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Abstract

Closed-end fund (CEF) prices often exhibit large and persistent deviations from their associated net asset values (NAVs), which is puzzling considering that NAVs are publicly observable for CEFs, which essentially represent repackaged financial assets. The persistence of these deviations is particularly notable when using linear models, suggesting the need for nonlinear models to comprehend this phenomenon known as the CEF discount puzzle. To unravel this puzzle, we employ the RALS-LM framework, enabling the identification of multiple endogenously chosen trend-breaks, and conduct an analysis utilizing data from 31 CEF discounts. Our findings reveal that CEF prices tend to fluctuate around time-varying trends, which aligns with the characteristics of regime switching models. Additionally, we demonstrate that incorporating non-normal errors through moment conditions enhances efficiency at the margin. Moreover, we establish that nonlinearity solely in the form of level shifts falls short in explaining the persistent nature of CEF discounts.

Keywords: Closed-End Fund; CEF Discount Puzzle; Residual Augmented Least Squares; Non-Normal Error; Trend Breaks

JEL Classification: C22; G12; G15

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1 Introduction

Fama (1970), in his influential survey article, posits that a market can be deemed informationally efficient when its prices fully incorporate all relevant available information pertaining to the underlying fundamental values. This implies that stock prices, for instance, should align with the present value of rationally expected future cash flows, such as dividend payments. While the efficient market hypothesis (EMH) offers a compelling and intuitive framework for asset pricing models, identifying the fundamental variables that drive and influence asset prices often presents a significant challenge.

In assessing the empirical relevance of the present value model of stock prices, several studies have made notable contributions. Campbell and Shiller (1988a, 1988b), Chen and Zhao (2009), and Chen, Da, and Zhao (2013) have conducted evaluations in this regard. Specifically, Chen and Zhao (2009) and Chen *et al.* (2013) emphasized the significance of cash flow components in stock prices, distinct from the more challenging quantification of discount rate news. In examining stock returns, Roll (1988) and Morck, Yeung, and Yu (2000) have decomposed them into firm-specific news and market-wide news, revealing that stock prices often fail to fully reflect individual firm-specific news relative to general market news.[1]

More recently, Brogaard, Nguyen, Putnins, and Wu (2022) reported similar evidence suggesting that approximately 30% of stock return variations can be attributed to noise, while firm-specific information plays a comparatively weaker role. Conversely, Bai, Philippon, and Savov (2016) and Farboodi, Matray, Veldkamp, and Venkateswaran (2022) have highlighted certain, albeit limited, improvements in the informativeness of prices specifically for large and growth stocks.

Closed-end funds (CEFs) offer an intriguing case in the ongoing debate surrounding the relationship between price and fundamentals, primarily due to their nature as repackaged financial assets. Unlike many other financial assets, CEFs possess a distinctive characteristic that simplifies the evaluation of their fundamental value. This is attributed to the net asset value (NAV) of a CEF, which represents the market value of the fund's portfolio divided by the number of shares. Notably, NAVs are readily available to the public without delay, enabling a direct and unambiguous

[1] For instance, when private property rights are inadequately protected, it can pose challenges to informed trading.

assessment of the fund's intrinsic valuation. In the absence of significant transaction costs, it is expected that the price of a CEF would closely align with its NAV. Consequently, any deviations between the CEF's fund price and NAV should be short-lived, as the market adjusts to reflect the underlying fundamental value of the fund.[2]

Yet it is well-known that CEFs typically trade at a discount, sometimes at a premium, to their NAV, and we often observe large and persistent deviations of CEF prices from their associated NAVs, presenting a challenge to conventional models of asset pricing.[3] See, among others, Lee, Shleifer, and Thaler (1990, 1991), Ahmed, Koppl, Rosser, and White (1997), Gemmill and Thomas (2002), Hughen and Wohar (2006), and Berk and Stanton (2007) who have shed light on the factors and mechanisms that contribute to the observed discounts and premiums in CEFs.[4]

Many researchers put forward theoretical hypotheses for such puzzling phenomena of the CEF Puzzle, such as investor sentiment (De Long, Shleifer, Summers, and Waldmann, 1990; Lee *et al*., 1991; Chopra, Lee, Shleifer, and Thaler, 1993; Anderson, Beard, Kim, and Stern, 2013); arbitrage costs (Pontiff, 1996; Gemmill and Thomas, 2002); accumulated tax liability effects (Malkiel, 1995; Day, Li, and Xu, 2011); the structure of management fees and compensation (Ross, 2002; Berk and Stanton, 2007); and asset price bubbles (Jarrow and Protter, 2019). These theoretical hypotheses represent different perspectives on the underlying drivers of the CEF puzzle, aiming to provide a comprehensive understanding of this intriguing phenomenon.

This paper revisits the CEF Puzzle by employing an econometric model framework that allows multiple trend-breaks at unknown dates. We further utilize moment conditions to enhance the efficiency of estimations when the error term obeys a non-normal distribution. In what follows,

[2] Malkiel (1977, 1995), Thompson (1978) and Pontiff (1995) put forth arguments suggesting that discount-based trading strategies could be profitable for investors. Their contention is that by purchasing shares of closed-end funds (CEFs) with high discounts to net asset value (NAV) and simultaneously short selling shares of CEFs with low discounts, investors can potentially exploit price discrepancies and generate positive returns.

[3] Abraham, Elan, and Marcus (1993) find that bond CEFs exhibit premium trades whereas equity CEFs are usually trade at a discount. Furthermore, Anderson, Beard, Kim, and Stern (2016) identified distinct short-term pricing behaviors between stock funds and bond funds. Their research indicated that stock CEFs exhibit different price dynamics compared to bond CEFs, further emphasizing the differences in pricing behavior across different types of CEFs. Regarding the initial public offerings (IPOs) of CEFs, Hanley, Lee, and Seguin (1996) explored the phenomenon of CEFs being offered at a premium during the IPO stage. They suggested that brokers may intentionally choose to market these funds to less informed traders, potentially contributing to the premium pricing observed during the IPOs.

[4] Using hand-collected data, Bradley, Briav, Goldstein, and Jian (2010) introduce the possibility that activist arbitrages may reduce fund discounts. Open-ending the target CEF can force the managers to take corrective actions, because the price of the fund's shares will be forced to converge to its NAV.

we demonstrate that prolonged deviations of the fund price from its fundamentals are mainly observed when linear models are employed, even though these discounts often exhibit multiple long swings over time.

Long swings can be associated with a sequence of segmented time trends that are difficult to distinguish from a directionless drift of a random walk process. Since the seminal work of Nelson and Plosser (1982), many research works have been carried out with an assumption that macroeconomic time series contain a unit root. However, an array of researchers suggested that those series are better characterized by trend break models that allow structural breaks in time trend. See, among others, Perron (1989, 1997), Zivot and Andrews (1992), and Hsu and Kuan (2001). In the context of the CEF Puzzle, the incorporation of multiple trend-breaks in the econometric model framework aligns with the idea of accounting for structural breaks and segmented time trends in time series analysis. This approach allows for a more nuanced understanding of the prolonged deviations of CEF prices and helps to overcome the limitations of linear models that may fail to capture the complexity of CEF pricing dynamics.

We also note that such trend break models can be motivated by a regime switching model framework. The concept of regime switching, where the underlying dynamics of a system can shift between different states or regimes, has been employed in various fields of research. In closely related work, Engel and Hamilton (1990) and Kaminsky (1993) employed Hamilton's (1989) framework for the dollar exchange rate that may be consistent with a sequence of segmented time trends. They show that such models can generate long swings that are often observed in the exchange rate data when the regimes have different signs and the transition probability matrix is close to be reducible.[5] Although the present paper does not employ regime switching models, our empirical model with trend breaks at endogenously chosen dates is consistent with such model specifications.

Recognizing that the CEF puzzle is mostly observed under the linear model framework, we investigate the role of nonlinear specifications to understand such highly persistent deviations of the CEF price from its fundamentals. For this purpose, we apply the residual augmented least squares (RALS) Lagrange multiplier (RALS-LM) unit root test framework for CEF discounts,

[5] For example, in a data generating process (DGP) with two regimes, where the mean of the process changes sign across the regimes, and the diagonal elements of the transition matrix are close to one, it is likely to generate dynamics characterized by long swings.

which yields the efficiency gain by utilizing moment conditions when the error terms obey a non-normal distribution. Being motivated by persistent dynamics of CEF discounts that exhibit long swings, we allow endogenously identified multiple trend-breaks at unknown dates following the approach of Meng, Lee, and Payne (2017).

The implementation of the nonlinear (trend breaks) RALS-LM test on the dataset consisting of 31 monthly frequency CEF discount data from January 1999 to April 2018 yielded compelling empirical evidence supporting the notion of nonlinear stationarity around time-varying trends. This finding contributes to explaining the long-swing dynamics observed in CEF discounts. The RALS approach yields robust and sizable power gains by utilizing moment conditions embodied in non-normal errors. Furthermore, it is crucial to allow for multiple trend breaks in the analysis, as it greatly enhances the efficiency of the test. This indicates that capturing trend shifts in the underlying dynamics of CEF discounts is essential for understanding and explaining the observed patterns.

On the other hand, the efficiency gains obtained from allowing only level shifts (without considering trend breaks) are found to be negligible. This suggests that the complexity of CEF pricing dynamics goes beyond simple level shifts and highlights the importance of considering the presence of multiple trend-breaks to obtain a more comprehensive understanding of CEF discounts. Overall, the results of this study provide strong empirical support for the nonlinear stationarity around time-varying trends and underscore the significance of incorporating multiple trend breaks in analyzing the long-swing dynamics of CEF discounts.

Organization of the paper is as follows. Section 2 introduces the dataset used in the study and provides a detailed description of the CEF discounts. This section also highlights the non-normal nature and high degree of persistence observed in CEF discounts. Section 3 carefully describes the methodology employed in the study. In Section 4, we present and discuss the major findings of the study. We also report the empirical evidence in favor of nonlinear stationarity around time-varying trends and its implications for understanding the long-swing dynamics of CEF discounts. Section 5 concludes.

2 Stochastic Properties of Closed-End Fund Discounts

2.1 Data Descriptions and Summary Statistics

We obtained 31 daily frequency closed-end fund (CEF) prices and their net asset value (NAV) series from the Closed-End Fund Center (cefa.com) and Morningstar (morningstar.com). We transformed the daily frequency data to monthly data by taking the end of the month observations to reduce noise in the data. Observations span from January 1999 through April 2018. Discounts are constructed by taking the log difference of the price from NAV.[6]

Our data includes CEF discounts of 14 core stock funds, 6 corporate debt funds, and 11 general bond funds. Bond funds are selected from the Closed-End Fund Association's *General Bond* and *Corporate Debt BBB Rated Funds* categories, while stock funds are selected from the *Core Funds* category. We selected the funds with assets that exceed $50 million (US) as of May 8, 2018.[7] See Table 1 for the complete list of these mutual funds.

Table 1 around here

Figure 1 reports nine CEF discount graphs, three from each of the three categories. As mentioned before, long swings are observed in most discounts which imply the existence of multiple trend-breaks in the data generating process (DGP). For example, ADX exhibits roughly two swings, including the longer one that is bottoming around 2001, accompanied by a shorter swing in 2010's. MGF exhibits a downward time trend until around 2010 followed by an upward trend since then. See Appendix C for the complete set of figures with estimated time trends that will be explained in what follows.

Recall that such long swings can be associated with a sequence of segmented time trends that is difficult to distinguish from directionless drift of a random walk process. See Engel and Hamilton (1990) and Kaminsky (1993) for related discussion on the dollar exchange rate dynamics using a regime switching model framework that can motivate our nonlinear trend break model in this paper.

[6] That is, positive (negative) values imply the fund is traded at a discount (premium).
[7] For more information on these Lipper classifications, see https://www.cefa.com.

Figure 1 around here

Table 2 presents summary statistics for these 31 CEF discounts. We first note that shares of most mutual funds are traded at a discount with exceptions of 7 out of 31 mutual funds, ranging from 14.99% average discount (ADX) to -17.23% average premium (CET). The skewness estimates don't exhibit any tendency, showing positively skewed distributions roughly as often as negatively skewed distributions. However, we observe leptokurtic distributions from all discounts that have kurtosis of 3 or higher, which implies a fat-tailed distribution. The Jarque-Bera test statistic (Jarque and Bera, 1980) implies strong evidence against the normal distribution based on the critical values from Deb and Sefton (1996).[8]

Table 2 around here

To help visualize the non-normal nature of the distribution of CEF discounts, Figure 2 presents kernel density estimates (solid lines) of 9 discounts along with their associated normal density function estimates (dashed lines), based on the estimated mean and standard deviation of the discounts.[9,10] Clearly, estimated kernel densities confirm the summary statistics in Table 2. They overall show fat-tail leptokurtic distributions. GAM and DUC even exhibit a bimodal distribution. That is, we observe strong evidence in favor of non-normal distributions in the CEF discounts, which provides solid justification for using the RALS-LM model specification in our paper.

Figure 2 around here

2.2 Persistence in CEF Discounts

[8] The asymptotic critical values suggested by Jarque and Bera (1987) are known to have a size distortion problem.
[9] We employed the Epanechnikov kernel to estimate the kernel density function.
[10] Recall that the first and second moments are sufficient statistics for the normal distribution.

Let $dc_{i,t}$ be the discount of a mutual fund i as a percent (%) deviation of the share price ($P_{i,t}$) from its net asset value ($NAV_{i,t}$), that is, $dc_{i,t} = (\ln NAV_{i,t} - \ln P_{i,t}) \times 100$. Note that a positive (negative) value implies that the fund is traded at a discount (premium).

Consider the following linear Augmented Dickey-Fuller (ADF) regression model for the discount with an intercept.

$$dc_{i,t} = c_i + \alpha_i dc_{i,t-1} + \sum_{j=1}^{k} \beta_{i,j} \Delta dc_{i,t-j} + \varepsilon_{i,t}$$

It is well-known that the ordinary least squares (OLS) estimator of the persistence parameter α_i is biased.[11] Thus, we obtain the median unbiased estimate for the persistence parameter for each $dc_{i,t}$ via the grid bootstrap method (Hansen, 1999) as follows.

We implemented 10,000 nonparametric bootstrap simulations at each of 30 fine grid points, $\alpha_{i,j} \in \{\alpha_{i,1}, \alpha_{i,2}, \ldots, \alpha_{i,30}\}$, in the vicinity of $\hat{\alpha}_{i,OLS}$, $[\hat{\alpha}_{i,OLS} \pm 4se(\hat{\alpha}_{i,OLS})]$, to generate the p^{th} quantile function estimates with T observations, $\hat{q}^*_{T,p}(\alpha_{i,j}, \varphi(\alpha_{i,j}))$, where φ denotes nuisance parameters such as $\beta_{i,j}$. We obtain the median unbiased estimate $\hat{\alpha}_{i,MUE}$ by matching the grid-t statistics, $t = (\hat{\alpha}_{i,OLS} - \alpha_{i,j})/se(\hat{\alpha}_{i,OLS})$, with $\hat{q}^*_{T,0.5}(\alpha_{i,j}, \varphi(\alpha_{i,j}))$. The 95% confidence bands are similarly constructed utilizing $\hat{q}^*_{T,0.025}(\alpha_{i,j}, \varphi(\alpha_{i,j}))$ and $\hat{q}^*_{T,0.975}(\alpha_{i,j}, \varphi(\alpha_{i,j}))$.

