Employment Protection and Labor Productivity: Positive or Negative?

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Abstract

Since the 1980s, many European countries have implemented labor market reforms, introducing more flexible labor contracts. This paper develops a matching model with heterogeneous matches in order to analyse the impact of employment protection on labor productivity. Several channels affect productivity. On one hand, flexible contracts reduce mismatching: low productive jobs are destroyed at no cost with a positive impact on the overall productivity. On the other hand, they imply lower human capital investment, reducing labor productivity. We analyze a third channel: the selection of the employees. Low costs of dismissal reduce the incentive of firms to invest in screening applicants, therefore increasing the uncertainty about their unobserved skills and productivity.

Keywords: Employment protection, Stochastic job matching model, Screening.

1 Introduction

Since the 1980s, many European countries have implemented labor market reforms aimed at reducing the level of employment protection. The idea behind this process is twofold: on one hand, labor market rigidities are regarded as a cause of the poor dynamic of employment in Europe as opposed to US. Furthermore, it is claimed that rigidities hinder the adjustment of workforce to shocks and, therefore, are a burden on the competitiveness of the European economy.

There’s a wide literature on the effects of employment protection legislation (EPL). It focuses mainly on the effects on unemployment and job turnover. The main findings are that EPL reduces both hiring and firing of workers with ambiguous effect on unemployment.\footnote{Among the most cited studies, see Bentolila and Bertola (1990) and Ljungqvist (2002). An overview is provided in Cahuc and Zylberberg (2004, chapter 12).} Little effort has been devoted to the analysis of the impact of EPL on productivity. Few exceptions are Alonso-Borrego et al. (2004), Aguirregabiria and Alonso-Borrego (2004) and Veracierto (2003) that calibrate and simulate general equilibrium models to assess the impact of labor market reforms on output and productivity. They generally find some positive effect of the introduction of temporary contracts. Their models focus on the role of EPL in increasing mismatching, that is the tendency of firms to retain unproductive workers instead of dismissing them and paying firing costs (institutional labor hoarding).

Other factors affect productivity: firing costs can stimulate investment in human capital,\footnote{See Arulampalam and Booth (1998), and Rix et al. (1999) for empirical evidence.} or may lead firms to implement stricter selection rules. We consider the latter channel: the employers’ recruitment choice. Low costs of dismissal reduce the incentive of firms to invest in screening applicants,
therefore increasing the uncertainty about their unobserved skills and productivity. Literature on employers' search provides evidence of the correlation between recruitment and workers quality.\(^3\) Furthermore, a study by Abowd et al (2002) reveals that productivity is positively related to soft skills, i.e. unobservable characteristics of employees. In light of this findings, the selection process turns out to be an important factor in the formation of firms outcome.

This paper develops a matching model with heterogeneous matches and endogenous job destruction capable to account for both mismatching and employee selection. On one side, EPL is associated with lower job destruction and more mismatching. On the other side, firing costs increase the average quality of workforce, inducing higher hiring standards. The net effect depends on the relative importance of job-specific and match-specific components of productivity.

Next Section reviews the previous literature on the relation between recruitment choice and productivity. Section 3 presents the model. Findings are discussed in Section 4 and Section 5 concludes.

## 2 Employment selection and productivity

The selection process consists in the set of activities through which the employer collects informations about potential employees. The choice of the recruitment method affects the quality of applicants,\(^4\) thereby the productivity of jobs. This relation is stricter the greater is the dependence of productivity on personal characteristics of workers.

Abowd et al. (2002) evaluate the contribution of human capital to business productivity and shareholder value. They find a substantial positive relation between human capital and market value that is primarily related to the unmeasured personal characteristics of the employees, called "soft skills".\(^5\) In the same spirit, Haskel, Hawkes and Pereira (2003) distinguish between hard skills - i.e. education, experience and, in general, formal observable skills - and soft skills - informal skills such as attitude, time keeping, etc. Exploiting UK data, they find positive correlation between both types of skill and productivity, measured as gross output per employee.

Nevertheless, empirical evidence relating personnel department expenditures to organizational outcomes is inconclusive. Abowd, Milkovich and Hannon (1990) find no significant correlation between human resource decisions and shareholder value. Eastwood, Rudin and Lee (1995) analyze the relation between previous year’s personnel department expenditures and total annual output, for eleven large railroads in US over six years. The rough correlation is positive, but after controlling for total assets and workforce size, no effect of personnel department expenditure is found. On the other side, Boundreau (1991) reviewed 19 empirical studies of improved selection techniques and concludes that <<virtually every study has produced dollar-valued payoffs that clearly exceeded costs>>.

Also Phillips (1988) finds positive significant correlation between HR expenditures and organizational outcomes.

In the following, we introduce the selection process in a matching model and study the influence of employment protection legislation on the selection choice and on productivity.

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\(^3\)See Devaro (2003, 2005).
\(^4\)See Devaro (2003, 2005) for a discussion.
\(^5\)Abowd et al. (2002) construct a measure of human capital using employer-employee data. Their measure includes not only education and experience, but also the person effect, which can capture unobservable component of skills.
3 The model

In this section we present a stochastic job matching model with endogenous job destruction. Our theoretical framework is an extension of the matching model developed by Pries (2004) and Pries and Rogerson (2005).

