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A Bayesian Perspective on Commodity Style Integration

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Abstract

Commodity style integration is appealing because by forming a unique long-short portfolio with exposure to K mildly correlated factors, a larger and more stable risk premium can be extracted than with any of the standalone styles. A key decision that a commodity style-integration investor faces at each rebalancing time is the relative weighting of the factors. We propose a Bayesian optimized style-integration (BOI) strategy with excellent out-of-sample performance. Focusing on the problem of a commodity investor that seeks exposure to the carry, hedging pressure, momentum, skewness, and basis-momentum factors, the evidence suggests that the BOI portfolio achieves better Sharpe ratios and certainty equivalent returns, among other performance metrics, than the $1/K$ style-weighted integrated portfolio, and a battery of sophisticated optimized integrations. The findings survive the consideration of longer estimation windows, various commodity score schemes, and alternative Bayesian priors.

Keywords: Commodity risk premia; Style integration; Long-short portfolio; Parameter estimation risk; Bayesian portfolio optimization.

JEL classifications: G13, G14

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"Probability is an orderly opinion and inference from data is nothing other than the revision of such opinion in the light of relevant new information."– Eliezer S. Yudkowsky

1. Introduction

In line with the theory of storage (Kaldor, 1939; Working, 1949; Brennan, 1958) and the hedging pressure hypothesis (Cootner, 1960; Hirshleifer, 1988), there is predictive content for the cross-section of commodity futures returns in the roll-yield and the hedging pressure signals, respectively, both of which contain information about the inexorable backwardation and contango cycle.[1] Accordingly, a term-structure or carry risk premium can be extracted by taking long positions in commodity futures with the most downward sloping forward curves and simultaneous short positions in the commodity futures with the most upward sloping forward curves (Erb and Harvey, 2006; Gorton and Rouwenhorst, 2006; Szymanowska et al., 2014; Koijen et al., 2018). Likewise, a hedging pressure premium can be captured by longing the commodities with the largest hedgers' hedging pressure and shorting those with the smallest hedgers' hedging pressure (Dewally et al., 2013; Basu and Miffre, 2013).

A popular commodity investment style that has been linked not only with the backwardation/contango phases but also with mispricing and behavioural biases is the cross-sectional momentum or trend-following strategy which longs (shorts) the commodities with the best (worse) past performance (Miffre and Rallis, 2007; Fuertes et al., 2010; Asness et al., 2015). Echoing a large empirical literature on equities, evidence has been adduced also in support of a commodity skewness style. Investors that are simultaneously long and short in the commodities with the most negative and positive skew, respectively, can capture a premium

[1] The theory of storage contends that commodity futures prices are driven by inventory levels and hence, it associates a backwardated or downward-sloping futures curve with scarce inventories and a high convenience yield. The hedging pressure hypothesis states that there is a risk transfer mechanism from hedgers or commercial traders (consumers or producers of the commodity) to speculators and thus, the futures price is set low (high) relative to the expected future spot price when hedgers are net short (long) so as to attract net long (short) speculation. The rise (fall) in the futures price as maturity approaches is the premium or compensation received by speculators for absorbing the net hedging demand.

which has been rationalized in terms of investors' preferences for lottery-type payoffs (Fernandez-Perez et al., 2018). Yet another celebrated commodity style uses the basis-momentum signal to capture a risk premium that relates to imbalances in the supply and demand of futures contracts that materialize when the market-clearing ability of speculators and intermediaries is impaired (Boons and Prado, 2019). Albeit not exhaustive, this catalogue of commodity investment styles or factors is fairly representative of the recent empirical literature.

The performance of a given standalone style may temporarily weaken and later recover because of investor crowding effects (e.g., Kang et al., 2021) or because the way the factor is priced by the market changes over time (e.g., Bhattacharya et al., 2017). The upshot is that the performance of an individual style is likely to be time-varying. One way to alleviate this issue is through simultaneous exposure to multiple style or factors. The idea is simple: form a unique long-short style-integrated portfolio which, in essence, is the application of the old adage *don't put all your eggs in the same basket* to factor investing. The style-integrated portfolio benefits from factor diversification by capturing a sizeable risk premium in a fairly stable manner. [2]

In practice, a key decision that an investor faces at each portfolio rebalancing time t is the relative importance or weight $\omega_{1,t}, \ldots, \omega_{K,t}$ to give to the sorting signals that underlie the K individual styles. With historical excess returns on each of the styles, the investor can obtain the style-weights as solutions of an optimization problem formulated according to her chosen economic criteria (e.g., mean-variance utility maximization). However, these optimized style-integrations (OIs) are bedevilled by parameter estimation risk and, for this reason, they are no

[2] The third generation commodity indices, which are also long-short, enable investors to gain exposure to commodities without concerns about the rolling of contracts, margin calls, and posting collateral. However, these indices rely, by construction, on one or two signals at most (see e.g., Fethke and Prokopczuk, 2018). By contrast, the signal-mixing style-integration framework laid out in Fernandez-Perez et al. (2019) offers full flexibility as regards the number of different style premia to capture. Sakkas and Tessaromatis (2020) adopt instead the two-step (or bottom up) approach where standalone style long-short portfolios are formed first and then they are combined into a multi-factor portfolio.

better than the naïve equal-weighted style integration (EWI) as shown by Fernandez-Perez et al. (2019). Style-integration is thus a problem that remains open for Bayesian notions.

The present paper contributes to the style-integration literature in various directions. We put forward a novel Bayesian optimized style-integration (BOI) method that controls for parameter estimation risk. Specifically, we embed the mean-variance optimized style-integration (OI) problem within a Bayesian framework that treats the style-weights at each portfolio formation time as random variables. The essence of the BOI is that the investor establishes a prior distribution for the style-weights according to her beliefs or information, and this distribution is mapped onto a prior distribution for the expected commodity excess returns. The priors are updated with new evidence to obtain the posterior distributions of interest. We document empirically the superiority of the Bayesian style-integration vis-à-vis the challenging EWI and a battery of alternative OIs for a wide cross-section of commodity futures contracts.

The findings suggest that, by contrast with the traditional "sophisticated" OI portfolios, the BOI portfolio is able to extract a significantly larger commodity risk premia (net of trading costs) than the EWI portfolio benchmark in a fairly consistent manner. This finding is not challenged in robustness tests that use longer estimation windows to lessen the OI parameter estimation risk or consider alternative commodity score schemes and Bayesian priors.

Our paper complements a literature on combining multiple asset characteristics to inform optimal portfolio choice (e.g., Brandt et al., 2009; Barroso and Santa-Clara, 2015; Fernandez-Perez et al., 2019; De Miguel et al., 2020). In a comparison across style-integration methods, Fernandez-Perez et al. (2019) study the performance of the naïve EWI and a battery of OIs which include, for instance, the Brandt et al. (2009) approach where the style-weights maximize the expected power utility of the style-integrated portfolio. Their evidence suggests that the naïve EWI portfolio is unrivalled in terms of risk-adjusted performance by the OIs. This evidence echoes the DeMiguel et al. (2009) finding in the portfolio choice problem that "of the various optimizing models in the literature, there is no single model that consistently delivers a Sharpe ratio or a certainty-equivalent-return (CER) higher than that of the 1/*N* portfolio, which also has a very low turnover, which suggests that the out-of-sample gain from optimal diversification is more than offset by estimation error.” Our study is the first to show that, once Bayesian principles are embedded into the style-integration problem, the BOI portfolio solution outperforms the challenging EWI portfolio.

Our paper complements also a prolific literature that has applied Bayesian principles to asset allocation and portfolio choice. Early theoretical studies examined optimal asset allocation and argued that parameter uncertainty should not be ignored (Zellner and Chetty, 1965; Klein and Bawa, 1976; Brown, 1979; Jobson and Korkie, 1980). These and other studies suggest using Bayesian principles to mitigate the noise that bedevils the optimal portfolio solution due to the estimation error in the assets covariance matrix and, especially, in the expected returns vector. From the different lens of commodity style-integration, our article complements recent advances in Bayesian portfolio allocation (Polson and Tew, 2000; Pástor and Stambaugh, 2000; Tu and Zhou, 2010; Bauder et al., 2021). For instance, Pástor and Stambaugh (2000) add Bayesian elements to the portfolio choices of quadratic utility investors who use sample evidence to update prior beliefs centred on risk-based or characteristic-based pricing models.

The remainder of the paper is organized as follows. Section 2 presents the methodology. Section 3 describes the data and discusses the main empirical results. Section 4 presents results from various robustness checks before concluding the study in a final section.

2. Methodology

2.1 Commodity Style-Integration Framework

We adopt the framework laid out in Fernandez-Perez et al. (2019) to conduct a structured study of alternative commodity style-integration approaches. Let $k = 1, \dots, K$ denote the standalone styles or factors considered by the commodity investor, and $i = 1, \dots, N$ the cross-section of commodity futures contracts available. The investor constructs at each time t a style-integrated long-short portfolio with allocations dictated by the $N \times 1$ asset *allocation* vector $\mathbf{\Phi}_t$ as

$$\mathbf{\Phi}_t \equiv \mathbf{\Theta}_t \times \boldsymbol{\omega}_t = \begin{pmatrix} \theta_{1,1,t} & \cdots & \theta_{1,K,t} \\ \vdots & \ddots & \vdots \\ \theta_{N,1,t} & \cdots & \theta_{N,K,t} \end{pmatrix} \begin{pmatrix} \omega_{1,t} \\ \vdots \\ \omega_{K,t} \end{pmatrix} = \begin{pmatrix} \phi_{1,t} \\ \vdots \\ \phi_{N,t} \end{pmatrix} \quad (1)$$

where $\mathbf{\Theta}_t$ is the $N \times K$ *commodity-score* matrix, and $\boldsymbol{\omega}_t$ the $K \times 1$ *style-weight* vector. The sign of the allocations indicates the type of position; a positive $\phi_{i,t} \equiv \phi_{i,t}^L > 0$ (negative $\phi_{i,t}^S < 0$) represents a long (short) position on the *i*th commodity futures contract at time *t*.

Following Brandt et al. (2009) and Barroso and Santa-Clara (2015) inter alia, we start by using directly the standardized-signals as commodity scores; thus, $\mathbf{\Theta}_t$ contains by column the predictive signals per style with zero mean and unit standard deviation, i.e. $\theta_{i,k,t} \equiv \tilde{x}_{i,k,t} = (x_{i,k,t} - \bar{x}_{k,t})/\sigma_{k,t}^x$ where $x_{i,k,t}$ is the kth characteristic or predictive signal for asset i at time t.

The commodity allocations vector $\mathbf{\Phi}_t = (\phi_{1,t}, \ldots, \phi_{N,t})'$ is given by Equation (1) as a weighted (by $\omega_{k,t}$) average of the commodity scores per style $\theta_{i,k,t}$. The allocations are normalized, $\tilde{\phi}_{i,t} = \phi_{i,t}/\sum_{i=1}^N |\phi_{i,t}|$, to ensure that 100% of the investor's mandate is invested $\sum_{i=1}^N |\phi_{i,t}| = 1$. The style-integrated portfolio thus constructed at time *t* is held until $t+1$ and, under full-collateralization of the futures positions, its excess return is given by

$$R_{P,t+1} = \tilde{\mathbf{\Phi}}_t' \boldsymbol{R}_{t+1} = \sum_{i=1}^N \tilde{\phi}_{i,t} R_{i,t+1} \quad (2)$$

where $\boldsymbol{R}_t \equiv (R_{1,t}, R_{2,t}, \ldots, R_{N,t})'$ is the $N \times 1$ vector of time t excess returns for the standalone styles; $R_{i,t} = \ln\left(\frac{f_{i,t}^{front}}{f_{i,t-1}^{front}}\right)$ with $f_{i,t}^{front}$ denoting the price of the front-end futures contract for the *i*th commodity. At time $t+1$ the investor rebalances the style-integrated portfolio by calculating $\tilde{\boldsymbol{\Phi}}_{t+1}$, and so forth. The framework encapsulated in Equation (1) is very flexible in that it nests not only a diversity of style-integration approaches but also each of the underlying standalone styles $k = 1, \ldots, K$ through a sparse vector $\boldsymbol{\omega}_t$ with the kth entry set at 1 and all other entries at 0. We discuss the standalone styles in Section 2.2 below.

Broadly speaking, two perspectives can be adopted to choose the style weights or relative importance of the factors in the style-integrated portfolio: *i*) time-constant and style-identical weights to sidestep estimation noise, and *ii*) time-varying and style-heterogeneous weights estimated from past data. We discuss further each of these perspectives in Section 2.3.

2.2 Traditional Style-Integration Strategies

Various investment styles or factors have been suggested in the commodity literature. Without loss of generality, in this paper we focus on the factors known as carry, momentum, hedging pressure, skewness, and basis-momentum In each standalone style, the corresponding long-short portfolio formed at each time t buys (sells) the commodity quintile which is expected to appreciate (depreciate) the most according to the underlying sorting (or predictive) signal.

The sorting signal in the *carry* (term-structure or basis) style is the futures roll-yield defined as the difference between the logarithmic prices of the front- and second-nearest maturity contract (Erb and Harvey, 2006; Gorton and Rouwenhorst, 2006). The *hedging pressure* style is based on the net short positions of hedgers or commercial traders calculated as the number of short minus long positions over total positions (Basu and Miffre, 2013). The *momentum* style is based on past performance as sorting signal given by the average of commodity futures returns (Erb and Harvey, 2006; Miffre and Rallis, 2007). The *skewness* style exploits the degree of asymmetry of the commodity futures return distribution estimated through the Pearson coefficient of skewness (Fernandez-Perez et al., 2018). Finally, the *basis-momentum* style (Boons and Prado, 2019) exploits the differential momentum between the first- and second-nearest commodity futures contracts. Appendix A provides detailed definitions for each signal.

The simplest approach to construct a long-short portfolio that benefits from factor or style diversification is the equal-weighted integration (EWI) that assigns time-constant, identical weights to all the sorting signals, i.e. $\boldsymbol{\omega}_t = (1/K, \ldots, 1/K)'$ in Equation (1). The EWI portfolio is appealing because the resulting allocations $\boldsymbol{\Phi}_t$ do not suffer from estimation error. Another reason is that EWI does not require a "ranking" of the standalone styles at each portfolio formation time using past data which entails choosing a length L for the estimation window. Last but not least, the N commodity allocations generated by the EWI strategy are likely to be relatively stable (low turnover) which lessens the impact of transaction costs.

An investor seeking simultaneous exposure to multiple factors can resort instead to an OI strategy. The key idea is that the style-weights vector is now treated as a fixed, unknown parameter to be estimated. The various OIs differ in the optimal estimator $\widehat{\boldsymbol{\omega}}_t$ used where "optimal" is broadly used to indicate that the style-weights are defined according to a criterion.

Let the first two moments of the distribution of the N commodity futures excess returns, $E_t(\boldsymbol{R}_{t+1})$ and $Var_t(\boldsymbol{R}_{t+1})$,, be parameterized as $\boldsymbol{\mu}_t$ and $\boldsymbol{V}_t$ denoting, respectively, the $N \times 1$ mean and corresponding $N \times N$ covariance. These parameters can be estimated at each portfolio formation time t through the corresponding sample estimators $\widehat{\boldsymbol{\mu}}_t$ and $\widehat{\boldsymbol{V}}_t$ using the observed excess returns $\{\boldsymbol{R}_{t-(L-1)}, \ldots, \boldsymbol{R}_{t-1}, \boldsymbol{R}_t\}$ over an length-L past window.

