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Abstract

We study a generalization of the static model of [11] with two risk neutral insiders to the case where each insider is partially informed about the value of the stock. First, we provide a necessary and sufficient condition for the uniqueness of the linear Bayesian equilibrium. Specifically, we show that, when the covariance matrix of the errors terms of the insiders' signals, is not singular, the linear Bayesian equilibrium is not unique. Then, we carry out a comparative statics analysis.

JEL classification: G14, D82

Keywords: Insider trading, Risk neutrality, Coefficient of Correlation, Partial Correlation, Market structure, Kyle model

1 Introduction

Information asymmetry in financial markets and in the presence of insider trading, has been extensively studied theoretically and empirically. The pioneering work of [11] on strategic trading is considered as the canonical model

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on insider trading that explains how markets can incorporate private information. [11] studied the single auction model in which one risky asset is exchanged for a risk-less asset among three kinds of traders: a single informed trader who has access to private observation of the ex-post liquidation value of the risky asset, uninformed noise traders and market makers who set prices conditional on the information they have about the quantities traded by others.

There is a large body of literature on the theoretical extensions of the model of [11].[1]. For instance, [8] extended [11] to the case of finite number of insiders, each of them, knowing perfectly the value of the risky asset. [18] extended [11] to include fully and partially informed traders. [9]) allowed for the market maker to observe a second signal that is correlated with the order flow. [1] studied [11]in which the noise traders are able to correlate their trade with the true price. Recently, [5] extended [11] to the case of two insiders, one risk-neutral and one risk-averse, while [10] studied the case of two insiders in which the first insider is risk-neutral while the second insider is overconfident. Among other extensions, are a group of papers interested in proving the existence (or not) and/or uniqueness of Kyle-type model equilibria (See for example, [3, 4, 5, 6, 12, 14, 15, 16, 19]). This paper belongs to the latter type of extensions, more specifically, it extended [11] to the general case of information correlation between the two insiders.

Our paper has two objectives. We first provide a necessary and sufficient condition for the uniqueness of the linear Bayesian equilibrium. Specifically, we show that when the covariance matrix of the errors terms of the insiders' signals is not singular, the linear Bayesian equilibrium is not unique. Indeed this result is based on the fact that the coefficient of correlation between the signals' errors, is not exogenously given and must be computed. Then, we shed light on the impact of the degree of correlation between the insiders' signals on the equilibrium outputs.

The paper is organized as follows: In Section 2, we present the model and provide the necessary and sufficient condition for the uniqueness of the linear Bayesian equilibrium. Moreover, we characterize the linear Bayesian equilibrium of the model. In Section 3, we conduct a comparative statics analysis of the equilibrium outcomes with respect to [18].

[1](e.g., See [2, 17, 20]).

2 The Model

We consider a modified version of [11] with three type of traders: two risk-neutral insiders, noise (or liquidity) traders and a Bertrand competitive market makers. The insiders and the noise traders submit market orders to the competitive market makers, and the latter are responsible for determining the asset (stock) price. The game goes as follows. First, nature moves by selecting a true value $\tilde{z}$ from a prior normal distribution $\mathcal{N}(\bar{z}, \sigma_z^2)$ for the traded asset, and by selecting a demand quantity $\tilde{u}$ from a prior normal distribution $\mathcal{N}(0, \sigma_u^2)$. As in [11] we assume that $\tilde{z}$ and $\tilde{u}$ are independent so that the noise trader's order, $\tilde{u}$, contains no information regarding the true value z of the stock.[2]

Next, we assume in this model, that each risk-neutral insider, i $(i = 1, 2)$ gets information about the true value of the stock, $\tilde{z}$, by observing the realization of the signal $\tilde{s}_i$ correlated to $\tilde{z}$, the value of the stock with coefficient of correlation ρ_i.[3] We assume that the signal $\tilde{s}_i, (i = 1, 2)$, is normally distributed with mean $\bar{s}_i$ and variance σ_i^2. Moreover, we assume that each of the variables $\tilde{z}$ and $\tilde{s}_i (i = 1, 2)$, is independent from $\tilde{u}$ and the couple $\tilde{z}$ and $\tilde{s}_i (i = 1, 2)$ is jointly normal.[4] Hence, we can summarize our variables using the standard representation of the normal distribution,[5]

$$\begin{pmatrix} \tilde{z} \\ \tilde{s}_1 \\ \tilde{s}_2 \\ \tilde{u} \end{pmatrix} \sim \mathcal{N} \left[\begin{pmatrix} \bar{z} \\ \bar{s}_1 \\ \bar{s}_2 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_z^2 & \rho_1 \sigma_z \sigma_1 & \rho_2 \sigma_z \sigma_2 & 0 \\ \rho_1 \sigma_z \sigma_1 & \sigma_1^2 & \rho \sigma_1 \sigma_2 & 0 \\ \rho_2 \sigma_z \sigma_2 & \rho \sigma_1 \sigma_2 & \sigma_2^2 & 0 \\ 0 & 0 & 0 & \sigma_u^2 \end{pmatrix} \right]$$

Where ρ is the coefficient of correlation between $\tilde{s}_1$ and $\tilde{s}_2$. Given the observation s_i of the signal $\tilde{s}_i$ and a pricing schedule $\tilde{p} = p(\Sigma_{i=1}^2 \tilde{x}_i(\tilde{s}_i) + \tilde{u})$ that each insider i $(i = 1, 2)$ expects to prevail in equilibrium, he selects his demand quantity $x_i(\tilde{s}_i)$ to maximize his conditional expected profit from trade, $E[\tilde{\pi}_i | \tilde{s}_i]$ where $\tilde{\pi}_i = (\tilde{z} - p(\tilde{x}_1 + \tilde{x}_2 + \tilde{u}))\tilde{x}_i$. Specifically,

[2] Random variables are denoted with a tilde. Realized values lack the tilde. The mean of the random variable is denoted with bar.

[3] Without loss of generality, we assume that ρ_1 and ρ_2 are either both strictly positive or both strictly negative. Moreover, the case of $\rho_i = 0$ $(i = 1, 2)$ is omitted since we focus on the impact of partial correlation on the model outcomes.

[4] Random variables are denoted with a tilde. Realized values lack the tilde. The mean of the random variable is denoted with bar.

[5] We thank the referee for suggesting this representation.

$$\tilde{x}_i(\tilde{s}_i) \in argmax\ E[(\tilde{z} - p(\tilde{x}_1 + \tilde{x}_2 + \tilde{u}))\tilde{x}_i|\tilde{s}_i]$$

where the price schedule $\tilde{p} = p(\Sigma_{i=1}^{2}\tilde{x}_i(\tilde{s}_i) + \tilde{u}))$ is correctly expected by the insider in equilibrium.

