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Abstract

This paper considers the possibility of technology licensing and tacit collusion between firms that produce homogeneous goods under asymmetric cost structures and compete in quantities. We discuss the possibility of collusion under Grim-Trigger strategies when technology may be licensed via fixed fee or royalty or two-part tariff. Irrespective of the type of licensing contract, the possibility that a stable cartel is formed is the same. In the no-licensing stage, the cartel formation is more likely if the cost difference between the firms is higher. In contrast to Lin (1996), all forms of licensing facilitate (obstruct) collusion, if the initial cost difference between the firms is less (more). Technology will always be licensed in the first stage and the optimal form of licensing is either fixed-fee or royalty or two-part tariff. The cartel will be formed if the firms are relatively patient and welfare either increases or decreases in the post-licensing stage.

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1 Introduction

Technology transfer between firms increases production and its efficiency by dissemination of superior technical know-how. Technology sharing can take various forms. It can be ex-ante (before innovation) as Research Joint-Venture (RJV), or ex-post as licensing. Patent licensing is the main channel of technology transfer in the majority of industries. Licensing allows the firms to use, modify or resell the intellectual property of the licensor and affects the welfare level in two distinct ways. On one hand, it disseminates knowledge among firms and thus makes production efficient. On the other hand, it can have anti-competitive effects. Lin (1996) points out that there has been a major concern among economists and policymakers regarding patent licensing and its role in facilitating anti-competitive effects such as collusion in the market. OECD Licensing of IP Rights and Competition Law (2019)[1] says "Licensing is a fundamental tool for diffusing innovation, for allowing innovators to be rewarded for their efforts, and to promote cooperation and follow-on innovation during IP rights' period of exclusivity. On the other hand, licensing agreements can also have anticompetitive effects, such as facilitating cartelisation or anticompetitive foreclosure. The main challenge for competition enforcers is to determine whether a particular agreement is likely to help or hurt competition."

The literature discusses either collusion or licensing among competing firms.[2] However, except Lin (1996), there is no other work that has studied the possibility of cartel formation and unilateral licensing in a single model. Eswaran (1994) however, studies the effects of cross-licensing on the incentives for collusion. In this paper, we study the possibility of tacit collusion between firms in the presence of unilateral licensing opportunities, where the firms produce a homogeneous good at different unit costs and compete in quantities. We discuss the possibility of tacit collusion (under Grim-Trigger strategies), when the technology may be licensed from the lower cost firm to the higher cost firm either through a fixed fee or royalty or two-part tariff. It is shown that if the technology is licensed in the first stage of the game, then all forms of licensing contract (fixed-fee, royalty, and two-part tariff) equally facilitate collusion. This is because after licensing not only does the actual unit cost of production of the firms become the same, but also for both the firms, the proportion of gain and loss from unilateral deviation from the cartel agreement is equal across fixed-fee, royalty and two-part tariff licensing (even though the effective cost of production differs under royalty and two-part tariff licensing as firm 2 pays a per-unit royalty to firm 1). However, if the technology is not licensed, then the possibility of cartel formation depends on the initial cost difference between the firms. In such a situation the possibility of the cartel first decreases if the initial cost difference increases, reaches the minimum and then

[1] See Licensing of IP Rights and Competition Law, OECD, 2019
https://one.oecd.org/document/DAF/COMP(2019)3/en/pdf

[2] See Donsimoni (1985), Rothschild (1999), Miklos-Thal (2011), etc. for collusion or for licensing see Shapiro (1985), Marjit (1990), Wang (1998), Fauli-Oller and Sandonis (2002), etc. Fauli-Oller and Sandonis (2003) has studied the optimal competition policy when licensing is an alternative to a merger to transfer a superior technology.

the possibility of cartel formation increases if the cost difference increases. Therefore, if the cost difference between the firms is low (high), then the possibility of cartel formation is more (less) under licensing than under no-licensing. Hence, contrary to Lin (1996) and Miyagiwa (2009), we show that technology licensing facilitates (obstructs) collusion if the initial cost difference between the firms is low (high).

Moreover, it is shown that in the first stage the technology will always be licensed either by fixed-fee or royalty or two-part tariff. Wang (1998) shows that royalty licensing is optimal for the licensor in a static framework. However, as shown in the present paper, given the firms can form a cartel, the optimal form of licensing is either fixed-fee or royalty or two-part tariff. If the cartel is not formed after licensing then royalty licensing is superior, since the licensor can control the effective unit cost of the licensee (as observed in Wang (1998)) and thereby charge the maximum royalty rate. On the other hand, if the cartel is formed after the technology is licensed, then either the licensor is indifferent between fixed-fee and royalty licensing or the optimal form of licensing is the two-part tariff. This is because if the cartel is formed in the post-licensing stage, then the licensor's profit not only depends on the licensing agreement but also on the cartel outcome that reduces competition in the output market.

It is shown that after licensing of technology, the firms will form a cartel if they are relatively patient (give more weight on their future profits). The firm's decisions of licensing and collusion also have a considerable impact on social welfare. Welfare always increases in the post-licensing stage if the cartel is not formed. However, welfare may either increase or decrease in the post-licensing stage if the cartel is formed.[3] It is observed that if the cost difference is low and the weights assigned to the future profits are moderate, then a cartel is formed only if the technology is licensed. In such cases, as effectively licensing facilitates the formation of a cartel (which is impossible under no-licensing), the welfare under no-licensing is more than under licensing. However, if the cost difference is high and the weights assigned to the future profits are moderate, then a cartel is formed only if the technology is not licensed. Hence, licensing can deter collusion and has a positive impact on welfare, as under licensing welfare is more than under no-licensing.

There are very few works in the literature that study the effects of technology licensing on collusion. Lin (1996) considers a Bertrand duopoly framework without R&D in which one of the firms has a cost advantage over its rival. After fixed-fee licensing of technology as licensing equalizes costs, the gains that the firms can expect if they deviate from collusion are lower and thus helps to sustain collusion. This happens as in the stage of unilateral deviation by undercutting price, the respective firm can get monopoly profit in that period and zero profit in the subsequent periods. Eswaran (1994) shows that cross-licensing of technology enhances the degree of collusion when the firms produce imperfect substitutes by credibly introducing the threat of increased rivalry in the market for each firm's product. On the other hand, Miyagiwa (2009) examines whether cooperation in R&D among firms producing similar products leads to product-market collusion, where the firms are engaged in a stochastic R&D race and maintain the collusive equilibrium in a repeated-game framework.[4] In every period, each firm decides whether to invest

[3] Baumol (1992) suggests that horizontal collaboration can be beneficial in contributing to social welfare by speeding productivity and growth and/or reducing the cost of growth.

[4] Miyagiwa (2009) considers repeated interaction between two a priori symmetric firms over an infinite time horizon that compete in prices and produce homogeneous good.

in R&D for the discovery of new technology which reduces the unit cost of production. It is shown that innovation under non-cooperative R&D leads to an inter-firm asymmetry, destabilizing collusion in pre-discovery and post-discovery periods. On the other hand innovation sharing under cooperative R&D preserves the symmetry and thereby facilitates collusion. Levy (2012) considers the possibility of product market collusion that arises between firms in the Bertrand duopoly model that maintain long term technology-sharing relationships, which reduces their marginal costs. The likelihood of collusion is compared when firms join together in an RJV, with the alternate situation when firms conduct independent research and license innovations to each other using fixed license fees. As shown in Levy (2012), only when innovations are quite substantial such that fixed-fee licensing would not be incentive compatible for the innovating firm, an RJV yields better collusive outcomes for the firms.

Rothschild (1999) analyses the cartel maintenance using standard grim trigger strategies under firm asymmetries and finds them to be successful in the absence of side payments. Donsimoni (1985) points out the problems of forming a cartel, especially within asymmetric costs. Fischer and Normann (2019) and Miklos-Thal (2011) find side payments as an important factor in increasing incentives for cooperation among firms. Miklos-Thal (2011) shows that: (i) without side payments, some collusion is sustainable under cost asymmetry whenever it is sustainable under cost symmetry and (ii) with side payments, cost asymmetries facilitate collusion. However, in our paper in contrast to Miklos-Thal (2011), in the absence of licensing we show that both without and with side payments, cost asymmetries facilitate collusion when the cost difference is high, while it obstructs collusion if the cost difference is low.

It is believed, among anti-trust agencies in particular, that cost-differences among competitors is a big obstacle to collusion. In this paper, technology licensing makes the costs symmetric between the two firms under fixed-fee licensing and does not change or reduce the cost structure under royalty or two-part licensing where the optimal royal rate equals the initial cost differential. Yet, licensing will obstruct collusion if the initial cost differential is high. This can be explained by the behaviour of the higher-cost firm to whom the technology is licensed, as this determines the stability of the cartel. First, under licensing, the higher-cost firm's profit is greater than its profit under no-licensing if under both these cases cartel is formed. This is because the higher-cost firm has an extra share of the gain that accrues to the benefits of licensing from the bargaining process at the time of cartel agreement, which is absent in the case of no-licensing. Hence, this gain is higher when the cost difference is high. Therefore, the gain from deviation from the cartel agreement is much lower than under the cartel agreement in the absence of licensing, when the cost difference is high for firm 2. As a result, licensing will obstruct collusion if the initial cost differential is high.

The paper is organized as follows: Section 2 sets out the basic set-up of the model, Section 3 discusses the possibilities of cartel formation under no-licensing. Section 4 deals with possibilities of cartel formation under fixed-fee, royalty and two-part tariff licensing respectively. In section 5, we discuss the basic results and section 6 solves the game through backward induction and comments on the licensing decisions. Finally, section 7 presents the concluding remarks.

2 Basic set-up

Consider a duopoly structure (Firm 1 and Firm 2) where both firms produce a homogeneous product. The firms are engaged in a repeated (static) quantity competition game over an infinite time horizon.[5] In contrast to Lin (1996), we assume that the firms compete in quantities (rather than in prices). The choice of Cournot instead of Bertrand may be realistically dictated by binding capacities (as in Kreps and Scheinkman (1983)). The cost function of firm i is given by $C(q_i) = c_i q_i$. Firm 1 is the technologically advanced firm (patent holder) and hence has a lower constant marginal cost of production ($c_1 < c_2$). The inverse demand function is $P = a - q$, where P is the price, $a > c_i$, $q = q_1 + q_2$ is the total output supplied in the market and $q_i,\ i = 1, 2$, is the output produced by the firm i. Without any loss of generality, we assume $c_1 = 0 < c_2$. Moreover, the innovation is non-drastic, such that after innovation firm 2 is not forced out of the market. This implies $p_1^m > c_2$ or $\frac{a}{2} > c_2$, i.e the price charged by firm 1 if it acts as a monopolist is greater than the marginal cost of firm 2. The structure of the market is assumed to remain unchanged over time. The information is complete and is known to all the firms. The periods are discounted by the factor $\delta \in (0, 1]$ [6] and both the firms are equally patient.

The sequence of the licensing game is as follows. This game is played at the beginning of period 1. In the first stage, firm 1 offers the licensing contract to firm 2 quoting a fixed-fee or royalty (or both if required, i.e. two-part tariff). In the second stage, firm 2 either accepts or rejects the licensing contract made by firm 1. The licensing stage gets over here. In the third stage, contingent on whether technology is licensed or not in the second stage, firms either tacitly collude and jointly produce the cartel output or do not collude and produce non-cooperative Nash equilibrium outputs in subsequent periods. We assume that licensing takes place (if at all) only at the beginning of period 1. This is because if licensing fails in period 1, then it will also fail in subsequent periods as the market structure and technology of the firms continues to remain unchanged. The game is solved using backward induction. We also assume that the fixed-fee is paid immediately after the licensing agreement is signed, while the royalty payment which depends on the level of the output produced using the licensed technology, is paid in every period (when the firms compete in the output market).

First, we discuss the possibilities of cartel under no-licensing. Then we discuss the possibilities of cartel under fixed-fee licensing and royalty licensing (as well as two-part tariff licensing) respectively. Finally, we analyze the decisions taken by the firms in regards to licensing and its nature (fixed-fee or royalty or both) in stage 1 and stage 2.

3 Cartel under No Licensing

We start with stage 3 of the game. Assume that the licensing fee or the royalty rate demanded by firm 1 is more than what firm 2 is ready to pay (This is similar to firm 1 not making any licensing offer). Thus, there is no licensing of technology and $c_2 > c_1 = 0$. Under no-licensing,

[5] As mentioned in Belleflame and Peitz (2015) "This is not to say that firms compete until the end of time; it just means that there is no known end date to the game, at each period there is a probability that firms will compete one more time."

[6] We assume $\delta \neq 0$ as it implies firms give weight to future profits and hence compete infinitely.

there are two possibilities, i.e. cartel may or may not be formed. If the cartel is not formed, then the unique static Cournot-Nash quantities are:

$$q_1^n = \frac{a+c_2}{3} \qquad and \qquad q_2^n = \frac{a-2c_2}{3}.$$

Here, q_i^n represents the static Nash equilibrium quantities, where $i = 1, 2$ and the Cournot-Nash profits for the firms are:

$$\pi_1^n = \frac{1}{9}(a+c_2)^2 \quad and \quad \pi_2^n = \frac{1}{9}(a-2c_2)^2. \tag{1}$$

The present discounted Nash-Cournot profits (such that cartel is not formed) of firm 1 and firm 2 are given by:

$$\pi_1^{NN} = \frac{1}{1-\delta}\frac{(a+c_2)^2}{9} \qquad and \qquad \pi_2^{NN} = \frac{1}{1-\delta}\frac{(a-2c_2)^2}{9} \tag{2}$$

respectively.[7]

Let us now analyze whether the implicit cartel between the firms will be formed or not if the technology is not licensed in the beginning. The static collusion profits are calculated by maximizing the total industry profits $\pi_1 + \pi_2$ w.r.t. to the cartel output q_c, where

$$\pi_1 + \pi_2 = (a - q_c)s_1 q_c + (a - q_c - c_2)s_2 q_c = aq_c - q_c^2 - c_2(1 - s_1)q_c,$$

and s_i is the share of firm i in q_c and $s_1 + s_2 = 1$. Harrington (2006) points out that "The majority of the cartels used sales quotas. This could take the form of a quantity that each firm was assigned to supply – which may be interpreted as a target quantity or a minimum quantity – or of an allocated share of the market." [8] We assume that $s_i > 0$, as otherwise, it becomes an explicit cartel. If $s_2 = 0$, then firm 1 will only produce the monopoly output in the market and firm 2 will not produce any output. A part of the profit of firm 1 now can be given to firm 2 as side payments, but this then becomes an explicit cartel that is generally prohibited under antitrust laws. In the *Appendix B* of the paper, we discuss the case when $s_2 = 0$ and introduce side payments. Before that we assume $s_2 > 0$. Specifically, (as in Bos and Marini (2019)) in the following paragraph, we consider the maximization of total cartel profits without side payments.

The optimal (total) cartel output is $q_c = \frac{1}{2}(a - c_2(1 - s_1))$, while the output of firm 1 and firm 2 are $q_1^c = \frac{s_1}{2}(a - c_2(1 - s_1))$ and $q_2^c = \frac{(1-s_1)}{2}(a - c_2(1 - s_1))$ respectively.[9] Hence, the profit of each firm under tacit collusion is given by:

$$\pi_1^c = \frac{s_1}{4}\left[a^2 - (1-s_1)^2 c_2^2\right] \quad and \quad \pi_2^c = \frac{(1-s_1)}{4}\left[a^2 - 2ac_2 + (1-s_1^2)c_2^2\right]. \tag{3}$$

[7] NN here denotes that no-licensing is not followed by a cartel.

[8] Harrington (2006) mentions that in 1991 each firm was entitled to a percentage of supply provided by the European Citric Acid Manufacturers Association to which all cartel members belonged.

[9] These quantities are Pareto-efficient. It satisfies the Pareto frontier as pointed by Fisher and Normann (2019). Fischer and Normann (2019) apply Kalai-Smorodinski, equal relative gains, equal split, etc. sharing structures in the analysis of tacit collusion. Each structure results in different joint pay-offs and individual gains that accrue to firms.

