2 + S_2(a,b,c,m)\, r + S_3(a,b,c,m)$$

where

$$S_1(a,b,c,m) = (1+m)(8-3b^2)$$

$$S_2(a,b,c,m) = 4ab^2 - (8-2b^2+b^3)cm - (8-2b^2)c - 8a - ab^3$$

$$S_3(a,b,c,m) = (4b-b^3)c^2 m + (8a-4ab-2ab^2+ab^3)c$$

Then for all $b \in [0,1]$ and $m \in [0,1]$, we have $S_1 > 0$, $S_2 < 0$ and $S_3 > 0$.

Since $S_1 > 0$, therefore H function is strictly convex in r, given (a,b,c,m).

Now consider any $m \in (m^*, 1)$. In the proof of Proposition 4 we have shown that there exists $\underline{r} \in (\mathrm{c}, \overline{r}(m))$ such that:

(i) $H(a,b,c,m,\underline{r}) = 0$,

(ii) $H(a,b,c,m,c) < 0$, and

(iii) $H(a,b,c,m,\overline{r}) > 0$.

Therefore, given that H is strictly convex and continuous in r, we must have $\frac{\partial H}{\partial r} > 0$ at some neighborhood of $\underline{r}$. This proves that $\left[\frac{\partial \underline{r}(m)}{\partial m}\right]_{\psi(m,r)} < 0$. □

Appendix 4: Proof of $\frac{\partial m^*}{\partial a} < 0$, $\frac{\partial m^*}{\partial b} > 0$ and $\frac{\partial m^*}{\partial c} > 0$

First note that m^* is solved from

$$I(a,b,c,m) = 0, \text{ i.e., from } J(a,b,c,m) = 0$$

Now given the expression of $J(a,b,c,m)$, we have

$$J_a = \frac{\partial J}{\partial a} = -(2-b)(3b^2m + b^2 + 4b - 8m)$$

Then $J_a > 0$ if and only if

$$(3b^2m + b^2 + 4b - 8m) < 0 \iff Y(b) \equiv \frac{b(4+b)}{(8-3b^2)} < m.$$

In Footnote 4 we have already shown that for all $m \in (Y(b),1)$, we have $\frac{\partial J}{\partial m} > 0$ and $Y(b) < m^* < 1$, hence $J_a > 0$. Therefore,

$$\frac{\partial m^*}{\partial a} = -\left[\frac{\partial J/\partial a}{\partial J/\partial m}\right]_{m=m^*} < 0$$

Again,

$$J_b = \frac{\partial J}{\partial b} = J_b = [-9b^2cm^2 + 8cm^2 - 3b^2cm + 8bcm + 8cm + 9ab^2m$$

$$-12abm - 8a + 3ab^2 + 4ab - 8a]$$

Note that

(i) $\frac{\partial J_b}{\partial a} = [9b^2m - 12bm + 3b^2 + 4b - 8 - 8m] < 0$ for all $b \in [0,1]$ and $m \in [0,1]$, and

(ii) $J_b(c,b,c,m) = c\,(1-m)\,(9b^2m - 8m + 3b^2 + 4b - 8) \leq 0$ for all $b \in [0,1]$ and $m \in [0,1]$.

This means, for all $a > c$, we have $J_b < 0$.

Therefore, the effect of a change of b is:

$$\frac{\partial m_*}{\partial b} = -\left[\frac{\partial J/\partial b}{\partial J/\partial m}\right]_{m=m^*} > 0.$$

Finally, note that

$$J_c = \frac{\partial J}{\partial c} = m[4b^2 + 8b - b^3 - 16 + 8bm - 3b^3m]$$
$$= -[(1-b)(8-b^2) + (1-bm)(8-3b^2)] < 0$$

Therefore, the effect of a change of c is:

$$\frac{\partial m^*}{\partial c} = -\left[\frac{\partial J/\partial c}{\partial J/\partial m}\right]_{m=m^*} > 0 \qquad \square$$

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