Secular Increase in Economic Inequality Accompanying a Constant Output/Capital Ratio

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Abstract

Many empirical studies have concluded that the level of economic inequality has increased in many countries since the mid-twentieth century, but the output/capital ratio has barely changed during the same period. This secular increase in economic inequality indicates the existence of some essential heterogeneity among households. In this paper, I attempt to uncover this essential heterogeneity by focusing on rate of time preference (RTP) and persistent economic rents. I find that if heterogeneous RTP were the cause behind the observed secular increase in economic inequality, it could not coexist with a constant output/capital ratio. On the other hand, I find that heterogeneous persistent economic rent can coexist simultaneously with both phenomena and consider it the most likely the cause behind the increase in economic inequality accompanied by a constant output/capital ratio. I support this conclusion with both a theoretical analysis and numerical examinations.

JEL Classification: D31, E23, I31
Keywords: Output/capital ratio, Heterogeneity, Inequality; Rents, Time preference

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1 INTRODUCTION

Many empirical studies have concluded that the level of economic inequality has increased in many countries since the mid-twentieth century (Piketty and Saez, 2003; Piketty, 2013; Saez and Zucman, 2016). Before the Great Depression and World War II, however, it was far higher than the current level. It sharply decreased in the era of the Great Depression and World War II, but in the mid-twentieth century, it again began to increase and has continued to do so.

During the same period, however, there is an important economic indicator that has barely changed: the output/capital ratio. The constancy of this ratio was first emphasized by Kaldor (1957) who argued that it is one of six remarkable historical constancies. Piketty (2013) showed that the output/capital ratio does not appear to have changed very much since the late nineteenth century.

The secular increase in economic inequality indicates that there is an essential heterogeneity among households because if there is no heterogeneity, no economic inequality can emerge. This essential heterogeneity should have the following properties: (1) its effect is not temporal but persistent and cannot easily disappear or be removed, (2) it makes the level of economic inequality increase considerably and continuously over a long period, and (3) at the same time, it has no power to greatly change the output/capital ratio. The purpose of this paper is to uncover this essential heterogeneity.

Nevertheless, only a few heterogeneities exhibit the above properties. The rate of time preference (RTP) exhibits at least properties (1) and (2). It can result in an extreme inequality in which the most advantaged household eventually owns all the capital in the economy (Becker, 1980). In addition, Harashima (2020b)[1] showed that these same two properties exhibited by heterogeneous RTP are also present in persistent economic rents. Harashima (2020b, 2020c) showed that there are two sources of persistent economic rents: those derived from ranking preference and ranking value (Harashima, 2016, 2017b, 2018b, 2018d)[2] and those derived from mistakes in business dealings (Harashima, 2020c). In this paper, I examine whether these two heterogeneities, RTP and persistent economic rents, exhibit not only properties (1) and (2) but also property (3), the inability to affect a major change in the output/capital ratio.

Harashima (2010, 2012, 2014)[3] also showed that even if RTPs and persistent economic rents are heterogeneous, there is a state called sustainable heterogeneity (SH) in which all optimality conditions of all heterogeneous households can be achieved, Although the “correct” and “true” SH may not be easily achieved, an approximate SH

[1] Harashima (2020b) is also available in Japanese as Harashima (2021d).
[2] Harashima (2016, 2018b) are also available in Japanese as Harashima (2018a, 2021b) respectively.
[3] Harashima (2010, 2012) are also available in Japanese as Harashima (2017a, 2020a) respectively.

can be achieved instead in which the number of votes cast in an election in response to increases in the level of economic inequality is equivalent to the number in response to decreases (Harashima 2018c [4]). However, an approximate SH has important vulnerabilities, and therefore it remains possible that the level of economic inequality increases considerably under SH because of heterogeneous RTPs and persistent economic rents (Harashima 2021c).

In this paper, I examine which heterogeneity, RTP or persistent economic rent, is most likely to be the cause of the considerable secular increase in economic inequality accompanying a constant output/capital ratio. I show that the cause is most likely heterogeneous persistent economic rents. If heterogeneous RTP were the cause, then the considerable increase in economic inequality could not coexist with the constancy of the output/capital ratio because a large amount of heterogeneity in RTP will result in both an increase in economic inequality and a decrease in the output/capital ratio. In contrast, heterogeneous persistent economic rents do not affect the output/capital ratio but can cause major increases in economic inequality. I confirm this theoretical conclusion by numerical examination. The results of this examination sufficiently support the theoretical conclusion.

2 TWO FUNDAMENTAL FACTS

2.1 Secular increase in economic inequality

Numerous empirical studies have concluded that income inequality has increased in many countries since the 1980s (Piketty, 2003, 2013; Piketty and Saez, 2003; Atkinson et al., 2011; Parker, 2014). In addition, within-country wealth inequality has increased in many countries during the same period (Piketty, 2013; Saez and Zucman, 2016). An important point is that this considerable increase in economic inequality is not a temporal phenomenon but rather a persistent phenomenon enduring over a half a century. This phenomenon may be a part of low frequency fluctuation of the level of inequality since the late nineteenth century.

Piketty and Saez (2003), Piketty (2013), and Saez and Zucman (2016) showed that in many industrialized countries, the level of economic inequality underwent a major decrease after the Great Depression and World War II, but then began to increase again in a few decades. It continued to increase for half a century as if it were returning to the level before the Great Depression and World War II. This long-term trend implies that the period after the Great Depression and World War II was a rather exceptional one and that a high level of economic inequality is the normal state.

[4] Harashima (2018c) is also available in Japanese as Harashima (2019a).

2.2 Constancy of output/capital ratio

Kaldor (1957) noted six remarkable historical constancies, which are known as Kaldor's facts. One of them is the fact that the output/capital ratio, i.e.,

$$\frac{Y}{K}$$

is roughly constant over long periods of time. Piketty (2013) showed that the output/capital ratio does not appear to have changed very much since the late nineteenth century. Indeed, it has continued to be basically constant from the end of World War II to now and has had no discernable increasing or decreasing trend (Fig. 1).

