Answering the social discount rate question

Szekeres, Szabolcs

IID Kft.

24 November 2022

ANSWERING THE SOCIAL DISCOUNT RATE QUESTION

by Szabolcs Szekeres[*]

Discounting project net flows with prescriptive rates fails to reflect costs of capital; discounting them with descriptive rates fails to reflect intertemporal preferences. A two-rate discounting method is described by which a descriptive rate is used to forecast costs of capital and a prescriptive rate is used to discount the all-inclusive net benefit flow. Using this method is the equivalent of discounting with the social time preference rate (SPTR) after having adequately shadow-priced investments, which satisfies in full the requirements of both discounting approaches. It also results in an easy to apply rule: for projects to be economically feasible their IRR should exceed both the STPR and the social opportunity cost rate (SOCR). The long-standing social discount-rate dilemma is thus solved, for in fact there is no choice. Both rates must be used. An agent-based capital market model with multiple agents and two financial instruments, one of them stochastic, illustrates and provides additional insights.

Keywords: Social discount rate; Prescriptive discounting; Descriptive discounting; Two-rate discounting; Declining discount rates; Ramsey rule.

JEL classification: D61; H43

1. Introduction

Referring to the familiar descriptive and prescriptive classification of approaches to discounting in cost-benefit analysis (CBA), William Nordhaus (2019) observed that the debate about discounting is “just as unsettled as it was when first raised three decades ago.”

The two approaches differ in their response to distortions in the capital market. In an undistorted market, the interest rate equals both the marginal rate of substitution (MRS) between present and future consumption of savers, from which the STPR is derived, and the marginal rate of transformation (MRT) of producers, from which the SOCR is derived. If the market is distorted, however, this equality no longer holds. Those who subscribe to the prescriptive approach to discounting rightly argue that to correctly gauge intertemporal welfare the SPTR must be used for discounting. Those who subscribe to the descriptive approach rightly argue that projects failing to cover their opportunity cost of capital, measured by the SOCR, are welfare destroying.

Beyond this fundamental discrepancy, there are further disagreements about the correct discount rate among adherents of both approaches. In the prescriptive camp some, but not all, view the setting of the STPR as primarily an ethical issue. Chichilnisky (1997) thinks that a constant discount rate embodies the “tyranny of the present over the future.” The descriptive camp faces the difficulty of identifying the correct cost of capital. According to Baumol (1968) “...we find ourselves forced to hunt for a solution in the dark jungles of the second best.” Freeman and Groom (2010) feel that these disagreements “raise the spectre of the near impossibility of reconciling” the prescriptive and the descriptive approaches to discounting.

---

[*] Independent researcher
https://orcid.org/0000-0003-3903-5377

This paper argues that what is needed is not to reconcile the two approaches, but to recognize that once the STPR and the SOCR diverge, neither, by itself, can compute correct net present values (NPV) of projects. This was already recognized by Baumol (1968) "We see now that no optimal rate exists. The rate that satisfies the one requirement cannot possibly meet the conditions of the other."

- The STPR (which could either correspond to the effective interest rate by reference to which inter-temporal consumption allocation is made or be politically determined) defines the correct intertemporal welfare weights but fails to correctly measure the costs of capital.
- The SOCR (which being derived from market interest rates with correction for market distortions and other indirect effects) defines the correct costs of capital but fails to assign correct intertemporal welfare weights.

This paper proposes a two-rate discounting method: costs of capital are calculated with the help of the SOCR, then added to the project net flow (replacing the initial capital outlay), and the resulting all-inclusive flow is discounted using the STPR. In this way, each discount rate performs that role, and only that role, for which it is suited. The NPV so obtained will then correctly establish whether a project is welfare-enhancing. The intertemporal weights will reflect the STPR, costs of capital will correspond to the SOCR. For a project to be welfare-enhancing its return must exceed the hurdle rates of both the STPR and the SOCR. As both are needed for welfare maximization, they are in fact complimentary, not competing.

These concepts are illustrated by a two-period agent-based capital market model in which the welfare consequences of alternative investments can be measured. The agents are risk-averse consumers who can borrow, lend, and invest in stocks offering stochastic returns. The numerical values used are not intended to reflect real-world magnitudes, but to provide a coherent set of values for the illustrative numerical examples. The model shows how CBA handles some of the most common distortions that call for its use in the first place, such as the displacement of private investment by fiscal borrowing and the taxing of interest income and profits. Two examples illustrate the two-rate discounting method and serve to compare the performance of alternative discounting methods.

Section 2 briefly describes the capital market model, which is explained in greater detail in the Appendix. The model itself is available as an Excel file[1]. Section 3 presents the calculation of the STPR and the SOCR used in the model. Section 4 analyzes two projects—one with a high return, the other with a low return—to explain two-rate discounting and compare it to its alternatives. Section 5 demonstrates why both the STPR and the SOCR are necessary hurdle rates, but sufficient only jointly. Section 6 extends the results of the two-period model to the consideration of the distant future. Section 7 explores the relationship between two-rate discounting and shadow pricing of capital. Section 8 presents conclusions. Section 9 discusses the implications of this paper's conclusions on the current literature on discounting.

2. Description of the capital market model

We assume that consumers aim to maximize a welfare function of the following form:

$$V(C) = \sum_0^t \frac{U(C_t)}{(1+\rho)^t} \tag{1}$$

where $\rho>0$ is the pure rate of time preference and $U(C_t)$ is a utility function of the constant-intertemporal-elasticity-of-substitution (CIES) type:

$$U(C) = \frac{C^{1-\sigma}-1}{1-\sigma} \tag{2}$$

---

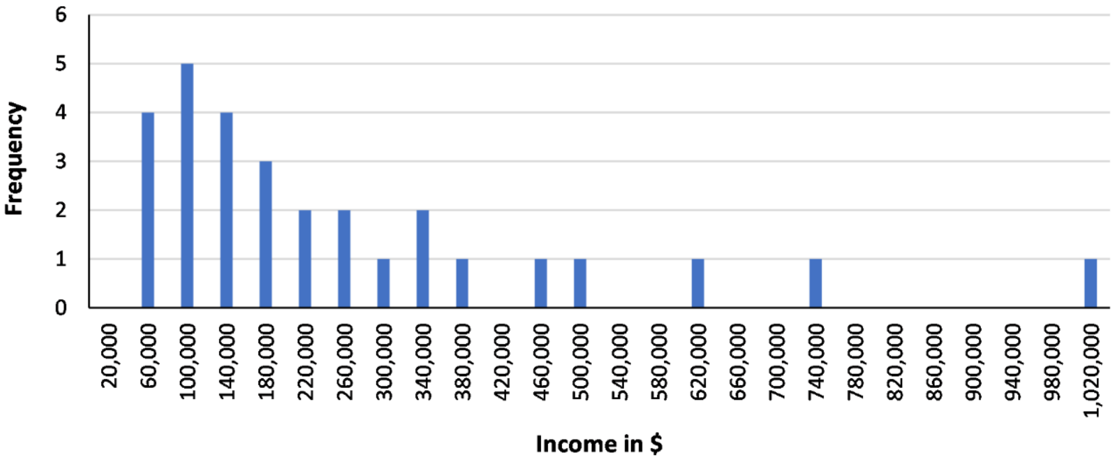
[1] http://doi.org/10.3886/E178081V2

where consumption C > 0, and the elasticity of marginal utility with respect to consumption σ > 0 but not equal to 1. When $\sigma = 1$, the utility function takes the form $U(C) = ln(C)$. This parameter is also the measure of the decision maker's constant proportional risk-aversion.

The optimization will only involve two time periods, the present, $t = 0$, and time $t = 1$. The degree of risk aversion chosen for the examples is $\sigma = 1$, following Gandelman and Hernández-Murillo (2014), who analyzed data from 75 countries and found that "the coefficient of relative risk aversion varies closely around one."[2] We assumed that the pure rate of time preference is $\rho = 2\%$ for all agents.

We first generated 29 agents with the following frequency distribution of Year 0 incomes, which is approximately log-normal:

Figure 1
Income distribution of agents



Next, we doubled their number, by assuming that half the agents would see their income grow by 2% between Year 0 and Year 1, while the other half would see it decline by 2%. This will naturally make half of them be inclined to lend or invest when attempting to smooth their consumption over time, while the other half would be inclined to borrow.

We computed the aggregate indifference curve of this society of 58 agents, as well as that of the representative agent derived from their average data. See details in the Appendix.

Agents optimize their welfare by choosing amounts to borrow or lend (by selling or buying discount notes) or investing in stocks that have an even chance of either yielding a profit of 40% or a loss of 20%. Their expected yield is 10%. The tax rate on interest income or capital gains is 40%.

A simulation algorithm finds the equilibrium price of the discount notes, thus defining the market rate of interest. The supply of stocks is assumed to be infinitely elastic.