Finally, the risk-neutral Bertrand-competitive market makers, given their correct expectation about the insiders aggregate trading strategy $\Sigma_{i=1}^{2}\tilde{x}_i$, compete in price upon seeing the total demand quantity submitted, i.e. the total order flow denoted by $\tilde{r} = \Sigma_{i=1}^{2}\tilde{x}_i(\tilde{s}_i) + \tilde{u}$. Note that in the Bertrand equilibrium of the subgame where market makers engage in price competition to absorb the order imbalance, each market maker must earn a zero expected profit conditional on his information $\tilde{r}$, which implies that the asset price $\tilde{p} = p(\tilde{z}|\tilde{r})$ fulfilling the semi-strong form efficiency of the asset market since $\tilde{r}$ is the public information at the time trading takes place.

A pure strategy Bayesian-Nash equilibrium is a vector of three functions $[x_1(.), x_2(.), p(.)]$ such that:

(a) Profit maximization of insider 1,

$$E[\tilde{z} - p(\tilde{x}_1 + \tilde{x}_2 + \tilde{u}))\tilde{x}_1|\tilde{s}_1] \geq E[\tilde{z} - p(\tilde{x}_1' + \tilde{x}_2 + \tilde{u}))\tilde{x}_1'|\tilde{s}_1]$$

for any level of trading order $\tilde{x}_1'$ decided by the insider;

(b) Profit maximization of insider 2,

$$E[\tilde{z} - p(\tilde{x}_1 + \tilde{x}_2 + \tilde{u}))\tilde{x}_1|\tilde{s}_2] \geq E[\tilde{z} - p(\tilde{x}_1 + \tilde{x}_2' + \tilde{u}))\tilde{x}_2'|\tilde{s}_2]$$

for any level of trading order $\tilde{x}_2'$ decided by the insider;

(c) Semi-Strong Market Efficiency: The pricing rule $p(.)$ satisfies,

$$p(\tilde{r}) = E[\tilde{z}|\tilde{r}].$$

An equilibrium is linear if the insiders' strategies are linear with respect to their observed signals and the pricing rule is linear with respect to the order flow signal. In other words, there exists constants $a_1, b_1, a_2, b_2, \mu, \lambda$ such that,

$$x_1(s_1) = a_0 + a_1 s_1, \qquad x_2(s_2) = b_0 + b_1 s_2$$

and

$$\forall\ r, \quad p(r) = \mu + \lambda r.$$

Information Structure: We assume that each partially informed trader $i, (i = 1, 2)$, observes only the realization of his/her signal, s_i, of $\tilde{s}_i$ and does not observe the values of $\tilde{u}, \tilde{r}$, before the order flow decisions are made. Moreover, market makers don't observe neither the realization z of $\tilde{z}$, x_i of $\tilde{x}_i$, u of $\tilde{u}$, nor the realization s_i of $\tilde{s}_i$ but only they know their distributions. Furthermore, we assume in this model that the coefficient of correlation ρ, between $\tilde{s}_1$ and $\tilde{s}_2$ is endogenously determined by the game players and not exogenously given as in [12]. In other words, the distributions of the value of the stock, $\tilde{z}$, the insider i' signal $\tilde{s}_i$ $(i = 1, 2)$ and the liquidity traders order $\tilde{u}$, are common knowledge to each player of the game, i.e., the two insiders and the market makers. Specifically, we assume the following exogenous variables $\bar{z}, \sigma_z^2, \bar{s}_1, \rho_1, \bar{s}_2, \rho_2, \sigma_{\varepsilon_1}^2, \sigma_{\varepsilon_2}^2$ and σ_η^2, are known to all the players. [6]

We turn now to present the main result of the paper. Lemma 1 shows that given the exogenous variables listed above, the coefficient of correlation of the insiders' signals, ρ, can take two values under a certain condition.

Lemma 1 *The coefficient of correlation between $\tilde{s}_1$ and $\tilde{s}_2$ is given by,*

$$\rho = \begin{cases} \rho_1\rho_2 & \textit{if the errors are independents.} \\ \rho_1\rho_2 + \sqrt{(1-\rho_1^2)}\sqrt{(1-\rho_2^2)} & \textit{if the errors' covariance matrix, is singular.} \\ \rho_1\rho_2 \pm \sqrt{\frac{\sigma_{\varepsilon_2}^2 - \sigma_\eta^2}{\sigma_{\varepsilon_2}^2}}\sqrt{(1-\rho_1^2)}\sqrt{(1-\rho_2^2)} & \textit{if the errors' covariance matrix, is} \\ & \textit{not singular.} \end{cases}$$

Proof: See the Appendix A.

Lemma 1 is the key result for the linear equilibrium uniqueness. Indeed, almost all the research papers which studied the static model of [11] and its

[6] σ_η^2 will be defined and explained in the proof of Lemma 1.

extensions under the assumption of normal distribution, obtained a unique linear equilibrium [7]. In these papers, one main reason for the uniqueness, is the relation between the insiders' signals that was quite specific. In other words, it was assumed that either the signals' errors were i.i.d., independents, or the coefficient of correlation between the signals' errors is given. Lemma 1 presents the novelty of our model and highlights the impact of the relation between the signals' errors on the uniqueness of the equilibrium. When the signals' errors are correlated (the coefficient of correlation between the insiders' signals errors ρ_ϵ is not given) with a non singular matrix, the coefficient of correlation between the insiders' signals, ρ, is not unique and more specifically, it takes two values.

Consequently, in the next proposition, we characterize the linear equilibrium outcomes which depend crucially on ρ and thus the linear equilibrium is not necessarily unique and each of the equilibrium outcomes that depends on ρ, takes also two values. It should be pointed out that many financial models using a times series panel structure, consider the case of cross sectional correlation of the errors. Despite the fact that our model discussed the one period case, it reveals the significance of considering the errors correlation and how it may impact the financial decisions.

Proposition 1 *In the Kyle type model with two insiders and partial correlation, a linear equilibrium exists and it is characterized by,*

$$a_1 = \frac{\sigma_z(2\rho_1 - \rho\rho_2)}{\lambda\sigma_1(4-\rho^2)} \quad \text{and} \quad a_0 = -\frac{\sigma_z\bar{s}_1(2\rho_1 - \rho\rho_2)}{\lambda\sigma_1(4-\rho^2)} \tag{1}$$

$$b_1 = \frac{\sigma_z(2\rho_2 - \rho\rho_1)}{\lambda\sigma_2(4-\rho^2)} \quad \text{and } b_0 = -\frac{\sigma_z\bar{s}_2(2\rho_2 - \rho\rho_1)}{\lambda\sigma_2(4-\rho^2)} \tag{2}$$

$$\lambda = \frac{\sigma_z\sqrt{(4+\rho^2)\left(\rho_1^2+\rho_2^2\right) - 8\rho\rho_1\rho_2}}{\sigma_u\left(4-\rho^2\right)}$$

$$R^2 = \frac{2(\rho_1^2 - \rho_1\rho\rho_2 + \rho_2^2)}{4-\rho^2}$$

$$E[\pi_1] = \frac{\sigma_z^2(2\rho_1 - \rho\rho_2)^2}{\lambda(4-\rho^2)^2} \quad \text{and } E[\pi_2] = \frac{\sigma_z^2(2\rho_2 - \rho\rho_1)^2}{\lambda(4-\rho^2)^2}$$

[7] (e.g., See [1, 5, 12, 14]).

Proof: See Appendix B.

