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Abstract

We present a model of Chinese growth in a two-country economy with the manufacturing and natural resource sectors to analyze the impacts of the reform and opening-up policy, which promotes free trade and technological progress, on the net capital flows, net export, and social welfare. We show that manufacturing capital in China initially decreased before increasing. This corresponds with the fact that China was a net importer of manufactured goods initially, and became a net exporter recently. These results are consistent with the data obtained after the reform and opening-up policy in 1978.

Keywords: trade liberalization; technological progress; the reform and opening-up policy; deindustrialization; home market effect.

JEL Classification: F12; O11; R12

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1 Introduction

China was a world economic and technological leader in the "premodern" era (circa 1200), whereas China missed the Industrial Revolution and had close-to-zero growth in per capita GDP from 1800 and 1950 (Zhu, 2012). Resources were misallocated, incentives were distorted, and the labor-intensive sectors in which China held a comparative advantage were repressed until 1978 (Lin, 2013). Nevertheless, the Chinese economy has dramatically transformed since the government began the reform and opening-up policy (ROP) in 1978 and established four economic zones, including Shenzhen in 1980. This was the beginning of rapid economic growth.

The ROP promoted privatization and marketization, which has led to the miracle of China: China has transformed from a backward agrarian country to the world's second-largest economy. The structural transformation from primary to secondary industries was spectacular. It was accompanied by rapid urbanization, which enables firms to enjoy the benefit of urban agglomeration economies (Fujita and Thisse, 2013). Consequently, China has overtaken Japan as the world's second-largest economy and replaced Germany as the world's largest exporter of merchandise in 2009 (Lin, 2013).

Chen et al. (2018) maintain that the Chinese policy of privatization in 1997 boosted firms' productivity and identified that the number of state-owned enterprises decreased by more than half in ten years. Song et al. (2011) construct a growth model to explain China's growth with distortion and reallocation between private firms and State-owned firms in the economic transition. Huang et al. (2017) documented the structural adjustments in Chinese manufacturing firms from 1999 to 2007 and found that production became more capital-intensive. China is now experiencing a transformation from investment-led growth to innovation-led growth (Zilibotti, 2017).

According to Wei et al. (2017), the economic growth between 1980 and 2015 was based on a sequence of market-oriented institutional reforms, including openness to international trade and direct investment, combined with low wages and a highly favorable demographic structure. Real wages increased 14-fold during this period.

The main drivers of economic growth due to the ROP are two-fold. The first is trade liberalization. China became a member of the Asia Pacific Economic Cooperation in 1991, established the China-ASEAN Free Trade Area in 2005, and started the Belt and Road Initiative in 2013, a massive China-led infrastructure project that aims to stretch around the globe. Consequently, the weighted mean tariff rate decreased from 32.2% in

1992 to 2.5% in 2020.[1] The second is technological progress through the introduction and absorption of advanced science and technology by attracting foreign direct investment (Huang, 1986; Zhang, 2019). The former gains from trade liberalization are well-known in trade theory (e.g., Helpman and Krugman, 1985). The latter gains from technological progress also contributed to economic growth (Tabuchi, Thisse, and Zhu, 2018).

These two policies are interdependent. "By expanding its imports and exports, attracting foreign direct investment, investing overseas, becoming involved in global governance, and, more recently, implementing the Belt and Road Initiative, China participates extensively in economic globalization. This process has helped China strengthen enterprise competitiveness, adopt advanced foreign technologies and management practices, capitalize on the demographic dividend's contribution to economic growth, and obtain a comparative advantage in industry development, as well as achieving a series of other reform and development goals" (Cai, 2019).[2]

Therefore, we considered the impacts of trade liberalization and technological progress from the ROP on the spatial distribution of economic activities between China and the rest of the world. For this purpose, we built a model of Chinese growth in a two-country economy with two sectors: the manufacturing sector producing a differentiated good with increasing returns to scale and the natural resource sector producing a homogeneous good with constant returns to scale. We examined how manufacturing capital relocates between countries in accordance with trade liberalization and technological progress and investigated changes in the net exports of the two sectors and in social welfare.

Our study is related to that of Yang and Zeng (2021) in that both studies consider the impacts of trade liberalization on international capital mobility. However, our study differs from theirs in that we introduced the resource sector, in addition to the manufacturing sector, and therefore focus on net exports in the two sectors. This enabled us to describe the structural transformation from the natural resource sector of primary industries to the manufacturing sector of secondary industries after the ROP in China.

We present a model of economic growth, show the existence and uniqueness of equilibrium, and explore the relationship between net capital inflows and net exports in the manufacturing and resource sectors in Section 2. The ROP in China involved trade liberalization and technological progress. The former policy impact is analyzed in Section 3,

[1] https://data.worldbank.org/

[2] Ferreira and Rossi (2003) confirmed the association between productivity growth and trade liberalization in 1988-90 in Brazil and showed that the impact was indeed substantial: The observed tariff reduction in the period brought a 6% estimated increase in total factor productivity growth rate and a similar impact on labor productivity.

while the latter is in Section 4. Since the two impacts are interrelated, we combine and study them together to evaluate the impacts of the ROP in Section 5. Section 6 concludes the study with future research directions.

2 The model

2.1 Consumers

Assume that the economy consists of two countries: The North (n) is a developed country and the South (s) is initially a lagged one such as China. The total number of the population is L, the total amount of natural resources is Z, and the total amount of capital is K, which are fixed in the economy. The share θ_c of both population and natural resources in country $c = n, s$ is fixed because they are immobile, whereas the share $\lambda_c \in [0, 1]$ of capital in country c is variable because capital is perfectly mobile between countries. Obviously, $\theta_s + \theta_n = 1$ and $\lambda_s + \lambda_n = 1$ hold. As China has large population, we assume that the Southern share is $\theta_s \in [1/2, 1)$.

Each consumer inelastically supplies one unit of labor. The natural resources are equally owned by consumers in each country. By contrast, the capital is equally owned by the total consumers, implying that the share θ_s of its employed capital belongs to the South and that θ_n of the employed capital comes from the North regardless of λ_c. This aligns with the assumption of Baldwin et al. (2003, p. 74).

The utility function of a representative consumer in country $c = n, s$ is given by:

$$U_c = U_0 M_c^{\mu} R_c^{1-\mu},$$

where M_c is the consumption of the composite differentiated good in the manufacturing sector ($\mathbb{M}$-sector) in country c, R_c is the consumption of the homogeneous good in the natural resource sector ($\mathbb{R}$-sector) in country c, $0 < \mu < 1$, and $U_0 \equiv \mu^{-\mu} (1-\mu)^{-(1-\mu)}$. The composite manufactured good in country c is given by:

$$M_c \equiv \left[\int_0^{N_c} q_{cc}(i)^{\frac{\sigma-1}{\sigma}} \mathrm{d}i + \int_0^{N_d} q_{dc}(i)^{\frac{\sigma-1}{\sigma}} \mathrm{d}i \right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution among different varieties, N_c is the mass of varieties produced in country c, and $q_{dc}(i)$ is the consumption of variety i produced in country d and sold in country c, where $c, d \in \{n, s\}$.

The budget constraint of a representative consumer in country c is

$$P_c M_c + p_{Rc} R_c = y_c,$$

where y_c is the income of a representative consumer in country c, p_{Rc} is the price of the homogeneous good in the $\mathbb{R}$-sector in country c, and

$$P_c \equiv \left\{ \int_0^{N_c} [p_{cc}(i)]^{1-\sigma} \mathrm{d}i + \int_0^{N_d} [p_{dc}(i)]^{1-\sigma} \mathrm{d}i \right\}^{\frac{1}{1-\sigma}}$$

is the price index of composite $\mathbb{M}$-good in country c, where $p_{dc}(i)$ is the consumer price of variety i produced in country d and sold in country c.

Accordingly, utility maximization yields the following demand functions of a representative consumer in country c:

$$q_{dc}(i) = \frac{p_{dc}(i)^{-\sigma}}{P_c^{1-\sigma}} \mu y_c, \qquad R_c = \frac{(1-\mu)\, y_c}{p_{Rc}}. \tag{1}$$

2.2 Natural resource firms

In the $\mathbb{R}$-sector, natural resource is the only production factor. Technology in this sector is constant returns to scale and is the same for all firms in a perfectly competitive market. Specifically, one unit of a natural resource is required to produce one unit of $\mathbb{R}$-good in each country. Perfect competition implies that the price of natural resource equals the price of $\mathbb{R}$-good in each country. Furthermore, with free trade of $\mathbb{R}$-good, the choice of this good as the numéraire implies that $p_{Rn} = p_{Rs} = 1$. Since the total supply of $\mathbb{R}$-good is Z and its total demand is $(1-\mu)(Y_n + Y_s)$, the market clearing condition in the $\mathbb{R}$-sector is

$$Z = (1-\mu)(Y_n + Y_s), \tag{2}$$

where $Y_c = y_c \theta_c L$ is the total income in country c.

2.3 Manufacturing firms

Firms are heterogeneous à la Melitz (2003) and there is the one-period timing proposed by Melitz and Ottaviano (2008). There are $\mathcal{N}_c$ potential firms in country c, which operate under increasing returns to scale in a monopolistically competitive market. Thus, each firm produces a single variety of a horizontally differentiated good ($\mathbb{M}$-good) and each variety is produced by a single firm, so that $\mathcal{N}_c$ is also the number of firms set up in country c.

To produce, a firm needs a fixed requirement f units of capital and a marginal requirement m units of labor. We assume no extra fixed requirement for export, so that each active firm exports its product.[3] Prior to entry, firms face uncertainty about their marginal requirement m. Entry requires a sunk cost of f^e units of capital. Once this cost is paid, firms observe their marginal requirement m drawn randomly from a common distribution $G_c(m)$; the density is denoted by $g_c(m)$ in country c. The support of the density in country c is $(0, m_{c\max})$, where $m_{c\max} \equiv m_{\max}/\gamma_c$, $\gamma_n = 1$ and $\gamma_s \equiv \gamma \in (0, 1]$. Accordingly, the South is said to exhibit a disadvantage in production in the $\mathbb{M}$-sector over the North on average whenever $\gamma < 1$. However, the South has an advantage in the large market as $\theta_s \in [1/2, 1)$. The advantage of population size stems from *the home market effect:* a country exports the good for which it has a relatively large local demand (Behrens, et al. 2007). We will see how the advantage and disadvantage change in accordance with the degrees of trade liberalization and technological progress.

The mobility of a good in the $\mathbb{M}$-sector is described by Samuelson's iceberg trade costs: $\tau \geq 1$ units of a variety have to be shipped for one unit of that variety to be available in the other country, while $\tau = 1$ in the same country.

The profits earned by an m-firm located in country c consist of the operating profits from the domestic market c and the foreign market d, respectively, given by

$$\pi_c(m) = \pi_{cc}(m) + \pi_{cd}(m) - fr_c,$$

where

$$\begin{aligned} \pi_{cc}(m) &= (p_{cc} - mw_c)\, q_{cc}(m)\theta_c L \\ \pi_{cd}(m) &= (p_{cd} - \tau_{cd} mw_c)\, q_{cd}(m)\theta_d L, \end{aligned}$$

r_c is the capital returns in country c, w_c is the wage rate in country c and $\tau_{cd} = 1$ if $c = d$, and $\tau_{cd} = \tau$ otherwise. It should be noted that each firm produces for both domestic and foreign markets in the absence of the fixed capital requirement for export. This is because each firm has an incentive to produce for another market without additional fixed costs.

[3]See Appendix A when there is a fixed requirement for export.

The profit-maximizing prices of an m-firm located in country c are

$$p_{cd}^{*}(m) = \frac{\sigma}{\sigma - 1} m \tau_{cd} w_c. \tag{3}$$

The cutoff $\bar{m}_c$-firm in country c earns zero profits gross of entry costs:

$$\pi_c(\bar{m}_c) - f r_c = 0. \tag{4}$$

Plugging (3) into (4) yields the total production of a cut-off firm in country c

$$q_{cc}^{*}(\bar{m}_c)\theta_c L + \tau q_{cd}^{*}(\bar{m}_c)\theta_d L = \frac{(\sigma - 1) f r_c}{\bar{m}_c w_c}. \tag{5}$$

Hence, the equilibrium consumption of an active m-firm located in country c is given by

$$q_{cc}^{*}(m) = \left(\frac{\bar{m}_c}{m}\right)^{\sigma} q_{cc}^{*}(\bar{m}_c) \quad \text{and} \quad q_{cd}^{*}(m) = \left(\frac{\bar{m}_c}{m}\right)^{\sigma} q_{cd}^{*}(\bar{m}_c) \tag{6}$$

Firms enter the market in country c until expected profits net of entry costs $f^e r_c$ are zero:

$$\int_0^{\bar{m}_c} \left[\pi_{cc}(m) + \pi_{cd}(m) - f r_c\right] g_c(m) \mathrm{d}m - f^e r_c = 0. \tag{7}$$

Substituting (3) into the operating profits and using (5) and (6), the zero expected profit condition (7) is rewritten as

$$f \int_0^{\bar{m}_c} \left[\left(\frac{\bar{m}_c}{m}\right)^{\sigma - 1} - 1\right] g_c(m) \mathrm{d}m - f^e = 0. \tag{8}$$

Since the LHS of (8) increases with $\bar{m}_c$, it has at most one solution $\bar{m}_c \in (0, m_{c\max})$. We assume that $g_c(m)$ is well-behaved such that such a solution exists. Since $g_c(m)$ depends on γ_c, two countries have different cutoff $\bar{m}_c$.