3.1 Description of the model

Labor market is characterized by frictions, in the spirit of Mortensen and Pissarides (1994), and by incomplete information, as in Jovanovic (1979). Firms with posted vacancies, $v$, and unemployed households, $u$, meet in the labor market at a frequency determined by the function $m(v,u)$. The probabilities that a firm meets a worker and that a worker meets a firm are, respectively, $q$ and $p$:

\begin{align}
q_t(\theta) &= \frac{m(v_t,u_t)}{v_t} \\
p_t(\theta) &= \frac{m(v_t,u_t)}{u_t}
\end{align}

where $\theta$ is the ratio between vacancies $v$ and unemployment $u$, and it is called market tightness.

But not all contacts lead to job matches. Production is characterized by stochasticity: jobs and workers have many unobservable characteristics that can influence the productivity of the job match. The outcome from a filled position is $\eta + y$, per unit of time, where $\eta$ is a job-specific component and $y$ is match specific. We assume that $\eta$ is observable and known by both parties, and is subject to productivity shocks that follow a Poisson process with arrival rate $\lambda$. When the match is initially formed, the technology component is equal to $\bar{\eta}$, but after a shock occurs, a new value for $\eta$ is drawn from a distribution $G(\eta)$ over the interval $[\underline{\eta}, \bar{\eta}]$. The match specific component is unobservable. Firms and workers learn about the true quality of the match through the selection process, before forming the match, and on-the-job, through monitoring. Following Pries (2004), we assume that

\begin{equation}
y = \bar{y} + \varepsilon
\end{equation}

where $\bar{y}$ is the true match quality and $\varepsilon$ is a zero-mean random variable. There are two types of matches: good, $\bar{y} = y^g$, and bad, $\bar{y} = y^b < y^g$. The noise term prevents workers and firms from perfectly inferring match quality immediately after observing the first production.

When a worker and a firm meet, each receives the same signal $\pi$ that correspond to the probability that the match will be good, $y^g$, if it is formed. The realization $\pi$ is a drawing from a known probability distribution $H(\pi)$. Low realization of $\pi$ may be rejected because of the prospect of a better job match in the future. There will be a minimum value, $\bar{\pi}$, such that it is optimal to form a match only if the signal is higher than the reservation probability $\bar{\pi}$. Once the match is formed and production is carried on, both firm and worker observe the output $\eta + y$, which is an imperfect signal of the true quality of the match. We assume that the noise $\varepsilon$ is uniformly distributed on $[-\omega, \omega]$. Therefore, whenever $y$ is lower than $y^g - \omega$, or higher than $y^b + \omega$, the quality of the match is revealed to be bad or good.

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6 We do not allow for on the job search.
7 It is standard to assume that $m(\cdot, \cdot)$ is of constant return to scale with positive first-order and negative second-order partial derivatives. See Petrongolo and Pissarides (2000) for a detailed discussion and empirical evidence.
respectively.\footnote{Note that we are assuming \( y^b + \omega > y^g - \omega \), otherwise, there would not be any uncertainty about the quality of the match after observing \( y \).} Call \( \alpha = \frac{y^g - y^b}{\omega} \) the probability that a match type is revealed. Jobs are destroyed either because the match is revealed to be bad, with probability \( \alpha (1 - \pi) \), or because they are hit by a negative productivity shock.

Following Kugler and Saint-Paul (2004), we assume that wages are composed by a fixed component, \( \bar{w} \), plus a constant fraction, \( \varphi \), of output:

\[
w(\eta, y) = \varphi (\eta + y) + (1 - \varphi) \bar{w}
\]

This assumption simplifies the solution of the model and allows to abstract from workers behavior: they are always willing to form and to continue a match as long as the wage is higher than their outside option, \( b \). A sufficient condition is \((1 - \varphi) \bar{w} \geq b\). However, the wage equation in 4 is not far from wage derived from the solution of a Nash bargaining problem, as proposed in standard matching models.\footnote{The standard wage equation is}

\[
w = (1 - \delta) b + \delta p + \delta k \theta
\]

where \( 0 \leq \delta \leq 1 \) is the bargaining power of workers, \( b \) is return from unemployment, \( p = \eta + y \) is job outcome, \( \theta \) is market tightness (i.e. number of vacancies over the number of unemployed workers) and \( k \) is the cost of posting vacancies. Note that the wage has a fixed component \((1 - \delta) b\) and a variable component which is increasing in productivity \( p \) and in market tightness \( \theta \). Our wage equation 4 as well has a fixed component \((1 - \varphi) \bar{w}\), and a variable component increasing in productivity, but it does not depend on market tightness.