Mean-variance utility maximization (MV)

Assuming an investor with constant relative risk aversion preferences (CRRA) and normally distributed portfolio returns, the weight estimator $\widehat{\boldsymbol{\omega}}_t$ is defined as the $K \times 1$ vector that maximizes the quadratic utility of the style-integrated portfolio, i.e. $\max_{\boldsymbol{\omega}_t} E_t[U(R_{P,t+1})]$ with

$$\begin{aligned} E_t[U(R_{P,t+1})] &= E_t(R_{P,t+1}) - \tfrac{1}{2}\gamma Var_t(R_{P,t+1}) \\ &= \boldsymbol{\Phi}_t'\boldsymbol{\mu}_t - \tfrac{\gamma}{2}\boldsymbol{\Phi}_t'\boldsymbol{V}_t\boldsymbol{\Phi}_t \\ &= (\boldsymbol{\Theta}_t\boldsymbol{\omega}_t)'\boldsymbol{\mu}_t - \tfrac{\gamma}{2}(\boldsymbol{\Theta}_t\boldsymbol{\omega}_t)'\boldsymbol{V}_t(\boldsymbol{\Theta}_t\boldsymbol{\omega}_t) \end{aligned} \quad (3)$$

where $R_{P,t+1} = \boldsymbol{\Phi}_t'\boldsymbol{\mu}_t = (\boldsymbol{\Theta}_t\boldsymbol{\omega}_t)'\boldsymbol{R}_{t+1}$ is the style-integrated portfolio return from time t to t+1, with $\boldsymbol{\Phi}_t$ denoting the $N \times 1$ allocation vector, Equation (1), and γ the coefficient of relative risk aversion. By solving the first-order maximization condition, $\frac{\partial E_t[U(R_{P,t+1})]}{\partial \boldsymbol{\omega}_t} = 0$, the MV optimized style-weights at each portfolio formation time *t* are given by

$$\widehat{\boldsymbol{\omega}}_t = \frac{1}{\gamma}\left(\boldsymbol{\Theta}_t'\widehat{\boldsymbol{V}}_t\boldsymbol{\Theta}_t\right)^{-1}\boldsymbol{\Theta}_t'\widehat{\boldsymbol{\mu}}_t \quad (4)$$

and the commodity allocations obtained by plugging (4) in Equation (1) are normalized $\tilde{\phi}_{i,t} = \phi_{i,t}/\sum_{i=1}^{N}|\phi_{i,t}|$; likewise, we conduct this normalization in all the subsequent OIs.

Mean-variance utility maximization under shrinkage covariance (MVshrinkage)

Forming a mean-variance efficient portfolio requires estimating the assets' covariance matrix. The sample covariance matrix $\widehat{\boldsymbol{V}}_t$ is typically not well-conditioned meaning that its inverse amplifies the estimation error, and may not even be invertible, when the number of assets N is large. To deal with this estimation issue Ledoit and Wolf (2003) propose imposing some structure in the covariance estimator. Specifically, they derive a shrinkage covariance matrix estimator $\boldsymbol{S}_t$ that is both well-conditioned and more accurate than the sample covariance matrix estimator. Seeking to reduce estimation uncertainty in the style-integration weights we implement a mean-variance OI method with shrinkage covariance matrix $\boldsymbol{S}_t$ defined as

$$\boldsymbol{S}_t = (1-\lambda)\,\widehat{\boldsymbol{V}}_t + \lambda\,\boldsymbol{I}_t \quad (5)$$

where $\lambda\,\boldsymbol{I}_t$ is a scalar multiple of the $N \times N$ identity matrix, and $\lambda \in (0,1)$ is the shrinkage intensity parameter $\boldsymbol{S}_t \approx \widehat{\boldsymbol{V}}_t$ if $\lambda \rightarrow 0$, and $\boldsymbol{S}_t \approx \boldsymbol{I}_t$ if $\lambda \rightarrow 1$. Ledoit and Wolf (2003) show that the optimal shrinkage parameter defined as the λ^* that minimizes the distance between the shrinkage covariance matrix estimator, $\boldsymbol{S}_t$, and the true covariance matrix, $\boldsymbol{V}_t$, is given by

$$\lambda^* = \max\{0, \min\{\tfrac{\kappa}{L}, 1\}\} \quad (6)$$

where L is the length of estimation window for $\widehat{\boldsymbol{V}}_t$, and κ is replaced by a consistent estimator as detailed in Appendix B.1. We replace the sample covariance estimator $\widehat{\boldsymbol{V}}_t$ in the MV style-weights solution, Equation (4), by $\boldsymbol{S}_t^*$ to derive the *MVshrinkage* style-weight estimator as

$$\widehat{\boldsymbol{\omega}}_t = \frac{1}{\gamma}(\boldsymbol{\Theta}_t' \boldsymbol{S}_t^* \boldsymbol{\Theta}_t)^{-1} \boldsymbol{\Theta}_t' \widehat{\boldsymbol{\mu}}_t \tag{7}$$

Appendix B.1 provides details on the implementation of the *MVshrinkage* style-integration.

<u>*Variance minimization (MinVar)*</u>

The *MinVar* weights $\boldsymbol{\omega}_t$ are defined as those that minimize the second moment or risk of the style-integrated portfolio. Accordingly, the style-integration investor solves at each portfolio formation time t the variance minimization problem $\min_{\boldsymbol{\omega}_t} E_t\left[\left(R_{P,t+1} - \bar{R}_P\right)^2\right]$ where

$$E_t\left[\left(R_{P,t+1} - \bar{R}_P\right)^2\right] = Var_t\left(R_{P,t+1}\right) = \boldsymbol{\Phi}_t' \boldsymbol{V}_t \boldsymbol{\Phi}_t = (\boldsymbol{\Theta}_t \boldsymbol{\omega}_t)' \boldsymbol{V}_t (\boldsymbol{\Theta}_t \boldsymbol{\omega}_t) \tag{8}$$

with $\boldsymbol{\Phi}_t = \boldsymbol{\Theta}_t \boldsymbol{\omega}_t$ denoting the style-integrated commodity allocations given by Equation (1). The solution of the first-order condition $\frac{\partial Var_t(R_{P,t+1})}{\partial \boldsymbol{\omega}_t} = 0$, defines the *MinVar* style-weights as

$$\widehat{\boldsymbol{\omega}}_t = \frac{\left(\boldsymbol{\Theta}_t' \widehat{\boldsymbol{V}}_t \boldsymbol{\Theta}_t\right)^{-1} \boldsymbol{\Theta}_t' \mathbf{1}}{\mathbf{1}' \boldsymbol{\Theta}_t \left(\boldsymbol{\Theta}_t' \widehat{\boldsymbol{V}}_t \boldsymbol{\Theta}_t\right)^{-1} \boldsymbol{\Theta}_t' \mathbf{1}} \tag{9}$$

where $\mathbf{1}$ is an $N \times 1$ vector of 1s. In essence, the *MinVar* approach reduces the dimensionality of the mean-variance parameter space by focusing on the risk of the style-integrated portfolio.

<u>*Style-volatility timing (StyleVol)*</u>

Kirby and Ostdiek (2012) develop a portfolio allocation method which also focuses on risk but with an additional dimensionality reduction assumption: zero covariances among asset returns. The solution, known as *volatility timing* in the literature, prescribes the allocation of wealth to each asset proportionally to the inverse of its risk as given by its past return variance. This approach can be adapted to the style-integration problem by shifting the focus to the risk of the standalone styles. Thus, the *StyleVol* style-weights estimator is defined as

$$\widehat{\boldsymbol{\omega}}_t = \left(1/\hat{\sigma}_{1,t}^2, \dots, 1/\hat{\sigma}_{K,t}^2\right) \tag{10}$$

with $\sigma_{k,t}^2$ denoting the kth entry of the $K \times K$ styles-covariance matrix $\boldsymbol{\Sigma}_t$ obtained from a window of past L-month data. Thus, according to the *StyleVol* approach, which assumes style independence, the more volatile a style has been in the recent past the less weight it receives.

Diversification-ratio maximization (MaxDiv)

Choueifaty and Coignard (2008) define the diversification ratio of a portfolio as the ratio of the aggregate individual assets' volatilities divided by the portfolio's volatility. Adapted to the present context, the diversification ratio of the style-integrated portfolio can be defined as

$$D(\boldsymbol{\Phi}_t) = \frac{\boldsymbol{\Phi}_t' \boldsymbol{\Omega}_t}{\sqrt{\boldsymbol{\Phi}_t' \boldsymbol{V}_t \boldsymbol{\Phi}_t}} \tag{11}$$

where $\boldsymbol{\Phi}_t \equiv \boldsymbol{\Theta}_t \boldsymbol{\omega}_t$ is the style-integrated commodity allocation, Equation (1), and $\boldsymbol{\Omega}_t = (\sigma_1^2, \dots, \sigma_N^2)$ is the diagonal of the commodity covariance matrix $\boldsymbol{V}_t$. Accordingly, the *MaxDiv* style-weights estimator $\widehat{\boldsymbol{\omega}}_t$ is defined as the solution of the maximization problem

$$max_{\boldsymbol{\omega}_t} \; D(\boldsymbol{\omega}_t) = \frac{(\boldsymbol{\Theta}_t \boldsymbol{\omega}_t)' \boldsymbol{\Omega}_t}{\sqrt{(\boldsymbol{\Theta}_t \boldsymbol{\omega}_t)' \boldsymbol{V}_t (\boldsymbol{\Theta}_t \boldsymbol{\omega}_t)}} \tag{12}$$

However, as no closed-form solution exists, we obtain the *MaxDiv* weights through the BFGS algorithm that belongs to the Quasi-Newton group of numerical optimization methods. To our best knowledge, the *MaxDiv* style-integration has not been studied as yet in the literature.

Power utility maximization (PowerU)

Deriving the style-weights under quadratic (mean-variance) utility has the advantage of tractability but it neglects the higher moments of the style-integrated portfolio return distribution, unlike the power utility. The *PowerU* style-weights estimator $\widehat{\boldsymbol{\omega}}_t$ is the $K \times 1$ vector that maximizes the expected power utility of the style-integrated portfolio. Formally,

$$max_{\omega_t}\left[\frac{\left(1+R_{P,t+1}\right)^{1-\gamma}-1}{1-\gamma}\right]=max_{\omega_t}\left[\frac{\left(1+(\mathbf{\Theta}_t\boldsymbol{\omega}_t)'\boldsymbol{R}_{t+1}\right)^{1-\gamma}-1}{1-\gamma}\right] \tag{13}$$

where $R_{P,t+1}=\sum_i^N\tilde{\phi}_{i,t}R_{i,t+1}$ with $\tilde{\phi}_{i,t}=\phi_{i,t}/\sum_{i=1}^N|\phi_{i,t}|=\frac{\sum_{k=1}^K\theta_{i,k,t}\omega_k}{\sum_{i=1}^N|\sum_{k=1}^K\theta_{i,k,t}\omega_k|}$ and γ is the coefficient of relative risk aversion. As no closed-form solution exists, we solve Equation (13) numerically, again via the Quasi-Newton BFGS algorithm, to find the *PowerU* style-weights.

We should note that in the original *PowerU* style-integration approach, as put forward by Brandt et al. (2009), the N asset allocations are defined as optimal deviations from the benchmark, e.g., value-weighted, equity market portfolio. The style-integration allocations derived from our Equation (1) can be rewritten as $\mathbf{\Phi}_t=\bar{\mathbf{\Phi}}_t+\mathbf{\Theta}_t\boldsymbol{\omega}_t$ with $\bar{\mathbf{\Phi}}_t=\left(\bar{\phi}_{1,t},\ldots,\bar{\phi}_{N,t}\right)$ denoting the benchmark allocations. Equation (13) is thus the Brandt et al. (2009) approach adapted to zero-net-supply assets ($\bar{\phi}_{i,t}=0$) such as futures and currencies, as implemented in Fernandez-Perez et al. (2019) and Barroso and Santa-Clara (2015), respectively.

Maximization of power utility under disappointment aversion (PowerDA)

Traditional portfolio choice models that build on the CRRA preferences assumption cannot generate non-participation in financial markets regardless of the value assumed for the coefficient of relative risk aversion, except in the presence of implausibly large trading costs (Liu and Loewenstein, 2002). Hence, these models tend to overpredict investors' equity positions and cannot generate the observed cross-sectional variation in portfolio choice. A way to mitigate this problem is to extend the expected utility framework to incorporate loss aversion.

Gul (1991) develops an axiomatic model of investors' preferences which can generate disappointment aversion (DA) and includes expected utility as a special case. Ang et al. (2005) utilize this model to reformulate the dynamic portfolio choice problem and show that it can explain better the actual cross-sectional variation in equity portfolio holdings, including optimal non-participation. These ideas can be utilized to design a *PowerDA* style-integration strategy. Let the expected utility framework be extended to incorporate DA as

$$\frac{(1+\delta)^{1-\gamma}-1}{1-\gamma} = \frac{1}{K}\left(\int_{-\infty}^{\delta} U(R_{P,t+1})dF(R_{P,t+1}) + A\int_{\delta}^{\infty} U(R_{P,t+1})dF(R_{P,t+1})\right) \quad (14)$$

where $F(R_{P,t+1})$ is the cumulative density of the style-integrated portfolio return $R_{P,t+1} = (\boldsymbol{\Theta}_t\boldsymbol{\omega}_t)'\boldsymbol{R}_{t+1}$, with the parameter *A* capturing the DA, and δ the certainty equivalent outcome. Thus, for the loss-averse investor $0 \le A < 1$, and good outcomes (above δ) are downweighted relative to bad outcomes (below δ).[3] The scaling parameter *K* is defined as

$$K = \Pr(R_{P,t+1} \le \delta) + A\Pr(R_{P,t+1} > \delta), \quad (15)$$

where $\Pr(\cdot)$ denotes probability. The *PowerDA* style-weights estimator $\widehat{\boldsymbol{\omega}}_t$ (alongside the $\hat{\delta}$ estimator) is obtained by solving simultaneously Equation (14) and the first-order condition

$$E_t\left[\frac{dU(R_{P,t+1})}{d\boldsymbol{\omega}_t}\mathbf{1}_{\{R_{P,t+1}\le\delta\}}\right] + AE_t\left[\frac{dU(R_{P,t+1})}{d\boldsymbol{\omega}_t}\mathbf{1}_{\{R_{P,t+1}>\delta\}}\right] = 0 \quad (16)$$

where $\mathbf{1}$ is an indicator function. Appendix B.2 details the *PowerDA* style-weights calculation.

2.3 Bayesian Optimized Style-Integration (BOI) Strategy

As shown by DeMiguel et al. (2009), parameter estimation uncertainty explains the inferior out-of-sample (OOS) performance of the Markowitz's mean-variance portfolio allocation versus the naïve constant $1/N$ allocation. Likewise, estimation risk can plausibly rationalize why a battery of OIs are unable to outperform the naïve EWI in the style-integration analysis of Fernandez-Perez et al. (2019). The purpose of this section is to design an optimized style-integration approach that incorporates Bayesian principles in order to account for parameter estimation risk – we refer to this approach as Bayesian optimized style-integration (BOI). One of the key contributions of Bayesian statistics is that by conceptualizing the parameters as

[3] The parameter value $A = 1$ generates the power utility with CRRA preferences.

