Stable cartel configurations: the case of multiple cartels

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Abstract

We develop a framework to analyse stability of cartels in differentiated Cournot oligopolies when multiple cartels may exist in the market. The consideration of formation of multiple cartels is in direct contrast to the existing literature which assumes, without further justification, that at most a single cartel may be formed, and we show that this consideration has markedly different implications for cartel stability. We define a cartel configuration to be stable if: (i) a firm in a cartel does not find it more profitable to leave the cartel and operate independently, (ii) a firm that operates independently does not find it more profitable to join an existing cartel, (iii) a firm in a cartel does not find it more profitable to join another existing cartel or form a new cartel with an independent firm, and (iv) two independent firms do not find it more profitable to form a new cartel. We show that now, when multiple cartels may exist in the market, a single cartel is never stable.

JEL Classification: C70; D43; L13.

Keywords: multiple cartels; stability; differentiated market.

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1 Introduction

The formation of cartels is a fundamental concern due to the inefficiency that it may cause by impeding competition in the market. In his influential paper, Stigler (1950) de-emphasised this concern by arguing that cartels are inherently unstable because the positive externalities generated by a cartel make it more profitable for each firm to not join the cartel but, instead, free-ride by remaining outside the cartel. For instance, in a market where firms choose the level of production, a cartel will attempt to reduce production in order to increase the price with the objective of obtaining a higher profit. However, this creates the incentive for an individual firm within the cartel to exit the cartel in order to not lower its quantity, thus making the cartel unstable. While persuasive, this argument ignores that the firms which remain in the cartel may also respond appropriately (for instance, by altering its quantity) to the extent that once this response is taken into consideration, it may not actually be profitable for a firm to exit the cartel. The vast extant literature on cartel stability, building on this game-theoretic consideration, has developed a more refined understanding of cartel stability. But, there is one crucial blind spot – the pre-supposition that at most one cartel may be formed. In this paper, we develop a framework for analysing cartel stability while admitting the possibility of formation of multiple cartels – with an individual firm being a part of at most one cartel – and show that this has markedly different implications for cartel stability.

We consider markets where firms simultaneously choose the level of production, and the total production determines the market price of each firm. If all firms are independent, then the objective of each firm is to maximise its profit by choosing the quantity produced. On the other hand, if a cartel is formed, then the cartel acts as a single unit, and it chooses a production level simultaneously with the independent firms that remain outside the cartel. The cartel's objective is to maximise the aggregate profit of the cartel members, and an independent firm's objective is to maximise individual profit. In the single cartel paradigm, where at most one cartel may be formed, d'Aspremont, Jacquemin, Gabszewicz and Weymark (1983) define a cartel to be stable if a firm in the cartel does not find it more profitable to leave the cartel, and a firm outside the cartel does not find it more profitable to be a part of the cartel. However, in the multiple cartels paradigm – when more than one cartel may be formed, with each firm being part of at most one cartel – two additional conditions become relevant: firstly, two independent firms should not find it more profitable to form a new cartel; secondly, a firm that belongs to a cartel should not find it more profitable to leave the cartel and, either join another existing cartel, or form a new cartel with an independent firm.

It is well-established that in homogenous markets (i.e. in homogenous Cournot competition), in the single cartel paradigm, a cartel is stable only when there are two firms. (See, for instance, Chapter 14 in Belleflamme and Peitz 2010.) The reason for cartel instability is that when firms attempt to form a cartel and collude by reducing their output, a firm in the cartel obtains a higher profit if it takes advantage of the other firms' reduced output by reneging on its own commitment to also decrease the output. However, in differentiated markets (i.e. in differentiated Cournot competition), the possibility of a stable cartel arises because reneging by a firm in the cartel may not be as attractive for the reason that the quantity produced by the other firms affects its market price to a lesser extent. That is, the positive externality generated by a reduction in other firms' output is not as significant – hence, once a firm commits to a cartel, it may not find it more profitable to renege. Consistent with this reasoning, we show that a stable cartel is more likely to exist when the market is more differentiated.