Discussion of the equilibrium: First, it should be pointed out that Proposition 1 characterizes the equilibrium outcomes as functions of the three coefficients of correlation, ρ, ρ_1 and ρ_2. In the case of independent, i.i.d. errors or in the case of singular errors' covariance matrix, Proposition 1 provides a unique expression of the equilibrium outcomes. Hence, the comparative statics of this model and the one period Kyle type models, become feasible.

Second, our model generalizes the case studied in [18]. Indeed, in [18], a Kyle model with finite number of insiders was considered, and where some of the insiders were perfectly informed about the underlying value of the stock, while the remaining insiders observed a specific signal about the underlying value. Hence, another aim of this paper is to further examine the effect of partial information correlation on equilibrium outcomes. In Lemma 2, we highlight the relation between our model and [18]. We point out that in [18], the authors assumed that all the signals' noises of the partially informed traders are i.i.d.

Lemma 2 *The two insiders' cases studied in Tighe and Michener (1994) correspond to the following values of ρ, ρ_1 and ρ_2 of our model. Specifically,*

a- The two insiders' case with perfect information of [18]($m = 2$ and $n = 0$) corresponds to our model case when $\rho = \rho_1 = \rho_2 = 1$.

b- The two insiders' case of [18] with one perfectly informed insider and one partially informed insider ($m = 1$ and $n = 1$) corresponds to our model case when $\rho_1 = 1$ and $\rho = \rho_2$.

c- The two insiders' case with two partially informed insiders of [18]($m = 0$ and $n = 2$) corresponds to our model case when $\rho_1 = \rho_2$ and $\rho = \rho_1^2 = \rho_2^2$

Proof: Recall that the partial correlation $\rho_{12.z}$ is exactly the net correlation between $\tilde{s}_1$ and $\tilde{s}_2$ after removing the effect of $\tilde{z}$. is given by

$$\rho_{12.z} = \frac{\rho - \rho_1\rho_2}{\sqrt{1-\rho_1^2}\sqrt{1-\rho_2^2}}.$$

Note that in Tighe and Michener (1994), each partially informed insider i, observes a signal of the form $\tilde{s}_i = \tilde{z} + \tilde{\varepsilon}_i$ where the $(\varepsilon_i)_{i=1,2}$ are i.i.d. In this case, $\rho_{12.z} = 0$ leading to the equality $\rho = \rho_1 \times \rho_2$. Hence, if one of the insiders, insider 1 for example, is perfectly informed, then $\rho_1 = 1$ and thus we

Table 1: Equilibrium Outcomes when $\sigma_z^2 = \sigma_u^2 = 1$

$\rho_1 = 0.5$, $\rho_2 = 0.75$, $\sigma_\eta^2 = 1$ and $\sigma_{\varepsilon_2}^2 = 1.25$	ρ	R^2	λ	$E[\pi_1]$	$E[\pi_2]$
Independents Errors	0.375	0.3481	0.3877	0.0894	0.2983
Singular Covariance Matrix	0.9478	0.2947	0.3437	0.02528	0.3184
Non Singular Covariance Matrix: Positive Sign Case	0.631	0.3197	0.3599	0.0594	0.3004
Non Singular Covariance Matrix: Negative Sign Case	0.1188	0.3853	0.4276	0.1221	0.3055

obtain parts a) and b). For part c), note that in the case of the two partially informed insiders of [18], $\rho_1 = \rho_2$, which completes the proof. ∎

Thirdly, in Table 1, we provide a numerical example in which we present the equilibrium outcomes under the different cases mentioned in Lemma 1. For the ease of presentation and without loss of generality, we consider the case where $\sigma_z^2 = \sigma_u^2 = 1$. The example in Table 1 shows that a linear equilibrium is not necessarily unique when the errors' covariance matrix is not singular. As stated in Lemma 1, when the errors' covariance matrix is not singular, the expression of ρ takes two values depending on whether the sign in middle of the expression, is positive or negative . Hence, in the last two rows of Table 1, we denote "Positive sign Case" ("Negative sign Case") to refer to the positive sign expression of ρ (negative sign expression of ρ). Consequently, all the dependent equilibrium outcomes, also take two values each and thus a linear equilibrium is no longer unique.

Finally, it should be pointed out that the combined assumptions of our model are crucial in characterizing Proposition 1 expressions. Indeed, many research papers which studied the existence and uniqueness of [11] type model, either considered the case of a single or multiple insider(s), with perfect observation of the stock value (see, [14, 15, 16]). Moreover, our model, similar to [12] model, studied [11] in a complex environment with partially informed traders.

3 Comparative Statics

In this section, we run a comparative statics analysis for three special cases. The first case was studied by [18].[8] In this case, the errors terms in the

[8] We compare our results to the results of part c) in Lemma 2.

signals are i.i.d. The second case corresponds to the case in which the assumption of i.i.d. errors is relaxed, and it is replaced by the assumption of having independent errors. The third case is the case where the errors are dependent. Note that in all these three cases, the linear equilibrium is unique (see Lemma 1). Hence, the comparative statics analysis is fundamental to understand the impact of information correlation on equilibrium outcomes when the equilibrium is unique.

In the following Proposition, we present the equilibrium outcomes in these three special cases. We will use the subscript "TM" to denote the case of the [18]. The subscript "I" will be used for the independent and the subscript "D" denotes the case of dependent errors.

Proposition 2 *When the signals' errors are i.i.d., independent or having a singular covariance matrix, we have,*[9]

1-

$$R^2_{TM} = \frac{2\rho_1^2}{2+\rho_1^2}, \ \lambda_{TM} = \frac{\sigma_z\sqrt{2}\rho_1}{\sigma_u(2+\rho_1^2)}, \ E[\pi_1]^{TM} = E[\pi_2]^{TM} = \frac{\sigma_z\sigma_u\rho_1}{\sqrt{2}(2+\rho_1^2)}.$$

2-

$$R^2_I = \frac{2(\rho_1^2 - \rho_1^2\rho_2^2 + \rho_2^2)}{4-\rho_1^2\rho_2^2}, \quad \lambda_I = \frac{\sigma_z\sqrt{(4+\rho_1^2\rho_2^2)(\rho_1^2+\rho_2^2) - 8\rho_1^2\rho_2^2}}{\sigma_u(4-\rho_1^2\rho_2^2)}$$

$$E[\pi_1]^I = \frac{\sigma_z^2(2\rho_1 - \rho_1\rho_2^2)^2}{\lambda_I(4-\rho_1^2\rho_2^2)^2} \quad \text{and } E[\pi_2]^I = \frac{\sigma_z^2(2\rho_2 - \rho_1^2\rho_2)^2}{\lambda_I(4-\rho_1^2\rho_2^2)^2}$$