In Table 3, we report $\hat{\alpha}_{i,MUE}$ and its associated 95% confidence band in addition to its implied half-life (HL) estimates.[12] $\hat{\alpha}_{i,MUE}$ point estimates are overall close to unity, ranging from 0.857 (INSI) to 1.001 (RVT), implying highly persistent dynamics of CEF discounts. Their associated half-life point estimates range from 4.49 months (INSI) to infinity (RVT). The median and the average half-life point estimates were 8.10 and 13.70 months, implying extremely long

[11] There exist both the median and the mean bias in the OLS estimator for the persistence parameter in the presence of deterministic terms. See Andrews (1993), Andrews and Chen (1994), and Hansen (1999) for median unbiased estimators and Kendall (1954), Shaman and Stine (1988), So and Shin (1999) for mean unbiased estimators. Mean unbiased estimates are quantitatively similar to the median unbiased estimates, although the confidence intervals tend to be finite more often.

[12] The half-life is defined as the time required for a deviation to half-way adjust to its long-run equilibrium. Half-lives are obtained by $\ln(0.5)/\ln(\hat{\alpha})$ assuming that deviations decay at a constant rate.

deviations of prices from their fundamentals. It should be noted that we obtain finite confidence bands only for 18 CEF discounts after correcting for median bias.[13]

Table 3 around here

3 The Econometric Model: Nonlinear RALS-LM Models

We investigate the observed high persistence of the closed-end fund (CEF) discounts by employing a RALS-LM model framework that allows not only level shifts (LS) but also trend breaks (TB) in the data generating process of the CEF discounts on endogenously identified structural break dates. Furthermore, this framework enhances the power by utilizing moment conditions based on non-normal errors.

We employ the following two-step procedure. We first estimate break-related parameters using the maximum *F* test (*max F*) proposed by Lee, Strazicich, and Meng (2012). Given the estimated number and the location of breaks, the second step implements the RALS-LM unit root test. Dropping the subscript *i* for simplicity, consider the following data generating process (DGP) for dc_t.

$$dc_t = \boldsymbol{\delta}'\boldsymbol{d}_t + \varepsilon_t, \tag{1}$$

where ε_t denotes potentially serially correlated errors. For now, denote T_{Bi} the *known* location of the i^{th} structural break ($i = 1, 2, \dots, R$), while $\boldsymbol{d}_t$, the vector of exogenous *deterministic* variables, is defined as follows.

$$\text{(LS) } \boldsymbol{d}_t = [1, t, d_{1t}, \dots, d_{Rt}]', \text{ or}$$
$$\text{(TB) } \boldsymbol{d}_t = [1, t, d_{1t}, \dots, d_{Rt}, dt_{1t}, \dots, dt_{Rt}]',$$

where

[13] We also implement panel estimations for the persistence parameters using a fixed effect model by applying the grid bootstrap approach for the data after removing the fixed effect intercept. Results yielded a finite confidence band for the half-life, however, the implied half-life point estimate was over 137 months, confirming a substantial degree of persistence of the CEF discounts.

$$d_{it} = \begin{cases} 1, \ t \geq T_{Bi} + 1 \\ 0, \ \text{otherwise} \end{cases}$$

$$dt_{it} = \begin{cases} t - T_{Bi}, t \geq T_{Bi} + 1 \\ 0 \quad , \text{otherwise} \end{cases}$$

Based on the LM (score) procedure suggested by Schmidt and Phillips (1992), one can test the null hypothesis of the unit root using the *t*-statistics on $\phi = 0$ from the following regression.

$$\Delta dc_t = \boldsymbol{\delta}' \Delta \boldsymbol{d}_t + \phi \widetilde{dc}_{t-1} + \sum_{j=1}^{k} \theta_j \, \Delta \widetilde{dc}_{t-j} + \upsilon_t, \tag{2}$$

where $\widetilde{dc}_t = dc_t - \tilde{\alpha} - \widetilde{\boldsymbol{\delta}}' \boldsymbol{d}_t$ is the LM detrended series (residuals) of dc_t. That is, $\widetilde{\boldsymbol{\delta}}$ is the coefficient vector that is estimated by a separate regression, $\Delta dc_t = \boldsymbol{\delta}' \Delta \boldsymbol{d}_t + \nu_t$. $\tilde{\alpha}$ is the restricted MLE of α which is given by $\tilde{\alpha} = dc_1 - \widetilde{\boldsymbol{\delta}}' \boldsymbol{d}_1$.[14] Note also that (2) includes $\Delta \widetilde{dc}_{t-j}, j = 1, \ldots, k$, to remove any existing serial correlations in the error term. The *t*-statistic for $\phi = 0$ from (2) is denoted by τ_{LM}.

As shown by Amsler and Lee (1995) and Lee and Strazicich (2003), the asymptotic distribution of τ_{LM} is invariant to the location and the magnitude of breaks when the model allows for breaks only in the level (LS). It should be noted, however, that the asymptotic distribution of τ_{LM} depends on the location of breaks in the model with trend breaks (TB). To control for such dependency on the nuisance parameter, Lee *et al*. (2012) suggest employing the transformed series $\widetilde{dc}_t^*$ for TB model as follows by dividing $\widetilde{dc}_t$ by the fraction of sub-samples in each regime: $\lambda_1^* = T_{B1}/T$; $\lambda_i^* = \left(T_{Bi} - T_{B(i-1)}\right)/T, i = 2, \ldots, R$; $\lambda_{R+1}^* = (T - T_{BR})/T$.[15]

$$\widetilde{dc}_t^* = \begin{cases} \dfrac{\widetilde{dc}_t}{\lambda_1^*} \ , \text{ for } t \leq T_{B1} \\ \dfrac{\widetilde{dc}_t}{\lambda_2^*} \ , \text{ for } T_{B1} < t \leq T_{B2} \\ \vdots \\ \dfrac{\widetilde{dc}_t}{\lambda_{R+1}^*} \ , \text{ for } T_{BR} < t \leq T \end{cases} \tag{3}$$

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[14] $\boldsymbol{d}_t = [1, t]'$ that allows no break gives the LM statistic of Schmidt and Phillips (1992).

[15] See Proposition 1 in Lee *et al*. (2012).

Replacing $\widetilde{dc}_{t-1}$ in the testing regression (2) with $\widetilde{dc}^*_{t-1}$, we obtain the following.

$$\Delta dc_t = \boldsymbol{\delta}'\Delta \boldsymbol{d}_t + \phi\widetilde{dc}^*_{t-1} + \sum_{j=1}^{k}\theta_j\,\Delta\widetilde{dc}_{t-j} + e_t \tag{4}$$

Let τ^*_{LM} be the *t*-statistic for $\phi = 0$ from (4). As Lee *et al*. (2012) indicated, under this transformation, the asymptotic distribution of τ^*_{LM} depends on the number of trend-breaks instead of the location of breaks. That is, one can use the critical values of τ^*_{LM} that correspond to the number of breaks instead of simulated critical values at the estimated optimal break points.[16]

We now turn back to the first stage of our testing procedure to endogenously identify the break parameters. Following Lee *et al*. (2012), we employ an *F*-statistic approach to identify the location of breaks along with the optimal number of lags as follows.

$$F = \frac{(SSR_0 - SSR_1(\lambda))/m}{SSR_1(\lambda)/(T-q)}, \tag{5}$$

where m is the number of restrictions (i.e., number of level shifts in the LS model and number of trend-breaks in the TB model) and q denotes the number of regressors in (4). SSR_1 is the sum of the squared residuals from (4), while SSR_0 is the sum of the squared residuals from the LM unit root test without break (Schmidt and Phillips, 1992) with $\boldsymbol{d}_t = [1, t]'$ in (2).

More specifically, we set a maximum number of breaks $\bar{R}$ and determine the number of lag augmentations using the general-to-specific procedure. Via a grid search, we identify the location of breaks where the *F*-statistic is maximized, simultaneously considering the estimated optimal lags and the corresponding number of breaks. Next, one can determine the number of structural changes by examining the significance of dummy coefficients with the usual *t*-test. If one or more number of the break dummies are insignificant based on the standard *t*-test, one can move to the first step with the break number of $\bar{R} - 1$. Once the number and the location of breaks are determined, one can implement the unit root test based on equation (4).

Finally, we employ the RALS (Im and Schmidt, 2008) approach-based procedure of Meng *et al*. (2017) which extended the RALS unit root test of Meng, Im, Lee, and Tieslau (2014) by

[16] The critical values of the transformed LM tests with trend shifts are presented in Im, Lee, and Tieslau (2010) for $R = 1, 2, 3$.

allowing for structural breaks. Meng *et al*. (2017) showed that the power of the RALS unit root procedure is superior to that of the two step-based LM tests in (4) when the error term follows a non-normal distribution. The RALS unit root test is implemented via the following two-step procedure.

First, we obtain the least square residual $\hat{e}_t$ by estimating equation (4) and construct $\hat{\boldsymbol{w}}_t$ given by,

$$\hat{\boldsymbol{w}}_t = \boldsymbol{h}(\hat{e}_t) - \frac{1}{T}\sum_{s=1}^{T} \boldsymbol{h}(\hat{e}_s) - \hat{e}_t \cdot \frac{1}{T}\sum_{s=1}^{T} \boldsymbol{h}'(\hat{e}_s), \tag{6}$$

where $\boldsymbol{h}(\cdot)$ is a $J \times 1$ *known* differentiable function. And For the RALS(2&3) test, define $\boldsymbol{h}(\hat{e}_t)$ as a vector of the function of the second and the third moments, $\boldsymbol{h}(\hat{e}_t) = [\hat{e}_t^2, \hat{e}_t^3]'$, which results in the following.

$$\hat{\boldsymbol{w}}_t = \begin{bmatrix} \hat{e}_t^2 - \hat{m}_2 \\ \hat{e}_t^3 - \hat{m}_3 - 3\hat{m}_2\hat{e}_t \end{bmatrix}, \tag{7}$$

where $\hat{m}_j$ denotes the j^{th} sample moment of its corresponding population moment, $m_j = E(e_t^j)$. The first term in (7) exploits the condition of *no heteroscedasticity* while the second term improves efficiency unless the *redundancy condition* is met. It turns out that augmenting equation (4) with $\hat{\boldsymbol{w}}_t$ in (7) increases efficiency and results in higher power in the RALS-LM tests relative to the LM tests in (4) unless the error term e_t follows a normal distribution.[17]

The RALS(t_v) test is based on an assumption that the error term obeys a *t*-distribution with v degrees of freedom. For this, we choose a scalar differentiable function $h(\hat{e}_t) = \frac{(v+1)\hat{e}_t}{v+\hat{e}_t^2}$ that yields the following.[18]

$$\hat{w}_t = \frac{(v+1)\hat{e}_t}{v+\hat{e}_t^2} - \frac{1}{T}\sum_{s=1}^{T}\frac{(v+1)\hat{e}_s}{v+\hat{e}_s^2} - \hat{e}_t \cdot \frac{1}{T}\sum_{s=1}^{T}\frac{(v+1)(v-\hat{e}_s^2)}{(v+\hat{e}_s^2)^2} \tag{8}$$

[17] It is known that the normal distribution of the error term satisfies the redundancy condition $m_{j+1} = j\sigma^2 m_{j-1}, j = 2, 3$, where m_j is the j^{th} central moment. See Breusch, Qian, Schmidt, and Wyhowski (1999) for details. Specifically, the normal distribution is the only distribution that satisfies $m_4 = 3\sigma^4$.

[18] Meng *et al*. (2014) provide a robustness check with alternative degrees of freedom.

We employ the *t*-distribution with *v* degrees of freedom to mimic fat-tailed distributions that are often observed in financial market data such as the mutual fund discount data used in this paper, as shown earlier. Meng *et al*. (2014) demonstrated that the additional term $\hat{w}_t$ results in efficiency gains under fat-tailed types of distributions.

Unlike the RALS(2&3) approach, the efficiency gain of using the RALS(t_v) procedure does not require any redundancy condition. Note that $h(\hat{e}_t) = \frac{(\nu+1)\hat{e}_t}{\nu+\hat{e}_t^2}$ is derived from the score vector of the MLE with an assumption that observations are randomly drawn from the generalized *t*-distribution (see Appendix A for its derivation). That is, we employ such score function as the moment condition to enhance the efficiency of the RALS-LM test in the spirit of the GMM estimator.

Finally, the RALS-LM test statistic with trend-breaks is obtained from the following regression by augmenting equation (4) with the constructed variable $\hat{\boldsymbol{w}}_t$ in (7) or (8).

$$\Delta dc_t = \boldsymbol{\delta}'\Delta\boldsymbol{d}_t + \phi\widetilde{dc}_{t-1}^* + \boldsymbol{\gamma}'\hat{\boldsymbol{w}}_t + \sum_{j=1}^{k}\theta_j\,\Delta\widetilde{dc}_{t-j} + u_t. \tag{9}$$

The null hypothesis of the unit root is tested using the *t*-statistics on $\phi = 0$ from (9). When the distribution of the error term follows a non-normal distribution, the added term $\hat{\boldsymbol{w}}_t$ works as the stationary covariate, similar as in Hansen (1995), and the error variance in the RALS-LM tests from (9) becomes smaller than that in equation (4). Accordingly, the RALS tests become more efficient and achieve higher power.

We denote the test statistic from the RALS-LM test with trend-shifts as $\tau_{RALS-LM}^*$. As shown by Meng *et al*. (2017), the asymptotic distribution of $\tau_{RALS-LM}^*$ depends on ρ^2 which can be estimated as,

$$\hat{\rho}^2 = \frac{\hat{\sigma}_R^2}{\hat{\sigma}^2}, \tag{10}$$

where $\hat{\sigma}_R^2$ and $\hat{\sigma}^2$ are the estimated variances of the regression in (9) and (4), respectively. With the transformation, the asymptotic distribution of the RALS-LM test statistic with trend-breaks no longer depends on the break location parameters. The RALS-LM tests with trend-breaks can be implemented using the appropriate critical values reported in Table 1 of Meng *et al*. (2017).[19]

4 The Empirics

4.1 Power Gains from Utilizing Non-Normal Errors: Linear RALS Unit Root Test

We first report benchmark linear unit root test results with no structural break in order to investigate the power gains from non-normal errors only. For this purpose, we employ the following RALS regression equation with a time-invariant intercept as in Meng *et al*. (2014).

$$\Delta dc_t = \alpha + \phi dc_{t-1} + \boldsymbol{\gamma}'\widehat{\boldsymbol{w}}_t + \sum_{j=1}^{k} \theta_j \Delta dc_{t-j} + u_t \tag{11}$$

which is equivalent to the Augmented Dickey-Fuller (ADF) regression equation in the absence of $\widehat{\boldsymbol{w}}_t$ term ($\boldsymbol{\gamma} = \boldsymbol{0}$). Note that this benchmark procedure requires neither any transformations nor LM detrending for dc_t.

We chose the optimal number of lags based on the general-to-specific rule with a maximum of 10 lags. The asymptotic critical values for the RALS tests depend on $\hat{\rho}^2 = \hat{\sigma}^2_{RALS}/\hat{\sigma}^2_{ADF}$ from (11), which is similarly defined as the ratio of the error variance estimates in (10). The critical values were obtained from Hansen (1995). Test results are reported in Table 4.