\[
J(\eta, \pi) = \eta + \pi y^g + (1 - \pi) y^b - w(\eta, \pi) + \beta \lambda \left[ \int_R \eta J(s, \pi) dG(s) + G(R)(V - F) \right] + \beta (1 - \lambda) \left[ \alpha \pi J(\eta, 1) + \alpha (1 - \pi) (V - F) + (1 - \alpha) J(\eta, \pi) \right]
\]

\[
V = -k + \beta \left[ q(\theta) \int_\tilde{\pi} J(\eta, \pi) dH(\pi) + q(\theta) H(\tilde{\pi}) V + (1 - q(\theta)) V \right]
\]

were \( \beta \) is the discount factor, \( F \) is the firing cost, and \( k \) is the cost of posting a vacancy. We assume that \( y^b \) is low enough so that bad matches are always destroyed and workers dismissed paying \( F \). We also assume that there is no learning about worker’s quality when there is a shock. Note that the value of a job, equation 5, is increasing in \( \eta \) and \( \pi \), and decreasing in \( F \).

### 3.2 Equilibrium

Let’s call \( e_g \) the mass of matches known to be of good quality, and \( e_n \) the mass of matches of unknown quality.

**Definition 1** A steady-state equilibrium is a list \( w(\eta, \pi), \theta, e_g, e_n, J(\eta, \pi), V \) such that:

- value functions 5 and 6 are satisfied,
- workers are paid according to the wage equation 4,
market tightness $\theta$ satisfies the free entry condition: $V = 0$,

- matches are formed only if $\pi \geq \bar{\pi}$,

- matches are destroyed if bad and if the productivity shock is such that $\eta < R_\pi$,

- the rate of unemployment, $e_g$ and $e_n$ are constant.

Knowing the value functions 5 and 6, the firm decides whether to post a vacancy, whether to form a match and, once the match is formed, under which condition continue or dismiss the worker. Firms post vacancies as long as their value is positive. Free entry ensures that, in equilibrium, the value of a vacant position is zero:

$$V = 0$$

therefore:

$$k = \beta q (\theta) \int_{\pi}^{1} J(\eta, \pi) \, dH(\pi)$$

Equation 8 gives the equilibrium market tightness $\theta$.

Then applicants arrive to the firm, but they are hired only if the probability to form a good match, $\pi$, is sufficiently high. A match is formed only if its value is positive:

$$J(\bar{\eta}, \bar{\pi}) = 0$$

Condition 9 implies that the hiring threshold $\bar{\pi}$ must satisfy the following:

$$(1 - \varphi) \left[ \bar{\eta} + \bar{\pi} y^g + (1 - \bar{\pi}) y^b - \bar{w} \right] + \beta (1 - \lambda) \alpha \bar{\pi} J_\eta(1) (\bar{\eta} - R_1)$$

$$+ \beta \lambda J_{\eta}(\bar{\pi}) \int_{R_\pi}^{\bar{\eta}} (s - R_\pi) \, dG(s) - \beta [(1 - \lambda) \alpha + \lambda] F = 0$$

where $J_{\eta}(x)$ is the derivative of $J(\eta, x)$ with respect to $\eta$, computed at value $\pi = x$, and $R_\pi$ is the job destruction threshold at $\pi = x$. Note that higher firing costs $F$ are associated with higher threshold $\bar{\pi}$. This result is found also in Pries and Rogerson (2005).

As anticipated above, a job is destroyed if the match is revealed to be bad quality, with probability $\alpha (1 - \pi)$, or if a productivity shock reduces its value below the firm’s outside option. Given firing costs, the outside option of a firm is $-F$ and the job destruction condition turns out:

$$J(R_\pi, \pi) = -F$$

The threshold $R$ depends on the match specific component $\pi$:

$$(1 - \varphi) \left[ R_\pi + \pi y^g + (1 - \pi) y^b - \bar{w} \right] + (1 - \beta) F$$

$$+ \beta (1 - \lambda) \alpha \pi J_\eta(1) (R_\pi - R_1) + \beta \lambda J_{\eta}(\pi) \int_{R_\pi}^{\bar{\eta}} (s - R_\pi) \, dG(s) = 0$$

higher signal $\pi$ means higher expected productivity, therefore the value of the match is larger and it is destroyed only if the productivity shock is particularly low. Also, $R$ is decreasing in $F$: firing costs protect workers by reducing job destruction.

\[\text{The derivative of } J(\eta, \pi) \text{ with respect to } \eta \text{ is increasing in } \pi.\]
Let’s consider workers’ flows. Every period, firms with vacancies and unemployed workers meet in the labor market according to the function \( m(v, u) = \theta q(\theta) u \). Depending on the signal \( \pi \), only some of these meeting lead to a match, so that job creation comes to \( \theta q(\theta) \left[ 1 - H(\pi) \right] u \). Job destruction depends on the match specific component \( \overline{\pi} \): matches are separated if they are detected to be bad, with probability \( (1 - \lambda) \alpha (1 - \overline{\pi}) \), and when they are hit by a negative productivity shock, at rate \( \lambda G(R_\pi) \). On average, job destruction occurs at rate \( (1 - \lambda) \alpha (1 - \overline{\pi}) + \overline{jd}(R) \), where \( \overline{\pi} \) is the probability that the match is good over the pool of existing matches of unknown quality and \( \overline{jd}(R) e = JD(R) \) is the mass of jobs destroyed by productivity shocks. Let’s call \( e_\pi \) the pool of matches with signal \( \pi \). Then, \( JD(R) \) is equal to the sum of \( \lambda G(R_\pi) e_\pi \) over \( \pi \in [\overline{\pi}, 1] \):

\[
JD(R) = \int_{\overline{\pi}}^{1} \lambda G(R_\pi) e_\pi dH(\pi)
\]