3-

$$R^2_D = \frac{2(\rho_1^2 - \rho_1\rho_D\rho_2 + \rho_2^2)}{4-\rho_D^2}, \quad \lambda_D = \frac{\sigma_z\sqrt{(4+\rho_D^2)(\rho_1^2+\rho_2^2) - 8\rho_D\rho_1\rho_2}}{\sigma_u(4-\rho_D^2)}$$

$$E[\pi_1]^D = \frac{\sigma_z^2(2\rho_1 - \rho_D\rho_2)^2}{\lambda_D(4-\rho_D^2)^2} \quad \text{and } E[\pi_2]^D = \frac{\sigma_z^2(2\rho_2 - \rho_D\rho_1)^2}{\lambda_D(4-\rho_D^2)^2}$$

where $\rho_D = \rho_1\rho_2 + \sqrt{(1-\rho_1^2)}\sqrt{(1-\rho_2^2)}$

[9] Recall that $R^2_{TM} = \frac{2\tau}{3\tau+2}$ ($m=0$ and $n=2$) where $\tau = \frac{\sigma_z^2}{\sigma_\varepsilon^2}$. Since $\rho_1^2 = \frac{\sigma_z^2}{\sigma_z^2+\sigma_\varepsilon^2} = \frac{\tau}{\tau+1}$ or equivalently $\tau = \frac{\rho_1^2}{1-\rho_1^2}$, we obtain the expression of R^2_{TM}. Moreover, for $m=0$ and $n=2$, we have $\lambda_{TM} = \frac{\sigma_z\sqrt{2\sigma_z^2\sigma_\varepsilon^2+2\sigma_z^4}}{\sigma_u(2\sigma_\varepsilon^2+3\sigma_z^2)} = \frac{\sigma_z\sqrt{\frac{2\sigma_z^2\sigma_\varepsilon^2+2\sigma_z^4}{(\sigma_\varepsilon^2+\sigma_z^2)^2}}}{\sigma_u\frac{2\sigma_\varepsilon^2+3\sigma_z^2}{\sigma_\varepsilon^2+\sigma_z^2}} = \frac{\sigma_z\sqrt{\frac{2\sigma_z^2(\sigma_\varepsilon^2+\sigma_z^2)}{(\sigma_\varepsilon^2+\sigma_z^2)^2}}}{\sigma_u\frac{2\sigma_\varepsilon^2+2\sigma_z^2+\sigma_z^2}{\sigma_\varepsilon^2+\sigma_z^2}} = \frac{\sigma_z\sqrt{\frac{2\sigma_z^2}{\sigma_\varepsilon^2+\sigma_z^2}}}{\sigma_u\frac{2(\sigma_\varepsilon^2+\sigma_z^2)+\sigma_z^2}{\sigma_\varepsilon^2+\sigma_z^2}}$. Since $\rho_1^2 = \frac{\sigma_z^2}{\sigma_z^2+\sigma_\varepsilon^2}$ we obtain $\lambda_{TM} = \frac{\sigma_z\sqrt{2}\rho_1}{\sigma_u(2+\rho_1^2)}$. Finally, for $m=0$ and $n=2, E[\pi_1]^{TM} = E[\pi_2]^{TM} = \frac{\tau(1+\tau)\sigma_z\sigma_u}{(2+3\tau)\sqrt{2\tau^2+2\tau}}$. Substituting τ by its corresponding expression, $\tau = \frac{\rho_1^2}{1-\rho_1^2}$, we obtained the required expression of the insider's unconditional profits.

3.1 Information Revelation

We begin our analysis with the study of the price revelation of information. In Lemma 3 we compare the information revelation measures for the three above mentioned cases.

Lemma 3 *The relations between R^2_{TM}, R^2_I and R^2_D are given by*

$$\begin{cases} a) \;\; R^2_I < R^2_{TM} \quad \text{if} \quad \rho_1 < \rho_2 \\ \\ R^2_I > R^2_{TM} \quad \text{if} \quad \rho_1 > \rho_2 \\ \\ b) \;\; R^2_I > R^2_D \quad \text{if} \quad \rho_1 < \rho_2 \quad \text{and} \quad 2\rho_1 > \rho_2 \end{cases}$$

Proof: See Appendix C.

Lemma 3 sheds light on the effect of partial correlation on the price information revelation. First, note that, when $\rho_1 < \rho_2$, $(\rho_1 > \rho_2)$, the stock price in the case of independent signals' errors, reveals less (more) information than in the case of i.i.d. errors, $R^2_I < R^2_{TM}$, $(R^2_I < R^2_{TM})$. This result is consistent with the result found in [18] when we compare the price revelation measure in the case of two perfectly informed traders to the case when one trader is informed and the other is partially informed.[10] In other words, we show that in the presence of strategic competition between the insiders, the higher the correlation between the insider's signal and the stock value, the lower the information revelation of the stock price is.

It should be pointed out that the relation between the price revelation measures in our model (when the signals' errors are independent) and the model of [18] (when the signals' errors are i.i.d.) is quite straightforward since in both models, the independency property of the errors is satisfied. However, the relation between the price revelation measures in the case of independent errors and in the case of dependent errors with singular covariance matrix, becomes more complex and ambiguous. Indeed, in Lemma 3, we show that when $\rho_1 < \rho_2$ and $2\rho_1 > \rho_2$, the stock price reveals more information in the case of independent errors than in the case of singular covariance matrix of the signals'errors. But, outside this indicated range, the relation is quite ambiguous, as highlighted graphically in Figure 1. Specifically, we graph in Figure 1, $\Delta_R = R^2_L - R^2_I$, the difference of the two measures of two cases. The graph shows that such difference can be positive or negative for many

[10] See the expression of R^2_{TM} for the cases a) and b) of Lemma 2.

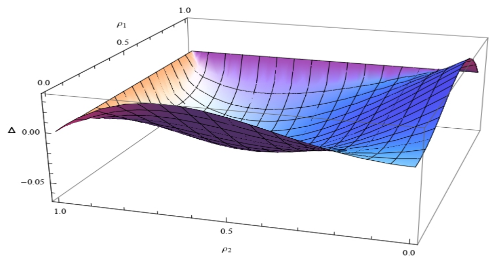

Figure 1: The graph of $\Delta_R = R_D^2 - R_I^2$ as function of the insiders' coefficients of correlation ρ_1 and ρ_2.

values of (ρ_1, ρ_2) outside the region defined by $\rho_1 < \rho_2$ and $2\rho_1 > \rho_2$. In other words, we show how the partial correlation structure, combined with the strategic competition of the insiders, affects positively or negatively the information revelation of the stock price.

3.2 Market Depth and Profits

In this section, we provide graphical and numerical analysis of the relation between the market depth's measures and the insiders' profits in the three particular models. The common feature in these analysis is the ambiguous impact of partial correlation on these outcomes. In Figure 2, we show the relation between the market depths' parameters λ in the case of independent errors and the case when the covariance of the signals' errors is singular. Figure 2 reveals the existence of regions for ρ_1 and ρ_2 in which the difference between the market depth's parameters is positive. Similarly, there are regions for ρ_1 and ρ_2 regions in which the difference between the market depth's parameters is negative. Numerically speaking, if $\rho_1 = 0.1$ and $\rho_2 = 0.3$ we find that $\lambda_I = 0.1567$ and $\lambda_D = 0.1679$ which leads to a negative difference. However, for $\rho_1 = 0.1$ and $\rho_2 = 0.2$ we find that $\lambda_I = 0.1109$ and $\lambda_D = 0.0988$ which leads to a positive difference. In other words, the impact of the correlation of the market depth is ambiguous.