Although $\mathcal{N}_c$ firms enter and pay the sunk cost, only $N_c = \mathcal{N}_c G_c(\bar{m}_c)$ firms are active in country c. Since capital is used for the fixed costs of entry and production, the market clearing condition for capital in country c is expressed as

$$\lambda_c K = \mathcal{N}_c \left[f^e + f \int_0^{\bar{m}_c} g_c(m) \mathrm{d}m\right] = \mathcal{N}_c \left[f^e + f G_c(\bar{m}_c)\right]. \tag{9}$$

Using (5) and (6), the market clearing condition for labor in country c is

$$\begin{aligned}\theta_c L &= \mathcal{N}_c \left[\int_0^{\bar{m}_c} m \left[q_{cc}^*(m)\theta_c L + \tau q_{cd}^*(m)\theta_d L \right] g_c(m)\mathrm{d}m \right] \\ &= \frac{(\sigma-1)r_c}{w_c}\mathcal{N}_c \left[f^e + f G_c(\bar{m}_c) \right]. \end{aligned} \tag{10}$$

Combining (9) and (10), we obtain

$$\lambda_c K = \frac{w_c}{(\sigma-1)r_c}\theta_c L. \tag{11}$$

The market clearing condition for a variety produced by $\bar{m}_c$-firm in country c is

$$\frac{(\sigma-1)f r_c}{\bar{m}_c w_c} = \frac{\mu Y_c \left[p_{cc}(\bar{m}_c)\right]^{-\sigma}}{P_c^{1-\sigma}} + \tau_{cd}\frac{\mu Y_d \left[p_{cd}(\bar{m}_c)\right]^{-\sigma}}{P_d^{1-\sigma}}. \tag{12}$$

The price index P_c is given by

$$\begin{aligned} P_c &\equiv \left[\int_0^{\bar{m}_c} \mathcal{N}_c \left[p_{cc}(m)\right]^{1-\sigma} g_c(m)\mathrm{d}m + \int_0^{\bar{m}_d} \mathcal{N}_d \left[p_{dc}(m)\right]^{1-\sigma} g_d(m)\mathrm{d}m \right]^{\frac{1}{1-\sigma}} \\ &= \frac{\sigma}{\sigma-1}\left(\frac{K}{f}\right)^{\frac{1}{1-\sigma}} \left[\lambda_c(\bar{m}_c w_c)^{1-\sigma} + \phi\lambda_d(\bar{m}_d w_d)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \end{aligned} \tag{13}$$

using (3), (8), and (9), where $\phi \equiv \tau_{cd}^{1-\sigma} = \tau_{dc}^{1-\sigma} \in (0,1)$ is the trade freeness.

Plugging (3), (9), and (13) into (12) yields

$$r_c = \frac{\mu}{\sigma K}\left[\frac{Y_c(\bar{m}_c w_c)^{1-\sigma}}{\lambda_c(\bar{m}_c w_c)^{1-\sigma} + \phi\lambda_d(\bar{m}_d w_d)^{1-\sigma}} + \phi \frac{Y_d(\bar{m}_c w_c)^{1-\sigma}}{\lambda_d(\bar{m}_d w_d)^{1-\sigma} + \phi\lambda_c(\bar{m}_c w_c)^{1-\sigma}} \right]. \tag{14}$$

The total income, which consists of the total wage, capital returns, and natural resource returns, in country c is given by

$$Y_c = \theta_c \left[w_c L + (\lambda_c r_c + \lambda_d r_d) K + Z \right]. \tag{15}$$

Free mobility of capital leads to $r_n = r_s = r$ in equilibrium.

Finally, in order to determine the cutoff $\bar{m}_c$, we have to specify the distribution of m. Specifically, we introduce the Pareto distribution of m given by

$$G_c(m) = \left(\frac{m}{m_{c\max}}\right)^{\kappa} \qquad m \in (0, m_{c\max}), \tag{16}$$

where κ is large enough such that $\kappa > \sigma - 1$ holds.

Then, we can solve (8) for $\bar{m}_c$, and obtain a unique equilibrium given by

$$\bar{m}_c = \left[\frac{(\kappa - \sigma + 1)\, f^e}{(\sigma - 1)\, f} \right]^{\frac{1}{\kappa}} m_{\text{cmax}}. \tag{17}$$

Thus, $\bar{m}_s = \bar{m}_n/\gamma \geq \bar{m}_n$ holds. The northern firms have a low ex-post cutoff cost because of their ex-ante advantage in the marginal cost distribution. Since $d\bar{m}_s/d\gamma < 0$, rising productivity intensifies competition between firms, resulting in a lower cutoff cost in the South.

2.4 Equilibrium

Given $r_n = r_s = r$ and $p_{Rn} = p_{Rs} = 1$, there are seven unknowns to Y_c, w_c, r, and λ_c for $c = n, s$, while there are eight equations (2), (11), (14), (15), and $\lambda_s + \lambda_n = 1$, one of which is redundant by Walras' law. We first solve five equations (2), (15), and (11) with respect to five unknowns Y_c, w_c, and r. We obtain

$$Y_c = \frac{\mu\,(\sigma - 1)\,\lambda_c + (\mu + \sigma - \mu\sigma)\,\theta_c}{(1 - \mu)\,\sigma} Z, \quad w_c = \frac{\mu\,(\sigma - 1)\,Z\lambda_c}{(1 - \mu)\,\sigma\theta_c L}, \quad r = \frac{\mu Z}{(1 - \mu)\,\sigma K}. \tag{18}$$

Then, substituting (18), $\lambda_s + \lambda_n = 1$ and $\theta_s + \theta_n = 1$ into (14) with $\lambda \equiv \lambda_s$ and $\theta \equiv \theta_s$, we have the following nonlinear equation

$$h(z) \equiv \left(\gamma^{1-\sigma} z - z_1\right) z^{\frac{\sigma-2}{\sigma-1}} + A\left(z - \gamma^{\sigma-1} z_2\right) = 0, \tag{19}$$

as an equilibrium condition, where

$$\begin{aligned} z &\equiv \left(\tfrac{w_s}{w_n}\right)^{\sigma-1} = \left[\tfrac{\lambda(1-\theta)}{\theta(1-\lambda)}\right]^{\sigma-1} > 0, \qquad & A &\equiv \tfrac{\theta\left\{\sigma-(1-\phi^2)[\theta\sigma+\mu(1-\theta)(\sigma-1)]\right\}}{\sigma\phi(1-\theta)} > 0, \\ z_1 &\equiv \tfrac{\sigma\phi^2+\theta(1-\phi^2)[\sigma-\mu(\sigma-1)]}{\sigma\phi} > 0, \qquad & z_2 &\equiv \tfrac{\sigma\phi}{\sigma-(1-\phi^2)[\theta\sigma+\mu(1-\theta)(\sigma-1)]} > 0. \end{aligned} \tag{20}$$

From (20), the share of capital in the South is expressed as

$$\lambda^* = 1 - \frac{1}{1 + \frac{\theta}{1-\theta}(z^*)^{\frac{1}{\sigma-1}}} \in (0, 1). \tag{21}$$

It shows that the wage ratio w_s/w_n monotonically increases with z^* and the share λ^* of capital in the South. As to the existence of equilibrium, we derive the following. The proof is contained in Appendix B.

Proposition 1 *There always exists a unique equilibrium $\lambda = \lambda^* \in (0, 1)$.*

2.5 Trade pattern

We now consider trade patterns. Net exports from the South in the $\mathbb{M}$-sector are defined by the difference between its gross exports and gross imports as:

$$\text{NetExport}_{Ms} \equiv N_s p_{sn} q_{sn} - N_n p_{ns} q_{ns}, \tag{22}$$

while the net export from the South in the $\mathbb{R}$-sector is the value of the total production minus the total expenditure of $\mathbb{R}$-good in the South:

$$\text{NetExport}_{Rs} \equiv p_{Rs} \theta Z - (1 - \mu) Y_s. \tag{23}$$

As the inflow of capital matches the outflows of trade in the South, we have the following balance of payments:

$$(\lambda^* - \theta) r^* K = \text{NetExport}_{Ms} + \text{NetExport}_{Rs}, \tag{24}$$

where the LHS is the capital account and the RHS is the current account. Equation (24) means that the South is a net exporter if $\lambda^* > \theta$ and a net importer if $\lambda^* < \theta$.

Plugging (18) into (22) and (23), we have

$$\text{NetExport}_{Ms} = \tfrac{\mu(\sigma+\mu-\mu\sigma)Z}{(1-\mu)\sigma} (\lambda^* - \theta), \qquad \text{NetExport}_{Rs} = \tfrac{\mu(\sigma-1)Z}{\sigma} (\theta - \lambda^*). \tag{25}$$

Equations (25) indicate that the net capital inflow ($\lambda^* > \theta$) in the South is accompanied by its net export of $\mathbb{M}$-good and net import of $\mathbb{R}$-good. Thus, the trade balance condition (24) implies that the South should have a net export of $\mathbb{M}$-good even larger than its net import of $\mathbb{R}$-good to match the net capital inflow.

Proposition 2 *When the net capital inflow is positive in a country, the net export is positive in the $\mathbb{M}$-sector and negative in the $\mathbb{R}$-sector. When there is no capital flow, there are no net exports in the $\mathbb{M}$- and $\mathbb{R}$-sectors.*

The trade balance equation (24) also indicates that the positive net capital inflow in the South implies that the value of the net trade of $\mathbb{M}$-good dominates that of $\mathbb{R}$-good. The trade domination of $\mathbb{M}$-good in the South means that the South, having a larger local demand, exports a good in the $\mathbb{M}$-sector with increasing returns to scale, therefore the home market effect is at work.

In the next three sections, we consider how the ROP has been affecting Chinese economic growth by conducting comparative statics. Specifically, we analyze the impacts of trade liberalization (rising ϕ) and technological progress (rising γ) on net capital inflow λ and the social welfare of the representative consumer in country c defined by

$$V_c \equiv \frac{y_c}{P_c^\mu}. \tag{26}$$

Note that the social welfare is regarded as the consumer surplus given by indirect utility V_c since the producer surplus defined by the equilibrium profits of all firms is zero.

3 Impact of trade liberalization

In this section, we examine the impact of trade liberalization on net capital flow by comparative statics $\mathrm{d}\lambda^*/\mathrm{d}\phi$. We first focus on the two extreme cases of autarky $\phi = 0$ and free trade $\phi = 1$.

When each country is in autarky $\phi = 0$, there is no trade in both the $\mathbb{M}$- and $\mathbb{R}$-sectors. From Proposition 2, there is no capital flow between countries, and thus, $\lambda^* = \theta$ holds. Substituting $\lambda^* = \theta$ into (18) yields $z = 1$, which means the same wage $w_s^* = w_n^*$ and the same income $y_s^* = y_n^*$ in autarky.

However, the price indices are shown to be different as $P_s^* \gtreqless P_n^* \Leftrightarrow \gamma \lesseqgtr \left(\frac{1-\theta}{\theta}\right)^{\frac{1}{\sigma-1}}$, therefore the social welfares are also different as

$$V_s^* \gtreqless V_n^* \;\Leftrightarrow\; \gamma \gtreqless \left(\frac{1-\theta}{\theta}\right)^{\frac{1}{\sigma-1}}. \tag{27}$$

Therefore, the autarkic welfare in the South is higher than that in the North for large γ and θ, whereas it is lower for small γ and θ. This implies that when the technological disadvantage is small (γ large) and the market size advantage is big (θ large), the social welfare is higher in the South. However, the social welfare is lower in the South due to significant technological disadvantages and the small market size.

The impact of trade opening on net capital flow can be shown as follows.

Lemma 1

$$\left.\frac{\mathrm{d}\lambda^*}{\mathrm{d}\phi}\right|_{\phi=0} \gtreqless 0 \;\Leftrightarrow\; \gamma \gtreqless \gamma_1 \equiv \left(\frac{1-\theta}{\theta}\right)^{\frac{1}{2(\sigma-1)}}.$$

In particular, $\mathrm{d}\lambda^*/\mathrm{d}\phi|_{\phi=0} < 0$ *when* γ *and* θ *are small.*

Proof. Solving $h(z) = 0$ for ϕ yields two solutions of $\phi = \phi_a(z)$ for $\gamma > \gamma_1$ and $\phi = \phi_b(z)$ for $\gamma \leq \gamma_1$.