The equilibrium between job inflows and outflows define the steady state unemployment:

\[
\theta q(\theta) \left[ 1 - H(\overline{\pi}) \right] u = (1 - \lambda) \alpha (1 - \overline{\pi}) e_n + \int_{\overline{\pi}}^{1} \lambda G(R_\pi) e_\pi dH(\pi)
\]

(13)

\[
u = \frac{\overline{jd}(R) + (1 - \lambda) \alpha (1 - \overline{\pi}) e_n}{\overline{jd}(R) + \theta q(\theta) \left[ 1 - H(\overline{\pi}) \right]} \]

(14)

where \( e_n \) is the mass of existing matches of unknown quality and \( e = 1 - u \).

Employment protection, in terms of firing costs \( F \), has several effects in this model: larger \( F \) is associated to higher hiring standard \( \overline{\pi} \), lower destruction threshold \( R_\pi \) for any given \( \pi \), and less job creation, i.e. smaller \( \theta \). The net effect on unemployment is ambiguous. Also the effect on the average labor productivity is ambiguous, as explained in the following.

3.3 Labor productivity

In this framework, labor productivity is determined by a technology component \( \eta \), and therefore on technological shocks, and a match specific component \( y \), which depends on the selection process of new employees and on the detection of bad matches. In order to assess the average productivity, we need to distinguish different groups of matches.

In steady state, there will be a mass of matches of unknown quality, \( e_n \), whose productivity is:

\[
ALP_n = \hat{\eta}_n + \overline{\pi} y^b + (1 - \overline{\pi}) y^b
\]

(15)

where \( \hat{\eta}_n \) is the average technology-specific component of \( e_n \). Note that \( \hat{\eta}_n \) is not equal to \( \int_{\overline{\pi}}^{1} \eta dG(\eta) \) over the pool of matches of unknown quality. Therefore, the distribution of the technology component within \( e_n \) has a mass point at \( \hat{\eta} \).

In steady state, outflows must be equal to inflows. We use this property to derive the steady state value for \( e_n \):

\[
\left[ \overline{jd}(R) + (1 - \lambda) \alpha \right] e_n = \theta q(\theta) \left[ 1 - H(\overline{\pi}) \right] u
\]

(16)

\[
e_n = \frac{\theta q(\theta) \left[ 1 - H(\overline{\pi}) \right] u}{\overline{jd}(R) + (1 - \lambda) \alpha}
\]

(17)
Equation 16 states the equality of inflows and outflows. A match exits the pool of unknown quality matches when it is hit by a bad shock and destroyed, \( JD_n (R) = j d_n (R) e_n \), and when it is detected, either good or bad, \( (1 - \lambda) \alpha e_n \). Note that \( JD_n (R) = \int_{\bar{\pi}}^{1-\xi} \lambda G (R) e_n dH (\pi) \) Entries into \( e_n \) are given by job creation flow, \( \theta_g (\theta) [1 - H (\bar{\pi})] u \).

Let’s consider good quality matches, \( e_g \). Their productivity is:

\[
ALP_g = \tilde{\eta}_g + y^g
\]

where \( \tilde{\eta}_g = \int_{\tilde{R}_1}^{\bar{\eta}} \eta dG (\eta) \). The mass of good workers is given by the following condition:

\[
\lambda G (R_1) e_g = (1 - \lambda) \alpha \tilde{\pi} e_n \quad \text{(19)}
\]

\[
e_g = \frac{(1 - \lambda) \alpha \tilde{\pi} e_n}{\lambda G (R_1)} \quad \text{(20)}
\]

Good type matches are destroyed only when hit by a negative productivity shock, \( \lambda G (R_1) e_g \), and they are generated by the detection of good matches among the pool of unknown quality ones, \( (1 - \lambda) \alpha \tilde{\pi} e_n \).

In the end, the average labor productivity is a weighted mean of \( ALP_n \) and \( ALP_g \):

\[
ALP = \frac{ALP_n e_n + ALP_g e_g}{e_n + e_g} = \frac{\tilde{\eta}_n + \tilde{\pi} y^g + (1 - \tilde{\pi}) y^g + (\tilde{\eta}_g + y^g) \star [(1 - \lambda) \alpha \tilde{\pi}] / [\lambda G (R_1)]}{1 + [(1 - \lambda) \alpha \tilde{\pi}] / [\lambda G (R_1)]}
\]

(21)

Lowering employment protection has an ambiguous effect on \( ALP \). On one side, smaller firing costs \( F \) are associated to larger job destruction after technology shocks: the threshold \( R \) increases and the technology specific component of productivity is raised, both for unknown quality matches, \( \tilde{\eta}_n \), and good matches, \( \tilde{\eta}_g \). On the other side, firms become less selective at the entry: the hiring standard \( \tilde{\pi} \) decreases and a higher share of potentially bad matches are formed, with negative effect on the match specific component of productivity.