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[19] Due to the invariance property of the level shifts (LS) model, indicated in Amsler and Lee (1995) and Lee and Starazicich (2003), the critical value of the LM unit root tests with the LS model is the same as that of Schmidt and Phillips (1992). Therefore, as Lemma 1 of Meng *et al.* (2014) shows, the limiting distribution of the RALS-LM tests with level shifts is as follows.

$$\tau_{RALS-LM} = \rho\tau_{LM} + \sqrt{1-\rho^2}Z \tag{FN.1}$$

where $\tau_{RALS-LM}$ is the *t*-statistic for $\phi = 0$ from the RALS regression (9), τ_{LM} is the *t*-statistic for $\phi = 0$ in (2) while assuming $\boldsymbol{d}_t = [1, t]'$ (Schmidt and Phillips, 1992), and ρ denotes the weighting parameter. Note that the invariance property allows the RALS-LM unit root tests with level shifts to be implemented without the transformation procedure in (3). In addition to this, the limiting distribution of (FN.1) is the same as that of the RALS-LM tests without breaks. This implies that the same set of critical values can be used for the RALS-LM unit root test for both the no-break and level shift models regardless of the number of shifts. Therefore, to implement both tests, one can obtain the estimated error variance of the RALS tests ($\hat{\sigma}^2_R$) from (9) without a transformation and $\hat{\sigma}^2$ from equation (2) while assuming $\boldsymbol{d}_t = [1, t]'$ and $\boldsymbol{d}_t = [1, t, D_{1t}, \ldots, D_{Rt}]'$ for the no-break model and for the level shifts model, respectively, and make a decision using the same set of critical values reported in Table 11.1 of Meng *et al*. (2014).

The ADF test rejects the null hypothesis of nonstationarity for 12 out of 31 mutual fund discounts at the 5% significance level (38.7%), while 4 more cases are rejected at the 10% level. The RALS(2&3) test rejects the null of nonstationarity for 18 out of 31 cases at the 5% significance level (61.3%). The RALS(t_5) test rejects the null for 16 and 18 mutual fund discounts at the 5% and the 10% significant levels, respectively, for all degrees of freedom we considered. See Table B1 for alternative degrees of freedom (ν) in Appendix B.

These findings imply considerable power gains by utilizing distribution information of non-normal errors in the mutual fund discounts. The RALS test rejects the null hypothesis for up to 6 more cases at the 5% significance level in comparison with the ADF test results, although both tests reject roughly the same number of cases at the 10% level. Put it differently, adding a distributional assumption seems to strengthen the power at the margin. In what follows, we investigate the possibility of adding further power gains by allowing structural breaks at unknown dates.

Table 4 around here

4.2 Power Gains from Time-Dependent Nonlinearity

This section reports the RALS-LM unit root test results allowing both the level shifts and the trend-breaks at unknown dates via the procedures explained in Section 3.

Tables B2 and B3 in Appendix B provide the LM and RALS-LM unit root test results allowing level shifts but not trend-breaks. We note that allowing level-shifts only fails to generate any power gains even when additional distributional assumptions are added via RALS procedures. As can be seen in Table B2, the LM test with one identified structural break in level rejects the null of nonstationarity only for 10 mutual fund discounts at the 10% significance level. The RALS-LM(2&3) and RALS-LM(t_ν) performed similarly. Allowing for two level-shifts doesn't change the performance of the test, rejecting up to 11 discounts. See Table B3.

These findings imply a possible misspecification problem in a model with time-varying intercepts. Given such findings, we switch our attention to the RALS-LM unit root test that allows trend-breaks. As seen in Figure 1, there exists strong visual evidence of time-varying time trends in our mutual fund discount data. To statistically evaluate such a possibility, we implement the

RALS-LM test based on equations (7), (8), and (9) that allows up to two trend breaks at unknown dates. We report substantial empirical evidence in favor of nonlinear stationarity in Tables 5, 6, and 7.

Table 5 reports results from benchmark models with $\boldsymbol{d}_t = [1, t]'$ but without allowing any structural break in time trend. The LM test by Schmidt and Phillips (1992) rejects the null of nonstationarity for 10 discounts at the 10% significance level, while adding distributional assumptions via the RALS principle improves the power by rejecting additional 2 or 4 more cases.

According to Figure 1, the visual inspections in favor of nonlinear stationarity and the results in Table 5 seem to provide strong evidence in favor of the initial findings of Perron (1989), who noted that unit root tests lose power if existing breaks are ignored. Lee and Hur (2021) indicate that the power of the test decreases when the RALS tests do not take into account the on-going structural breaks in the data. In this regard, we believe that extending the RALS unit root tests to explicitly consider ongoing structural breaks in the testing regression would be a meaningful contribution in examining the CEF discount data.

Table 5 around here

Indeed, allowing one deterministic trend break yields substantial power gains as can be seen in Table 6. The LM test rejects the null hypothesis for 16 mutual fund discounts at the 10% significance level. The RALS-LM tests reject the null for up to 19 out of 31 discounts at the 10% level. The RALS-LM(t_5) test rejects the null for 20 discounts at the 10% level. Comparing these findings with the ones in Tables 5, B2, and B3, we see that allowing nonlinearity with a trend-break yields significant improvement with or without adding information on non-normal errors.

We further implement our investigation with two trend breaks. Results are reported in Table 7. Note that the LM test rejects the null for 13 additional discounts, that is a total of 29 out of 31 discounts. The RALS-LM tests yield similar evidence of nonlinear stationarity by rejecting the null for almost all cases no matter what RALS methodologies are used.

It should be noted that this by no means is the result of allowing additional flexibility in modelling the time trend. As one can see in Tables B2 and B3, we obtained roughly the same weak evidence of nonlinear stationarity even when two level-shifts were allowed. These findings imply an important role of allowing time-varying time trends in the mutual fund discount dynamics over time, which is also consistent with regime switching models that generate long swings in the financial data.

Tables 6 and 7 around here

5 Concluding Remarks

The net asset value (NAV) of a closed-end fund (CEF) allows for direct information on the intrinsic value of the fund, providing a straightforward case as to the identification of the underlying fundamental forces that drive and move the asset prices. Therefore, when informed trading is feasible and if the transaction costs are not substantial, deviations of CEF prices from their associated NAVs should be short-lived. However, we often observe substantial and highly persistent deviations, which is hard to reconcile with regards to the reasonings based on the efficient market hypothesis (EMH). This paper presents an alternative empirical framework to understand this well-known CEF Puzzle.

We employ a nonlinear RALS-LM framework for 31 monthly frequency CEF discount data from January 1999 to April 2018, allowing multiple trend-breaks at endogenously identified dates. Our analysis reveals that introducing nonlinearity through level shifts does not adequately explain the highly persistent dynamics of CEF discounts. Instead, the presence of trend breaks emerges as a key factor in understanding the persistent deviations of CEF prices from their fundamentals.

The findings suggest that CEF prices tend to fluctuate around time-varying trends, leading to the generation of long-swing dynamics in discounts. These patterns are consistent with regime-switching models, particularly those with a near reducible transition matrix. Additionally, our findings demonstrate the efficiency gains achieved by incorporating moment conditions based on non-normal errors, further enhancing the understanding of CEF pricing dynamics on the margin.

The practical implications of the research lie in investment strategies related to CEFs. The study highlights that deviations of CEF prices from their NAVs do not necessarily indicate an immediate change in direction. Price reversals are unlikely to occur within short investment horizons. Instead, the persistent deviations are associated with long-swing dynamics, providing empirical support for momentum investment strategies that exploit identified trends expected to continue.

This paper contributes to the understanding of the CEF Puzzle by proposing a nonlinear empirical framework, revealing the importance of trend-breaks in understanding the persistence of CEF discounts. Additionally, it provides insights for investment strategies and highlights the potential benefits of utilizing momentum-based approaches.

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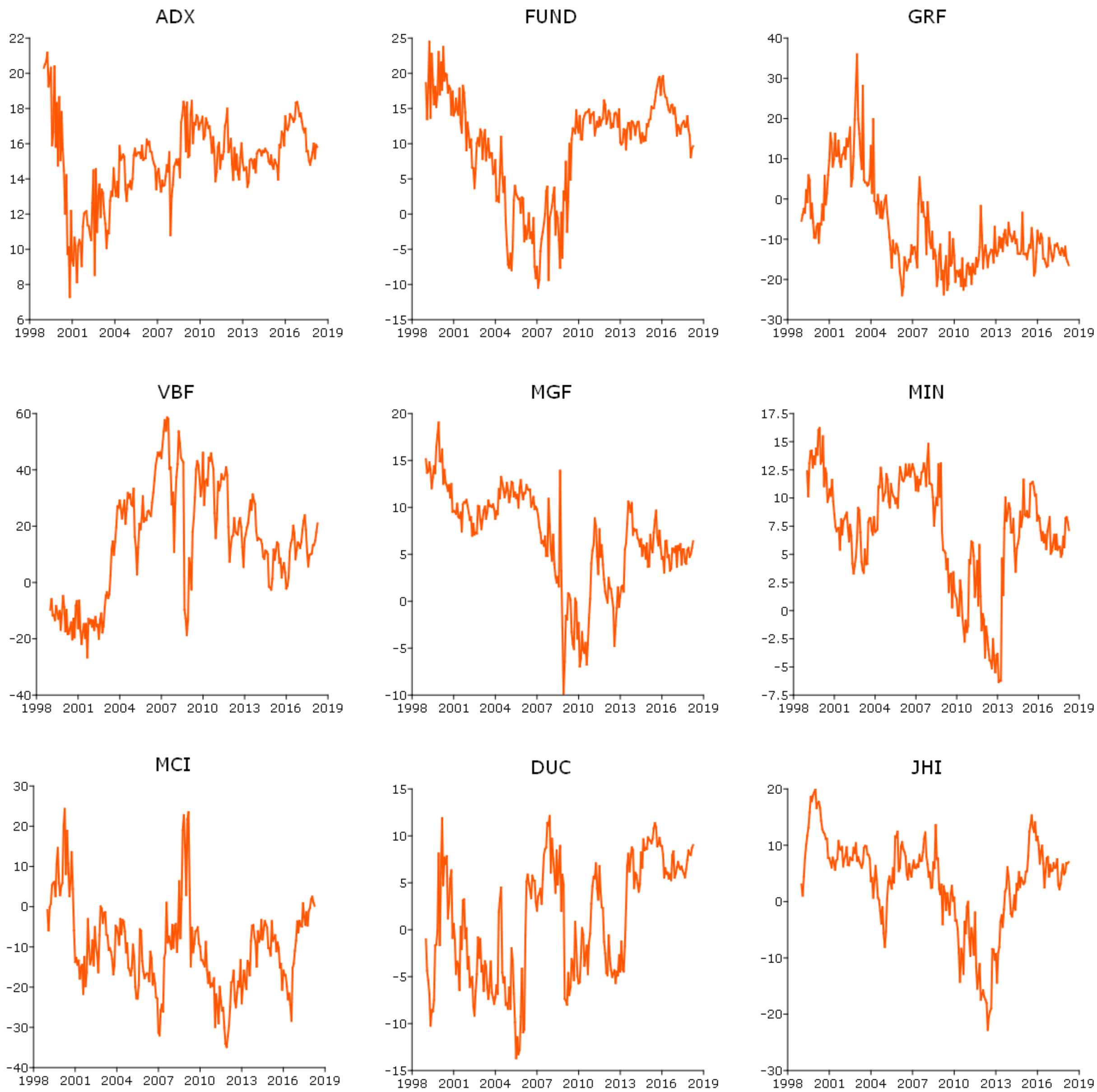
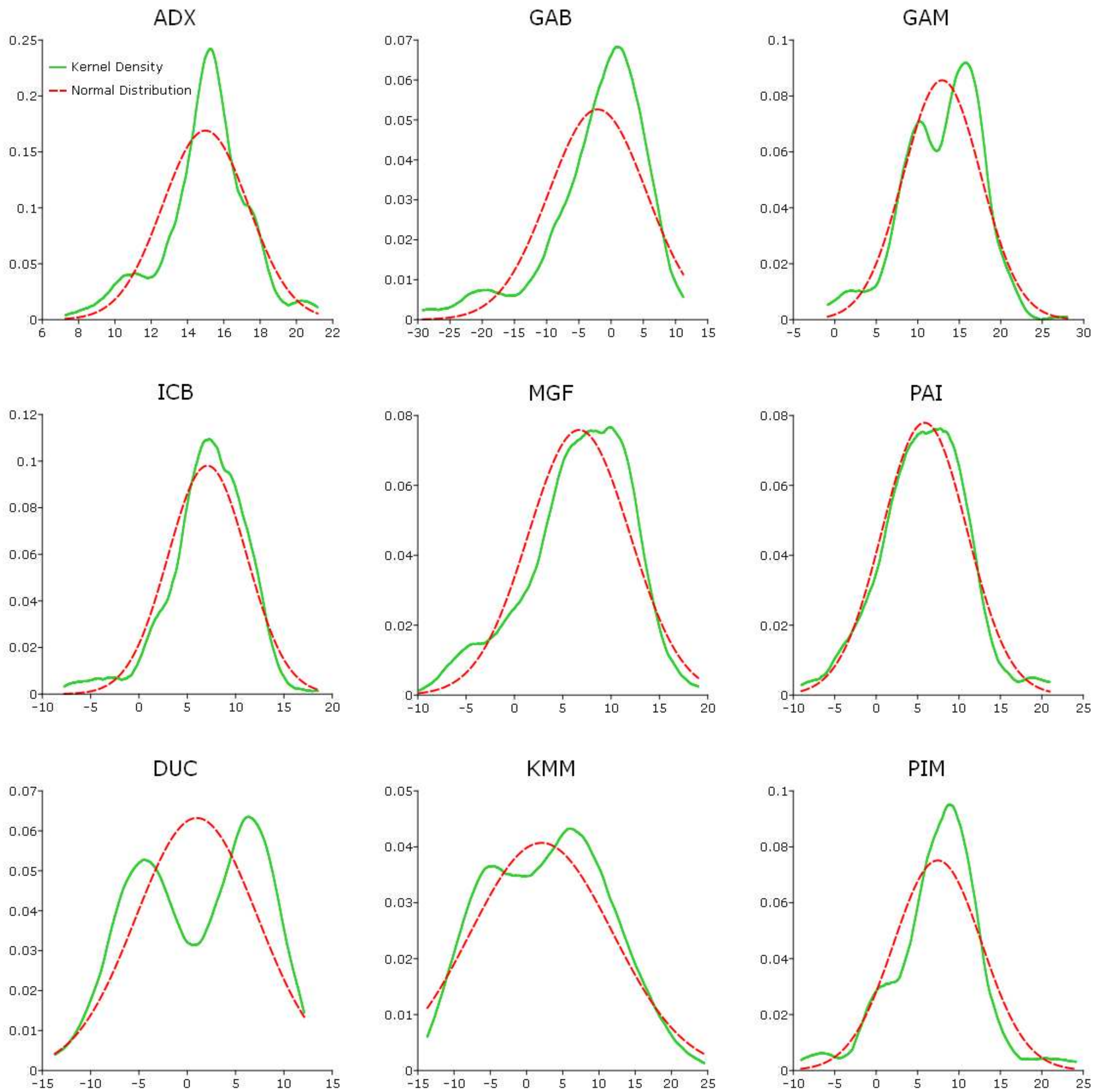
Figure 1. Mutual Fund Discounts



Note: We obtained the data from the Closed-End Fund Center (cefa.com) and Morningstar (morningstar.com). We transform the daily frequency data to monthly frequency data by taking end of the period values. The discount is defined as a percent (%) deviation of the fund price from its net asset value. The sample period is from January 1999 to April 2018. See the Appendix for all discount graphs.