The results of the simulations, in the absence of any fiscal borrowing, are described by the following figures. In both figures the vertical axis is dollars borrowed and lent and the horizontal axis is the price of discount notes. The low price of 0.967 corresponds to an annual interest rate of 3.4%, while the high price of 0.971 corresponds to 3.0%. We see in the following Figure 2 that

---

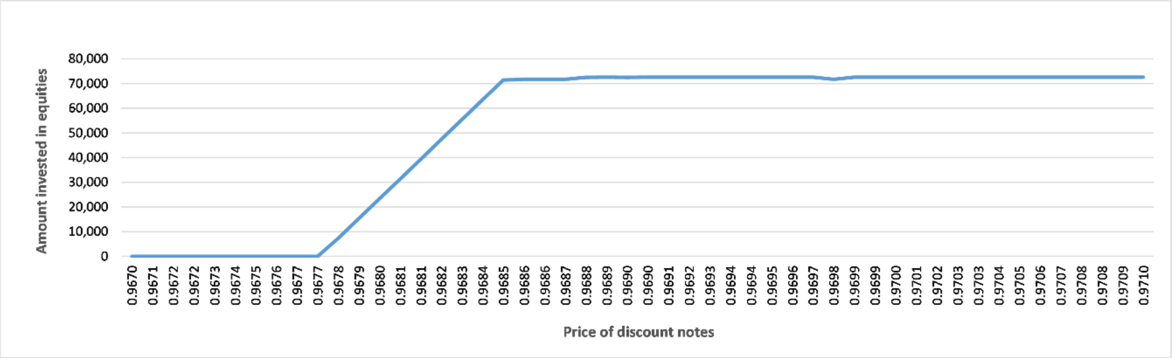
[2] Exhibiting logarithmic utility may be advantageous. Mark Rubinstein (1976) argues that logarithmic utility implies an optimal degree of risk aversion, resulting in the maximization of the expected value of terminal wealth, regardless of initial wealth.

lending declines as the interest rate declines (price of notes increases) while the converse is true for borrowing.

Figure 2
Lending and Borrowing

Figure 3 shows a point at which the amount of equity investment begins to appreciably increase. It occurs when interest rates decline to the point at which it becomes worth running the risk of investing in equities.

Figure 3
Equity Investment



Absent fiscal borrowing, the 58 agents borrow and lend \$27,200 among each other in the aggregate and invest a total of \$45,988 in equities. The equilibrium interest rate at which the amount lent equals the amount borrowed is 3.28%.

3. Calculation of the STPR and the SOCR

There are many approaches to defining the STPR. Some are derived from ethical considerations, such as proposing that there should be no discounting of future values at all, or positing an authoritarian social planner, who aims to maximize some welfare function. In this paper we compute the STPR from the preferences of the agents making up the experimental society of our model. We could therefore call it the revealed preference STPR. We believe that this is what is consistent with the objectives of CBA, namely, to measure the welfare consequences of alternative allocations of resources on members of society, preferably measured by revealed preferences.

Having found the market equilibrium interest rate in our model, all agents have optimized their welfare by either borrowing or lending, and possibly also investing. The Appendix shows that agents who borrow have a MRS that corresponds to the market interest rate of 3.28%. Because they incur no tax liability, there is no market distortion in their case, so this correspondence is a consequence of their consumption path optimization. The MRS of agents who lend, however, always corresponds to 1.97%, which shows that they too optimize to the interest rate that they effectively receive, which in their case is the (distorted) after-tax rate.

But CBA needs an STPR that reflects the MRS of the entire society. We computed it by aggregating the compensating variation in Year 0 income of all agents (an increase in income) such that their welfare is the same as when given (alternatively) $1 in Year 1. The result of this calculation is that this society requires a return higher than 2.62% to consider an investment to be welfare-enhancing, which is therefore the revealed preference STPR. This rate equals the weighted average of the rates implicit in the MRSs of the two groups of agents. Within the confines of this simple model, therefore, the weighted average of the rates by reference to which members of society optimize their consumption path equals the SRTP of society as a whole.

Based on the aggregated optimization results obtained in the model, we evaluated the well-known Ramsey equation[3], which also yielded an STPR of 2.62%.

Our model is a good vehicle with which the operation of the Ramsey equation can be explored. Recall that the MRS is a locally variable property of the welfare function; it is not a constant but varies with the quantity saved or borrowed. Examining the aggregate indifference curve of our modeled society (see the *Income distribution* worksheet of the Excel file) we see that for a +/– 20% deviation of Year 0 income, MRS values range widely, corresponding to interest rates between –28% and 58%, as measured by the Ramsey equation. As the Ramsey rate can be calculated for every possible point of a saver's/borrower's indifference curve, on the assumption that it identifies the optimum allocation, the equation is, in effect, an identity.

The Ramsey equation is generally used to calculate the discount rate from its remaining variables: the pure rate of time preference, the elasticity of marginal utility and the growth of consumption. But this is not the real direction of causation for savers/borrowers. For them, the rate is exogenous, and it is the growth of consumption (or rather, the action that leads to it) that is the variable to optimize. We assume an income growth in the model for each agent but cannot predict his consumption growth before finding the equilibrium interest rate. A good indication of the fact that the Ramsey equation is not a predictive equation can be had by examining the effect of reducing the pure rate of time preference of all agents from 2% to 1.5%. This does not reduce the Ramsey discount rate by a half percentage point, as its defining expression might lead one to believe, but results in a decline from 2.62% to 2.59%, because the change in parameters leads to a different optimization result. A change in the tax rate from 40% to 38% causes the Ramsey rate to increase from 2.62% to 3.06%, an effect that the Ramsey equation cannot predict. But in all cases, no matter what the result, the Ramsey equation will always hold, as it does for any possible equilibrium point, on the assumption that it is the point of optimality[4].

To measure the SOCR we must compute the welfare impact of fiscal borrowing. For this calculation we assumed that the state would raise $1,000 by issuing discount notes, and then compared the ensuing equilibrium with that of the base-case. The results are shown in Table 1 below, which shows key results of the two scenarios and their differences.

Table 1

Changes attributable to fiscal borrowing

(Dollar amounts)	**Base-case**	**Fiscal borrowing case**	**Attributable changes**
Public borrowing	0	1,000	1,000
Private borrowing	27,200	27,163	-37

---

[3] $r = \rho + \eta g$, where r is the Ramsey discount rate, ρ is the pure rate of time preference, η is the elasticity of marginal utility and g is the growth of consumption.

[4] This is the assumption on the basis of which the equation is derived. See Greaves (2017) Appendix A.

Private lending	27,200	28,163	963
Private investments	45,988	45,047	-941
Expected Tax	4,036	3,974	-63

To calculate the SOCR we use the method suggested by Harberger (1972), which is to calculate a weighted average cost of funds. It is an application of the standard CBA shadow-pricing methodology to the situation we face.

We see that the $1,000 borrowed by the state resulted in a decline of $941 in investments, a decline of $37 in consumer borrowing and a net increase of savings of $22 (increase in lending of $963 less the diversion from investment in equities of $941). Table 2 shows how the SOCR can be computed from this information and some other considerations that will be mentioned presently.

Table 2
Calculation of the SOCR

(Dollar amounts)	**Source**	**Opportunity cost**	**Cost**
Displaced equity investment	941	10%	94.11
Displaced consumer borrowing	37	3.28%	1,21
Incremental savings	22	1.97%	0.43
Financial services impact			–28.23
Compensating variation			0.11
Total welfare cost			**67.63**

The opportunity cost of displacing investment in equities is the expected yield of the investment. Agents that borrow to smooth their consumption over time pay the market interest rate of 3.28%, so that is the opportunity cost of this source. Finally incremental lending is valued at the after tax interest rate, which is 1.97%.

The financial services impact measures the change in the cost of the financial services needed to aggregate investors' funds and make them available to producers, taken to be 3% of the amount invested in equity. Its value is negative in this case because it reflects the value of financial sector services rendered redundant by the fall in the volume of private investments. Being a freeing-up of resources, it is a benefit.

The compensating variation cost measures the welfare change brought about by fiscal borrowing, which altered the market equilibrium. While all agents optimized their consumption path under the new circumstances, their welfare was nonetheless changed. This cost, the calculation of which is explained in the Appendix, reflects the reduction in aggregate welfare due to the state having intervened in the market, beyond what has already been directly accounted for.

Adding the values in Table 2 we get that the total welfare cost of borrowing $1,000 by the state is $67.63, which makes the SOCR equal to 6.76%.

A remarkable result of the model is that the sum of the effects of the classical components of the SOCR calculation (displaced productive investments, displaced consumer borrowing and induced savings) exactly corresponds to the sum of two financial items: the interest paid by the public sector on the amount it borrowed, and the amount of the taxes forgone because of its intervention in the capital market. In this simple model the result of a welfare impact calculation equals the computed out-of-pocket expense of the public sector. This result is displayed on the *Opportunity Cost* worksheet of the Excel workbook.

It is interesting to observe that if our model is run assuming no taxes and no equity investment opportunities then the market interest rate is 2.00%, which equals the STPR (and also the average pure rate of time preference). If it becomes possible to invest in equity with no taxes, then both the market interest rate and the STPR become 9.70%. This is because even agents that borrow invest in equities, financing their investments with loans. The possibility of equity investment makes borrowers bid up the interest rate but does not introduce a breach between the market interest rate and the STPR. When fiscal borrowing takes place under these circumstances the SOCR declines to 6.80% (below the value of the STPR), because private investments are crowded out, which results in resource savings in the financial services sector. (These results can be corroborated by running the model with the appropriate parameters.)