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unknown random variables (instead of as fixed, unknown quantities) the parameters have probability distributions attached to them, and therefore the parameter uncertainty is explicitly tackled. Bayes' theorem encapsulates this idea as $\Pr(\theta|y) \propto \Pr(\theta) \times \Pr(y|\theta)$ where $\propto$ indicates proportionality, y denotes the evidence, and θ the unknown parameter. Bayes' theorem can be read as "posterior = prior × evidence" which says that the prior distribution for the parameter, denoted $\Pr(\theta)$, often simply called the prior, can be updated with the loglikelihood or probability of observing the data given the model, denoted $\Pr(y|\theta)$, to obtain the posterior distribution $\Pr(\theta|y)$ of interest. Thus, while non-Bayesian statistics is built on asymptotics ($T \rightarrow \infty$) to "deal" with parameter estimation risk, Bayesian statistics embeds such risk into the problem by updating a chosen prior probability (for the parameter) with new data.

According to the MV style-integration approach, the investor adopts at each time t the style weights that maximize the expected quadratic utility of the style-integrated portfolio, $\widehat{\boldsymbol{\omega}}_t = \frac{1}{\gamma}\left(\boldsymbol{\Theta}_t'\widehat{\boldsymbol{V}}_t\boldsymbol{\Theta}_t\right)^{-1}\boldsymbol{\Theta}_t'\widehat{\boldsymbol{\mu}}_t$. These weights hinge on the mean vector and covariance matrix estimates, $\widehat{\boldsymbol{\mu}}_t$ and $\widehat{\boldsymbol{V}}_t$, obtained from a history of L past excess returns on the N commodities. The larger the parameter estimation risk the more the sample utility $U(\widehat{\boldsymbol{\omega}}_t)$ deviates from the true utility $U(\boldsymbol{\omega}_t)$ and, in turn, the more sub-optimal the OI portfolios become in practice.

In order to account for parameter estimation risk in optimal portfolio allocation, Zellner and Chetty (1965) embed the expected utility maximization problem into a Bayesian framework. We adapt this approach to the problem of commodity style-integration. Let $\mathcal{I}_t$ denote the information available at time t, and $U(\boldsymbol{\omega}_t)$ the quadratic utility of the style-integrated portfolio. The BOI style-weight estimator is defined as the solution of the maximization problem

$$max_{\omega_t}\int_{-\infty}^{\infty}U(\boldsymbol{\omega}_t)\Pr(\boldsymbol{R}_{t+1}|\,\mathcal{I}_t)d\boldsymbol{R}_{t+1} \tag{17}$$

which combines the quadratic utility of the style-integrated portfolio and the predictive density $\Pr(\boldsymbol{R}_{t+1}|\mathcal{I}_t)$. The latter can be obtained by integrating out the unknown $\boldsymbol{\mu}_t$ and $\boldsymbol{V}_t$ as

$$pr(\boldsymbol{R}_{t+1}|\mathcal{I}_t) = \int_{\boldsymbol{\mu}_t}\int_{\boldsymbol{V}_t} \Pr(\boldsymbol{R}_{t+1}, \boldsymbol{\mu}_t, \boldsymbol{V}_t|\mathcal{I}_t)\, d\boldsymbol{\mu}_t d\boldsymbol{V}_t$$
$$= \int_{\boldsymbol{\mu}_t}\int_{\boldsymbol{V}_t} \Pr(\boldsymbol{R}_{t+1}|\boldsymbol{\mu}_t, \boldsymbol{V}_t, \mathcal{I}_t)\Pr(\boldsymbol{\mu}_t, \boldsymbol{V}_t|\mathcal{I}_t)\, d\boldsymbol{\mu}_t d\boldsymbol{V}_t \quad (18)$$

where $\Pr(\boldsymbol{R}_{t+1}|\boldsymbol{\mu}_t, \boldsymbol{V}_t, \mathcal{I}_t)$ is the conditional probability, and $\Pr(\boldsymbol{\mu}_t, \boldsymbol{V}_t|\mathcal{I}_t)$ the posterior probability.[4] By applying the Bayes rule, the posterior probability can be obtained from priors for the commodity expected excess returns $\boldsymbol{\mu}_t$ and corresponding covariance matrix $\boldsymbol{V}_t$ using the Markov Chain Monte Carlo (MCMC) simulation method as detailed in Appendix B.3.

A key idea behind the specific BOI approach proposed is that investors do not need explicitly to formulate a prior for $\boldsymbol{\mu}_t$. Their beliefs (or information) on the past relative performance of the styles can be harnessed to form a prior for $\boldsymbol{\omega}_t$ which can next be mapped onto a prior for $\boldsymbol{\mu}_t$. To accomplish this, we adhere to the MV style-integration solution, Equation (4), which establishes a one-to-one relation between $\boldsymbol{\omega}_t$ and $\boldsymbol{\mu}_t$ as

$$\boldsymbol{\mu}_t = \gamma \boldsymbol{V}_{\boldsymbol{\Theta}} \boldsymbol{\omega}_t \quad (19)$$

where $\boldsymbol{V}_{\boldsymbol{\Theta}} = (\boldsymbol{\Theta}_t')^{-1}(\boldsymbol{\Theta}_t' \boldsymbol{V}_t \boldsymbol{\Theta}_t)$. The BOI method that we propose is based on a prior distribution for the style-weights $\boldsymbol{\omega}_t$ which is subsequently mapped into a prior distribution for the commodity excess returns $\boldsymbol{\mu}_t$. Specifically, we begin by formulating the prior for $\boldsymbol{\omega}_t$ as

$$\boldsymbol{\omega}_t \sim N\left(\boldsymbol{\omega}_0, \frac{1}{\gamma}\boldsymbol{V}_{\boldsymbol{\Theta}}^{-\boldsymbol{1}}\boldsymbol{V}_\mu\right) \quad (20)$$

where $\boldsymbol{V}_\mu$ is the covariance of the prior distribution for $\boldsymbol{\mu}_t$. Equation (20) says that the prior distribution for $\boldsymbol{\omega}_t$ is Normal with mean $\boldsymbol{\omega}_0$ and covariance $\frac{1}{\gamma}\boldsymbol{V}_{\boldsymbol{\Theta}}^{-\boldsymbol{1}}\boldsymbol{V}_\mu$ which, in essence, represents the uncertainty about the prior. The prior for $\boldsymbol{\mu}_t$, as implied from Equation (19), is

$$\boldsymbol{\mu}_t \sim N\left(\gamma \boldsymbol{V}_{\boldsymbol{\Theta}} \boldsymbol{\omega}_0, \boldsymbol{V}_{\boldsymbol{\mu}}\right) \quad (21)$$

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[4] The BOI approach proposed can be easily generalized to any non-quadratic utility.

where $\boldsymbol{V}_\mu$ captures how disperse $\boldsymbol{\mu}_t$ is around $\gamma \boldsymbol{V}_{\boldsymbol{\Theta}} \boldsymbol{\omega}_0$, namely, the confidence on the prior.

We begin by adopting $\boldsymbol{\omega}_0 = (\frac{1}{K}, \dots, \frac{1}{K})'$ as our uninformative prior for the style-weights. The confidence on the prior for $\boldsymbol{\mu}_t$ is modelled as $\boldsymbol{V}_\mu = \frac{\boldsymbol{V}_t}{s^2}$ where s^2 is the average of the commodity return variances (diagonal elements of $\boldsymbol{V}_t$) based on a past window of L months. The prior for $\boldsymbol{V}_t$ is the inverse Wishart distribution as it is typical in Bayesian estimation of a covariance matrix (e.g., Gelman et al., 2015, Tu and Zhou, 2010; Pástor and Stambaugh, 2000). Specifically, we adopt $\boldsymbol{V}_t \sim IW(\boldsymbol{\Lambda}_0, \nu)$ with scale matrix $\boldsymbol{\Lambda}_0 = \boldsymbol{I}_N$ and degrees of freedom parameter $\nu = N + 1$ = 29. The choice of priors for the style-weights and covariance matrix is revisited in the robustness tests section after the main empirical results are discussed.

The Markov Chain Monte Carlo (MCMC) simulation method is widely used in Bayesian statistics to derived the posterior distribution when a closed-form expression is hard to derive. Accordingly, sampling from the commodity return history we obtain M simulated commodity excess return sequences $\{\boldsymbol{R}_{m,t-(L-1)}, \dots, \boldsymbol{R}_{m,t-1}, \boldsymbol{R}_{m,t}\}_{m=1}^{M}$. These simulated returns are the key inputs to obtain the posterior density $pr(\boldsymbol{\mu}_t, \boldsymbol{V}_t | \mathcal{I}_t)$ using the Gibbs sampling algorithm which is one of the most popular MCMC methods (e.g., Chen et al., 2012). In our empirical analysis, we generate $M = 10{,}000$ sequences. Finally, with the posterior density at hand, $pr(\boldsymbol{\mu}_t, \boldsymbol{V}_t | \mathcal{I}_t)$, the MV portfolio optimization problem, Equation (3), is solved at each portfolio rebalancing time t to obtain the BOI style-weight estimator $\boldsymbol{\omega}_t$. Appendix B.3 provides further details on the BOI implementation. Appendix C provides a list of all the style-integrations.

2.4. Performance Evaluation Metrics and Tests

All the style-integration strategies are deployed sequentially with rolling windows of past data for the N commodities. Specifically, at the end of the each month t, the commodity excess return histories $\{\boldsymbol{R}_{t-(L-1)}, \dots, \boldsymbol{R}_t\}$ are used to estimate the style-weights $\boldsymbol{\omega}_t$. The style-integrated portfolio formed according to Equation (1) is held for one month, the portfolio formation process is repeated at month-end *t*+1, and so on. This enables a monthly sequence of OOS excess returns for each of the strategies. Seeking to reduce the parameter uncertainty that afflicts the traditional OIs, in robustness tests we consider longer windows.

The OOS performance of the style-integration strategies is appraised through various metrics. We calculate the mean excess return or risk premium, alongside the total risk (variance of excess returns), downside risk (semi-deviation and maximum drawdown), and left-tail risk (1% Value-at-Risk). Risk-adjusted performance is gauged with the Sharpe ratio (SR) and non-normality robust Sortino and Omega ratios. To provide statistical significance to our findings as regards the comparison of Sharpe ratios between the *j*th optimized-integration strategy and the EWI benchmark, we deploy the Ledoit and Wolf (2008) and Opdyke (2007) tests for the one-sided hypotheses $H_0\colon \Delta SR_j \leq 0$ versus $H_A\colon \Delta SR_j > 0$, where $\Delta SR_j = SR_j - SR_{EWI}$. We also deploy the Jobson and Korkie (1980) test with the Memmel (2003) correction for the two-sided hypotheses $H_0\colon \Delta SR_j = 0$ versus $H_A\colon \Delta SR_j \neq 0$, as in DeMiguel et al. (2009, 2020).

Another useful metric is the certainty equivalent return (CER) defined as the guaranteed or risk-free return that an investor is willing to accept instead of deploying a portfolio strategy that promises a higher but uncertain return. As in prior studies, we calculate the CER as

$$CER_P = \bar{R}_{P,t} - \frac{\gamma}{2} Var(R_{P,t}) \tag{22}$$

where $\bar{R}_{P,t}$ and $Var(R_{P,t})$ are, respectively, the mean and variance of the style-integrated portfolio OOS excess returns, and γ is the relative risk aversion coefficient.[5] The significance

[5]The CER of the portfolio strategy (or "gamble") that will pay the random return R_P is the return that gives the investor the same utility as the expected utility of the portfolio $U(CER) = E\big(U(R_P)\big)$. The CER metric, Equation (22), as deployed in DeMiguel et al. (2009), Anderson and Cheng (2016), and Fernandez-Perez et al. (2019) inter alia assumes that investors exhibit quadratic utility for the evaluation.

of the CER differential between a (B)OI portfolio and the EWI benchmark, $H_0: \Delta CER_j = 0$ vs $H_A: \Delta CER_j \neq 0$ with $\Delta CER_j = CER_j - CER_{EWI}$ is assessed through a GMM test using the Newey-West spectral density (e.g., DeMiguel et al., 2009, Anderson and Cheng, 2016).[6]

To assess the trading intensity of different style-integration methods, which can impact the risk premium captured net of transaction costs, we calculate the portfolio turnover (TO)

$$TO_j = \frac{1}{T-L} \sum_{t=L}^{T-1} \sum_{i=1}^{N} \left(\left| \tilde{\phi}_{j,i,t+1} - \tilde{\phi}_{j,i,t^+} \right| \right) \tag{23}$$

where $t = 1, \ldots, T$ denotes each of the portfolio rebalancing times (month-end), $\tilde{\phi}_{j,i,t+1}$ is the ith commodity allocation weight at t+1 by the jth style-integrated portfolio, while $\tilde{\phi}_{j,i,t^+} = \tilde{\phi}_{j,i,t} e^{R_{i,t+1}}$ is the actual portfolio weight right before the rebalancing at $t+1$ with $R_{i,t+1}$ denoting the monthly return of the ith commodity constituent from t to $t+1$. Thus, the TO metric, Equation (23), provides the average of all the trades incurred and embeds the mechanical evolution of the allocation weights due to within-month price dynamics.

Moreover, we measure how the transaction costs implied by the different TOs of the style-integrated portfolios impact on their performance. Marshall et al. (2012) provide a round-trip estimate of 8.6 basis points (bp) representing the spread of an investor who is prepared to wait up to 60 minutes for execution. For more impatient investors that require immediate execution the transaction cost is 25.8 bp. The net excess return of the style-integrated portfolios

[6] This test defines the CER differential as a parameter in a GMM system with four unknowns and four equations: $E[R_{j,t+1} - \mu] = 0$ and $E\left[\mu - \frac{\gamma}{2}(R_{j,t+1} - \mu)^2 - q\right] = 0$, with $R_{j,t+1}$ denoting the excess return of the jth portfolio strategy, alongside $E[R_{t+1}^{EWI} - \mu^{EWI}] = 0$ and $E\left[q - \mu^{EWI} + \frac{\gamma}{2}(R_{t+1}^{EWI} - \mu^{EWI})^2 - \Delta\right] = 0$. The t-statistic for $H_0: \Delta = 0$ vs $H_A: \Delta \neq 0$ is based on autocorrelation robust Newey-West standard errors and so only return stationarity is required.

$$\tilde{R}_{j,t+1} = \sum_{i=1}^{N} \tilde{\Phi}_{j,i,t+1} R_{i,t+1} - TC \sum_{i=1}^{N} \left| \tilde{\Phi}_{j,i,t+1} - \tilde{\Phi}_{j,i,t^{+}} \right| \quad (24)$$

is calculated, first, using $TC = 8.6$ bp for consistency with extant commodity risk premia studies (Fernandez-Perez et al., 2019; Prokopczuk et al., 2023).[7] Second, to reflect the costs faced by more impatient traders we adopt the conservative $TC = \{17.2, 25.8\}$ bp which represent the mid-point and upper limit of the range of costs provided by Marshall et al. (2012).

For the estimation of the style-weights according to the *MV*, *MVshrinkage*, *PowerU*, and *PowerDA* style-integration strategies, Equations (3), (7), (13) and (14), respectively, and the CER calculation, Equation (22), we assume a moderate risk aversion level $\gamma = 5$; see e.g., Brandt et al., 2009; Gao and Nardari, 2018; Fernandez-Perez et al., 2019). As in Fernandez-Perez et al. (2019), we adopt the disappointment aversion level A=0.6 in Equation (15).