Figure 2. Kernel Density Estimates of Mutual Fund Discounts



Note: We estimated the Kernel density function of mutual fund discounts assuming an Epanechnikov kernel. Dashed lines are the normal density function estimates utilizing the first and second moments of each discount. Kernel density estimates of all discounts are available upon request.

Table 1. Closed-End Mutual Fund Information

Categories	Fund Name	Total Net Assets ($ Mil)
Core Funds	Adams Diversified Equity (ADX)	1,771.6
	Central Securities Corp (CET)	821.4
	Cornerstone Strat Value (CLM)	563.3
	Cornerstone Total Return (CRF)	277.2
	Gabelli Equity Trust (GAB)	2,044.7
	General Amer Investors (GAM)	1,037.1
	Eagle Capital Growth (GRF)	33.3
	Royce Micro-Cap Trust (RMT)	412.1
	Royce Value Trust (RVT)	1,446.2
	Source Capital (SOR)	382.6
	Special Opportunities Fd (SPE)	142.5
	Tri-Continental Corp (TY)	1,609.7
	Sprott Focus Trust (FUND)	220.3
	Liberty All-Star Equity (USA)	1,302.0
Corp Debt BBB	Morg Stan Income Sec (ICB)	168.3
	MFS Govt Markets Inc Tr (MGF)	157.9
	MFS Intermediate Income (MIN)	494.0
	Insight Select Income (INSI)	216.8
	Western Asset Income (PAI)	139.6
	Invesco Bond Fund (VBF)	219.9
General Bond	Duff & Phelps Util&Corp (DUC)	261.5
	J Hancock Investors (JHI)	241.0
	Deutsche Mlti-Mkt Inc Tr (KMM)	203.1
	Deutsche Strat Inc Tr (KST)	55.2
	Barings Corporate Inv (MCI)	303.5
	MFS Charter Income Trust (MCR)	428.8
	MFS Multimkt Inc Tr (MMT)	432.9
	Barings Part Investors (MPV)	145.5
	PCM Fund (PCM)	118.4
	Putnam Mstr Intmdt Incom (PIM)	271.3
	Putnam Premier Income (PPT)	602.1

Note: We obtained the data from the Closed-End Fund Center (cefa.com) and Morningstar (morningstar.com). Total net asset values shown in million dollars are as of May 8, 2018.

Table 2. Summary Statistics

Categories	Fund	Mean	SD	Skew	Kurt	JB
Core Funds	ADX	14.99	2.36	-0.58	6.12	106.62
	CET	-17.23	3.73	0.33	5.85	82.32
	CLM	14.65	20.11	-1.70	13.86	1,245.02
	CRF	8.63	40.39	-1.72	21.99	3,583.00
	FUND	9.51	7.51	-0.46	5.42	64.34
	GAB	-2.08	7.58	-0.01	4.56	23.44
	GAM	12.93	4.66	0.26	11.45	689.64
	GRF	-7.27	10.97	0.43	5.90	88.14
	RMT	10.35	7.18	0.18	4.03	11.48
	RVT	8.50	8.16	0.09	4.01	10.16
	SOR	2.73	10.77	-0.14	4.87	34.28
	SPE	-11.53	3.53	0.08	4.62	25.55
	TY	14.84	3.34	0.28	8.47	291.09
	USA	8.77	8.81	0.42	5.97	91.47
Corp Debt BBB	ICB	7.09	4.07	-0.32	6.16	99.92
	INSI	8.96	3.72	0.94	7.36	217.11
	MGF	6.70	5.26	0.06	10.24	505.06
	MIN	7.14	4.88	0.63	6.23	115.87
	PAI	5.85	5.12	-0.31	4.71	31.79
	VBF	7.04	5.00	-0.11	4.28	16.19
General Bond	DUC	0.99	6.31	-0.29	4.82	35.11
	JHI	3.47	8.21	-0.04	4.12	12.16
	KMM	2.90	8.11	0.33	4.59	28.35
	KST	2.07	9.80	-0.10	7.79	220.92
	MCI	-9.94	10.95	0.47	4.19	22.19
	MCR	9.49	3.58	0.05	4.95	36.55
	MMT	9.78	3.68	0.27	4.10	14.44
	MPV	-8.02	10.54	-0.39	6.89	151.55
	PCM	-5.23	7.16	1.17	6.69	183.56
	PIM	7.44	5.31	0.17	6.20	99.83
	PPT	7.69	5.36	-0.04	8.77	320.59

Note: The discount is defined as a percent (%) deviation of the fund price from its net asset value. The sample period is from January 1999 to April 2018. We transform the daily frequency data to monthly frequency data by taking end of the period values. JB refers to the normality test statistics by Jarque and Bera (1980, 1987), of which the null hypothesis is that the discount is normally distributed. The null hypothesis was rejected for all fund discounts at the 5% significance level. The critical values were taken from Deb and Sefton (1996) to overcome a size distortion problem using an asymptotic chi-square distribution with the two degrees of freedom.

Table 3. Persistence Properties of Closed-End Mutual Fund Discounts

Categories	Fund	α_{MUE}	LB	UB	HL	HLB	HUB
Core Funds	ADX	0.910	0.835	1.001	7.344	3.840	∞
	CET	0.914	0.833	1.013	7.704	3.792	∞
	CLM	0.947	0.899	1.006	12.732	6.516	∞
	CRF	0.979	0.946	1.012	32.664	12.492	∞
	GAB	0.916	0.857	0.980	7.896	4.488	34.308
	GAM	0.980	0.915	1.028	34.308	7.800	∞
	GRF	0.955	0.886	1.023	15.060	5.724	∞
	RMT	0.971	0.918	1.019	23.556	8.100	∞
	RVT	1.001	0.950	1.025	∞	13.512	∞
	SOR	0.987	0.944	1.019	52.968	12.024	∞
	SPE	0.869	0.789	0.954	4.932	2.928	14.724
	TY	0.901	0.832	0.981	6.648	3.768	36.132
	FUND	0.981	0.920	1.026	36.132	8.316	∞
	USA	0.975	0.928	1.017	27.372	9.276	∞
Corp Debt BBB	ICB	0.892	0.817	0.976	6.060	3.432	28.536
	MGF	0.940	0.886	1.008	11.208	5.724	∞
	MIN	0.959	0.906	1.016	16.560	7.020	∞
	INSI	0.857	0.775	0.943	4.488	2.724	11.808
	PAI	0.859	0.780	0.941	4.560	2.784	11.400
	VBF	0.917	0.859	0.982	8.004	4.560	38.160
General Bond	DUC	0.925	0.861	1.006	8.892	4.632	∞
	JHI	0.968	0.917	1.016	21.312	8.004	∞
	KMM	0.951	0.891	1.015	13.800	6.000	∞
	KST	0.935	0.885	0.995	10.308	5.676	138.288
	MCI	0.918	0.844	1.010	8.100	4.092	∞
	MCR	0.900	0.833	0.974	6.576	3.792	26.316
	MMT	0.900	0.829	0.982	6.576	3.696	38.160
	MPV	0.890	0.812	0.976	5.952	3.324	28.536
	PCM	0.916	0.848	0.999	7.896	4.200	692.796
	PIM	0.921	0.857	1.004	8.424	4.488	∞
	PPT	0.900	0.830	0.980	6.576	3.720	34.308

Note: (a) α_{MUE} denotes the persistence parameter estimate from AR(p) that is corrected for median bias following Hansen's (1999) grid bootstrap approach. We implemented 10,000 bootstrap iterations at 30 fine grid points in the vicinity of the OLS estimate, $[\hat{\alpha}_{OLS} - 4 \times se, \hat{\alpha}_{OLS} + 4 \times se]$. The 95% confidence band [LB,UB] is similarly constructed. HL denotes the implied half-life estimate in month, and its 95% confidence band is [HLB,HUB]. The median and the average HL estimates were 8.10 and 13.70 months, respectively.

Table 4. ADF and RALS Unit Root Test Results

Categories	Fund	τ_{ADF}	$\tau_{RALS(2\&3)}$	$\hat{\rho}^2_{(2\&3)}$	$\tau_{RALS(t_5)}$	$\hat{\rho}^2_{(t_5)}$	$\hat{k}$
Core Funds	ADX	-2.719*	-3.475‡	0.861	-3.470‡	0.778	8
	CET	-2.470	-1.854	0.786	-2.080	0.630	5
	CLM	-1.843	-0.629	0.701	-1.768	0.854	10
	CRF	-2.248	-1.115	0.642	-1.888	0.728	0
	FUND	-1.892	-2.259	0.854	-2.596*	0.750	2
	GAB	-3.419†	-2.825†	0.858	-3.316†	0.830	0
	GAM	-2.045	-3.845‡	0.623	-3.286‡	0.475	9
	GRF	-1.510	-2.747†	0.786	-1.760	0.856	10
	RMT	-1.675	-1.609	0.898	-1.744	0.823	10
	RVT	-1.721	-1.854	0.912	-1.808	0.814	7
	SOR	-1.335	-1.753	0.812	-1.509	0.616	3
	SPE	-3.769‡	-2.995†	0.844	-3.973‡	0.657	1
	TY	-3.588‡	-3.153†	0.608	-3.746‡	0.397	6
	USA	-2.104	-1.985	0.742	-1.895	0.510	1
Corp Debt BBB	ICB	-3.308†	-3.951‡	0.794	-4.383‡	0.650	7
	INSI	-3.603‡	-3.836‡	0.840	-3.821‡	0.763	5
	MGF	-2.305	-3.159†	0.710	-3.048†	0.623	8
	MIN	-2.290	-0.981	0.843	-0.804	0.799	5
	PAI	-3.377†	-2.943†	0.849	-3.305†	0.823	8
	VBF	-3.418†	-3.387‡	0.891	-3.111†	0.842	3
General Bond	DUC	-2.990†	-1.894	0.854	-0.788	0.754	1
	JHI	-2.254	-2.233	0.925	-2.134	0.875	1
	KMM	-2.760*	-3.257†	0.871	-3.248‡	0.676	7
	KST	-2.530	-2.343	0.727	-2.934†	0.626	10
	MCI	-2.899†	-3.535‡	0.886	-2.827†	0.959	2
	MCR	-3.528‡	-3.253†	0.876	-2.700*	0.826	1
	MMT	-3.362†	-3.712‡	0.906	-3.445‡	0.843	1
	MPV	-3.352†	-4.090‡	0.782	-2.804†	0.750	8
	PCM	-2.535	-2.855†	0.665	-1.510	0.915	4
	PIM	-2.843*	-2.847†	0.760	-3.245‡	0.586	8
	PPT	-2.719*	-2.118	0.816	-1.487	0.674	10

Note: (a) τ_{ADF}, $\tau_{RALS(2\&3)}$, and $\tau_{RALS(t_5)}$ are the test statistics for the ADF, RALS(2&3), and RALS(t_5) tests, respectively. (b) $\hat{\rho}^2_{(2\&3)}$ and $\hat{\rho}^2_{(t_5)}$ indicate the ratio of the estimated error variances for RALS(2&3) and RALS(t_5) tests, respectively. (c) We chose the optimal number of lags ($\hat{k}$) based on the general-to-specific rule with a maximum 10 lags. (d) *, † and ‡ denote a rejection of the null hypothesis of nonstationarity at the 10%, 5% and 1% significance level, respectively. (e) The critical values of RALS tests are dependent on $\hat{\rho}^2$ and were obtained from Hansen (1995).

Table 5. LM and RALS-LM Test Results with No Trend Break

Categories	Fund	τ_{LM}	$\tau_{RALS-LM(2\&3)}$	$\hat{\rho}^2_{(2\&3)}$	$\tau_{RALS-LM(t_5)}$	$\hat{\rho}^2_{(t_5)}$
Core Funds	ADX	-1.627	-2.481	0.870	-2.127	0.777
	CET	-2.714	-1.953	0.795	-2.217	0.635
	CLM	-1.492	-0.207	0.711	-1.270	0.880
	CRF	-2.157	-1.021	0.644	-1.742	0.577
	FUND	-1.535	-1.642	0.869	-1.673	0.767
	GAB	-3.563†	-2.987†	0.867	-3.523†	0.832
	GAM	-2.569	-4.349‡	0.621	-4.027‡	0.476
	GRF	-1.846	-3.054†	0.793	-2.247	0.853
	RMT	-1.219	-0.975	0.912	-0.971	0.831
	RVT	-1.333	-1.387	0.923	-1.303	0.820
	SOR	-1.839	-2.160	0.825	-2.045	0.618
	SPE	-3.726‡	-5.367‡	0.286	-6.328‡	0.230
	TY	-2.728	-1.987	0.594	-2.711†	0.361
	USA	-1.959	-1.787	0.748	-1.632	0.513
Corp Debt BBB	ICB	-3.221†	-3.873‡	0.802	-4.280‡	0.653
	INSI	-3.719‡	-3.931‡	0.850	-4.028‡	0.766
	MGF	-2.423	-3.485‡	0.706	-3.529‡	0.617
	MIN	-2.287	-1.004	0.852	-0.780	0.807
	PAI	-2.470	-2.074	0.848	-2.570	0.806
	VBF	-3.255†	-3.264†	0.896	-2.949*	0.844
General Bond	DUC	-3.592†	-2.793*	0.879	-2.416	0.825
	JHI	-2.115	-2.059	0.933	-2.029	0.873
	KMM	-2.438	-2.748*	0.883	-2.534	0.697
	KST	-2.468	-2.196	0.737	-2.400	0.699
	MCI	-2.865*	-3.497†	0.893	-2.800*	0.964
	MCR	-2.864*	-2.635	0.867	-2.212	0.795
	MMT	-3.229†	-3.443†	0.917	-3.269†	0.843
	MPV	-3.305†	-4.039‡	0.788	-2.843*	0.746
	PCM	-2.063	-2.325	0.677	-1.119	0.890
	PIM	-2.603	-2.784*	0.760	-3.126†	0.577
	PPT	-2.218	-1.789	0.812	-1.530	0.633

Note: (a) τ_{LM}, $\tau_{RALS-LM(2\&3)}$, $\tau_{RALS-LM(t_5)}$ are the test statistics for the LM, RALS-LM(2&3), and RALS-LM(t_5) tests, respectively. (b) $\hat{\rho}^2_{(2\&3)}$ and $\hat{\rho}^2_{(t_5)}$ indicate the ratio of the estimated error variances for RALS-LM (2&3) and RALS-LM (t_5) tests, respectively. (c) The 1%, 5%, and 10% critical values for the LM test (T = 200) are: -3.61, -3.04, -2.76. (d) The critical values for RALS-LM tests are reported in Table 11.1 of Meng *et al*. (2014). (e) ‡, †, and * represent 1%, 5%, and 10% rejection, respectively.