It is also interesting to note that the expected yield of equity investments only has an indirect influence on the STPR. If we run our model returning to the original assumptions but making the expected yield of equities 10.5% instead of 10%, by increasing the probability of gains from 50% to 50.8%, we find the following in the base-case: (1) investment in equities increases by 72%; (2) lending/borrowing declines by 76%; (3) the market interest rate increases from 3.28% to 3.84%; and (4) the STPR grows from 2.62% to 3.05%. The STPR is again the weighted average of the effective cost/yield of safe instruments utilized by agents. The Ramsey rule, being an identity, confirms the new STPR value, reflecting the increased consumption in Year 1 that results from the higher yield of equities. The SOCR increases from 6.72% to 7.23%.

4. Alternative NPV calculations

In this Section we show the evaluation of two projects: Project A and Project B.

Project A has an investment cost of $1,000 and yields a financial return of 5% (at market prices) and an economic return of 8% (at accounting prices, because of, say, additional external benefits). We will first show three alternative ways of computing this project's financial net present value (FNPV). These correspond to the CBA calculation in the absence of any market distortion or externality. The calculations illustrate a well-known fact that is central to the argument of this paper, namely that, besides providing intertemporal valuation, the act of discounting implicitly imputes costs of capital equal to the discount rate used.

Table 3
Alternative FNPV calculations

(Dollar amounts)	Year 0	Year 1
Project net flow	–1,000.00	1.050.00
Borrowing	1,000.00	
Repayment	0.00	–1,000.00
Interest	0.00	–32.84
Net flow to equity	0.00	17.16
FNPV1[5]	16.61	
FNPV2[6]	16.61	

The first line of Table 3 shows the net flow of the project, that is, the difference between revenues and expenditures for investment and operating costs. Notice that the interest cost is explicitly left out, even though it is a project expenditure, precisely because the act of discounting will impute it and we do not want to count it twice. Discounting this net flow at the market interest rate of 3.28% gives us our first FNPV value of $16.61.

---

[5] – 1,000 + 1,050 / 1.03284 = 16.61

[6] 17.16 / 1.03284 = 16.61

The second FNPV is computed from the flow labeled “Net flow to equity.” This is the cashflow of the project owner and equals the project net flow less the financing flow. We see that the Year 0 value is zero, as the capital cost is entirely financed, while in Year 1 the interest due is explicitly subtracted from the net flow, along with the repayment of the loan’s principal. In other words, the flow to equity is the project net flow after financing, referred to earlier as the all-inclusive project flow.

FNPV2 = FNPV1 = $16.61, which illustrates the fact that discounting the project net flow before financing imputes the entire interest cost implicitly, expressing the result as a present value (FNPV1), while using the interest rate to define intertemporal weights. The interest rate both establishes the costs of capital and discounts the future.

There is a third FNPV that can be computed, which derives directly from the definition of present value, namely, it is the sum that will compound to a given future value at the going rate of interest. FNPV3 is also $16.61, because $16.61× 1.0328 = $17.16. This shows the coherence that results from using the same rate for both functions of discounting and illustrates that the discount rate measures the opportunity cost of capital.

Dropping the assumption that there are neither price distortions nor externalities, we analyze the behavior of the SOCR and find that it also displays this coherence. In Table 4 ENPV means economic net present value. The SOCR measures the opportunity cost of capital, it is not just the market interest rate.

Table 4
Alternative ENPV calculations

(Dollar amounts)	**Year 0**	**Year 1**
Project net flow	–1,000.00	1,080.00
Borrowing	1,000.00	
Repayment		–1,000.00
Cost of borrowing		–67.63
Net welfare effect	0.00	12.37
ENPV1	11.59	
ENPV2	11.59	

ENPV1 = $11.59 is obtained by discounting the project net flow in Table 4 at the SOCR = 6.76%, and ENPV2 = $11.59 is obtained by discounting the net welfare effect flow at the same rate.

ENPV3 is the amount to be invested in Year 0 that will compound to the net welfare effect of Year 1. Using the SOCR to compound, the required amount is 11.59. Coherently, ENPV3 = ENPV1 = ENPV2.

This coherence of discounting/compounding is exclusively possible when the discount rate both defines the costs of capital and provides the intertemporal weighting. In the no-distortions case of the first example this is natural: there is no discount rate choice. When distortions exist, however, there will be two rates: the STPR and the SOCR. The SOCR, which reflects the costs of capital, shares with the undistorted interest rate the property that the same rate is used both to determine the costs of capital and to define intertemporal weights. Thus, it displays the coherence shown above.

Not so the STPR, or any other set of intertemporal welfare weights that are unequal to the costs of capital. This has consequences, because in their cases:

- ENPV1 ≠ ENPV2, which means that the imputed costs of capital are incorrect, leading to incorrect NPV results. If STPR < SOCR, the costs of capital will be understated.

- ENPV3 does not exist. ENPV1 compounded to the future at the STPR will not equal the all-inclusive future net benefit and therefore does not comply with the definition of present value.

Table 5 compares alternative NPV calculations for Project A, the high-return project. Four alternative cashflows are discounted at the rates identified in the first column: the project net flow and the net flow after financing, at either market or CBA accounting prices (abbreviated Acc).

Table 5

Alternative NPV calculations, high-return project

Discount Rates		**Project net flow (Dollar amounts)**		**Project net flow less Financing flow (Dollar amounts)**	
		Market	**Acc**	**Market**	**Acc**
Market interest rate	3.28%	16.61		16.61	
SOCR	6.76%		11.59		11.59
STPR	2.62%		52.40		**12.05**
Zero	0.00%		80.00		12.37

We see that the same NPV is obtained from both the before and after financing flows only with the first two discount rates, the market rate and the SOCR, which being descriptive in nature, use the cost of capital to define intertemporal weights.

In the case of prescriptive discount rates, including or excluding financing in the net flow makes for a significant NPV difference, which proves that such rates do not impute the correct capital costs. The STPR discounted NPV of the project net flow imputes interest cost at accounting prices of $1,000 × 0.0262 = $26.20, rather than the actually incurred one of $67.63 ($1,000 × 0.06763), thereby understating project costs and overstating project net benefits in the amount of $41.43. If a zero discount rate is used, as has been advocated on normative grounds, it results in the total omission of capital costs from the analysis. (Notice that in that case the computed NPV is the undiscounted aggregation of the project net flow, row 1 of Table 4). *Because intertemporal preferences are not a measure of capital costs, it follows that prescriptive discount rates cannot be used to discount project cash flows that have not accounted for the costs of capital explicitly.*

On the other hand, the SOCR rate does not reflect of the time preferences of society, as it is derived from the market interest rates and corrections thereof, and therefore does not correspond to the weighted average optimal consumption path MRS, or whatever rate is politically adopted. Therefore, the SOCR rate should not be used for discounting future net benefits. It understates/overstates NPV if it is greater/lesser than the STPR.

The insight that this paper offers is that the twin functions of discounting, (1) establishing costs of capital and (2) providing intertemporal weights, cannot be accomplished using the same discount rate. By performing each step using the discount rate that is appropriate for it, the welfare impact of projects can be calculated correctly. The two-rate discounting method proposed in this paper takes the cost of capital defined by the SOCR but does not use the SOCR for discounting. Once the all-inclusive welfare net flow has been computed[7] it is discounted at the STPR, which is not used to calculate costs of capital.

Applying this method to the project example we see that SOCR = 6.76% defines the interest cost of $67.63, which after taking principal repayment into account leaves a net future welfare effect of $12.37 (see Table 4). Then this value is discounted at the STPR = 2.62% to yield a NPV of $12.05,

---

[7] As our example only has two time periods, the cost of capital accrues in the single future period. In real life they would accrue at various points of the project flow as a function of how the benefits of the project make capital cost amortization possible.

which correctly reflects the welfare impact of this project, because all the welfare costs have been accounted for and the future net welfare effect has been discounted at the STPR. The calculations show Project A to be welfare-enhancing, as this value is positive. All other NPV calculations on Table 6 are incorrect, some very seriously so.

The capital market model corroborates this result (see the Project A worksheet of the associated Excel file). If $12.05 of net welfare increase is distributed equally among the 58 agents in Year 1 and we compute the utility preserving compensating variation in their income of Year 0 to which this benefit is equivalent, we obtain approximately $0.21 for each, which aggregate to $12.05. This shows that two-rate discounting with both the SOCR and the STPR accurately predicts the welfare consequence of undertaking this project. As each rate is used for its intended purpose, the SOCR and STPR are not conflicting but complementary, and only their joint use yields the correct CBA result.

Discounting primary (unadjusted) project net flows by the STPR is wrong. A NPV of $52.40 (see Table 5) requires a Year 1 net welfare value of $52.40 × 1.0262 = $53.77, which as we see in Table 4 is just not available. The project's net welfare flow in Year 1 is $12.05, so $52.40 cannot be its present value.

The same analysis is presented in the Excel file for Project B, the low-return project, which has a financial yield of 5% and an economic yield of 5%. Table 6 shows the economic net flows that correspond to these assumptions.