4 Discussion

The model predicts that selection of employees is more accurate the higher is employment protection (EPL). This is consistent with the empirical evidence provided in Chapter 2. The net effect on productivity is ambiguous: on one side, small firing costs are associated with low average quality of matches, on the other side there is larger adjustment to exogenous shocks and the average job specific productivity turns higher. Which effect dominates depends on the characteristics of the economy: a dynamic environment, characterized by high volatility, is likely to benefit from the lessening of labor market regulation, as long as the outcome is mainly related to job-specific factors. When the match-specific component is prevailing, the relaxation of EPL may damper labor productivity.

A similar argument is stressed in Felstead and Gallie (2004). They distinguish between numerical flexibility, i.e. the ability of an organization workforce to be quickly and easily increased or decrease, and functional flexibility, which relates with adaptability of the workforce to change and take on new roles. The former is assured by temporary contracts, the latter is mainly provided by permanent workers.

Employment protection affects also the investment in human capital. Using a matching model, Wasmer (2005) shows that EPL enhances the investment in specific skills to the detriment of general
skills. Lamo, Messina and Wasmer (2007) provide evidence that the large investment in specific human capital slow down labor reallocation, lengthening the duration of unprofitable jobs. On the other side, reducing employment protection is not proved to be beneficial. Empirical studies by Arulampalam and Booth (1998), and Rix et al. (1999) reveals that temporary workers are less likely to be involved in training activities to improve their skills. Marinescu (2006) examines the 1999 British reform that lowered the tenure necessary for a worker to be protected from unfair dismissal. The calibration of the model reveals an increase in both recruitment and a small increase in monitoring effort, hence match quality. Furthermore, she finds that low tenure workers are more likely to receive training after the reform.

Whether employment protection enhances or depress productivity is an empirical issue. Only a few studies try to assess the impact of employment protection. Autor, Kerr and Kugler (2007) exploit the adoption of wrongful discharge protection by US state courts to estimate the effect of dismissal costs on firms' productivity. Their results show that the introduction of good faith exception augmented the investment in capital, decreased total factor productivity, computed as the residual of the production function, and raised substantially labor productivity. This increase reflects capital deepening and compositional shifts in labour quality. Diaz-Mayans and Sanchez (2004) used the stochastic frontier approach to measure the technical efficiency of Spanish manufacturing firms and find a negative relation between the proportion of fixed-term contracts and technical efficiency.

A different approach has been used by macroeconomists. Aguirregabiria an Alonso-Borrego (2004) calibrate and simulate a dynamic labor demand model on Spain to study the effects of the introduction of temporary contracts. Their framework allows for endogenous job destruction and human capital accumulation, i.e. newly hired employee are less productive than tenured ones. The use of temporary contracts is associated with higher turnover, i.e. lower mismatching, but also to a higher share of low tenure workers. Simulation shows a small positive effect on output and firms' value. Similar results are found in Alonso-Borrego et al. (2004).

5 Conclusion

The past two decades have been characterized by a series of labor market reforms in several European countries. In particular, the use of temporary contracts, subject to low or null dismissal costs, has been extended. The effects of such reforms have been widely analyzed, but only as regard labor outcomes, such as unemployment level and job flows.

Only a few papers study the relation between employment protection and productivity with controversial results. Studies using micro data provide empirical evidence of a negative effect of lowering EPL on labor productivity. On the contrary, small positive effects are found in macro analysis simulating general equilibrium models.

A complete understanding of the influence of EPL on productivity requires the identification of all the channels through which EPL operates. The size of firing costs affect firms' production choice at many level: high EPL give rise to institutional labor hoarding, with negative effect on productivity, but also increase hiring standard, thereby increasing the average quality of matches. We study both effects in a matching model with heterogeneous matches. The net impact on productivity is ambiguous and depends on the incidence of exogenous shocks relative to the contribution of the match-specific component to the overall productivity. Further analysis is needed in order to quantify the impact of each factor on labor productivity.
References


A The model

Let’s recall the firm’s value function:

\[
J(\eta, \pi) = \eta + \pi y^g + (1 - \pi) y^b - w(\eta, \pi) + \beta \lambda \left[ \int_{R}^{\eta} J(s, \pi) dG(s) + G(R) (V - F) \right] \\
+ \beta (1 - \lambda) [\alpha \pi J(\eta, 1) + \alpha (1 - \pi) (V - F) + (1 - \alpha) J(\eta, \pi)] 
\] (22)

\[
V = -k + \beta \left[ q(\theta) \int_{\pi}^{1} J(\eta, \pi) dH(\pi) + q(\theta) H(\pi) V + (1 - q(\theta)) V \right] 
\] (23)