Table 6. LM and RALS-LM Test Results with One Trend Break

Categories	Fund	τ^*_{LM}	$\tau^*_{RALS-LM(2\&3)}$	$\hat{\rho}^2_{(2\&3)}$	$\tau^*_{RALS-LM(t_5)}$	$\hat{\rho}^2_{(t_5)}$	$\hat{T}_B$
Core Funds	ADX	-3.944†	-5.305‡	0.837	-6.167‡	0.703	2002:09
	CET	-5.167‡	-3.632†	0.786	-4.294‡	0.704	2008:09
	CLM	-2.651	-2.064	0.655	-2.409	0.992	2011:08
	CRF	-3.024	-2.113	0.653	-2.445	0.719	2011:08
	FUND	-4.416‡	-4.169‡	0.888	-4.118‡	0.833	2008:11
	GAB	-3.652*	-2.943	0.874	-3.087	0.906	2014:07
	GAM	-5.411‡	-7.033‡	0.616	-6.982‡	0.553	2000:12
	GRF	-3.045	-3.690†	0.819	-3.308*	0.851	2004:04
	RMT	-3.214	-2.910	0.905	-3.150	0.826	2007:05
	RVT	-3.181	-3.390*	0.918	-3.706†	0.759	2007:01
	SOR	-3.473*	-3.855†	0.829	-4.031‡	0.631	2007:01
	SPE	-12.37‡	-21.874‡	0.304	-25.732‡	0.208	2002:06
	TY	-4.554‡	-3.362*	0.666	-4.050‡	0.427	2008:12
	USA	-3.867†	-3.676†	0.735	-3.767†	0.485	2001:01
Corp Debt BBB	ICB	-3.478*	-3.902†	0.787	-4.905‡	0.673	2000:12
	INSI	-3.292	-3.708†	0.860	-4.529‡	0.746	2000:11
	MGF	-3.236	-2.865	0.793	-3.475†	0.676	2008:08
	MIN	-2.521	-1.549	0.848	-1.275	0.788	2013:01
	PAI	-3.528*	-2.903	0.846	-3.005	0.836	2008:10
	VBF	-3.499*	-3.740†	0.873	-3.793†	0.782	2015:07
General Bond	DUC	-3.471*	-2.732	0.868	-2.186	0.859	2015:12
	JHI	-2.705	-2.386	0.928	-2.492	0.812	2012:05
	KMM	-2.905	-3.390*	0.871	-3.557†	0.659	2000:12
	KST	-5.226‡	-4.831‡	0.818	-5.534‡	0.724	2001:03
	MCI	-3.901†	-4.755‡	0.855	-4.469‡	0.928	2000:11
	MCR	-3.093	-3.867†	0.828	-4.038†	0.759	2001:01
	MMT	-3.111	-3.130	0.918	-2.759	0.862	2015:08
	MPV	-3.033	-3.575†	0.782	-2.107	0.661	2012:06
	PCM	-3.866†	-4.363‡	0.683	-3.438*	0.778	2006:12
	PIM	-3.120	-3.004	0.812	-3.132*	0.673	2009:03
	PPT	-3.038	-2.780	0.810	-2.085	0.640	2007:11

Note: (a) τ^*_{LM}, $\tau^*_{\text{RALS-LM(2\&3)}}$, $\tau^*_{\text{RALS-LM}(t_5)}$ are the test statistics for the LM, RALS-LM(2&3), and RALS-LM(t_5) tests with transformation, respectively. (b) $\hat{\rho}^2_{(2\&3)}$ and $\hat{\rho}^2_{(t_5)}$ indicate the ratio of the estimated error variances for RALS-LM (2&3) and RALS-LM (t_5) tests, respectively. (c) $\hat{T}_B$ denote the estimated break point. (d) The 1%, 5%, and 10% critical values for the one break LM test with transformation (T = 200) are: - 4.261, - 3.716, -3.443. (e) The critical values for RALS-LM tests are reported in Table 1 of Meng *et al*. (2017). (f) ‡, †, and * represent 1%, 5%, and 10% rejection, respectively.

Table 7. LM and RALS-LM Test Results with Two Trend Breaks

Categories	Fund	τ^*_{LM}	$\tau^*_{RALS-LM(2\&3)}$	$\hat{\rho}^2_{(2\&3)}$	$\tau^*_{RALS-LM(t_5)}$	$\hat{\rho}^2_{(t_5)}$	$\hat{T}_B$	
Core	ADX	-8.232‡	-9.235‡	0.878	-9.846‡	0.785	2000:11	2002:09
Funds	CET	-4.416†	-4.314†	0.815	-4.347†	0.656	2008:09	2008:12
	CLM	-6.184‡	-6.208‡	0.869	-6.449‡	0.759	2008:07	2008:10
	CRF	-8.280‡	-9.020‡	0.783	-9.692‡	0.58	2008:07	2008:10
	FUND	-6.953‡	-6.835‡	0.864	-7.321‡	0.751	2004:08	2009:05
	GAB	-6.085‡	-6.398‡	0.810	-6.996‡	0.737	2001:01	2002:09
	GAM	-6.467‡	-7.535‡	0.782	-8.000‡	0.638	2008:09	2008:12
	GRF	-3.165	-3.582	0.830	-3.668*	0.696	2003:05	2003:08
	RMT	-4.612†	-3.731	0.863	-4.091†	0.710	2007:01	2007:12
	RVT	-4.179*	-4.232†	0.899	-4.510‡	0.778	2007:01	2007:11
	SOR	-6.237‡	-7.347‡	0.826	-7.821‡	0.663	2002:09	2003:09
	SPE	-12.88‡	-14.50‡	0.848	-15.91‡	0.705	2001:02	2001:05
	TY	-7.074‡	-10.18‡	0.669	-10.80‡	0.505	2008:10	2009:07
	USA	-7.799‡	-8.416‡	0.812	-9.511‡	0.628	2008:07	2008:12
Corp Debt	ICB	-5.830‡	-6.295‡	0.833	-6.743‡	0.722	2008:08	2009:01
BBB	INSI	-5.688‡	-6.217‡	0.953	-6.286‡	0.906	2008:08	2009:02
	MGF	-5.735‡	-5.674‡	0.931	-5.902‡	0.872	2008:08	2009:01
	MIN	-6.039‡	-6.135‡	0.970	-6.165‡	0.928	2013:03	2013:09
	PAI	-3.776	-3.808	0.940	-3.676	0.934	2008:10	2009:02
	VBF	-4.553†	-4.462†	0.919	-4.247†	0.903	2008:08	2009:02
General	DUC	-6.255‡	-6.840‡	0.858	-6.423‡	0.785	2006:01	2009:01
Bond	JHI	-5.442‡	-5.568‡	0.907	-5.699‡	0.844	2011:06	2012:10
	KMM	-5.087‡	-4.787‡	0.936	-4.657‡	0.784	2007:02	2009:01
	KST	-5.582‡	-5.608‡	0.787	-5.431‡	0.779	2000:11	2001:04
	MCI	-5.500‡	-5.823‡	0.959	-5.736‡	0.935	2007:04	2009:04
	MCR	-5.623‡	-5.625‡	0.867	-5.600‡	0.805	2008:08	2009:02
	MMT	-4.009*	-4.110†	0.889	-4.088†	0.812	2013:03	2013:09
	MPV	-4.287†	-4.624‡	0.821	-3.951†	0.723	2006:11	2007:02
	PCM	-5.733‡	-6.424‡	0.726	-5.586‡	0.867	2008:07	2008:10
	PIM	-4.027*	-4.078†	0.802	-3.931†	0.666	2007:09	2007:12
	PPT	-8.188‡	-8.633‡	0.876	-10.02‡	0.641	2007:09	2007:12

Note: (a) τ^*_{LM}, $\tau^*_{\text{RALS-LM}(2\&3)}$, $\tau^*_{\text{RALS-LM}(t_5)}$ are the test statistics for the LM, RALS-LM (2&3), and RALS-LM (t_5) tests with transformation, respectively. (b) $\hat{\rho}^2_{(2\&3)}$ and $\hat{\rho}^2_{(t_5)}$ indicate the ratio of the estimated error variances for RALS-LM (2&3) and RALS-LM (t_5) tests, respectively. (c) $\hat{T}_B$ denote the optimal number of lags and the estimated break points. (d) The 1%, 5%, and 10% critical values for the two breaks LM test with transformation (T = 200) are: - 4.799, - 4.261, - 3.997. (e) The critical values for RALS-LM tests are reported in Table 1 of Meng *et al*. (2017). (f) ‡, †, and * represent 1%, 5%, and 10% rejection, respectively.

Appendix A: Derivation of $h(t)$ in RALS(t$_\nu$) Test

Let $\{y_1, y_2, \cdots, y_T\}$ be *i.i.d.* observations from a generalized Student's t-distribution with the following pdf:

$$f(y_t; \nu, \mu, \sigma) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sigma\Gamma\left(\frac{\nu}{2}\right)\sqrt{\nu\pi}}\left(1 + \frac{(y_t-\mu)^2}{\nu\sigma^2}\right)^{-\frac{1}{2}(\nu+1)},$$

where μ is a location parameter, σ is a scale parameter, and ν denotes degrees of freedom. The log-likelihood function is the following.

$$\ln L_T(\nu, \mu, \sigma) = \frac{1}{T}\sum_{t=1}^{T} \ln f(y_t; \nu, \mu, \sigma)$$

$$= \frac{1}{T}\sum_{t=1}^{T}\left[\ln\Gamma\left(\frac{\nu+1}{2}\right) - \ln\sigma - \ln\Gamma\left(\frac{\nu}{2}\right) - \frac{1}{2}\ln\nu\pi - \frac{\nu+1}{2}\ln\left(1 + \frac{(y_t-\mu)^2}{\nu\sigma^2}\right)\right]$$

$$= \frac{1}{T}\cdot T\left[\ln\Gamma\left(\frac{\nu+1}{2}\right) - \ln\sigma - \ln\Gamma\left(\frac{\nu}{2}\right) - \frac{1}{2}\ln\nu\pi\right]$$

$$-\frac{1}{T}\sum_{t=1}^{T}\frac{(\nu+1)}{2}\ln\left(\frac{\nu\sigma^2 + (y_t-\mu)^2}{\nu\sigma^2}\right)$$

Differentiating $\ln L_T(\cdot)$ with respect to μ yields,

$$\frac{\partial \ln L_T}{\partial\mu} = -\frac{1}{T}\sum_{t=1}^{T}\frac{(\nu+1)}{2}\left(\frac{\nu\sigma^2}{\nu\sigma^2 + (y_t-\mu)^2}\right)\frac{2(y_t-\mu)(-1)}{\nu\sigma^2}$$

$$= \frac{1}{T}\sum_{t=1}^{T}\frac{(\nu+1)(y_t-\mu)}{\nu\sigma^2 + (y_t-\mu)^2} = \frac{1}{T}\sum_{t=1}^{T} h(y_t)$$

The first order condition is $\frac{1}{T}\sum_{t=1}^{T} h(y_t) = 0$. Replacing y_t with $e_t \sim (\mu = 0, \sigma^2 = 1)$, we obtain,

$$h(e_t) = \frac{(\nu+1)e_t}{\nu + e_t^2}$$

where ν is degrees of freedom. ■

Appendix B: Additional Tables and Figures

Table B1. RALS(t_v) Test Results with Alternative Degrees of Freedom

Categories	Fund	$\tau_{RALS(t_3)}$	$\hat{\rho}^2_{(t_3)}$	$\tau_{RALS(t_7)}$	$\hat{\rho}^2_{(t_7)}$	$\hat{k}$
Core Funds	ADX	-3.470‡	0.753	-3.476‡	0.792	8
	CET	-2.211	0.612	-2.009	0.646	5
	CLM	-1.878	0.865	-1.684	0.852	10
	CRF	-1.959	0.793	-1.825	0.681	0
	FUND	-2.426*	0.789	-2.688*	0.731	2
	GAB	-3.367†	0.852	-3.274†	0.817	0
	GAM	-3.395‡	0.451	-3.220‡	0.492	9
	GRF	-1.696	0.913	-1.804	0.820	10
	RMT	-1.713	0.819	-1.769	0.825	10
	RVT	-1.740	0.828	-1.846	0.811	7
	SOR	-1.601	0.585	-1.471	0.631	3
	SPE	-4.190‡	0.615	-3.835‡	0.681	1
	TY	-3.851‡	0.385	-3.669‡	0.410	6
	USA	-1.861	0.486	-1.914	0.530	1
Corp Debt BBB	ICB	-4.419‡	0.630	-4.354‡	0.661	7
	INSI	-3.701‡	0.780	-3.904‡	0.756	5
	MGF	-3.027†	0.630	-3.050†	0.622	8
	MIN	-0.878	0.794	-0.766	0.802	5
	PAI	-3.356†	0.829	-3.266†	0.820	8
	VBF	-3.141†	0.847	-3.104†	0.842	3
General Bond	DUC	-0.736	0.753	-0.830	0.752	1
	JHI	-2.145	0.874	-2.138	0.876	1
	KMM	-3.187†	0.676	-3.289‡	0.681	7
	KST	-2.933†	0.631	-2.936†	0.624	10
	MCI	-2.848†	0.979	-2.817†	0.945	2
	MCR	-2.735*	0.831	-2.706*	0.825	1
	MMT	-3.372‡	0.837	-3.486‡	0.847	1
	MPV	-2.809†	0.777	-2.818†	0.722	8
	PCM	-1.749	0.956	-1.409	0.889	4
	PIM	-3.130†	0.594	-3.306‡	0.587	8
	PPT	-1.496	0.684	-1.495	0.672	10

Note: (a) $\tau_{RALS(t_v)}$ are the test statistics for the RALS(t_v) test. (b) $\hat{\rho}^2_{(t_v)}$ indicates the ratio of the estimated error variances for the RALS(t_v). (c) We chose the optimal number of lags ($\hat{k}$) based on the general-to-specific rule with a maximum 10 lags. (d) *, † and ‡ denote a rejection of the null hypothesis of nonstationarity at the 10%, 5% and 1% significance level, respectively. (e) The critical values of RALS tests are dependent on $\hat{\rho}^2$ and were obtained from Hansen (1995).