Table 6

Economic net flows, low-return project

(Dollar amounts)	**Year 0**	**Year 1**
Project net flow	–1,000.00	1,050.00
Financing flow	1,000.00	–1,067.63
Net welfare effect	0.00	–17.63

The Year 1 financing cost can be quickly calculated using the SOCR, as follows: $1,000 × (1+0.06763) = $1,067.63, which leaves a net welfare effect in Year 1 of $–17.63.

Table 7 shows the results of the alternative discounting methods for Project B.

Table 7

Alternative NPV calculations, low-return project

Discount Rates		**Project Net flow before financing (Dollar amounts)**		**Project net flow less Financing flow (Dollar amounts)**	
		Market	**Acc**	**Market**	**Acc**
Financial market rate	3.28%	16.61		16.61	
SOCR	6.76%		–16.51		–16.51
STPR	2.62%		23.17		**–17.18**
Zero	0.00%		50.00		–17.63

We conclude that Project B is not economically feasible because its primary net benefit is insufficient to cover its costs of capital. Its correctly calculated NPV is $–17.18. If we distribute this project's Year 1 welfare loss in the form of a lump sum tax of $0.30 to each member of society, the aggregation of their Year 0 utility preserving compensating variation will be –17.18., which is the same as its NPV when discounting the project net flow after financing by the STPR. We see again that conventional discounting of the project's primary net flow at the STPR is wrong.

Using the SOCR to correctly measure the costs of capital of any project is an indispensable first step of any proper CBA. Only thereafter can correct intertemporal comparisons be made. Serious errors can be avoided by the simple expedient of adjusting flows with the costs of capital explicitly. The choice of STPR matters but, at least for the values of this example, the resulting differences are not large, provided that the calculation basis is correct.

Finally, we note that while for any given project it is possible to replace two-rate discounting with a single discount rate that yields the same NPV when applied to a project's primary net flow, this rate would only be valid for that specific flow[8]. Rules combining the STPR and the SOCR to get a single discount rate can produce good results only by chance.

5. Optimality conditions

The first-order condition for optimal intertemporal resource allocation is that the MRS between present and future consumption equal the rate of transformation between them, that is, the effective rate at which present day savings can be converted into future consumption.

Because savers optimize by reference to effective interest rates, the revealed preference STPR reflects the market distortions in place. It is, nevertheless, the hurdle rate that any investment project must surpass to be welfare-enhancing because having optimized their consumption path by reference to it, it measures how savers trade-off present and future income. Therefore, a necessary condition of welfare improvement for any public sector investment is that its return exceed the STPR.

Though necessary, this condition is not sufficient, as the STPR carries no information about the welfare cost of providing capital for the project. That is measured by the SOCR. If the project returns less than the SOCR, its net primary benefits are insufficient to cover its costs of capital, which leads to a welfare loss. Thus, another necessary condition for an investment to be welfare-enhancing is that its rate of return (at accounting prices) exceed the SOCR.

Therefore, for a project to be welfare-enhancing its rate of return must exceed *both* the STPR and SOCR hurdle rates. Neither condition is sufficient by itself. Jointly, however, they are.

The answer to the SDR question is that the STPR and the SOCR are not each other's alternatives. Absent a willingness to disregard the opportunity cost of capital, which would invalidate the CBA results, there is no choice to be made between them: both rates must be used.

Given that when a project clears the SOCR hurdle rate, it will also have cleared the STPR's, doesn't this make the STPR redundant? Not at all. The STPR is needed to compute the correct NPV and is therefore useful in choosing among projects that have cleared the SOCR rate. As IRRs cannot be used for project ranking, the STPR dependent B/C ratio will help in this task. There might be a choice of STPR to be made. Using revealed preference STPR would maximize the welfare of those alive today, which is the usual CBA goal. But policymakers could choose a lower rate to give greater weight to benefits accruing in the more distant future.

6. Discounting the distant future

The examples discussed above are set in a one year apart two-period model. In this section we show how two-rate discounting can throw light on the key issue on which disputes about discounting center: the valuation of the distant future.

---

[8] That rate is 6.67% for the net flow of project A, but if we change the Year 1 flow value to 1,100, the two-rate NPV becomes 31.92 and the conventional equivalent discount rate becomes 8.65%.

Advocates of prescriptive discounting complain that discounting by the SOCR gives insufficient weight to benefits accruing to future generations, thereby "tyrannizing them." Pigou (1932) famously characterized this as having "defective telescopic faculty." Two-rate discounting solves this problem, because it does not use the SOCR for discounting but rather the STPR, just as advocates of prescriptive discounting propose. In the following Table 8 a simple project is used to illustrate the application of two-rate discounting to long-lived projects.

Table 8

Net flow of a long-lived project

(Dollar amounts)

Years	Primary net benefits	Financing flow	Net benefits after financing
0	–100.00	100.00	0.00
100	1,921.86	–5,050.49	–3,128.63

Assume that the project's net flows are at accounting prices, with all externalities considered and, for the sake of the example, that STPR = 2.5% for year 100 (Green Book, 2020:122) and that SOCR = 4%. We can then compute the NPVs shown in Table 9.

Table 9

Alternative NPVs of a long-lived project

(Dollar amounts)

Conventional NPV @ STPR = 2.5%	62.68
Conventional NPV @ SOCR = 4%	–61.95
Two-rate NPV @ STPR = 2.5%, SOCR = 4%	–264.83
Two-rate NPV @ STPR = 0%, SOCR = 4%	–3,228.63

If the project's primary net benefits are discounted at the Green Book recommended STPR, the project has a positive NPV of $62.68 and would receive approval. The NPV discounted at the SOCR is –61.95, however, which means that this project should be rejected because its benefits are not sufficient to cover its costs of capital. The correct welfare-equivalent NPV, computed by the two-rate discounting method, is $–364.83, which confirms that the correct CBA recommendation is rejection.

This example illustrates a key finding of this paper: discounting projects' primary net flows at prescriptive discount rates fails to properly account for the costs of capital and hence yields incorrect results. Trying to remedy the perceived excessive discounting of the future by lowering the conventionally used discount rate is incorrect because it comes at the price of invalidating the result. Indeed, those conventionally discounting with the STPR can be faulted for having a "defective telescopic faculty" of their own by failing to correctly account for the costs of capital. It is the two-rate NPV discounted at the super egalitarian STPR of 0%—which, setting the telescope to full magnification, values the welfare of future generations on a par with that of present ones—that illustrates the travesty that would be perpetrated on future generations by undertaking this project. They would be forced to pay $3,228.63 in incremental taxes above the value of the benefits generated by the project[9]. Accepting this project would truly be an act of tyranny of the present over the future, born from the failure to properly account for the costs of capital. The telescopic faculty metaphor supports the contention of prescriptive advocates only if the telescope trains exclusively on benefits. Once costs of capital come into its view as well, the picture changes, for it's not just benefits that accrue in the future, opportunity costs of capital do so as well.

---

[9] To illustrate: savers today buy 100-year government bonds and bequeath them to their heirs. When the notes come due, the heirs redeem them, and future taxpayers make payment. From an inter-generational point of view the transfer is between the savers and their heirs, but because current day government made the wrong decision, a welfare loss is induced in the future, because the welfare cost of the taxes raised to pay for the interest due exceeds the welfare benefits of the project.

Nothing prevents the CBA analyst from looking at the very distant future. The value of an infinite series of benefits can be easily ascertained. Having $3 per annum forever has a present value of $120, discounted at the STPR of the last example (3 / 0.025 = 120). If the investment required to generate that benefit stream is $100 (implying an IRR of 3%) the project would clear the hurdle rate of the STPR but would not clear the hurdle rate of the SOCR (3% < 4%), again using the rate of the last example. Therefore, the project would not be welfare enhancing according to the conclusions of the previous section.

This is confirmed by computing the two-rate NPV of the project. Annual cost of capital would be $4 (100 × 0.04 = 4) leaving a post financing net flow of $–1, the present value of which, discounted at the STPR, is $–40 (–1 / 0.025 = –40). This result is not due to excessive discounting, as all NPV calculations were carried out at the STPR. Further lowering the discount rate would not change this, as there is no positive discount rate at which a stream of future net losses can have a positive present value.

In the discounting debate these questions arise when discussing the economic feasibility of environmental projects. It is often argued that distant benefits are undervalued by discounting. Two-rate discounting undervalues nothing, as we have seen. Having correctly accounted for the costs of capital, the dilemma remains: when is it worth investing in the present for the sake of benefits in the distant future? Using a different metaphor: when would it make sense to invest in an ounce of prevention that would prevent a pound of cure in the distant future? Whenever the method of prevention is so effective that compounding its cost at the SOCR would result in a future value that is less than the cost of the damage it prevents. Notice that the comparison to be made is between two alternative future amounts and not a comparison between a present cost and a future benefit. Therefore, intertemporal weighting plays no role in finding the answer to the question. What needs to be ascertained is whether a proposed project is more efficient at transforming present day resources into future benefits than other available investment possibilities. The welfare of future generations would only be enhanced by investing in projects that are more effective at converting present-day resources into future benefits than the capital market. As Becker, Murphy and Topel (2011) stated: “Future generations would not thank us for investing in a low-return project.”