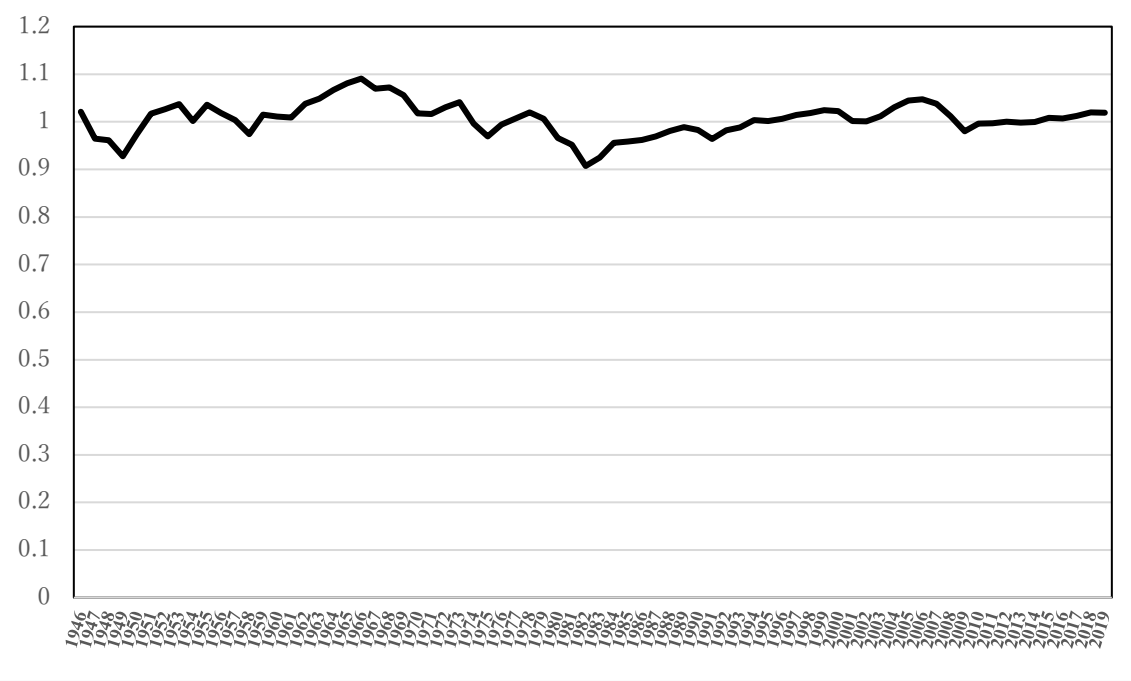

Fig. 1: Output/capital ratio of the United States[5]

Furthermore, the constancy of the output/capital ratio has been regarded as an essential element, not only in familiar Ramsey-type growth models but also in many other types of economic growth models (e.g., Solow, 1956; Romer, 1986). In this sense, the constancy of the output/capital ratio is generally accepted not only as an empirical fact but also as a quality required by economic theory.

[5] Data of output and capital are "Real Gross Domestic Product, Quantity Indexes" and "Chain-Type Quantity Indexes for Net Stock of Private Nonresidential Fixed Assets" in GDP statistics published by the Bureau of Economic Analysis and U.S. Department of Commerce, respectively.

3 THE MECHANISM OF INCREASING ECONOMIC INEQUALITY

3.1 Essential heterogeneity

Several explanations for the recent rise in income inequality have been presented. Among them, "skill-biased technological change" (SBTC) was the most favored explanation until the early 2000s (Katz and Murphy, 1992; Autor et al., 1998, 2003). However, SBTC has not been sufficiently supported by empirical studies (Card and DiNardo, 2002). On the other hand, explanations based on globalization have also gained wide acceptance; in particular, those based on the Stolper-Samuelson theorem (Stolper and Samuelson, 1941) were favored before the 21st century. These explanations were also not sufficiently supported by the empirical studies conducted around the decade of the 2000s (Leamer, 1998; Goldberg and Pavcnik, 2007). Since then, the main underlying mechanisms of globalization-based explanations changed to heterogeneity of firms, labor market frictions, and offshoring of tasks (Helpman, 2016).

In any case, a factor that can generate a secular increase in economic inequality for half a century should not be temporal but persistent and should continue to have an effect for a long period. The factor should be robust and should not disappear soon or be easily removed, and it should rarely be influenced by various kinds of shocks. In addition, because inequality cannot exist if people are homogeneous, the existence of economic inequality requires a certain amount of heterogeneity among people. Also, because this heterogeneity generates considerable increases in economic inequality, it should have a large and broad affect upon economic activities. Given these conditions, such a factor should be an essential heterogeneity in economic activities that does not disappear quickly and cannot be easily removed.

However, such essential heterogeneities are few. One of them is the heterogeneity in RTP that results in an extreme inequality such that the most advantaged household eventually owns all the capital in an economy (Becker 1980). Another is the heterogeneity in persistent economic rents, which also exhibits some of the same properties as heterogeneous RTPs (Harashima, 2020b). I therefore focus on these two heterogeneities as I examine what gives rise to the fundamental facts (i.e., the secular increase in economic inequality and constant output/capital ratio).

3.2 Consequences of heterogeneity

3.2.1 Heterogeneous RTPs

Becker (1980) showed that having heterogeneous RTPs among households generates extreme economic inequalities; i.e., all capital will eventually be owned by the most

advantaged (lowest RTP) household. This means that a small degree of heterogeneity in certain characteristics of people can lead to a huge disparity in outcomes. Because RTP is one of the most fundamental and essential preferences of people in economic activities and its heterogeneity among people will continue almost indefinitely, its effect can prevail over a long period, and it can generate a long-lasting phenomenon. Therefore, the heterogeneity in RTPs is a promising candidate for the cause of the secular increase in economic inequality.

Note that Harashima (2018c) posited a procedure under which households keep their capital-wage ratio (CWR) at the maximum degree of comfortability (MDC) and showed that the behavior of households based on rational expectations under the RTP-based procedure is equivalent to that under the MDC-based procedure. Also, if the CWRs kept by households at MDC ("CWR at MDC") are heterogeneous, there is no guarantee of a steady state like the one in the case of heterogeneous RTPs (Harashima 2018c). Because behaviors under the MDC- and RTP-based procedures are equivalent, the consequences of heterogeneity in CWR at MDC are basically identical to the consequences of heterogeneity in RTPs. In the following sections of this paper, I generally use models based on the RTP-based procedure for simplicity, but where necessary, I add explanations based on the MDC-based procedure.

3.2.2 Heterogeneous persistent economic rents

Harashima (2020b) showed that an amplification mechanism similar to that for RTP exists for persistent economic rents and that a different type of economic rent exists that had not been discussed previously: monopoly profits (rents) derived from people's ranking preferences (Harashima 2016, 2018b). These rents enable some individuals to be superstars in the world of sport, art, or music (Harashima, 2016, 2018b) and enable some corporate executives to earn extremely high compensation (Harashima, 2018d). Ranking preference is an important element of product differentiation that allows companies to accrue large amounts of monopoly rent (Harashima, 2017b), and product differentiation is one of the most important and commonly pursued strategies companies use to prosper (Porter, 1980, 1985). Consequently, the monopoly rents derived from product differentiation that was designed to benefit from ranking preference are highly likely to be found across different economies.

Harashima (2020c) has furthermore discussed the importance of another kind of economic rent; that which arises from the heterogeneity in mistakes made in business. Here, a "mistake" means, for example, that a household purchases a product at a price that is higher than the cost to produce it plus a normal margin, or that a worker accepts a wage that is lower than their marginal productivity would indicate is appropriate. Because there is certainly some heterogeneity in the ability to make fewer mistakes in business dealings

among people, the economic rents from such mistakes are likely to be ubiquitous and present at a large scale across an economy (Harashima 2020c).