Table B2. LM and RALS-LM Test Results with One Level Shift

Categories	Fund	τ_{LM}	$\tau_{RALS-LM(2\&3)}$	$\hat{\rho}^2_{(2\&3)}$	$\tau_{RALS-LM(t_5)}$	$\hat{\rho}^2_{(t_5)}$	$\hat{T}_B$	$\hat{k}$
Core Funds	ADX	-1.952	-1.787	0.827	-1.741	0.723	2002:07	8
	CET	-3.531†	-2.671	0.843	-2.806*	0.714	2008:10	8
	CLM	-2.357	-1.906	0.869	-2.831*	0.805	2008:08	0
	CRF	-1.803	-0.886	0.793	-2.123	0.712	2008:08	0
	FUND	-1.480	-1.204	0.861	-0.992	0.776	2007:10	3
	GAB	-2.312	-1.222	0.878	-1.562	0.830	2009:04	1
	GAM	-2.854*	-2.580	0.720	-2.378	0.632	2008:10	9
	GRF	-2.315	-3.467†	0.827	-2.221	0.985	2003:06	9
	RMT	-1.273	-1.672	0.920	-1.401	0.844	2005:12	10
	RVT	-1.505	-1.699	0.914	-1.423	0.814	2009:04	7
	SOR	-2.567	-2.016	0.856	-2.481	0.615	2008:10	6
	SPE	-1.911	-0.614	0.306	-1.211	0.200	2001:02	0
	TY	-3.671‡	-3.230†	0.617	-2.896†	0.433	2008:11	6
	USA	-2.564	-1.995	0.773	-2.180	0.590	2008:10	5
Corp Debt BBB	ICB	-2.769*	-2.745*	0.843	-2.935†	0.741	2000:11	1
	INSI	-2.481	-2.060	0.914	-1.698	0.876	2008:08	10
	MGF	-2.135	-2.660*	0.790	-3.094†	0.663	2008:08	8
	MIN	-2.004	-1.570	0.975	-1.466	0.916	2013:04	5
	PAI	-1.936	-1.888	0.922	-1.948	0.878	2009:09	9
	VBF	-3.210†	-3.739‡	0.893	-3.433†	0.832	2003:06	3
General Bond	DUC	-2.202	-1.597	0.914	-1.211	0.781	2008:12	10
	JHI	-2.178	-2.531	0.951	-2.429	0.873	2012:09	1
	KMM	-2.647	-2.503	0.906	-2.406	0.676	2008:10	7
	KST	-2.035	-1.746	0.713	-1.520	0.726	2000:12	10
	MCI	-2.546	-2.884*	0.927	-2.426	0.978	2009:04	2
	MCR	-2.836*	-2.646	0.879	-2.258	0.847	2014:11	10
	MMT	-2.794*	-2.541	0.950	-2.280	0.856	2008:08	1
	MPV	-2.734	-2.822*	0.824	-1.990	0.772	2006:12	8
	PCM	-3.078†	-3.085†	0.820	-2.450	0.905	2008:08	8
	PIM	-3.546†	-3.566‡	0.794	-3.832‡	0.614	2007:09	8
	PPT	-3.143†	-3.059†	0.834	-3.335†	0.582	2007:10	10

Note: (a) τ_{LM}, $\tau_{RALS-LM(2\&3)}$, $\tau_{RALS-LM(t_5)}$ are the test statistics for the LM, RALS-LM(2&3), and RALS-LM(t_5) tests, respectively. (b) $\hat{\rho}^2_{(2\&3)}$ and $\hat{\rho}^2_{(t_5)}$ indicate the ratio of the estimated error variances for RALS-LM(2&3) and RALS-LM(t_5) tests, respectively. (c) $\hat{k}$ and $\hat{T}_B$ denote the optimal number of lags and the estimated break point, respectively. Since the number of lags and the break date determined using *max F* statistic are used in both LM and RALS-LM unit root tests, we report them one time. (d) The critical values for LM and RALS-LM tests are reported in Table 11.1 of Meng *et al.* (2014). (e) *, †, and ‡ denote a 10%, 5%, and 1% rejection, respectively.

Table B2. Continued

Categories	Fund	$\tau_{RALS-LM(t_3)}$	$\hat{\rho}^2_{(t_3)}$	$\tau_{RALS-LM(t_7)}$	$\hat{\rho}^2_{(t_7)}$	$\hat{T}_B$	$\hat{k}$
Core Funds	ADX	-1.911	0.695	-1.649	0.740	2002:07	8
	CET	-3.012†	0.702	-2.874*	0.725	2008:10	8
	CLM	-1.954	0.850	-1.848	0.817	2008:08	0
	CRF	-2.066	0.789	-2.124	0.679	2008:08	0
	FUND	-0.717	0.829	-0.776	0.785	2007:10	3
	GAB	-2.179	0.864	-2.075	0.856	2009:04	1
	GAM	-2.371	0.635	-2.373	0.635	2008:10	9
	GRF	-2.144	0.994	-2.052	0.947	2003:06	9
	RMT	-1.322	0.848	-1.463	0.843	2005:12	10
	RVT	-1.336	0.832	-1.479	0.809	2009:04	7
	SOR	-2.859†	0.589	-2.525	0.654	2008:10	6
	SPE	-1.025	0.210	-0.843	0.201	2001:02	0
	TY	-2.849†	0.415	-2.755†	0.444	2008:11	6
	USA	-1.997	0.575	-1.992	0.605	2008:10	5
Corp Debt BBB	ICB	-2.578	0.730	-2.478	0.755	2000:11	1
	INSI	-2.143	0.905	-2.128	0.876	2008:08	10
	MGF	-3.004†	0.669	-3.128†	0.663	2008:08	8
	MIN	-1.487	0.896	-1.462	0.927	2013:04	5
	PAI	-1.681	0.875	-1.724	0.892	2009:09	9
	VBF	-3.399†	0.833	-3.457†	0.835	2003:06	3
General Bond	DUC	-1.597	0.757	-1.683	0.778	2008:12	10
	JHI	-2.443	0.854	-2.434	0.881	2012:09	1
	KMM	-2.383	0.661	-2.419	0.689	2008:10	7
	KST	-1.419	0.777	-1.606	0.701	2000:12	10
	MCI	-2.448	0.977	-2.409	0.976	2009:04	2
	MCR	-2.676	0.860	-2.612	0.839	2014:11	10
	MMT	-2.247	0.836	-2.306	0.867	2008:08	1
	MPV	-2.125	0.834	-1.910	0.733	2006:12	8
	PCM	-2.059	0.924	-1.933	0.888	2008:08	8
	PIM	-3.797‡	0.617	-3.851‡	0.617	2007:09	8
	PPT	-3.434‡	0.545	-3.274†	0.605	2007:10	10

Note: (a) $\tau_{RALS-LM(t_3)}$, $\tau_{RALS-LM(t_7)}$ are the test statistics for the RALS-LM tests with degrees of freedom of 3 and 7, respectively. (b) $\hat{\rho}^2_{(t_3)}$ and $\hat{\rho}^2_{(t_7)}$ indicate the ratio of the estimated error variances for RALS-LM (t_3) and RALS-LM (t_7) tests, respectively. (c) $\hat{k}$ and $\hat{T}_B$ denote the optimal number of lags and the estimated break point, respectively. Since the number of lags and the break date determined using *max F* statistic are used in both LM and RALS-LM unit root tests, we report them one time. (d) The critical values for RALS-LM tests are reported in Table 11.1 of Meng *et al.* (2014). (e) *, †, and ‡ denote a 10%, 5%, and 1% rejection, respectively.

Table B3. LM and RALS-LM Test Results with Two Level Shifts

Categories	Fund	τ_{LM}	$\tau_{RALS-LM(2\&3)}$	$\hat{\rho}^2_{(2\&3)}$	$\tau_{RALS-LM(t_5)}$	$\hat{\rho}^2_{(t_5)}$	$\hat{T}_B$		$\hat{k}$
Core Funds	ADX	-1.780	-1.356	0.875	-1.499	0.772	2002:07	2007:11	8
	CET	-3.180†	-2.887*	0.839	-3.361†	0.684	2001:01	2008:10	5
	CLM	-2.163	-1.981	0.871	-2.598	0.825	2008:08	2010:01	0
	CRF	-2.118	-2.172	0.880	-2.788*	0.801	2002:09	2008:08	0
	FUND	-1.523	-1.081	0.893	-0.910	0.773	2007:10	2009:02	4
	GAB	-2.453	-1.654	0.857	-2.213	0.861	2008:11	2009:04	7
	GAM	-2.658	-3.289†	0.752	-3.341†	0.620	2008:10	2008:12	9
	GRF	-3.474†	-3.897‡	0.812	-3.895‡	0.698	2003:05	2004:03	1
	RMT	-1.194	-0.699	0.949	-0.997	0.898	2008:12	2009:04	10
	RVT	-1.903	-2.112	0.932	-1.850	0.828	2008:08	2009:04	7
	SOR	-2.127	-1.960	0.858	-2.067	0.675	2008:10	2008:12	6
	SPE	-2.010	-0.851	0.289	-1.478	0.198	2001:02	2009:12	0
	TY	-2.024	-1.489	0.654	-0.297	0.464	2008:11	2009:02	10
	USA	-2.579	-2.367	0.813	-1.773	0.630	2004:04	2008:10	0
Corp Debt BBB	ICB	-2.982*	-2.911*	0.858	-3.233†	0.736	2000:11	2008:11	1
	INSI	-1.856	-1.384	0.937	-0.929	0.811	2008:08	2009:01	1
	MGF	-2.342	-2.826*	0.803	-3.508‡	0.677	2008:08	2008:10	8
	MIN	-1.412	-1.033	0.981	-1.039	0.907	2010:11	2013:04	5
	PAI	-2.600	-2.880*	0.959	-2.909*	0.926	2008:12	2009:09	9
	VBF	-3.406†	-3.665‡	0.916	-3.719‡	0.835	2003:06	2008:12	6
General Bond	DUC	-3.173†	-3.018†	0.905	-1.976	0.815	2006:02	2008:12	1
	JHI	-2.309	-2.467	0.977	-2.286	0.918	2011:07	2012:09	1
	KMM	-2.540	-2.499	0.931	-2.647*	0.631	2000:11	2008:10	7
	KST	-2.289	-2.042	0.761	-2.110	0.791	2000:12	2001:07	10
	MCI	-2.673	-3.066†	0.939	-2.661	0.991	2008:09	2009:04	2
	MCR	-2.341	-1.723	0.886	-1.513	0.859	2008:08	2014:11	10
	MMT	-3.126†	-3.239†	0.956	-3.078†	0.874	2008:08	2008:12	1
	MPV	-2.202	-1.971	0.872	-1.692	0.819	2003:11	2006:12	10
	PCM	-2.078	-2.044	0.888	-2.101	0.895	2007:02	2008:08	4
	PIM	-3.389†	-3.222†	0.787	-3.455‡	0.605	2007:09	2008:09	8
	PPT	-2.759	-2.275	0.827	-2.098	0.563	2007:10	2008:09	10

Note: (a) τ_{LM}, $\tau_{RALS-LM(2\&3)}$, $\tau_{RALS-LM(t_5)}$ are the test statistics for the LM, RALS-LM (2&3), and RALS-LM (t_5) tests, respectively. (b) $\hat{\rho}^2_{(2\&3)}$ and $\hat{\rho}^2_{(t_5)}$ indicate the ratio of the estimated error variances for RALS-LM (2&3) and RALS-LM (t_5) tests, respectively. (c) $\hat{k}$ and $\hat{T}_B$ denote the optimal number of lags and the estimated break points, respectively. Since the number of lags and the break dates determined using *max F* statistic are used in both LM and RALS-LM unit root tests, we report them one time. (d) The critical values for LM and RALS-LM tests are reported in Table 11.1 of Meng *et al.* (2014). (e) *, †, and ‡ denote a 10%, 5%, and 1% rejection, respectively.

Table B3. Continued

Categories	Fund	$\tau_{RALS-LM(t_3)}$	$\hat{\rho}^2_{(t_3)}$	$\tau_{RALS-LM(t_7)}$	$\hat{\rho}^2_{(t_7)}$	$\hat{T}_B$		$\hat{k}$
Core Funds	ADX	-1.668	0.747	-1.410	0.787	2002:07	2007:11	8
	CET	-3.386†	0.668	-3.328†	0.696	2001:01	2008:10	5
	CLM	-1.901	0.858	-1.764	0.842	2008:08	2010:01	0
	CRF	-2.682*	0.835	-2.840*	0.782	2002:09	2008:08	0
	FUND	-0.342	0.814	-0.487	0.819	2007:10	2009:02	4
	GAB	-3.021*	0.971	-2.923*	0.917	2008:11	2009:04	7
	GAM	-3.402†	0.603	-3.291†	0.633	2008:10	2008:12	9
	GRF	-2.732*	0.714	-2.807*	0.669	2003:05	2004:03	1
	RMT	-0.938	0.882	-1.027	0.903	2008:12	2009:04	10
	RVT	-1.767	0.834	-1.897	0.830	2008:08	2009:04	7
	SOR	-2.210	0.670	-2.140	0.702	2008:10	2008:12	6
	SPE	-1.331	0.206	-1.074	0.200	2001:02	2009:12	0
	TY	-0.897	0.454	-1.272	0.480	2008:11	2009:02	10
	USA	-1.291	0.610	-1.455	0.635	2004:04	2008:10	0
Corp Debt BBB	ICB	-2.790*	0.720	-2.707*	0.746	2000:11	2008:11	1
	INSI	-0.985	0.812	-1.106	0.834	2008:08	2009:01	1
	MGF	-3.465‡	0.676	-3.523‡	0.681	2008:08	2008:10	8
	MIN	-1.072	0.889	-1.023	0.919	2010:11	2013:04	5
	PAI	-2.417	0.955	-2.504	0.945	2008:12	2009:09	9
	VBF	-3.199†	0.855	-3.257†	0.858	2003:06	2008:12	6
General Bond	DUC	-1.811	0.791	-2.083	0.823	2006:02	2008:12	1
	JHI	-2.288	0.907	-2.293	0.923	2011:07	2012:09	1
	KMM	-2.742*	0.585	-2.595*	0.663	2000:11	2008:10	7
	KST	-1.997	0.843	-2.189	0.762	2000:12	2001:07	10
	MCI	-2.654	0.997	-2.668	0.986	2008:09	2009:04	2
	MCR	-1.752	0.871	-1.675	0.849	2008:08	2014:11	10
	MMT	-3.034†	0.859	-3.101†	0.884	2008:08	2008:12	1
	MPV	-1.948	0.834	-1.817	0.804	2003:11	2006:12	10
	PCM	-2.050	0.928	-2.133	0.875	2007:02	2008:08	4
	PIM	-3.408†	0.603	-3.484‡	0.613	2007:09	2008:09	8
	PPT	-2.176	0.510	-2.072	0.592	2007:10	2008:09	10

Note: (a) $\tau_{RALS-LM(t_3)}$, $\tau_{RALS-LM(t_7)}$ are the test statistics for the RALS-LM tests with degrees of freedom of 3 and 7, respectively. (b) $\hat{\rho}^2_{(t_3)}$ and $\hat{\rho}^2_{(t_7)}$ indicate the ratio of the estimated error variances for RALS-LM (t_3) and RALS-LM (t_7) tests, respectively. (c) $\hat{k}$ and $\hat{T}_B$ denote the optimal number of lags and the estimated break points, respectively. Since the number of lags and the break dates determined using *max F* statistic are used in both LM and RALS-LM unit root tests, we report them one time. (d) The critical values for RALS-LM tests are reported in Table 11.1 of Meng *et al.* (2014). (e) *, †, and ‡ denote a 10%, 5%, and 1% rejection, respectively.