Some may feel that the opportunity cost of capital is a hypothetical concept, somehow less real and tangible than incurred cost. This is not so, however, for in CBA all costs are opportunity costs, given the requirement of establishing the difference in aggregate welfare levels between the with-project and the without-project states of the world. Furthermore, at least in our model, interestingly, the opportunity cost nearly equals the directly and indirectly incurred fiscal cost.

Investment projects incur costs of capital because the difference between the flows of benefits and costs is initially negative and this difference must be financed by someone who provides resources that have alternative uses, hence an opportunity cost. This gives rise to a stock of debt that keeps growing as long as accumulated all-inclusive project costs exceed accumulated project benefits. The welfare cost of providing this funding is the lenders’ opportunity cost of funds: the interest/dividend income they would have earned by investing elsewhere the amount lent to the project.

This line of reasoning shows why the costs of capital must be computed at the SOCR using compound interest rules, as is done in two-rate discounting. It also shows that a project that is unable to cover its cost of capital from its benefits is welfare destroying. This can be detected by its failure to clear both the STPR and the SORC hurdle rates, or equivalently, by yielding a negative two-rate NPV.

The situation is conceptually similar if a project is financed by taxation rather than by borrowing because the welfare cost of taxation is not just the forgone consumption that directly follows. The ensuing re-optimization of consumption paths will likely cause investment in productive activities to decline, which results in additional opportunity losses.

7. Shadow pricing capital

The OECD CBA Manual (2018:221) states “Using the Shadow Price of Capital approach (SPC) is advisable when using the [STPR], so that the opportunity cost of public capital can be reflected in the NPV calculation. This rarely happens in practice due to onerous informational requirements”. Not using the SPC correction means that projects’ primary flows are simply discounted with the STPR, which results in incorrect NPV results, as seen in the previous Sections. But what happens if the SPC correction is used and how does that result compare to the result of applying the two-rate discounting method?

The need for computing a shadow price of capital to measure the opportunity cost of displacing private investment was raised by Marglin (1963), who formulated six alterative models, all measuring effects of displaced private sector investments reaching to infinity, “as a function of displacement, reinvestment, and yield rates”. Groom *et al* (2022) state that “The STP approach can be augmented by ensuring that any impact of public investment on private investment (crowding out), and subsequent perturbations to consumption possibilities, are valued and reflected” and that “another way in which the STP approach can be augmented is via the Marginal Cost of public Funds (MCF)”

These considerations all center on answering the need to measure the welfare costs of raising funds for the undertaking of public projects, which is crucial to CBA, in its attempt to measure the welfare impact of public sector investments. It is possible to identify those effects (such as direct and indirect costs of raising funds, opportunity cost of forgone investments, cost of raising public funds, dead weight losses and costs of collection attributable to taxing, etc.) and express them as an interest rate, the SOCR, as was done in Section 3 above, or convert them into a shadow price factor, to be applied to the capital costs of projects, as will be done in this Section.

Formulations of the SPC generally use the private return of capital as a departure point. For the reasons described above, however, we will use the SOCR instead, as this is what is required in CBA. A good SPC formulation can be found in Broughel (2020), which just like Marglin (1963) uses an infinite time horizon. It is a function of the STPR, the return on investment and the fraction of the investment that is reinvested in each period.

Using a factor that quantifies effects extending to infinity for finitely lived projects requires adjusting residual values and estimating reinvestment rates. A simpler formulation proposed by Cline (1992) and explained in the OECD CBA Manual (2018:207) limits the time horizon of the calculation to the life of the project to which it applies and dispenses with the reinvestment parameter, assuming that project net benefits and repayments of capital will equally be available for consumption or savings, as beneficiaries/lenders see fit. On this basis the SPC factor is defined by the following expression:

$$SPC = \frac{r}{STPR} \cdot \frac{1-(1+STPR)^{-N}}{1-(1+r)^{-N}} \tag{3}$$

where r is the private return on investment, but which we will take to be the SOCR, and N is the life of the project in years. What this equation calculates is the present value of the stream of equal repayments that amortize a loan of \$1 yielding r over N years, discounted at the STPR. It is thus the present value of the opportunity cost of capital for each invested dollar.

Evaluating this expression with the Table 5 data of Project A of section 4 above (STPR = 2.6225%, SOCR = 6.7631%, N = 1) results in SPC = 1.040347.

The conventional NPV of Project A, adjusting the investment with this SPC factor and discounting with the SPTR, can be computed as follows (flow data from Table 4):

$$– 1000 \cdot 1.040347 + 1080 / 1.026225 = 12.0533 \tag{4}$$

which is exactly the same NPV as was obtained by the two-rate discounting method in Table 5.

We can conclude, therefore, that the two-rate discounting method is fully equivalent to discounting SPC adjusted project flows by the STPR. This is so because the two-rate method does the same thing as the SPC adjustment: computes capital costs with compound interest rules and then discounts them with the SPTR.

Repeating the exercise for the long-lived project of Section 6 (STPR = 2. 5%, SOCR = 4%, N = 100) we get SPC = 1.494148409. The consequent conventional SPC adjusted NPV is as follows (flow data from Table 9):

$$– 100 \cdot 1.494148409+ 1921.86 / 1.025^{100} = 13.26555051 \tag{5}$$

which, however, is not the same as the NPV of –264.83 computed by the two-rate method (Table 9). This is because the assumption of even loan amortization over the life of the project implicit in Cline's expression (3) is not met by this project. The correct SPC factor in this case can be computed from the data in Table 8 as follows: $(5050.49 / 1.025^{100)}) / 100 = 4.275106876$. Recomputing (5) with this result we get:

$$– 100 \cdot 4.275106876 + 1921.86 / 1.025^{100} = –264.83 \tag{6}$$

which is the same as the NPV computed by the two-rate method, see Table 9.

Thus, by not using a generic formulation, but reckoning with the actual use of capital of the specific project, the two-rate discounting method provides a more accurate SPC adjusted, STPR discounted NPV. These examples illustrate that the correct SPC factor is always project specific, and that there is no single SPC value that will correctly shadow price the capital expense of any project in a given country. The SOCR is project independent, however, which makes using the two-rate discounting method easy. The SPC adjustment will implicitly be correctly calculated thereby, without any additional "onerous" data requirements.

An important corollary that can be deduced is the practical shortcut already arrived at: for projects to be economically feasible their IRR must exceed both the STPR and the SOCR. The justification for the first condition is obvious: not meeting it will result, by definition, in a negative NPV. The justification for the second is that if IRR < SOCR then the project will have been unable to repay its capital costs by the end of its useful life making its residual value negative, the present value of which will also be negative, regardless of what STPR is used to discount it.

Given that typically SOCR > STPR, it is true that for an SPC adjusted, STPR discounted NPV to be positive, the project IRR must exceed the SOCR, which becomes the effective hurdle rate.

8. Conclusions

This paper answers the choice of discounting-approach question by stating that there is no need to choose between the approaches, nor is there a need to reconcile them. There is no choice to be made: both must be used and the higher of the two, generally the SOCR, is the effective hurdle rate. This does not make the STPR redundant, it is needed to compute the correct NPV of projects and is useful in ranking them.

The two-rate discounting method proposed meets the objectives of both approaches. It can accommodate any STPR, including the zero-discount rate desired by some adherents of the prescriptive approach, while simultaneously taking full account of the appropriate opportunity cost of capital. Furthermore, this method is equivalent to conventional discounting with the SPTR after having adequately shadow-priced investments.

The most important operative conclusions of this paper are:

- The two-rate discounting method, using both the STPR and the SOCR, yields an accurate welfare value NPV of projects.
- The STPR must not be used to conventionally discount project net cash flows unless the SPC correction has been previously correctly applied, for it could result in investing in projects that fail to recover their capital costs.
- The SPC correction is not a generic economy-wide factor but must be computed specifically for each project. The two-rate discounting method implicitly performs the SPC adjustment better than standard SPC formulas because it reflects better the actual use of capital of projects.
- A rule of thumb to follow for conventional discounting is that for projects to be welfare-enhancing, their internal rate of return must exceed *both* the STPR and SOCR hurdle rates.
- A corollary of the foregoing is that if STPR < SOCR, as is generally the case, then the SOCR is the effective feasibility hurdle rate.

The proposed two-rate discounting method will ensure that public sector projects be no less effective at converting present consumption into future consumption than what the public can already manage and that the benefits of projects exceed all their direct and indirect costs.

The above conclusions of this paper do not derive from the general equilibrium capital market model it uses to illustrate its arguments. Rather, they derive from applying well-known compounding and discounting rules and complying with the requirement that discounting be done at the rate of fall of the consumption numeraire. However, the model does provide interesting insights concerning the estimation of both types of discount rate.