When $\gamma > \gamma_1$, substituting $\phi = \phi_a(z)$ into $\mathrm{d}\lambda^*/\mathrm{d}\phi$ yields

$$\left.\frac{\mathrm{d}\lambda^*}{\mathrm{d}\phi}\right|_{\phi=\phi_a(z)} = -\left.\frac{\partial h/\partial\phi}{\partial h/\partial z}\right|_{\phi=\phi_a(z)} \cdot \frac{\mathrm{d}\lambda^*}{\mathrm{d}z} = \frac{k_1(z)}{k_2(z)}, \tag{28}$$

where $k_1(1) = k_2(1) = 0$. However, using L'Hôpital's rule,

$$\lim_{z\to1}\frac{k_1(z)}{k_2(z)} = \lim_{z\to1}\frac{k_1'(z)}{k_2'(z)} = \frac{\gamma^{1-\sigma}\sigma\left[\left(1+\gamma^{2(\sigma-1)}\right)\theta - 1\right]}{\sigma(1-\mu)+\mu} > 0,$$

where $z = 1$ holds when $\phi = 0$.

When $\gamma \leq \gamma_1$, plugging $\phi = \phi_b(z)$ into into $\mathrm{d}\lambda^*/\mathrm{d}\phi$ yields

$$\lim_{z\to1}\frac{k_1(z)}{k_2(z)} = \lim_{z\to1}\frac{k_1'(z)}{k_2'(z)} = \frac{\gamma^{1-\sigma}\sigma\left[1-\left(1+\gamma^{2(\sigma-1)}\right)\theta\right]}{\sigma(1-\mu)+\mu} \geq 0.$$

Since $\mathrm{d}\lambda^*/\mathrm{d}z > 0$ always holds, we have shown the lemma. ■

Lemma 1 implies that when trade opens, the South experiences capital outflows if the South is underdeveloped (γ small) and the market in the South is not large (θ small). That is, the least developed countries with small population sizes would lose firms at the opening of trade owing to the double disadvantages. Conversely, developed countries with large populations would gain firms due to the double advantages. This is because trade opening with increasing returns to scale in the $\mathbb{M}$-sector magnifies the technological gap between lagged and developed countries, and the market size difference between large and small countries. As conditions between (27) and Lemma 1 are similar, we may say that the net capital inflow is associated with a welfare differential between the countries.

Next, we consider the opposite extreme case of full integration $\phi = 1$. Solving $h(z) = 0$ with $\phi = 1$ for z yields $z^* = \gamma^{\sigma-1} < 1$, which leads to $\lambda^* = \frac{\gamma\theta}{\gamma\theta+1-\theta} < \theta$ from (21). This means that there is capital outflow in the South. Plugging this λ^* into (18), we have $w_s^* < w_n^*$ and $y_s^* < y_n^*$, whereas $P_s^* = P_n^*$ in full integration. Thus, we derive $V_s^* < V_n^*$: the social welfare in the South is lower than that in the North. Given free trade, the access to consumers in both countries is the same, whereas the technological difference $\gamma < 1$ persists. The latter causes wage differences, income differences, and hence, social welfare differences.

When trade opens at $\phi = 0$, $\lambda^* = \theta$ holds, but when the countries are fully integrated $\phi = 1$, $\lambda^* < \theta$ holds. The comparison implies that full integration does not allow the capital in the South as the same level as the autarky. This is attributed to the existence of technological differences $\gamma < 1$.

When the countries are about to be fully integrated $\phi \approx 1$, the impact of trade freeness on the net capital flow can be shown as follows.

Lemma 2

$$\left.\frac{\mathrm{d}\lambda^*}{\mathrm{d}\phi}\right|_{\phi=1} \gtreqless 0 \;\Leftrightarrow\; \gamma \lesseqgtr \gamma_2 \equiv \frac{(1-\theta)\left[\sigma - 2\theta\left(\mu + \sigma - \mu\sigma\right)\right]}{\theta\left[2\mu\left(\sigma - 1\right) - \sigma + 2\theta\left(\mu + \sigma - \mu\sigma\right)\right]}.$$

In particular, $\mathrm{d}\lambda^*/\mathrm{d}\phi|_{\phi=1} > 0$ *when* γ *and* θ *are small.*

The proof is straightforward by computing the sign of $\mathrm{d}\lambda^*/\mathrm{d}\phi|_{\phi=1,\ z=\gamma^{\sigma-1}}$.

In summary, if the technology is low and the market size is small in the South (small γ and θ), the South loses capital at trade opening $\phi \approx 0$ because opening trade magnifies the double disadvantages of low technology and non-big market. In contrast, the South attracts capital near economic integration $\phi \approx 1$ because trade freeness softens the technological disadvantages and reduces the international price differential.

Examining the parameters in γ_1 and γ_2, we show in Appendix C that $\gamma_1 > \gamma_2$ always holds. Therefore, using Lemmas 1 and 2, we establish the following.

Proposition 3 *When trade costs fall, three cases may arise.*

(i) If $\gamma_1 < \gamma$*, the capital in the South initially increases, and then decreases.*

(ii) If $\gamma_2 \leq \gamma \leq \gamma_1$*, the capital in the South monotonically decreases.*

(iii) If $\gamma < \gamma_2$*, the capital in the South initially decreases, and then increases.*

Proof. $h(z)$ in (31) can be expressed as

$$h(\phi, z) = \frac{B_0(z) + B_1(z)\phi + B_2(z)\phi^2}{(1-\theta)\,\sigma\phi} = 0. \tag{29}$$

where B's are functions of z and parameters except ϕ. The numerator of (29) is quadratic in ϕ implying that given an equilibrium value of z^*, there are at most two ϕ's that satisfy the equality in (29). In case (i), as $\mathrm{d}\lambda^*/\mathrm{d}\phi > 0$ at $\phi = 0$ and $\mathrm{d}\lambda^*/\mathrm{d}\phi < 0$ at $\phi = 1$, the curve $h(\phi, z) = 0$ is an inverted U-shape on the (ϕ, z)-coordinates. In case (iii), as $\mathrm{d}\lambda^*/\mathrm{d}\phi < 0$ at $\phi = 0$ and $\mathrm{d}\lambda^*/\mathrm{d}\phi > 0$ at $\phi = 1$, the curve is U-shaped. In case (ii), $\mathrm{d}\lambda^*/\mathrm{d}\phi < 0$ holds at $\phi = 0$ and 1. However, because the curve $h(\phi, z) = 0$ passes through

$(0,1)$ and $(1,\gamma^{\sigma-1})$ on the (ϕ,z)-coordinates, it does not have an extremum for $\phi \in [0,1]$. Hence, the curve monotonically decreases. ■

Note that the direction of the net capital flow in the North is opposite to that in the South.

The U-shaped relationship between λ^* and ϕ in case (iii) would express the situation in China as we will show in Section 5.1. The U-shape occurs if goods are close substitutes (σ large), the expenditure share of $\mathbb{M}$-good is large (μ large), the South is underdeveloped (γ small), and the population in the South is not very large (θ close to $1/2$). The last two conditions imply that the U-shape appears when the disadvantage of low technology outweighs the advantage of large demand. In fact, the sufficient condition of the U-shape is given by $\theta = 1/2$. Since there is no difference in market size between the countries, only the disadvantage in the South $\gamma < 1$ matters.

In case (iii), as trade costs fall (ϕ increases), the capital moves from the South to North in the early stage of development. However, the capital moves back from the North to South in the late stage. The reasons are as follows.

In the early stage, because the trade costs are high, the export revenues defined by the second term of the LHS in (5) are small relative to the domestic revenues given by the first term of the LHS in (5) in the $\mathbb{M}$-sector. Since the market in the South is larger than that in the North ($\theta > 1/2$), trade opening is more beneficial for firms in the North. Therefore, the North attracts capital from the South in the early stage of development. This is the industrialization stage in the North.

In the late stage, because trade costs reduce, the price index does not differ much between the countries. As productivity continues to be lower in the South $\gamma < 1$, the wage is lower in the South. Thus, it is more profitable for $\mathbb{M}$-firms to relocate to the South for lower labor costs, which results in industrialization in the South as well as deindustrialization in the North.

Substituting (3) and (18) into (26), we can express the indirect utility as $V_c(\lambda,\phi)$. The impact of rising trade freeness ϕ on social welfare V_c is given by

$$\frac{\mathrm{d}V_c}{\mathrm{d}\phi} = \frac{\partial V_c}{\partial \phi} + \frac{\partial V_c}{\partial \lambda_c}\frac{\mathrm{d}\lambda_c}{\mathrm{d}\phi}. \tag{30}$$

The first term in (30) is the direct effect of falling trade costs, while the second term is the indirect effect of falling trade costs through the change in capital flows λ_c. The direct effect is better access for consumers due to trade liberalization. This is because the consumer index decreases in trade freeness ϕ, which always increases the social welfare in both countries. However, the indirect effect reduces capital in one of the two countries, i.e., $\mathrm{d}\lambda_c/\mathrm{d}\phi < 0$ in (30), which may worsen the social welfare.

Nevertheless, we can show that $\mathrm{d}V_c/\mathrm{d}\phi|_{\phi=0 \text{ or } 1} > 0$ at the two extreme cases. We can also show that $\mathrm{d}V_c/\mathrm{d}\phi|_{\lambda=\theta=1/2,\ \gamma=1} > 0$ holds for all ϕ in the special case of symmetric country setting. Furthermore, $\mathrm{d}V_c/\mathrm{d}\phi > 0$ also holds according to our numerical analysis with various parameter values. Thus, it is tempting to conclude that trade liberalization is always beneficial for each symmetric country even though one country experiences capital outflows.

4 Impact of technological progress

We next consider a rise in γ, which is a productivity increase in the South due to technological progress. Since $\bar{m}_s = \bar{m}_n/\gamma$, rising γ means falling variable labor requirement of $\mathbb{M}$-firms in the South on average. Since this leads to an increasing capital-labor ratio, the $\mathbb{M}$-sector in the South becomes more capital intensive and approaches the North.

The impact of technological progress in the South on net capital flow is investigated by comparative statics $\mathrm{d}\lambda^*/\mathrm{d}\gamma$. We have the following clear result.

Lemma 3

$$\frac{\mathrm{d}\lambda^*}{\mathrm{d}\gamma} = \frac{\mathrm{d}\lambda^*}{\mathrm{d}z} \cdot \left[-\frac{\partial h(z^*)/\partial\gamma}{h'(z^*)}\right] > 0. \tag{31}$$

Proof. The proof is as follows. First, $\partial\lambda^*/\partial z > 0$ is obvious from (21). Second, $\partial h(z)/\partial\gamma < 0$ is straightforward from (19). Finally, $h'(z^*) > 0$ always holds at $z = z^*$ because $z = z^*$ is unique and because $h(\gamma^{\sigma-1}z_2) < 0 < h(\gamma^{\sigma-1}z_1)$ holds for $\gamma^{\sigma-1}z_2 < z < \gamma^{\sigma-1}z_1$ and $h(\gamma^{\sigma-1}z_1) < 0 < h(\gamma^{\sigma-1}z_2)$ holds for $\gamma^{\sigma-1}z_1 < z < \gamma^{\sigma-1}z_2$ from (19). ∎

Lemma 3 shows that increasing productivity always increases the net capital flow in a country, implying that technological progress is an important policy issue.

When there is no net capital flow $\lambda^* = \theta$, we know that $z^* = 1$. Plugging it into (19), we derive a unique

$$\gamma = \overline{\gamma} \equiv \left[\sqrt{\left(1 - \frac{1}{2\theta}\right)^2 \phi^2 + \frac{1}{\theta} - 1} + \left(1 - \frac{1}{2\theta}\right)\phi\right]^{\frac{1}{\sigma-1}} > 0. \tag{32}$$

Meanwhile, we have

$$\overline{\gamma}|_{\theta=1/2} = 1, \qquad \overline{\gamma}|_{\theta=1} = \phi^{\frac{1}{\sigma-1}},$$

and

$$\frac{\partial \overline{\gamma}}{\partial \theta} = -\frac{\overline{\gamma}}{2\theta(1-\theta)(\sigma-1)}\left[1 - \frac{\phi}{\sqrt{\phi^2 + 4\theta(1-\theta)(1-\phi^2)}}\right] < 0. \tag{33}$$

Therefore, there exists a unique threshold $\overline{\gamma} \in \left(\phi^{\frac{1}{\sigma-1}}, 1\right]$.

Two cases may arise when $\gamma \neq \overline{\gamma}$.

(a) If $\gamma > \overline{\gamma}$, then $\lambda^* > \theta$ holds because $\mathrm{d}\lambda^*/\mathrm{d}\gamma > 0$ from Lemma 3. Thus, the South is a net importer of $\mathbb{R}$-good and a net exporter of $\mathbb{M}$-good from (25). Furthermore, the value of net export of $\mathbb{M}$-good dominates that of its net import of $\mathbb{R}$-good in the South, which implies net capital inflow.