An important nature of these heterogeneities is that some households or family lines will persistently obtain these economic rents with a higher probability than others because the intrinsic abilities for obtaining these rents are likely exogenously given and unevenly distributed. This makes it highly likely that abilities such as those related to obtaining economic rents are exogenously and unevenly given (Harashima, 2020d, 2020e[6]). Therefore, the average abilities of people in a given group (or family line) will remain different from those in other groups (Harashima, 2020d, 2020e), which means that there are groups (or family lines) that obtain persistent economic rents and hold them indefinitely. The result is that many economic rents will be enjoyed persistently by only a small number of households and family lines; that is, the persistent economic rents will be distributed very unevenly.

Due to the persistent nature of these economic rents, the heterogeneity among these rents can generate long-lasting phenomena such as the secular increase in economic inequality. For this reason, the heterogeneity in persistent economic rents is also a promising candidate for the cause behind the secular increase in economic inequality.

3.3 Sustainable heterogeneity

However, the heterogeneities in these elements, i.e., RTP (or equivalently, CWR at MDC) and persistent economic rents, do not necessarily result in extreme economic inequality. This is because of the existence of SH (see the Appendix) as demonstrated by Harashima (2010, 2012, 2014). Although the state of SH is not necessarily achieved naturally, it can be achieved if a government intervenes appropriately, with the result that extreme economic inequality can be avoided.

Harashima (2018c) also showed that under typical circumstances, an approximate SH can be formed by a government rather than a "pure" one (see the Appendix), and it is not necessarily guaranteed that an approximate SH would always keep the level of economic inequality unchanged. This occurs because such an SH is merely an approximation and reflects the state in which the number of votes cast in an election in response to increases in the level of economic inequality is equivalent to those cast in response to decreases in the level of inequality. Households cast votes in accordance with their subjective, and probably systematically biased, considerations about their surrounding situations. Furthermore, there are important vulnerabilities in approximate SH (Harashima 2021c) because households cannot know the "true" and "correct" SH and may misunderstand the surrounding economic situation, possibly severely so. In particular three types of misunderstandings are possible. Hence,

[6] Harashima (2020e) is also available in Japanese as Harashima (2022).

democracy does not automatically guarantee that the level of economic inequality does not increase considerably. Therefore, the two types of essential heterogeneities (RTP and persistent economic rents) can still cause a major increase in economic inequality even if an approximate SH is achieved.

4 THEORETICALLY MOST LIKELY CAUSE

In this section, I conduct a theoretical examination of which heterogeneity (RTP or persistent economic rent) is the most likely cause of the observed continual increase in economic inequality.

4.1 Effect of heterogeneous RTP (CWR at MDC)

To begin with, I examine whether the heterogeneity in RTP (or equivalently CWR at MDC) is the cause, but before doing so, I examine the case of homogeneous households for comparison. Consider a Ramsey-type growth model in which each of homogeneous households maximizes its expected utility

$$E\int_0^\infty \exp(-\theta t)u(c_t)dt$$

in the same way subject to

$$\frac{dk_t}{dt} = f(A, k_t) - c_t \ ,$$

where y_t, k_t, and c_t are production, capital, and consumption per capita, respectively, in period t; A represents the state of technology; θ (> 0) is RTP; u is the utility function; $y_t = f(A, k_t)$ is the production function; and E is the expectation operator. The production function is assumed to be Harrod neutral such that $y_t = A^\alpha k_t^{1-\alpha}$, where α ($0 < \alpha < 1$) is a constant.

At steady state,

$$\theta = \frac{\partial y_t}{\partial k_t}$$

holds. By the production function,

$$\frac{\partial y_t}{\partial k_t} = (1-\alpha)A^{\alpha}k_t^{-\alpha} = (1-\alpha)\frac{A^{\alpha}k_t^{1-\alpha}}{k_t} = (1-\alpha)\frac{y_t}{k_t} \quad , \tag{1}$$

so

$$\theta = (1-\alpha)\frac{y_t}{k_t} \tag{2}$$

at steady state; that is, RTP (θ) is equivalent to $\frac{y_t}{k_t}$ times $(1-\alpha)$.

I next examine the case of heterogeneous households. Suppose that households are identical except for RTP. Because approximate SH is vulnerable to certain weaknesses, more capital will come to be owned over time by lower RTP households (who are most advantaged), thereby increasing the level of economic inequality. Taking equations (1) and (2) into consideration, I assume that heterogeneous households behave independently according to their own RTPs, which vary by household.

Given equation (2), let

$$\tilde{\theta}_t = (1-\alpha)\frac{\tilde{y}_t}{\tilde{k}_t} \tag{3}$$

where $\tilde{y}$ and $\tilde{k}_t$ are the average values of y_t and k_t in the economy. $\tilde{\theta}_t$ can be interpreted as the average RTP in the economy in period *t*. Because greater amounts of capital will come to be owned over time by lower RTP households, the value of $\tilde{\theta}_t$ in equation (3) becomes more reflective over time of lower RTP households (or becomes better represented by their RTPs) and likewise becomes lower over time. Eventually, the value of $\tilde{\theta}_t$ will become equal to the RTP of the lowest RTP household.

Because the output/capital ratio in the economy has a positive linear correlation with $\tilde{\theta}_t$ as indicated in equation (3), the ratio also decreases as $\tilde{\theta}_t$ decreases over time. That is, if the heterogeneity in RTP is the cause for the secular increase in economic inequality, the output/capital ratio should also have decreased as $\tilde{\theta}_t$ approaches the RTP of the lowest RTP household. However, as indicated in Section 2, in actuality, the ratio has been basically unchanged for half a century or more.

Conversely, the constancy of output/capital ratio may mean that heterogeneity in RTP is in actuality quite low. If this heterogeneity is indeed low, the increase in economic inequality must also be low. In other words, it is difficult for the heterogeneity in RTP to be associated simultaneously with both an increase in economic inequality and a constant output/capital ratio.

Note that because the two procedures are equivalent, the result under the MDC-based procedure is basically the same as that under the RTP-based procedure, (Harashima,

2018c).

4.2 Effect of persistent economic rents

I now examine whether the cause is the heterogeneity in persistent economic rents. Suppose that only persistent economic rents are heterogeneous among households and that the level of economic inequality increases persistently because of the vulnerability of approximate SH. Hence, more capital will come to be owned by households that obtain a greater amount of persistent economic rents over time by a similar mechanism to the case of heterogeneous RTP (or CWR at MDC) (Harashima, 2021a).

An important difference from the case of heterogeneous RTP (or CWR at MDC) is that the increase in economic inequality is independent of the output/capital ratio. Because the output/capital ratio is closely related to $\tilde{\theta}_t$ that will eventually become equal to the RTP (or CWR at MDC) of the lowest RTP (or CWR at MDC) household, as shown in equation (3), but not to persistent economic rents, the output/capital ratio does not change even if heterogeneous persistent economic rents induce a major change in economic inequality. That is, unlike heterogeneous RTP, heterogeneous persistent economic rents can coexist with and be consistent with both the secular increase in economic inequality and the constancy of the output/capital ratio.