Moreover, we assume that s_i is determined through Nash Bargaining.[10] As in Roberts (1985), two firms face a standard co-operative bargaining problem which would be expected to entail a resolution that embodied efficiency and symmetry, e.g. the Nash bargaining solution which involves the maximization of the product of the gains of the agents as compared to some status quo position. In our scenario, through bargaining, firms can earn π_i^c and their fallout option is π_i^n per period if licensing does not happen. Thus optimal sharing agreement is determined by

$$\max_{s_1} V = (\pi_1^c - \pi_1^n)(\pi_2^c - \pi_2^n)$$
$$= \left(\frac{s_1}{4}[a^2 - (1-s_1)^2 c_2^2] - \frac{1}{9}(a+c_2)^2\right)\left(\frac{(1-s_1)}{4}\left[a^2 - 2ac_2 + (1-s_1^2)c_2^2\right] - \frac{1}{9}(a-2c_2)^2\right), \tag{4}$$

such that $\pi_i^c > \pi_i^n$ for $i = 1,2$. The optimal s_1 (we call it s_1^*) exists (See *Appendix A.1* for details) and the F.O.C. and S.O.C. of the maximization problem as stated in equation (4) at s_1^* are respectively[11]

$$(\pi_1^c - \pi_1^n)\frac{\partial \pi_2^c}{\partial s_1} + (\pi_2^c - \pi_2^n)\frac{\partial \pi_1^c}{\partial s_1} = 0 \quad and \tag{5}$$

$$(\pi_1^c - \pi_1^n)\frac{\partial^2 \pi_2^c}{\partial s_1^2} + (\pi_2^c - \pi_2^n)\frac{\partial^2 \pi_1^c}{\partial s_1^2} + 2\frac{\partial \pi_2^c}{\partial s_1}\frac{\partial \pi_1^c}{\partial s_1} < 0. \tag{6}$$

Using the F.O.C. and S.O.C. of the maximization problem as stated above we come to the following lemma.

Lemma 1 $\frac{\partial s_1^*}{\partial c_2} > 0$ *and* $s_1^* \in (0.5, 1)$ *for* $c_2 \in (0, \frac{a}{2})$.

Proof. See *Appendix A.2.* ■

Firstly, if $c_2 = 0$ (firms are identical), then after solving equation (4) we get $s_1^* = 0.5$ and if $c_2 = \frac{a}{2}$ (firm 1 is the monopolist as $q_2^n = 0$), then $s_1^* = 1$. We observe from Lemma 1 that $s_1^* > 0.5$ and s_1^* increases with c_2 for $c_2 \in (0, \frac{a}{2})$. Therefore, the output share under the cartel agreement is more for firm 1 than for firm 2 ($s_1^* > s_2^*$) as firm 1 is the efficient firm in the no-licensing stage.[12]

The incentives of both firms should be compatible to sustain collusive outcomes for an infinite period. For this, firms follow punishment strategies and we assume firms to follow *Grim-Trigger*

[10] It involves maximizing the Nash Product with respect to the shares of firms.

[11] $\frac{\partial \pi_1^n}{\partial s_1} = \frac{\partial \pi_2^n}{\partial s_1} = 0$.

[12] In our paper the cartel output, as well as the output shares of the firms (s_i), are determined simultaneously, such that the firms attain an equitable division of the gains from collusion (like Schmalensee (1987)), as well as it maximizes the industry profit given $s_i > 0$ (in the absence of side-payments, such that both firms produce positive outputs under the cartel agreement). As in Schmalensee (1987) in our paper also the solution concepts discussed produce collusive outcomes at which all parties are at least as well off as at the status quo (Nash-bargaining solution) and on top of that it also maximizes the industry profit given $s_i > 0$. It is also important to note that in terms of our methodology under royalty licensing (discussed later) the firms can produce the monopoly output (that gives the maximum industry profit), but if we $\max_{q_1,q_2}(\pi_1^c - \pi_1^n)(\pi_2^c - \pi_2^n)$ then the firms are not able to produce the monopoly output. Under no-licensing in our model as in Schmalensee (1987) when side payments are not possible, total industry profit is reduced to attain an equitable division of the gains from collusion. In our model for that reason in the absence of side-payments, the cartel output is q_c under no-licensing and is it less than the monopoly output ($\frac{a}{2}$). Producing the monopoly output and thereby getting the maximum profit is possible in the presence of side-payments as discussed in *Appendix B.*

Strategy (as in Lin (1996), Symeonidis (2018), Bos and Marini (2019), etc.) and tacitly cooperate. Under this strategy, both firm 1 and firm 2 agree to produce $q_1^c(s_1^*) = \frac{s_1^*}{2}(a - c_2(1 - s_1^*))$ and $q_2^c(s_1^*) = \frac{1-s_1^*}{2}(a - c_2(1 - s_1^*))$ that maximizes total profits (i.e., sum of firm 1 and firm 2 profits) and keep choosing this strategy till the other firm has done the same in all the previous periods. This is the *cooperation phase*. However, if one firm deviates i.e chooses some other action, this triggers the *punishment phase* wherein, from the subsequent periods, both firms produce outputs that correspond to the Nash equilibrium outputs of the static game. This implies that the firms will cooperate so long as the present value of their total profits from deviating from the strategy is lower than the present value of the profits from collusion, or

$$\pi_i^d + \frac{\delta}{1-\delta}\pi_i^n < \frac{1}{1-\delta}\pi_i^c, \quad i = 1, 2 \tag{7}$$

where π_i^d is the deviation profit from the cartel agreement for a particular period, π_i^c is the collusion profit per period and π_i^n is the static Cournot-Nash profit for firm i.

The deviation profit π_1^d for firm 1 is the maximum profit in any period, such that the firm 2 sticks to the cartel output, but firm 1 produces its best response output. Best response function of firm 1 is given by $q_1^{BR} = \frac{1}{2}(a - q_2)$. Substituting $q_2^c(s_1^*) = \frac{1-s_1^*}{2}(a - c_2(1 - s_1^*))$, gives the following deviation quantity: $q_1^d = \frac{1}{4}\Big[(1 + s_1^*)a + (1 - s_1^*)^2 c_2\Big]$. Hence, the deviation profit of firm 1 is $\pi_1^d = \frac{1}{16}\big[(1 + s_1^*)a + (1 - s_1^*)^2 c_2\big]^2 = (q_1^d)^2$. Moreover, $\pi_1^d > \pi_1^c$ as $(1 - s_1^*)^2 a^2 + 2ac_2(1 + s_1^*)(1 - s_1^*)^2 + c_2^2(1 - s_1^*)^2(1 + s_1^*)^2 > 0$. Similarly for firm 2, the deviation profit $\pi_2^d = \frac{1}{16}\big[(2 - s_1^*)a + s_1^*(1 - s_1^*)c_2 - 2c_2\big]^2$ is greater than π_2^c.[13] Therefore $\pi_i^d > \pi_i^c > \pi_i^n$ for firm i, $i = 1, 2$. Moreover, using Lemma (1) we observe that

Lemma 2 $\frac{d\pi_1^c}{dc_2} > 0, \frac{d\pi_1^d}{dc_2} > 0, \frac{d\pi_2^c}{dc_2} < 0$ *and* $\frac{d\pi_2^d}{dc_2} < 0$.

Proof. See *Appendix A.3.* ■ [14]

Condition (7) holds if,

$$\delta_i > \frac{\pi_i^d - \pi_i^c}{\pi_i^d - \pi_i^n} \equiv \delta_i^{min}. \tag{8}$$

Given s_1^* (calculated using equation (5)),

$$\begin{aligned} \delta_1^{min} &\equiv \frac{\frac{1}{16}\Big[(1 + s_1^*)a + (1 - s_1^*)^2 c_2\Big]^2 - \frac{s_1^*}{4}(a^2 - (1 - s_1^*)^2 c_2^2)}{\frac{1}{16}\Big[(1 + s_1^*)a + (1 - s_1^*)^2 c_2\Big]^2 - \frac{1}{9}(a + c_2)^2} \quad \textit{and} \\ \delta_2^{min} &\equiv \frac{\frac{1}{16}\Big[(2 - s_1^*)a + s_1^*(1 - s_1^*)c_2 - 2c_2\Big]^2 - \frac{(1-s_1^*)}{4}\Big[a^2 - 2ac_2 + (1 - s_1^{*2})c_2^2\Big]}{\frac{1}{16}\Big[(2 - s_1^*)a + s_1^*(1 - s_1^*)c_2 - 2c_2\Big]^2 - \frac{1}{9}(a - 2c_2)^2}. \end{aligned} \tag{9}$$

[13] π_2^d is calculated similar to π_1^d calculated above.

[14] We also observe that *at* $c_2 = 0, s_1^* = 0.5, \pi_1^d > \pi_1^c > \pi_1^n$; *at* $c_2 = \frac{a}{2}, s_1^* = 1, \pi_1^d = \pi_1^c = \pi_1^n$; *and for* $c_2 \in (0, \frac{a}{2}), s_1^* \in (0.5, 1), \pi_1^d > \pi_1^c > \pi_1^n$. Therefore, using Lemma (1) and (2), it can be shown that for $c_2 \in (0, \frac{a}{2})$ $0 < \frac{d\pi_1^d}{dc_2} < \frac{d\pi_1^c}{dc_2}$.

Hence, the cartel will be formed if condition (8) holds for both the firms. This implies that the firms must put sufficient weight on future losses to offset the temptation of securing an immediate gain by deviating. As we are assuming that the degree of the patience of both the firms is the same, hence δ must be greater than δ_1^{min} as well as δ_2^{min}, such that the cartel is stable. Moreover, we observe that δ_1^{min} decreases in c_2, whereas δ_2^{min} initially increases in c_2, reaches the maximum and then falls with increase in c_2 from the following lemma. It is also shown that $\delta_1^{min} < \delta_2^{min}$ when $c_2 \in (0, \frac{a}{2})$.

Lemma 3
i) $\frac{d\delta_1^{min}}{dc_2} < 0$ *for* $c_2 \in (0, \frac{a}{2})$.
ii) δ_2^{min} *initially increases in* c_2*, reaches the maximum and then falls with increase in* c_2 *for* $c_2 \in (0, \frac{a}{2})$.
iii) $\delta_1^{min} < \delta_2^{min}$ *when* $c_2 \in (0, \frac{a}{2})$.

Proof. See *Appendix A.4* [15], *Appendix A.5* and *Appendix A.6* respectively. ■

From the above lemma it can be argued that the minimum weight to be put by firm 1 on the future profits, such that there is no incentive to deviate (i.e. δ_1^{min}), decreases in the cost of the other firm (c_2). This is because the deviation pays less and punishment hurts more if the cost of the rival firm increases. When the cost of firm 2 increases, the output share of firm 1 increases and thereby gets a higher profit under the cartel agreement. Hence, the increase in the cost of firm 2 reduces the incentives to deviate for firm 1. More is the cost of firm 2, less is the temptation to deviate for firm 1 as the gain from deviation is always being offset by the loss from being punished. On the other hand, we observe that δ_2^{min} initially increases and then falls in c_2. The minimum weight to be put by firm 2 on the future profits such that there is no incentive to deviate (i.e. δ_2^{min}), initially increases in c_2. For firm 2 initially, the deviation pays more and punishment hurts less if its cost increases. This happens when c_2 is marginally greater than $c_1 (= 0)$, as the gain from deviation is much higher given the firms are almost the same in terms of the cost, but firm 2's output share is lower than that of firm 1. However, for a higher cost, δ_2^{min} decreases in c_2, as deviation starts paying less and punishment hurts more as the loss of future profits is always being offset by the immediate gains from deviation. However, at $c_2 = 0$ (firms are identical such that $s_1^* = 0.5$), from equation (9) we get that $\delta_1^{min} = \delta_2^{min} = \frac{9}{17}$.

It is also shown that $\delta_1^{min} < \delta_2^{min}$ when $c_2 \in (0, \frac{a}{2})$. This means that the minimum weight on future losses to offset temptation is less for firm 1 than for firm 2. This happens as for firm 1 the gain from deviation is always lower than the future losses in comparison to firm 2. This can be explained in terms of the share of output under the cartel agreement. First, firm 1 has a higher output share under the cartel agreement than firm 2, as the unit cost of the second firm is higher than the first firm. This gives a higher profit to firm 1 than firm 2 under the cartel agreement. Hence, firm 2 has a higher temptation to deviate for grabbing a higher profit than firm 1 under the cartel agreement. Hence, $\delta_1^{min} < \delta_2^{min}$.

Therefore, using Lemma 3, it can be argued that if $\delta > \delta_2^{min}$, then the cartel is stable.

[15] It can also be checked after substituting the values that at $c_2 = 0$, $\frac{d\delta_1^{min}}{dc_2} = \frac{1}{(\pi_1^d - \pi_1^c)^2}\Big[(\pi_1^d - \pi_1^n)(\frac{d\pi_1^d}{dc_2} - \frac{d\pi_1^c}{dc_2}) - (\pi_1^d - \pi_1^c)(\frac{d\pi_1^d}{dc_2} - \frac{d\pi_1^n}{dc_2})\Big] < 0$.

Lemma 4 *If and only if $\delta > \delta_2^{min}$, then the firms will form the cartel and firm 1 and firm 2 will produce $q_1^c(s_1^*)$ and $q_2^c(s_1^*)$ respectively in the absence of technology licensing.*

This implies that the present discounted profits of firm 1 and firm 2, if the cartel is formed under no-licensing, are[16]

$$\pi_1^{NC} = \frac{1}{1-\delta}\frac{s_1^*}{4}(a^2 - (1-s_1^*)^2 c_2^2) \quad \textit{and} \quad \pi_2^{NC} = \frac{1}{1-\delta}\frac{(1-s_1^*)}{4}(a^2 - 2ac_2 + (1-s_1^{*2})c_2^2) \tag{10}$$

respectively.

4 Cartel under Licensing

4.1 Cartel under Fixed Fee Licensing

Consider now the scenario when the technology is licensed from firm 1 to firm 2 via a fixed-fee (F) and post-licensing both firms have the same technology i.e $c_1 = c_2 = 0$. Static Cournot Nash equilibrium outputs are given by $q_i^n = \frac{a}{3}$ and the profits are $\pi_i^n = \frac{a^2}{9}$. The present discounted Nash-Cournot profits of firm 1 and firm 2, if after licensing cartel is not formed, are given by:

$$\pi_1^{FN} = \frac{1}{1-\delta}\frac{a^2}{9} + F \quad \textit{and} \quad \pi_2^{FN} = \frac{1}{1-\delta}\frac{a^2}{9} - F \tag{11}$$

respectively.[17] Both firms follow the *Grim-Trigger Strategy.* As the marginal cost of both firms are the same in the post-licensing stage, they share the collusion profits and outputs equally ($s_1 = s_2 = \frac{1}{2}$, where s_i is the market share of firm i under the cartel agreement)[18] and produce monopoly output. Static monopoly output and profit per period are given by $q^m = \frac{a}{2}$ and $\pi^m = \frac{a^2}{4}$ respectively. Both the firms therefore produce $q_i^c = \frac{a}{4}$ and the per firm profit is $\pi_i^c = \frac{a^2}{8}$ in each period under the cartel agreement.

For the stability of the cartel, condition (8) must be satisfied as before. If firm 2 produces the cartel output (given by $q_2^c = \frac{a}{4}$), the deviation output of firm 1 is $q_1^d = \frac{1}{2}(a - \frac{a}{4}) = \frac{3a}{8}$. Hence, the deviation profit is given by $\pi_1^d = (q_1^d)^2 = \frac{9a^2}{64}$, which is also equal to the deviation profit of firm 2, π_2^d (as both the firms are identical). Here, we also observe that $\pi_i^d > \pi_i^c > \pi_i^n$ for $i = 1, 2$, and after substituting these values in condition (8) we get

$$\delta_{iF}^{min} \equiv \frac{\pi_i^d - \pi_i^c}{\pi_i^d - \pi_i^n} = \frac{9a^2/64 - a^2/8}{9a^2/64 - a^2/9} = \frac{9}{17} = 0.52941 \quad (\textit{approx}). \tag{12}$$

Therefore, if the technology is licensed through a fixed fee, then the cartel will be formed in the next stage if and only if $\delta \in (\frac{9}{17}, 1]$. This implies that the present discounted profits of firm 1 and firm 2 if the cartel is formed after fixed-fee licensing are[19]

$$\pi_1^{FC} = \frac{1}{1-\delta}\frac{a^2}{8} + F \quad \textit{and} \quad \pi_2^{FC} = \frac{1}{1-\delta}\frac{a^2}{8} - F \tag{13}$$

respectively.

[16] NC here denotes that no-licensing is followed by a cartel.
[17] FN here denotes that fixed-fee licensing is not followed by a cartel.
[18] This result can also be derived by using Nash Bargaining which is discussed in detail in section 3.
[19] FC here denotes that fixed-fee licensing is followed by the cartel.

4.2 Cartel under Royalty Licensing

Under royalty licensing, the licensee (firm 2) pays a per-unit royalty r for using the licensed technology of firm 1 in each period. Therefore, under royalty licensing, the effective marginal cost of firm 2 is r and of firm 1 is $c_1 = 0$. The maximum royalty rate that firm 1 can charge cannot exceed c_2 i.e. $r < c_2$. This is because the anti-trust authorities will be able to detect collusive behaviour as assumed in the literature. (See Shapiro (1985) etc.)