(b) If $\gamma < \overline{\gamma}$, then $\lambda^* < \theta$ holds. In this case, the South is a net exporter of $\mathbb{R}$-good and a net importer of $\mathbb{M}$-good from (25). The value of the net import of $\mathbb{M}$-good dominates that of its net export of $\mathbb{R}$-good in the South, which implies net capital outflows.

In summarizing the above, we have the following.

Proposition 4 *There exists a unique* $\overline{\gamma} \in (0, 1)$ *such that the South is a net importer of* $\mathbb{M}$*-good while a net exporter of* $\mathbb{R}$*-good for* $\gamma \in (0, \overline{\gamma})$*. However, the South is a net exporter of* $\mathbb{M}$*-good while a net importer of* $\mathbb{R}$*-good for* $\gamma \in (\overline{\gamma}, 1]$.

Next, we investigate comparative statics for wages, price indices, and social welfares. We can readily show

$$\frac{\mathrm{d}w_n^*}{\mathrm{d}\gamma} < 0, \qquad \frac{\mathrm{d}w_s^*}{\mathrm{d}\gamma} > 0 \tag{34}$$

from (18) and we have already shown $\mathrm{d}\lambda^*/\mathrm{d}\gamma > 0$ by Lemma 3. That is, as productivity increases in the South, it steadily attracts capital and raises the nominal wage in the South, while decreasing the nominal wage in the North.

We also show that the price index in the South is initially higher than that in the North, but is reversed later as productivity rises in the South. The proof is contained in Appendix D.

Lemma 4 *There exists a unique* $\gamma_3 \in (0, \overline{\gamma}]$ *such that* $P_s^* > P_n^*$ *for* $\gamma \in (0, \gamma_3)$ *and* $P_s^* < P_n^*$ *for* $\gamma \in (\gamma_3, 1]$.

Despite lower productivity in the South, such a reversal in the price index is achieved owing to the larger population in the South. Phrased differently, small disadvantages in production technology can be overwhelmed by a larger market size.

Thus, we establish the following proposition.[4] The proof is contained in Appendix E.

Proposition 5 *There exists a unique* $\gamma_4 \in (0,1]$ *such that* $V_s^* < V_n^*$ *for* $\gamma \in (0, \gamma_4)$ *and* $V_s^* > V_n^*$ *for* $\gamma \in (\gamma_4, 1]$.

We know from Lemma 3 that rising productivity γ in the South increases the share λ^* of capital in the South. We also know from (21) that rising γ also increases the relative wage $w_s/w_n (= z^{\frac{1}{\sigma-1}})$. By using (25), we can further ascertain that rising productivity in the South increases the net export of $\mathbb{M}$-good while decreasing the net import of $\mathbb{R}$-good in the South. In summary, we have the following.

Proposition 6 *Technological progress in the South increases net capital inflows, relative wage, net exports of* $\mathbb{M}$*-good and the net imports of* $\mathbb{R}$*-good in the South.*

Finally, we consider the impact of technological progress on the social welfare in each country. The impact of rising manufacturing technology in the South on the social welfare in country c is given by

$$\frac{\mathrm{d}V_c}{\mathrm{d}\gamma} = \frac{\partial V_c}{\partial \gamma} + \frac{\partial V_c}{\partial \lambda}\frac{\mathrm{d}\lambda}{\mathrm{d}\gamma}. \tag{35}$$

First, we can easily show $\partial V_c/\partial \gamma > 0$ for $c = n, s$ because V_c in (26) involves only one $\bar{m}_s (= \bar{m}_n/\gamma)$, which is in the price indices P_c for $c = n, s$. Rising γ decreases $\bar{m}_s$, which decreases p_{ss} and p_{sn}, which decrease both price indices. Second, we know $\mathrm{d}\lambda/\mathrm{d}\gamma > 0$ from Lemma 3. Therefore, it is tempting to say that $\mathrm{d}V_c/\mathrm{d}\gamma > 0$. Third, however, the sign of $\partial V_c/\partial \lambda$ can be negative depending on parameter values, and hence, the sign of $\mathrm{d}V_c/\mathrm{d}\gamma$ depends on parameter values.

In particular, when γ approaches 0, we have the following. The proof is contained in Appendix F.

Lemma 5

$$\lim_{\gamma \to 0} \frac{\mathrm{d}V_s}{\mathrm{d}\gamma} > 0,$$

$$\lim_{\gamma \to 0} \frac{\mathrm{d}V_n}{\mathrm{d}\gamma} \begin{cases} < 0 & \textit{for large } \theta, \textit{ small } \phi, \textit{ or small } \sigma \\ \geq 0 & \textit{otherwise.} \end{cases}$$

[4] In Arkolakis et al. (2012), the change in the welfare is explained by the change in the share of domestic expenditure on $\mathbb{M}$-good. Such a straightforward relationship does not hold in our model due to the presence of the $\mathbb{R}$-sector.

We expect that $\mathrm{d}V_c/\mathrm{d}\gamma > 0$ holds in both countries because rising γ increases productivity in the South, which decreases the prices of a good produced in the South, which decreases the price indices in both countries. This positive effect may be called *the price-decreasing effect*. However, rising γ promotes capital movement from the North to South, which decreases the number of firms and the wage, and increases the price index in the North. This negative effect may be called *the deindustrialization effect* due to the loss of capital.

The effect that is dominant depends on the parameter values. If the latter dominates the former, the social welfare in the North decreases due to technological progress in the South. From Lemma 5, this happens when the population in the South is large (θ large) because the capital outflow from the North is larger for a larger market in the South. This effect is magnified when trade is very costly (ϕ small) because large differences in the price index augment the deindustrialization effect. It is also magnified when goods are bad substitutes (small σ) because weak competition among $\mathbb{M}$-firms in the large market accelerates to relocate Northern capital to the South.

It is noted that γ does not necessarily approach 0 to be $\mathrm{d}V_n/\mathrm{d}\gamma < 0$. For example, when $\theta = \mu = 1/2$ and $\sigma = 2$, we can show that $\mathrm{d}V_n/\mathrm{d}\gamma < 0$ when γ and ϕ are small, for example, for all $\gamma < 1/2$ and $\phi < 1/2$. This suggests that the deindustrialization effect dominates the price-decreasing effect in the North, especially when Southern technology is lagging behind and trade is costly.

Symmetric countries: $\theta = 1/2$ and $\gamma = 1$ To gain further insight on the impact of technological progress, we focus on the special case of symmetric countries $\theta = 1/2$ and $\gamma = 1$. Evaluating $\mathrm{d}V_s/\mathrm{d}\gamma$ in (35) at the symmetric equilibrium $\lambda^* = 1/2$, we can obtain

$$\mathrm{sgn}\left(\left.\frac{\mathrm{d}V_s}{\mathrm{d}\gamma}\right|_{\lambda^*=\theta=\frac{1}{2},\gamma=1}\right) = \mathrm{sgn}\left(\frac{X_1}{X_2}\right),$$

where

$$X_1 \equiv (1-\mu)(\sigma-1) + \mu\phi(\sigma-1) + 1 - \phi + 4\phi\left[\left(\sigma - \frac{7}{8}\right)^2 - \frac{1}{64}\right] > 0,$$

$$X_2 \equiv (1-\phi)^2 + 4\phi(\sigma-1)^2 + (\sigma-1)(1+\phi)(1-\mu+\phi+\mu\phi) > 0.$$

Therefore, $\mathrm{d}V_s/\mathrm{d}\gamma\big|_{\lambda^*=\theta=\frac{1}{2},\gamma=1} > 0$ is satisfied. That is, rising productivity in the South always increases the social welfare in the South. However, evaluating $\mathrm{d}V_n/\mathrm{d}\gamma$ at the symmetric equilibrium yields

$$\mathrm{sgn}\left(\left.\frac{\mathrm{d}V_n}{\mathrm{d}\gamma}\right|_{\lambda^*=\theta=\frac{1}{2},\gamma=1}\right)=\mathrm{sgn}\left[\frac{(1-\phi)(\sigma-1)}{X_2}\left(\frac{2}{1-\phi}-\frac{\sigma}{\sigma-1}-\mu\right)\right].$$

Thus, we have

$$\left.\frac{\mathrm{d}V_n}{\mathrm{d}\gamma}\right|_{\lambda^*=\theta=\frac{1}{2},\gamma=1}\gtreqless 0 \;\Leftrightarrow\; \mu \lesseqgtr \frac{2}{1-\phi}-\frac{\sigma}{\sigma-1}. \tag{36}$$

Inspecting the second inequality of (36), we have $\mathrm{d}V_n/\mathrm{d}\gamma < 0$ when σ and ϕ are small while μ is large. This implies that when the goods are bad substitutes, trade is costly, and the expenditure share of $\mathbb{M}$-good is large, *rising productivity in the South decreases the social welfare in the North.* In this case, the deindustrialization effect dominates the price-decreasing effect as before.

However, rising productivity in the South increases the social welfare in the North if the goods are close substitutes, trade is less costly, and the expenditure share of $\mathbb{M}$-good is small. In this case, the price-decreasing effect due to technological progress in the South dominates the deindustrialization effect. Even though σ is large and equal to 8,[5] we have $\mathrm{d}V_n/\mathrm{d}\gamma < 0$ when ϕ are small and μ is large, say, $(\phi, \mu) = (1/50, 9/10)$. Thus, we can hardly maintain that generally, rising productivity in the South increases the social welfares in both countries when the two countries are symmetric.

5 Policy impacts on economic growth in China

5.1 Empirical evidence

We confirm whether the propositions obtained thus far describe Chinese economic development after the ROP in 1978. Figure 1 depicts the productivity growth in China after World War II: (i) the GDP per capita in China relative to that in the world average and (ii) patent applications per capita from National Bureau of Statistics of China (NBSA).[6] It shows that the GDP per capita relative to the world (blue graph) was initially low, but gradually grew from the 1990s, and then rapidly grew from 2006. This economic growth is consistent with the rapid growth in patent applications per capita (orange graph).[7] Therefore, we confirm that the ROP has been substantially promoting technological progress

[5] The estimate of σ varies from 4 to 8 according to Anderson and van Wincoop (2004).

[6] https://data.stats.gov.cn/english/

[7] According to Wei et al. (2017), not only the number of patents has exploded, but also patent quality shows a real and robust improvement over time.

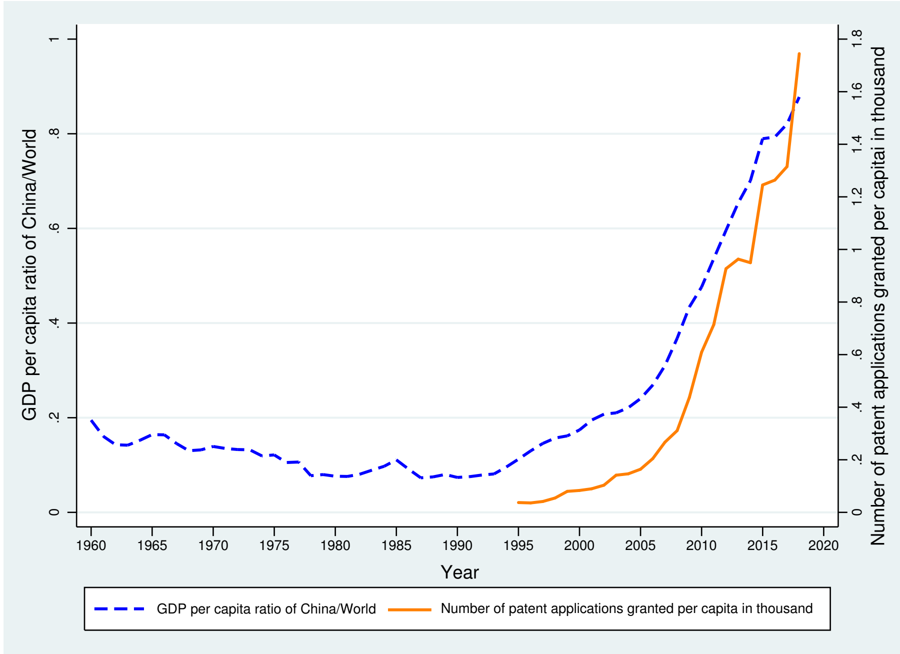

Figure 1: Productivity growth in China

in China.

Figure 2a describes the trade patterns in China using the UN Comtrade Database.[8] It shows that the net capital inflow calculated by net trade (black graph) and the net export (green graph) in the $\mathbb{M}$-sector were initially growing slowly until 2004, and then grew rapidly. The net export (blue graph) in the $\mathbb{R}$-sector behaves the opposite, which is in agreement with Proposition 2.[9] We observe that these line graphs are synchronized with those of the GDP per capita relative to the world and per capita patent applications in Figure 1, which vindicates Proposition 6.