However, more central to the primary objective of this paper, in the 'multiple cartels paradigm', we find that firstly, a single cartel is never stable, and secondly, a stable configuration of multiple cartels may exist even in differentiated markets where a stable cartel does not exist in the single cartel paradigm. The intuition for the first finding is that in a cartel that is stable in the single cartel paradigm, a firm in the cartel finds it more profitable to leave the cartel and form a new cartel with an independent firm outside the cartel. The intuition for the second result rests on the fact that a firm in a cartel has to lower its output to a much greater extent in the single cartel paradigm than when multiple cartels are formed – this makes it more profitable for the firm to renege in the single cartel paradigm.

The implication of our results for the detection of cartelisation from market outcomes is as follows. The quantity chosen by a firm in a cartel in the multiple cartels paradigm is higher than the quantity chosen by a firm in a cartel in the single cartel paradigm. Correspondingly, the market price of a firm in a cartel in the multiple cartel paradigm is lower than the market price of a firm in a cartel in the single cartel paradigm. Hence, the market outcome is more competitive in case of multiple cartels than in case of a single cartel. Consequently, if one uses market quantities and market prices to detect cartelisation, then a stable multiple cartel may go undetected if one only considers the possibility of the existence of at most one cartel.

2 Model

There are $n \geq 2$ firms that compete simultaneously in quantities. Each firm has a constant marginal cost c of production. Let q_i denote the quantity produced by firm i. A profile of non-negative quantities $q = (q_j)_{j=1}^n$ determines firm i's price $p_i(q) = \alpha - \beta q_i - \delta \sum_{j \neq i} q_j$, with $\alpha > 0$ and $\beta \geq \delta > 0$. This leads to firm i obtaining profit $\pi_i(q) = (p_i(q) - c)q_i$, where $\alpha > c \geq 0$. The market is homogenous when $\delta = \beta$ – here, each firm's market price is identical at any profile of quantities. The market is differentiated when $\delta < \beta$ – now, there exist profiles of quantities where the market price of two firms differ. We interpret $\gamma \equiv \frac{\delta}{\beta}$ as an inverse measure of market differentiation – given β, a lower value of γ is associated with a lower value of δ, implying the other firms' quantities affects a firm's price to a lesser extent.

3 The single cartel paradigm

In the single cartel paradigm, at most one cartel may be formed. Let k firms, with $1 \leq k \leq n$, be part of a cartel. The situations corresponding to $k = n$ and $2 \leq k < n$ represent a complete cartel and a single incomplete cartel, respectively, while $k = 1$ represents the situation where all firms are independent. Whenever a k-firm incomplete cartel is formed, we will assume (without loss of generality) that the last k firms are in the cartel, and the remaining firms, if any, are the independent firms that remain outside the cartel. The k-firm cartel and the $n - k$ independent firms compete simultaneously in quantities. The cartel's objective is to maximise the aggregate profit of the cartel members while an independent firm's objective is to maximise its own profit. We assume that, because of symmetry, the quantity chosen by the cartel is split equally amongst the firms in the cartel, and that each firm in the cartel receives the profit generated from its own output.

Let kq^C be the aggregate quantity chosen by the k-firm cartel so that each firm in the cartel produces q^C, i.e. for any $i \in \{n-k+1, \ldots, n\}$, the quantity produced by firm i is $q_i = q^C$. The profit of each firm in the cartel equals $(\{\alpha - \beta q^C - \delta[(k-1)q^C + \sum_{j=1}^{n-k} q_j]\} - c)q^C$. So, a k-firm cartel chooses q^C to maximise $\sum_{i=n-k+1}^{n} \pi_i(q) = k(\{\alpha - \beta q^C - \delta[(k-1)q^C + \sum_{j=1}^{n-k} q_j]\} - c)q^C$. Similarly, an independent firm i, if it exists, chooses q_i with the objective of maximising its profit $(\{\alpha - \beta q_i - \delta[kq^C + \sum_{j=1, j\neq i}^{n-k} q_j]\} - c)q_i$. Assuming, because of symmetry, that each independent firm produces the same quantity, we obtain the equilibrium quantities $q^C(n, k)$

and $q^I(n,k)$ chosen by a cartel member and an independent firm as function of the pair (n,k): $q^C(n,k) = \frac{[2\beta-\delta](\alpha-c)}{[2\beta+2\delta(k-1)][2\beta+\delta(n-k-1)]-\delta^2 k(n-k)}$ and $q^I(n,k) = \frac{[2\beta+\delta(k-2)](\alpha-c)}{[2\beta+2\delta(k-1)][2\beta+\delta(n-k-1)]-\delta^2 k(n-k)}$.