Moreover, comparing the market depth parameters for the i.i.d errors and the independent errors, is not straightforward. Indeed, similar to the case

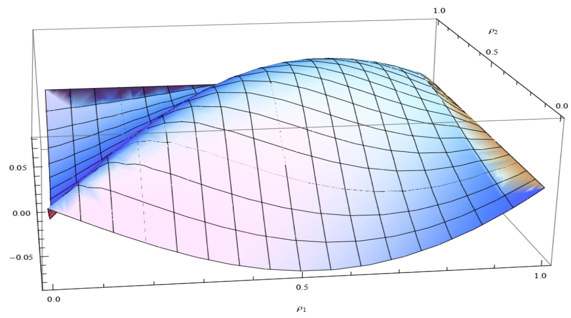

Figure 2: The graph of $\Delta_\lambda = \lambda_I - \lambda_D$ as function of the insiders' coefficients of correlation ρ_1 and ρ_2.

studied above, we found regions of ρ_1 and ρ_2 regions in which the difference between the market depth's parameters is either positive or negative.

On the other hand, the relation between the insiders' profits between the three models is quite similar to the relation of the market depth parameters. Specifically, we find graphically and numerically regions for ρ_1 and ρ_2 in which the difference between the insiders' profits is either positive or negative. Moreover, proposition 2 shows that in the case of independent errors or in the case of singular covariance matrix of the errors, the profits of insider 1 is greater (less) than the profits of insider 2 if $\rho_1 > \rho_2$ ($\rho_1 < \rho_2$).

Consequently, this model provides a fundamental result on the impact of private information on equilibrium outcomes. In other words, with this market microstructure setting, we can conclude that better private information of the insider does not necessarily lead to higher profits.

4 Conclusion

In this paper, we provide a necessary and sufficient conditions for the existence and uniqueness of linear equilibria in a static model of [11] and in the case of two risk neutral insiders, each of whom, observes a correlated signal with the value of the stock. As we have explained, the assumption that the insiders' signal errors are either i.i.d. or independent, which is the key for the linear equilibrium uniqueness, is realistic and well used in the field of market microstructure. However, in the case of correlated signals errors, a linear equilibrium is not necessarily unique. This assumption is more appropriate for the case of security trading with multiple trading periods and multiple insiders. We leave this as an open question for future research.

Moreover, our analysis illustrates the impact and the important role that partial correlation between the insiders' signals, has on the insiders' decisions and the equilibrium variables. In particular, we showed that the relation between the insiders' unconditional profits is quite ambiguous and depends crucially on the degree of correlation between the insiders' signals. Therefore, understanding the role of partial correlation and its implications to the regulatory aspects in the world of insider trading should be examined more closely.

Declarations

Author contribution statement

Wassim Daher: Conceived and designed the analysis; Analyzed and interpreted the data; Contributed analysis tools or data; Wrote the paper.

Elias G. Saleeby: Conceived and designed the analysis; Analyzed and interpreted the data; Contributed analysis tools or data; Wrote the paper.

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Appendices

Appendix A: proof of Lemma 1

First, we consider the case in which the error's covariance matrix is not singular. Recall that the variables $\tilde{z}$ and $\tilde{s}_1$ are jointly normally distributed. Thus, we have,

$$\tilde{s}_1 = \alpha_1 + \beta_1 \tilde{z} + \tilde{\varepsilon}_1$$

where $\tilde{\varepsilon}_1 \sim N(0, \sigma^2_{\varepsilon_1})$ and $\tilde{z}$ and $\tilde{\varepsilon}_1$ are independent. Similarly, the variables $\tilde{z}$ and $\tilde{s}_2$ are jointly normally distributed. Thus, we have,

$$\tilde{s}_2 = \alpha_2 + \beta_2 \tilde{z} + \tilde{\varepsilon}_2$$

where $\tilde{\varepsilon}_2 \sim N(0, \sigma^2_{\varepsilon_2})$ and $\tilde{z}$ and $\tilde{\varepsilon}_2$ are independent.

Since the errors are normally jointly distributed with a non singular covariance matrix, we can assume that $\tilde{\varepsilon}_2 = h\tilde{\varepsilon}_1 + \tilde{\eta}$, where h is a constant and $\tilde{\eta}$ is a noise term $N\left(0, \sigma^2_{\eta}\right)$. Assume also that $\tilde{\eta}$ is not correlated with $\tilde{\varepsilon}_1$, that is, $Cov\left(\tilde{\varepsilon}_1, \tilde{\eta}\right) = 0$.

Recall that the partial correlation between $\tilde{s}_1$ and $\tilde{s}_2$ net of the effect of $\tilde{z}$, is defined by

$$\rho_{12.z} = \frac{Cov\left(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2\right)}{\sigma_{\varepsilon_1}\sigma_{\varepsilon_2}}$$

For convenience set $\rho_{12.z} = \rho_\varepsilon$, i.e. $\rho_\varepsilon = \frac{Cov(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2)}{\sigma_{\varepsilon_1}\sigma_{\varepsilon_2}} = \frac{Cov(\tilde{\varepsilon}_1, h\tilde{\varepsilon}_1 + \tilde{\eta})}{\sigma_{\varepsilon_1}\sigma_{\varepsilon_2}} = \frac{h\sigma_{\varepsilon_1}}{\sigma_{\varepsilon_2}}$, as $Cov\left(\tilde{\varepsilon}_1, h\tilde{\varepsilon}_1 + \tilde{\eta}\right) = hVar\left(\tilde{\varepsilon}_1\right) = h\sigma^2_{\varepsilon_1}$. Hence,

$$h = \rho_\varepsilon \frac{\sigma_{\varepsilon_2}}{\sigma_{\varepsilon_1}} \tag{3}$$

On the other hand, $\tilde{\varepsilon}_2 = h\tilde{\varepsilon}_1 + \tilde{\eta}$, then,

$$Var\left(\tilde{\varepsilon}_2\right) = h^2 Var\left(\tilde{\varepsilon}_1\right) + Var\left(\tilde{\eta}\right) = h^2\sigma^2_{\varepsilon_1} + \sigma^2_\eta. \tag{4}$$

Combining (3) and (4), we obtain,

$$\rho_\varepsilon = \pm\sqrt{\frac{\sigma^2_{\varepsilon_2} - \sigma^2_\eta}{\sigma^2_{\varepsilon_2}}}$$

By the equality $\rho_{12.z} = \rho_\varepsilon$, we obtain that[11]

$$\frac{\rho - \rho_1\rho_2}{\sqrt{1-\rho_1^2}\sqrt{1-\rho_2^2}} = \pm\sqrt{\frac{\sigma^2_{\varepsilon_2} - \sigma^2_\eta}{\sigma^2_{\varepsilon_2}}};$$

which gives us

$$\rho = \rho_1\rho_2 \pm \sqrt{\frac{\sigma^2_{\varepsilon_2} - \sigma^2_\eta}{\sigma^2_{\varepsilon_2}}}\sqrt{(1-\rho_1^2)}\sqrt{(1-\rho_2^2)}.$$

Secondly, we consider the case in which the errors are independents. In this case, $\rho_{12.z} = \rho_\varepsilon = 0$. Consequently, we obtain $\rho = \rho_1\rho_2$.