Proposition 3(iii) shows that when the country size is not much different, rising trade freeness ϕ initially decreases the share λ^* of capital in the South, and increases λ^* later. The initial period of decreasing λ^* may be regarded as the period until 1993 in China. This is because the net capital inflow and net export of $\mathbb{M}$-good are often negative until 1993 in Figure 2b, which is an enlarged figure of the 20th-century section in Figure 2a. In contrast, the late period of increasing λ^* would correspond to the period after 1993, when both the net capital inflow and net export of $\mathbb{M}$-good are positive.

A longer time series of net trade values (percentage of GDP) in China is drawn in

[8]https://comtrade.un.org/

[9]Following the definitions by Lall (2000), the $\mathbb{R}$-sector includes primary products and resource-based manufacturing, while the $\mathbb{M}$-sector comprises the balance of manufacturing.

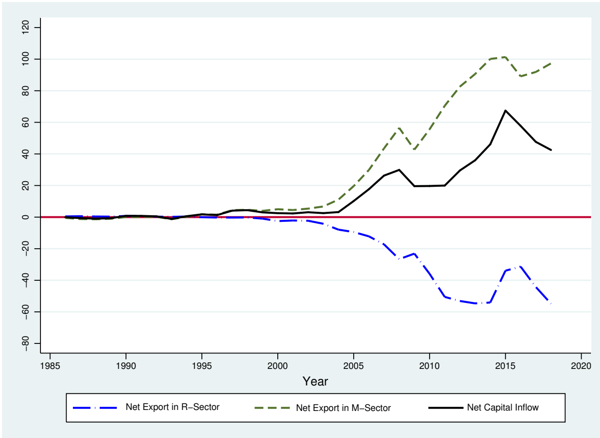

Figure 2a

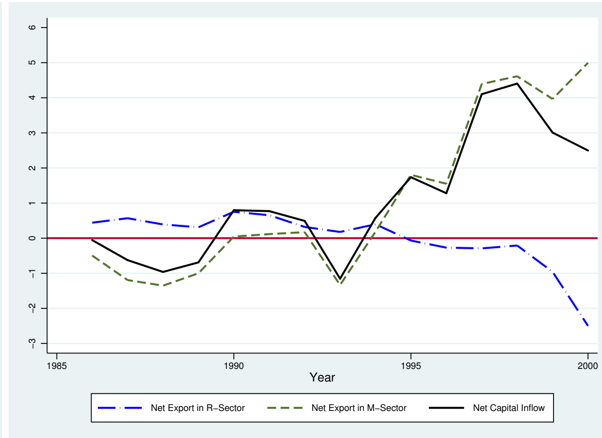

Figure 2b

Figure 2: Net trade values in China for 1986-2018

Figure 3 using OECD data.[10] It illustrates that the net trade values are often negative especially at the beginning of the ROP from 1978 to 1993, which is consistent with Figure 2b. The sign of the net trade values should be the same as that of the net capital inflow and net export of $\mathbb{M}$-good from Proposition 2. However, Lemma 3 shows that net capital inflow increases according to technological progress in the South. This would explain the net capital inflow after 1994 in China, where the net trade values are positive in Figure 3. That is, GDP growth was largely driven by net capital inflow. This increased the capital/labor ratio extremely quickly, which in turn led to an increase in labor productivity (Zheng et al., 2009)

Summarizing these results, we may conclude first, that China experienced net capital outflows until 1993 because of rising trade freeness ϕ due to the introduction of the ROP in 1978. Second, we may conclude that China experienced net capital inflows and attained rapid growth from 1994 mainly because of technological progress γ in the $\mathbb{M}$-sector. That is, increasing trade freeness ϕ would explain the initial stage of Chinese economic growth, while technological progress γ would explain the late stage of Chinese economic growth. To evaluate the impact of the ROP, we combine the two increasing parameters in the next section.

5.2 Impact of the ROP

The ROP decreases trade costs and increases Chinese productivity concurrently. To analyze its impact, we assume that both parameters ϕ and γ linearly increase with a

[10]https://data.oecd.org/trade/trade-in-goods-and-services.htm#indicator-chart.

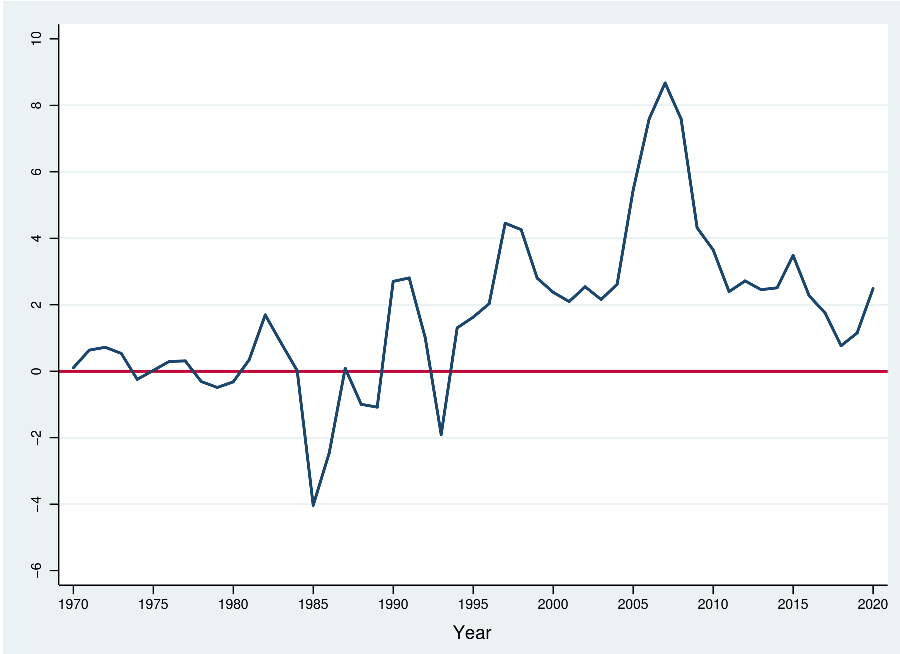

Figure 3: Net trade values (% of GDP) in China for 1970-2020

single parameter ρ as follows:

$$\phi = \phi_1 \frac{\rho - \rho_0}{1 - \rho_0} \text{ and } \gamma = \rho \qquad \text{for } \rho \in [\rho_0, \rho_1] \tag{37}$$

where ρ is the degree of ROP, $\rho_0 > 0$ is sufficiently small, $\rho_1 \equiv \frac{1-\rho_0(1-\phi_1)}{\phi_1} > 1$, and $\phi_1 \in (0, 1)$. This enables us to conduct comparative statics with respect to ρ as an ad hoc combination of ϕ and γ in Sections 3 and 4.

Assumption (37) means that (i) the initial stage is autarky with $\phi = 0$ and very low productivity ($\rho = \rho_0 \approx 0$) in the South relative to the North, (ii) the catch-up stage is equal productivity $\bar{m}_s = \bar{m}_n$ ($\rho = 1$) with $\phi = \phi_1 < 1$, and (iii) the final stage is free trade with $\phi = 1$ and higher productivity in the South than the North $\bar{m}_s < \bar{m}_n$ ($\rho = \rho_1 > 1$). We may be able to see the overview of the impact of the ROP by examining these extreme stages.[11]

We are ready to conduct comparative statics using the assumption of (37). It should be noted that changing ρ with (37) means a simultaneous rise in productivity in the South and a fall in trade costs. We call the simultaneous changes *the impact of the ROP*.

In the initial stage of autarky $\rho = \rho_0$, plugging (37) into (19) yields a unique equilibrium $z^* = 1$, which leads to $\lambda^* = \theta$. Thus, the wages are equal and the trade balances

[11]In reality, international trade and productivity before the ROP in China were not extremely low.

between the two countries. Furthermore, we have the following.

Lemma 6 *In the vicinity of autarky $\rho = \rho_0$, we have* $\mathrm{d}\lambda^*/\mathrm{d}\rho < 0$ *unless σ or θ is close to* 1.

Proof. Substituting (37) into $h(z) = 0$ yields a nonlinear equation with respect to ρ and z. The Taylor series of the zeroth- and first-order term about $z = 1$ is a linear equation of z. Solving it for z, differentiating it with respect to ρ, evaluating it at $\rho = \rho_0$, and computing $\mathrm{d}\lambda^*/\mathrm{d}z$ by (21), we have

$$\left.\frac{\mathrm{d}\lambda^*}{\mathrm{d}\rho}\right|_{\rho=\rho_0} \approx \frac{\phi_1\sigma\left[\rho_0^{2\sigma}\theta - \rho_0^2\left(1-\theta\right)\right]}{\left(1-\rho_0\right)\rho_0^{\sigma+1}\left(\mu+\sigma-\mu\sigma\right)}.$$

Therefore,

$$\mathrm{sgn}\left(\left.\frac{\mathrm{d}\lambda^*}{\mathrm{d}\rho}\right|_{\rho=\rho_0}\right) = \mathrm{sgn}\left(\theta - \frac{1}{1+\rho_0^{2(\sigma-1)}}\right).$$

Since ρ_0 is sufficiently small, the RHS is negative unless $\sigma \approx 1$ or $\theta \approx 1$. ■

We know that $\mathrm{d}\lambda^*/\mathrm{d}\gamma > 0$ always holds from (31) in the absence of assumption (37). We also know that $\mathrm{d}\lambda^*/\mathrm{d}\phi$ depends on parameter values from Proposition 3 in the absence of (37). In the presence of (37), $\mathrm{d}\lambda^*/\mathrm{d}\rho < 0$ normally holds near autarky from Lemma 6. This is because neither $\sigma \approx 1$ nor $\theta \approx 1$ is a realistic value. Thus, we may say that technological progress is dominated by the trade opening in the initial stage of the ROP.

In contrast, in the final stage of free trade $\rho = \rho_1 \approx 1/\phi_1$, plugging (37) into (19) also yields $z^* = \phi_1^{1-\sigma} > 1$, which leads to $\lambda^* > \theta$. Therefore, the nominal wage is higher in the South, which is a net exporter of $\mathbb{M}$-good and a net importer of $\mathbb{R}$-good. In the final stage, we obtain the following.

Lemma 7 *In the vicinity of free trade $\rho = \rho_1$, we have* $\mathrm{d}\lambda^*/\mathrm{d}\rho > 0$ *for all* $\sigma \geq 2$.

Proof. Substituting (37) into $h(z) = 0$ yields a nonlinear equation with respect to ρ and z. The Taylor series of the zeroth- and first-order term about $z = \phi_1^{1-\sigma}$ is a linear equation of z. Solving it for z, differentiating it with ρ, evaluating it at $\rho = \rho_1$, and computing $\mathrm{d}\lambda^*/\mathrm{d}z$, we get

$$\begin{aligned}\left.\frac{\mathrm{d}\lambda^*}{\mathrm{d}\rho}\right|_{\rho=\rho_1} &= \frac{\phi_1^2\theta(1-\theta)}{\left(\sigma-1\right)\sigma\left(\phi_1+\theta-\phi_1\theta\right)^3}X_3 \\ X_3 &\equiv \left(\sigma-2\theta\right)\sigma\left(\phi_1+\theta-\phi_1\theta\right) - 2\left(1-\phi_1\right)\left(\sigma-1\right)\left(1-\theta\right)\theta\mu\end{aligned}$$

for $\rho_0 \approx 0$. Since X_3 is decreasing in μ, we have

$$\begin{aligned} X_3 > X_3|_{\mu=1} &= (\phi_1 + \theta - \phi_1\theta)(\sigma - 2)^2 + 2(2\phi_1 + \theta - 2\phi_1\theta)(\sigma - 2) + 2(2\phi_1 + \theta - \phi_1\theta)(1 - \theta) \\ &> 0 \qquad \forall \sigma \geq 2. \end{aligned}$$

Hence, $\mathrm{d}\lambda^*/\mathrm{d}\rho$ is positive at $\rho = \rho_1$ for all $\sigma \geq 2$. ■

We know that in the absence of assumption (37), $\mathrm{d}\lambda^*/\mathrm{d}\gamma > 0$ always holds from (31), whereas $\mathrm{d}\lambda^*/\mathrm{d}\phi < 0$ holds at $\phi = 1$ for large $\gamma > \gamma_2$ from Lemma 2. From Lemma 7, it turns out that the former effect of technological progress dominates the latter effect of trade integration in the final stage of free trade in the presence of (37).

Since the estimated value of σ varies from 4 to 8 in developed countries (Anderson and van Wincoop, 2004), we can safely assume $\sigma \geq 2$ and verify $\mathrm{d}\lambda^*/\mathrm{d}\rho > 0$ in China. Then, we have the U-shaped relationship between capital λ^* in the South and ROP ρ. This is the main proposition of this study.