Similar to fixed-fee licensing, firms have the opportunity to tacitly collude following the Grim-Trigger strategy. Let us find the condition on δ such that the cartel is formed (sustained) given that the technology has been licensed via royalty in the first stage of the game. If firms do not form a cartel, then the quantities produced by the firms are $q_1^n = \frac{a+r}{3}$ and $q_2^n = \frac{a-2r}{3}$, while the profit of the firms are $\pi_1^n = (\frac{a+r}{3})^2 + r(\frac{a-2r}{3})$ and $\pi_2^n = \frac{a-2r}{3}$. The discounted Cournot-Nash profits of firm 1 and firm 2, if the cartel is not formed are respectively [20] :

$$\pi_1^{RN} = \frac{1}{1-\delta}\Big[\Big(\frac{a+r}{3}\Big)^2 + r\Big(\frac{a-2r}{3}\Big)\Big] \;\; and \;\; \pi_2^{RN} = \frac{1}{1-\delta}\Big[\frac{a-2r}{3}\Big]^2. \tag{14}$$

Now, if the firms enter into a cartel agreement, they share the industry output q_c. Let the sharing agreement be given by s_1, the share of firm 1, and $s_2 = 1 - s_1$, the share of firm 2, where $s_1 \in (0, 1)$. The industry output q_c must be maximizing the total industry profits (as discussed in section 3). Solving

$$\max_{q_c} \;\; \pi_1 + \pi_2 = (a - q_c)s_1 q_c + r(1 - s_1)q_c + (a - q_c - r)(1 - s_1)q_c,$$

we get the optimal cartel output as $q_c = \frac{a}{2}$, which is also the monopoly output, similar to what has been observed under fixed-fee licensing.[21] Hence, the outputs produced by firm 1 and firm 2 per period under the cartel agreement are $q_1^c = s_1\frac{a}{2}$ and $q_2^c = (1 - s_1)\frac{a}{2}$ respectively. This gives the per period profit of firm 1 and firm 2 respectively under cartel as $\pi_1^c = \frac{s_1 a^2}{4} + \frac{a(1-s_1)r}{2}$ and $\pi_2^c = \frac{a(1-s_1)}{2}(\frac{a}{2} - r)$. Moreover, as in case of no-licensing (see Section 3), assume that s_i is determined through Nash Bargaining. Therefore, the optimal value of s_1 (call s_1^*) maximizes the Nash-product V:

$$\begin{aligned} &\max_{s_1} \;\; V = (\pi_1^c - \pi_1^n)(\pi_2^c - \pi_2^n) \\ = &\;\Big[\frac{s_1 a^2}{4} + \frac{a(1-s_1)r}{2} - \frac{(a+r)^2}{9} - r\frac{a-2r}{3}\Big]\Big[\frac{a(1-s_1)}{2}(\frac{a}{2} - r) - \frac{(a-2r)^2}{9}\Big]. \end{aligned} \tag{15}$$

After solving the above problem we get $s_1^* = \frac{a+2r}{2a}$.[22] If firm 1 deviates from the cartel agreement in a particular period given firm 2 produces the cartel output, it will produce $q_1^d = \frac{a(1+s_1^*)}{4}$ and get $\pi_1^d = \frac{(a(1+s_1^*))^2}{16} + r(1 - s_1^*)\frac{a}{2}$ as profit in that particular period. Similarly for firm 2, we get $q_2^d = \frac{(2-s_1^*)a-2r}{4}$ and $\pi_2^d = \frac{((2-s_1^*)a-2r)^2}{16}$. Hence, $\pi_i^d > \pi_i^c$ for both firms. Moreover, for firm 1

[20] RN here denotes that royalty licensing is not followed by a cartel.

[21] As under the cartel agreement, the firms are jointly producing the monopoly output (output if firm 1 acts as a monopolist), we desist from introducing side payments in this context.

[22] From the first order condition, we get $\frac{\partial v}{\partial s_1} = \Big[\frac{a^2}{4} - \frac{ar}{2}\Big]\Big[\frac{a(1-s_1)}{2}(\frac{a}{2} - r) - \frac{(a-2r)^2}{9}\Big] + \Big[-\frac{a}{2}(\frac{a}{2} - r)\Big]\Big[\frac{s_1 a^2}{4} + \frac{a(1-s_1)r}{2} - \frac{(a+r)^2}{9} - r\frac{a-2r}{3}\Big] = 0 \implies 9a^2 - 36r^2 = 18a^2 s_1 - 36ars_1$.

$\pi_1^c > \pi_1^n$ as $\frac{s_1^* a^2}{4} + \frac{a(1-s_1^*)r}{2} > \frac{(a+r)^2}{9} + r\frac{a-2r}{3}$. Similarly, for firm 2, $\pi_2^c > \pi_2^n$ as $\frac{a(1-s_1^*)}{2}(\frac{a}{2} - r) > \frac{(a-2r)^2}{9}$.

Substituting the value of s_1^* in the deviation, Cournot-Nash and collusion profit functions of firm 1 we get:

$$\pi_1^d = \frac{1}{64}\Big[9a^2 + 28ar - 28r^2\Big] \ , \quad \pi_1^c = \frac{1}{8}\Big[a^2 + 4ar - 4r^2\Big] \ and \ \pi_1^n = \frac{1}{9}\Big[a^2 + 5ar - 5r^2\Big].$$

Therefore,

$$\delta_{1R}^{min} = \frac{\pi_1^d - \pi_1^c}{\pi_1^d - \pi_1^n} = \frac{\frac{1}{64}(a-2r)^2}{\frac{17}{576}(a-2r)^2} = \frac{9}{17}. \tag{16}$$

Substituting $1 - s_1^* = \frac{a-2r}{2a}$ in the profit functions of firm 2, we get

$$\pi_2^d = \frac{1}{64}(3a - 6r)^2 \ , \quad \pi_2^c = \frac{1}{8}(a-2r)^2 \ , \quad \pi_2^n = \frac{1}{9}(a-2r)^2$$

and

$$\delta_{2R}^{min} = \frac{\pi_2^d - \pi_2^c}{\pi_2^d - \pi_2^n} = \frac{\frac{1}{64}(a-2r)^2}{\frac{17}{576}(a-2r)^2} = \frac{9}{17}. \tag{17}$$

It is important to note that as in fixed-fee licensing, the critical discount factors are the same for both the firms under royalty licensing ($\delta_{1R}^{min} = \delta_{2R}^{min}$), even though the effective unit costs of production are not the same for both the firms. Under royalty licensing the actual unit cost of production is zero for both the firms, however, the effective unit cost of production of firm 2 is r (the royalty rate). Hence, the firms under the cartel agreement can produce the monopoly output and can get the maximum profit (as in fixed-fee licensing) irrespective of the royalty rate (as the royalty payment is a transfer of revenue from firm 2 to firm 1). Therefore, the critical discount factors and thereby the temptation to deviate for both the firms are the same. To explain further, in our model, the profits of the firms, if they compete (π_i^n) or deviate (π_i^d) as well as the profits of the firms under the cartel agreement (π_i^c) are not the same as the effective unit cost of production are different. However, the immediate gain from deviation $\pi_i^d - \pi_i^c$ as well as the discounted loss from deviation ($\frac{\delta}{1-\delta}(\pi_i^c - \pi_i^n)$) are the same for both the firms. Therefore, the minimum weights to be assigned on the future profits to offset the temptation to deviate (the critical discount factors) are the same for both the firms in this context. Interestingly, for the same reason, the sustainability of collusion is independent of the licensing conditions, i.e. same under fixed-fee, royalty and two-part tariff (discussed later) even when the firms remain asymmetric after licensing under the latter two cases. Some works in the literature show that two asymmetric firms may have the same discount factor. However, this happens, for instance, when firms adopt the "balanced temptation solution" (Friedman, 1971; Bae, 1987; Correia-da-Silva and Pinho, 2016; Sabbatini, 2016), according to which the cartel profit is shared in a way that the critical discount factor is (by construction) the same for the two firms.

Comparing this result with section 3, we observe that the firms will tacitly collude with each other after licensing via fixed fee or royalty if and only if $\delta > \frac{9}{17}$. Moreover, the present discounted value of the profits of firm 1 and firm 2 under cartel in royalty licensing are respectively [23] :

$$\pi_1^{RC} = \frac{1}{8(1-\delta)}\left[a^2 + 4ar - 4r^2\right] \ \ and \ \ \pi_2^{RC} = \frac{1}{8(1-\delta)}\left[a^2 - 4ar + 4r^2\right]. \tag{18}$$

4.3 Cartel under Two-part tariff licensing

Let us briefly mention what happens under two-part tariff licensing, which includes both fixed-fee and royalty. The present discounted profits of firm 1 and firm 2 under Cournot-Nash equilibrium, if the cartel is not formed are respectively:

$$\pi_1^{TN} = \frac{1}{1-\delta}\left[\left(\frac{a+r}{3}\right)^2 + r\left(\frac{a-2r}{3}\right)\right] + F \ \ and \ \ \pi_2^{TN} = \frac{1}{1-\delta}\left[\frac{a-2r}{3}\right]^2 - F. \tag{19}$$

As observed in royalty licensing, under two-part tariff licensing, the stable cartel will be formed in the next stage if $\delta \in (\frac{9}{17}, 1]$. Moreover, the present discounted value of the profits of firm 1 and firm 2 under cartel in case of two-part licensing are respectively:

$$\pi_1^{TC} = \frac{1}{8(1-\delta)}\left[a^2 + 4ar - 4r^2\right] + F \ \ and \ \ \pi_2^{TC} = \frac{1}{8(1-\delta)}\left[a^2 - 4ar + 4r^2\right] - F. \tag{20}$$

4.4 Findings

From the discussion in sections 4.1, 4.2 and 4.3, we observe that irrespective of the type of licensing contract the possibility that a stable cartel is formed is the same as for both the firms, proportion of gain and loss from unilateral deviation from the cartel agreement are equal across fixed-fee, royalty and two-part tariff licensing.

Lemma 5 *If technology is licensed via fixed-fee or royalty or two-part tariff, the stable cartel will be formed in the next stage if and only if* $\delta \in (\frac{9}{17}, 1]$.

5 Basic Result

In this section, we discuss the basic results. These results are used in the next section to determine the possibility of licensing in the first stage of the game. From Lemma 4 and Lemma 5 we observe that:

Observation 1
i) if technology is not licensed in the first stage, then cartel is formed if and only if $\delta > \delta_2^{min}$ *and ii) if technology is licensed in the first stage via fixed-fee or royalty or two-part tariff, then cartel is formed if and only if* $\delta > \frac{9}{17}$.

In section 3, it has been observed from Lemma 3 that under no-licensing if $c_2 = 0$, then $\delta_2^{min} = \frac{9}{17}$; and δ_2^{min} initially increases in c_2, reaches the maximum and then falls with increase in

[23] RC here denotes that royalty licensing is followed by a cartel.

c_2 for $c_2 \in (0, \frac{a}{2})$. Hence, δ_2^{min} may either be greater than, equal to or lower than $\frac{9}{17}$. Comparing with equation (9), we get some value of c_2 say $\bar{c_2} \in (0, \frac{a}{2})$, such that $\delta_2^{min} = \frac{9}{17}$ or

$$\frac{\frac{1}{16}\left[(2-s_1^*)a + s_1^*(1-s_1^*)\bar{c_2} - 2\bar{c_2}\right]^2 - \frac{(1-s_1^*)}{4}\left[a^2 - 2a\bar{c_2} + (1-s_1^{*2})\bar{c_2}^2\right]}{\frac{1}{16}\left[(2-s_1^*)a + s_1^*(1-s_1^*)\bar{c_2} - 2\bar{c_2}\right]^2 - \frac{1}{9}(a-2\bar{c_2})^2} = \frac{9}{17}. \qquad (21)$$

Therefore, when $c_2 < \bar{c_2}$, $\delta_2^{min} > \frac{9}{17}$ and when $c_2 \geq \bar{c_2}$, $\delta_2^{min} \leq \frac{9}{17}$.

Proposition 1 *If the cost difference between the firms is low (high), then the possibility of cartel formation is more (less) under licensing than under no-licensing.*

This is one of the most fundamental results of this paper. Moreover, this result is in contrast to Lin (1996), where licensing facilitates collusion since it enlarges the parameter (δ) space that supports collusion. In Lin (1996) as the firms compete in prices and produce homogeneous goods, after licensing the firms earn zero profit in the absence of cartel. Hence, in Lin (1996) if $\delta \geq \frac{1}{2}$, after licensing cartel is always formed. Moreover, if $\delta \geq \frac{1}{2}$ and the technology is not licensed, then the cartel is formed if the cost difference is low, as the lost profit from deviation is not large enough for firm 1 to deter deviation. Moreover, Lin (1996) argues that if $\delta < \frac{1}{2}$, the cartel is neither formed after licensing nor in the no-licensing stage. On the other hand in the present paper, licensing facilitates collusion since it enlarges the parameter (δ) space that supports collusion only if $c_2 < \bar{c_2}$, such that $\delta_2^{min} > \frac{9}{17}$. Otherwise, when $c_2 \geq \bar{c_2}$, such that $\delta_2^{min} \leq \frac{9}{17}$, licensing obstructs collusion since it reduces the parameter (δ) space that supports collusion. After licensing cartel is always formed if $\delta \geq \frac{9}{17}$. However, if $\delta \geq \frac{9}{17}$ then in the absence of licensing agreement, the cartel is formed if the cost difference is more ($c_2 \geq \bar{c_2}$) since the lost profit from deviation is large enough (than the gain) for firm 2 to deter deviation. Lin (1996) mentions in footnote (see footnote 4 of page 448 of Lin (1996)) that licensing makes collusion harder if firms compete in quantities. However, Lin (1996) did not provide formal proof of this result. In contrast to this observation by Lin (1996) in regards to Cournot competition, we show that licensing makes collusion harder (easier) if the cost difference is more (less).

This result can also be related to the papers that study the possibility of cartel formation under cost asymmetry (in the absence of technology licensing). If we assume that $c_2 = 0$ (such that the firms are symmetric), then $s_i^* = 0.5$ (as mentioned in section 3) and thereby we get that $\delta_i^{min} = \frac{9}{17}$ for $i = 1, 2$. Therefore, in the absence of licensing if the firms are symmetric without side payments, the stable cartel will be formed in the next stage if $\delta \in (\frac{9}{17}, 1]$. However, in the absence of licensing we know from Lemma 4 that, if the cost is different (firms are asymmetric), the cartel will be formed if $\delta > \delta_2^{min}$. Hence, similar to Proposition 1, it can be said, in contrast to Miklos-Thal (2011), that *in the absence of licensing without side payments, cost asymmetries facilitate collusion when the cost difference between the firms is high, while it obstructs collusion if the cost difference is low.*[24] Collusion becomes more likely under no-licensing as the cost of firm 2 is much higher (firms are more asymmetric). This is because the high-cost firm is less likely to defect: as the loss from deviation is too high when the unit cost of firm 2 is very high,

[24]As shown in Appendix B, with side payments too we get a similar result in this context.

in comparison to the gains from deviation. This makes collusion easier to sustain if the firms are more asymmetric.

It is believed, among anti-trust agencies in particular, that cost-differences among competitors are a big obstacle to collusion. In this paper, technology licensing makes the costs symmetric between the two firms under fixed-fee licensing and does not change or reduce (to be discussed in the next section) the cost structure under royalty or two-part licensing where the optimal royal rate equals the initial cost differential. Yet, licensing will obstruct collusion if the initial cost differential is high. This can be explained by the behaviour of the higher-cost firm to whom the technology is licensed, as this determines the stability of the cartel. First, under licensing the higher-cost firm's profit is greater than its profit under no-licensing if under both these cases the cartel is formed. This is because the higher-cost firm has some extra gain that accrues to the benefits of licensing from the bargaining process at the time of cartel agreement, which is absent in the case of no-licensing. Hence, this gain is higher when the cost difference is high. Therefore, the gain from deviation under licensing from the cartel agreement is much lower than under the cartel agreement in the absence of licensing, when the cost difference is high for firm 2. As a result, licensing will obstruct collusion if the initial cost differential is high.

6 Licensing Decision

This section discusses what happens in the first stage of the game when firm 1 offers the contract (either Fixed-fee or Royalty or both)[25] to firm 2 for licensing its technology. In this stage firm 1 may not offer any form of contract to firm 2.

6.1 Consider $c_2 < \bar{c_2}$

When $c_2 < \bar{c_2} \implies \delta_2^{min} > \frac{9}{17}$. This implies that we have three cases as follows:
i) Case 1: $\delta_2^{min} > \frac{9}{17} \geq \delta$.
ii) Case 2: $\delta_2^{min} \geq \delta > \frac{9}{17}$ and
iii) Case 3: $\delta > \delta_2^{min} > \frac{9}{17}$.