3. Data and Empirical Results

3.1 Data

The empirical analysis uses settlement prices and open interest data for a cross-section of futures contracts on 28 commodities pertaining to various sectors: agriculture (cocoa, coffee, corn, cotton, frozen concentrated orange juice, oats, rough rice, soybeans, soybean meal, soybean oil, sugar 11, wheat, and lumber), energy (PJM electricity, gasoline RBOB, heating oil, light sweet crude oil, natural gas of Henry hub, and unleaded gasoline), livestock (feeder cattle, frozen pork bellies, lean hogs, and live cattle), and metal (high-grade copper, gold, palladium, platinum, and silver 5000). In additional tests to assess the robustness of the main findings, we expand the cross-section which several contracts some of which are rather less liquid.

[7] Prokopczuk et al. (2023) report a very close average TC estimate of 10 bp across 27 commodities over the 1959-2018 period and additionally document a downward time-trend in the trading costs.

Daily prices are obtained from *Refinitiv Datastream*. Open interest data is available weekly from the Commitment of Traders report of the U.S. Commodity Futures Trading Commission (CFTC). The sample period is January 1992 to December 2021. Futures returns are computed with the price of the front-end futures contract up to the month preceding the maturity month when positions are rolled to the next contract to avoid any erratic price behaviour near maturity.

3.2 Preliminary Data Analysis

Table 1 summarizes the distribution of excess returns for the 28 commodity futures contracts and cross-correlations. The mean excess returns vary widely across commodities and time. The individual excess returns are generally insignificant on average over the sample period. Moreover, there is no evidence of monthly return predictability from the prior returns as borne out by very small first-order autocorrelations (e.g., the largest AR(1) is obtained for gasoline RBOB at 0.217). The distributions of returns are broadly symmetrical – exceptions are sugar, RBOB gasoline, and platinum with a large negative skew. Some futures contracts exhibit the heavy-tailed property such as sugar, electricity, RBOB gasoline and platinum with kurtosis coefficients of 8.707, 6.829, 10.622 and 5.235, respectively. The average pairwise correlations of each commodity futures excess returns with the excess returns of the commodities in the same (and different) sector(s) indicate that the within-sector price dynamics is highly similar, but far more unrelated across sectors.

[Insert Table 1 around here]

Table 2 summarizes the out-of-sample performance of the basis, hedging pressure, momentum, skewness and basis-momentum styles statically over the full sample period (Panel A), and dynamically over non-overlapping 6-year subperiods (Panel B).

[Insert Table 2 around here]

The reward-to-risk profiles suggested by the Sortino, Omega, and Sharpe ratios, together with the crash risk profiles suggested by the maximum drawdown, 1% VaR, and semi-deviation, Panel A, endorse primarily the skewness and basis-momentum styles. However, the picture changes over subperiods as the skewness style ranks last in the second subperiod and the basis-momentum ranks almost last in the third subperiod. The fact that their relative performance is unstable – no individual style emerges as consistently superior – poses a challenge for investors in terms of choosing an individual style to adhere to. This motivates the idea of signal diversification that can be harnessed through a style-integration strategy; namely, style-integration can be used as "hedge" against temporary style under-performance.

Figure 1 plots the cumulative out-of-sample Sharpe ratio of the standalone styles. The first point in the graph is based on monthly excess returns within the initial 60-month window, and the last point is based on the monthly excess returns over the entire sample period. The graph confirms the instability in individual style rankings over time which calls for style-integration.

[Insert Figure 1 around here]

Next we examine the extent of dependence among the standalone styles. In order to provide a complete picture, Table 3 reports three different measures of (non)linear dependence. The widely-used Pearson correlation (Panel A) suggests that the five styles are mildly overlapping. This is confirmed by the Spearman rank-order correlation (Panel B), and the Kendall correlation (Panel C) that additionally capture nonlinear dependence.[8] All three statistics concur in suggesting that the excess returns of the five styles under consideration can be ascribed to different sources. Thus, seeking to benefit from factor exposure diversification we construct style-integrated portfolios using various strategies that we compare next.

[Insert Table 3 around here]

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[8] The Spearman correlation between two variables is the standard correlation between their rankings. While Pearson's correlation assesses linear relationships, Spearman's correlation captures monotonic (linear or not) relationships. Kendall's correlation is analogous to Spearman's correlation but outperforms it because it is more robust to outliers and has better small-sample properties.

3.3 Out-of-Sample Performance of Style-Integrated Portfolios

Table 4 reports summary statistics for the distribution of OOS excess returns obtained with the traditional optimized style-integrations (OIs) and the new Bayesian approach (BOI), alongside the EWI benchmark. Panel A reports the results over the entire sample period (static evaluation), and Panel B over non-overlapping 6-year subsample periods (dynamic evaluation).

The easy-to-deploy EWI strategy stands out as very effective at capturing risk premia. With an excess return of 8.0% p.a., the EWI portfolio surpasses each one of the standalone-style portfolios' excess returns ranging from 3.6% p.a. (hedging pressure) to 5.1% p.a. (basis-momentum). The Sharpe ratio of the EWI portfolio at 0.815 represents a pervasive gain in reward-to-risk of 40% across all styles on average, and between 18% (basis-momentum) and 65% (hedging pressure) individually. The reward-risk gain of EWI versus the standalone styles suggested by the Sharpe ratio is emphasised by the Sortino and Omega ratios. Last but not least, the EWI portfolio has a favourable crash risk profile vis-à-vis the standalone styles as suggested by the semi-deviation, maximum drawdown and 1% VaR measures. These findings overall confirm that exposure to multiple factors via an EWI long-short portfolio approach is beneficial.

[Insert Table 4 around here]

Our next task is to investigate whether an OI strategy that conceptualizes the style-weights as a fixed but unknown parameter vector can outperform the naïve EWI strategy. Table 4 provides summary statistics for the excess returns of the battery of OI portfolios outlined in Section 2.2.

It is noticeable that, with a mean excess return of 5.2% p.a., Sharpe ratio of 0.588, maximum drawdown of -0.296 and 1% VaR of -0.058, the *PowerU* style-integration strategy (Brandt et al., 2009) fails to outperform the EWI with corresponding values of 8.0% p.a., 0.815, -0.243 and -0.061, respectively. The naïve EWI portfolio is thus not only able to extract a larger commodity risk premium, but also exhibits less crash risk. Introducing the disappointment aversion parameter $A = 0.6$ in the commodity style-integration with power utility (*PowerDA*) does not improve upon the baseline *PowerU* approach and hence, the EWI portfolio remains unchallenged.[9] Likewise, the OI strategies that focus on quadratic utility (*MV*), and those that reduce the dimensionality of the mean-variance parameter space (*MVshrinkage*, *MinVar*, and *StyleVol*), or bring diversification into the objective function (*MaxDiv*) are unable to challenge the simple EWI strategy. The style-integrated portfolios formed by the *MinVar*, *StyleVol* and *MaxDiv* methods are quite competitive but not superior to the EWI portfolio. The calculated *p*-values to assess the statistical significance of Sharpe ratio differentials – Ledoit and Wolf (2008) and Opdyke (2007) one-sided tests, and Jobson and Korkie (1980) two-sided test – suggest that the Sharpe ratio of each of the OI portfolios is similar (or worse) than that of the EWI benchmark. This finding is confirmed by the *p*-values of the two-sided GMM test for the significance of differences in CERs between each OI portfolio and the EWI. Altogether these findings suggest that none of the OI portfolios succeeds at capturing multiple risk premia more effectively than the EWI portfolio which, despite sample differences (cross-section *N* and time period *T*), are well aligned with those in Fernandez-Perez et al. (2019).

By contrast, the Bayesian approach to parameter estimation risk overlaid to the traditional mean-variance utility setting delivers a style-integrated portfolio with excellent OOS performance. The BOI portfolio affords larger Sharpe ratios and certainty equivalent returns, among other performance metrics, than the challenging EWI portfolio and the alternative OI portfolios. The *p*-values of the Sharpe ratio tests and CER test shown in Table 4, Panel A, suggest at the 5% significance level or better that the BOI portfolio outperforms the challenging EWI portfolio.[10] This evidence based on a static (full sample) analysis of portfolio performance is confirmed by the dynamic analysis over non-overlapping 6-year subperiods in Panel B of

[9] With a Sharpe ratio of 0.371, maximum drawdown of -0.225, and 1% VaR of -0.055 the *PowerDA* style-integrated portfolio deployed with larger disappointment aversion ($A = 0.2$) is still unable to outperform the EWI. Additional performance measures for the latter are available from the authors.

[10] The findings from the CER comparison are qualitatively similar when we adopt power utility.

Table 4. The sequential Sharpe ratios and associated ranking of style-integrated portfolios reiterates the superiority of the BOI approach. Thus, embedding the mean-variance style integration approach into a Bayesian framework to account for parameter estimation risk allows investors to harness multiple commodity factor exposures rather efficiently.

Further to illustrate the dynamic performance of the BOI portfolio vis-à-vis the EWI and OI portfolios, we plot in Figure 2 their cumulative Sharpe ratio, mean and volatility.

[Insert Figure 2 around here]

The cumulative Sharpe ratio of the BOI portfolio, Panel A, surpasses rather steadily over time the Sharpe ratio of the EWI benchmark and alternative OI portfolios. This stems from its ability to capture a superior combined risk premia, as shown in Panel B, with relatively low risk, as shown in Panel C. The ability of the BOI strategy to improve the style-weights $\boldsymbol{\omega}_t$ decision (versus the EWI and alternative OIs) leads to a more efficient joint exposure to multiple factors.

What explains the outperformance of the BOI strategy versus the EWI and traditional OIs? The EWI approach can be cast as "atheoretical" in that it is fully non-parametric, i.e. the weights assigned to the K styles are time constant and identical. Instead, the mean-variance optimized style-integration (and the other OIs we consider) are parametric and follow the classical or frequentist approach of treating the style-weights as fixed, unknown values. They produce point estimates for the style-weights at each portfolio formation time. Thus, although the traditional approaches to style-integration (OIs) are built on finance principles (e.g., mean-variance framework) to determine the style-weights, they build on asymptotics ($T \to \infty$) to "deal" with parameter estimation uncertainty. As shown in extant portfolio choice studies, too large samples are required in practice to overcome the noise that contaminates the estimates (see e.g., DeMiguel et. al., 2009). This explains why the OIs are unable to outperform the naïve EWI in our analysis, as shown also in Fernandez-Perez et al. (2019). While also building on the quadratic utility framework, the BOI approach introduces the Bayesian notion of treating the style-weights as random variables with specific prior distributions that are updated with data. The upshot is that the BOI portfolio benefits both from the "sophistication" of an optimized style-weighing approach (by contrast with the naïve EWI) and from the powerful Bayesian approach to estimation risk that treats parameters as random variables (by contrast with the OIs).

It is well known that estimation error in expected returns is far more costly than estimation error in the covariance matrix. Our findings suggest that by transforming a meaningful prior of style weights $\boldsymbol{\omega}_t$ into a prior of expected returns $\boldsymbol{\mu}_t$, it is possible to improve the performance of the traditional mean-variance OI significantly and outperform the EWI.

So far we have assumed zero transaction costs in the analysis of style-integrated portfolio performance. However, since different style-integration strategies imply a different turnover, as shown in Panel A of Figure 3, it is important to consider transaction costs.

[Insert Figure 3 around here]

Figure 3 reveals that, even though the style-integrated portfolios can potentially include all N commodities while the standalone styles focus on the extreme quintiles and thus, by construction, include only in 40% of the N commodities, the style-integrated portfolios are not more trading intensive than the standalone styles. Among the standalone styles, the highest TO is exhibited by the *carry* portfolio and the lowest by the *hedging pressure* portfolio. Among the style-integrated portfolios, *MinVar* has the lowest TO followed by *MaxDiv* and *StyleVol*. Most importantly for the present purposes, the BOI portfolio exhibits a relatively low TO which suggests prima facie its outperformance is unlikely to be wiped out by transaction costs. To assess this, we report in Table 5 summary statistics for the style-integrated portfolios' net excess returns. First, we adopt $TC = 8.9$ bp, as in prior studies (e.g., Fernandez-Perez et al, 2019).

[Insert Table 5 around here]

It is noticeable that the erosion of performance due to transaction costs does not undermine the effectiveness of the style-integration solution versus standalone-style investing. Furthermore, the BOI portfolio still offers the best performance among all the style-integrated portfolios. These findings withstand the introduction of notably higher trading costs $TC = \{17.2, 25.8\}$ bp as borne out by the net Sharpe ratios in Figure 3, Panel B, and the full array of performance metrics gathered in Appendix Table D.1. Thus we can assert that, unlike the traditional OI portfolios, the BOI portfolio outperforms the EWI benchmark by delivering superior risk-adjusted returns net of transaction costs. Thus, adopting a Bayesian approach to estimation risk in the commodity style-integrated portfolio problem is quite fruitful.

4. Robustness Tests

In this section we investigate whether our main findings are robust to various aspects of the style-integrated portfolio construction such as, for instance, the commodity score scheme, length of estimation windows, and Bayesian priors. We discuss each of them in turn.

4.1 Commodity Score Schemes

Thus far we have implemented the style-integrated (long-short) portfolios using as elements of the score matrix $\mathbf{\Theta}_t$ in Equation (1) the individual-style signals $k = 1, \ldots .K$ appropriately standardized; namely, $\theta_{i,k,t} \equiv \tilde{x}_{i,k,t} = (x_{i,k,t} - \bar{x}_{k,t})/\sigma_{k,t}^{x}$ with $x_{i,k,t}$ denoting, say, the *roll-yield* signal for commodity i at time t (see Appendix A). Now we consider three alternative score matrices that could mitigate the biases induced by outliers in the signal measurement.

As in Fernandez-Perez et al. (2019) we consider a binary score scheme with short-versus-long entries $\theta_{i,k,t} \in \{-1,1\}$, a standardized rank scheme $\theta_{i,k,t} \equiv \tilde{z}_{i,k,t} = (z_{i,k,t} - \bar{z}_{k,t})/\sigma_{k,t}^{z}$ where $z_{i,k,t} \in \{N, \ldots, 1\}$ is the rank given to commodity i as candidate for a long position (N denotes top, and 1 denotes bottom) according to the kth predictive signal. Inspired by DeMiguel et al. (2020), we consider a commodity scoring approach where each commodity characteristic is winsorized cross-sectionally $\{x_{i,k,t}\}_{i=1}^{N}$; that is, we set as bottom (top) threshold the first (third) quartile minus (plus) three times the interquartile range, any observation outside those thresholds is shrank towards the corresponding threshold. Table 6 summarizes the performance of style-integrated portfolios implemented with the aforementioned score schemes.

[Insert Table 6 around here]

The earlier finding that the BOI method is unsurpassed by the challenging EWI benchmark and alternative OI methods remains unchanged. As a by-product we observe that for most style-integrated portfolios the binary {-1,1} scores and the standardized rank scores mitigate the noise (outliers) in the signals $x_{i,k,t}$ as borne out by their larger reward-to-risk ratios.

4.2 Longer Estimation Windows

The (B)OI style-integration portfolios constructed at each month-end *t* rely on *L*-month rolling windows of excess returns $\{\boldsymbol{R}_{t-(L-1)}, \dots, \boldsymbol{R}_t\}$ for the *N* commodities to estimate the style-weights $\boldsymbol{\omega}_t$; thus far, we have used $L = 60$ months. Asymptotically, as the estimation sample grows ($L \to \infty$) the parameter estimation risk ought to decrease and hence, the merit of the BOI method versus extant OIs may be diluted and likewise, the superiority of the non-parametric EWI versus OIs may fade away. We deploy now the (B)OI strategies using: *i*) recursive estimation windows starting from $L = 60$ months at the first portfolio formation time and then $L + 1$, $L + 2$, and so on, and *ii*) rolling estimation windows of fixed length $L = 120$ months.