- The model indicates that the SOCR derived from direct and indirect financing costs of fiscal borrowing closely corresponds to that calculated from the forgone expected yield of displaced private sector investments. Therefore, the opportunity cost of displacing private sector investments has nearly equivalent, tangible budgetary consequences.
- The STPR directly measured as a utility preserving compensating variation equals the weighted average interest rate that savers/borrowers effectively face. An STPR defined in any other way (based on philosophical considerations or determined politically, as by a “social planner”, or derived from some theoretical model) will not lead to the calculation of accurate welfare value NPVs, as usually required in CBA.
- Interestingly, in our capital market model the expected yield of stochastic investment opportunities only plays an indirect role in the determination of the STPR, through its effect on the market interest rate. Agents, even those that do invest in stocks, optimize their consumption path by reference to the effective yield of the safe instruments available to them. This is worthy of further research.
- The model illustrates the fact the revealed preference STPR cannot be predicted by the Ramsey equation as conventionally used. Changes in its parameters do not lead to the expected changes in the computed STPR.

9. Implications for selected discounting approaches found in the literature

In choosing what to focus on in this Section, we looked at two comprehensive survey articles: “Discounting for public policy: A survey,” by Greaves (2017) and “The Future, Now: A Review of Social Discounting,” by Groom *et al* (2022). In this Section we address the impact of the conclusions of this paper on a selection of proposed discount rate setting methods discussed in these surveys.

Groom *et al* (2022) described early attempts to reconcile the prescriptive and descriptive approaches by considering a weighted average of the alternative discount rates, with the weights depending on the source of funding. According to the finding of this paper, weighted average of the STPR and SOCR discount rates are unlikely to produce the right result, except by chance, given that the weighted average rate will neither correctly measure the relative value of money across time periods nor adequately quantify the cost of capital. It should be noted that the weighted average of interest rates proposed by Harbberger (1972) is a method to estimate the SOCR, and not a weighted average of the STPR and the SOCR.

Also mentioned are methods whereby the STPR would be used for discounting, with the cost of capital reflected by the SPC. This is congruent with the findings of this paper, but there does not seem to be any awareness of an important conclusion of this paper, namely that applying the SPC correction will result in the SOCR becoming the effective feasibility hurdle rate.

Perhaps because of the perceived problem of excessive discounting of the distant future, the bulk of recent literature concerns the prescriptive approach, which tends to result in lower discount rates. Ben Groom et al (2005:459) observe that it is "because we are uncertain about the long-run market rate of return that the social rate of time preference is frequently used for CBA." This practice, nearly always coupled with the already quoted one of foregoing the SPC adjustment, paves the way for wrong investment decisions, according to the conclusions of this paper. No STPR, regardless of how it is defined, can be used as the sole discount rate to determine project feasibility in conventional discounting, unless a correct SPC adjustment had been applied first.

Greaves (2017) stated that "the standard approach to determining the [STPR] discount rate is via the Ramsey equation," and discusses at length the possible values attributed to its parameters by, among others, Nicholas Stern, William Nordhaus and Martin Weitzman, computing discount rate values from their estimations with results ranging from 1.4% to 6%. This paper questions the validity of this practice, at least as far as defining a revealed preference STPR goes. The revealed preference SPTR is the reflection of a market equilibrium, which the parameters of the utility functions of savers/borrowers are insufficient to describe, and which the Ramsey identity cannot predict.

Groom *et al* (2022) also discussed the "Ramsey Rule" but derived it from a more general formulation based on "a time-discounted Utilitarianism approach." It showed that the equation given in footnote 3 above, which they called the "Simple Ramsey Rule," is the first term of a more general expression derived from a welfare function. Next, they presented the "extended Ramsey Rule," which contains an additional term that reflects "the prudence of the social planner." "The social planner is concerned that future consumption levels may be below their expected value and therefore is prepared to save out of precaution in projects that pay off in the future. This translates into a lower [social discount rate (SDR)]," which, if consumption growth displays positive autocorrelation, will result in declining discount rates (DDR). Should this effort be aimed at estimating a revealed preference STPR, we can just restate that no welfare function contains all the information needed to predict it. Nothing is gained by tinkering with the Ramsey expression, for once agents optimize their consumption path by reference to the effective interest rate they face, their MRS will correspond to that rate, regardless of how their welfare function is defined. This is the consequence of complying with the first-order optimality condition. Our model results suggest that the best way to determine a welfare measuring STPR would be by direct empirical observation of effective interest rates. The Ramsey equation is not useful for this task.

Groom et al (2022) cited another important strand of recent research that analyzes the impact of uncertainty on discounting. Even though this paper does not address this topic, it must be mentioned because of its importance in recent literature. Weitzman (1998) asserted that when market interest rates show positive autocorrelation, their certainty equivalent should be a declining function of time.

This assertion does not pertain to STPRs, but rather to market interest rates, or by extension to SOCRs. Greaves (2017) also described Weitzman's work and cited Weitzman's argument that the certainty equivalent discount factor should be derived from the expected value of all possible discount factors, which results in declining discount rates (DDR) if interest rates are positively autocorrelated. Greaves (2017) presciently pointed out that "Weitzman did not, however, supply any fundamental justification for his assumption that the effective discount factor is the expectation value of the true discount factor." As it turns out, Weitzman's assertion is a seductive fallacy, which when corrected, results in certainty equivalent discount rates that are increasing, rather than declining[10]. See Szekeres (2020a). Some of the empirical evidence for DDRs cited by Groom et al (2022), *e.g.*, Newell and Pizer (2003), is spurious because it applies empirical data to Weitzman's fallacious premise. See Szekeres (2020b).

An interesting light is cast on the state of discourse on the SDR by "Philosophers and Economists Can Agree on the Intergenerational Discount Rate and Climate Policy Paths" by Nesje *et al* (2022). The authors conducted a survey of philosophers and economists, to which 29 of the former and 627 of the latter responded. They were asked to propose an SDR, and remarkably "the mean responses are almost identical at 2.27%." This article is interesting not so much because of the estimates that it provides, but rather for the light it shines on the SDR debate.

- "…the recommendations of Nobel laureate William Nordhaus and Lord Stern on the urgency of climate action were largely attributable to disagreement on how much the SDR should reflect prescriptive versus descriptive factors, notably intergenerational justice versus opportunity costs reflected in interest rates." According to the conclusion of this paper there is no conflict between intergenerational justice and the opportunity cost of capital. The economic feasibility of very long-lived projects depends on the relative efficiency of proposed projects in creating benefits in the future, as compared to what available alternative investments can achieve in the same future. Ascertaining this requires forecasting, not intergenerational comparison.
- Concerning the simple Ramsey rule of STPR determination, the views of economists and philosophers differed concerning estimates of parameters such as the pure rate of time preference and the marginal utility of consumption. "They also provided a number of criticisms of the model itself; alongside proposed extensions ... one broad critique was that 'there is no empirical support' [for this model]." This paper also questions the usefulness of the Ramsey expression for revealed preference STPR determination. All these considerations, however, relate to the STPR, and are irrelevant to the question whether the benefits of projects suffice to cover their opportunity costs of capital.
- Extensions and alternatives to discounting are mentioned. "Rather than mainly proposing alternatives to Discounted Utilitarianism per se, economists predominantly recommended myriad technical extensions that lie squarely within the consequentialist Utilitarian framework." Inasmuch as these pertain to specifying the STPR, the principal decision rule proposed by this paper is unaffected. No utility function is needed to compare a project's IRR to the SOCR. Many of the other problems cited, such as uncertainty, income distribution, sustainability or substitutability issues, can be dealt with within the CBA context by adequately specifying costs and benefit flows and need not alter the effective

---

[10] A Weitzman present value is computed by multiplying a future sum to be discounted by the expected value of all possible discount factors, while calculating the correct present value requires multiplying the future sum by the inverse of the expected value of all possible compound factors. It is easy to verify that the latter method complies with the definition of present value, whereas the former does not. This discrepancy gave rise to what was called the Weitzman-Gollier puzzle, which engendered a large literature of its own. As shown in Szekeres (2020a) a Weitzman present value is equal to the correct present value times one minus the covariance between the compound and discount factors. As this covariance is always negative, because the factors are each other's reciprocals, the Weitzman present value will always be larger than the correct present value. The Weitzman formulation results in certainty equivalents that are declining as a function of time, whereas the correct formulation yields increasing ones, assuming positively autocorrelated interest rates.

feasibility threshold SDR, the SOCR. The only notion raised that might break the CBA framework of analysis is the precautionary principle, which eschews quantification.

It can be concluded that the current literature on social discounting focuses principally on the STPR and shows less interest in the SOCR. Authors might have discarded the need to account for the opportunity cost of capital or might think that once the SPC adjustment has been applied to the capital costs of projects (reportedly seldom done in practice), it is the STPR that will constitute the sole hurdle rate of return of projects. The already cited expression "because we are uncertain about the long-run market rate of return that the social rate of time preference is frequently used for CBA." This is effectively saying that "as the cost of capital is hard to estimate, assume it's zero." Given that we can be nearly certain that SOCR>STPR, this is very poor advice.

The poor advice has been widely heeded. The academic literature succeeded in leading some policy makers to adopt the STPR as the sole relevant discount rate, or to subscribe to the theory of DDRs. "The successful deployment and dissemination of DDRs suggests that, for better or worse, academic economists can enslave practical men with economic ideas." (Groom and Cameron, 2017) It was for the worse, judging by the conclusions of this paper. Most of the recent literature on the SDR (that calls for discounting unadjusted project flows with STPRs or DDRs) results in recommendations that are unsuitable for use in CBA, because they seriously understate the capital cost of projects.