6.1.1 Case 1

Let us first consider the case when $\delta_2^{min} > \frac{9}{17} \geq \delta$. As mentioned before, δ is exogenous, so if $\delta \leq \frac{9}{17}$, from Observation 1 we say that both in case of licensing and no-licensing of technology, cartel formation is not possible. The profit (present value) of firm 1 in case of fixed-fee licensing is given by $\pi_1^{FN} = \frac{1}{1-\delta}\frac{a^2}{9} + F$ and of firm 2 is $\pi_2^{FN} = \frac{1}{1-\delta}\frac{a^2}{9} - F$ (refer to equation (11)). As cartel is not formed under royalty licensing, the present discounted profit of firm 1 is given by $\pi_1^{RN} = \frac{1}{1-\delta}\left[\frac{(a+r)^2}{9} + r\frac{(a-2r)}{3}\right]$, while for firm 2 is $\pi_2^{RN} = \frac{1}{1-\delta}\frac{(a-2r)^2}{9}$ (refer to equation (14)). On the other hand, the profits for firm 1 and firm 2 in case of no-licensing are given by $\pi_1^{NN} = \frac{1}{1-\delta}\frac{(a+c_2)^2}{9}$ *and* $\pi_2^{NN} = \frac{1}{1-\delta}\frac{(a-2c_2)^2}{9}$ respectively (refer to equation (2)).

[25] In this section we discuss the two-part tariff licensing when required for finding the optimal form of licensing.

Under **fixed-fee licensing**, the fixed-fee F is charged by firm 1 such that firm 2 is indifferent between licensing and no-licensing of the new technology (see Marjit (1990) and Wang (1998))[26]. Hence, F^* is determined such that the profits of firm 2 are the same in both licensing and no licensing i.e. $\frac{1}{1-\delta}\frac{a^2}{9} - F^* = \frac{1}{1-\delta}\frac{(a-2c_2)^2}{9} \implies F^* = \frac{1}{1-\delta}\left[\frac{a^2}{9} - \frac{(a-2c_2)^2}{9}\right]$ which is the maximum licensing fee that firm 1 can charge. Now, after substituting F^* and comparing the profits of firm 1 in both the scenarios, we get: $\pi_1^{FN} = \frac{1}{1-\delta}\frac{a^2}{9} + \frac{1}{1-\delta}[\frac{a^2}{9} - \frac{(a-2c_2)^2}{9}] > \pi_1^{NN} = \frac{1}{1-\delta}\frac{(a+c_2)^2}{9} \implies c_2 < \frac{2a}{5}$.

Similarly, under **royalty licensing**, the per-unit royalty r is charged by firm 1 such that, firm 2 is indifferent between licensing and no-licensing of new technology (see Wang (1998)). Hence, the optimal royalty rate r^* is solved by equating $\pi_2^{RN} = \pi_2^{NN}$: $\frac{1}{1-\delta}\frac{(a-2r)^2}{9} = \frac{1}{1-\delta}\frac{(a-2c_2)^2}{9} \implies r^* = c_2$. Now, comparing the profits of firm 1 in both the scenarios after substituting $r^* = c_2$, we find that $\pi_1^{RN} > \pi_1^{NN}$, i.e. firm 1 will always license its technology via royalty when cartel will not be formed in the following stages. Moreover, using Wang (1998), it can be said that **royalty licensing is superior (optimal form of licensing) to fixed-fee and technology will always be licensed if $c_2 < \bar{c_2}$ and $\delta_2^{min} > \frac{9}{17} \geq \delta$.**[27]

As cartel is not formed in the post-licensing stage, the welfare also increases after licensing in this context as in Wang (1998). This happens as in the post-licensing stage both consumer surplus and industry profit increase.

Lemma 6 *If $c_2 < \bar{c_2}$ and $\delta_2^{min} > \frac{9}{17} \geq \delta$, then technology will be licensed via royalty and post-licensing the welfare will increase.*

6.1.2 Case 2

Let us consider the second case when $\delta_2^{min} \geq \delta > \frac{9}{17}$. From Observation 1, we argue that the cartel is possible if the technology is licensed via fixed-fee or royalty or two-part tariff, but the cartel is not formed if the technology is not licensed. The profit (present value) of firm 1 in case of fixed-fee licensing followed by cartel is given by $\pi_1^{FC} = \frac{1}{1-\delta}\frac{a^2}{8} + F$ and of firm 2 is $\pi_2^{FC} = \frac{1}{1-\delta}\frac{a^2}{8} - F$ (refer to equation (13)). Similarly under royalty licensing the cartel is formed and the present discounted profit of firm 1 is given by $\pi_1^{RC} = \frac{1}{8(1-\delta)}\left[a^2 + 4ar - 4r^2\right]$, while for firm 2 is $\pi_2^{RC} = \frac{1}{8(1-\delta)}\left[a^2 - 4ar + 4r^2\right]$ (refer to equation (18)). On the other hand, as the cartel is not formed under no-licensing, the profits for firm 1 and firm 2 in case of no-licensing are given by $\pi_1^{NN} = \frac{1}{1-\delta}\frac{(a+c_2)^2}{9}$ *and* $\pi_2^{NN} = \frac{1}{1-\delta}\frac{(a-2c_2)^2}{9}$ respectively (refer to equation (2)).

Under **fixed-fee licensing** we find that, $F^* = \frac{1}{1-\delta}\left[\frac{a^2}{8} - \frac{(a-2c_2)^2}{9}\right]$. Here also fixed-fee is charged such that firm 2 is indifferent between licensing and no-licensing, i.e. $\pi_2^{FC} = \pi_2^{NN}$. Now, comparing the profits of firm 1 after substituting F^*, we get:

$$\begin{aligned} &\pi_1^{FC} = \frac{1}{1-\delta}\frac{a^2}{8} + \frac{1}{1-\delta}\left[\frac{a^2}{8} - \frac{(a-2c_2)^2}{9}\right] > \pi_1^{NN} = \frac{1}{1-\delta}\frac{(a+c_2)^2}{9} \\ &\implies (a-2c_2)(a+10c_2) > 0. \end{aligned} \tag{22}$$

[26]Marjit (1990) and Wang (1998) discuss the licensing of technology in a static framework.
[27]Two-part tariff is not possible in this context.

This is always true as $c_2 < \frac{a}{2}$. Therefore, the technology will be licensed and a cartel will be formed under fixed-fee licensing.

Let us now consider **royalty licensing**. Firm 1 sets r^* as high as possible (as π_1^{RC} increases in r) such that $\pi_2^{RC} \geq \pi_2^{NN}$ or $\frac{1}{8(1-\delta)}\left[a^2 - 4ar + 4r^2\right] \geq \frac{1}{1-\delta}\frac{(a-2c_2)^2}{9}$. Hence, firm 1 will charge $r^* = c_2$ from firm 2 as $r^* > c_2$ is not possible. Now, comparing the profits of firm 1, after substituting $r^* = c_2$ we get:

$$\begin{aligned} &\pi_1^{RC} = \frac{1}{8(1-\delta)}\left[a^2 + 4ac_2 - 4c_2^2\right] > \pi_1^{NN} = \frac{1}{1-\delta}\frac{(a+c_2)^2}{9} \\ &\implies (a-2c_2)(a+22c_2) > 0 \end{aligned} \tag{23}$$

This is always true as $c_2 < \frac{a}{2}$ and the technology will be licensed via royalty and a cartel will be formed. After comparing we also observe that $\pi_1^{RC} > \pi_1^{FC}$.

However, as at $r^* = c_2$, $\pi_2^{RC} > \pi_2^{NN}$, **the optimal form of licensing is two-part tariff in this context**, such that $r^* = c_2$ and $F^* = \pi_2^{RC} - \pi_2^{NN}$.

After licensing via two-part tariff the per period consumers' surplus (CS) is $CS_T = \frac{(q_m)^2}{2} = \frac{a^2}{8}$, the industry profit ($IP$) is $IP_T = \frac{a^2}{4}$ and the welfare (W) is $W_T = \frac{3a^2}{8}$. On the other hand, as the cartel is not formed under no-licensing the consumer surplus is $CS_N = \frac{4a^2-4ac_2+c_2{}^2}{18}$ and the industry profit is $IP_N = \frac{4a^2-4ac_2+10c_2{}^2}{18}$. Therefore, the welfare is $W_N = \frac{8a^2-8ac_2+11c_2{}^2}{18}$. Moreover, $W_N > W_T$, if $Z = 5a^2 - 32ac_2 + 44c_2{}^2 > 0$. The expression Z initially falls in c_2, becomes negative and then starts rising in c_2. It is seen that if c_2 is relatively low then $W_N > W_T$. Therefore, if c_2 is low, after licensing as the cartel is formed the welfare is lower than in case of no-licensing.

However, as we cannot solve for $\bar{c_2}$ explicitly and thereby state whether $W_N > W_T$ or not for $c_2 < \bar{c_2}$, we discuss the effect on welfare in terms of examples. From the Example ($a = 120$ as discussed in Appendix A.1) for $c_2 = 1, 2, ..., 22$ we observe $\delta_2^{min} > \frac{9}{17}$ and for this cost range $W_N > W_T$.

Therefore, after licensing (as the cartel is formed), welfare is likely to fall after licensing. This is because in the no-licensing stage cartel is not formed and the firms maintain (duopoly structure) competition in the market even though one firm produces the good at a higher cost.

Lemma 7 *If $c_2 < \bar{c_2}$ and $\delta_2^{min} \geq \delta > \frac{9}{17}$, then technology will be licensed via two-part tariff, but post-licensing the welfare is likely to fall.*

6.1.3 Case 3

Now assume $\delta > \delta_2^{min} > \frac{9}{17}$. This means that tacit collusion is possible in both the scenarios (after licensing or if technology is not licensed). The profit (present value) of firm 1 in case of fixed-fee licensing is given by $\pi_1^{FC} = \frac{1}{1-\delta}\frac{a^2}{8} + F$ and of firm 2 is $\pi_2^{FC} = \frac{1}{1-\delta}\frac{a^2}{8} - F$ (refer to equation (13)). Similarly under royalty licensing the present discounted profit of firm 1 is given by $\pi_1^{RC} = \frac{1}{8(1-\delta)}\left[a^2 + 4ar - 4r^2\right]$, while for firm 2 is $\pi_2^{RC} = \frac{1}{8(1-\delta)}\left[a^2 - 4ar + 4r^2\right]$ (refer to equation (18)).

The present discounted profits of firm 1 and firm 2 as the cartel is formed under no-licensing are $\pi_1^{NC} = \frac{1}{1-\delta}\frac{s_1^*}{4}(a^2-(1-s_1^*)^2c_2^2) = \frac{1}{1-\delta}B_1$ (say) and $\pi_2^{NC} = \frac{1}{1-\delta}\frac{(1-s_1^*)}{4}\Big[a^2-2ac_2+(1-s_1^{*2})c_2^2\Big] = \frac{1}{1-\delta}B_2$ (say) respectively (See equation (10)), where B_1 and B_2 are the per-period profits of firm 1 and firm 2, if the cartel is formed in the absence of licensing.

As in the previous section under **fixed-fee licensing**, $F^* = \frac{1}{1-\delta}(\frac{a^2}{8} - B_2)$, such that $\pi_2^{FC} = \pi_2^{NC}$. Now, comparing the profits of firm 1 after substituting F^*, we get:

$$\begin{aligned} \pi_1^{FC} &= \frac{1}{1-\delta}\frac{a^2}{8} + \frac{1}{1-\delta}(\frac{a^2}{8} - B_2) > \pi_1^{NC} = \frac{1}{1-\delta}B_1 \\ \implies \frac{a^2}{4} &> B_1 + B_2 = \frac{\Big(a - c_2(1-s_1^*)\Big)^2}{4}, \end{aligned} \tag{24}$$

which is always true, hence firm 1 will license its technology to firm 2.

Under **royalty licensing**, firm 1 sets r^* as high as possible (as π_1^{RC} increases in r) such that $\pi_2^{RC} \geq \pi_2^{NC}$ or $\frac{1}{8(1-\delta)}\Big[a^2 - 4ar + 4r^2\Big] \geq \frac{1}{1-\delta}\frac{(1-s_1^*)}{4}\Big[a^2 - 2ac_2 + (1 - s_1^{*2})c_2^2\Big] = \frac{1}{1-\delta}B_2$. If, r^* is set such that $\pi_2^{RC} = \pi_2^{NC}$, then we get $r^* = \frac{a-2\sqrt{2B_2}}{2}$. Therefore, two-part tariff licensing is not possible in this context. Now, comparing the profits of firm 1 after substituting r^* we get:

$$\begin{aligned} \pi_1^{RC} &= \frac{1}{8(1-\delta)}[2a^2 - 8B_2] > \pi_1^{NC} = \frac{1}{1-\delta}B_1 \\ \implies \frac{a^2}{4} &> B_1 + B_2, \end{aligned} \tag{25}$$

which is always true and hence firm 1 will license its technology. Moreover, as $\pi_1^{RC} = \pi_1^{FC}$, **the optimal form of licensing is either fixed-fee or royalty**, i.e. firm 1 is indifferent between fixed-fee licensing and royalty licensing.

As the firms produce monopoly output ($q_m = \frac{a}{2}$) after licensing (as the cartel is formed), consumer surplus will increase after licensing. This is because in the no-licensing stage cartel is formed but the collusive output produced ($q_c = \frac{1}{2}(a - c_2(1 - s_1^*))$) is less than q_m. Hence, welfare always increases after licensing as the industry profit and the consumer surplus increase.

Lemma 8 *If $c_2 < \bar{c}_2$ and $\delta > \delta_2^{min} > \frac{9}{17}$, then technology will be licensed via either fixed-fee or royalty and post-licensing the welfare will increase.*

6.2 Consider $c_2 \geq \bar{c}_2$

When $c_2 \geq \bar{c}_2 \implies \delta_2^{min} \leq \frac{9}{17}$. This implies that we have three cases as follows:
i) Case 1: $\delta \leq \delta_2^{min} \leq \frac{9}{17}$.
ii) Case 2: $\delta_2^{min} < \delta \leq \frac{9}{17}$ and
iii) Case 3: $\delta_2^{min} \leq \frac{9}{17} < \delta$.

6.2.1 Case 1

Let us first consider the case when $\delta \leq \delta_2^{min} \leq \frac{9}{17}$. From Observation 1 we argue that both in case of licensing and no licensing of technology, cartel formation is not possible. As discussed in the previous section, under **fixed-fee licensing**, we get $F^* = \frac{1}{1-\delta}\left[\frac{a^2}{9} - \frac{(a-2c_2)^2}{9}\right]$ and $\pi_1^{FN} \geq \pi_1^{NN}$ if $c_2 \leq \frac{2a}{5}$. Hence, firm 1 will license its technology via fixed-fee if and only if $\bar{c_2} \leq c_2 \leq \frac{2a}{5}$, but cartel will not be formed in the following stage.

Similarly, under **royalty licensing**, the per-unit royalty r is charged by firm 1 such that, $\pi_2^{RN} = \pi_2^{NN} : \implies r^* = c_2$, and after substituting $r^* = c_2$, we find that $\pi_1^{RN} > \pi_1^{NN}$. Hence, firm 1 will always license its technology via royalty. Moreover, from Wang (1998), it can be said that **royalty licensing is superior to fixed-fee and technology will always be licensed if $c_2 \leq \bar{c_2}$ and $\delta \leq \delta_2^{min} \leq \frac{9}{17}$.** As cartel is not formed in the post-licensing stage, as observed in Wang (1998) the welfare increases as in the post-licensing stage both consumer surplus and industry profit are more than in case of no-licensing.

Lemma 9 *If $c_2 \geq \bar{c_2}$ and $\delta \leq \delta_2^{min} \leq \frac{9}{17}$, then technology will be licensed via royalty and post-licensing the welfare will increase.*

6.2.2 Case 2

Let us first consider the second case when $\delta_2^{min} < \delta \leq \frac{9}{17}$. From Observation 1 we say that only under no-licensing of technology, cartel formation is possible. The profit (present value) of firm 1 in case of fixed-fee licensing is given by $\pi_1^{FN} = \frac{1}{1-\delta}\frac{a^2}{9} + F$ and of firm 2 is $\pi_2^{FN} = \frac{1}{1-\delta}\frac{a^2}{9} - F$ (refer to equation (11)). As cartel is not formed under royalty licensing, the present discounted profit of firm 1 is given by $\pi_1^{RN} = \frac{1}{1-\delta}\left[\frac{(a+r)^2}{9} + r\frac{(a-2r)}{3}\right]$, while for firm 2 is $\pi_2^{RN} = \frac{1}{1-\delta}\frac{(a-2r)^2}{9}$ (refer to equation (14)). The present discounted profits of firm 1 and firm 2 as the cartel is formed under no-licensing are $\pi_1^{NC} = \frac{1}{1-\delta}\frac{s_1^*}{4}(a^2 - (1-s_1^*)^2 c_2^2) = \frac{1}{1-\delta}B_1$ (say) and $\pi_2^{NC} = \frac{1}{1-\delta}\frac{(1-s_1^*)}{4}\left[a^2 - 2ac_2 + (1-s_1^{*2})c_2^2\right] = \frac{1}{1-\delta}B_2$ (say) respectively (See equation (10)), where B_1 and B_2 are the per-period profits of firm 1 and firm 2, if the cartel is formed in the absence of licensing.