Using the free entry condition, \( V = 0 \), we can rewrite the job value function as:

\[
J(\eta, \pi) = \eta + \pi y^g + (1 - \pi) y^b - w(\eta, \pi) + \beta \lambda \left[ \int_{R}^{\eta} J(s, \pi) dG(s) - G(R) F \right] \\
+ \beta (1 - \lambda) [\alpha \pi J(\eta, 1) - \alpha (1 - \pi) F + (1 - \alpha) J(\eta, \pi)] 
\] (24)

Note that, for any given \( \pi \), \( J(\eta, \pi) \) is a linear function of \( \eta \) with slope:

\[
J_\eta(\pi) = \frac{\partial J(\eta, \pi)}{\partial \eta} = \frac{1 - \varphi + \beta (1 - \lambda) \alpha \pi J_\eta(1)}{1 - \beta (1 - \lambda) (1 - \alpha)} 
\] (25)

where

\[
J(\eta, 1) = (1 - \varphi) [\eta + y^g - \omega] + \beta (1 - \lambda) [\alpha J(\eta, 1) + (1 - \alpha) J(\eta, 1)] + \beta \lambda \left[ \int_{R}^{\eta} J(s, 1) dG(s) - G(R) F \right] 
\] (26)

\[
\rightarrow J_\eta(1) = \frac{(1 - \varphi)}{1 - \beta (1 - \lambda)} 
\] (27)

therefore:

\[
J_\eta(\pi) = (1 - \varphi) \frac{1 - \beta + \beta \lambda + \beta \alpha \pi - \beta \alpha \pi \lambda}{1 - \beta + \beta \lambda + \beta \alpha - \beta \alpha \lambda} [1 - \beta + \beta \lambda] 
\] (28)

The linearity of \( J(\eta, \pi) \) in \( \eta \) allow to solve the integral in equation 24:

\[
\int_{R}^{\eta} J(s, \pi) dG(s) = J_\eta(\pi) \int_{R_{\pi}}^{\eta} (s - R_{\pi}) dG(s) - [1 - G(R_{\pi})] F 
\] (29)

and we can rewrite the job value function as:

\[
J(\eta, \pi) = \eta + \pi y^g + (1 - \pi) y^b - w(\eta, \pi) + \beta \lambda \left[ J_\eta(\pi) \int_{R_{\pi}}^{\eta} (s - R_{\pi}) dG(s) - F \right] \\
+ \beta (1 - \lambda) [\alpha \pi J(\eta, 1) - \alpha (1 - \pi) F + (1 - \alpha) J(\eta, \pi)] 
\] (30)
A.1 Equilibrium conditions

JOB DESTRUCTION:

The job destruction condition is:

\[ J(R, \pi) = -F \]  

or, equivalently:

\[ [1 - \beta (1 - \lambda) (1 - \alpha)] J(R, \pi) = -[1 - \beta (1 - \lambda) (1 - \alpha)] F \]  

Using equation 30 valued at \( \eta = R_\pi \), we get:

\[ -[1 - \beta (1 - \lambda) (1 - \alpha)] F = (1 - \varphi) [R_\pi + \pi y^q + (1 - \pi) y^b - \bar{w}] + \beta \lambda \left[ J_\eta(\pi) \int_{R_\pi}^{\bar{\eta}} (s - R_\pi) dG(s) - F \right] + \beta (1 - \lambda) [\alpha \pi J(R_\pi, 1) - \alpha (1 - \pi) F] \]  

We can further simplify the equation by deriving the value for \( J(R_\pi, 1) \):

\[ J(R_\pi, 1) = \frac{(1 - \varphi) [R_\pi - R_1]}{1 - \beta (1 - \lambda)} \]  

we used condition 31 to pass from 34 to 35. Now equation 33 reads:

\[ -[1 - \beta (1 - \lambda) (1 - \alpha)] F = (1 - \varphi) [R_\pi + \pi y^q + (1 - \pi) y^b - \bar{w}] - \beta [\lambda + (1 - \lambda) \alpha] F \]

\[ + \beta (1 - \lambda) \alpha \pi J_\eta(1) (R_\pi - R_1) + \beta \lambda J_\eta(\pi) \int_{R_\pi}^{\bar{\eta}} (s - R_\pi) dG(s) \]  

Simplifying \( F \), we get the job destruction equation:

\[ (1 - \varphi) [R_\pi + \pi y^q + (1 - \pi) y^b - \bar{w}] + (1 - \beta) F \]

\[ + \beta (1 - \lambda) \alpha \pi J_\eta(1) (R_\pi - R_1) + \beta \lambda J_\eta(\pi) \int_{R_\pi}^{\bar{\eta}} (s - R_\pi) dG(s) = 0 \]  

Let’s compute the derivative of \( R_\pi \) with respect to \( F \):

\[ \frac{\partial R_\pi}{\partial F} = -\frac{\partial J D/\partial F}{\partial J D/\partial R_\pi} = -\frac{1 - \beta}{1 - \varphi - \beta \lambda J_\eta(\pi) [1 - G(R_\pi)] + \beta (1 - \lambda) \alpha \pi J_\eta(1)} < 0 \]  