However, the cited enslavement has only been partial. Figure 1 of Groom *et al* (2022) shows that out of 21 countries and institutions selected, 13 still exclusively use the SOCR for discounting. These have not lost sight of the basic fact that projects failing to cover their opportunity cost of capital are welfare destroying.

REFERENCES

Baumol, William J. (1968) "On the Social Rate of Discount", *American Economic Review, Volume 58*, No. 4 (September 1968), https://www.jstor.org/stable/1815533

Becker, Gary, Kevin Murphy and Robert Topel. (2011). "On the Economics of Climate Policy." *The B.E. Journal of Economic Analysis & Policy*. 10. 19-19. 10.2202/1935-1682.2854. https://doi.org/10.2202/1935-1682.2854

Broughel, James (2020). "Accounting for the opportunity cost of capital in cost-benefit analysis: public interest comment on the marginal excess tax burden as a potential cost under Executive Order 13771." Docket No. OMB-2017-0002. https://www.mercatus.org/publications/regulation/accounting-opportunity-cost-capital-cost-benefit-analysis-public-interest

Cline, W. R. (1992) "The Economics of Global Warming", Institute for International Economics, Washington, DC.Chichilnisky, Graciela. (1997). "What is Sustainable Development?" *Land Economics* 73, 467–491. http://www.jstor.org/stable/pdfplus/3147240

Freeman, Mark C. and Ben Groom, "Gamma Discounting and the Combination of Forecasts" (2010). http://dx.doi.org/10.2139/ssrn.1676793

Gandelman, Néstor and Rubén Hernáandez-Murillo (2014). "Risk Aversion at the Country Level," Federal Reserve Bank of St. Louis, http://research.stlouisfed.org/wp/2014/2014-005.pdf

Greaves, Hilary R. G. (2017) Discounting for public policy: A survey. Economics and Philosophy 33 (3):391-439, https://philarchive.org/archive/GREDFP-2

"Green Book:" HMT (2020). *Guidelines on Cost-benefit Analysis*. HM Treasury, UK, https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/938046/The_Green_Book_2020.pdf

Groom, Ben, Cameron Hepburn, Phoebe Koundouri, and David Pearce (2005). "Declining Discount Rates: The Long and the Short of it." *Environ Resource Economics* 32, 445–493 https://doi.org/10.1007/s10640-005-4681-y

Groom, Ben and Hepburn, Cameron. (2017) "Looking back at social discounting policy: The influence of papers, presentations, political preconditions, and personalities." *Review of Environmental Economics and Policy* 11, (2) pp.336-356. ISSN 1750-6824. https://www.journals.uchicago.edu/doi/10.1093/reep/rex015

Groom, Ben, Moritz A. Drupp, Mark C. Freeman, Frikk Nesje (2022) "The Future, Now: A Review of Social Discounting," *Annual Review of Resource Economics* 2022 14:1 https://doi.org/10.1146/annurev-resource-111920-020721

Harberger, A. C. (1972) Chapter 4 in Project Evaluation, collected papers of A. C. Harberger. Chicago: University of Chicago Press

Marglin, Stephen A. (1963) "The Opportunity Costs of Public Investment." *The Quarterly Journal of Economics*, May, 1963, Vol. 77, No. 2 (May, 1963), pp. 274-289 https://www.jstor.org/stable/1884403

Nesje, Frikk and Drupp, Moritz A. and Freeman, Mark and Groom, Ben (2022). "Philosophers and Economists Can Agree on the Intergenerational Discount Rate and Climate Policy Paths" *CESifo* Working Paper No. 9930, http://dx.doi.org/10.2139/ssrn.4219434

Newell R G, Pizer W A. 2003. "Discounting the distant future: how much do uncertain rates increase valuations?" *Journal of Environmental Economics and Management*, 46(1):52–71. https://doi.org/10.1016/S0095-0696(02)00031-1

Nordhaus, William (2019). "Climate Change: The Ultimate Challenge for Economics." *American Economic Review,* 2019, 109(6): 1991–2014, https://doi.org/10.1257/aer.109.6.1991

OECD (2018), Cost-Benefit Analysis and the Environment: Further Developments and Policy Use, OECD Publishing, Paris, https://doi.org/10.1787/9789264085169-en.

Pigou, Arthur C. (1932). *The Economics of Welfare*, 4th edition. Macmillan, London

Rubinstein, Mark, (1976), "The Strong Case for the Generalized Logarithmic Utility Model as the Premier Model of Financial Markets," *Journal of Finance*, 31, issue 2, p. 551-71, https://doi.org/10.1111/j.1540-6261.1976.tb01906.x

Szekeres, Szabolcs (2020a) "Correcting the Error in Gamma Discounting." MPRA Paper No. 102232, https://mpra.ub.uni-muenchen.de/102232/

Szekeres, Szabolcs (2020b). "Checking the Evidence for Declining Discount Rates," MPRA Paper No. 102233, https://mpra.ub.uni-muenchen.de/102233/

Weitzman, Martin L. (1998). "Why the Far Distant Future Should Be Discounted at its Lowest Possible Rate", *Journal of Environmental Economics and Management* 36, 201–208. https://doi.org/10.1006/jeem.1998.1052

APPENDIX

1. Introduction

This appendix provides details of the agent-based model used to generate the numerical examples in this paper. It is best read in conjunction with the accompanying Excel workbook so as to view the results discussed, which, because of their availability there, are not presented here.

An overview of the model is that 58 agents are endowed with pairs of incomes in Years 0 and Year 1, and each finds his own optimal intertemporal resource reallocation action maximizing the welfare function described in the main text. The interest rate on lending/borrowing is determined endogenously as the market-clearing rate.

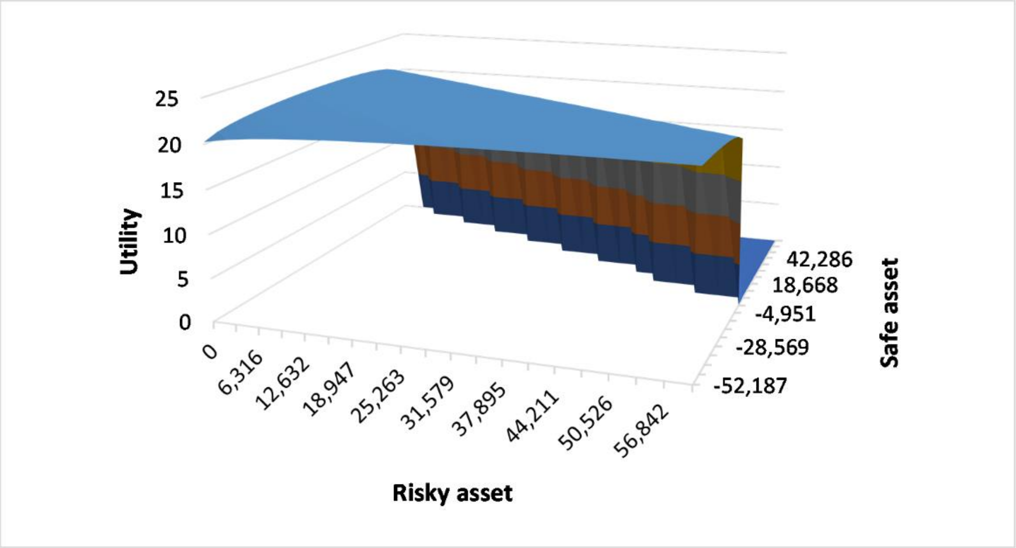
This requires two levels of optimization. First, for any given interest rate, the optimal welfare maximizing action must be found for each agent. If at the first tested interest rate there is a disequilibrium between the demand for loanable funds and their supply, then the interest rate must be appropriately changed, which is done iteratively until the imbalance between lending and borrowing falls below $0.5.

The algorithms doing the optimizations are written in the MS Visual Basic language in which Excel macros are written and can be viewed by clicking on the *Visual Basic* icon in the *Developer* tab of Excel. Most of the macros are in Module 1, but there are also some in Sheets 2, 5 and 7.

The optimization macros are straightforward, without recourse to sophisticated search methods, and the code is documented, so it is easy to follow. The individual agent's optimization routine evaluates the utility function at all feasible combinations of lending/borrowing and investing, arrayed in a grid of 50 rows and columns, and finds the best combination.

Figure 1A

Utility of lending/borrowing and investing



In Figure 1A the vertical axis shows utilities, invested amounts are shown in the breadth of the plot, and lending (negative if borrowing) in the depth.

Once the best combination of assets is found, the immediate neighborhood of the optimal value is subdivided into a new 50 × 50 grid to again find the optimum. The process is repeated until the desired precision is achieved.

A simplified version of this search that has a coarser grid and operates without macros is available on the *Explore* worksheet for illustrative purposes. The utilities are computed by formulas that can be traced without any programming knowledge. (The case is simplified: it includes no taxes, and the utility is logarithmic only). By manually changing the possible ranges of risky and safe asset amounts, the reader can find the optimal allocation to any desired degree of precision. The cell containing the utility of the best combination is highlighted in orange.