5 NUMERICALLY MOST LIKELY CAUSE

To confirm the theoretical conclusion in Section 4, I now use a numerical example to examine the effects of RTP (or CWR at MDC) and persistent economic rents on increases in economic inequality.

5.1 Heterogeneous RTPs (CWR at MDC)

To begin with, I examine the effect of heterogeneous RTPs on economic inequality. Suppose that each household has either a Low, Middle, or High RTP, the values of which are 0.02, 0.04, and 0.06 respectively (I call each of these households a Low, Middle, or High RTP household, respectively). It is assumed that the distribution of these three types of households is not skewed; for example, the Low, Middle, and High RTP households may account for 10%, 80%, and 10% of all households, respectively. In addition, it is assumed that households are identical except for their RTPs (or CWRs at MDC) and that the initial consumption and capital are identical, i.e., initially there is no economic inequality.

5.1.1 Consumption

In non-scale models of endogenous economic growth, the growth rate of consumption on a balanced growth path can be most simply described as

$$\frac{\dot{c}_t}{c_t} = \varepsilon^{-1}(\Gamma - \theta) \tag{4}$$

where Γ is a positive constant and includes the factor that generates endogenous growth, ε is the degree of risk aversion and remains constant, and θ is RTP as before (Harashima, 2013)[7]. Let $c_{g,Low}$, $c_{g,Middle}$, and $c_{g,High}$ be the growth rates of consumption ($\frac{\dot{c}_t}{c_t}$) of Low, Middle, and High RTP households, respectively.

For simplicity, I simply apply equation (4) to $c_{g,Low}$, $c_{g,Middle}$, and $c_{g,High}$ separately (i.e., correlations among them are ignored), and therefore it is assumed that approximately $c_{g,Low} = \varepsilon^{-1}(\Gamma - 0.02)$, $c_{g,Middle} = \varepsilon^{-1}(\Gamma - 0.04)$, and $c_{g,High} = \varepsilon^{-1}(\Gamma - 0.06)$. Hence, the disparity of consumption between Low and Middle RTP households and that between High and Middle RTP households after T years can be described approximately by

$$\frac{c_{Low,\,T}}{c_{Middle,\,T}} = \frac{[1 + \varepsilon^{-1}(\Gamma - 0.02)]^T}{[1 + \varepsilon^{-1}(\Gamma - 0.04)]^T}$$

and

$$\frac{c_{High,\,T}}{c_{Middle,\,T}} = \frac{[1 + \varepsilon^{-1}(\Gamma - 0.06)]^T}{[1 + \varepsilon^{-1}(\Gamma - 0.04)]^T}$$

where $c_{Low,\,T}$, $c_{Middle,\,T}$, and $c_{High,\,T}$ are the level of consumption of Low, Middle, and High RTP households after T years, respectively.

5.1.2 Capital

The numerical examination of heterogeneous capital accumulation presents one problem: the difficulty in reasonably assuming heterogeneous households' initial levels of consumption. If a household's RTP is lower, it will set a lower level of initial consumption than a household with higher RTP because it then increases consumption at higher rates than the higher RTP household. However, because households do not reach a steady state other than corner solutions when their RTPs are heterogeneous, it is not easy to set or

[7] Harashima (2013) is also available in Japanese as Harashima (2019b).

assume the reasonable initial levels of consumption when calibrating the model. In other words, it is not easy to know or assume how much lower the initial consumption of lower RTP households should be for any level can be interpreted as "reasonable." Therefore, amounts of households' savings in each period cannot be calculated under the assumption of heterogeneity.

There is a solution, however: to calibrate the growth rates and not the amounts of consumption. Under the MDC-based procedure, households behave in order to keep their CWRs at MDC until their capital becomes too small. Because a household's CWR has a positive linear correlation with its output/capital ratio, the consumption and capital of households will basically change at the same rate. Thus, given the equivalency of the RTP- and MDC-based procedures, we can reasonably assume that the growth rate of consumption equals the growth rate of capital for each household, even though we cannot know the initial levels of consumption. Hence, I assume that

$$\begin{aligned} c_{g,Low} &= k_{g,Low} \\ c_{g,Middle} &= k_{g,Middle} \text{ , and} \\ c_{g,High} &= k_{g,High} \end{aligned}$$

where $k_{g,Low}$, $k_{g,Middle}$, and $k_{g,High}$ are the growth rates of the capital of Low, Middle, and High RTP households, respectively. In addition, let $k_{Low\ RTP,\ T}$, $k_{Middle\ RTP,\ T}$, and $k_{High\ RTP,\ T}$ be the capital of Low, Middle, and High RTP households after T years, respectively, and assume for simplicity that $k_{Low\ RTP,\ 0} = k_{Middle\ RTP,\ 0} = k_{High\ RTP,\ 0} = 1$.

To begin with, I examine the case of no technological progress (i.e., the economic growth rate is zero) and assume that $c_{g,Middle} = k_{g,Middle} = 0$. In this case, by equation (4), $\Gamma - 0.04 = 0$ and thereby $\Gamma = 0.04$ for any ε. Hence, by equation (4),

$$\begin{aligned} c_{g,Low} = k_{g,Low} &= \varepsilon^{-1}(0.04 - 0.02) = \varepsilon^{-1}0.02 \quad \text{and} \\ c_{g,High} = k_{g,High} &= \varepsilon^{-1}(0.04 - 0.06) = -\varepsilon^{-1}0.02 \text{ .} \end{aligned}$$

With these rates of increase (or decrease), the capital of Low, Middle, and High RTP households increases or decreases differently over the 50 years.

The results indicate that if the value of ε is sufficiently small (e.g., $\varepsilon = 1$), the heterogeneity in RTP causes an increase in economic inequality 50 years later (Table 1). Specifically, the ratio of the capital of Low RTP households to that of Middle RTP households is 2.69, but a magnitude of increase in which a Low RTP household can be only two to three times wealthier than a Middle RTP household does not appear to be dramatic. On the other hand, if the value of ε is sufficiently large such as $\varepsilon = 10$, the

heterogeneity in RTP barely causes any increase in economic inequality in 50 years (the ratio is 1.11).

Table 1: Ratios of capital of Low and High RTP households to those of Middle RTP households after 50 years, economic growth rate = 0%

	Low RTP household / Middle RTP household	High RTP household / Middle RTP household
$\varepsilon = 1$	2.69	0.36
$\varepsilon = 2$	1.64	0.61
$\varepsilon = 10$	1.11	0.90

Next, I examine the case of a 2% annual economic growth rate. In this case, I assume that $c_{g,Middle} = k_{g,Middle} = 0.02$. Hence, by equation (4), $\Gamma = 0.06$ if $\varepsilon = 1$, $\Gamma = 0.08$ if $\varepsilon = 2$, and $\Gamma = 0.24$ if $\varepsilon = 10$. The results after 50 years are summarized in Table 2.

The results in the case of 2% economic growth are almost identical to those of 0% growth (Table 2). If the value of ε is sufficiently small, the heterogeneity in RTP causes an increase in economic inequality over 50 years to some extent, but barely causes such an increase if sufficiently large.