Figures 4 and 5 present the cumulative risk-adjusted performance of the (B)OI portfolios and the EWI benchmark based on expanding and 120-month rolling windows, respectively.

[Insert Figures 4 and 5 around here]

Since longer estimation windows ought to alleviate the parameter uncertainty problem, it is not surprising to see that the performance of some sophisticated OI portfolios such as *MaxDiv* and *StyleVol* has been enhanced relative to EWI. However, the BOI approach remains the most attractive by delivering long-short portfolios with the best Sharpe ratio which stems from the larger excess returns captured and the lower risk. In other words, neither the expanding windows nor the 120-month rolling windows can fully "hedge" the parameter estimation risk.

This is rather plausible since, as demonstrated by DeMiguel et al. (2009), unfeasibly large estimation windows of 3,000 months are needed to overcome estimation risk in the optimal portfolio allocation problem towards outperforming the equal-allocations benchmark.

4.3 Alternative Bayesian Priors

Since Bayesian estimation methods require the choice of a prior distribution for the parameters, in this section we deploy the BOI approach using alternative priors. In Section 3 we adopted the uninformative prior $\boldsymbol{\omega}_0 = (\frac{1}{K}, \dots, \frac{1}{K})'$ as the mean of the Normal distribution for the style-weights $\boldsymbol{\omega}_t$. Likewise, we adopted the uninformative prior $\boldsymbol{I}_N$ (identity matrix) as the scale matrix of the inverse Wishart distribution for the commodity returns covariance $\boldsymbol{V}_t$.

We consider now informative priors for the style-weights and the covariance. Let us suppose that the style-integrated investor places more "value" on the *carry* and *hedging pressure* styles because their underlying signals (roll-yield and hedger's hedging pressure, respectively) are strongly motivated by fundamental commodity market theory – the theory of storage and the hedging pressure hypothesis, respectively. To reflect this investor's principle, we adopt $\boldsymbol{\omega}_0 = (0.23, 0.23, 0.18, 0.18, 0.18)$ as prior mean weights for the carry, HP, momentum, skewness, and basis-momentum styles, respectively.

Following Anderson and Cheng (2016), the prior distribution for the $N \times N$ covariance matrix $\boldsymbol{V}_t$ at each month-end *t* is tied to the historical excess returns for the *N* commodities $\{\boldsymbol{R}_1, \dots, \boldsymbol{R}_t\}$ with $\boldsymbol{R}_t = (R_{1,t}, \dots, R_{N,t})'$. Specifically, the prior for $\boldsymbol{V}_t$ is the inverted Wishart distribution, as in Section 2.3, but with the scale matrix $\bar{\lambda}_t \boldsymbol{I}_N$ (instead of the uninformative prior $\boldsymbol{I}_N$) with $\bar{\lambda}_t$ a scalar that represents the common variance across the *N* commodities; namely, $\bar{\lambda}_t = \frac{1}{N}\sum_{i=1}^{N} \bar{\lambda}_{i,t}$ with $\bar{\lambda}_{i,t} = \frac{1}{t-1}\sum_{\tau=1}^{t}\left(R_{i,\tau} - \mu_{i,t}\right)^2$ and $\mu_{i,t} = \frac{1}{t-1}\sum_{\tau=1}^{t} R_{i,\tau}$.

The BOI portfolios obtained with these alternative priors are summarized in Table 7.

[Insert Table 7 around here]

The results do not change qualitatively. The BOI2 portfolio based on the former prior $\boldsymbol{\omega}_0 = (\frac{1}{K}, \dots, \frac{1}{K})'$ for the style-weights together with the scale matrix $\bar{\lambda}_t \boldsymbol{I}_N$ for the inverse Wishart prior of the return covariance, and the BOI3 portfolio that combines the "value" weighted prior $\boldsymbol{\omega}_0$ and the identity matrix prior $\boldsymbol{I}_N$ for the inverse scale of the Wishart covariance, also outperform significantly the EWI. The results of the BOI strategy with these alternative priors, as shown in the new Table 7, do not change qualitatively. Of course, there is no guarantee that the BOI will work satisfactorily for any other prior – since a Bayesian method generates a posterior as a kind of weighted average between the prior and the new evidence (data), the posterior will only be accurate if the prior is well chosen. A comprehensive study of the impact of many different priors on the BOI performance goes beyond the scope of the paper. However, we provide readers with useful evidence to suggest that the BOI strategy based on the various priors entertained in the paper obtains significantly better performance than the EWI strategy.

4.4 Lower Rebalancing Frequency and Broader Cross-section

Now we address the portfolio holding period or equivalently, the rebalancing frequency. The analysis thus far focused on the monthly frequency. We deployed the style-integration strategy encapsulated in Equation (1) at each month-end *t* to form a unique (long-short) style-integrated portfolio that is held for one month. We assume now that the investor lets three months elapse before rebalancing the style-integrated portfolio; that is, the style-integrated portfolio return is measured from *t* to *t*+3, and so on. This approach ensures that even though the holding period is larger than one month, we do not incur the overlapping portfolio returns issue.

As shown previously in the case of the monthly holding period (c.f., Figure 3) the BOI portfolio incurs a relatively low turnover which favours it as regards net performance. By reducing the style-integrated portfolio rebalancing frequency from monthly to quarterly, the superior performance of BOI versus alternative OIs and the EWI could be diluted. Table 8 reports summary statistics for the net excess returns of the quarterly rebalanced portfolios neglecting transaction costs in Panel A and using the conservative TC=17.2 bp in Panel B.

[Insert Table 8 around here]

The results suggest that the earlier evidence in support of the Bayesian commodity style-integration is not challenged by this lower (quarterly) portfolio holding period.

Our analysis has hitherto relied on over two dozen commodity markets ($N = 28$) which is a typical cross-section in empirical studies; see e.g., Prokopczuk et al. (2023), Bakshi et al. (2019), Szymanowska et al. (2014), Fernandez-Perez et al. (2018), and Basu and Miffre (2013). However, to cater for investors that may wish to consider a wider cross-section of commodity futures contracts (even though some of them are less liquid) we added BFP Milk, Brent Crude Oil, Butter Cash, Cheese Cash, Coal, Silver 5000 oz, and White Wheat. The style-integrations based on this cross-section ($N = 35$) are summarized in Appendix Table D.2. The findings are qualitatively similar, namely, while the OI portfolios perform either significantly worse or not significantly better than the EWI portfolio, the BOI portfolio emerges as clearly superior.

4.5 Enlarging the Set of Alternative OI Strategies.

Finally, we deploy the following four style-integrations, as in Fernandez-Perez et al. (2019), to enlarge the "universe" of OIs that we confront the EWI benchmark and the proposed BOI with.

Rotation-of-Styles Integration (RSI). The style-weights vector is sparse with one entry at 1 and 0s elsewhere. At each month-end *t*, we form a long-short portfolio according to the style with the largest Sharpe ratio ($\omega_{j,t}$ = 1) in the preceding L-month window and ignore the remaining styles, $\omega_{k,t}$ = 0, $k = 1, \ldots, K$ ($k \neq j$). The RSI strategy is motivated by the theoretical style-switching model of Barberis and Shleifer (2003).

Cross-Sectional Pricing Integration (CSI). At each month-end t, we estimate a univariate time-series OLS regression per futures contract $i = 1, \ldots, N$ and style $k = 1, \ldots, K$ (a total of $N \times K$ regressions) using the past L-month window of data $s = t - (L - 1), \ldots t$

$$r_{i,s} = a_{i,k} + b_{i,k} r_{k,s} + \epsilon_{i,s} \tag{25}$$

where $r_{i,s}$ is the month s excess return of the ith futures contract, $r_{k,s}$ is the month s excess return of the long-short portfolio formed according to the kth style, and $\epsilon_{i,s}$ is an error term. At step two, we estimate in each of those prior L months a cross-section OLS regression

$$r_{i,s} = \lambda^0_{k,s} + \lambda^1_{k,s}\, \hat{b}_{i,k} + e_{i,s}, i = 1,2, \ldots, N \tag{26}$$

with $\hat{b}_{i,k}$ the estimate from Equation (25); a total of $L \times K$ regressions. The CSI weights estimator is $\widehat{\boldsymbol{\omega}}_t = (\frac{1}{L}\sum_{j=0}^{L-1} R^2_{1,t-j}, \ldots, \frac{1}{L}\sum_{j=0}^{L-1} R^2_{K,t-j})$ with $R^2_{k,t-j}$ capturing the explanatory power or pricing ability of the kth factor in the month $t - j$ cross-section OLS regression.

Principal Components Integration (PCI). The PCI style-weights estimator is a function of the eigenvectors pertaining to the first m principal components of the K style premia ($m < K$); namely, $\widehat{\boldsymbol{\omega}}_t \equiv \frac{e_{1,t}\boldsymbol{L}_{1,t} + e_{2,t}\boldsymbol{L}_{2,t} + \ldots + e_{m,t}\boldsymbol{L}_{m,t}}{e_{1,t} + e_{2,t} + \ldots + e_{m,t}}$ where $e_{j,t}$ is the jth eigenvalue that represents the explanatory power of the jth principal component, $\boldsymbol{L}_{j,t}$ is the corresponding K-vector of loadings (or jth eigenvector of $\boldsymbol{\Sigma}_t$, the $K \times K$ correlation matrix of standalone style returns) and m is the number of principal components that explain at least τ of the total variation in the standalone-style premia. We adopt the conservative threshold value $\tau = 90\%$.

Style Momentum Integration (SMI). The SMI style-weights are given by the average excess returns of the standalone-style portfolios over an L-month lookback period, $\widehat{\boldsymbol{\omega}}_t = (\frac{1}{L}\sum_{j=0}^{L-1} r_{1,t-j}, \ldots, \frac{1}{L}\sum_{j=0}^{L-1} r_{K,t-j})'$ to exploit any continuation in their relative performance.

The performance of the above OIs (with $L = 60$ months) is summarized in Table 9.

[Insert Table 9 around here]

The main findings remain unchallenged. By contrast with the BOI proposed, none of these additional OI portfolios is able to outperform the EWI benchmark, consistent with Fernandez-Perez et al. (2019), as suggested by the significance tests for Sharpe ratio and CER differentials.

5. Conclusions

Commodity style-integration is an intuitive and clearcut proposition to capture a superior and fairly stable risk premium by forming a unique long-short portfolio with simultaneous exposure to several factors. This factor diversification idea requires, in practice, choosing an appropriate blend of factor exposures at each portfolio rebalancing time. Extant strategies for this purpose are the equal-weights integration (EWI) that sets equal exposures constantly over time, and "sophisticated" style-integrations where the style-weights are the solution of an optimization problem. Echoing the portfolio allocation literature, the EWI strategy has proven very resilient vis-à-vis optimized integrations because it does not suffer from parameter estimation risk. This paper designs an optimized style-integration that overlays to the quadratic utility-based style integration the Bayesian notion of conceptualizing the style-weights parameter vector as a random variable with a prior distribution that is subsequently updated with evidence.

Using data on a cross-section of 28 commodities from January 1992 to December 2021, and focusing on the carry, hedging pressure, momentum, skewness and basis-momentum styles, we confront the Bayesian optimized style-integration (BOI), with the EWI and various optimized integrations (OIs) inspired from the portfolio optimization literature.

The findings indicate that it is beneficial to adopt Bayesian principles to deal with parameter estimation risk in commodity style-integration. By contrast with the battery of OI strategies, the BOI strategy outperforms the EWI benchmark by extracting a significantly larger premia consistently over time with less crash risk. This finding survives trading costs, various commodity scoring schemes, longer estimation windows, and alternative Bayesian priors.

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APPENDIX A. Standalone styles.

The table outlines five standalone styles that have been advocated in the literature to extract commodity risk premia. For each style, we indicate the underlying sorting or predictive signal, the long-short portfolio strategy, and a few key references. The notation $f_{1,t}$ ($f_{2,t}$) denotes the month-end *t* logarithmic settlement price of the front- (second-nearest) futures contracts. The HP signal is based on the short (long) open interest of large hedgers or commercial traders, denoted $Short_t$ ($Long_t$), as reported weekly by the CFTC in the Commitment of Traders' Report. The parameters μ_t and σ_t in the skewness signal denote the mean and standard deviation of commodity futures excess returns calculated at time *t* using daily data over the preceding year (*D* days). The subscripts *j*, *w*, and *d* denote months, weeks, and days, respectively.

Style	Signal	Portfolio formation	Key references
Carry or term-structure (TS)	$f_{1,t} - f_{2,t}$ (roll-yield)	long high, short low	Erb and Harvey (2006), Gorton and Rouwenhorst (2006), Bakshi et al. (2019)
Hedging pressure (HP)	$\frac{1}{52}\sum_{w=0}^{51}\frac{Short_{t-w} - Long_{t-w}}{Short_{t-w} + Long_{t-w}}$	long high, short low	Basu and Miffre (2013)
Momentum	$\frac{1}{12}\sum_{j=0}^{11}\Delta f_{1,t-j}$	long high, short low	Erb and Harvey (2006), Miffre and Rallis (2007), Bakshi et al. (2019)
Skewness	$\frac{1}{D}\frac{\sum_{d=0}^{D-1}(\Delta f_{1,t-d} - \mu_t)^3}{\sigma_t^3}$	long low, short high	Fernandez-Perez et al. (2018)
Basis-momentum	$\frac{1}{12}\sum_{j=0}^{11}\Delta f_{1,t-j} - \frac{1}{12}\sum_{j=0}^{11}\Delta f_{2,t-j}$	long high, short low	Boons and Prado (2019)

APPENDIX B. Implementation details for MVshrinkage, PowerDA and BOI strategies.

B.1. Mean-variance with shrinkage style-integration (MVshrinkage).