As in the previous case under **fixed-fee licensing** we find that, $F^* = \frac{1}{1-\delta}\left[\frac{a^2}{9} - B_2\right]$. Here also fixed-fee is charged such that firm 2 is indifferent between licensing and no-licensing, i.e. $\pi_2^{FN} = \pi_2^{NC}$. Now, comparing the profits of firm 1 after substituting F^*, we get[28]:

$$\begin{aligned} \pi_1^{FC} = \frac{1}{1-\delta}\left[\frac{2a^2}{9} - B_2\right] > \pi_1^{NC} = \frac{1}{1-\delta}B_1 \\ \implies \frac{2a^2}{9} > B_1 + B_2. \end{aligned} \tag{26}$$

However, under **royalty licensing**, the per-unit royalty r is charged by firm 1 such that, $\pi_2^{RN} \geq \pi_2^{NC} : \implies \frac{1}{1-\delta}\frac{(a-2r)^2}{9} \geq \frac{1}{1-\delta}B_2$. Firm 1 will set r as high as possible as its post-licening profit increases in the royalty rate. Therefore, firm 1 will set $r^* < c_2$, in this context as

[28] As the optimal form of licensing is royalty in the present context (discussed later), we desist to check whether this inequality holds or not.

$\frac{1}{1-\delta}\frac{(a-2c_2)^2}{9} < \frac{1}{1-\delta}B_2$, such that $\implies \frac{1}{1-\delta}\frac{(a-2r^*)^2}{9} = \frac{1}{1-\delta}B_2$ or $r^* = \frac{a-3\sqrt{B_2}}{2}$. After substituting r^* we get

$$\begin{aligned} \pi_1^{RN} &= \frac{1}{1-\delta}\Big[\frac{(a+r^*)^2}{9} + r^*\frac{(a-2r^*)}{3}\Big] > \pi_1^{NC} = \frac{1}{1-\delta}B_1 \\ \implies \frac{2a^2}{9} &+ \frac{ar^* - r^{*2}}{9} = L(say) > B_1 + B_2, \end{aligned} \tag{27}$$

or $L - B_1 - B_2 > 0$, where $L = \frac{a^2 - B_2}{4}$. We observe that if c_2 tends to zero then the above inequality (27) will not hold as $L - B_1 - B_2 < 0$. Moreover, if c_2 increases, then L increases. However, $B_1 + B_2$ which is the joint profit of the firms (if the cartel is formed under no-licensing), initially decreases in c_2, reaches the minimum and then starts rising. It is also true that at $c_2 = \frac{a}{2}$, $L = B_1 + B_2$. This therefore, implies that if c_2 is relatively higher then L is likely to be greater than $B_1 + B_2$. Therefore, it is more plausible that firm 1 will license its technology via royalty, but the cartel will not be formed in the following stage. This pattern is also observed from the examples as discussed in the following paragraph for $c_2 \geq \bar{c_2}$ (higher costs) and we find that $M = L - B_1 - B_2 > 0$.

As we cannot solve for s_1^* *and* $\bar{c_2}$ explicitly, we discuss about the possibility of royalty licensing in terms of examples for $c_2 \geq \bar{c_2}$. From the Example ($a = 120$) (see Table 3 in the Appendix A.7) for $c_2 = 23, 24, ..., 59$ we observe $\delta_2^{min} \leq \frac{9}{17}$ as well as the inequality (27) holds or $M = L - B_1 - B_2 > 0$.

Therefore, it is more likely that firm 1 will license its technology via royalty, but the cartel will not be formed in the following stages. Moreover as $\pi_1^{RN} > \pi_1^{FN}$, **royalty licensing is superior to fixed-fee (as well as optimal form of licensing) and it is plausible that technology will be licensed if $c_2 \geq \bar{c_2}$ and $\delta_2^{min} < \delta \leq \frac{9}{17}$**. The firms produce $q = \frac{2a-r^*}{3}$ after licensing, whereas in the no-licensing stage the cartel is formed and the firms jointly produce $q_c = \frac{1}{2}(a - c_2(1-s_1^*))$. As $\frac{2a-r^*}{3} > \frac{1}{2}(a - c_2(1-s_1^*))$, consumers surplus is more in the post-licensing stage than in case of no-licensing. Hence, welfare always increases after licensing as the industry profit and the consumers surplus increase.

Lemma 10 *If $c_2 \geq \bar{c_2}$ and $\delta_2^{min} < \delta \leq \frac{9}{17}$, then technology is likely to be licensed via royalty and post-licensing the welfare will increase.*

6.2.3 Case 3

As in Case 3 of the earlier section[29], here too for $\delta_2^{min} \leq \frac{9}{17} < \delta$ we observe that firm 1 will always license its technology. Moreover, **firm 1 is indifferent between fixed-fee and royalty licensing**, as in the post-licensing stage it gets the same profit if the technology is licensed via fixed-fee or royalty. The firms produce monopoly output ($q_m = \frac{a}{2}$) after licensing, whereas in the no-licensing stage the cartel is formed and the firms jointly produce $q_c = \frac{1}{2}(a - c_2(1 - s_1^*))$. Hence, consumer surplus is more in post-licensing than under no-licensing. Hence, welfare always increases after licensing as the industry profit and the consumer surplus increase.

Lemma 11 *If $c_2 \geq \bar{c_2}$ and $\delta_2^{min} \leq \frac{9}{17} < \delta$, then technology will be licensed via either fixed-fee or royalty and post-licensing the welfare will increase.*

[29] As the analysis is the same as that of Case 3 of the previous section, in this part we mention the final result.

6.3 Results

The main results of this discussion are presented in the following table:

Table A

Cost	δ	**Licensing**	**Cartel**	**Welfare**
$c_2 < \bar{c_2}$	$\delta_2^{min} > \frac{9}{17} \geq \delta$	Royalty	No	Increase
$c_2 < \bar{c_2}$	$\delta_2^{min} \geq \delta > \frac{9}{17}$	Two-part tariff	Yes	Fall
$c_2 < \bar{c_2}$	$\delta > \delta_2^{min} > \frac{9}{17}$	Fixed-fee or Royalty	Yes	Increase
$c_2 \geq \bar{c_2}$	$\delta \leq \delta_2^{min} \leq \frac{9}{17}$	Royalty	No	Increase
$c_2 \geq \bar{c_2}$	$\delta_2^{min} < \delta \leq \frac{9}{17}$	Royalty	No	Increase
$c_2 \geq \bar{c_2}$	$\delta_2^{min} \leq \frac{9}{17} < \delta$	Fixed-fee or Royalty	Yes	Increase

In the above table (Table A), we observe that in the first stage of the game technology will always be transferred as shown in the third column. In the next stage, the cartel may or may not form as shown in the fourth column. Finally, except for two-part tariff licensing, welfare always increases after licensing, as shown in the fifth column.

Proposition 2 *Technology will always be licensed either by fixed-fee or royalty or two-part tariff in the first stage. However, i) if $c_2 < \bar{c_2}$ and $\delta_2^{min} \geq \delta > \frac{9}{17}$ then cartel is formed only if technology is licensed and ii) if $c_2 \geq \bar{c_2}$ and $\delta_2^{min} < \delta \leq \frac{9}{17}$ then cartel is formed only if technology is not licensed. Otherwise, either the cartel is formed both under licensing or no-licensing or the cartel is not formed in both the cases.*

In a static framework Wang (1998) shows that royalty licensing is optimal for the licensor, whereas in the dynamic settings given that the firms can form a cartel, we observe that the optimal form of licensing is either fixed-fee or royalty or two-part tariff. It is observed that if the cartel is not formed after licensing then royalty licensing is superior, as the licensor charges the maximum royalty rate such that the effective cost of production for the licensee remains unchanged after licensing. Otherwise, if the cartel is formed after the technology is licensed, then either the licensor is indifferent between fixed-fee and royalty licensing or the optimal form of licensing is the two-part tariff. This is because if the cartel is formed in the post-licensing stage then the licensor's profit not only depends on the revenue from the licensing agreement but also from the cartel outcome that reduces competition in the output market.[30]

Now to understand, how exactly licensing facilitates or obstructs cartel formation we consider the second and the fifth case of the table. Firstly, from the second case, i.e. $c_2 < \bar{c_2}$ and $\delta_2^{min} \geq \delta > \frac{9}{17}$, it is observed that the cartel is formed only if technology is licensed. This

[30] As we have assumed that fixed-fee is paid once in the first period, even though the firms compete in the subsequent periods, the licensor may not charge F^* if the cartel is likely to be formed after licensing. The anti-trust authorities after knowing that F^* (for the 3rd and the 5th case of the table) is too high in comparison to any F that usually happens in absence of tacit collusion, may charge an additional penalty or disallow licensing deal to take place. If this is the case then for the 3rd and the 5th case of the table, the firm should prefer royalty licensing. However, this problem does not arise for the 2nd case of the table, as both the firms are better off even if the fixed-fee is zero.

shows that effectively licensing is facilitating collusion and it thereby harms welfare. In such cases, the welfare under no-licensing is more than under licensing. Licensing partially facilities collusion as from the first case of the table it can be argued that if the firms put very little weight on the future profits (δ very low), then even under the licensing cartel will not be formed. However, if $c_2 \geq \bar{c_2}$ and $\delta_2^{min} < \delta \leq \frac{9}{17}$ (fifth case) then cartel is formed only if technology is not licensed. This effectively shows that licensing can deter collusion. Hence, licensing has a positive impact on welfare as the welfare under licensing is more than the welfare under no-licensing. Otherwise, either the cartel is formed both under licensing or no-licensing or the cartel is not formed. Moreover, in these cases (apart from second and fifth), welfare is more under licensing than under no-licensing, as the firms produce more output in the case of licensing than when technology is not licensed.

Proposition 3 *Welfare decreases after licensing such that the technology is licensed and post-licensing the cartel is formed. Otherwise, after licensing welfare always increases.*

7 Conclusion

This work studies the possibility of tacit collusion between firms that produce homogeneous goods and compete in quantities in the presence of licensing opportunities, where one firm produces output at a lower unit cost than its rival. Specifically, we discuss the possibility of tacit collusion, after the technology is licensed from the lower cost firm to the higher cost firm either through a fixed fee or royalty payment or two-part tariff at the beginning of the game. We analyze the model by first observing the possibility of cartel formation using the Grim-Trigger strategy if in the preceding stage technology licensing contract is either signed or not. Finally, we point out whether in the initial stage the firms sign the licensing contract or not and thereby whether they form a cartel subsequently in the next stage.

Firstly, it has been observed that under fixed-fee, royalty and two-part tariff licensing the possibilities of the formation of a stable cartel among firms, are the same. All the forms of licensing equally facilitate collusion. On the other hand, if the technology is not licensed in the first stage, then the possibility of cartel formation initially decreases if the cost difference increases and then reduces if the cost difference increases. Hence, if the cost difference between the firms is low, then the possibility of cartel formation is more under licensing than under no-licensing and vice versa. Therefore, in contrast to Lin (1996), all the forms of licensing facilitate collusion, if the initial cost difference between the firms is less since it enlarges the parameter space that supports collusion. However, licensing obstructs collusion, if the cost difference is more, as it reduces the parameter space that supports collusion. Finally, using backward induction it is shown that in the first stage, technology will always be licensed either by fixed-fee or royalty or two-part tariff. However, the cartel will be formed if the firms are relatively patient and welfare will either increase or decrease in the post-licensing stage in comparison to what happens in the no-licensing stage. Moreover, if we assume that in the no-licensing stage, the cartel agreement is such that only the efficient firm produces the monopoly output and the inefficient firm shuts down its production (and receives a lump-sum payment from the other firm), then the results change slightly. In such a scenario, in the first stage of the game technology will not always be transferred and in the following stage, the cartel may or may not be formed. Licensing here too facilitates collusion and it harms the welfare, but it cannot deter collusion. Similarly, one can look into the possibilities of licensing as well as the formation of cartels if the firms produce differentiated goods as a possible extension of this work.

A Appendix

A.1 Optimal share of output

The optimal sharing agreement as discussed in the main text (see Section 3) is determined by

$$\max_{s_1} V = (\pi_1^c - \pi_1^n)(\pi_2^c - \pi_2^n)$$
$$= \left(\frac{s_1}{4}[a^2 - (1-s_1)^2c_2^2] - \frac{1}{9}(a+c_2)^2\right)\left(\frac{(1-s_1)}{4}[a^2 - 2ac_2 + (1-s_1^2)c_2^2] - \frac{1}{9}(a-2c_2)^2\right),$$

such that $\pi_i^c > \pi_i^n$ for $i = 1, 2$. We observe that $\pi_1^c > \pi_1^n$, if

$$T_1(s_1) = \frac{s_1}{4}(a^2 - (1-s_1)^2c_2^2) - \frac{1}{9}(a+c_2)^2 > 0. \tag{28}$$

At $s_1 = 0$, the above inequality doesn't hold. At $s_1 = 1$, the inequality reduces to $\frac{a^2}{4} > (\frac{a+c_2}{3})^2 \implies \frac{a}{2} > c_2$.[31] Hence, the inequality (28) is always satisfied at $s_1 = 1$. We observe that $\frac{dT_1}{ds_1} = \frac{1}{4}[2s_1(1-s_1){c_2}^2 + (a^2 - (1-s_1)^2{c_2}^2)] > 0$, so for some $s_1 = \bar{s_1}$ $(\bar{s_1} \in (0,1))$, we can say that $T_1(\bar{s_1}) = 0$. Hence, for $s_1 \in (\bar{s_1}, 1], \pi_1^c > \pi_1^n$ is satisfied. Similarly, $\pi_2^c > \pi_2^n$, if

$$T_2(s_1) = \frac{1-s_1}{4}(a^2 - 2ac_2 + (1-s_1^2)c_2^2) - \frac{1}{9}(a-2c_2)^2 > 0. \tag{29}$$

At $s_1 = 0$, inequality (29) is always satisfied and never satisfied at $s_1 = 1$. As $\frac{dT_2}{ds_1} = \frac{1}{4}[-2s_1(1-s_1){c_2}^2 - (a^2 - 2ac_2 + (1-s_1)^2{c_2}^2)] < 0$, for $s_1 = \tilde{s_1}$ $(\tilde{s_1} \in (0,1))$, we have $T_2(\tilde{s_1}) = 0$. Hence, for $s_1 \in [0, \tilde{s_1})$, $\pi_2^c > \pi_2^n$ is satisfied.

Moreover, $\frac{d^2T_1}{ds_1^2} = \frac{c_2^2(2-3s_1)}{2}$ and $\frac{d^2T_2}{ds_1^2} = \frac{c_2^2(-1+3s_1)}{2}$. At, $s_1 = 0.5$, $T_1(0.5) = \frac{4a^2-64ac_2-41c_2^2}{288}$, $T_2(0.5) = \frac{4a^2+56ac_2-101c_2^2}{288} > 0$ and $T_2(0.5) > T_1(0.5)$ as $a > \frac{c_2}{2}$. However, $T_1(0.5)$ is either positive (first case) or negative (second case).

I) Therefore, if we assume that $T_2(0.5) > T_1(0.5) > 0$ (first case), then it demands that $\bar{s_1} < 0.5 < \tilde{s_1}$ (as $\frac{dT_1}{ds_1} > 0$ and $\frac{dT_2}{ds_1} < 0$) such that $T_1(\bar{s_1}) = T_2(\tilde{s_1}) = 0$.

II) However, if we assume that $T_2(0.5) > 0 > T_1(0.5)$ (second case), then it demands that $0.5 < \tilde{s_1}$ and $0.5 < \bar{s_1}$ (as $\frac{dT_1}{ds_1} > 0$ and $\frac{dT_2}{ds_1} < 0$), but as $\frac{d^2T_1}{ds_1^2} > 0$ and $\frac{d^2T_2}{ds_1^2} > 0$ at $s_1 = 0.5$, hence, for that reason, we should have $\bar{s_1} < \tilde{s_1}$ such that $T_1(\bar{s_1}) = T_2(\tilde{s_1}) = 0$.

Therefore, $\bar{s_1} < \tilde{s_1}$.

Hence, it follows that V has a maxima for a certain share of firm 1, s_1^*, such that $\bar{s_1} < s_1^* < \tilde{s_1}$. At this maxima, firm 2's share is $s_2^* = 1 - s_1^*$. Finding a general, closed-form analytical expression of s_1^*, however, turns out to be impossible as it requires calculating roots of a fifth-order

[31] This is assumed in the beginning of the paper.

polynomial equation. Abel's Impossibility Theorem tells us that no general formulas, like the quadratic formula, exist for roots of fifth or higher degree polynomial equations. To circumvent this problem, we have numerically solved the optimization problem using Wolfram Mathematica for a wide range of a and c_2 values and found that s_1^* exists and is unique for all of them.