Now, because $q^I(n,k) \geq q^C(n,k)$, with the inequality being strict when $k \geq 2$, an independent firm produces a higher quantity than a cartel firm. These equilibrium quantities result in equilibrium prices $p^C(n,k)$ and $p^I(n,k)$ for a cartel firm and an independent firm, respectively, where $p^C(n,k) = \alpha - \beta q^C(n,k) - \delta[(k-1)q^C(n,k) + (n-k)q^I(n,k)]$ and $p^I(n,k) = \alpha - \beta q^I(n,k) - \delta[kq^C(n,k) + (n-k-1)q^I(n,k)]$.

In case of a differentiated product market (i.e. $\delta < \beta$), since $p^I(n,k) \leq p^C(n,k)$ with the inequality being strict in case $k \geq 2$, an independent firm's market price is lower than that of a cartel firm. Finally, the equilibrium profit of a cartel firm is $\pi^C(n,k) = (p^C(n,k) - c)q^C(n,k)$ and the equilibrium profit of an independent firm is $\pi^I(n,k) = (p^I(n,k) - c)q^I(n,k)$.

In homogenous markets ($\delta = \beta$), since profit margins (i.e. market price less marginal cost of production) are equal and positive for all firms, the higher quantity produced by an independent firm makes it more profitable than a cartel firm. In differentiated markets ($\delta < \beta$), even though the profit margin of an independent firm is lower, its higher output more than compensates for this and results in it obtaining a higher profit than a cartel firm.

We use the definition of cartel stability proposed in d'Aspremont, Jacquemin, Gabszewicz, and Weymark (1983). A cartel of size $2 \leq k \leq n$ is *internally stable* if none of the firms in the cartel has an incentive to leave the cartel; that is, if $\pi^C(n,k) \geq \pi^I(n,k-1)$. A cartel of size $1 \leq k \leq n-1$ is *externally stable* if none of the independent firms has an incentive to join the cartel; that is, if $\pi^I(n,k) \geq \pi^C(n,k+1)$. We follow the convention that internal stability is trivially satisfied in case $k = 1$, and external stability is trivially satisfied in case $k = n$. A cartel of size $1 \leq k \leq n$ is *stable* if it is both internally and externally stable.

The computation of the stability conditions show that in a homogenous market, incomplete cartels (where $2 \leq k < n$) are never internally stable. Complete cartels, however, are internally stable if and only if there are only two firms; this, along with the convention that complete cartels are always externally stable, gives the following result that already exists in the literature. (See, for instance, Chapter 14 in Belleflamme and Peitz 2010.)

Proposition 1 *In case of a homogenous product market (where $\delta = \beta$), a cartel is stable in the single cartel paradigm if and only if $k = n = 2$.*

The reason behind the internal instability of both incomplete cartels, and complete cartels comprising of more than two firms, is that a firm in the cartel finds it more profitable to leave the cartel and free-ride on the reduced output of the other firms in the cartel. In addition, in the case of incomplete cartels, the independent firms exploit the reduced production by the cartel by increasing their own production, and this undercuts, to a certain extent, the cartel's attempt to increase the price (which is common for all firms in a homogeneous market). However, when there are only two firms, and a firm in the cartel leaves the cartel, then it cannot free-ride off the reduced output of the other firms in the cartel simply because the cartel ceases to exist; furthermore, there does not exist any independent firm that exploits the reduced quantity of the cartel; these factors result in a firm in the cartel not having an incentive to leave the cartel, thereby making the cartel stable.