Finally, we consider the case in which the errors' covariance matrix is singular. In this case, there exist a normal random variable $\tilde{\varepsilon} \sim N(0, \sigma^2_\varepsilon)$ and two constants k_1 and k_2 such that $\tilde{\varepsilon}_1 = k_1\tilde{\varepsilon}$ and $\tilde{\varepsilon}_2 = k_2\tilde{\varepsilon}$. Hence, the insiders' signals can be written as

$$\tilde{s}_1 = \alpha_1 + \beta_1\tilde{z} + \tilde{\varepsilon}_1 = \alpha_1 + \beta_1\tilde{z} + k_1\tilde{\varepsilon} \;\; and \; \tilde{s}_2 = \alpha_2 + \beta_2\tilde{z} + \tilde{\varepsilon}_2 = \alpha_2 + \beta_2\tilde{z} + k_2\tilde{\varepsilon}$$

Consequently, the expressions of the coefficients of correlation ρ_1, ρ_2 and ρ, become

$$\rho_1 = \frac{Cov(\tilde{s}_1, \tilde{z})}{\sigma_{s_1}\sigma_z} = \frac{Cov(\alpha_1 + \beta_1\tilde{z} + k_1\tilde{\varepsilon}, \tilde{z})}{\sigma_{s_1}\sigma_z} = \frac{\beta_1\sigma_z}{\sigma_{s_1}} = \frac{\beta_1\sigma_z}{\sqrt{\beta_1^2\sigma_z^2 + k_1^2\sigma^2_\varepsilon}}.$$

[11] The left side expression is simply the expression of the partial correlation between $\tilde{s}_1$ and $\tilde{s}_2$ in terms of the given coefficients of correlations. See [13].

$$\rho_2 = \frac{Cov(\tilde{s_2}, \tilde{z})}{\sigma_{s_2}\sigma_z} = \frac{Cov(\alpha_2 + \beta_2\tilde{z} + k_2\tilde{\varepsilon}, \tilde{z})}{\sigma_{s_2}\sigma_z} = \frac{\beta_2\sigma_z}{\sigma_{s_2}} = \frac{\beta_2\sigma_z}{\sqrt{\beta_2^2\sigma_z^2 + k_2^2\sigma_\varepsilon^2}}.$$

and

$$\rho = \frac{Cov(\tilde{s_1}, \tilde{s_2})}{\sigma_{s_1}\sigma_{s_2}} = \frac{Cov(\alpha_1 + \beta_1\tilde{z} + k_1\tilde{\varepsilon}, \alpha_2 + \beta_2\tilde{z} + k_2\tilde{\varepsilon})}{\sigma_{s_1}\sigma_{s_2}} = \frac{\beta_1\beta_2\sigma_z^2 + k_1k_2\sigma_\varepsilon^2}{\sqrt{\beta_1^2\sigma_z^2 + k_1^2\sigma_\varepsilon^2}\sqrt{\beta_2^2\sigma_z^2 + k_2^2\sigma_\varepsilon^2}}$$

Applying simple algebra manipulations to the expression of ρ, we obtain,

$$\rho = \rho_1\rho_2 + \sqrt{(1-\rho_1^2)}\sqrt{(1-\rho_2^2)}$$

■

Appendix B: proof of Proposition 1

Insider 1 solves

$$\underset{\tilde{x}_1}{\text{Max}}\, E[(\tilde{z} - p(\tilde{r}))\tilde{x}_1|\tilde{s}_1] = \underset{\tilde{x}_1}{\text{Max}}\, E[(\tilde{z} - \mu - \lambda(\tilde{x}_1 + \tilde{x}_2 + \tilde{u}))\tilde{x}_1|\tilde{s}_1]$$

Taking into account that insider 2's strategy is of the form $b_0 + b_1s_2$, insider 1 problem becomes

$$\underset{\tilde{x}_1}{\text{Max}}\, E[(\tilde{z} - \mu - \lambda\tilde{x}_1 - \lambda(b_1\tilde{s}_2 + b_0) - \lambda\tilde{u}))\tilde{x}_1|\tilde{s}_1]$$

The F.O.C implies that

$$\tilde{x}_1(\tilde{s}_1) = \frac{E[\tilde{z}|\tilde{s}_1] - \mu - \lambda b_0 - \lambda b_1 E[\tilde{s}_2|\tilde{s}_1]}{2\lambda} \tag{5}$$

Recall that $E[\tilde{z}|\tilde{s}_1] = \bar{z} + \frac{\rho_1\sigma_z}{\sigma_1}(\tilde{s}_1 - \bar{s}_1)$ and $E[\tilde{s}_2|\tilde{s}_1] = \bar{s}_2 + \frac{\rho\sigma_2}{\sigma_1}(\tilde{s}_1 - \bar{s}_1)$. Hence, equation (5) becomes

$$\tilde{x}_1(\tilde{s}_1) = \frac{\bar{z} - \mu - \lambda b_0 - \lambda b_1\bar{s}_2}{2\lambda} + \frac{-\bar{s}_1(\rho_1\sigma_z - \lambda b_1\rho\sigma_2)}{2\lambda\sigma_1} + \frac{(\rho_1\sigma_2 - \lambda b_1\rho\sigma_2)}{2\lambda\sigma_1}\tilde{s}_1$$

Thus, we get

$$a_1 = \frac{(\rho_1\sigma_2 - \lambda b_1\rho\sigma_2)}{2\lambda\sigma_1} \quad \text{and} \quad a_0 = \frac{\bar{z} - \mu - \lambda b_0 - \lambda b_1\bar{s}_2}{2\lambda} + \frac{-\bar{s}_1(\rho_1\sigma_z - \lambda b_1\rho\sigma_2)}{2\lambda\sigma_1} \quad (6)$$

We turn now to insider 2's problem. She solves

$$\underset{\tilde{x}_2}{\text{Max}}\, E[(\tilde{z} - p(\tilde{r}))\tilde{x}_2|\tilde{s}_2] = \underset{\tilde{x}_2}{\text{Max}}\, E[(\tilde{z} - \mu - \lambda(\tilde{x}_1 + \tilde{x}_2 + \tilde{u}))\tilde{x}_2|\tilde{s}_2]$$

Taking into account that insider 1's strategy is of the form $a_0 + a_1s_1$, insider 2 problem becomes

$$\underset{\tilde{x}_2}{\text{Max}}\, E[(\tilde{z} - \mu - \lambda(a_0 + a_1\tilde{s}_1) - \lambda\tilde{x}_2 - \lambda\tilde{u})\tilde{x}_2|\tilde{s}_2]$$

The F.O.C implies that

$$\tilde{x}_2(\tilde{s}_2) = \frac{E[\tilde{z}|\tilde{s}_2] - \mu - \lambda a_0 - \lambda a_1 E[\tilde{s}_1|\tilde{s}_2]}{2\lambda} \quad (7)$$