Following Ledoit and Wolf (2003), the shrinkage estimator of the commodities covariance matrix $\boldsymbol{V}_t$ is a linear combination of the standard estimator $\widehat{\boldsymbol{V}}_t$ and the identity matrix $\boldsymbol{I_N}$

$$\boldsymbol{S}_t = (1-\lambda)\,\widehat{\boldsymbol{V}}_t + \lambda \boldsymbol{I_N} \tag{B.1}$$

where the parameter $\lambda \in (0,1)$ dictates the shrinkage intensity. Let $||Z||_F$ denote the Frobenius norm of the $N \times N$ symmetric matrix Z with entries $\left(z_{ij}\right)_{i,j=1,..,N}$ defined as

$$||Z||_F = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} z_{ij}^2} \tag{B.2}$$

The optimal λ minimizes the expected Frobenius norm of the difference between the shrinkage covariance estimator and the true covariance $E(||\widehat{\boldsymbol{S}}_t - \boldsymbol{V}_t||_F)$. For a fixed N and T going to infinity, Ledoit and Wolf (2003) prove that the optimal λ^* shrinkage intensity is given by

$$\lambda^* = \max\{0, \min\{\tfrac{\kappa}{L}, 1\}\} \tag{B.4}$$

where L is the length of the estimation window used to obtain $\widehat{\boldsymbol{V}}_t$, and κ is a constant given by

$$\kappa = \frac{\pi-\rho}{\gamma} \tag{B.3}$$

A consistent estimator of π is

$$\hat{\pi} = \sum_{i=1}^{N}\sum_{j=1}^{N}\hat{\pi}_{ij} \quad \text{with} \quad \hat{\pi}_{ij} = \frac{1}{L}\sum_{t=1}^{L}\left\{\left(R_{it}-\bar{R}_i\right)\left(R_{jt}-\bar{R}_j\right)-\sigma_{ij}^2\right\}^2 \tag{B.5}$$

where R_{it} is the excess return of commodity i, and σ_{ij}^2 is an off-diagonal element of the standard covariance estimator $\widehat{\boldsymbol{V}}_{\boldsymbol{t}}$. A consistent estimator of ρ is given by

$$\hat{\rho} = \sum_{i=1}^{N}\hat{\pi}_{ii} + \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}\frac{\bar{\eta}}{2}\left(\sqrt{\frac{\sigma_{jj}^2}{\sigma_{ii}^2}}\ \hat{v}_{ii,ij} + \sqrt{\frac{\sigma_{ii}^2}{\sigma_{jj}^2}}\ \hat{v}_{jj,ij}\right) \tag{B.6}$$

where

$$\hat{v}_{ii,ij} = \frac{1}{T}\sum_{t=1}^{T}\{(R_{it}-\bar{R}_i)^2-\sigma_{ii}^2\}\{(R_{i,t}-\bar{R}_i)(R_{j,t}-\bar{R}_j)-\sigma_{ij}^2\}, \tag{B.7}$$

and

$$\bar{\eta} = \frac{2}{(N-1)N}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\eta_{ij} \tag{B.8}$$

with $\eta_{ij} = \frac{\sigma_{ij}^2}{\sqrt{\sigma_{ii}^2\sigma_{jj}^2}}$. Finally, a consistent estimator of γ is given by

$$\hat{\gamma} = \sum_{i=1}^{N}\sum_{j=1}^{N}\left(f_{ij}-\sigma_{ij}^2\right)^2 \tag{B.9}$$

where $f_{ij} = \bar{\eta}\sqrt{\sigma_{ii}^2\sigma_{jj}^2}$ with σ_{ii}^2 (commodity variance) represents the ith diagonal entry of $\widehat{\boldsymbol{V}}_t$.

B.2. Power utility with disappointment aversion style-integration (PowerDA).

Let $R_{P,t,s} = \boldsymbol{\Theta}_t \boldsymbol{\omega}_t \boldsymbol{R}_{t,s}$ denote the excess return of the style-integrated portfolio associated with the potential state s, and let $p_s \equiv pr(R_{P,t,s})$ denote the likelihood of this excess return. To solve equations (14) and (16) simultaneously, the concept of *quadrature* is used to approximate the certainty equivalent outcome of the style-integrated portfolio, δ, as follows

$$(1+\delta)^{1-\gamma} = \frac{1}{K}\left(\sum_{s:R_{P,t,s}\leq\delta} p_s R_{P,t,s}^{1-\gamma} + \sum_{s:R_{P,t,s}>\delta} A p_s R_{P,t,s}^{1-\gamma}\right) \tag{B.10}$$

and the first-order-condition in (16) as

$$\sum_{s:R_{P,t,s}\leq\delta} p_s R_{P,t,s}^{-\gamma} \exp(\boldsymbol{R}_{t,s}) + \sum_{s:R_{P,t,s}>\delta} A p_s R_{P,t,s}^{-\gamma} \exp(\boldsymbol{R}_{t,s}) = 0 \tag{B.11}$$

Let the commodity futures excess return vector in any state s out of S possible states be denoted $\{\boldsymbol{R}_{t,s}\}_{s=1}^{S}$ with probability weights $\{p_s\}_S^S = 1$. Assuming that the commodity futures excess return vector $\boldsymbol{R}_t \equiv (R_{1,t}, \dots, R_{N,t})$ follows a multivariate Normal distribution: $\boldsymbol{R}_t \sim MVN(\boldsymbol{\mu}_t, \boldsymbol{V}_t)$, the vector of returns from the MVN distribution can be sorted from low to high across the S states. The certainty equivalent outcome δ^* corresponding to the optimal style-weights vector $\boldsymbol{\omega}_t^*$ (and style-integrated portfolio return $\boldsymbol{\Theta}_t \boldsymbol{\omega}_t^* \boldsymbol{R}_t$) could lie within any interval

$$\begin{array}{ll} [\boldsymbol{\Theta}_t \boldsymbol{\omega}_t^* \boldsymbol{R}_{t,1}, & \boldsymbol{\Theta}_t \boldsymbol{\omega}_t^* \boldsymbol{R}_{t,2}), \\ [\boldsymbol{\Theta}_t \boldsymbol{\omega}_t^* \boldsymbol{R}_{t,2}, & \boldsymbol{\Theta}_t \boldsymbol{\omega}_t^* \boldsymbol{R}_{t,3}), \\ \vdots & \vdots \\ [\boldsymbol{\Theta}_t \boldsymbol{\omega}_t^* \boldsymbol{R}_{t,N-1}, & \boldsymbol{\Theta}_t \boldsymbol{\omega}_t^* \boldsymbol{R}_{t,N}) \end{array}$$

where $\boldsymbol{\Theta}_t \boldsymbol{\omega}_t^*$ is the $N \times 1$ optimal commodity allocation, Equation (1), and thus $R_{P,t,s}^* = \boldsymbol{\Theta}_t \boldsymbol{\omega}_t^* \boldsymbol{R}_{t,s}$ is the excess return of the style-integrated portfolio associated with state s. Suppose δ^* lies within $[\boldsymbol{\Theta}_t \boldsymbol{\omega}_t^* \boldsymbol{R}_{t,i},\ \boldsymbol{\Theta}_t \boldsymbol{\omega}_t^* \boldsymbol{R}_{t,i+1})$, then $\boldsymbol{\omega}_t^*$ is the solution of the first-order condition

$$\sum_{s:R_{P,t,s}\leq\Theta_t\omega_t^*\boldsymbol{R}_{t,i}} p_s R_{P,t,s}^{1-\gamma} \exp(\boldsymbol{R}_{t,s}) + \sum_{s:R_{P,t,s}>\Theta_t\omega_t^*\boldsymbol{R}_{t,i}} A p_s R_{P,t,s}^{1-\gamma} \exp(\boldsymbol{R}_{t,s}) = 0 \tag{B.12}$$

Equation (B.12) can be interpreted as the first-order condition of a maximization problem with probabilities π_i that are linked to the original portfolio return probabilities as follows

$$\pi_i = \frac{(p_1, \dots, p_i, Ap_{i+1}, \dots, A_{p_N})}{(p_1 + \dots + p_i) + A(p_{i+1} + \dots + p_N)} \tag{B.13}$$

The certainty equivalent outcome δ^* can thus be written as

$$\delta^* = \left(\sum_{s=1}^{N} (R_{P,t,s}^*)^{1-\gamma} \pi_{i,s}\right) \tag{B.14}$$

The algorithm of bisection search can be used to find the optimal style-weight vector as follows:

1. Start with an arbitrary choice of state i, for example, a value of 0.001 for the style-integrated portfolio return. Solve ω_t^* by equation (B.12).

2. Compute δ^* for the style-integrated portfolio through Equation (B.14).

3. If $\delta^* \in [\boldsymbol{\Theta}_t \boldsymbol{\omega}_t^* R_{t,i}, \boldsymbol{\Theta}_t \boldsymbol{\omega}_t^* R_{t,i+1})$ then $\boldsymbol{\omega}_t^*$ is the optimal style-weight vector at time t and the algorithm ends. If δ^* is instead larger (smaller) than the above upper (lower) bound, go back to step 1 and search within the upper (lower) half of the state space, and so on.

B.3. Bayesian Optimized Integration (BOI).

Let $\boldsymbol{R}_t$ denote the $N \times 1$ vector of commodity future excess returns with mean vector $\boldsymbol{\mu}_t$ and covariance matrix $\boldsymbol{V}_t$. The prior distribution for the style-weights is $\boldsymbol{\omega}_t \sim N\left(\boldsymbol{\omega}_{t,0}, \frac{1}{\gamma}\boldsymbol{V}_{\boldsymbol{\Theta}}^{-1}\boldsymbol{V}_{\mu}\right)$ and for the mean vector $\boldsymbol{\mu}_t \sim N\left(\gamma \boldsymbol{V}_{\boldsymbol{\Theta}}\boldsymbol{\omega}_{t,0}, \frac{\boldsymbol{V}_t}{s^2}\right)$ where $\boldsymbol{\omega}_{t,0}$ is the mean of the style-weights prior, $\boldsymbol{V}_{\boldsymbol{\Theta}}$ is a direct function of $\boldsymbol{V}_t$ given by $\boldsymbol{V}_{\boldsymbol{\Theta}} = (\boldsymbol{\Theta}_t')^{-1}(\boldsymbol{\Theta}_t'\boldsymbol{V}_t\boldsymbol{\Theta}_t)$ with $\boldsymbol{\Theta}_t$ the commodity score matrix in Equation (1), and s^2 is the average of the commodity excess return variances (diagonal entries of $\boldsymbol{V}_t$). The prior distribution for $\boldsymbol{\mu}_t$ is thus determined by the choice $\boldsymbol{\omega}_{t,0}$ and $\boldsymbol{V}_t$. We adopt $\boldsymbol{\omega}_{t,0} = (\frac{1}{K}, \ldots, \frac{1}{K})'$ and $\boldsymbol{V}_t \sim IW(\boldsymbol{\Lambda}_{\mathbf{0}}, \nu)$ where IW denotes inverse Wishart distribution with scale matrix $\boldsymbol{\Lambda}_{\mathbf{0}}$ and degrees of freedom parameter $\nu > N-1$. In the main analysis we set up $\boldsymbol{\Lambda}_{\mathbf{0}} = \boldsymbol{I}_N$ and $\boldsymbol{\Lambda}_{\mathbf{0}} = 29$.

As it is often the case in empirical Bayesian statistics, we circumvent the challenging problem of deriving a closed-form expression for the posterior density of the parameters by adopting the Markov Chain Monte Carlo (MCMC) simulation method. Accordingly, we construct empirically the posterior distribution $pr(\boldsymbol{\mu}_t, \boldsymbol{V}_t|\mathcal{T}_t)$ at each portfolio rebalancing time (month-end t) using a length-L window of commodity future returns $\{\boldsymbol{R}_{t-(L-1)}, \ldots, \boldsymbol{R}_t\}$. We adopt $L = 60$ months in the main analysis.

The MCMC method is built upon the property that, under mild assumptions, a Markov chain initiated at an arbitrary point will converge to the stationary posterior distribution $pr(\boldsymbol{\mu}_t, \boldsymbol{V}_t|\mathcal{T}_t)$. The method unfolds in three steps: (1) The prior distribution is used to generate $M_0 + M$ sequences of commodity excess returns $\{\boldsymbol{R}_{t-(L-1)}, \ldots, \boldsymbol{R}_{t-1}, \boldsymbol{R}_t\}_{m=1}^{M_0+M}$, (2) The first M_0 "burn-in" sequences are discarded to ensure that the chain has converged; (3) The remaining M sequences are used to obtain the mean and covariance parameters that approximate the posterior distribution, namely, $\{\boldsymbol{\mu}_t, \boldsymbol{V}_t\}_{m=1}^{M}$. We adopt $M = 10{,}000$ and $M_0 = 2{,}000$.

These steps can be directly performed using the PyMC3 package of *Python*. After setting the priors of $\boldsymbol{\mu}_t$ and $\boldsymbol{V}_t$, and the historical monthly excess returns for the N commodities from $t - L + 1$ to t as inputs, $\{\boldsymbol{R}_{t-(L-1)}, \ldots, \boldsymbol{R}_{t-1}, \boldsymbol{R}_t\}$, the package generates the posterior densities $\{\boldsymbol{\mu}_t\}_{m=1}^{M}$ $\{\boldsymbol{V}_t\}_{m=1}^{M}$ using the Gibbs sampling (MCMC) algorithm. The posterior means, $\overline{\boldsymbol{\mu}}_t$ and $\overline{\boldsymbol{V}}_t$, are then plugged into the mean-variance optimization Equation (4) as $\widehat{\boldsymbol{V}}_t$ and $\widehat{\boldsymbol{\mu}}_t$ to derive the style-weights for the Bayesian optimized-integration $\boldsymbol{\omega}_t^{BOI}$.

APPENDIX C. Commodity style-integration strategies.

The table summarizes the style-integration approaches with key references. $R_{P,t+1} = (\boldsymbol{\Theta}_t \boldsymbol{\omega}_t)' \boldsymbol{R}_{t+1}$ is the style-integrated portfolio excess return from t to t+1 based on the underlying N commodity excess returns $\boldsymbol{R}_{t+1}$, the $N \times N$ commodity scores or characteristics matrix $\boldsymbol{\Theta}_t$, and the $N \times 1$ style-weights $\boldsymbol{\omega}_t$. The approaches differ in the style-weight determination method. $\widehat{\boldsymbol{\mu}}_t$ and $\widehat{\boldsymbol{V}}_t$ denote the $N \times 1$ sample mean excess return vector and $N \times N$ sample covariance matrix. $\boldsymbol{S}_t^*$ is the shrinkage covariance estimator. $\hat{\sigma}_{1,t}^2, \ldots, \hat{\sigma}_{K,t}^2$ are the diagonal elements of the sample covariance matrix for the standalone style portfolios. $\widehat{\boldsymbol{\Omega}}_t$=*diag*($\widehat{\boldsymbol{V}}_t$). γ is the coefficient of relative risk aversion.