The derivative is negative because:

\[ \beta \lambda J_\eta(\pi) = \frac{\beta \lambda [1 - \beta + \beta \lambda + \beta \alpha \pi - \beta \alpha \pi] (1 - \varphi)}{(1 - \beta + \beta \lambda) [1 - \beta + \beta \lambda + \beta \alpha - \beta \alpha \lambda]} < (1 - \varphi) \]  

\[ \rightarrow 1 - \varphi - \beta \lambda J_\eta(\pi) [1 - G(R_\pi)] > 0 \]  

\[ \rightarrow 1 - \varphi - \beta \lambda J_\eta(\pi) [1 - G(R_\pi)] + \beta (1 - \lambda) \alpha \pi J_\eta(1) > 0 \]  

\[ \rightarrow \frac{1 - \beta}{1 - \varphi - \beta \lambda J_\eta(\pi) [1 - G(R_\pi)] + \beta (1 - \lambda) \alpha \pi J_\eta(1)} < 0 \]
We can show that $R_\pi$ is decreasing in $\pi$:

$$\frac{\partial R_\pi}{\partial \pi} - \frac{\partial JD}{\partial \pi} = \frac{\beta (1-\lambda) \alpha J_\eta (1) (R_\pi - R_1) + \beta \lambda \frac{\partial J_\eta (\pi)}{\partial \pi} \int_{R_\pi}^{\bar{\eta}} (s - R_\pi) \, dG(s)}{1 - \varphi - \beta \lambda J_\eta (\pi) [1 - G(R_\pi)] + \beta (1-\lambda) \alpha \pi J_\eta (1)} < 0$$

(43)

because $\frac{\partial J_\eta (\pi)}{\partial \pi} > 0$

**MATCH FORMATION:**

The match formation condition is:

$$J(\bar{\eta}, \bar{\pi}) = 0$$

(44)

or, equivalently:

$$[1 - \beta (1-\lambda) (1-\alpha)] J(\bar{\eta}, \bar{\pi}) = 0$$

(45)

Let’s value equation 30 at $\bar{\pi}$:

$$[1 - \beta (1-\lambda) (1-\alpha)] J(\bar{\eta}, \bar{\pi}) = (1 - \varphi) \left[ \bar{\eta} + \bar{\pi} y^g + (1 - \pi) y^b - \bar{w} \right] + \beta \lambda \left[ J_\eta (\bar{\pi}) \int_{R_\pi}^{\bar{\eta}} (s - R_\pi) \, dG(s) - F \right] + \beta (1-\lambda) \left\{ \alpha \bar{\pi} J(\bar{\eta}, 1) - \alpha (1 - \bar{\pi}) F \right\}$$

$$= (1 - \varphi) \left[ \bar{\eta} + \bar{\pi} y^g + (1 - \pi) y^b - \bar{w} \right] + \beta \lambda J_\eta (\bar{\pi}) \int_{R_\pi}^{\bar{\eta}} (s - R_\pi) \, dG(s) - \beta \lambda F + \beta (1-\lambda) \left\{ \alpha \bar{\pi} J_\eta (1) (\bar{\eta} - R_1) - F \right\}$$

$$= (1 - \varphi) \left[ \bar{\eta} + \bar{\pi} y^g + (1 - \pi) y^b - \bar{w} \right] + \beta \lambda J_\eta (\bar{\pi}) \int_{R_\pi}^{\bar{\eta}} (s - R_\pi) \, dG(s) + \beta (1-\lambda) \alpha \bar{\pi} J_\eta (1) (\bar{\eta} - R_1) - \beta [(1-\lambda) \alpha + \lambda] F$$

(46)

We combine equation 46 with the job destruction function 37:

$$\frac{\partial \bar{\pi}}{\partial F} = \frac{- \partial M F / \partial F}{\partial M F / \partial \bar{\pi}}$$

(48)

$$\frac{\partial M F}{\partial \bar{\pi}} = - (1 - \varphi) \frac{\partial R_\pi}{\partial \bar{\pi}} + \beta (1-\lambda) \alpha J_\eta (1) \left[ (\bar{\eta} - R_\pi) - \frac{\partial R_\pi}{\partial \bar{\pi}} \right] > 0$$

(49)

because $\frac{\partial R_\pi}{\partial \bar{\pi}} < 0$, as we showed in 43.

$$\frac{\partial M F}{\partial F} = - (1 - \varphi) \frac{\partial R_\pi}{\partial F} + \beta (1-\lambda) \alpha J_\eta (1) \bar{\pi} \frac{\partial R_\pi}{\partial F} - [1 - \beta (1-\lambda) (1-\alpha)]$$

$$= \frac{(1 - \beta) [1 - \varphi + \beta (1-\lambda) \alpha \bar{\pi} J_\eta (1)]}{1 - \varphi - \beta \lambda J_\eta (\bar{\pi}) [1 - G(R_\pi)] + \beta (1-\lambda) \alpha \bar{\pi} J_\eta (1)} - [1 - \beta (1-\lambda) (1-\alpha)]$$