Table B4. No Trend Break RALS-LM Test Results with Alternative Degrees of Freedom

Categories	Fund	$\tau_{RALS-LM(t_3)}$	$\hat{\rho}^2_{(t_3)}$	$\tau_{RALS-LM(t_7)}$	$\hat{\rho}^2_{(t_7)}$	$\hat{k}$
Core Funds	ADX	-2.086	0.751	-2.153	0.793	8
	CET	-2.372	0.614	-2.134	0.651	5
	CLM	-1.388	0.890	-1.179	0.874	10
	CRF	-1.909	0.624	-1.604	0.557	0
	FUND	-1.574	0.802	-1.737	0.750	2
	GAB	-3.571‡	0.853	-3.481†	0.819	0
	GAM	-4.121‡	0.453	-3.965‡	0.493	9
	GRF	-2.142	0.915	-2.310	0.814	10
	RMT	-0.937	0.828	-0.997	0.832	10
	RVT	-1.234	0.833	-1.344	0.817	7
	SOR	-2.171	0.583	-1.987	0.634	3
	SPE	-6.225‡	0.235	-6.400‡	0.230	3
	TY	-2.944†	0.342	-2.538*	0.378	4
	USA	-1.587	0.490	-1.660	0.533	1
Corp Debt BBB	ICB	-4.311‡	0.633	-4.254‡	0.664	7
	INSI	-3.897‡	0.786	-4.112‡	0.757	5
	MGF	-3.515‡	0.624	-3.516‡	0.616	8
	MIN	-0.849	0.802	-0.747	0.810	5
	PAI	-2.658	0.801	-2.515	0.808	8
	VBF	-2.965*	0.848	-2.952*	0.845	3
General Bond	DUC	-2.463	0.825	-2.397	0.825	3
	JHI	-2.046	0.870	-2.028	0.874	1
	KMM	-2.491	0.695	-2.569	0.702	7
	KST	-2.320	0.740	-2.444	0.679	10
	MCI	-2.817*	0.985	-2.792*	0.949	2
	MCR	-2.257	0.802	-2.207	0.794	1
	MMT	-3.206†	0.835	-3.301†	0.849	1
	MPV	-2.841*	0.771	-2.864*	0.720	8
	PCM	-1.281	0.929	-1.044	0.866	4
	PIM	-3.037†	0.579	-3.173†	0.580	8
	PPT	-1.557	0.630	-1.511	0.639	10

Note: (a) $\tau_{RALS-LM(t_3)}$, $\tau_{RALS-LM(t_7)}$ are the test statistics for the RALS-LM tests with degrees of freedom of 3 and 7, respectively. (b) $\hat{\rho}^2_{(t_3)}$ and $\hat{\rho}^2_{(t_7)}$ indicate the ratio of the estimated error variances for RALS-LM (t_3) and RALS-LM (t_7) tests, respectively. (c) $\hat{k}$ denotes the optimal number of lags. Since the number of lags determined using *max F* statistic is used in both LM and RALS-LM unit root tests, we report this one time. (d) The critical values for RALS-LM tests are reported in Table 11.1 of Meng *et al*. (2014). (e) ‡, †, and * represent 1%, 5%, and 10% rejection, respectively.

Table B5. One Trend Break RALS-LM Test Results with Alternative Degrees of Freedom

Categories	Fund	$\tau^*_{RALS-LM(t_3)}$	$\hat{\rho}^2_{(t_3)}$	$\tau^*_{RALS-LM(t_7)}$	$\hat{\rho}^2_{(t_7)}$	$\hat{T}_B$	$\hat{k}$
Core Funds	ADX	-6.331‡	0.671	-6.050‡	0.724	2002:09	8
	CET	-3.492†	0.733	-3.416*	0.711	2008:09	10
	CLM	-2.248	0.993	-2.100	0.980	2011:08	8
	CRF	-2.630	0.741	-2.313	0.697	2011:08	0
	FUND	-3.156	0.887	-3.121	0.855	2008:11	4
	GAB	-3.177	0.931	-3.028	0.889	2014:07	0
	GAM	-6.958‡	0.543	-6.981‡	0.561	2000:12	9
	GRF	-3.158	0.920	-3.417*	0.806	2004:04	10
	RMT	-3.085	0.839	-3.182	0.824	2007:05	10
	RVT	-3.664†	0.731	-3.601†	0.763	2007:01	4
	SOR	-3.729†	0.641	-3.817†	0.651	2007:01	6
	SPE	-15.23‡	0.207	-14.581‡	0.213	2002:06	0
	TY	-4.052‡	0.402	-3.682†	0.442	2008:12	3
	USA	-2.664	0.491	-2.643	0.517	2001:01	6
Corp Debt BBB	ICB	-4.946‡	0.660	-4.884‡	0.667	2000:12	7
	INSI	-4.513‡	0.751	-4.523‡	0.747	2000:11	5
	MGF	-3.516†	0.687	-3.426†	0.675	2008:08	8
	MIN	-0.886	0.777	-0.786	0.789	2013:01	3
	PAI	-2.561	0.870	-2.443	0.855	2008:10	8
	VBF	-3.772†	0.794	-3.781†	0.803	2015:07	3
General Bond	DUC	-2.081	0.881	-1.997	0.845	2015:12	3
	JHI	-2.540	0.791	-2.473	0.824	2012:05	1
	KMM	-3.560†	0.663	-3.552†	0.663	2000:12	7
	KST	-5.510‡	0.749	-5.527‡	0.712	2001:03	10
	MCI	-4.449‡	0.934	-4.494‡	0.919	2000:11	2
	MCR	-3.994†	0.775	-4.050‡	0.755	2001:01	1
	MMT	-2.715	0.855	-2.794	0.865	2015:08	1
	MPV	-2.424	0.686	-2.402	0.650	2012:06	8
	PCM	-2.473	0.828	-2.535	0.792	2006:12	9
	PIM	-2.807	0.694	-3.010	0.670	2009:03	9
	PPT	-1.566	0.626	-1.631	0.632	2007:11	9

Note: (a) $\tau^*_{\mathrm{RALS-LM}(t_3)}$, $\tau^*_{\mathrm{RALS-LM}(t_7)}$ are the test statistics for the RALS-LM tests with degrees of freedom of 3 and 7, respectively. (b) $\hat{\rho}^2_{(t_3)}$ and $\hat{\rho}^2_{(t_7)}$ indicate the ratio of the estimated error variances for RALS-LM (t_3) and RALS-LM (t_7) tests, respectively. (c) $\hat{k}$ and $\hat{T}_B$ denote the optimal number of lags and the estimated break point, respectively. Since the number of lags and the break date determined using *max F* statistic are used in both LM and RALS-LM unit root tests, we report them one time. (d) The critical values for RALS-LM tests are reported in Table 1 of Meng *et al.* (2017). (e) ‡, †, and * represent 1%, 5%, and 10% rejection, respectively.

Table B6. Two Trend Break RALS-LM Test Results with Alternative Degrees of Freedom

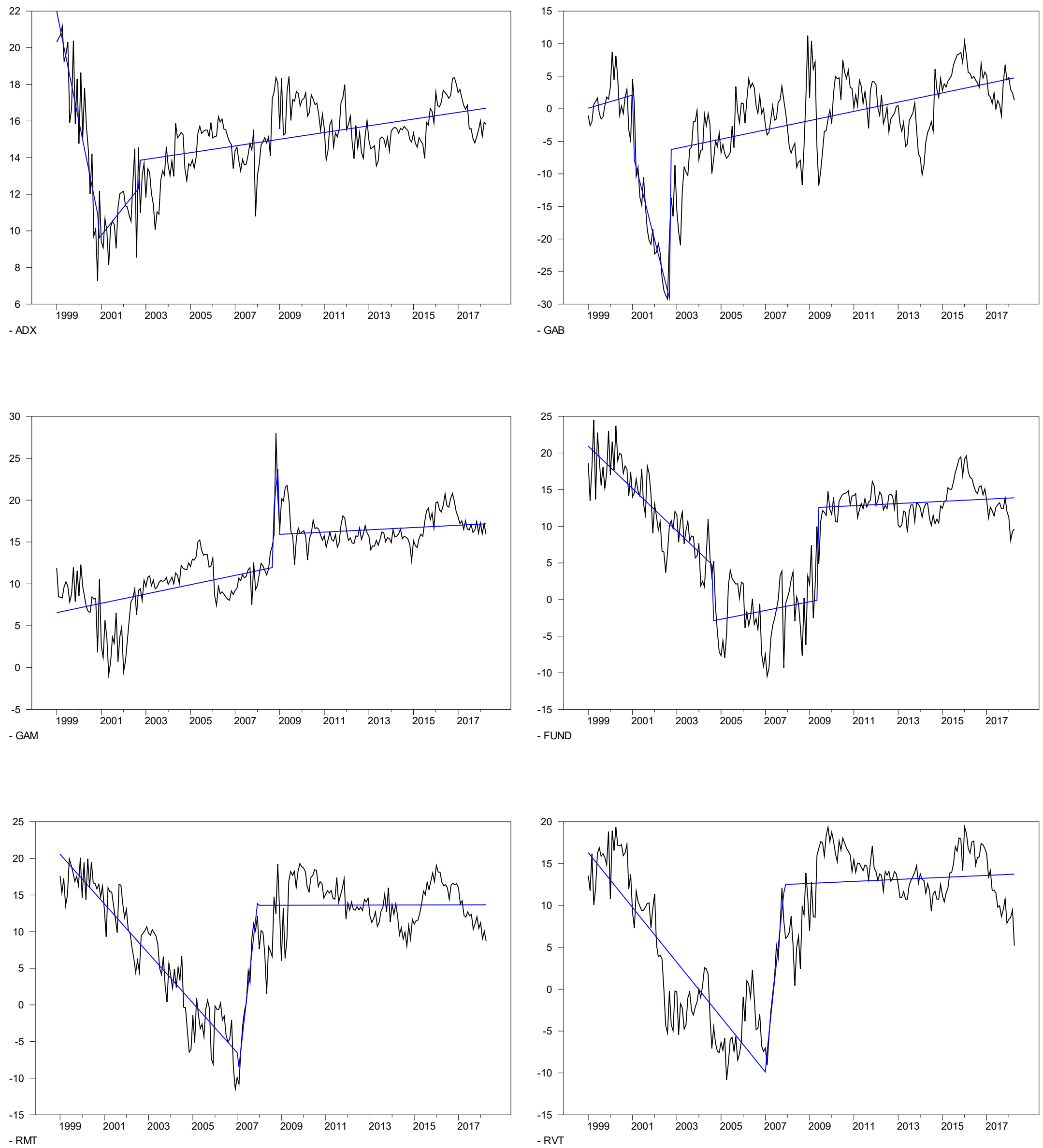
Categories	Fund	$\tau^*_{RALS-LM(t_3)}$	$\hat{\rho}^2_{(t_3)}$	$\tau^*_{RALS-LM(t_7)}$	$\hat{\rho}^2_{(t_7)}$	$\hat{T}_B$		$\hat{k}$
Core Funds	ADX	-9.938‡	0.764	-9.772‡	0.799	2000:11	2002:09	8
	CET	-4.319†	0.637	-4.356†	0.670	2008:09	2008:12	5
	CLM	-5.257‡	0.812	-5.547‡	0.767	2008:07	2008:10	0
	CRF	-9.526‡	0.581	-9.795‡	0.579	2008:07	2008:10	0
	FUND	-6.289‡	0.797	-6.164‡	0.807	2004:08	2009:05	4
	GAB	-6.936‡	0.749	-7.039‡	0.728	2001:01	2002:09	0
	GAM	-7.991‡	0.614	-7.986‡	0.654	2008:09	2008:12	9
	GRF	-3.458	0.739	-3.753*	0.683	2003:05	2003:08	6
	RMT	-3.370	0.693	-3.184	0.732	2007:01	2007:12	1
	RVT	-4.429†	0.780	-4.366†	0.783	2007:01	2007:11	9
	SOR	-5.018‡	0.612	-4.884‡	0.651	2002:09	2003:09	0
	SPE	-16.43‡	0.672	-15.94‡	0.718	2001:02	2001:05	5
	TY	-10.75‡	0.483	-10.62‡	0.524	2008:10	2009:07	6
	USA	-8.181‡	0.690	-7.916‡	0.717	2008:07	2008:12	10
Corp Debt BBB	ICB	-6.896‡	0.696	-6.665‡	0.736	2008:08	2009:01	7
	INSI	-6.495‡	0.932	-6.659‡	0.913	2008:08	2009:02	5
	MGF	-5.612‡	0.865	-5.529‡	0.877	2008:08	2009:01	8
	MIN	-6.158‡	0.914	-6.171‡	0.936	2013:03	2013:09	5
	PAI	-3.432	0.951	-3.480	0.942	2008:10	2009:02	8
	VBF	-4.232†	0.91	-4.263†	0.901	2008:08	2009:02	3
General Bond	DUC	-6.225‡	0.778	-6.151‡	0.809	2006:01	2009:01	3
	JHI	-5.488‡	0.828	-5.457‡	0.849	2011:06	2012:10	1
	KMM	-4.287†	0.803	-4.393†	0.797	2007:02	2009:01	7
	KST	-5.422‡	0.831	-5.419‡	0.749	2000:11	2001:04	10
	MCI	-5.721‡	0.938	-5.741‡	0.933	2007:04	2009:04	2
	MCR	-5.223‡	0.803	-5.137‡	0.808	2008:08	2009:02	1
	MMT	-4.094†	0.795	-4.092†	0.820	2013:03	2013:09	1
	MPV	-3.928†	0.748	-3.998†	0.707	2006:11	2007:02	8
	PCM	-5.480‡	0.892	-5.651‡	0.854	2008:07	2008:10	4
	PIM	-3.768*	0.679	-4.022†	0.664	2007:09	2007:12	8
	PPT	-10.53‡	0.576	-9.777‡	0.676	2007:09	2007:12	10

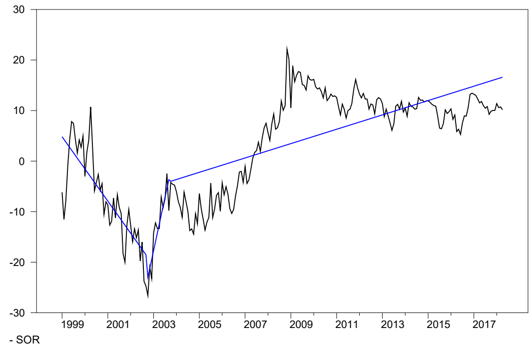
Note: (a) $\tau^*_{RALS-LM(t_3)}$, $\tau^*_{RALS-LM(t_7)}$ are the test statistics for the RALS-LM tests with degrees of freedom of 3 and 7, respectively. (b) $\hat{\rho}^2_{(t_3)}$ and $\hat{\rho}^2_{(t_7)}$ indicate the ratio of the estimated error variances for RALS-LM (t_3) and RALS-LM (t_7) tests, respectively. (c) $\hat{k}$ and $\hat{T}_B$ denote the optimal number of lags and the estimated break points, respectively. Since the number of lags and the break dates determined using *max F* statistic are used in both LM and RALS-LM unit root tests, we report them one time. (d) The critical values for RALS-LM tests are reported in Table 1 of Meng *et al*. (2017). (e) ‡, †, and * represent 1%, 5%, and 10% rejection, respectively.