Strategy's name and abbreviation		Style-weights definition and estimator	
Equal-weighted integration	EWI	Time-constant, identical weights	$\boldsymbol{\omega}_t = (\frac{1}{K}, \ldots, \frac{1}{K})'$
Mean-variance utility maximization	MV	Maximizes expected quadratic utility of style-integrated portfolio	$\widehat{\boldsymbol{\omega}}_t = \frac{1}{\gamma}\left(\boldsymbol{\Theta}_t'\widehat{\boldsymbol{V}}_t\boldsymbol{\Theta}_t\right)^{-1}\boldsymbol{\Theta}_t'\widehat{\boldsymbol{\mu}}_t$
Mean-variance utility maximization with shrinkage	MVshrinkage	Maximizes style-integrated portfolio's expected utility with shrinkage covariance	$\widehat{\boldsymbol{\omega}}_t = \frac{1}{\gamma}\left(\boldsymbol{\Theta}_t'\widehat{\boldsymbol{S}}_t^*\boldsymbol{\Theta}_t\right)^{-1}\boldsymbol{\Theta}_t'\widehat{\boldsymbol{\mu}}_t$
Variance-minimization	MinVar	Minimizes risk of style-integrated portfolio	$\widehat{\boldsymbol{\omega}}_t = \frac{\left(\boldsymbol{\Theta}_t'\widehat{\boldsymbol{V}}_t\boldsymbol{\Theta}_t\right)^{-1}\boldsymbol{\Theta}_t'\mathbf{1}}{\mathbf{1}'\boldsymbol{\Theta}_t\left(\boldsymbol{\Theta}_t'\widehat{\boldsymbol{V}}_t\boldsymbol{\Theta}_t\right)^{-1}\boldsymbol{\Theta}_t'\mathbf{1}}$
Style-volatility timing	StyleVol	Inverse of past style's volatilities	$\widehat{\boldsymbol{\omega}}_t = \left(1/\hat{\sigma}_{1,t}^2, \ldots, 1/\hat{\sigma}_{K,t}^2\right)$
Power utility maximization	PowerU	Maximize expected power utility of style-integrated portfolio via BFGS numerical optimization	$max_{\boldsymbol{\omega}_t}\left[\frac{\left(1+R_{P,t+1}\right)^{1-\gamma}-1}{1-\gamma}\right.$

APPENDIX C. List of commodity style-integration strategies considered. (*Cont.*)

Strategy's name and abbreviation		Style-weights definition and estimator	
Power utility maximization under disappointment aversion	PowerDA	Maximize expected power utility under disappointment aversion degree parameterized by $A \in [0,1)$	$max_{\boldsymbol{\omega}_t}\left[\frac{1}{K}\left(\int_{-\infty}^{\delta} U(R_{P,t+1})dF(R_{P,t+1}) + A\int_{\delta}^{\infty} U(R_{P,t+1})dF(R_{P,t+1})\right)\right]$
Bayesian optimized style-integration	BOI	Maximize expected utility under predictive density via Gibbs sampling	$max_{\boldsymbol{\omega}_t}\int_{-\infty}^{\infty} U(\boldsymbol{\omega}_t)\Pr(\boldsymbol{R}_{t+1}\vert\, \mathcal{T}_t)d\boldsymbol{R}_{t+1}$
Rotation-of-styles integration	RSI	Adopt the style with the past largest Sharpe ratio	$\widehat{\omega}_{j,t} = 1,\ \widehat{\omega}_{k,t} = 0,\ k = 1,\dots,K\ \ (k \neq j)$
Cross-sectional pricing integration	CSI	Style-weights reflect the relative ability of the underlying factors to explain the cross-sectional variation in the *N* commodity futures returns	$\widehat{\boldsymbol{\omega}}_t = (\frac{1}{L}\sum_{j=0}^{L-1} R_{1,t-j}^2,\dots,\frac{1}{L}\sum_{j=0}^{L-1} R_{K,t-j}^2)$ with $R_{k,t-j}^2$ the coefficient of determination of past month $t-j$ cross-sectional regression
Principal components integration	PCI	Style-weights reflect the role of each style as determinants of the total variation in style premia	$\widehat{\boldsymbol{\omega}}_t \equiv \frac{e_{1,t}\boldsymbol{L}_{1,t}+e_{2,t}\boldsymbol{L}_{2,t}+\dots+e_{m,t}\boldsymbol{L}_{m,t}}{e_{1,t}+e_{2,t}+\dots+e_{m,t}}$ with $\boldsymbol{L}_{j,t}$ the *j*th $K\times 1$ eigenvector of $\boldsymbol{\Sigma}_t$ ($K\times K$ correlation matrix of the standalone style returns) and $e_{j,t}$ the *j*th eigenvalue (explanatory power of *j*th eigenvector)
Style-momentum integration	SMI	Style-weights capture continuation in relative performance of standalone styles	$\widehat{\boldsymbol{\omega}}_t = (\frac{1}{L}\sum_{j=0}^{L-1} r_{1,t-j},\dots,\frac{1}{L}\sum_{j=0}^{L-1} r_{K,t-j})$ with $r_{k,t-j}$ the month $t-j$ excess return of the *k*th standalone-style portfolio

APPENDIX D. Additional Evidence.

Table D.1. Net Performance of Commodity Style-Integrated Portfolios.

The table reports summary statistics for the out-of-sample net excess returns of the style-integrated portfolios using two conservative transaction cost estimates: Panel A is based on TC=17.2 bp which is the mid-point of the range provided by Marshall et al. (2012), Panel B uses TC=25.8 bp which is the upper bound of the Marshall et al. (2012) range that may apply to investors requiring immediate execution.

		Optimized Style-Integrations (OIs)							
	EWI	MV	MVshrinkage	MinVar	StyleVol	MaxDiv	PowerU	PowerDA	BOI
Panel A: *Conservative transaction costs TC=17.2 bps*									
Mean	0.070	0.044	0.041	0.067	0.072	0.073	0.042	0.042	0.085
StDev	0.101	0.091	0.099	0.087	0.093	0.095	0.090	0.089	0.087
Semi-deviation	0.278	0.255	0.284	0.226	0.246	0.255	0.256	0.252	0.218
Max Drawdown	-0.248	-0.225	-0.303	-0.177	-0.203	-0.221	-0.238	-0.224	-0.175
1% VaR	-0.062	-0.057	-0.063	-0.053	-0.056	-0.058	-0.057	-0.056	-0.051
Sharpe Ratio (SR)	0.721	0.522	0.456	0.789	0.798	0.792	0.498	0.504	0.985
Sortino ratio	1.206	0.850	0.730	1.390	1.381	1.353	0.805	0.816	1.813
Omega ratio	1.763	1.512	1.424	1.856	1.872	1.869	1.482	1.488	2.171
ΔSR (gain vs EWI)		-0.199	-0.265	0.068	0.078	0.071	-0.223	-0.216	0.265
Ledoit-Wolf test *p* -value		0.925	0.964	0.229	0.147	0.125	0.935	0.935	0.001
Opdyke test *p* -value		0.911	0.953	0.129	0.202	0.213	0.924	0.924	0.016
Jobson-Korkie test *p* -value		0.137	0.067	0.458	0.090	0.053	0.119	0.123	0.003
Cert. equiv. return (CER)	0.004	0.002	0.002	0.004	0.004	0.004	0.002	0.002	0.006
ΔCER (gain vs EWI)		-0.002	-0.002	0.000	0.000	0.000	-0.002	-0.002	0.002
GMM test *p* -value		0.112	0.064	0.804	0.269	0.166	0.098	0.098	0.021
Panel B: *Very conservative transaction costs TC=25.8 bps*									
Mean	0.065	0.039	0.036	0.063	0.068	0.068	0.037	0.037	0.081
StDev	0.102	0.091	0.099	0.087	0.093	0.095	0.090	0.089	0.087
Semi-deviation	0.281	0.258	0.288	0.229	0.249	0.258	0.260	0.255	0.220
Max Drawdown	-0.251	-0.235	-0.311	-0.179	-0.205	-0.223	-0.253	-0.227	-0.176
1% VaR	-0.062	-0.057	-0.063	-0.053	-0.057	-0.058	-0.057	-0.056	-0.052
Sharpe Ratio (SR)	0.674	0.468	0.407	0.744	0.751	0.746	0.443	0.449	0.940
Sortino ratio	1.115	0.752	0.643	1.296	1.285	1.262	0.706	0.716	1.710
Omega ratio	1.699	1.448	1.370	1.791	1.802	1.802	1.418	1.423	2.093
ΔSR (gain vs EWI)		-0.205	-0.267	0.070	0.078	0.073	-0.231	-0.225	0.266
Ledoit-Wolf test *p* -value		0.932	0.966	0.221	0.146	0.123	0.943	0.942	0.001
Opdyke test *p* -value		0.921	0.956	0.119	0.189	0.195	0.934	0.932	0.013
Jobson-Korkie test *p* -value		0.124	0.064	0.443	0.088	0.048	0.105	0.108	0.003
Cert. equiv. return (CER)	0.004	0.002	0.001	0.004	0.004	0.004	0.002	0.002	0.005
ΔCER (gain vs EWI)		-0.002	-0.002	0.000	0.000	0.000	-0.002	-0.002	0.002
GMM test *p* -value		0.110	0.064	0.730	0.232	0.133	0.095	0.096	0.016

Table D.2. Broad cross-section of commodity markets.

The table provides summary statistics for the annualized excess returns of 7 additional commodity futures contracts in Panel A and summarizes the performance of the style-integrated portfolios deployed on the wider cross-section ($N = 35$) in Panel B. Panel B.1 is based on the raw excess returns of the strategies and Panel B.2 on the net excess returns using the conservative transaction cost estimate of 17.2 bp (Marshall et al., 2012).

Panel I. *Summary statistics for excess returns of additional commodity futures contracts*

	Mean	StDev	AR(1)	Skew	Kurt	Obs	First obs YYYYMM	Last obs YYYYMM
BFP milk	0.059 (0.198)	0.146	0.231	1.074	4.805	136	199604	201506
Brent crude oil	0.020 (0.202)	0.369	0.240	-2.186	13.519	219	200310	202112
Butter cash	-0.087 (-2.133)	0.152	0.101	-0.223	3.520	195	200510	202112
Cheese cash	0.010 (1.268)	0.178	0.083	-0.942	3.761	231	201007	202112
Coal	0.065 (0.732)	0.287	0.166	0.360	3.972	185	200608	202112
Silver 5000 oz	-0.073 (-2.571)	0.249	-0.046	-4.215	34.321	135	199201	200210
White wheat	-0.063 (-0.414)	0.212	0.201	0.025	1.261	130	199201	200303

				Optimized Style-Integrations (OIs)					
	EWI	MV	MVshrinkage	MinVar	StyleVol	MaxDiv	PowerU	PowerDA	BOI
Panel II(a). *Style-integrated portfolios with N=35 commodity futures contracts (TC=0)*									
Mean	0.077	0.041	0.045	0.071	0.077	0.081	0.043	0.042	0.089
StDev	0.091	0.083	0.083	0.077	0.081	0.087	0.083	0.082	0.079
Semi-deviation	0.239	0.236	0.227	0.187	0.197	0.220	0.231	0.228	0.187
Max Drawdown	-0.203	-0.343	-0.315	-0.147	-0.152	-0.187	-0.349	-0.360	-0.155
1% VaR	-0.055	-0.052	-0.052	-0.046	-0.048	-0.052	-0.052	-0.052	-0.046
Sharpe Ratio (SR)	0.864	0.529	0.569	0.928	0.968	0.945	0.551	0.545	1.127
Sortino ratio	1.512	0.856	0.949	1.749	1.810	1.717	0.907	0.901	2.179
Omega ratio	1.950	1.509	1.548	2.070	2.088	2.090	1.545	1.541	2.410
ΔSR (gain vs EWI)		-0.335	-0.295	0.064	0.103	0.081	-0.314	-0.319	0.262
Ledoit-Wolf test p-value		0.979	0.975	0.295	0.059	0.091	0.978	0.977	0.003
Opdyke test p-value		0.971	0.961	0.331	0.189	0.240	0.967	0.968	0.031
Jobson-Korkie test p-value		0.043	0.051	0.582	0.118	0.179	0.045	0.042	0.004
Cert. equiv. return (CER)	0.005	0.002	0.003	0.005	0.005	0.005	0.002	0.002	0.006
ΔCER (gain vs EWI)		-0.003	-0.002	0.000	0.000	0.000	-0.002	-0.003	0.001
GMM test p-value		0.024	0.029	0.879	0.532	0.330	0.025	0.023	0.052
Panel II(b). *Style-integrated portfolios with N=35 commodity futures contracts (TC=17.2 bps)*									
Mean	0.067	0.032	0.035	0.061	0.068	0.071	0.033	0.032	0.080
StDev	0.091	0.084	0.083	0.077	0.081	0.087	0.083	0.082	0.079
Semi-deviation	0.246	0.244	0.234	0.193	0.203	0.226	0.238	0.235	0.192
Max Drawdown	-0.209	-0.392	-0.368	-0.150	-0.155	-0.192	-0.398	-0.412	-0.158
1% VaR	-0.056	-0.053	-0.052	-0.046	-0.048	-0.052	-0.053	-0.052	-0.046
Sharpe Ratio (SR)	0.755	0.413	0.456	0.814	0.859	0.833	0.435	0.425	1.018
Sortino ratio	1.288	0.650	0.739	1.488	1.561	1.473	0.696	0.682	1.914
Omega ratio	1.791	1.379	1.419	1.888	1.919	1.914	1.409	1.399	2.208
ΔSR (gain vs EWI)		-0.342	-0.299	0.058	0.104	0.078	-0.320	-0.330	0.263
Ledoit-Wolf test p-value		0.982	0.978	0.309	0.058	0.097	0.981	0.981	0.003
Opdyke test p-value		0.977	0.968	0.336	0.165	0.226	0.974	0.976	0.023
Jobson-Korkie test p-value		0.037	0.046	0.611	0.114	0.192	0.039	0.034	0.006
Cert. equiv. return (CER)	0.004	0.001	0.002	0.004	0.004	0.004	0.002	0.002	0.005
ΔCER (gain vs EWI)		-0.003	-0.002	0.000	0.000	0.000	-0.002	-0.003	0.001
GMM test p-value		0.026	0.033	0.968	0.410	0.310	0.028	0.024	0.042

Figure 1. Cumulative Sharpe ratio of standalone styles.

The figure plots the cumulative Sharpe ratio of long-short commodity futures portfolios or standalone styles based on the basis, hedgers' hedging pressure, momentum, skewness and basis-momentum signals as return predictors. The first feasible 60-month excess returns window is expanded by one month at a time. The analysis is based on commodity futures data from January 1992 to December 2021.

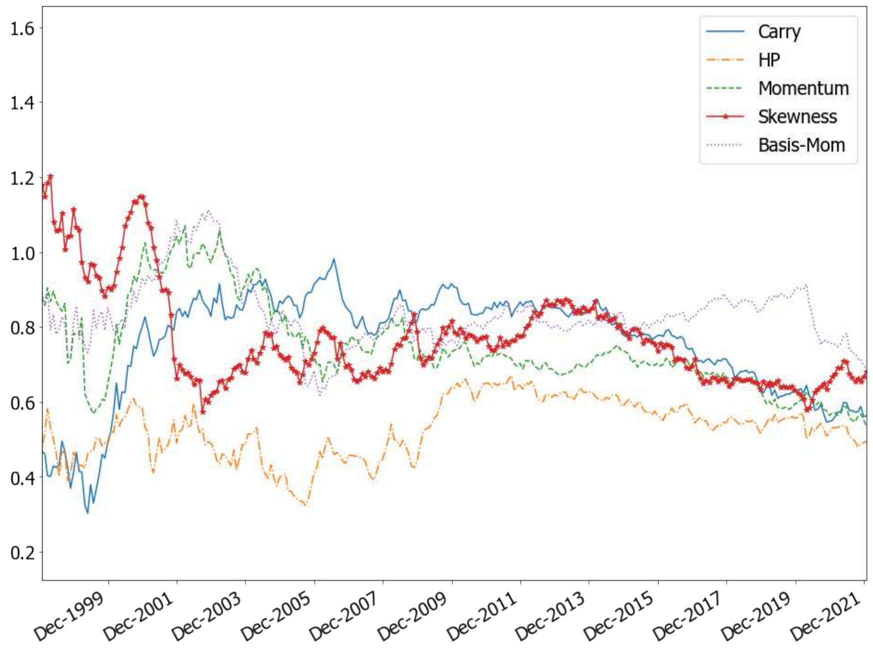
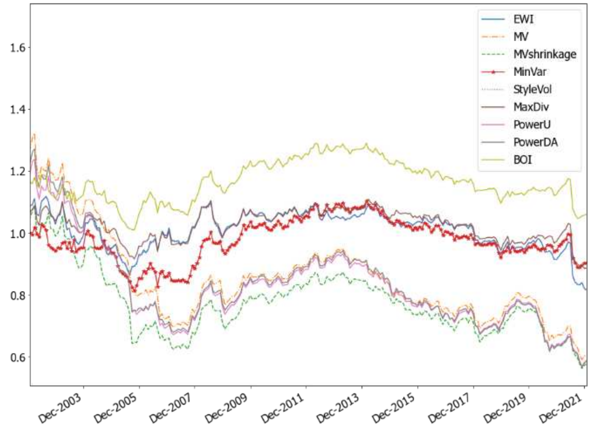


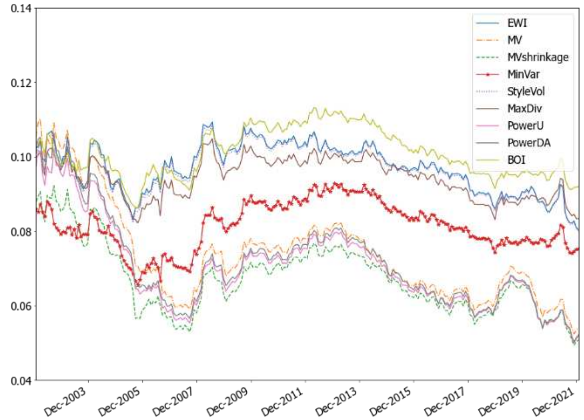
Figure 2. Cumulative reward and risk of commodity style-integrated portfolios.