Table 2: Ratio of capital of Low and High RTP households to that of Middle RTP household after 50 years, economic growth rate = 2%

	Low RTP household / Middle RTP household	High RTP household / Middle RTP household
$\varepsilon = 1$	2.64	0.37
$\varepsilon = 2$	1.63	0.61
$\varepsilon = 10$	1.10	0.91

5.1.3 Output/capital ratio

As equations (2) and (3) indicate, the output/capital ratio basically changes at the same rate as RTP. Hence, changes in the output/capital ratio can be measured by the changes in average RTP. Because a greater amount of investment is undertaken by households that have a greater amount of capital, the average RTP in the economy ($\tilde{\theta}$) can be calculated approximately as the average RTP per household, weighted by the capital possessed by each household.

Suppose that Low, Middle, and High RTP households account for 10%, 80%, and 10% of all households, respectively. Hence, using the data assumptions in Section 5.1.2, I calculate the average RTP after 50 years as follows. In the case of no technological progress, the average RTP is

$$
\begin{array}{ll}
0.0357 & \text{if } \varepsilon = 1, \\
0.0379 & \text{if } \varepsilon = 2, \text{ and} \\
0.0396 & \text{if } \varepsilon = 10 \ ,
\end{array}
$$

and in the case of 2% economic growth, it is

$$
\begin{array}{ll}
0.0359 & \text{if } \varepsilon = 1, \\
0.0380 & \text{if } \varepsilon = 2, \text{ and} \\
0.0396 & \text{if } \varepsilon = 10 \ .
\end{array}
$$

If the value of ε is sufficiently small, such as $\varepsilon = 1$, heterogeneity in RTP causes a decrease in the output/capital ratio in 50 years, from 0,04 to 0.036, but if sufficiently large, such as $\varepsilon = 10$, it barely causes any decrease.

5.1.4 RTP (CWR at MDC) is highly unlikely the cause

The numerical examinations in Sections 5.1.2 and 5.1.3 indicate that if the value of ε (the degree of risk aversion) is sufficiently small, the level of inequality increases in 50 years while the output/capital ratio decreases over the same period. But if the value of ε is sufficiently large, both the amount of inequality and the output/capital ratio barely change in 50 years. This means that when an increase in economic inequality is present, it cannot coexist with a constant output/capital ratio. Therefore, it is unlikely that RTP (or CWR at MDC) is the primary cause of the increase in economic inequality over the past half century.

5.2 Heterogeneous persistent economic rents

Next, I numerically examine the effect of heterogeneous persistent economic rents. Here, it is useful to remember that the output/capital ratio is not affected by heterogeneous persistent economic rents—a consequence of the output/capital ratio being closely related to $\tilde{\theta}_t$ as shown in equations (2) and (3), whereas no such relationship exists for persistent economic rents. Hence, the question to be answered in this case is whether heterogeneous persistent economic rents can cause a major increase in economic inequality in 50 years.

5.2.1 Inequality in capital

Suppose for simplicity that rich households consist of $x\%$ of all households and monopolize all persistent economic rents, and the remaining ordinary households consist of $100 - x\%$ of all households and that part of their incomes is extracted by rich households because of the persistent economic rents. It is assumed that each rich

household obtains an equal amount of persistent economic rents and that an equal amount of income is extracted from each ordinary household.

Let μ ($0 < \mu < 1$) be the ratio of extracted income to all income for an ordinary household. Because the sum of persistent economic rents of all rich households, z, is equal to that of all extracted income from all ordinary households,

$$z = \frac{\mu(100 - x)}{x} = \mu\left(\frac{100}{x} - 1\right) .$$

For example, if $x = 2$ (i.e., rich households consist of 2% of the population) and $\mu = 0.01$ (i.e., 1% of the income of ordinary households is extracted),

$$z = \frac{\mu(100 - 2)}{2} = 0.01\left(\frac{100}{2} - 1\right) = 0.01 \times 49 = 0.49 .$$

In this case, each rich household can obtain persistent economic rents that are equivalent to 49% of its (original) income; i.e., the income of a rich household increases 49% thanks to persistent economic rents, while the income of each ordinary household decreases 1%.

A wide range of values of x and μ can be assumed as reasonable for calibration. In the following numerical examinations, I choose five values of x: 1%, 2%, 5%, 10%, and 20%, and three values of μ: 0.01, 0.005, and 0.0001. However, considering the observed economic inequality between rich and ordinary households, the cases that x = 20% and μ = 0.0001 may be somewhat unreasonable and unnatural. In addition to the above values, I choose two values of annual economic growth rate, $g = 0$ (no technological progress) and $g = 0.02$ (2% growth).

Suppose that initially there are no differences or inequality among households. This scenario entails a ratio of capital of a rich household to that of an ordinary household that is initially unity (1.0), but because rich households obtain persistent economic rents and ordinary households have economic rents extracted every year, differences emerge between rich and ordinary households and are exacerbated over time, causing the capital ratio between rich households and ordinary households to increase continuously and at a steady rate.

The results after 50 years are summarized in Table 3. They indicate that as the rich population x becomes smaller and extracted income μ becomes larger, the rich-to-ordinary capital ratio increases by a greater amount, and that unless x is unreasonably large (such as $x = 20$) and μ is unreasonably small (such as $\mu = 0.001$), the ratio shows a sizeable increase over 50 years, sometimes as high as 100 or more. In general, heterogeneity in persistent economic rents generates large-scale economic inequality over the long run.

Table 3: Ratio of capital of rich households to that of ordinary households after 50 years

		μ= 0.01	μ= 0.005	μ= 0.001
g = 0	x = 1	14.5×10^{14}	89.1×10^{7}	185
	x = 2	75.4×10^{7}	94.8×10^{3}	18.1
	x = 5	98.9×10^{2}	154	4.24
	x = 10	122	14.9	2.59
	x = 20	11.7	4.45	2.02
g = 0.02	x = 1	87.8×10^{13}	63.7×10^{7}	168
	x = 2	54.0×10^{7}	77.3×10^{3}	17.1
	x = 5	83.8×10^{2}	140	4.12
	x = 10	305	38.5	6.9
	x = 20	30.4	11.7	5.41

Heterogeneous persistent economic rents therefore cause a large increase in economic equality while not affecting the output/capital ratio, which means that the heterogeneity in persistent economic rents is basically consistent with the two fundamental facts described in Section 2.

5.2.2 Skewed distribution

The result of heterogeneous persistent economic rents bears a sharp contrast to the case of heterogeneous RTPs. One reason for this contrast is the different assumptions regarding the distribution of persistent economic rents and RTP among households: the distribution is assumed to be largely skewed for persistent economic rents but not so skewed for RTP. In addition, the variance is assumed to be large for persistent economic rents, but small for RTPs.