For finding s_1^*, we have numerically maximized V when $a = 10, 20, \cdots, 150$, and for each a we let c_2 take values in the range $[1, 2, \cdots, a/2]$ (see figure 1). Note that the maximum value of V and the corresponding s_1^* both vary smoothly as we change a and c_2. This numerical result led us to conjecture that s_1^* indeed has finite values when $0 < c_2 < \frac{a}{2}$.

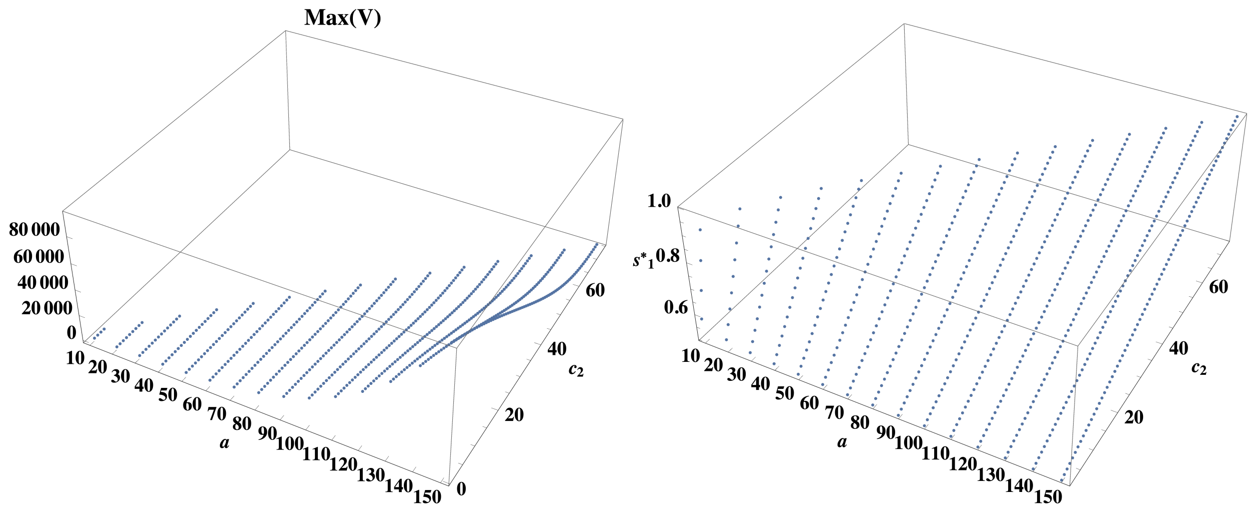

Figure 1: Maximum value of V (left) and corresponding s_1^* (right) for $a = 10, 20, \cdots, 150$ and $c_2 = 1, 2, \cdots, a/2$ (for each a).

To be concrete, we further analyzed a specific values of a. In the Example (see Table 1 on the next page), we took $a = 120$ and considered 59 c_2 value in the range $[1, 2, \cdots, 59]$ (remember that $0 < c_2 < \frac{a}{2}$). Note that they are also present in the above figure, in addition to several other a and c_2 values.

In Table 1, we have shown s_1^*, along with δ_1^{min} and δ_2^{min}, obtained by substituting s_1^* in equation (9). Figure 2 shows that s_1^* increases with c_2 as the cost difference between the two firms increases. (In the examples the values of s_1^* are solved using Wolfram Mathematica as stated before.)

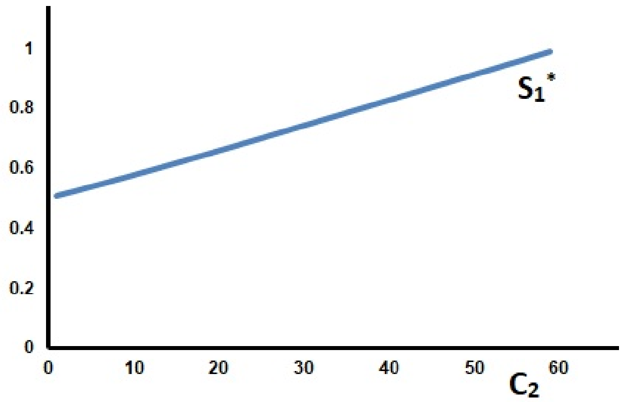

Figure 2: s_1^* as function of c_2, $a = 120$ and $c_2 = 1, 2, ..., 59$

Table 1 For $a = 120$ and $c_2 = 1, 2, ..., 59$

a	c_2	s_1^*	δ_1^{min}	δ_2^{min}		a	c_2	s_1^*	δ_1^{min}	δ_2^{min}
120	1	0.50744	0.5282	0.53052		120	30	0.74297	0.42823	0.51576
120	2	0.51494	0.52688	0.53154		120	31	0.75149	0.42176	0.51342
120	3	0.52249	0.52544	0.53246		120	32	0.76002	0.41501	0.51095
120	4	0.53011	0.52389	0.53328		120	33	0.76855	0.40797	0.50835
120	5	0.53778	0.52221	0.53401		120	34	0.77709	0.40063	0.50561
120	6	0.54551	0.5204	0.53464		120	35	0.78563	0.39297	0.50274
120	7	0.55329	0.51846	0.53517		120	36	0.79417	0.38498	0.49973
120	8	0.56113	0.51639	0.53559		120	37	0.80272	0.37665	0.49659
120	9	0.56901	0.51419	0.53591		120	38	0.81127	0.36794	0.49331
120	10	0.57695	0.51184	0.53612		120	39	0.81982	0.35885	0.4899
120	11	0.58493	0.50935	0.53623		120	40	0.82836	0.34935	0.48635
120	12	0.59296	0.50671	0.53622		120	41	0.83691	0.33942	0.48266
120	13	0.60103	0.50391	0.5361		120	42	0.84546	0.32903	0.47883
120	14	0.60914	0.50096	0.53587		120	43	0.854	0.31815	0.47487
120	15	0.61729	0.49785	0.53553		120	44	0.86255	0.30674	0.47077
120	16	0.62548	0.49457	0.53507		120	45	0.87109	0.29478	0.46652
120	17	0.6337	0.49112	0.53449		120	46	0.87964	0.28222	0.46214
120	18	0.64196	0.4875	0.5338		120	47	0.88818	0.26901	0.45761
120	19	0.65025	0.48369	0.53298		120	48	0.89672	0.2551	0.45294
120	20	0.65857	0.4797	0.53204		120	49	0.90527	0.24043	0.44813
120	21	0.66692	0.47552	0.53098		120	50	0.91382	0.22494	0.44318
120	22	0.67529	0.47114	0.5298		120	51	0.92238	0.20854	0.43807
120	23	0.68369	0.46656	0.52849		120	52	0.93094	0.19115	0.43282
120	24	0.69211	0.46177	0.52706		120	53	0.93951	0.17267	0.42743
120	25	0.70055	0.45677	0.5255		120	54	0.94809	0.15299	0.42188
120	26	0.709	0.45154	0.52381		120	55	0.95669	0.13196	0.41618
120	27	0.71747	0.44608	0.52199		120	56	0.9653	0.10942	0.41033
120	28	0.72596	0.44038	0.52005		120	57	0.97394	0.08521	0.40432
120	29	0.73446	0.43443	0.51797		120	58	0.9826	0.05908	0.39815
						120	59	0.99128	0.03078	0.39182

A.2 Proof of Lemma 1

Lemma 1: $\frac{\partial s_1^*}{\partial c_2} > 0$ and $s_1^* \in (0.5, 1)$ for $c_2 \in (0, \frac{a}{2})$.

Proof. Differentiating F.O.C., equation (5) of the main text w.r.t. c_2, we observe that

$$\frac{\partial s_1^*}{\partial c_2}\left[(\pi_1^c - \pi_1^n)\frac{\partial^2 \pi_2^c}{\partial s_1^2} + (\pi_2^c - \pi_2^n)\frac{\partial^2 \pi_1^c}{\partial s_1^2} + 2\frac{\partial \pi_2^c}{\partial s_1}\frac{\partial \pi_1^c}{\partial s_1}\right] =$$
$$\frac{\partial \pi_2^c}{\partial s_1}\left[\frac{\partial \pi_1^n}{\partial c_2} - \frac{\partial \pi_1^c}{\partial c_2}\right] + \frac{\partial \pi_1^c}{\partial s_1}\left[\frac{\partial \pi_2^n}{\partial c_2} - \frac{\partial \pi_2^c}{\partial c_2}\right] - (\pi_1^c - \pi_1^n)\frac{\partial^2 \pi_2^c}{\partial s_1 \partial c_2} - (\pi_2^c - \pi_2^n)\frac{\partial^2 \pi_1^c}{\partial s_1 \partial c_2}. \quad (30)$$

Moreover, i) $\frac{\partial \pi_1^n}{\partial c_2} = \frac{2(a+c_2)}{9} > 0$, ii) $\frac{\partial \pi_1^c}{\partial c_2} = \frac{-s_1(1-s_1)^2 c_2}{2} < 0$, iii) $\frac{\partial \pi_1^c}{\partial s_1} = \frac{a^2+(1-s_1)(3s_1-1)c_2{}^2}{4} > 0$, iv) $\frac{\partial \pi_2^n}{\partial c_2} = \frac{-4(a-2c_2)}{9} < 0$, v) $\frac{\partial \pi_2^c}{\partial c_2} = -\frac{(1-s_1)[a-(1-s_1)^2 c_2]}{2} < 0$, vi) $\frac{\partial \pi_2^c}{\partial s_1} = -\frac{a^2-2ac_2+(1+2s_1-3s_1^2)c_2^2}{4} < 0$, vii) $\frac{\partial^2 \pi_2^c}{\partial s_1^2} = \frac{s_1 c_2^2}{2} > 0$, viii) $\frac{\partial^2 \pi_1^c}{\partial s_1^2} = \frac{c_2^2}{4} > 0$, ix) $\frac{\partial^2 \pi_1^c}{\partial s_1 \partial c_2} = \frac{2c_2(1-s_1)(3s_1-1))}{4}$ and x) $\frac{\partial^2 \pi_2^c}{\partial s_1 \partial c_2} = \frac{a}{2} - \frac{c_2(1+2s_1-3s_1^2)}{2} > 0$.

If $c_2 = 0$, then both the firms become identical hence we observe that the solution of (4) is $s_1^* = 0.5$. Hence, if $c_2 = 0$ such that optimal $s_1 = 0.5$, then $\frac{\partial^2 \pi_1^c}{\partial s_1 \partial c_2} > 0$ and $\frac{\partial \pi_2^n}{\partial c_2} < \frac{\partial \pi_2^c}{\partial c_2} < 0$. Using S.O.C. equation (6) of the main text and equation (30) we therefore get $\frac{\partial s_1^*}{\partial c_2} > 0$ at $c_2 = 0$. Moreover, for $s_1 \in (0.5, 1)$ $\frac{\partial^2 \pi_1^c}{\partial s_1 \partial c_2} > 0$ and $\frac{\partial \pi_2^n}{\partial c_2} < \frac{\partial \pi_2^c}{\partial c_2} < 0$. This means that if c_2 increases then π_2^n falls at a faster rate than π_2^c for any cartel output share given by s_2 $(= 1 - s_1)$ (when the cartel output remains unchanged.) Using S.O.C. equation (6) and equation (30) we therefore get $\frac{\partial s_1^*}{\partial c_2} > 0$ and $s_1^* \in (0.5, 1)$ for $c_2 \in (0, \frac{a}{2})$. ∎

We note here that at $c_2 = 0$, using equation (30) and substituting the values, we observe that $\frac{\partial s_1^*}{\partial c_2} = \frac{8}{9a}$.

A.3 Proof of Lemma 2

Lemma 2: $\frac{d\pi_1^c}{dc_2} > 0, \frac{d\pi_1^d}{dc_2} > 0, \frac{d\pi_2^c}{dc_2} < 0$ *and* $\frac{d\pi_2^d}{dc_2} < 0$.

Proof. From the previous Lemma we observe that, $\frac{\partial s_1^*}{\partial c_2} > 0$ and $s_1^* \in (0.5, 1)$ for $c_2 \in (0, \frac{a}{2})$. Therefore, using the the expressions used in the proof of the previous lemma, we argue that

$$\frac{d\pi_1^c}{dc_2} = \frac{\partial \pi_1^c}{\partial s_1^*}\frac{\partial s_1^*}{\partial c_2} + \frac{\partial \pi_1^c}{\partial c_2} > 0 \quad and \quad \frac{d\pi_1^d}{dc_2} = \frac{\partial \pi_1^d}{\partial s_1^*}\frac{\partial s_1^*}{\partial c_2} + \frac{\partial \pi_1^d}{\partial c_2} > 0$$
$$\frac{d\pi_2^c}{dc_2} = \frac{\partial \pi_2^c}{\partial s_1^*}\frac{\partial s_1^*}{\partial c_2} + \frac{\partial \pi_2^c}{\partial c_2} < 0 \quad and \quad \frac{d\pi_2^d}{dc_2} = \frac{\partial \pi_2^d}{\partial s_1^*}\frac{\partial s_1^*}{\partial c_2} + \frac{\partial \pi_2^d}{\partial c_2} < 0,$$

as i) $\frac{\partial \pi_1^d}{\partial s_1^*} = \frac{1}{8}[(1+s_1^*)a + (1-s_1^*)^2 c_2)][a - 2s_1^* c_2] > 0$, ii) $\frac{\partial \pi_1^d}{\partial c_2} = \frac{1}{8}[(1+s_1^*)a + (1-s_1^*)^2 c_2)](1-s_1^*)^2 > 0$, iii) $\frac{\partial \pi_2^d}{\partial s_1^*} = \frac{1}{8}[(2-s_1^*)a + s_1^*(1-s_1^*)c_2 - 2c_2][-a + c_2(1-2s_1^*)] < 0$ and iv) $\frac{\partial \pi_2^d}{\partial c_2} = \frac{1}{8}[(2-s_1^*)a + s_1^*(1-s_1^*)c_2 - 2c_2][s_1^*(1-s_1^*) - 2] < 0$. ∎

In Table 2, for the Example ($a = 120$ and $c_2 = 1, 2, ..., 59$ such that $c \in (0, \frac{a}{2})$) we mention Π_1^d, Π_1^c, Π_1^n, Π_2^d, Π_2^c and Π_2^n. With the help of Table 2 we have drawn Figure 3.