When the product market is differentiated, we find, for similar reasons outlined above, that a complete cartel is stable if and only if $k = n = 2$, and an incomplete cartel comprising of $k \geq 3$ firms is always internally unstable. However, an incomplete cartel with $k = 2$ is stable if and only if the inverse measure of market differentiation γ is low enough. The intuition behind this rests on two factors. Firstly, when $k = 2$, a firm in the cartel realises that since the cartel ceases to exist if it leaves the cartel, it cannot leave the cartel and yet free-ride off the reduced output of the other firms in the cartel. Secondly, when γ is low enough, the other firms' output affects a firm's market price to a lesser extent. Consequently, when the firms in the cartel attempt to increase their market price by reducing their output, the independent firms, being not as affected, do not increase their production as substantially. This, in turn, implies that the cartel firms' efforts to increase the market price is not as significantly undercut by the independent firms. Both these factors combine to bring about internal stability. This, along with our finding that any incomplete cartel is externally stable, gives the next proposition.[1]

Proposition 2 *In the single cartel paradigm in a differentiated product market (where $\delta < \beta$) with n firms, a single complete cartel is stable if and only if $k = n = 2$, and a single incomplete cartel is stable if and only if $k = 2$ and $\gamma(3 - \gamma)n \leq 2\gamma + (2 - \gamma)[2\sqrt{1 + \gamma} - (1 - \gamma)]$, which implies that the inverse index of product differentiation γ is low enough.*[2]

[1]Using the expressions for $q^C(n, k)$ and $q^I(n, k)$ provided earlier, it can be verified that, for any k-firm cartel with $k < n - 1$, the external stability $(p^I(n, k) - c)q^I(n, k) \geq (p^C(n, k+1) - c)q^C(n, k+1)$ always holds. On the other hand, the internal stability condition $(p^C(n, k) - c)q^C(n, k) \geq (p^I(n, k-1) - c)q^I(n, k-1)$ holds if and only if the conditions detailed in the proposition hold.

[2]The inequality in the proposition can be re-written as $n \leq \frac{2\gamma + (2-\gamma)[2\sqrt{1+\gamma} - (1-\gamma)]}{\gamma(3-\gamma)}$. This implies that, given

The multiple cartel paradigm

We now consider the possibility that more than one cartel may be formed. Naturally, a particular firm is part of at most one cartel. Let $m \geq 1$ denote the number of cartels. For any $\ell \in \{1, \ldots, m\}$, let the number of firms in the ℓ^{th} cartel be equal to k_ℓ. If $\sum_{\ell=1}^{m} k_\ell < n$, then the number of independent firms is $n^I = n - \sum_{\ell=1}^{m} k_\ell$. Whenever an independent firm exists, we assume, without loss of generality, that firm i, for any $i \in \{1, \ldots, n^I\}$, is an independent firm; firm i, for any $i \in \{n^I + 1, \ldots, n^I + k_1\}$, belongs to the first cartel; and, more generally, firm i, for any $i \in \{n^I + \sum_{j=1}^{\ell-1} k_j + 1, \ldots, n^I + \sum_{j=1}^{\ell} k_j\}$ belongs to the ℓ^{th} cartel. Each cartel acts as a single decision-making unit. The cartels and the independent firms (if any) simultaneously choose quantities. A cartel's objective is to maximise the aggregate profit of its member firms, and an independent firm's objective is to maximise its own profit. As before, we assume that the quantity produced by a cartel is split equally amongst its members, and each cartel member obtains the profit from its own quantity.