(50)

Use equation 25:

$$J_\eta (\bar{\pi}) = \frac{1 - \varphi + \beta (1-\lambda) \alpha \bar{\pi} J_\eta (1)}{1 - \beta (1-\lambda) (1-\alpha)}$$

(51)

$$\rightarrow 1 - \varphi + \beta (1-\lambda) \alpha \bar{\pi} J_\eta (1) = J_\eta (\bar{\pi}) [1 - \beta (1-\lambda) (1-\alpha)]$$

(52)
therefore:

\[
1 - \varphi - \beta \lambda J_\eta (\bar{\pi}) [1 - G (R_\pi)] + \beta (1 - \lambda) \alpha \bar{\pi} J_\eta (1) = J_\eta (\bar{\pi}) [1 - \beta (1 - \alpha + \alpha \lambda) + \beta \lambda G (R_\pi)]
\]  

(53)

\[
\frac{\partial MF}{\partial F} = \frac{(1 - \beta) J_\eta (\bar{\pi}) [1 - \beta (1 - \lambda) (1 - \alpha)]}{J_\eta (\bar{\pi}) [1 - \beta (1 - \alpha + \alpha \lambda) + \beta \lambda G (R_\pi)]} - [1 - \beta (1 - \lambda) (1 - \alpha)]
\]

\[
= \frac{(1 - \beta) [1 - \beta (1 - \lambda) (1 - \alpha)] - [1 - \beta (1 - \lambda) (1 - \alpha)] [1 - \beta (1 - \alpha + \alpha \lambda) + \beta \lambda G (R_\pi)]}{1 - \beta (1 - \lambda)(1 - \alpha) - \beta \lambda [1 - G (R_\pi)]}
\]

(54)

and the derivative \( \frac{\partial \bar{\pi}}{\partial F} \) turns out to be positive: larger firing costs imply higher hiring standards.

**JOB CREATION:**

Job creation is derived substituting the free entry condition, \( V = 0 \), into the value function of vacancies:

\[
k = \beta q (\theta) \int_\bar{\pi}^1 J (\bar{\eta}, \pi) dH (\pi)
\]

(55)

We showed that an increase in \( F \) implies a decrease in \( R_\pi \) and a rise in \( \bar{\pi} \). To compute the effect on market tightness \( \theta \) we have first to derive the relation between \( F \) and the job value:

\[
\frac{\partial J (\eta, \pi)}{\partial F} = \frac{1}{1 - \beta (1 - \lambda)(1 - \alpha)} \left\{ \beta (1 - \lambda) \left[ \alpha \pi \frac{\partial J (\eta, 1)}{\partial F} - \alpha (1 - \pi) \right] + \beta \lambda \left[ -J_\eta (\pi) \frac{\partial R_\pi}{\partial F} [1 - G (R_\pi)] - 1 \right] \right\}
\]

(56)

\[
\frac{\partial J (\eta, 1)}{\partial F} = \frac{\beta \lambda}{1 - \beta (1 - \lambda)} \left[ -J_\eta (1) \frac{\partial R_1}{\partial F} [1 - G (R_1)] - 1 \right]
\]

\[
= \frac{\beta \lambda}{1 - \beta (1 - \lambda)} \left[ \frac{J_\eta (1) (1 - \beta) [1 - G (R_1)]}{1 - \beta (1 - \lambda)} \right] - \frac{\beta \lambda J_\eta (1) [1 - G (R_1)] + \beta (1 - \lambda) \alpha J_\eta (1)}{1 - \beta (1 - \lambda)(1 - \alpha) - \beta \lambda [1 - G (R_\pi)]} < 0
\]

(57)

\[
-J_\eta (\bar{\pi}) \frac{\partial R_\pi}{\partial F} [1 - G (R_\pi)] - 1 = J_\eta (\bar{\pi}) \frac{1 - \beta}{J_\eta (\bar{\pi}) [1 - \beta (1 - \lambda) (1 - \alpha)] - \beta \lambda J_\eta (\bar{\pi}) [1 - G (R_\pi)] [1 - G (R_\pi)] - 1}
\]

\[
= \frac{1 - \beta}{J_\eta (\bar{\pi}) [1 - \beta (1 - \lambda) (1 - \alpha)] - \beta \lambda J_\eta (\bar{\pi}) [1 - G (R_\pi)] [1 - G (R_\pi)] - 1} - \frac{\beta \lambda J_\eta (\bar{\pi}) [1 - G (R_\pi)] + \beta (1 - \lambda) \alpha J_\eta (1)}{1 - \beta (1 - \lambda)(1 - \alpha) - \beta \lambda [1 - G (R_\pi)]} < 0
\]

(58)

Using results in 57 and 58, we proved that the job value is decreasing in \( F \).

Given that \( F \) reduces \( \bar{\pi} \) and \( J (\eta, \pi) \), for equation 55 to hold is necessary that \( \theta \) decreases as \( F \) increases.