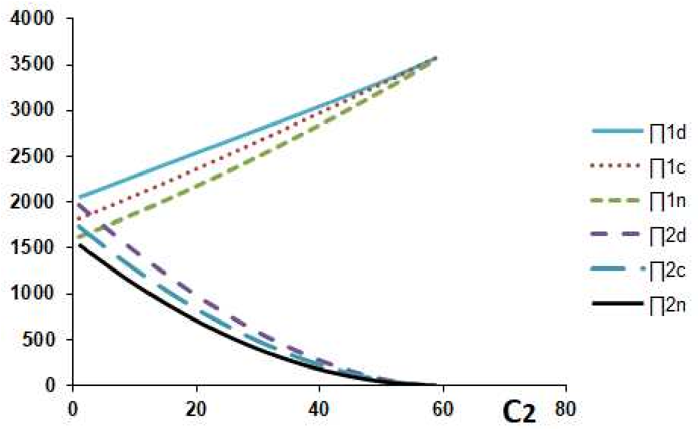

Figure 3: Profits of the firms, $a = 120$ and $c_2 = 1, 2, ..., 59$

Table 2 For $a = 120$ and $c_2 = 1, 2, ..., 59$

c_2	Π_1^d	Π_1^c	Π_1^n	Π_2^d	Π_2^c	Π_2^n
1	2050.622	1826.746	1626.778	1965.977	1743.761	1547.111
2	2076.237	1853.651	1653.778	1907.662	1688.376	1495.111
3	2101.842	1880.713	1681	1850.056	1633.849	1444
4	2127.435	1907.932	1708.444	1793.164	1580.178	1393.778
5	2153.013	1935.304	1736.111	1736.989	1527.366	1344.444
6	2178.575	1962.83	1764	1681.535	1475.413	1296
7	2204.118	1990.508	1792.111	1626.807	1424.32	1248.444
8	2229.642	2018.335	1820.444	1572.809	1374.089	1201.778
9	2255.144	2046.31	1849	1519.547	1324.719	1156
10	2280.623	2074.431	1877.778	1467.025	1276.212	1111.111
11	2306.079	2102.696	1906.778	1415.249	1228.569	1067.111
12	2331.511	2131.103	1936	1364.224	1181.789	1024
13	2356.919	2159.65	1965.444	1313.958	1135.875	981.7777
14	2382.302	2188.334	1995.111	1264.456	1090.827	940.4444
15	2407.66	2217.154	2025	1215.725	1046.645	900
16	2432.995	2246.105	2055.111	1167.773	1003.331	860.4444
17	2458.307	2275.186	2085.444	1120.606	960.8845	821.7777
18	2483.597	2304.394	2116	1074.232	919.3073	784
19	2508.866	2333.726	2146.778	1028.66	878.5999	747.1111
20	2534.116	2363.18	2177.778	983.8972	838.7634	711.1111
21	2559.35	2392.751	2209	939.9533	799.7986	676
22	2584.569	2422.437	2240.444	896.8369	761.7067	641.7777
23	2609.777	2452.234	2272.111	854.5574	724.4888	608.4444
24	2634.976	2482.14	2304	813.1246	688.1463	576
25	2660.17	2512.15	2336.111	772.5484	652.6804	544.4444
26	2685.362	2542.261	2368.444	732.8392	618.0929	513.7777
27	2710.558	2572.47	2401	694.0077	584.3854	484
28	2735.761	2602.773	2433.778	656.065	551.5598	455.1111
29	2760.977	2633.167	2466.778	619.0226	519.6183	427.1111
30	2786.211	2663.646	2500	582.892	488.5632	400

Table 2 continue...

c_2	Π_1^d	Π_1^c	Π_1^n	Π_2^d	Π_2^c	Π_2^n
31	2811.468	2694.209	2533.444	547.6854	458.3972	373.7777
32	2836.756	2724.85	2567.111	513.4154	429.1231	348.4444
33	2862.08	2755.566	2601	480.0946	400.7442	324
34	2887.448	2786.354	2635.111	447.7364	373.2641	300.4444
35	2912.867	2817.208	2669.444	416.3544	346.6866	277.7777
36	2938.345	2848.126	2704	385.9625	321.0161	256
37	2963.891	2879.103	2738.778	356.5752	296.2573	235.1111
38	2989.514	2910.136	2773.778	328.2074	272.4155	215.1111
39	3015.224	2941.22	2809	300.8744	249.4965	196
40	3041.03	2972.353	2844.444	274.592	227.5065	177.7777
41	3066.943	3003.529	2880.111	249.3765	206.4525	160.4444
42	3092.976	3034.746	2916	225.2448	186.342	144
43	3119.139	3065.999	2952.111	202.2143	167.1832	128.4444
44	3145.445	3097.286	2988.444	180.3031	148.9852	113.7777
45	3171.909	3128.603	3025	159.5299	131.7579	100
46	3198.543	3159.946	3061.778	139.914	115.5118	87.1111
47	3225.365	3191.312	3098.778	121.4757	100.2587	75.1111
48	3252.388	3222.698	3136	104.2358	86.0113	64
49	3279.631	3254.101	3173.444	88.2164	72.7834	53.7777
50	3307.112	3285.518	3211.111	73.4402	60.59	44.4444
51	3334.85	3316.947	3249	59.9311	49.4475	36
52	3362.865	3348.385	3287.111	47.7143	39.3738	28.4444
53	3391.18	3379.829	3325.444	36.816	30.3883	21.7777
54	3419.818	3411.279	3364	27.2641	22.512	16
55	3448.805	3442.732	3402.778	19.0878	15.7681	11.1111
56	3478.168	3474.186	3441.778	12.3182	10.1816	7.1111
57	3507.935	3505.64	3481	6.9883	5.7801	4
58	3538.14	3537.094	3520.444	3.1332	2.5935	1.7777
59	3568.816	3568.548	3560.111	0.7903	0.6548	0.4444

From Table 2 we also observe that $\frac{d\pi_1^c}{dc_2} > 0, \frac{d\pi_1^d}{dc_2} > 0, \frac{d\pi_2^c}{dc_2} < 0 \;\; and \;\; \frac{d\pi_2^d}{dc_2} < 0$. With the help of the example where $a = 120$ and $c_2 = 1, 2, ..., 59.$, after substituting s_1^* for different values of a and c_2 as shown in the Table 2 in the Appendix A.3, we observe that (See Figure 3) that π_1^d, π_1^c and π_1^n increases in c_2, while π_2^d, π_2^c and π_2^n decreases in c_2.

A.4 Proof of Lemma 3. i)

Lemma 3. i): $\frac{d\delta_1^{min}}{dc_2} < 0$ for $c_2 \in (0, \frac{a}{2})$.

Proof. Let, $\delta_1^{min} = [1 - (\frac{\pi_1^c}{\pi_1^d})]/[1 - (\frac{\pi_1^n}{\pi_1^d})]$, we call $\frac{\pi_1^c}{\pi_1^d}$ *as* X and $\frac{\pi_1^n}{\pi_1^d}$ *as* Y. Now, $\frac{d\delta_1^{min}}{dc_2} = \frac{1}{(1-Y)^2}\Big[(1-X)\frac{dY}{dc_2} - (1-Y)\frac{dX}{dc_2}\Big]$, where $0 < 1 - X < 1 - Y$, as $X > Y$. Moreover, $\frac{d^2\delta_1^{min}}{dc_2^2} = \frac{1}{(1-Y)^4}\Big[(1-Y)^2\Big((1-X)\frac{d^2Y}{dc_2^2} - (1-Y)\frac{d^2X}{dc_2^2}\Big) + 2(1-Y)\frac{dY}{dc_2}\Big((1-X)\frac{dY}{dc_2} - (1-Y)\frac{dX}{dc_2}\Big)\Big]$. Let us now check, whether δ_1^{min} attains it maximum or minimum in the interval $(0, \frac{a}{2})$. If it attains the maximum or minimum then $\frac{d\delta_1^{min}}{dc_2} = 0$ (F.O.C.) and at that point $\delta_1^{min} = \frac{1-X}{1-Y} = \frac{\frac{dX}{dc_2}}{\frac{dY}{dc_2}}$. Now differentiating $\delta_1^{min} = \frac{\frac{dX}{dc_2}}{\frac{dY}{dc_2}}$ at the maxima or minima the following condition should hold $\frac{d\delta_1^{min}}{dc_2} = \frac{1}{(\frac{dY}{dc_2})^2}\left[\frac{dY}{dc_2}\frac{d^2X}{dc_2^2} - \frac{dX}{dc_2}\frac{d^2Y}{dc_2^2}\right] = 0$ or $\frac{\frac{dX}{dc_2}}{\frac{dY}{dc_2}} = \frac{\frac{d^2X}{dc_2^2}}{\frac{d^2Y}{dc_2^2}}$.

Hence, at maxima or minima we should have $\delta_1^{min} = \frac{1-X}{1-Y} = \frac{\frac{dX}{dc_2}}{\frac{dY}{dc_2}} = \frac{\frac{d^2X}{dc_2^2}}{\frac{d^2Y}{dc_2^2}}$. Therefore, at the maximum or minimum given F.O.C is satisfied $\frac{d\delta_1^{min}}{dc_2} = 0$; S.O.C. is not satisfied as $\frac{d^2\delta_1^{min}}{dc_2^2} = 0$. Hence, maximum or minimum does not exit in the interval $(0, \frac{a}{2})$. After substituting we observe that at $c_2 = 0$ as $s_1^* = 0.5$, $\delta_1^{min} = \frac{9}{17}$ and if c_2 tends to $\frac{a}{2}$ then $s_1^* = 1$ and $\delta_1^{min} = 0$, as firm 1 becomes the monopolist. Hence, $\frac{d\delta_1^{min}}{dc_2} < 0$ for $c_2 \in (0, \frac{a}{2})$. ∎

From Table 1: $a = 120$ and $c_2 = 1, 2, ..., 59$; we observe that $\frac{d\delta_1^{min}}{dc_2} < 0$ for $c_2 \in (0, \frac{a}{2})$.

A.5 Proof of Lemma 3. ii)

Lemma 3. ii): δ_2^{min} initially increases in c_2, reaches the maximum and then falls with increase in c_2 for $c_2 \in (0, \frac{a}{2})$.

Proof. Using the expressions used in the proof of Lemma (2), it is observed that at $c_2 = 0$ such that $s_1^* = 0.5$, $\frac{d\delta_2^{min}}{dc_2} = \frac{1}{(\pi_2^d - \pi_2^c)^2}\left[(\pi_2^d - \pi_2^n)(\frac{d\pi_2^d}{dc_2} - \frac{d\pi_2^c}{dc_2}) - (\pi_2^d - \pi_2^c)(\frac{d\pi_2^d}{dc_2} - \frac{d\pi_2^n}{dc_2})\right] > 0$ (after substituting the values). Hence, $\frac{d\delta_2^{min}}{dc_2} > 0$ at $c_2 = 0$. However, we know that if c_2 tends to $\frac{a}{2}$, then δ_2^{min} tends to 0 as firm 1 becomes the monopolist. As δ_2^{min} is continuous in c_2, it initially increases in c_2, reaches the maximum and then falls with increase in c_2 for $c_2 \in (0, \frac{a}{2})$. From Table 1: $a = 120$ and $c_2 = 1, 2, ..., 59$; we observe that for $c_2 \in (0, \frac{a}{2})$, δ_2^{min} initially increases in c_2, reaches the maximum and then falls with increase in c_2 (also shown in Figure 4).

Numerically we have proved that δ_2^{min} will have one critical point (where it attains maxima). As we can't solve s_1^*, hence getting a proper expression of δ_2^{min}, which is a function of s_1^*, explicitly in terms of only a and c_2 (the parameters of the model) is not possible. Hence, for understanding the shape of the δ_2^{min} curve one can only analyze numerically. Schmalensee (1987) in the conclusion of the paper says that "Four technologies for effecting collusion that are essentially equivalent in the symmetric case are quite distinct when sellers' costs differ, and plausible bargaining outcomes can only be analyzed numerically. Numerical analysis of collusive optima implied by axiomatic bargaining theory reveals a variety of distinctions and effects that are neither present in the symmetric case nor sensitive to the axiomatic solution concept employed." ∎

A.6 Proof of Lemma 3. iii)

Lemma 3. iii): $\delta_1^{min} < \delta_2^{min}$ when $c_2 \in (0, \frac{a}{2})$.

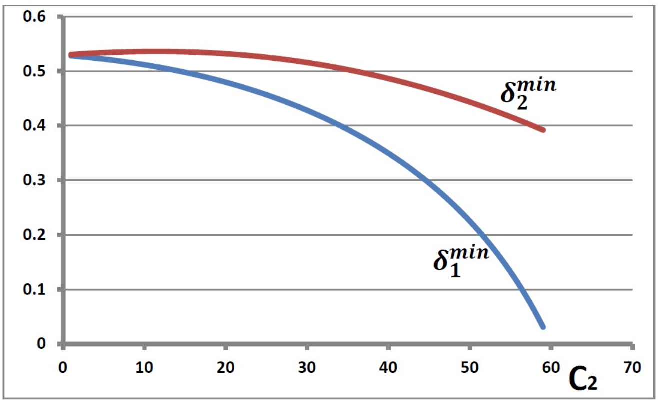

Figure 4: δ_1^{min} and δ_2^{min}, for $a = 120$ and $c_2 = 1, 2, ..., 59$

Proof. At $c_2 = 0$ such that $s_1^* = 0.5$; from equation (9) we get $\delta_1^{min} = \delta_2^{min} = \frac{9}{17}$ as then the firms are identical. From Lemma 3. i) and Lemma 3. ii), it is observed that δ_1^{min} falls in c_2, while δ_2^{min} initially increases in c_2 reaches the maximum and then falls for $c_2 \in (0, \frac{a}{2})$. Therefore, $\delta_1^{min} < \delta_2^{min}$ for $c_2 \in (0, \frac{a}{2})$, as otherwise, we would have $\delta_1^{min} = \delta_2^{min}$ for any $c_2 \in (0, \frac{a}{2})$, which is not possible as the unit cost of both the firms are different, i.e. one firm has a lower marginal cost, while the other firm has a higher marginal cost if the technology is not licensed.[32] ■

From Table 1: $a = 120$ and $c_2 = 1, 2, ..., 59$; we observe that for $c_2 \in (0, \frac{a}{2})$, $\delta_1^{min} < \delta_2^{min}$. In Figure 4 we have plotted the values of δ_1^{min} *and* δ_2^{min} for this example.

A.7 For Licensing Decision

For understanding whether equation (27) (see section 6.2.2 of the main text) holds or not, we use the Example and check it from Table 3. From the following table, we observe that equation (27) always holds as $L - B1 - B2 = M$ is always positive.

[32] Later in the paper under royalty licensing it is shown that as the unit cost of production becomes the same for the firms (even though the effective cost of production are different) and the firms produce the monopoly output under the cartel agreement and the industry profit is maximum, the critical discount factor for both the firms will be the same. However, under no-licensing the firms are not able to produce the monopoly output under the cartel agreement as the unit costs of production are different.

Table 3

a	c_2	s_1^*	$B1$	$B2$	δ_2^{min}	r^*	L	M
120	23	0.683689	2452.234	724.4889	0.528491291	19.6255	3418.878	242.155
120	24	0.692108	2482.14	688.1463	0.527057448	20.65119	3427.963	257.6776
120	25	0.700546	2512.15	652.6805	0.525496878	21.67858	3436.83	271.9996
120	26	0.709002	2542.261	618.0929	0.523808637	22.70779	3445.477	285.1225
120	27	0.717474	2572.47	584.3854	0.521991844	23.7389	3453.904	297.0478
120	28	0.725961	2602.773	551.5598	0.520045662	24.77203	3462.11	307.7768
120	29	0.73446	2633.167	519.6183	0.517969307	25.80729	3470.095	317.3104
120	30	0.74297	2663.646	488.5633	0.515762028	26.8448	3477.859	325.6495
120	31	0.751489	2694.209	458.3972	0.513423124	27.88468	3485.401	332.7945
120	32	0.760015	2724.85	429.1232	0.510951925	28.92707	3492.719	338.7458
120	33	0.768549	2755.566	400.7443	0.508347795	29.9721	3499.814	343.5032
120	34	0.777087	2786.354	373.2641	0.50561014	31.01993	3506.684	347.0661
120	35	0.785629	2817.208	346.6866	0.502738364	32.07072	3513.328	349.4334
120	36	0.794174	2848.126	321.0161	0.499731914	33.12462	3519.746	350.6038
120	37	0.802721	2879.103	296.2574	0.496590235	34.18181	3525.936	350.5751
120	38	0.811269	2910.136	272.4156	0.493312787	35.24247	3531.896	349.3446
120	39	0.819817	2941.22	249.4965	0.489899029	36.30681	3537.626	346.909
120	40	0.828365	2972.353	227.5066	0.486348414	37.37502	3543.123	343.2642
120	41	0.836912	3003.529	206.4525	0.482660376	38.44732	3548.387	338.4053
120	42	0.845458	3034.746	186.342	0.478834357	39.52393	3553.414	332.3267
120	43	0.854004	3065.999	167.1833	0.474869726	40.60509	3558.204	325.0216
120	44	0.862548	3097.286	148.9853	0.470765845	41.69107	3562.754	316.4823
120	45	0.871092	3128.603	131.7579	0.466522023	42.78212	3567.061	306.7
120	46	0.879635	3159.946	115.5118	0.462137511	43.87853	3571.122	295.6647
120	47	0.888179	3191.312	100.2587	0.457611496	44.98061	3574.935	283.365
120	48	0.896724	3222.698	86.01132	0.452943088	46.08866	3578.497	269.7883
120	49	0.905271	3254.101	72.7834	0.448131303	47.20302	3581.804	254.9201
120	50	0.913822	3285.518	60.59003	0.44317506	48.32406	3584.852	238.7446
120	51	0.922378	3316.947	49.44757	0.43807316	49.45215	3587.638	221.2438
120	52	0.930941	3348.385	39.37386	0.432824272	50.58771	3590.157	202.3981
120	53	0.939512	3379.829	30.3883	0.427426917	51.73116	3592.403	182.1853
120	54	0.948095	3411.279	22.51204	0.421879451	52.88297	3594.372	160.581
120	55	0.956691	3442.732	15.7681	0.416180044	54.04364	3596.058	137.5583
120	56	0.965305	3474.186	10.18165	0.410326656	55.2137	3597.455	113.0872
120	57	0.973938	3505.64	5.780128	0.404317015	56.39372	3598.555	87.13455
120	58	0.982596	3537.094	2.593566	0.398148586	57.58432	3599.352	59.66362
120	59	0.991281	3568.548	0.654837	0.391818541	58.78617	3599.836	30.63379

B Cartel under no-licensing with side-payments

Here, we slightly desist from section 3 and discuss the case under no-licensing such that $s_2 = 0$, i.e. if the cartel is formed then only firm 1 produces monopoly output $(\frac{a}{2})$ that maximizes the industry profit and firm 2 doesn't produce any output, but receives a side payment in return from firm 1. This can be thought of as the scenario where the anti-trust agencies are not active enough to detect collusion. If the cartel is not formed then the amount of outputs produced by firm 1 and firm 2 in each period are respectively $q_1^n = \frac{a+c_2}{3}$ *and* $q_2^n = \frac{a-2c_2}{3}$ and the profits for the firms are $\pi_1^n = \frac{1}{9}(a+c_2)^2 \quad and \quad \pi_2^n = \frac{1}{9}(a-2c_2)^2$. The present discounted Nash-Cournot profits (such that cartel is not formed) of firm 1 and firm 2 are $\pi_1^{NN} = \frac{1}{1-\delta}\frac{(a+c_2)^2}{9}$ *and* $\pi_2^{NN} = \frac{1}{1-\delta}\frac{(a-2c_2)^2}{9}$ respectively (as mentioned in equation (2) of the main text).