The aggregate quantity chosen by the ℓ^{th} cartel is denoted by $k_\ell q_\ell^C$ so that each firm in this cartel produces q_ℓ^C units. That is, for any $i \in \{n^I + \sum_{j=1}^{\ell-1} k_j + 1, \ldots, n^I + \sum_{j=1}^{\ell} k_j\}$, firm i produces $q_i = q_\ell^C$. This firm's profit equals $(\{\alpha - \beta\, q_\ell^C - \delta[(k_\ell - 1)q_\ell^C + \sum_{j=1, j\neq\ell}^{m} k_j q_j^C + \sum_{j=1}^{n^I} q^I]\} - c)q_\ell^C$, and so, the ℓ^{th} cartel chooses q_ℓ^C to maximise $k_\ell\,(\{\alpha - \beta q_\ell^C - \delta[(k_\ell - 1)q_\ell^C + \sum_{j=1, j\neq\ell}^{m} k_j q_j^C + \sum_{j=1}^{n^I} q^I]\} - c)q_\ell^C$. An independent firm i chooses q_i to maximise its profit $(\{\alpha - \beta q_i - \delta[\sum_{j=1}^{m} k_j q_j^C + \sum_{j=1, j\neq i}^{n^I} q_j]\} - c)q_i$.

It follows that, in a homogenous market (where $\delta = \beta$), a cartel behaves exactly like an independent firm. So, a homogenous market with m cartels (i.e. k_ℓ firms in the ℓ^{th} cartel and n^I independent firms) is exactly equivalent to a homogenous market with a single k_ℓ firm cartel and $n - k_\ell$ independent firms for any $\ell \in \{1, \ldots, m\}$. We will make use of this shortly.

A multiple cartel configuration $(n^I, k_1, \ldots, k_m)$, that comprises of n^I independent firms and m cartels with k_ℓ firms in the ℓ^{th} cartel, is stable if the following conditions are satisfied:

1. A firm belonging to a cartel does not find it more profitable to leave the cartel to become an independent firm.

γ, stability of a two-firm incomplete cartel is obtained when the number of firms n is below a threshold value, say $\overline{n}(\gamma)$, that is strictly decreasing in γ starting from asymptotic $+\infty$ at $\gamma = 0$ to $1 + \sqrt{2}$ at $\gamma = 1$. At the same time, when written in terms of γ, the condition is a fourth degree polynomial in γ, and there is only one relevant root of this inequality that gives a threshold value of γ that corresponds to the relation as specified by $\overline{n}(\gamma)$ on the domain $[0, 1]$ for γ. Then, the statement that the number of firms $n \leq \overline{n}(\gamma)$ is equivalent to $\gamma \leq \overline{n}^{-1}(\gamma)$ if $n \geq 3$, and for all γ if $n = 2$.

2. An independent firm does not find it more profitable to join an existing cartel.

3. A firm belonging to a cartel does not find it more profitable to leave the cartel to join another cartel or form a new cartel with an independent firm.

4. An independent firm does not find it more profitable to form a new cartel with another independent firm.

Thus, the stability conditions in the single cartel paradigm are necessary for stability in the multiple cartels paradigm. This is because the first/second condition above is similar to the internal/external stability condition in the single cartel paradigm. The last two conditions are necessitated by the possibility of the formation of multiple cartels.

We present three results. The first result mirrors the instability of a cartel in homogenous markets in the single cartel paradigm (Proposition 1), and states that there is no stable multiple cartel configuration in homogenous markets apart from the case where $n = 2$. The second result shows that the incomplete two-firm cartel that was stable in differentiated markets in the single cartel paradigm (Proposition 2) are no longer stable when multiple cartels may be formed. That is, in the multiple cartels paradigm, there does not exist any single cartel – complete or incomplete – that is stable in differentiated markets. Finally, we show in an example that a stable multiple cartel configuration may exist, even in differentiated markets where a stable cartel does not exist in the single cartel paradigm.

Proposition 3 *In a homogenous product market (where $\delta = \beta$), a configuration of cartels is stable in the multiple cartels paradigm if and only if $n = 2$, and hence $m = 1$ and $k_1 = 2$.*

The argument is as follows. As mentioned earlier, in a homogenous market, each cartel behaves exactly like an independent firm. Hence, a homogenous market with m cartels, where there are k_ℓ firms in the ℓ^{th} cartel and n^I independent firms, is exactly equivalent to a homogenous market with a single k_ℓ-firm cartel and $n-k_\ell$ independent firms. Now, we obtain from Proposition 1 that this k_ℓ-firm single cartel is stable in the single cartel paradigm if and only if $k_\ell = n = 2$. Since the stability criteria of the single cartel paradigm are necessary conditions for stability in the multiple cartels paradigm, this k_ℓ-firm single cartel is stable in the single cartel paradigm if and only if $k_\ell = n = 2$. The proposition above follows because this must hold for all $\ell \in \{1, \ldots, m\}$.