Proposition 7 *Assume $\sigma \geq 2$. As trade costs fall and productivity in the South rises simultaneously, the capital in the South initially decreases, and then increases.*

Proof. Substituting (37) and $\rho_0 = 0$ into (19) yields

$$\begin{aligned} h(z) &\equiv h_1(\rho, z) = \frac{h_2}{\rho\phi_1\sigma(1-\theta)} \\ h_2 &\equiv \left\{\sigma - \left(1 - \rho^2\phi_1^2\right)\left[\sigma\theta + (\sigma - 1)(1-\theta)\mu\right]\right\}\theta z - \rho^\sigma\phi_1\sigma\theta \\ &- (1-\theta)\left[\left(1 - \rho^2\phi_1^2\right)(\sigma + \mu - \mu\sigma)\theta + \rho^2\phi_1\sigma\left(\phi_1 - \rho^{-\sigma}z\right)\right] z^{\frac{\sigma-2}{\sigma-1}}. \end{aligned} \tag{38}$$

We have

$$\frac{\partial^3 h_2}{\partial \rho^3} = -\rho^{-\sigma-3}\phi_1(\sigma-1)(\sigma-2)\sigma^2\left[\rho^{2\sigma}\theta + (1-\theta)\rho^2 z^{\frac{2\sigma-3}{\sigma-1}}\right] \leq 0 \qquad \text{for } \sigma \geq 2.$$

This implies that given an equilibrium value $z = z^*$, there is at most one ρ that satisfies $\partial^2 h_2/\partial\rho^2 = 0$. This means that there is at most two ρ's that satisfy $\partial h_2/\partial\rho = 0$, which means that there are at most three ρ's that satisfy $h_1(\rho, z) = 0$ given z on the (ρ, z)-coordinates. However, we know from Lemmas 6 and 7 that $\mathrm{d}\lambda^*/\mathrm{d}\rho < 0$ at $\rho = \rho_0$ and $\mathrm{d}\lambda^*/\mathrm{d}\rho > 0$ at $\rho = \rho_1$ for all $\sigma \geq 2$. Since λ^* monotonically increases in z^* from (21), it must be that $\mathrm{d}\rho/\mathrm{d}z^* < 0$ at $\rho = \rho_0$ and $\mathrm{d}\rho/\mathrm{d}z^* > 0$ at $\rho = \rho_1$ for all $\sigma \geq 2$.

Hence, there exist two values of ρ in $h_1(\rho, z) = 0$ for $\rho \in \mathbb{R}^+$ given z and there exists a unique minimum z on the (ρ, z)-coordinates for $\rho \in (\rho_0, \rho_1)$. That is, $h_1(\rho, z) = 0$ is

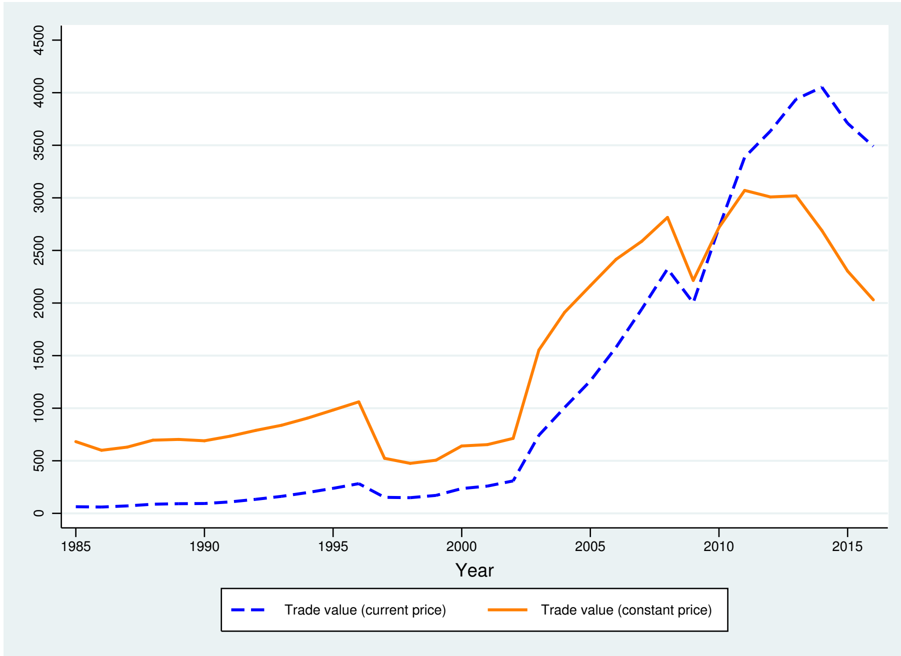

Figure 4: Trade values in China for 1985-2016

U-shaped on the (ρ, z)-coordinates, and thus, $h_1(\rho, \lambda^*) = 0$ is U-shaped on the (ρ, λ^*)-coordinates. ■

As $z^* = 1$ holds in the initial stage $\rho = \rho_0$, we have $\lambda^* = \theta$. This is because there are no net capital flows in autarky. In contrast, in the final stage of free trade $\rho = \rho_1$, we get $\lambda^* > \theta$ so that the South exports $\mathbb{M}$-good and imports $\mathbb{R}$-good. In the catch-up stage $\rho = 1$, we show in Appendix G that $\lambda^* > \theta$ also holds. That is, when the South catches up with the North $\rho = 1$, the South with larger local demand $\theta > 1/2$ exports $\mathbb{M}$-good and imports $\mathbb{R}$-good. This is due to the home market effect. Finally, because $h_1(\rho, z) = 0$ is U-shaped, it must be that there exists a unique $\rho_2 \in (0, 1)$ that satisfies $z^* = 1$. Stated differently, there exists ρ_2 between the initial autarky stage and catch-up stage that satisfies $\lambda^* = \theta$, when the wages are equal and trade balances.

According to Figures 2b and 3, China experienced net capital outflows mostly in the 1980s directly after the ROP in 1978, whereas it experienced net capital inflows from the 1990s to the present. The former period corresponds to $\rho \in (0, \rho_2)$ when the South imported $\mathbb{M}$-good and exported $\mathbb{R}$-good. The latter corresponds to $\rho \in (\rho_2, \rho_1)$ when the South exported $\mathbb{M}$-good and imported $\mathbb{R}$-good.

Figure 4 depicts the growth of the total trade value, which is the sum of exports and imports in billion US dollars. The total trade value has been growing over time since the ROP like (see capita GDP in Figure 1). To check this with our model, we calculate

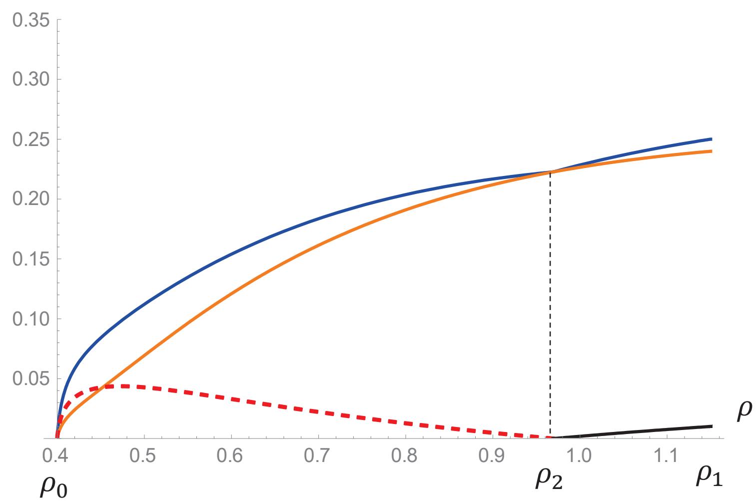

Figure 5: Total trade value (blue), gross trade value in M-sector (orange), and net trade value in R-sector (dashed red and black)

the total trade value, which consists of the gross trade value in the $\mathbb{M}$-sector, $N_s p_{sn} q_{sn} + N_n p_{ns} q_{ns}$, and the net trade value in the $\mathbb{R}$-sector, $\frac{\mu(\sigma-1)Z}{\sigma}|\theta - \lambda^*|$, using equations in Section 2.5.[12]

Given parameter values: $\sigma = 5$, $\mu = 1/2$, $\theta = 3/4$, $K = L = Z = \bar{m}_n = 1$, $\rho_0 = 2/5$, and $\phi_1 = 4/5$, we numerically compute and plot the total trade values, the gross trade values in the $\mathbb{M}$-sector, and the net trade values in the $\mathbb{R}$-sector for $\rho \in [\rho_0, \rho_1]$ in Figure 5. We observe that the orange curve of the gross trade value in the $\mathbb{M}$-sector and the blue curve of the total trade value have been increasing in Figure 5, the latter of which is in agreement with the real data of the orange curve in Figure 4. The dashed red curve is the net trade value in the $\mathbb{R}$-sector for $[\rho_0, \rho_2)$ when $\lambda^* < \theta$ and the black curve is the net trade value in the $\mathbb{R}$-sector for $(\rho_2, \rho_1]$ when $\lambda^* > \theta$. That is, the South is a net exporter of $\mathbb{R}$-good for $[\rho_0, \rho_2)$ and a net exporter of $\mathbb{M}$-good for $(\rho_2, \rho_1]$. The dashed red curve is an inverted U-shape and the black curve is decreasing because λ^* is U-shaped from Proposition 7. These results are consistent with the changes in the net trade values in both sectors in Figures 2a and 2b.

Finally, when ϕ and γ are independent, we have $\mathrm{d}V_n/\mathrm{d}\phi > 0$ at $\phi = 0$ in Section 3, whereas $\mathrm{d}V_n/\mathrm{d}\gamma < 0$ for small ϕ and γ in Lemma 5 of Section 4. In this section, however, ϕ

[12]Since $\mathbb{R}$-good is homogeneous in our model, the trade value in the $\mathbb{R}$-sector is net rather than gross. However, some $\mathbb{R}$-goods are heterogeneous in reality and the trade value would be larger in the $\mathbb{R}$-sector.

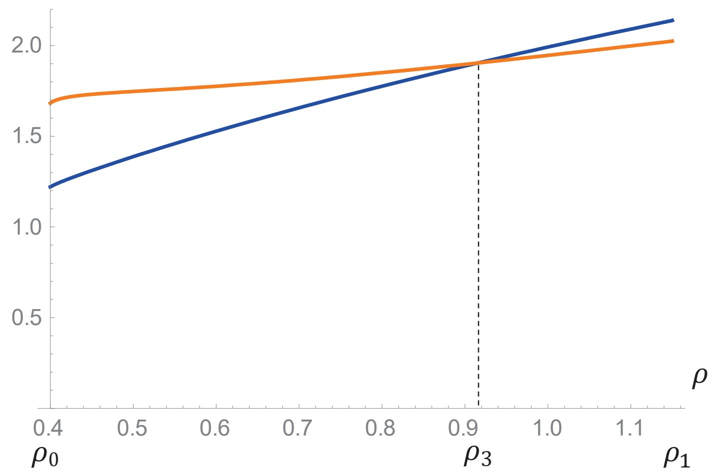

Figure 6: Social welfares in the South (blue) and North (orange)

and γ are mutually dependent. What happens if $\rho \to \rho_0$, which means $\phi \to 0$ and $\gamma \to \rho_0$ from the assumption of (37)? We can show that $\mathrm{d}V_n/\mathrm{d}\rho > 0$ at $\rho \to \rho_0 \approx 0$. That is, the social welfare in the North rises when both trade freeness and productivity are low. This suggests that the gains from trade opening are larger than the losses from capital outflows. In other words, the price-decreasing effect is stronger than the deindustrialization effect.

Using the same set of the above parameter values, we also conducted numerical simulations. We found that the social welfare in each country monotonically increases with ρ. That is, the simultaneous increase in the trade freeness and productivity in the South benefits both countries. However, there exists a social welfare differential between the countries as illustrated in Figure 6. When trade is costly and Southern technology is poor, the social welfare in the North is higher than that in the South. However, as trade costs reduce and Southern technology improves due to the ROP, the South gets ahead of the North when ρ exceeds threshold $\rho_3 \in (\rho_0, \rho_1)$.

6 Conclusion

In this study, we considered the impacts of ROP on net capital flows, net exports, and the social welfare in an international trade context. We developed a model of Chinese growth in a two-country economy with the manufacturing and natural resource sectors.

We analyzed how net capital flows of the manufacturing sector in China changed due to trade liberalization and technological progress and examined the net exports in the two sectors as well as the social welfare.

First, we showed that given low productivity in the South, as trade costs fall, the manufacturing capital in the South initially decreases, and then increases. Second, given the level of trade freeness, we showed that technological progress in the South increases net capital flows, relative wage, net exports in the manufacturing sector, and net imports in the resource sector in the South. We also showed that technological progress in the South decreases the social welfare in the North if the deindustrialization effect dominates the price-decreasing effect.

Finally, we analyzed the impacts of the ROP through the promotion of trade freeness and technological progress in the South. We showed that the capital in the South initially decreases, and then increases, and that the South is a net importer of the manufactured good in the early period, and a net exporter of the good in the late period, which concurs with the Chinese data after the ROP in 1978.