I believe these differences in assumptions to be reasonable. RTPs appear to approximate a normal distribution and their variance seems to be relatively small, similar to other preferences, which is the nature of human preferences. Preferences do differ among people, but the magnitude of these difference will be small because the most suitable preferences would have been selected and inherited through the process of natural selection in which only a few traits out of many survive the test of time. At the same time, however, a certain amount of diversity will be necessary for the survival of the species in frequently changing environment. I therefore find it likely that human preferences have something close to a normal distribution while their variances are relatively small.

For economic rents on the other hand, it seems highly likely that only a few

households and family lines can obtain persistent economic rents monopolistically, and that the ordinary households that comprise the vast majority of the population whose incomes are extracted even though the amount of income extracted is small for each household. This situation has come about because of the nature of ranking preference and value, in which most economic rents derived therefrom can be obtained by only a few top-ranked individuals. It is therefore natural that the distribution of persistent economic rents has a large skew.

5.2.3 Persistent economic rents derived from mistakes

As shown in Section 3.2.2, there are two kinds of persistent economic rents: those derived from ranking preference and value and those derived from mistakes in business dealings. As discussed in Section 5.2.2, the former are likely to have a noticeably large skew, but it is unclear whether the latter also have a large skew. The ability to avoid mistakes in business dealings may have a normal distribution (e.g., their distribution may be similar to that of people's intelligences, which seem to have an approximately normal distribution).

Although the distribution of such abilities may approximate a normal distribution, the amount of persistent economic rents derived from mistakes will have a positive correlation—not to the distribution of individual abilities but to their cumulative distribution. The higher the individual abilities, the more these rents can be obtained. If this is true, as in the case of the persistent economic rents derived from ranking preference, only a few people will be able to obtain an exceptionally large amount of persistent economic rents derived from mistakes.

Suppose for simplicity that a given household's abilities in avoiding mistakes in business dealings can be divided into three categories: Low, Middle, and High. Assume there are N households, and that the households with Low, Middle and High abilities comprise 10%, 80%, and 10%, respectively, of the N households (I will call these households Low-ability, Middle-ability, and High-ability households). I assume further that households are identical except for these abilities. I also posit that there is no technological progress.

A High-ability household obtains rents ηk_{Middle} per business dealing from a Middle-ability household and $2\eta k_{Low}$ per business dealing from a Low-ability household, and a Middle-ability household obtains rents ηk_{Low} per business dealing from a Low-ability household, where $\eta(>0)$ is a constant and k_{Middle} and k_{Low} are capital per household of Middle- and Low-ability households, respectively. It is assumed for simplicity that any household transacts with each of the other households once a year. Hence, in every year, the amount of a Low-ability household's incomes that are extracted by all Middle-ability households is in total $\eta k_{Low} \times 0.8N$ and that by all High-ability

households is in total $2\eta k_{Low} \times 0.1N$, and the amount of a Middle-ability household's incomes that are extracted by all High-ability households is in total $\eta k_{Middle} \times 0.1N$. Let $k_{Low,T}$, $k_{Middle,T}$, and $k_{High,T}$ be the capital of Low-, Middle-, and High-ability households, respectively, after *T* years. Let $k_{Low,0} = k_{Middle,0} = k_{High,0} = 1$.

The capital of a Low-ability household after *T* years is

$$k_{Low,T} = k_{Low,0}(1 - \eta \times 0.8N - 2\eta \times 0.1N)^T = k_{Low,0}(1 - \eta N)^T. \quad (5)$$

Because $k_{Low,0} = k_{Middle,0} = k_{High,0} = 1$, by equation (5),

$$k_{Low,T} = (1 - \eta N)^T. \quad (6)$$

Similarly, the capital of a Middle-ability household after a year is

$$\begin{aligned} k_{Middle,1} &= \left(k_{Middle,0} + k_{Low,0} \times 2\eta \times 0.1N\right) \times (1 - \eta \times 0.1N) \\ &= \left(k_{Middle,0} + 0.2\eta N k_{Low,0}\right)(1 - 0.1\eta N)\ . \end{aligned}$$

By recursion, the capital after *T* years becomes

$$k_{Middle,T} = k_{Middle,0}(1 - 0.1\eta N)^T + 0.2\eta N \sum_{i=1}^{T} k_{Low,T-i}(1 - 0.1\eta N)^i\ . \quad (7)$$

By equations (6) and (7),

$$\begin{aligned} k_{Middle,T} = {} & k_{Middle,0}(1 - 0.1\eta N)^T \\ & + 0.2\eta N k_{Low,0} \sum_{i=1}^{T} (1 - \eta N)^{T-i}(1 - 0.1\eta N)^i\ . \end{aligned} \quad (8)$$

Here, if the value of ηN is sufficiently smaller than unity,

$$(1 - \eta N)^{T-i}(1 - 0.1\eta N)^i = \frac{\exp(0.9\eta N i)}{\exp(\eta N T)}\ . \quad (9)$$

Hence, by equations (8) and (9),

$$k_{Middle,T} = k_{Middle,0}(1 - 0.1\eta N)^T + 0.2\eta N k_{Low,0} \sum_{i=1}^{T} \frac{\exp(0.9\eta N i)}{\exp(\eta N T)}. \quad (10)$$

Because approximately

$$\sum_{i=1}^{T} \frac{\exp(0.9\eta N i)}{\exp(\eta N T)} = \int_0^T \frac{\exp(0.9\eta N i)}{\exp(\eta N T)} di = \frac{0.9\eta N}{\exp(\eta N T)} [\exp(0.9\eta N T) - 1] , \quad (11)$$

by equations (10) and (11), approximately

$$k_{Middle,T} = k_{Middle,0}(1 - 0.1\eta N)^T + k_{Low,0} \frac{0.18(\eta N)^2}{\exp(\eta N T)} [\exp(0.9\eta N T) - 1] . \quad (12)$$

Because $k_{Low,0} = k_{Middle,0} = k_{High,0} = 1$, then by equation (12) approximately

$$k_{Middle,T} = (1 - 0.1\eta N)^T + \frac{0.18(\eta N)^2}{\exp(\eta N T)} [\exp(0.9\eta N T) - 1] . \quad (13)$$

Finally, as for the capital of High-ability households,

$$\begin{aligned} k_{High,T} &= k_{High,0} + \frac{(k_{Low,0} - k_{Low,T})0.1N + (k_{Middle,0} - k_{Middle,T})0.8N}{0.1N} \\ &= k_{High,0} + (k_{Low,0} - k_{Low,T}) + 8(k_{Middle,0} - k_{Middle,T}) . \end{aligned} \quad (14)$$

Because $k_{Low,0} = k_{Middle,0} = k_{High,0} = 1$, then by equation (14),

$$k_{High,T} = 10 - k_{Low,T} - 8k_{Middle,T} . \quad (15)$$

With equations (6), (13), and (15), I calculate the effects of persistent economic rents derived from mistakes in the use of capital after 50 years ($k_{Low,0} = k_{Middle,0} = k_{High,0} = 1$ as assumed above) (see Table 4). These results indicate that the inequality between Middle-ability households (which consist of 80% of all households) and High-ability households (which consist of 10% of all households) can increase by a large amount in 50 years. Hence, similar to the persistent economic rents derived from ranking preference and value, those from mistakes in business dealings may be a cause behind the considerable increase in economic inequality.