Proposition 4 *In case of a differentiated product market (where $\delta < \beta$), in the multiple cartels paradigm, a single stable cartel exists only if it is a two-firm cartel and $n = 3$.*

Since at most one cartel can be formed when there are at most three firms, the corollary of this proposition is that a single firm cartel is stable in the multiple cartels paradigm only when it is not possible to form multiple cartels. We establish this proposition in the following manner. Since the stability conditions for the single cartel paradigm are necessary for stability in the multiple cartels paradigm, when it comes to stability of a single cartel in the multiple cartels paradigm, due to Proposition 2, we only need to consider a single cartel comprising of two firms. It follows that two independent firms exist if and only if $n \geq 4$, in which case it is easily verified that if the internal stability condition holds (i.e. a firm in the cartel does not find it more profitable to leave the cartel), then any two independent firms in the two-firm single cartel configuration find it profitable to form a second cartel, and vice-versa.[3] Furthermore, when $n < 4$, the possibility of a second cartel does not arise. So, the firms being symmetric, the stability conditions for the single cartel paradigm coincides with the stability criteria for the multiple cartels paradigm, and the result follows from Proposition 2.

Example 1. In context of Proposition 4, we show that in a differentiated market with number of firms $n = 4$: *(i)* a unique stable multiple cartel configuration exists, *(ii)* this multiple cartel configuration is stable whenever a two-firm cartel is stable in the single cartel paradigm, and *(iii)* this multiple cartel configuration is stable even if there does not exist a two-firm cartel that is stable in the single cartel paradigm. In light of Proposition 2, and in regard to point *(ii)* and point *(iii)* above, recall that two-firm cartels are the only ones which may be stable in the single cartel paradigm.

The table below presents the profit of a cartel firm ($\pi^C(\cdot)$) and an independent firm ($\pi^I(\cdot)$)

[3] Using the notation introduced earlier, the quantity produced by an independent firm in a n-firm differentiated market where there is a single two-firm cartel is $q^I(n,2) = \frac{2\beta(\alpha-c)}{[2\beta+2\delta][2\beta+\delta(n-3)]-2\delta^2(n-2)} = \frac{\beta(\alpha-c)}{2\beta^2-\beta\delta-\delta^2+\beta\delta n}$ and its profit is $\pi^I(n,2) = \frac{\beta^3(\alpha-c)^2}{(2\beta^2-\beta\delta-\delta^2+\beta\delta n)^2}$. A cartel firm produces $q^C(n,2) = \frac{(2\beta-\delta)(\alpha-c)}{[2\beta+2\delta][2\beta+\delta(n-3)]-2\delta^2(n-2)} = \frac{(2\beta-\delta)(\alpha-c)}{2\beta^2-\beta\delta-\delta^2+\beta\delta n}$ and its profit is $\pi^C(n,2) = \frac{(\beta+\delta)(-2\beta+\delta)^2(\alpha-c)^2}{4(2\beta^2-\beta\delta-\delta^2+\beta\delta n)^2}$. If a firm in the two-firm single cartel leaves the cartel, then all firms are independent firms, and the quantity and profit of each firm is $q^C(n,1) = q^I(n,1) = \frac{\alpha-c}{2\beta-\delta+\delta n}$ and $\pi^C(n,1) = \pi^I(n,1) = \frac{\beta(\alpha-c)^2}{(2\beta-\delta+\delta n)^2}$. If two independent firms form a cartel, so that now there are two cartels comprised of two firms each, then, using $m = 2$ in the profit-maximisation problem of the firm, along with symmetry of the firms in the cartels, gives that the quantity produced by a firm in this newly formed cartel is $\frac{2\alpha\beta-2\beta c-\alpha\delta+c\delta}{2(2\beta^2-\beta\delta-2\delta^2+\beta\delta n)}$ and its profit is $\frac{(\alpha-c)^2(2\beta-\delta)^2(\beta+\delta)}{4(2\beta^2-\beta\delta-2\delta^2+\beta\delta n)^2}$. It can then be verified that $\pi^I(n,2) \geq \frac{(\alpha-c)^2(2\beta-\delta)^2(\beta+\delta)}{4(2\beta^2-\beta\delta-2\delta^2+\beta\delta n)^2}$ and $\pi^C(n,2) \geq \pi^I(n,1)$ cannot hold simultaneously.