Let us call $s \in (0,1)$ as the share of firm 1 in the industry (monopoly) profit $\frac{a^2}{4}$ under the cartel agreement. The cartel agreement is such that firm 2 completely shut down its production, but receives a share $(1-s)$ of the monopoly profit in each period. Hence, the profit of firm 1 and firm 2 respectively under collusion is given by

$$\pi_1^c = \frac{sa^2}{4} \quad and \quad \pi_2^c = \frac{(1-s)a^2}{4}.$$

Moreover, s is determined through Nash Bargaining. Thus optimal sharing agreement is determined by

$$\max_s V = (\pi_1^c - \pi_1^n)(\pi_2^c - \pi_2^n)$$

such that $\pi_i^c > \pi_i^n$ for $i = 1,2$. The optimal value of s is $s^* = \frac{3a^2+8ac_2-4c_2{}^2}{6a^2}$ and it increases in c_2. At $c_2 = 0$ we have $s^* = 0.5$ and when $c_2 = \frac{a}{2}$ then $s^* = 1$.

Under Grim-Trigger Strategy, the tacit collusion is stable if $\delta_i > \frac{\pi_i^d - \pi_i^c}{\pi_i^d - \pi_i^n} \equiv \delta_i^{min}$. For firm 1, it can be said that the maximum gain under deviation is by setting $s = 0$, not paying anything to firm 2. Hence, $\pi_1^d = \frac{a^2}{4}$. Therefore, $\delta_1^{min} = \frac{3(3a^2-8ac_2+4c_2{}^2)}{2(5a^2-8ac_2-4c_2{}^2)}$. Moreover, $\frac{\partial \delta_1^{min}}{\partial c_2} = \frac{-24a(a^2-4ac_2+4c_2{}^2)}{(5a^2-8ac_2-4c_2{}^2)^2} < 0$. At $c_2 = 0$, $\delta_1^{min} = \frac{9}{10}$ and when c_2 tends to $\frac{a}{2}$, then δ_1^{min} tends to zero. Similarly for firm 2, given firm 1 produces $\frac{a}{2}$ amount of output solely, if it wants to deviate it should produce $\frac{a-2c_2}{4}$ units of output and earn $\pi_2^d = \frac{(a-2c_2)^2}{16}$. However, for $c_2 \in (0, \frac{a}{2})$, $\pi_2^d - \pi_2^c = \frac{-3a^2+4ac_2+4c_2^2}{48} < 0$. Hence, firm 2 will never deviate from the cartel agreement and not produce in each periods. Therefore, if technology is not licensed firms will tacitly collude if $\delta > \delta_1^{min}$. The present discounted profits of firm 1 and firm 2 if the cartel is formed under no-licensing are

$$\pi_1^{NC} = \frac{s^* a^2}{4(1-\delta)} \qquad and \qquad \pi_2^{NC} = \frac{(1-s^*)a^2}{4(1-\delta)}$$

respectively. *Therefore, if technology is not licensed in the first stage, then cartel is formed if $\delta > \delta_1^{min}$, whereas if technology is licensed in the first stage via fixed-fee or royalty or two-part tariff, then cartel is formed if $\delta > \frac{9}{17}$* (as observed in the main text Lemma 5).

Moreover, $\delta_1^{min} = \frac{3(3a^2-8ac_2+4c_2{}^2)}{2(5a^2-8ac_2-4c_2{}^2)} > \frac{9}{17}$ if $c_2 < \frac{21a}{46}$, and $\delta_1^{min} \leq \frac{9}{17}$ if $c_2 \in [\frac{21a}{46}, \frac{a}{2})$. *Hence, if the cost difference between the firms is low (high), then the possibility of cartel formation is more (less) under licensing than under no-licensing.* This is similar to what has been observed in the main text, where we have assumed s_2 as positive (see Proposition 1 of the main text). In the following part of this section we discuss what happens in the first stage.

B.1 Consider $c_2 < \frac{21a}{46}$

When $c_2 < \frac{21a}{46} \implies \delta_1^{min} > \frac{9}{17}$. This implies that we have three cases as follows:
i) Case 1: $\delta_1^{min} > \frac{9}{17} \geq \delta$.

ii) Case 2: $\delta_1^{min} \geq \delta > \frac{9}{17}$ and
iii) Case 3: $\delta > \delta_1^{min} > \frac{9}{17}$.

B.1.1 Case 1

Let us first consider $\delta_1^{min} > \frac{9}{17} \geq \delta$, such that both in case of licensing and no-licensing of technology, cartel formation is not possible. The profit (present value) of firm 1 in case of fixed-fee licensing is given by $\pi_1^{FN} = \frac{1}{1-\delta}\frac{a^2}{9} + F$ and of firm 2 is $\pi_2^{FN} = \frac{1}{1-\delta}\frac{a^2}{9} - F$. As cartel is not formed under royalty licensing, the present discounted profit of firm 1 is given by $\pi_1^{RN} = \frac{1}{1-\delta}\left[\frac{(a+r)^2}{9} + r\frac{(a-2r)}{3}\right]$, while for firm 2 is $\pi_2^{RN} = \frac{1}{1-\delta}\frac{(a-2r)^2}{9}$. On the other hand, the profits for firm 1 and firm 2 in case of no-licensing are given by $\pi_1^{NN} = \frac{1}{1-\delta}\frac{(a+c_2)^2}{9}$ *and* $\pi_2^{NN} = \frac{1}{1-\delta}\frac{(a-2c_2)^2}{9}$ respectively.

Under **fixed-fee licensing**, the fixed-fee charged by firm 1 is $F^* = \frac{1}{1-\delta}\left[\frac{a^2}{9} - \frac{(a-2c_2)^2}{9}\right]$, such that firm 2 is indifferent between licensing and no-licensing. Now, after substituting F^* we get $\pi_1^{FN} > \pi_1^{NN} \implies c_2 < \frac{2a}{5}$. However, this is not always true. Firm 1 will license its technology via fixed-fee only if $c_2 < \frac{2a}{5}$, while technology will not be licensed if $c_2 \in (\frac{2a}{5}, \frac{21a}{46})$. Similarly, under **royalty licensing**, the per-unit royalty that is charged is $r^* = c_2$. As in the main text section 6.1.1, it can be said that royalty licensing is superior (the optimal form of licensing) to fixed-fee and technology will always be licensed. As cartel is not formed in the post-licensing stage, the welfare also increases after licensing. *Therefore, if* $c_2 < \frac{21a}{46}$ *and* $\delta_1^{min} > \frac{9}{17} \geq \delta$, *then technology will be licensed via royalty and post-licensing cartel will not be formed, but the welfare will increase.*

B.1.2 Case 2

Consider the second case when $\delta_1^{min} \geq \delta > \frac{9}{17}$. This means that the cartel is formed if the technology is licensed, but the cartel is not formed if the technology is not licensed. Therefore, as in the main text section 6.1.2 we observe that *if* $c_2 < \bar{c}_2$ *and* $\delta_1^{min} \geq \delta > \frac{9}{17}$, *then technology will be licensed via two-part tariff and post-licensing cartel will be formed, but the welfare will fall.*

B.1.3 Case 3

Now assume $\delta > \delta_1^{min} > \frac{9}{17}$. This means that tacit collusion is possible in both the scenarios (after licensing or if technology is not licensed). The profit (present value) of firm 1 in case of fixed-fee licensing is given by $\pi_1^{FC} = \frac{1}{1-\delta}\frac{a^2}{8} + F$ and of firm 2 is $\pi_2^{FC} = \frac{1}{1-\delta}\frac{a^2}{8} - F$. Similarly under royalty licensing the present discounted profit of firm 1 is given by $\pi_1^{RC} = \frac{1}{8(1-\delta)}\left[a^2 + 4ar - 4r^2\right]$, while for firm 2 is $\pi_2^{RC} = \frac{1}{8(1-\delta)}\left[a^2 - 4ar + 4r^2\right]$. The present discounted profits of firm 1 and firm 2 as the cartel is formed under no-licensing are $\pi_1^{NC} = \frac{s^* a^2}{4(1-\delta)}$ *and* $\pi_2^{NC} = \frac{(1-s^*)a^2}{4(1-\delta)}$ respectively.

Under **fixed-fee licensing**, the optimal fixed-fee charged is $F^* = \frac{a^2}{4(1-\delta)}(s^* - \frac{1}{2})$, such that $\pi_2^{FC} = \pi_2^{NC}$. Now, comparing the profits of firm 1 after substituting F^*, we get $\pi_1^{FC} = \pi_1^{NC}$. Hence, firm 1 is indifferent between licensing and no-licensing. Under **royalty licensing**, firm 1 sets r^* as high as possible such that $\pi_2^{RC} \geq \pi_2^{NC}$ or $\frac{1}{8(1-\delta)}\Big[a^2 - 4ar + 4r^2\Big] \geq \frac{1}{1-\delta}\frac{(1-s^*)a^2}{4} = \frac{1}{1-\delta}H_2$ (say). If, r^* is set such that $\pi_2^{RC} = \pi_2^{NC}$, then we get $r^* = \frac{a-2\sqrt{2H_2}}{2}$. Therefore, two-part tariff licensing is not possible in this context. Now, comparing the profits of firm 1 after substituting r^* we get $\pi_1^{RC} = \pi_1^{NC}$. *Hence, firm 1 is indifferent between licensing and no-licensing, but the cartel will be formed.*

B.2 Consider $c_2 \geq \frac{21a}{46}$

When $c_2 \geq \frac{21a}{46} \implies \delta_1^{min} \leq \frac{9}{17}$. This implies that we have three cases as follows:
i) Case 1: $\delta \leq \delta_1^{min} \leq \frac{9}{17}$.
ii) Case 2: $\delta_1^{min} < \delta \leq \frac{9}{17}$ and
iii) Case 3: $\delta_1^{min} \leq \frac{9}{17} < \delta$.

B.2.1 Case 1

Let us first consider the case when $\delta \leq \delta_1^{min} \leq \frac{9}{17}$. Hence, both in case of licensing and no licensing of technology, cartel formation is not possible. As discussed in the previous section, under **fixed-fee licensing**, we get $F^* = \frac{1}{1-\delta}\Big[\frac{a^2}{9} - \frac{(a-2c_2)^2}{9}\Big]$ and $\pi_1^{FN} \geq \pi_1^{NN}$ if $c_2 \leq \frac{2a}{5}$. Hence, technology will not be licensed via fixed-fee as $c_2 \geq \frac{21a}{46}$. Similarly, under **royalty licensing**, the per-unit royalty r is charged by firm 1 such that, $\pi_2^{RN} = \pi_2^{NN} \implies r^* = c_2$, and therefore $\pi_1^{RN} > \pi_1^{NN}$. *Therefore, if* $c_2 \geq \frac{21a}{46}$ *and* $\delta \leq \delta_2^{min} \leq \frac{9}{17}$, *then technology will be licensed via royalty. Post-licensing the cartel will not be formed and the welfare will increase.*

B.2.2 Case 2

Let us first consider the second case when $\delta_1^{min} < \delta \leq \frac{9}{17}$, i.e. under no-licensing of technology, cartel formation is possible. The profit (present value) of firm 1 in case of fixed-fee licensing is given by $\pi_1^{FN} = \frac{1}{1-\delta}\frac{a^2}{9} + F$ and of firm 2 is $\pi_2^{FN} = \frac{1}{1-\delta}\frac{a^2}{9} - F$. As cartel is not formed under royalty licensing, the present discounted profit of firm 1 is given by $\pi_1^{RN} = \frac{1}{1-\delta}\Big[\frac{(a+r)^2}{9} + r\frac{(a-2r)}{3}\Big]$, while for firm 2 is $\pi_2^{RN} = \frac{1}{1-\delta}\frac{(a-2r)^2}{9}$. The present discounted profits of firm 1 and firm 2 as the cartel is formed under no-licensing are $\pi_1^{NC} = \frac{1}{1-\delta}\frac{s^*a^2}{4} = \frac{1}{1-\delta}H_1$ (say) and $\pi_2^{NC} = \frac{1}{1-\delta}\frac{(1-s^*)a^2}{4} = \frac{1}{1-\delta}H_2$ (say) respectively, where H_1 and H_2 are the per-period profits of firm 1 and firm 2, if the cartel is formed in the absence of licensing.

Under **fixed-fee licensing** we find that, optimal fixed-fee charged by firm 1 is $F^* = \frac{1}{1-\delta}\Big[\frac{a^2}{9} - H_2\Big]$, such that $\pi_2^{FN} = \pi_2^{NC}$. Now, comparing the profits of firm 1 after substituting F^*, we get $\pi_1^{FC} = \frac{1}{1-\delta}\Big[\frac{2a^2}{9} - H_2\Big] > \pi_1^{NC} = \frac{1}{1-\delta}H_1 \implies \frac{2a^2}{9} > H_1 + H_2$, which is never true. Therefore, the technology will not be licensed via fixed-fee and cartel will be formed. While, under **royalty licensing**, the per-unit royalty r is charged by firm 1 such that, $\pi_2^{RN} \geq \pi_2^{NC} :\implies \frac{1}{1-\delta}\frac{(a-2r)^2}{9} \geq \frac{1}{1-\delta}H_2$. Firm 1 will set r as high as possible as its post-licensing profit increases in the royalty rate. Therefore, firm 1 will set r^* such that $\frac{1}{1-\delta}\frac{(a-2r^*)^2}{9} = \frac{1}{1-\delta}H_2$ or $r^* = \frac{a-3\sqrt{H_2}}{2}$. After substituting r^* we get $\pi_1^{RN} = \frac{1}{1-\delta}\Big[\frac{(a+r^*)^2}{9} + r^*\frac{(a-2r^*)}{3}\Big] > \pi_1^{NC} = \frac{1}{1-\delta}H_1 \implies \frac{2a^2}{9} + \frac{ar^* - r^{*2}}{9} > H_1 + H_2$. This is never possible. *Therefore, if* $c_2 \geq \frac{21a}{46}$ *and* $\delta_1^{min} < \delta \leq \frac{9}{17}$*, then technology will not be licensed via royalty, but the cartel will be formed.*

B.2.3 Case 3

As in Case 3, section B.1.3, firm 1 is indifferent between licensing and no-licensing both in the case of fixed-fee and royalty licensing. *Hence, firm 1 is indifferent between licensing and no-licensing, but the cartel will be formed.*

B.3 Results

The main results of this discussion are presented in the following table:

Table B

Cost	δ	**Licensing Yes/No**	**Cartel**	**Welfare**
$c_2 < \frac{21a}{46}$	$\delta_1^{min} > \frac{9}{17} \geq \delta$	Royalty	No	Increases
$c_2 < \frac{21a}{46}$	$\delta_1^{min} \geq \delta > \frac{9}{17}$	Two-part tariff	Yes	Falls
$c_2 < \frac{21a}{46}$	$\delta > \delta_1^{min} > \frac{9}{17}$	Indifferent	Yes	...
$c_2 \geq \frac{21a}{46}$	$\delta \leq \delta_1^{min} \leq \frac{9}{17}$	Royalty	No	Increases
$c_2 \geq \frac{21a}{46}$	$\delta_1^{min} < \delta \leq \frac{9}{17}$	No	Yes	...
$c_2 \geq \frac{21a}{46}$	$\delta_1^{min} \leq \frac{9}{17} < \delta$	Indifferent	Yes	...

In the above table (Table B), we observe that in the first stage of the game technology will not always be transferred as either firm 1 is indifferent between licensing and no-licensing or sometimes no-licensing is optimal. In the next stage, the cartel may or may not be formed as shown in the fourth column. Moreover, after licensing welfare may fall. For firm 1 the optimal form of licensing is either royalty or two-part tariff. For understanding how licensing facilitates or obstructs cartel formation we look into the second and the fifth case of the table. Firstly, from the second case, i.e. $c_2 < \frac{21a}{46}$ and $\delta_1^{min} \geq \delta > \frac{9}{17}$, it is observed that the cartel is formed only if technology is licensed. Here effectively licensing is facilitating collusion and it hurts welfare. In such cases, the welfare under no-licensing is more than under licensing. However, if $c_2 \geq \frac{21a}{46}$ and $\delta_1^{min} < \delta \leq \frac{9}{17}$ (fifth case) then cartel is formed only if technology is not licensed. However, in such a situation firm 1 will not license its technology, but the cartel will be formed. This shows that licensing cannot deter collusion.

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