Table 4: Effect on capital of persistent economic rents derived from mistakes after 50 years

Capital of a household after 50 years	ηN = 0.1	ηN = 0.05	ηN = 0.03	ηN = 0.02	ηN = 0.01
Low-ability household	0.01	0.08	0.22	0.36	0.61
Middle-ability household	0.61	0.78	0.86	0.90	0.95
High-ability household	5.13	3.69	2.90	2.40	1.79
High-ability household/Middle-ability household	8.42	4.74	3.37	2.65	1.88
Low-ability household/Middle-ability household	0.01	0.10	0.25	0.40	0.64

5.3 A cause that is numerically consistent with the two facts

The numerical examinations in Sections 5.1 and 5.2 confirm the conclusion from the theoretical examinations in Section 4. From both the theoretical and numerical points of view, the cause for the observed considerable increase in economic inequality is highly likely to be persistent economic rents, not RTP (or CWR at MDC), because the former is consistent with the fundamental facts, whereas the latter is not.

6 CONCLUDING REMARKS

The level of economic inequality has increased since the mid-twentieth century, but the output/capital ratio has barely changed during the same period. The secular increase in economic inequality indicates the material existence of heterogeneity among households. At the same time, however, this heterogeneity should not have affected the output/capital ratio. Furthermore, the heterogeneity should not be temporal but a persistent and important one that cannot easily disappear or be removed. The purpose of this paper is to uncover this essential heterogeneity that has caused the secular increase in economic inequality accompanying a constant output/capital ratio.

There are few essential heterogeneities that have the potential to lead to a major increase in economic inequality. Among them, heterogeneous RTPs and persistent economic rents are particularly important. In this paper, I examined which heterogeneity is most likely to be the cause for the increase in economic inequality accompanying a constant output/capital ratio. I first used a theoretical examination to conclude that the cause is most likely heterogeneous persistent economic rents after eliminating heterogeneous RTP as a likely cause: although it is consistent with the increase in economic inequality, it cannot coexist with a constant output/capital ratio. Conversely, heterogeneous persistent economic rents can cause considerable increases in economic inequality while also not affecting the output/capital ratio. I confirm this theoretical

conclusion by use of numerical examinations. The conclusions from the theoretical and numerical examinations are basically identical.

The conclusion in this paper suggests that in order to curb increases in economic inequality, persistent economic rents should be controlled and redistributed among households by governments to a greater extent than at present.

APPENDIX

Sustainable heterogeneity (SH)

Heterogeneities in RTP and persistent economic rents do not always result in extreme economic inequality. Indeed, Harashima (2010, 2012, 2014) has shown that even if SH does not arise naturally, it can be achieved via government intervention. In this appendix, I briefly explain the mechanism through which appropriate government intervention enables SH to be achieved in an economy with heterogeneous households; this discussion is based on Harashima (2010, 2012, 2014).

A1 SH

Here, three heterogeneities—RTP, degree of risk aversion (DRA), and productivity—are considered. Suppose that there are two economies (Economy 1 and Economy 2) that are identical except for RTP, DRA, and productivity. Each economy is interpreted as representing a group of identical households, and the population in each economy is constant and sufficiently large. The economies are fully open to each other, and goods, services, and capital are freely transacted between them, but labor is immobilized in each economy. Households also provide laborers whose abilities are one of the factors that determine the productivity of each economy. Each economy can be interpreted as representing either a country or a group of identical households in a country. Usually, the concept of the balance of payments is used only for international transactions, but in this paper, this concept and the associated terminology are used even if each economy represents a group of identical households in a country.

The production function of Economy i (= 1, 2) is

$$y_{i,t} = A_t^{\alpha} k_{i,t}^{1-\alpha} \ ,$$

where $y_{i,t}$ and $k_{i,t}$ are the production and capital of Economy i in period t, respectively; A_t is technology in period t; and α ($0 < \alpha < 1$) is a constant and indicates the labor share. All variables are expressed in per capita terms. The current account balance in Economy 1 is τ_t and that in Economy 2 is $-\tau_t$. The accumulated current account balance

$$\int_0^t \tau_s ds$$

mirrors capital flows between the two economies. The economy with current account

surpluses invests them in the other economy. Since $\frac{\partial y_{1,t}}{\partial k_{1,t}}\left(=\frac{\partial y_{2,t}}{\partial k_{2,t}}\right)$ is returns on investments,

$$\frac{\partial y_{1,t}}{\partial k_{1,t}}\int_0^t \tau_s ds \quad \text{and} \quad \frac{\partial y_{2,t}}{\partial k_{2,t}}\int_0^t \tau_s ds$$

represent income receipts or payments on the assets that an economy owns in the other economy. Hence,

$$\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}}\int_0^t \tau_s ds$$

is the balance on goods and services of Economy 1, and

$$\frac{\partial y_{1,t}}{\partial k_{1,t}}\int_0^t \tau_s ds - \tau_t$$

is that of Economy 2. Because the current account balance mirrors capital flows between the economies, the balance is a function of capital in both economies such that

$$\tau_t = \kappa\left(k_{1,t}, k_{2,t}\right).$$

This two-economy model can be easily extended to a multi-economy model. Suppose that a country consists of H economies that are identical except for RTP, DRA, and productivity (Economy 1, Economy 2, … , Economy H). Households within each economy are identical. $c_{i,t}$, $k_{i,t}$, and $y_{i,t}$ are the per capita consumption, capital, and output of Economy i in period t, respectively; and θ_i, $\varepsilon_q = -\frac{c_{1,t}u_i''}{u_i'}$, ω_i, and u_i are the RTP, DRA, productivity, and utility function of a household in Economy i, respectively (i = 1, 2, …, H). The production function of Economy i is

$$y_{i,t} = \omega_i A_t^{\alpha} k_{i,t}^{1-\alpha}.$$

In addition, $\tau_{i,j,t}$ is the current account balance of Economy i with Economy j, where i, j = 1, 2, … , H and $i \neq j$.

Harashima (2010) showed that if, and only if,

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}}=\left(\frac{\sum_{q=1}^{H}\varepsilon_q\omega_q}{\sum_{q=1}^{H}\omega_q}\right)^{-1}\left\{\left[\frac{\varpi\alpha\sum_{q=1}^{H}\omega_q}{Hmv(1-\alpha)}\right]^{\alpha}-\frac{\sum_{q=1}^{H}\theta_q\omega_q}{\sum_{q=1}^{H}\omega_q}\right\} \tag{A1}$$

for any i (= 1, 2, … , H), all the optimality conditions of all heterogeneous economies are satisfied, where m, v, and ϖ are positive constants. Furthermore, if, and only if, equation (A1) holds,

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}}=\lim_{t\to\infty}\frac{\dot{k}_{i,t}}{k_{i,t}}=\lim_{t\to\infty}\frac{\dot{y}_{i,t}}{y_{i,t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=\lim_{t\to\infty}\frac{\dot{\tau}_{i,j,t}}{\tau_{i,j,t}}=\lim_{t\to\infty}\frac{\frac{d\int_0^t\tau_{i,j,s}ds}{dt}}{\int_0^t\tau_{i,j,s}ds}$$

is satisfied for any i and j ($i\neq j$). Because all the optimality conditions of all heterogeneous economies are satisfied, the state at which equation (A1) holds is SH by definition.