The market-clearing interest rate is found by an algorithm that uses interval halving, which will stop when the market-clearing interest rate is found.

There are several additional macros, some that help audit the optimization results, and some that perform operations that are necessary to derive needed outputs. These will be described in the following sections.

2. Income distribution

The Year 0 income levels considered are shown in Column A of the *Income distribution* worksheet starting at row 6. Thirty income levels are used, growing from $20,000 in increments of $40,000 to reach $1,220,000. These are converted to natural logs in Column B. Column C contains the normal cumulative probability distribution of these log income values, with a mean of 12 and a variance of 1[11]. In Column D we have the cumulative number of agents up to the income level of the corresponding row, and finally Colum E has the number of agents for each income level. However, the number of agents so generated is only 29, with the lowest income being $60,000 and the highest $1,020,000. These are the values plotted in Figure 1 of the main text and at the top of this worksheet.

The number of agents is then doubled because half see their Year 1 income grow and half see it decline. The rates are given in Rows 1 and 2 of Column S. There is a macro that can be launched by clicking on the button labeled *Populate* that will fill[12] Columns R-T starting with Row 5 as appropriate, creating the required Income 0 and income growth combinations. The other parameters of the agents are computed by formulas. The pure rate of time preference and coefficient of risk aversion of each agent will be the ones assigned to the first agent, but these can be changed if desired. Income 1 is computed from the growth rate of income assigned to each agent.

This worksheet contains a second plot, that of an indifference curve. It is possible to request the indifference curve of any one of the agents, or of the entire market, or of the representative agent, by entering a number in Cell J21. The indifference curve is constructed by placing in Column G the Income 0 range of 20 steps up and 20 steps down from the endowment value of the chosen agent in percentage steps that can be specified in cell L21 (or the aggregation of incomes for the entire market option, or their average for the representative agent). Column H contains the similarly defined Income 1 for the endowment case (row 58). For all other rows of this column a macro, launched by clicking on the *Generate Indiff* button[13], finds an Income 1 value such that the total utility of it and the corresponding Income 0 value leaves the agent's utility unchanged from the endowment base-case. From these changes the slope of the defined arc of the indifference curve can be computed, and from that, the corresponding interest rate.

Column I contains the slope defined by adjacent combinations of Income 0 and Income 1 values, which defines the MRS, and Column J contains the corresponding interest rate. Be mindful of the fact that these interest rates are only approximately correct because they correspond to arcs defined

---

[11] There is no particular reason to have adopted these values other than to obtain the skewed density function shown in Figure 1 of the main text.

[12] Sub Populate in Sheet2

[13] Sub IndifferenceMap in Module1.

in discrete steps of Income 0. The exact value is computed by the macro using the familiar Ramsey equation[14], and placed in column K.

When the representative agent option is exercised, the same output is copied to the adjacent light blue shaded area, so that this value will not be overwritten by subsequent requests. Looking at the values for the entire market and those of the representative agent, we see that the interest rates corresponding to the MRS values are identical, which means that the representative agent is indeed representative.

3. Simulation

The *Simulation* worksheet is where simulations are launched. The first two rows specify the amount of fiscal borrowing (negative if the state buys bonds) and the parameters of the risky asset (percent of gain and loss of the invested amount and the probability of gain, from which the expected yield is computed). The tax rate is specified, as well as the time elapsed in years between the first and the second modeled periods. This is set to 1 and it should not be changed, because higher values have not been tested yet.

In row 3 a lump sum tax raised to finance a project can be specified. Be sure that both C2 and C3 contain numeric values.

Simulations are launched by clicking on the *Simulate* button[15]. Simulations take a while, during which progress report messages appear. Usually around 40 rounds are needed to find a market-clearing interest rate. The results of the simulation appear in Columns A-C below Row 5. Aggregate values are given, as well as the capital market actions of all agents. Negative amounts of the safe asset indicate borrowings.

The equilibrium safe security price can be verified by clicking on the *Verify equilibrium* button[16]. This launches a new equilibrium search by a different search method. It uses a hill-climbing algorithm that starts with a low price, goes in steps until it overshoots the solution, and then turns around in smaller steps. After each sign-change of the difference between saved and borrowed amounts is shown and the user is given the option of quitting or continuing. Click on *Clear*[17] to erase this.

There is a third market equilibrium algorithm as well that operates on the *Plot* worksheet. This subdivides a price search range specified by the user into 50 segments and computes, for each price, the supply and demand of loanable funds as well as the amount invested in equities and plots the results (this is the source of Figures 1 and 2 in the main text). By running this over ever-narrowing ranges, the market-clearing price can be found to any desired degree of precision. Click on *Run*[18] to start. Runs take a while. *It is advisable to use this option to determine if a market-clearing interest rate exists when choosing a new set of model parameter values*. For some combination of values there might be no lending or borrowing and, in such cases, the primary market equilibrium finding algorithm will fail to converge, as there would be nothing to converge to.

There is also the option of verifying the equilibrium of any single agent. Specify the agent number in Cell J5 on the *Simulation* worksheet and click on *Start*[19]. This will show the 50 × 50 grid mentioned in section 1 of this Appendix, identify the optimal cell as well as the optimal row and column. All the values required to compute the utility value of the optimal combination will be

---

[14] Defined in footnote 6.

[15] Sub Simulate in Module1

[16] Sub VerifyEquilibrium in Module1

[17] Sub Clear in Module1

[18] Sub Plot in Module1

[19] Sub VerifyInvestor

presented below the grid to permit an audit of the result. To continue to zoom in on the answer with greater precision click on *Continue*[20]. This can be done until the desired precision has been reached.

Click on *Clear* to clear the verification area.

4. Opportunity cost

The Opportunity cost worksheet is where the model's key outputs can be seen. There are three blocks of data: results of a reference case run, those of a public borrowing/lending or taxing case, and the difference between the two. In a fourth area the direct and indirect costs of fiscal borrowing/lending/taxing are calculated. To make the correct comparison, the two cases must be run independently.

First, the reference simulation without public borrowing or lump sum taxing is run, as described in the previous section. Next, on the Opportunity cost worksheet click on the *Copy*[21] button found in the reference case area. This transcribes the results from the just finished simulation from the Simulation worksheet, including the yields of the safe and risky asset, the amount borrowed, lent or invested both globally and individually by each agent. Lending is rationed pro-rata to the extent to which lending exceeds borrowing, to ensure that they match exactly.

The routine also computes agents' expected tax payments, attained utility and MRS, which is expressed as the corresponding interest rate. The latter are analytically computed from the agents' welfare function. This procedure also aggregates Year 0 and Year 1 consumption across all agents, from which a cell formula computes the discount rate that corresponds to the Ramsey rule.

Clicking on the *Indiff rate* button[22] computes the STPR by finding the compensating variation in Year 0 income that leaves each agent's utility unchanged after receiving $1 in Year 1. The aggregated compensating variations and amount of added Year 1 income define the social MRS, from which the corresponding STPR is derived.

The Ramsey rate shown is computed by its formula from the average parameter values of all investors and the observed change in total consumption between the time periods. It is interesting to note that this always coincides with the STPR, and it is also quite close to the weighted average of the individual MRS values expressed as interest rates.

After performing a second simulation with fiscal borrowing or taxing (borrowing $1,000 in our example), the copying procedure just described is performed for the borrowing/lending case by clicking on the homologous buttons in the area that corresponds to this case.

Once both cases have been copied, the differences in their results are automatically displayed in the third area that is labeled as CHANGES. Changes in expected tax payments and attained utility are displayed for each agent. Clicking on the *Compensating Variation* button[23] found in the CHANGES area quantifies the welfare gain or loss suffered by each agent because of the changes induced by fiscal borrowing, which induced them to change their behavior. The aggregation of the compensations needed (negative in some cases, as some agents benefit) gives the welfare cost of fiscal borrowing that is not compensated by the interest paid.

The final panel shows how the SOCR is derived from the detected changes, as described in the main text. Cell Q8 contains the percent of equity investments that the cost of financial services represents. In our example we assumed this to be 3%.

---

[20] Sub Continue in Module1

[21] Sub RefCase for the reference case and Sub BorrowingCase for the Public borrowing/lending case, both in Module1

[22] Sub Refindiff for the reference case and Sub Pubindiff for the Public borrowing/lending case, both in Module1

[23] Sub CompensatingVariation in Module1

An alternative SOCR calculation method is shown below that. The first line shows the actual interest expenditure of the public sector and the second the forgone tax revenues. The subsequent adjustments are the same as in the first SOCR estimation.

5. Project appraisal

Worksheets *Project A* and *Project B* contain the analyses pertaining to the projects. These are the sources of Tables 3-7 in the main text, where they have been explained.

Each project worksheet has a button labeled *Com Var NPV*[24]. Clicking on this, computes for each agent the equivalent compensating variation in Year 0 that leaves his utility on a par with receiving, in Year 1, his share (1/58) of the net future welfare effect of the project in question (Cell C5). The aggregation of these compensating variations is the welfare equivalent NPV of the project. It is the same as discounting the net welfare effect by the STPR.

The individual compensating variation amounts are shown, as well as the interest rate corresponding to each agent's MRS.

---

[24] The macro code is found on the corresponding sheet.