corresponding to each possible multiple cartel configurations when there are four firms.

situation		$\pi^C(\cdot)$	$\pi^I(\cdot)$
a	no-cartel	—	$\frac{\beta(\alpha-c)^2}{(2\beta+3\delta)^2}$
b	one two-firm cartel	$\frac{(\beta+\delta)(2\beta-\delta)^2(\alpha-c)^2}{4(2\beta^2+3\beta\delta-\delta^2)^2}$	$\frac{\beta^3(\alpha-c)^2}{(2\beta^2+3\beta\delta-\delta^2)^2}$
c	one three-firm cartel	$\frac{(\beta+2\delta)(2\beta-\delta)^2(\alpha-c)^2}{(4\beta^2+8\beta\delta-3\delta^2)^2}$	$\frac{\beta(2\beta+\delta)^2(\alpha-c)^2}{(4\beta^2+8\beta\delta-3\delta^2)^2}$
d	complete cartel	$\frac{(\alpha-c)^2}{4(\beta+3\delta)}$	—
e	two two-firm cartels	$\frac{(\beta+\delta)(\alpha-c)^2}{4(\beta+2\delta)^2}$	—

We use notation such as $\pi^C(b)$ and $\pi^I(b)$ to denote a cartel firm's profit and an independent firm's profit in situation b where there is one two-firm cartel. It is then easy to verify that:
(i) Neither the complete cartel nor the three-firm cartel is internally stable as $\pi^C(d) < \pi^I(c)$ and $\pi^C(c) < \pi^I(b)$, respectively.
(ii) The no-cartel situation is externally stable, and hence stable (since internal stability is satisfied trivially), if and only if $\pi^I(a) \geq \pi^C(b)$, or $\beta\delta(8\beta + 19\delta) \geq 4\beta^3 + 9\delta^3$.
(iii) The single two-firm cartel situation is internally stable only if $\pi^C(b) \geq \pi^I(a)$ or $4\beta^3 + 9\delta^3 \geq \beta\delta(8\beta + 19\delta)$, and externally stable only if $\pi^I(b) \geq \pi^C(e)$, or $\beta\delta(\beta + 5\delta) \geq \beta^3 + \delta^3$. However, both $4\beta^3 + 9\delta^3 \geq \beta\delta(8\beta + 19\delta)$ and $\beta\delta(\beta + 5\delta) \geq \beta^3 + \delta^3$ cannot be satisfied simultaneously (recall Footnote 3). Hence, a single two-firm cartel is never stable.
(iv) The two two-firm cartel situation is stable if and only if $\pi^C(e) \geq \pi^I(b)$ (recall stability condition 1) and $\pi^C(e) \geq \pi^C(c)$ (recall stability condition 3). The condition $\pi^C(e) \geq \pi^C(c)$ is always satisfied while $\pi^C(e) \geq \pi^I(b)$ is satisfied if and only if $\beta^3 + \delta^3 \geq \beta\delta(\beta + 5\delta)$.

Hence, there are three possible situations, solely depending on the values of δ and β, which can be re-interpreted in terms of the inverse index of market differentiation γ:

1. Only the no-cartel situation is stable if and only if $\beta^3 + \delta^3 < \beta\delta(\beta + 5\delta)$, implying that γ is above a threshold value $\overline{\gamma}$.

2. Only the two two-firm cartel is stable if and only $\beta\delta(8\beta + 19\delta) < 4\beta^3 + 9\delta^3$, implying that γ is below a threshold value $\underline{\gamma}$, where $\underline{\gamma} < \overline{\gamma}$.