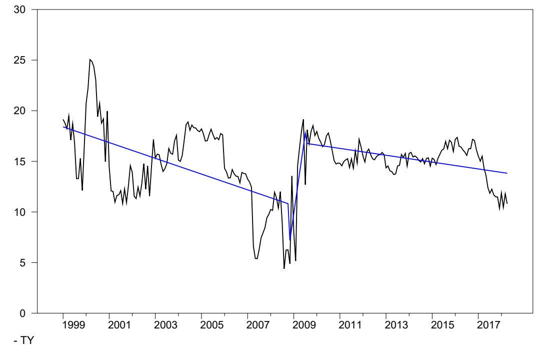
Appendix C: Additional Figures

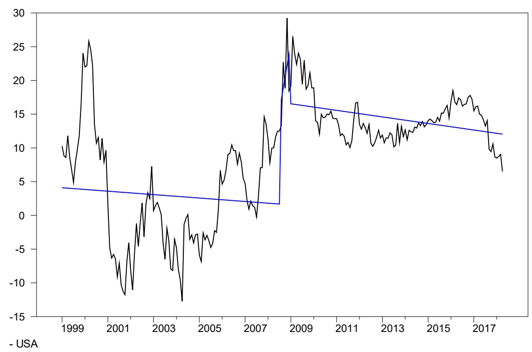
Figure C1: Discount Data with Estimated Time-Varying Intercepts and Trend

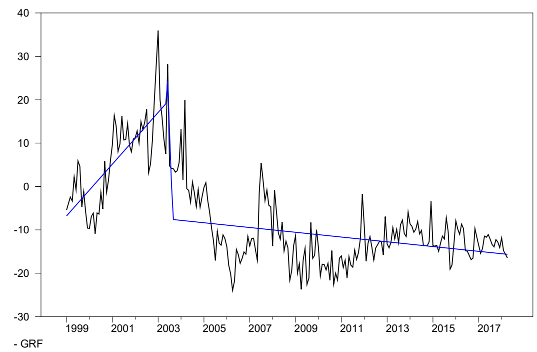
(a) Core Funds

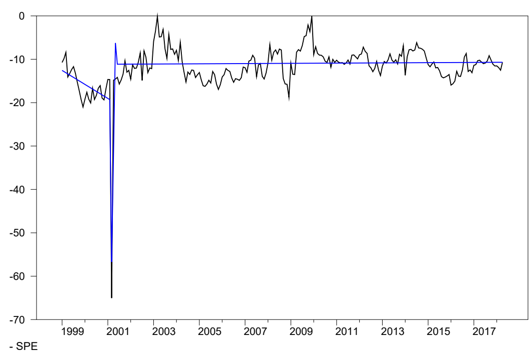


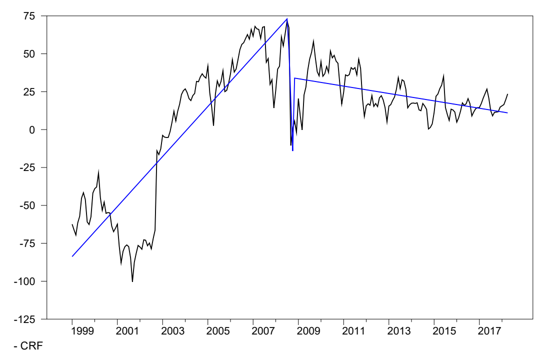

30 20 10 0 -10 -20 -30
1999 2001 2003 2005 2007 2009 2011 2013 2015 2017
- SOR

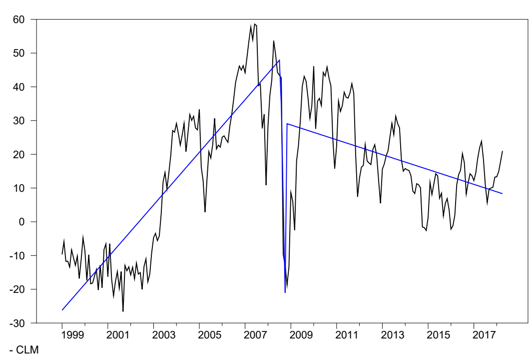

30 25 20 15 10 5 0
1999 2001 2003 2005 2007 2009 2011 2013 2015 2017
- TY


30 25 20 15 10 5 0 -5 -10 -15
1999 2001 2003 2005 2007 2009 2011 2013 2015 2017
- USA


40 30 20 10 0 -10 -20 -30
1999 2001 2003 2005 2007 2009 2011 2013 2015 2017
- GRF


0 -10 -20 -30 -40 -50 -60 -70
1999 2001 2003 2005 2007 2009 2011 2013 2015 2017
- SPE


75 50 25 0 -25 -50 -75 -100 -125
1999 2001 2003 2005 2007 2009 2011 2013 2015 2017
- CRF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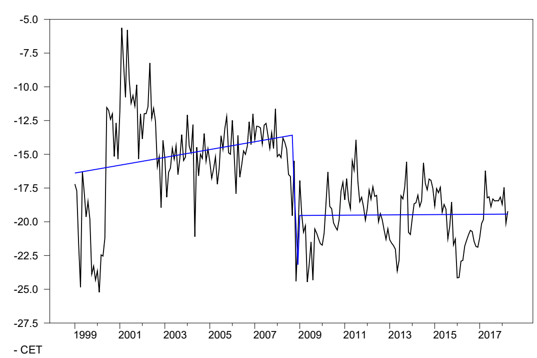
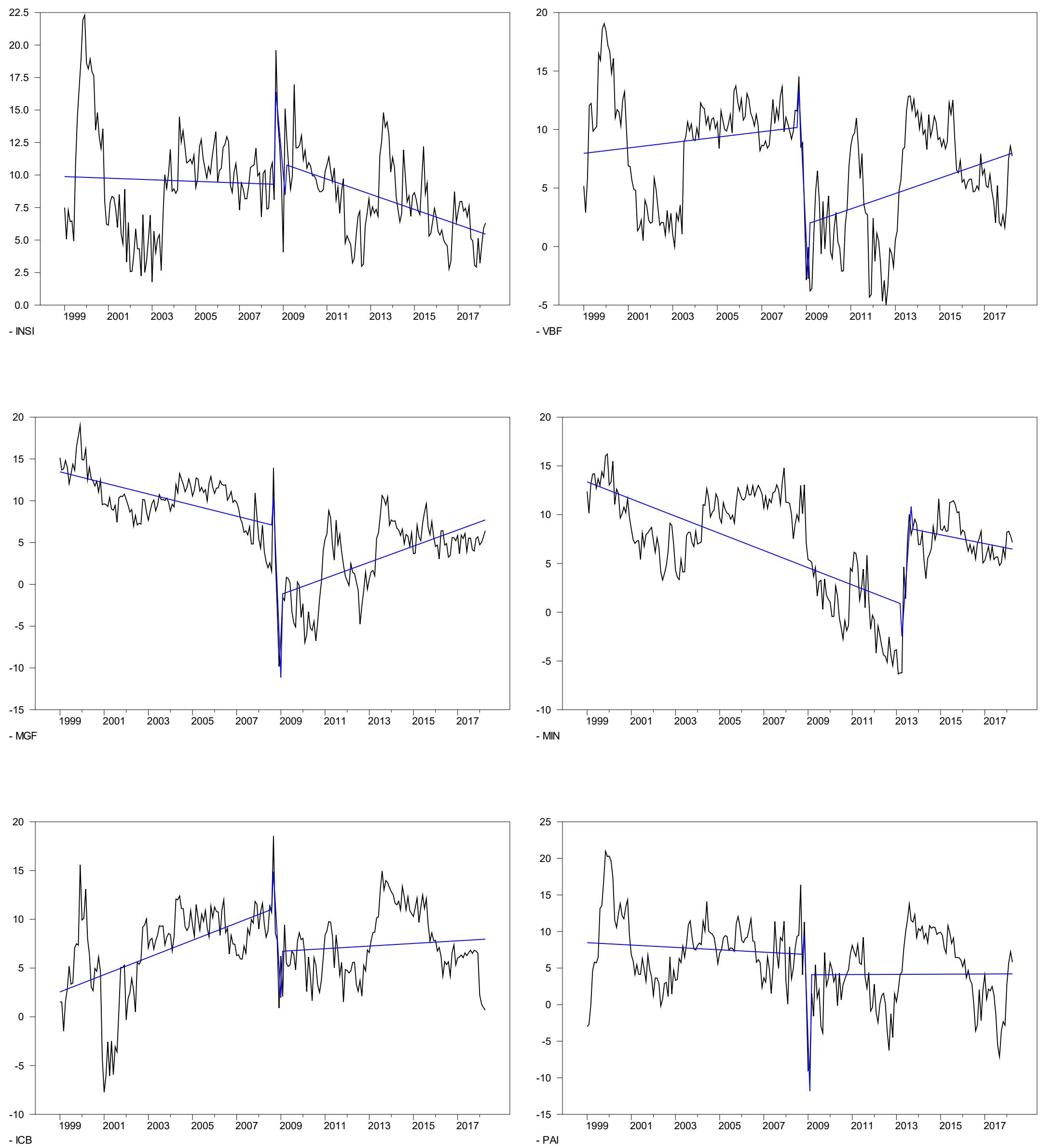
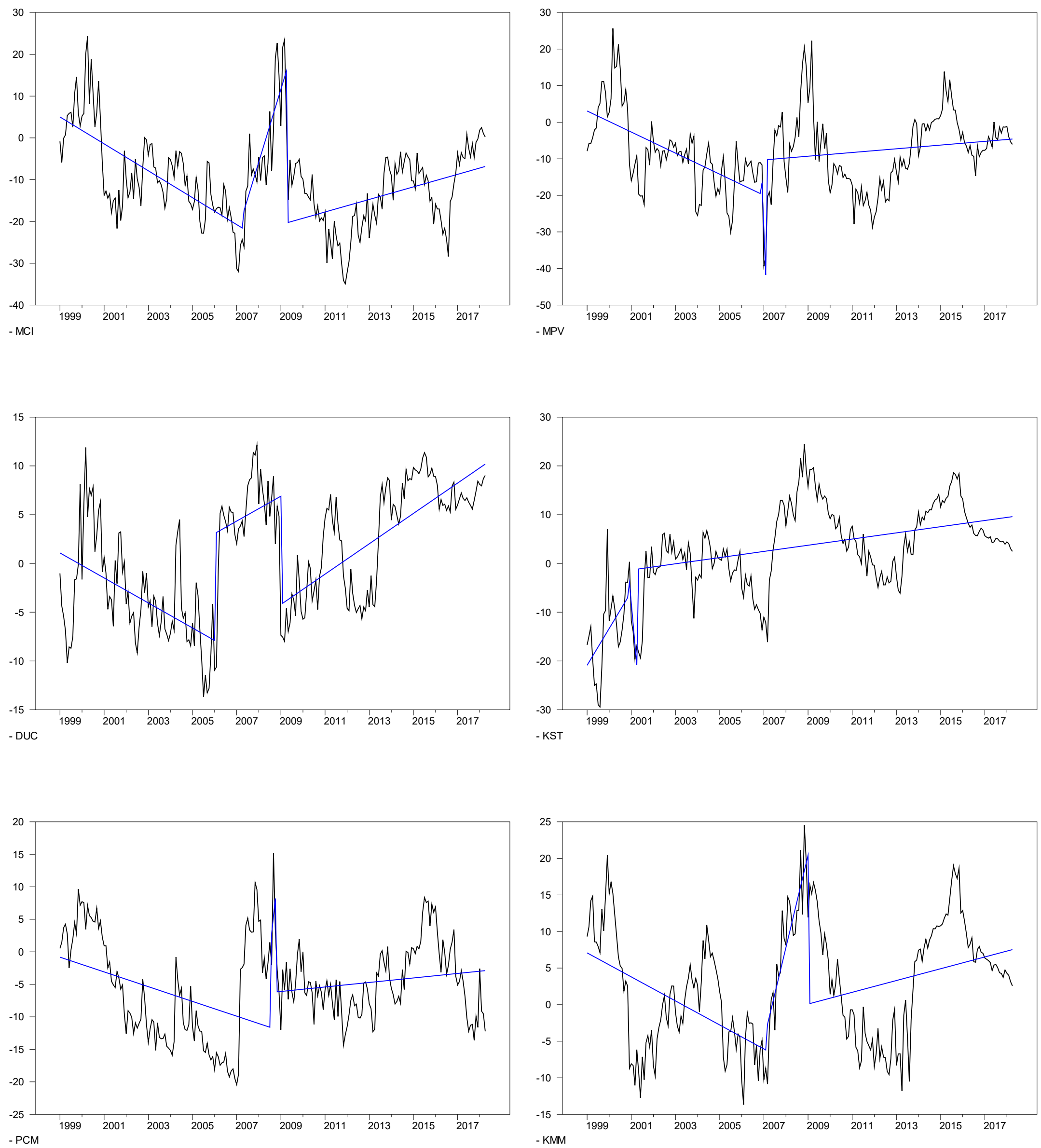
1999 2001 2003 2005 2007 2009 2011 2013 2015 2017
- CLM

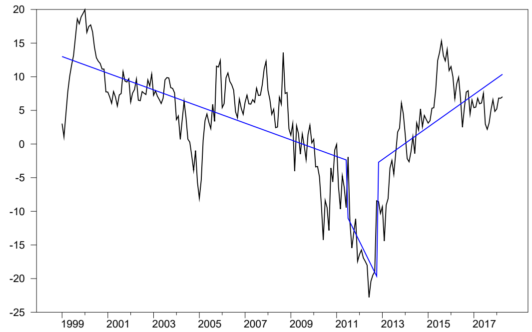

-5.0 -7.5 -10.0 -12.5 -15.0 -17.5 -20.0 -22.5 -25.0 -27.5
1999 2001 2003 2005 2007 2009 2011 2013 2015 2017
- CET

(b) Core Debt BBB Funds

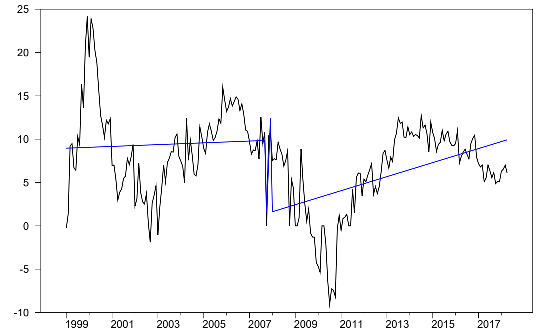


(c) General Bond Funds

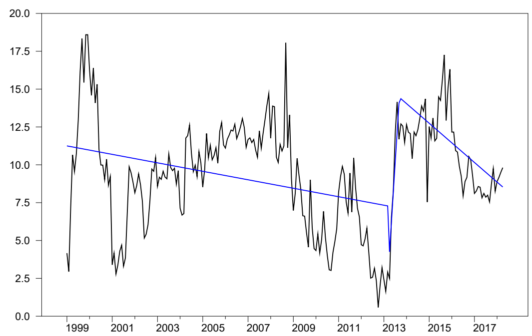




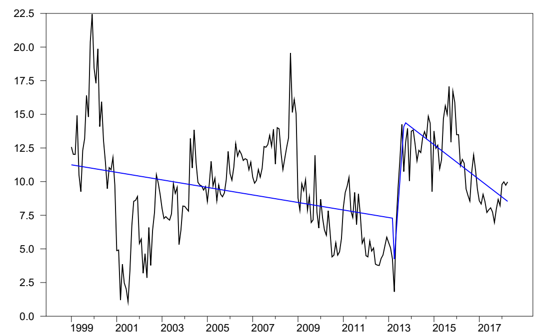
- JHI



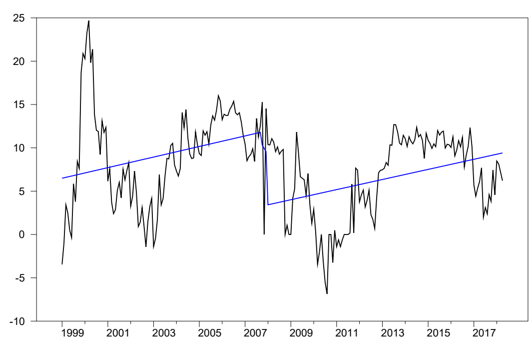
- PIM



- MCR



- MMT



- PPT