Recall that $E[\tilde{z}|\tilde{s}_2] = \bar{z} + \frac{\rho_2\sigma_z}{\sigma_2}(\tilde{s}_2 - \bar{s}_2)$ and $E[\tilde{s}_1|\tilde{s}_2] = \bar{s}_1 + \frac{\rho\sigma_1}{\sigma_2}(\tilde{s}_2 - \bar{s}_2)$. Hence, equation (7) becomes

$$\tilde{x}_2(\tilde{s}_2) = \frac{\bar{z} - \mu - \lambda a_0 - \lambda a_1\bar{s}_1}{2\lambda} + \frac{-\bar{s}_2(\rho_2\sigma_z - \lambda a_1\rho\sigma_1)}{2\lambda\sigma_2} + \frac{(\rho_2\sigma_2 - \lambda a_1\rho\sigma_1)}{2\lambda\sigma_2}\tilde{s}_2$$

Thus, we get

$$b_1 = \frac{\bar{z} - \mu - \lambda a_0 - \lambda a_1\bar{s}_1}{2\lambda} \quad \text{and} \quad b_0 = \frac{-\bar{s}_2(\rho_2\sigma_z - \lambda a_1\rho\sigma_1)}{2\lambda\sigma_2} + \frac{(\rho_2\sigma_2 - \lambda a_1\rho\sigma_1)}{2\lambda\sigma_2} \quad (8)$$

Combining equations (6) and (8), we obtain

$$a_1 = \frac{\sigma_z(2\rho_1 - \rho\rho_2)}{\lambda\sigma_1(4 - \rho^2)} \quad \text{and} \quad a_0 = -\frac{\sigma_z\bar{s}_1(2\rho_1 - \rho\rho_2)}{\lambda\sigma_1(4 - \rho^2)} + \frac{\bar{z} - \mu}{3\lambda} \quad (9)$$

$$b_1 = \frac{\sigma_z(2\rho_2 - \rho\rho_1)}{\lambda\sigma_2(4 - \rho^2)} \quad \text{and}\ b_0 = -\frac{\sigma_z\bar{s}_2(2\rho_2 - \rho\rho_1)}{\lambda\sigma_2(4 - \rho^2)} + \frac{\bar{z} - \mu}{3\lambda} \quad (10)$$

We turn now to find the expressions of μ and λ. First, recall that the market efficiency condition together with the price linearity imply that

$$p(\tilde{r}) = E[\tilde{z}|\tilde{r}] = \mu + \lambda\tilde{r}$$

taking the expectation on both sides of the above equation we obtain,

$$\bar{z} = \mu + \lambda[\bar{x}_1 + \bar{x}_2] = \mu + [a_0 + b_0 + a_1\bar{s}_1 + b_1\bar{s}_2]$$

Substituting the expressions of a_0, a_1, b_0 and b_1 in (9) and (10), we find that $\mu = \bar{z}$. Consequently, the expressions of a_0 and b_0 become

$$a_0 = -\frac{\sigma_z\bar{s}_1(2\rho_1 - \rho\rho_2)}{\lambda\sigma_1(4-\rho^2)} \text{ and } b_0 = -\frac{\sigma_z\bar{s}_2(2\rho_2 - \rho\rho_1)}{\lambda\sigma_2(4-\rho^2)}$$

Hence, we obtain (1) and (2). It remains to find the value of λ. Since the orders of the insiders and the liquidity traders are normally distributed, applying the projection theorem for normal random variables, we have

$$\lambda = \frac{Cov(\tilde{z},\tilde{r})}{Var(\tilde{r})} = \frac{a_1\rho_1\sigma_z\sigma_1 + b_1\rho_2\sigma_z\sigma_2}{a_1^2\sigma_1^2 + b_1^2\sigma_2^2 + 2a_1b_1\rho\sigma_1\sigma_2 + \sigma_u^2} \tag{11}$$

Combining equations (1), (2), and (11) we obtain,

$$\lambda = \frac{\sigma_z\sqrt{4\rho_1^2 + 4\rho_2^2 + \rho^2\rho_1^2 + \rho^2\rho_2^2 - 8\rho\rho_1\rho_2}}{\sigma_u(4-\rho^2)}$$

or equivalently

$$\lambda = \frac{\sigma_z\sqrt{(4+\rho^2)(\rho_1^2+\rho_2^2) - 8\rho\rho_1\rho_2}}{\sigma_u(4-\rho^2)}$$

We turn now to compute the coefficient of determination R^2 to measure the amount of information contained in the order flow.

$$R^2 = \frac{Cov(\tilde{z},\tilde{r})^2}{Var(\tilde{z})Var(\tilde{r})} = \frac{(a_1\rho_1\sigma_z\sigma_1 + b_1\rho_2\sigma_z\sigma_2)^2}{\sigma_z^2(a_1^2\sigma_1^2 + b_1^2\sigma_2^2 + 2a_1b_1\rho\sigma_1\sigma_2 + \sigma_u^2)}$$

After simplification we obtain

$$R^2 = \frac{2(\rho_1^2 - \rho_1\rho\rho_2 + \rho_2^2)}{4-\rho^2}$$

Finally, note that the order of each insider can be written as $\tilde{x}_1(\tilde{s}_1) = a_1(\tilde{s}_1 - \bar{s}_1)$ and $\tilde{x}_2(\tilde{s}_2) = b_1(\tilde{s}_2 - \bar{s}_2)$. The conditional expected profits of insider 1