3. Both the no-cartel situation and the two two-firm cartel are stable if and only if $\beta\delta(8\beta + 19\delta) \geq 4\beta^3 + 9\delta^3$ and $\beta^3 + \delta^3 \geq \beta\delta(\beta + 5\delta)$, implying $\gamma \in [\underline{\gamma}, \overline{\gamma}]$.

The above result can be understood in terms of the intuition provided earlier. If market differentiation is not sufficiently high, then a cartel is not stable due to the free-riding incentive – this makes the no-cartel situation stable. If market differentiation is sufficiently high, then the multiple cartel configuration is stable as the free-riding incentive is not as pronounced. For intermediate values of market differentiation, both the no-cartel situation and the multiple cartel configuration are stable; however, compared to the no-cartel situation, the multiple cartel configuration provides a higher profit to each firm – hence, if firms possess sufficient foresight, one may expect the firms to coordinate on the stable multiple cartel configuration.

Finally, one can use $n = 4$ in Proposition 2 to determine that in the single cartel paradigm, a two-firm single cartel is stable if and only if $\gamma \leq \overline{\overline{\gamma}}$, where $\overline{\overline{\gamma}}$ follows from the condition in Proposition 2. No other cartel is stable in the single cartel paradigm (recall Proposition 2). Since $\overline{\overline{\gamma}} < \overline{\gamma}$, the two two-firm cartel configuration is stable in the multiple cartel paradigm if a single two-firm cartel is stable in the single cartel paradigm. However, when $\gamma \in (\overline{\overline{\gamma}}, \overline{\gamma}]$, the two two-firm cartel is stable in the multiple cartel paradigm but a single two-firm cartel is not stable in the single cartel paradigm. Hence, there exist differentiated markets where a stable single cartel in the single cartel paradigm does not exist but a stable multiple cartel configuration exists. The implication is that if one assumes the single cartel paradigm, then cartelisation would not be expected in these markets – yet, one actually needs to consider this possibility because a stable multiple cartel configuration exists.

4 Conclusion

We develop a framework to analyse cartel stability when firms in the market may form more than one cartel. This is in direct contrast to the existing literature which assumes – without further justification – that only a single cartel may form. We show that a single cartel is stable only when it is not possible to form multiple cartels (for instance, when there are at most three firms in the market). That is, whenever it is possible to form multiple cartels (i.e. whenever there are at least four firms in the market), a stable cartel configuration must involve multiple cartels. Furthermore, a stable multiple cartel configuration may exist even in differentiated markets where a stable cartel in the single cartel paradigm does not exist. The implication is that if one attempts to detect cartels with the preconception that only a single cartel may be formed, then cartels may go undetected. This could be either because a stable multiple cartel configuration results in market outcomes that are more competitive than what would be obtained with a single cartel, or because a stable multiple cartel configuration exists in differentiated markets where one expects a single stable cartel to not exist.

We conclude with a conjecture. In Example 1, we have provided conditions under which the two two-firm cartel configuration is stable, and shown that this is the only possible stable cartel configuration when there are four firms in the market. We have been able to derive that when there are five firms, the only possible stable cartel configuration involves two two-firm cartels and an independent firm; and when there are six firms, the only possible stable cartel configuration involves three two-firm cartels. Determining stability beyond this has been intractable. However, the conjecture is that when the number of firms n is even, then a stable cartel configuration, whenever it exists, is described by $\frac{n}{2}$ two-firm cartels; and when n is odd, then a stable cartel configuration, whenever it exists, is described by $\frac{n-1}{2}$ two-firm cartels and one independent firm.

References

1. d'Aspremont C, A Jacquemin, JJ Gabszewicz and JA Weymark (1983). On the stability of collusive price leadership. The Canadian Journal of Economics 16(1): 17-25.

2. Belleflamme P and M Peitz (2010). Industrial Organisation: Markets and Strategies. Cambridge University Press.

3. Stigler G (1950). Monopoly and oligopoly by merger. The American Economic Review 40(2): 23-34.