A2 SH with government intervention

As shown above, SH is not necessarily naturally achieved, but if the government properly transfers money or other types of economic resources from some economies to other economies, SH is achieved.

Let Economy 1+2+…+ (H – 1) be the combined economy consisting of Economies 1, 2, …, and (H – 1). The population of Economy 1+2+… + (H – 1) is therefore (H – 1) times that of Economy i (= 1, 2, 3, …, H). $k_{1+2+\cdots+(H-1),t}$ indicates the capital of a household in Economy 1+2+…+ (H – 1) in period t. Let g_t be the amount of government transfers from a household in Economy 1+2+…+ (H – 1) to households in Economy H, and $\bar{g}_t$ be the ratio of g_t to $k_{1+2+\cdots+(H-1),t}$ in period t to achieve SH. That is,

$$g_t=\bar{g}_t k_{1+2+\cdots,+(H-1),t}\ .$$

$\bar{g}_t$ is solely determined by the government and therefore is an exogenous variable for households.

Harashima (2010) showed that if

$$\lim_{t\to\infty}\bar{g}_t=\left(\frac{\sum_{q=1}^{H}\varepsilon_q\omega_q}{\omega_H}\right)^{-1}\left\{\frac{\varepsilon_H\sum_{q=1}^{H}\omega_q-\sum_{q=1}^{H}\varepsilon_q\omega_q}{\sum_{q=1}^{H-1}\omega_q}\left[\frac{\varpi\alpha\sum_{q=1}^{H}\omega_q}{Hmv(1-\alpha)}\right]^{\alpha}-\frac{\varepsilon_H\sum_{q=1}^{H}\theta_q\omega_q-\theta_H\sum_{q=1}^{H}\varepsilon_q\omega_q}{\sum_{q=1}^{H-1}\omega_q}\right\}$$

is satisfied for any i (= 1, 2, …, H) in the case that Economy H is replaced with Economy i, then equation (A1) is satisfied (i.e., SH is achieved by government interventions even if households behave unilaterally). Because SH indicates a steady state, $\lim_{t\to\infty} \bar{g}_t$ = constant.

Note that the amount of government transfers from households in Economy 1+2+ … + (H – 1) to a household in Economy H at SH is

$$(H-1)g_t = (H-1)\, k_{1+2+\cdots+(H-1),t} \lim_{t\to\infty} \bar{g}_t \ .$$

Note also that a negative value of g_t indicates that a positive amount of money or other type of economic resource is transferred from Economy H to Economy 1+2+ · · · + (H – 1) and vice versa.

A3 SH for heterogeneous RTP with government intervention

Suppose that RTP is heterogeneous among households. If the government's transfers from a household in economy 1+2+ · · · + (H – 1) to households in economy H are such that

$$\lim_{t\to\infty} \bar{g}_t = \frac{\theta_H - \dfrac{\sum_{q=1}^{H-1} \theta_q}{H-1}}{H} \ , \tag{A2}$$

then

$$\lim_{t\to\infty} \frac{\dot{c}_{i,t}}{c_{i,t}} = \varepsilon^{-1} \left[\left(\frac{\varpi\alpha}{mv} \right)^{\alpha} (1-\alpha)^{-\alpha} - \frac{\sum_{q=1}^{H} \theta_q}{H} \right] \tag{A3}$$

for any i (= 1, 2, · · ·, H). If equation (A2) is satisfied for any i (= 1, 2, · · ·, H) in the case that Economy H is replaced with Economy i, then equation (A3) is satisfied (i.e., SH is achieved by government intervention). Because SH indicates a steady state, $\lim_{t\to\infty} \bar{g}_t$ = constant.

A4 SH for heterogeneous RTP and economic rents with government intervention

Next, suppose that not only RTP but also persistent economic rents are heterogeneous among households, as shown in Harashima (2020b). First, I examine this case using the two-economy model. A household in Economy 1 obtains rent income z_t in period t, and conversely, the income of a household in Economy 2 is reduced by z_t in period t. Suppose, for simplicity, that a household in Economy 1 does not consume z_t in period t but lends the money equivalent to z_t to a household in Economy 2 in period t. It is assumed that z_t

is proportional to $k_{i,t}$ such that

$$z_t = \bar{z} k_{1,t} \ ,$$

where $\bar{z}$ (> 0) is a constant. A positive value of $\bar{z}$ means that the mean of rents that households (family lines) in Economy 1 obtain over generations is positive.

In the case of multiple economies such that there are H economies (Economy 1, Economy 2, …, Economy H) that are identical except for RTP and rent income and that only Economy H obtains rent income (z_t), as Harashima (2012) showed for an analogous case, SH requires government (positive or negative) transfers from a household in Economy 1+2+ · · · + (H – 1) to households in Economy H by

$$\lim_{t\to\infty} \bar{g}_t = \frac{\theta_H - \frac{\sum_{q=1}^{H-1} \theta_q}{H}}{H-1} - \frac{\bar{z}}{H-1} \ ,$$

where Economy 1+2+ · · · + (H – 1) is the combined economy of Economy 1, Economy 2, …, and Economy (H − 1), and SH is satisfied among these economies; that is, equation (A3) is satisfied.

A5 Approximate SH

SH can be achieved by appropriate government intervention, but as Harashima (2018c) showed, households cannot know the true SH; therefore, a government will adjust the amounts of transfers among households to achieve an approximate SH as a substitute for the true SH. That is, a situation in which the number of votes cast in response to increases in economic inequality is equivalent to that in response to decreases in economic inequality will be pursued.

The reason why households cannot know the true SH can be easily understood from the maximum degree of comfortability (MDC)–based procedure presented by Harashima (2018c). There are two possible procedures through which a household can reach steady state: (1) the conventional RTP-based procedure in which households reach steady state by generating rational expectations using RTP and (2) an alternative MDC-based procedure in which households self-assess their value from the combination of earned (labor) income and wealth (capital) (the capital–wage ratio; CWR) and then adjust its consumption to the point at which it feels most comfortable. Harashima (2018c) proved that both procedures are equivalent and thereby a household can reach the same steady state whichever procedure is used. Nevertheless, under the MDC-based procedure, a household is not required to do anything equivalent to computing a complex, large-scale, macro-econometric model to generate rational expectations; in fact, it is not even required

to be aware of any sort of economic model. Thus, the MDC-based procedure is extremely easy for a household to use.

An important result of using the MDC-based procedure is that even though households cannot know the true SH, an approximate SH can still be achieved. This approximate SH will be not necessarily be equal to the true SH, but it can result in a steady state forming in a heterogeneous population because the votes relating to economic inequality are balanced.

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