There are several directions for future research. First, it would be possible to extend our model by incorporating the service sector. This is because not only the manufacturing sector but also the service sector has been significantly expanding in China. Digital economic activities have especially been growing rapidly in recent years and becoming one of the leading industries, which has boosted productivity growth and reshaped the spatial distribution of economic activities in China. Second, it would also be possible to consider the impact of special economic zones, which is also an important strategy of the ROP. For example, after being designated as a special economic zone, Shenzhen's population grew from 59,000 in 1980 to 12.4 million in 2020 (more than 200 times in forty years). Third, China is a large country characterized by considerable regional disparities. Differences in endowments and economic policies across provinces have contributed to the persistence of spatial inequality, which is worth an in-depth examination for attainment of an equal society.

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Appendices

A Positive fixed requirement for export

Assume $f^x > 0$ units of capital are required for a firm to export. Then, domestic and export firms coexist in both countries. The former firms produce exclusively for their domestic market, whereas the latter earn profits from their domestic and foreign sales.

The cutoff conditions are

$$\pi_{cc}(\bar{m}_c) - fr_c = \frac{\mu Y_c}{\sigma}\frac{p_{cc}^{1-\sigma}(\bar{m}_c)}{P_c^{1-\sigma}} - fr_c = 0 \tag{A.1}$$

$$\pi_{dc}(\bar{m}_d^x) - f^x r_d = \frac{\mu Y_c}{\sigma}\frac{p_{dc}^{1-\sigma}(\bar{m}_d^x)}{P_c^{1-\sigma}} - f^x r_d = 0. \tag{A.2}$$

Because $r_c = r_d = r$, we have $p_{cc}(\bar{m}_c)/p_{dc}(\bar{m}_d^x) = (f/f^x)^{\frac{1}{1-\sigma}}$, and thus,

$$\bar{m}_c w_c = (f/f^x)^{\frac{1}{1-\sigma}} \tau \bar{m}_d^x w_d \tag{A.3}$$

holds for $c, d = s, n$ and $c \neq d$.

The net profit of an m-firm is defined by

$$\Pi_c(m) = \begin{cases} \pi_{cc}(m) + \pi_{cd}(m) - fr_c - f^x r_c - f^e r_c, & \text{for } 0 < m < \bar{m}_c^x \\ \pi_{cc}(m) - fr_c - f^e r_c, & \text{for } \bar{m}_c^x \leq m < \bar{m}_c \\ -f^e r_c, & \text{otherwise.} \end{cases}$$

That is, the active domestic firms and exporting firms in country c have their types $m \in (\bar{m}_c^x, \bar{m}_c)$ and $m \in (0, \bar{m}_c^x)$, respectively.

Then, the zero expected profit conditions are given by

$$f\int_0^{\bar{m}_c}\left[\left(\frac{\bar{m}_c}{m}\right)^{\sigma-1} - 1\right]g_c(m)\mathrm{d}m + f^x\int_0^{\bar{m}_c^x}\left[\left(\frac{\bar{m}_c^x}{m}\right)^{\sigma-1} - 1\right]g_c(m)\mathrm{d}m - f^e = 0 \tag{A.4}$$

for $c = s, n$. Solving (A.3) and (A.4) yields a unique solution

$$\bar{m}_c = \left\{\frac{(\kappa-\sigma+1)\, f^e\left[1 - \left(\frac{f}{f^x}\right)^{\frac{\kappa}{\sigma-1}-1}\left(\frac{\gamma_c w_d}{\tau\gamma_d w_c}\right)^{\kappa}\right]}{(\sigma-1)\, f\left[1 - \left(\frac{f}{f^x}\right)^{\frac{2\kappa}{\sigma-1}-2}\tau^{-2\kappa}\right]}\right\}^{\frac{1}{\kappa}} m_{c\max} \tag{A.5}$$

for $c, d = s, n$ and $c \neq d$.

Plugging (18) and (A.5) into (14) yields a single equation like $h(z) = 0$ in (20). However, because (A.5) involves w_d/w_c, the equation is much more highly nonlinear function than $h(z)$. Nevertheless, we can show that there always exists an equilibrium when f^x is sufficiently large.[13]

B Proof of Proposition 1

If $z_1 = z_2$ holds, $z^* = \gamma^{\sigma-1} z_1$ is the unique solution of $h(z^*) = 0$ from (19). Let $\widehat{z} \equiv \gamma^{1-\sigma} z$. Plugging it into (19) and manipulating it yields

$$a(\widehat{z}) \equiv \log \frac{(z_1 - \widehat{z})\widehat{z}^{\frac{\sigma-2}{\sigma-1}}}{A\gamma\,(\widehat{z} - z_2)} = 0 \qquad \text{for } z_{\min} < \widehat{z} < z_{\max}, \tag{B.1}$$

where $z_{\min} \equiv \min\{z_1, z_2\}$ and $z_{\max} \equiv \max\{z_1, z_2\}$.

If $z_1 \neq z_2$ holds, $a(\widehat{z}) = 0$ has no solution for all $\widehat{z} \notin (z_{\min}, z_{\max})$. We have

$$\begin{aligned} a'(\widehat{z}) &= \frac{j(\widehat{z})}{(\sigma-1)\,\widehat{z}\,(z_1 - \widehat{z})\,(\widehat{z} - z_2)}, \\ j(\widehat{z}) &\equiv (2-\sigma)\,\widehat{z}^2 + (2\sigma z_2 - 3z_2 - z_1)\,\widehat{z} + (2-\sigma)\,z_1 z_2. \end{aligned}$$

Since the denominator of $a'(\widehat{z})$ is positive, we pay attention to the sign of $j(\widehat{z})$.

(i) When $\sigma > 3/2$, we get

$$j(z_1) = (\sigma-1)\,z_1\,(z_2 - z_1) \qquad \text{and} \qquad j(z_2) = (\sigma-1)\,z_2\,(z_2 - z_1)\,, \tag{B.2}$$

so that $j(z_1)$ and $j(z_2)$ have the same sign. Since

$$j'(z_1) = (2\sigma - 3)\,(z_2 - z_1) \qquad \text{and} \qquad j'(z_2) = (z_2 - z_1)\,, \tag{B.3}$$

$j'(z_1)$ and $j'(z_2)$ have the same sign. Because $j(\widehat{z})$ is quadratic in $\widehat{z}$, $j(\widehat{z})$ and $a'(\widehat{z})$ do not change its sign in the interval of $(z_{\min}, z_{\max})$.

(ii) When $1 < \sigma \leq 3/2$, there are two subcases.

If $z_1 > z_2$, the above $a'(\widehat{z})$ can be rewritten as

$$a'(\widehat{z}) = -\left[\frac{2-\sigma}{(\sigma-1)\,\widehat{z}} + \frac{z_1 - z_2}{(z_1 - \widehat{z})\,(\widehat{z} - z_2)}\right] < 0, \tag{B.4}$$

[13] A proof is available upon request from the authors.

which holds for all $\widehat{z} \in (z_2, z_1)$.

If $z_1 < z_2$, the above $a'(\widehat{z})$ can be rewritten as

$$a'(\widehat{z}) = \frac{\left[2\left(2-\sigma\right)\widehat{z} - z_1 + \left(2\sigma - 3\right) z_2\right]^2 + \left(z_2 - z_1\right)\left[z_1 - \left(2\sigma - 3\right)^2 z_2\right]}{4\left(\sigma - 1\right)\left(2-\sigma\right)\widehat{z}\left(\widehat{z} - z_1\right)\left(z_2 - \widehat{z}\right)} > 0, \qquad \text{(B.5)}$$

which holds for all $\widehat{z} \in (z_1, z_2)$. This is because

$$z_1 - (2\sigma - 3)^2 z_2 = \frac{\theta(1-\theta)(1-\phi^2)^2[\sigma - \mu(\sigma-1)]^2 + \sigma\phi^2(\sigma-1)[4\sigma(2-\sigma) - \mu(1-\phi^2)]}{\sigma\phi\{\sigma - (1-\phi^2)[\theta\sigma + \mu(1-\theta)(\sigma-1)]\}} > 0$$

holds for all $1 < \sigma \leq 3/2$.

Thus, we have shown that $a'(\widehat{z})$ does not change its sign in the interval of $(z_{\min}, z_{\max})$. Meanwhile, $a(z_1)a(z_2) < 0$ holds. Therefore, there exists a unique solution $\widehat{z}^* \in (z_{\min}, z_{\max})$ of $a(\widehat{z}) = 0$, and hence, a unique $\lambda = \lambda^* \in (0, 1)$.

C Proof of $\gamma_1 > \gamma_2$

Since $\partial\gamma_2/\partial\mu \geq 0$ holds for $1/2 \leq \theta < 1$ and $0 < \mu < 1$, we only show $\gamma_1 \geq \gamma_2|_{\mu=1} \geq \gamma_2$. Define $v \equiv (1-\theta)/\theta \in (0, 1)$ and

$$b(v) \equiv \log \frac{\gamma_1}{\gamma_2|_{\mu=1}} = \log \frac{v^{\frac{3-2\sigma}{2(\sigma-1)}}\left(\sigma v - 2v + \sigma\right)}{\sigma v - 2 + \sigma},$$

so that we show $b(v) \geq 0$ for all $v \in (0, 1)$. We know that $\gamma_1 > 0$ always holds and

$$\gamma_2|_{\mu=1} \lesseqgtr 0 \text{ for } v \lesseqgtr v_0 \equiv \frac{2-\sigma}{\sigma}.$$

Therefore, it is sufficient to show $b(v) \geq 0$ $\forall v \in (v_0, 1)$ when $v_0 > 0$.

(i) When $1 < \sigma \leq 3/2$, we have

$$b'''(v) < 0 \ \forall v \in (v_0, 1), \text{ and } b''(1) > 0.$$

Thus, $b''(v) > 0$ is satisfied. Since $b'(1) < 0$, $b'(v) < 0$ holds. Because $b(1) = 0$, we get $b(v) > 0$ $\forall v \in (v_0, 1)$.

(ii) When $\sigma > 3/2$, we can show that $b''(v) > 0$ always holds. We also have $b'(1) < 0$ and $b(1) = 0$, and thus, we have $b(v) \geq 0$ $\forall v \in (v_0, 1)$.

D Proof of Lemma 4

The price index ratio is given by

$$\frac{P_n}{P_s} = \left(\frac{N_s p_{ss}^{1-\sigma} + N_n p_{ns}^{1-\sigma}}{N_n p_{nn}^{1-\sigma} + N_s p_{sn}^{1-\sigma}}\right)^{\frac{1}{\sigma-1}} = \left(\frac{X_6 + \phi}{1 + \phi X_6}\right)^{\frac{1}{\sigma-1}},$$

where

$$X_6 \equiv \frac{\lambda w_s^{1-\sigma}}{(1-\lambda)(\gamma w_n)^{1-\sigma}} = \left(\frac{\gamma\theta}{1-\theta}\right)^{\sigma-1}\left(\frac{\lambda}{1-\lambda}\right)^{2-\sigma}.$$

When $\gamma \to 0$, $\lim_{\gamma\to 0} X_6 = 0$, which means $\lim_{\gamma\to 0}\frac{X_6+\phi}{1+\phi X_6} = \phi < 1$, should hold. On the other hand, when $\gamma = \overline{\gamma} \in [0,1)$, $\lambda^* = \theta$ is an equilibrium. Since

$$X_6|_{\gamma=\overline{\gamma},\ \lambda^*=\theta} = \frac{\sqrt{\phi^2(\theta-1/2)^2 + (1-\theta)\theta} + \phi(\theta-1/2)}{1-\theta}$$

is increasing in θ and is equal to 1 at $\theta = 1/2$, we have

$$X_6|_{\gamma=\overline{\gamma},\ \lambda^*=\theta} \geq 1 \qquad \text{and} \qquad X_6|_{\gamma=\overline{\gamma},\ \lambda^*=\theta} \geq 1,$$

where the two inequalities are strict when $\theta > 1/2$. By continuity, there exists $\gamma_3 \in (0, \overline{\gamma}]$ that satisfies $P_n = P_s$ for $\theta > 1/2$.