$$
\begin{aligned}
E[(\tilde{z} - p(\tilde{r}))\tilde{x}_1|\tilde{s}_1] &= E[(\tilde{z} - \mu - \lambda(\tilde{x}_1 + \tilde{x}_2 + \tilde{u}))\tilde{x}_1|\tilde{s}_1] \\
&= (E[(\tilde{z}|\tilde{s}_1] - \mu - \lambda\tilde{x}_1 - \lambda E[\tilde{x}_2|\tilde{s}_1])\tilde{x}_1 \\
&= (E[(\tilde{z}|\tilde{s}_1] - \bar{z} - \lambda a_1(\tilde{s}_1 - \bar{s}_1) - \lambda E[b_1(\tilde{s}_2 - \bar{s}_2)|\tilde{s}_1])(a_1(\tilde{s}_1 - \bar{s}_1)) \\
&= (\bar{z} + \frac{\rho_1\sigma_z}{\sigma_1}(\tilde{s}_1 - \bar{s}_1) - \bar{z} - \lambda a_1(\tilde{s}_1 - \bar{s}_1) - \lambda b_1 E[\tilde{s}_2 - \bar{s}_2)|\tilde{s}_1])(a_1(\tilde{s}_1 - \bar{s}_1)) \\
&= (\frac{\rho_1\sigma_z}{\sigma_1}(\tilde{s}_1 - \bar{s}_1) - \lambda a_1(\tilde{s}_1 - \bar{s}_1) - \lambda b_1(\frac{\rho\sigma_2}{\sigma_1}(\tilde{s}_1 - \bar{s}_1))(a_1(\tilde{s}_1 - \bar{s}_1)) \\
&= a_1(\tilde{s}_1 - \bar{s}_1)^2\left[\frac{\rho_1\sigma_z}{\sigma_1} - \lambda a_1 - \lambda b_1\frac{\rho\sigma_2}{\sigma_1}\right] \\
&= a_1(\tilde{s}_1 - \bar{s}_1)^2\left[\frac{\rho_1\sigma_z}{\sigma_1} - \frac{\sigma_z(2\rho_1 - \rho\rho_2)}{\sigma_1(4-\rho^2)} - \frac{\sigma_z(2\rho_2 - \rho\rho_1)}{\sigma_2(4-\rho^2)}\frac{\rho\sigma_2}{\sigma_1}\right] \\
&= a_1(\tilde{s}_1 - \bar{s}_1)^2\sigma_z\left[\frac{\rho_1}{\sigma_1} - \frac{(4-\rho^2)}{\sigma_1(4-\rho^2)} - \frac{\rho(2\rho_2 - \rho\rho_1)}{\sigma_1(4-\rho^2)}\right] \\
&= a_1(\tilde{s}_1 - \bar{s}_1)^2\sigma_z\left[\frac{\rho_1(4-\rho^2) - (2\rho_2 - \rho\rho_1) - \rho(2\rho_2 - \rho\rho_1)}{\sigma_1(4-\rho^2)}\right] \\
&= a_1(\tilde{s}_1 - \bar{s}_1)^2\frac{\sigma_z(2\rho_1 - \rho\rho_2)}{\sigma_1(4-\rho^2)} = \frac{\sigma_z^2(2\rho_1 - \rho\rho_2)^2}{\lambda\sigma_1^2(4-\rho^2)^2}(\tilde{s}_1 - \bar{s}_1)^2
\end{aligned}
$$

Computing the unconditional expected profits for insider 1, we obtain

$$
\begin{aligned}
E[\pi_1] &= E[\frac{\sigma_z^2(2\rho_1 - \rho\rho_2)^2}{\lambda\sigma_1^2(4-\rho^2)^2}(\tilde{s}_1 - \bar{s}_1)^2] \\
&= \frac{\sigma_z^2(2\rho_1 - \rho\rho_2)^2}{\lambda\sigma_1^2(4-\rho^2)^2}E[(\tilde{s}_1 - \bar{s}_1)^2] = \frac{\sigma_z^2(2\rho_1 - \rho\rho_2)^2}{\lambda\sigma_1^2(4-\rho^2)}\sigma_1^2 \\
E[\pi_1] &= \frac{\sigma_z^2(2\rho_1 - \rho\rho_2)^2}{\lambda(4-\rho^2)^2}
\end{aligned}
$$

Similarly, we obtain for insider 2,

$$
E[\pi_2|\tilde{s}_2] = \frac{\sigma_z^2(2\rho_2 - \rho\rho_1)^2}{\lambda\sigma_2^2(4-\rho^2)^2}(\tilde{s}_2 - \bar{s}_2)^2 \quad \text{and} \quad E[\pi_2] = \frac{\sigma_z^2(2\rho_2 - \rho\rho_1)^2}{\lambda(4-\rho^2)^2}
$$

■

Appendix C: proof of Lemma 3

We start by proving part a) of the Lemma. Let $\rho_2 = c\rho_1$ (we assume $\rho_1, \rho_2 \geq 0$,) and consider the two cases where $0 < c < 1$ or $c > 1$. Note that in this case, R_I^2 becomes

$$R_I^2 = \frac{2\left(\rho_1^2 - c^2\rho_1^4 + c^2\rho_1^2\right)}{4 - c^2\rho_1^4} = 2\rho_1^2 \frac{-1 + c^2\rho_1^2 - c^2}{-4 + c^2\rho_1^4}.$$

Then

$$\begin{aligned} R_I^2 - R_{TM}^2 &= 2\rho_1^2 \frac{-1 + c^2\rho_1^2 - c^2}{-4 + c^2\rho_1^4} - \frac{2\rho_1^2}{\rho_1^2 + 2} \\ &= 2\rho_1^2 \left(c - 1\right)\left(c + 1\right) \frac{\rho_1^2 - 2}{\left(c\rho_1^2 - 2\right)\left(c\rho_1^2 + 2\right)\left(\rho_1^2 + 2\right)}. \end{aligned}$$

Case 1: $0 < c < 1$

$$R_I^2 - R_{TM}^2 = \frac{2\rho_1^2\left(c + 1\right)\left(1 - c\right)\left(2 - \rho_1^2\right)}{\left(c\rho_1^2 + 2\right)\left(\rho_1^2 + 2\right)}\left(\frac{1}{c\rho_1^2 - 2}\right)$$

The sign $R_I^2 - R_{TM}^2$ depends on the last right term in the bracket which is obviously negative for $0 < c = \frac{\rho_2}{\rho_1} < 1$.

Case 2: $c > 1$

$$\begin{aligned} R_I^2 - R_{TM}^2 &= \frac{2\rho_1^2\left(c - 1\right)\left(c + 1\right)\left(2 - \rho_1^2\right)}{\left(c\rho_1^2 + 2\right)\left(\rho_1^2 + 2\right)}\left(\frac{1}{2 - c\rho_1^2}\right). \\ &= \frac{2\rho_1^2\left(c - 1\right)\left(c + 1\right)\left(2 - \rho_1^2\right)}{\left(c\rho_1^2 + 2\right)\left(\rho_1^2 + 2\right)}\left(\frac{1}{2 - \rho_1\rho_2}\right) \end{aligned}$$

Note that in this case $R_I^2 - R_{TM}^2$ is always negative since ρ_1 ρ_2 each is less than 1.

Proof of part b): we will show that R^2 is decreasing with respect to ρ when $\rho_1 < \rho_2$ and $2\rho_1 > \rho_2$, since $\rho_I = \rho_1\rho_2 < \rho_L = \rho_1\rho_2 + \sqrt{(1-\rho_1^2)(1-\rho_2^2)}$. Indeed, let $\rho = x, \rho_1 = y, \rho_2 = z$. Then $\frac{dR^2}{d\rho}$ is given by

$$2\frac{d\left(\frac{y^2 - xyz + z^2}{4 - x^2}\right)}{dx} = 2\frac{-4yz - yzx^2 + 2xy^2 + 2xz^2}{\left(-4 + x^2\right)^2}$$

The bottom is > 0. So look at the numerator

$$-4yz - yzx^2 + 2xy^2 + 2xz^2 = -yzx^2 + \left(2y^2 + 2z^2\right)x - 4yz \qquad (12)$$

Thus, we find $x = 2\frac{z}{y}$ and $x = 2\frac{y}{z}$ as critical points. Note that the graph of equation (12) is a parabola open down. In other words, the sign of (12) is negative outside the critical points and positive inside. Suppose that $0 < y < z$, then $\frac{y}{z} < \frac{z}{y}$. In this case, the critical point $x = 2\frac{z}{y} > 1$ which complete the proof.

$\blacksquare$