Next, since P_n/P_s is increasing in X_6, we have

$$\operatorname{sgn}\left(\frac{\mathrm{d}}{\mathrm{d}\gamma}\frac{P_n}{P_s}\right) = \operatorname{sgn}\left(\frac{\mathrm{d}X_6}{\mathrm{d}\gamma}\right).$$

The derivative is

$$\frac{\mathrm{d}X_6}{\mathrm{d}\gamma} = \frac{\partial X_6}{\partial\gamma} + \frac{\partial X_6}{\partial\lambda^*}\frac{\mathrm{d}\lambda^*}{\mathrm{d}\gamma}.$$

Using (31), we get

$$\frac{\mathrm{d}X_6}{\mathrm{d}\gamma} = -\frac{\gamma^{\sigma-3} z^{\frac{3-2\sigma}{\sigma-1}}\theta(\sigma-1)}{1-\theta}\frac{A\gamma^\sigma\left[\gamma^\sigma(\sigma-2)z_2 - \gamma z(\sigma-1)\right] + \gamma z^{\frac{\sigma-2}{\sigma-1}}\left[\gamma^\sigma(\sigma-2)z_1 - \gamma z(\sigma-1)\right]}{A(\sigma-1)\gamma^\sigma - z^{\frac{1}{1-\sigma}}\left[\gamma^\sigma(\sigma-2)z_1 - \gamma(2\sigma-3)z\right]}. \tag{D.1}$$

Solving $h(z) = 0$ with respect to A and plugging it into (D.1), we obtain

$$\left.\frac{\mathrm{d}X_6}{\mathrm{d}\gamma}\right|_{h(\widehat{z})=0} = \frac{\theta(\sigma-1)\widehat{z}^{\frac{1}{\sigma-1}}}{(1-\theta)X_7}, \tag{D.2}$$

where

$$X_7(\widehat{z}) \equiv \frac{(\sigma-2)(\widehat{z}^2+z_1z_2)+[z_1-(2\sigma-3)z_2]\,\widehat{z}}{z_1-z_2}.$$

We have

$$X_7(z_1)=(\sigma-1)z_1,\;\; X_7(z_2)=(\sigma-1)z_2,\;\; X_7'(z_1)=2\sigma-3,\;\text{ and }\;\; X_7'(z_2)=1.$$

There are two cases. (i) If $z_1<z_2$, then $0<X_7(z_1)<X_7(z_2)$. Therefore, $X_7(\widehat{z})>0$ holds for all $\widehat{z}\in(z_1,z_2)$ if $X_7'(z_1)=2\sigma-3\geq 0$. This is because $X_7(\widehat{z})$ is a quadratic function. (ii) If $z_1\geq z_2$, then $X_7(z_1)\geq X_7(z_2)>0$. Therefore, $X_7(\widehat{z})>0$ holds for all $\widehat{z}\in(z_2,z_1)$ because $X_7'(z_2)=1>0$.

We have shown that $X_7(\widehat{z})>0$ (i) if $z_1<z_2$, and $\sigma\geq 3/2$; or (ii) if $z_1\geq z_2$. Thus, we finally examine the sign of $X_7(\widehat{z})$ when $z_1<z_2$, and $1<\sigma<3/2$. In order to prove $X_7(\widehat{z})>0$, it is sufficient to show that $X_7(\widetilde{z})>0$, where $\widetilde{z}$ is the extremum of the quadratic function and is a solution of $X_7'(\widetilde{z})=0$. Substituting z_1 and z_2 in (20), we get

$$X_7(\widetilde{z})=\frac{(1-\theta)\,\theta\,[(1-\mu)\,\sigma+\mu]^2\,(1-\phi^2)^2+\sigma\phi^2(\sigma-1)\,[4\sigma\,(3/2-\sigma)+2\sigma-\mu+\mu\phi^2]}{4\,(2-\sigma)\,\phi\sigma\{(1-\theta)+\theta\phi^2+(\sigma-1)\,[\phi^2+(1-\theta)(1-\mu)(1-\phi^2)]\}},$$

which is positive for all $1<\sigma<3/2$.

Hence,

$$\frac{\mathrm{d}}{\mathrm{d}\gamma}\frac{P_n}{P_s}>0, \tag{D.3}$$

and thus, there exists a unique $\gamma_3\in(0,\overline{\gamma}]$ that satisfies $P_n^*=P_s^*$.

E Proof of Proposition 5

We have

$$\frac{\partial}{\partial\gamma}\left(\frac{V_s}{V_n}\right)=\frac{\partial}{\partial\gamma}\left[\frac{y_s/y_n}{(P_s/P_n)^{\mu}}\right]=\frac{1}{(P_s/P_n)^{2\mu}}\left[\frac{\partial\,(y_s/y_n)}{\partial\gamma}\left(\frac{P_s}{P_n}\right)^{\mu}-\frac{\partial\,(P_s/P_n)^{\mu}}{\partial\gamma}\frac{y_s}{y_n}\right]. \tag{E.1}$$

Since $y_c=w_c+\frac{(\mu+\sigma-\mu\sigma)Z}{(1-\mu)\sigma L}$ from (18), $\partial y_c/\partial\gamma=\partial w_c/\partial\gamma$ holds for $c=n,s$. Then, the first term of the RHS in (E.1) is positive because

$$\frac{\partial\,(y_s/y_n)}{\partial\gamma}=\frac{1}{y_n^2}\left(\frac{\partial y_s}{\partial\gamma}y_n-\frac{\partial y_n}{\partial\gamma}y_s\right)=\frac{1}{y_n^2}\left(\frac{\partial w_s}{\partial\gamma}y_n-\frac{\partial w_n}{\partial\gamma}y_s\right)>0,$$

where $\partial w_s/\partial\gamma > 0$ and $\partial w_n/\partial\gamma < 0$ hold from (34). The second term of the RHS is also positive because $\partial\left(P_s/P_n\right)/\partial\gamma < 0$ holds from (D.3). Hence, $\partial\left(V_s/V_n\right)/\partial\gamma > 0$ is satisfied for all $\gamma \in (0,1)$.

We know from (B.1) that $z_{\min} < \gamma^{1-\sigma} z < z_{\max}$, where $z_{\min}$ and $z_{\max}$ are positive and finite. When $\gamma \to 0$, we get $z_{\min} \to z_{\max}$, so that $z \to 0$. That is, $w_s < w_n$ and $y_s < y_n$, whereas $P_s > P_n$ holds, and hence, $V_s/V_n < 1$ is satisfied at $\gamma \to 0$. On the other hand, when $\gamma \to 1$, we have $y_s > y_n$ and $P_s < P_n$ hold, which imply $V_s/V_n > 1$ at $\gamma \to 1$. Therefore, there exists a unique γ_4 that satisfies $V_s^*/V_n^*\big|_{\gamma=\gamma_4} = 1$.

F Proof of Lemma 5

Solving $h(z) = 0$ in (19) for γ yields two solutions given by

$$\gamma = \left[\frac{2z}{z_1 - Az^{\frac{1}{\sigma-1}} \pm \sqrt{\left(z_1 - Az^{\frac{1}{\sigma-1}}\right)^2 + 4Az_2 z^{\frac{1}{\sigma-1}}}}\right]^{\frac{1}{\sigma-1}}. \tag{F.1}$$

Since only $+$ of the $\pm$ sign is reasonable because of $4Az_2z^{\frac{1}{\sigma-1}} > 0$, we choose the $+$ one in (F.1). Plugging it and z in (20) into V_c for $c = s, n$, we can express V_c as a function of λ only. Then, differentiating it with respect to λ and evaluating it at $\lambda \to 0$, we obtain $\mathrm{d}V_s/\mathrm{d}\gamma > 0$.

We also obtain

$$\operatorname{sgn}\left(\frac{\mathrm{d}V_n}{\mathrm{d}\gamma}\right) = \operatorname{sgn}\left[\phi^2(\sigma-1)\sigma - \left(1-\phi^2-\phi^2\sigma+\phi^2\sigma^2\right)(\mu+\sigma-\mu\sigma)\theta + \left(1-\phi^2\right)(2-\sigma)(\mu+\sigma-\mu\sigma)^2\theta^2\right].$$

The RHS is negative when $\theta \to 1$, $\phi \to 0$, and $\sigma \to 1$, which correspond to large θ, small ϕ, and small σ, respectively.

G Proof of $\lambda^* > \theta$ at $\rho = 1$

When $\rho = 1$ and $\rho_0 = 0$, h_2 in (38) is rewritten as

$$\begin{aligned} h_3(z) \equiv h_2\big|_{\rho=1} \;=\; & \left\{\sigma - \left(1-\phi_1^2\right)\left[\sigma\theta + (\sigma-1)(1-\theta)\mu\right]\right\}\theta z - \phi_1\sigma\theta \\ & - (1-\theta)\left[\left(1-\phi_1^2\right)(\sigma+\mu-\mu\sigma)\theta + \phi_1\sigma(\phi_1 - z)\right] z^{\frac{\sigma-2}{\sigma-1}}. \end{aligned}$$

We show the solution of $h_3(z) = 0$ is unique and exceeds 1, which implies $\lambda^* > \theta$.

(i) When $1 < \sigma \leq 3/2$, we have

$$h_3''(z) \lesseqgtr 0 \qquad \text{for } z \lesseqgtr z_3 \equiv \frac{\phi_1^2\sigma + (1-\phi_1^2)(\sigma+\mu-\mu\sigma)\theta}{\phi_1(3-2\sigma)\sigma} > 1.$$

We also have

$$h_3'(z_3) = \phi_1 h_4(\theta) + \theta(1-\theta)(\sigma+\mu-\mu\sigma) > \phi_1 h_4(\theta),$$

where

$$h_4(\theta) = \phi_1\theta\left[\theta\sigma + \mu(\sigma-1)(1-\theta)\right] - \sigma(1-\theta)(3-2\sigma)\left[\frac{\sigma\phi_1^2 + \theta(1-\phi_1^2)(\sigma+\mu-\mu\sigma)}{\sigma\phi_1(3-2\sigma)}\right]^{\frac{\sigma-2}{\sigma-1}}.$$

Because $h_4'(\theta) > 0$, we get

$$h_4(\theta) \geq h_4(1/2) = \sigma h_5(\mu) + \frac{\phi_1\mu(\sigma-1)}{4} > \sigma h_5(\mu),$$

where

$$h_5(\mu) \equiv \frac{\phi_1}{4} - \frac{3-2\sigma}{2}\left[\frac{\sigma(1+\phi_1^2) - \mu(1-\phi_1^2)(\sigma-1)}{2\sigma\phi_1(3-2\sigma)}\right]^{\frac{\sigma-2}{\sigma-1}}$$

Since $h_5'(\mu) < 0$, we have

$$h_5(\mu) > h_5(1) = \frac{\phi_1}{4}\left[1 - \frac{1}{h_6(\phi_1)}\right],$$

where

$$h_6(\phi_1) \equiv \frac{\phi_1}{2(3-2\sigma)}\left[\frac{1-\phi_1^2+2\sigma\phi_1^2}{2\sigma\phi_1(3-2\sigma)}\right]^{\frac{2-\sigma}{\sigma-1}}.$$

It is verified that $h_6(\phi_1) \lesseqgtr 0$ for $\phi_1 \lesseqgtr \sqrt{\frac{3-2\sigma}{2\sigma-1}}$, so that

$$h_6(\phi_1) \geq h_6\left(\sqrt{\frac{3-2\sigma}{2\sigma-1}}\right) = \frac{1}{2\sqrt{4(2-\sigma)\sigma-3}}\left[\frac{(2-\sigma)\sqrt{2\sigma-1}}{\sigma(3-2\sigma)^{3/2}}\right]^{\frac{2-\sigma}{\sigma-1}}.$$

Replacing σ with $x \equiv \frac{2-\sigma}{\sigma-1} \in (1,\infty)$, h_6 can be rewritten as

$$h_7(x) = \frac{x+1}{2\sqrt{(x-1)(x-3)}}\left[\frac{x(x+1)\sqrt{x+3}}{(x+2)(x-1)^{3/2}}\right]^x.$$

Since $\mathrm{d}^2 \log h_7(x)/\mathrm{d}x^2 > 0$, $\mathrm{d}\log h_7(x)/\mathrm{d}x$ is increasing in x. Because $\lim_{x\to+\infty} \mathrm{d}\log h_7(x)/\mathrm{d}x = 0$, we have $\mathrm{d}\log h_7(x)/\mathrm{d}x < 0$. Since $\lim_{x\to+\infty} \log h_7(x) = 2 - \log 2$, we get $h_7(x) = e^2 - 2 > 1$ for all $x \in (1, +\infty)$. Thus, $h_3'(z) > 0$ holds, which means $h_3(z)$ is increasing in $z \in (0, +\infty)$. Since $h_3(1) < 0$ holds, there exists a unique $z^* \in (1, +\infty)$.

(ii) When $3/2 < \sigma \leq 2$, $h_3''(z) \leq 0$ always is satisfied. Since $\lim_{z\to+\infty} h_3'(z) > 0$, we get $h_3'(z) > 0$, so that $h_3(z)$ is increasing in $z \in (0, +\infty)$. Because $h_3(1) < 0$, there exists a unique $z^* \in (1, +\infty)$.

(iii) When $\sigma > 2$, $h_3''(z) > 0$ always holds. Because $\lim_{z\to 0} h_3'(z) = -\infty$ and $\lim_{z\to+\infty} h_3'(z) = +\infty$, there exists a unique $z_4 \in (0, +\infty)$ such that $h_3'(z) \lesseqgtr 0$ for $z \lesseqgtr z_4$. Since $h_3(1) < 0$ and $\lim_{z\to+\infty} h_3(z) = +\infty$, it follows that there exists a unique $z^* \in (1, +\infty)$.