The figure plots the Sharpe ratio, mean excess return and volatility of style-integrated portfolios based on their annualized monthly excess returns within expanding windows. The strategies are naïve equal-weights integration (EWI), and optimized integrations (OIs) formed according to mean-variance utility maximization (MV), mean-variance with shrinkage maximization (MVshrinkage), variance minimization (MinVar), style-volatility timing (StyleVol), diversification ratio maximization (MaxDiv), power utility maximization (PowerU), maximized power utility with disappointment aversion (PowerDA), and Bayesian optimized integration (BOI). The analysis is based on commodity futures data from January 1992 to December 2021.

Panel A: Sharpe ratio



Panel B: Mean excess return



Panel C: Volatility

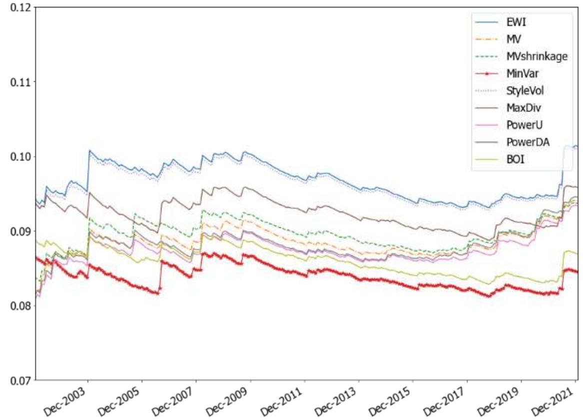
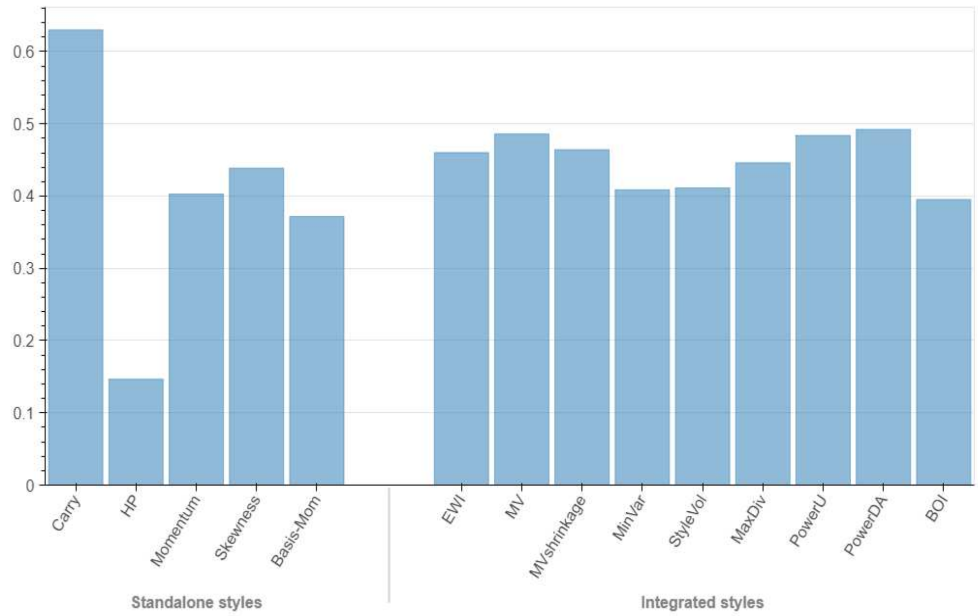


Figure 3. Turnover and net Sharpe ratio.

Panel A reports the monthly turnover averaged over the entire sample period. Panel B reports the net Sharpe ratio of each long-short portfolio strategy using as transaction cost proxies the lower bound of the Marshall et al. (2012) range of estimates at 8.6 bp, the middle point at 17.2 bp and the upper bound at 25.8 bp which is applicable to impatient investors that require immediate execution. The analysis is based on commodity futures data from January 1992 to December 2021.

Panel A: Portfolio turnover (TO)



Panel B: Net Sharpe ratio with TC = {8.6, 17.2, 25.8} bp

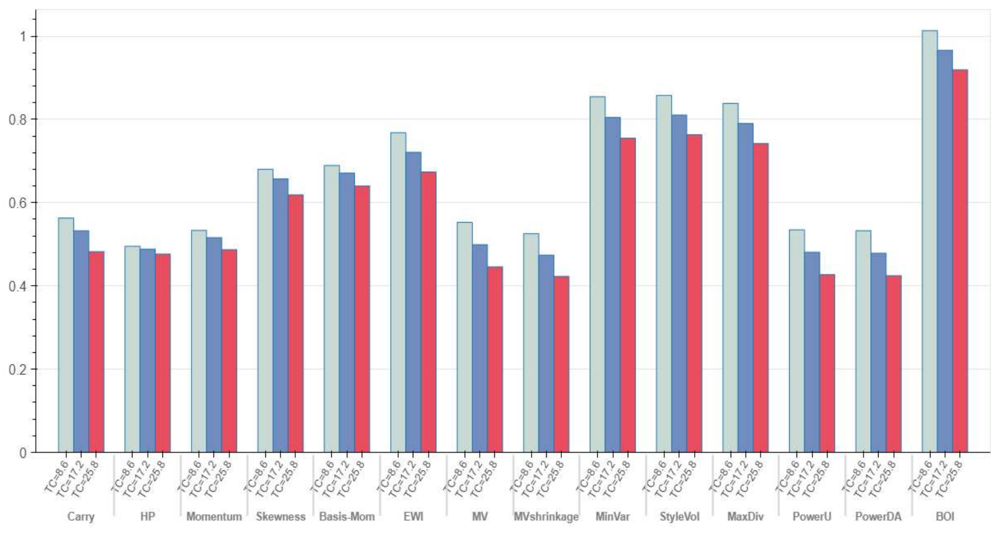


Figure 4. Cumulative risk and reward of style-integrated portfolios with style weights estimated over expanding windows.

This figure plots the cumulative Sharpe ratio, mean excess return and volatility of the style-integrated portfolios with style-weights estimated at each month-end using recursive windows. The strategies are equal-weights integration (EWI), and optimized integrations (OIs) formed according to mean-variance utility maximization (MV), mean-variance with shrinkage maximization (MVshrinkage), variance minimization (MinVar), style-volatility timing (StyleVol), diversification ratio maximization (MaxDiv), power utility maximization (PowerU), maximized power utility with disappointment aversion (PowerDA), and Bayesian optimized integration (BOI). The analysis is based on commodity futures data from January 1992 to December 2021.

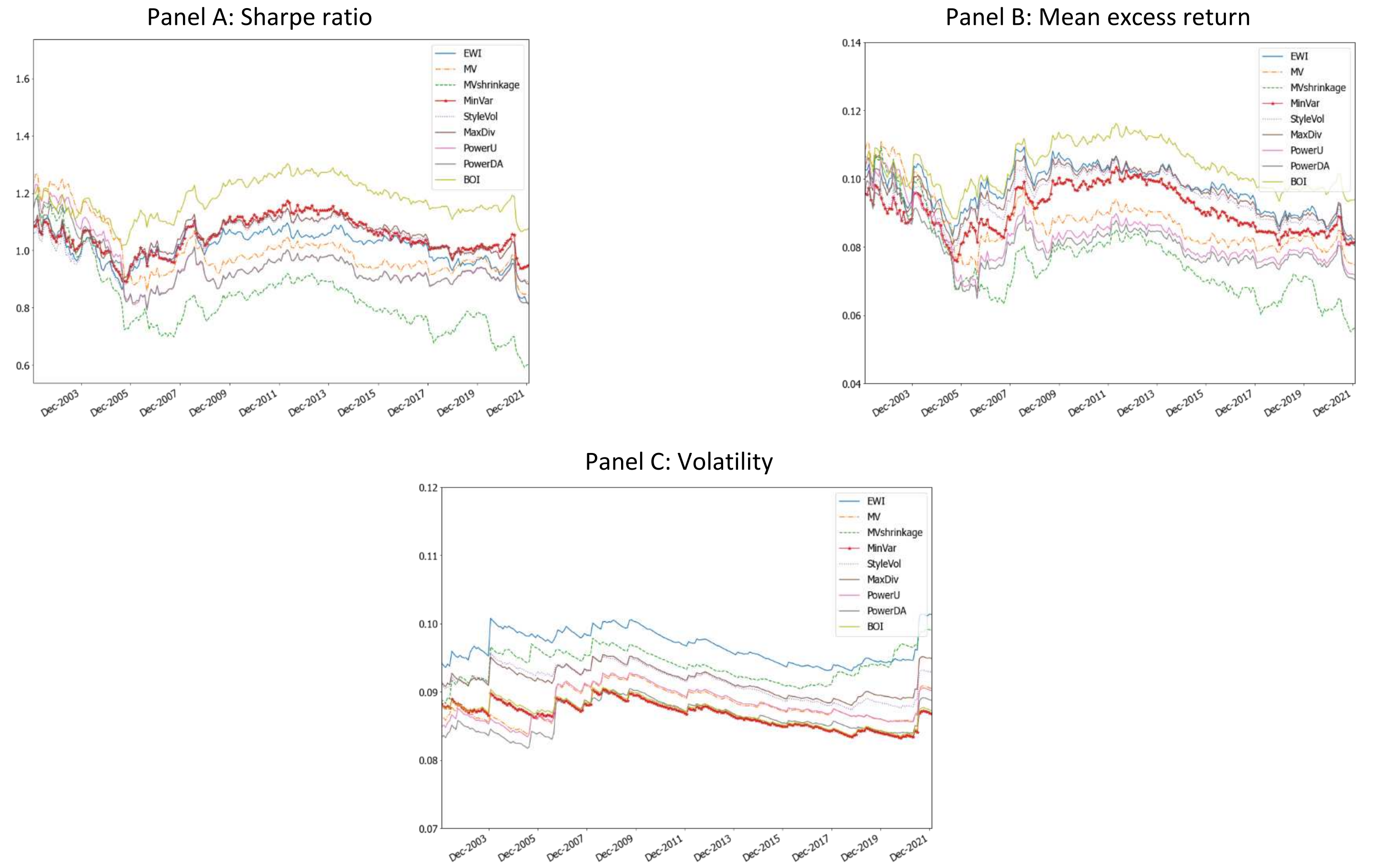
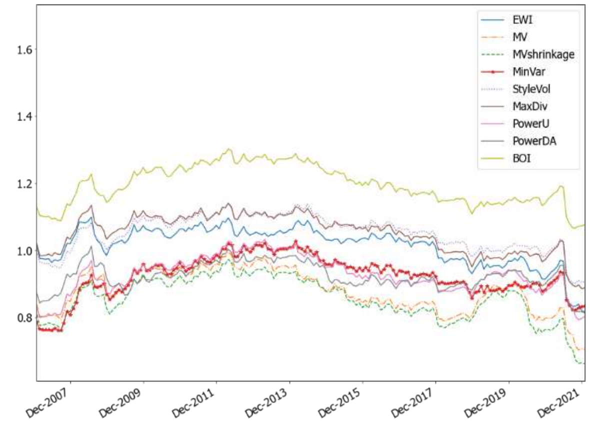
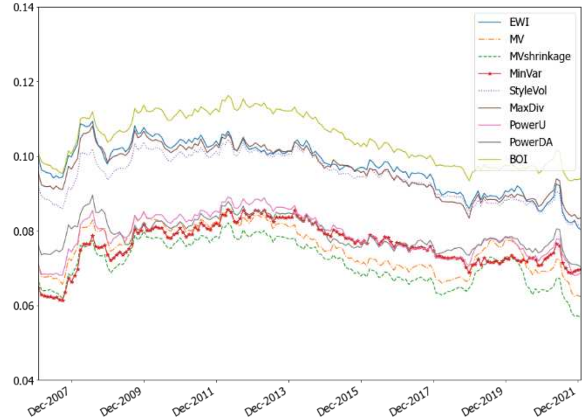


Figure 5. Cumulative risk and reward of style-integrated portfolios with weights estimated over 120-month rolling windows.
This figure plots the cumulative Sharpe ratio, mean excess return and volatility of the style-integrated portfolios with style-weights estimated at each month-end using 120-month rolling windows. The strategies are equal-weights integration (EWI), and optimized integrations (OIs) formed according to mean-variance utility maximization (MV), mean-variance with shrinkage maximization (MVshrinkage), variance minimization (MinVar), style-volatility timing (StyleVol), diversification ratio maximization (MaxDiv), power utility maximization (PowerU), maximized power utility with disappointment aversion (PowerDA), and Bayesian optimized integration (BOI). The analysis is based on commodity futures data from January 1992 to December 2021.

Panel A: Sharpe ratio



Panel B: Mean excess return



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Panel C: Volatility

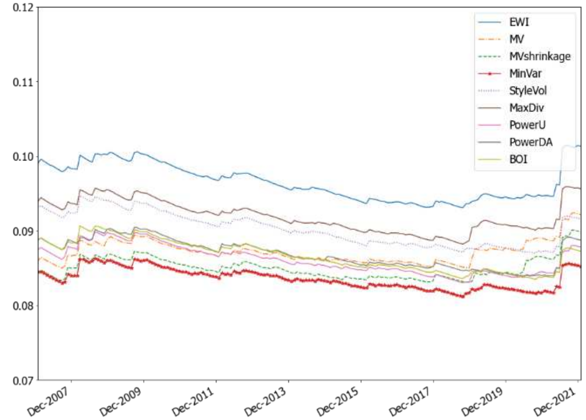

Table 1. Summary Statistics for Commodity Futures Returns.

The table summarizes the monthly excess returns of 28 commodity futures contracts through the mean, standard deviation, first-order autocorrelation, skewness, kurtosis, and average correlations between the commodity at hand and the commodities within each sector. Newey-West robust *t*-statistics for the significance of the mean excess returns are reported in parentheses. The returns are annualized. The sample period is shown in the last two columns.

Sector	Commodity	Mean	StDev	AR(1)	Skew	Kurt	Average Correlations				Obs	First obs	Last obs
							Agriculture	Energy	Livestock	Metal		YYYYMM	YYYYMM
Agriculture	Cocoa	-0.054 (-0.267)	0.293	-0.203	0.152	0.727	0.372	0.074	-0.057	0.212	360	199201	202112
	Coffee	-0.116 (-0.845)	0.357	-0.046	0.657	1.744	0.450	0.026	-0.039	0.252	360	199201	202112
	Corn	-0.107 (-1.513)	0.261	0.021	-0.125	0.846	0.743	0.156	-0.021	0.241	360	199201	202112
	Cotton	-0.077 (-0.725)	0.276	-0.023	-0.079	0.541	0.529	0.113	0.029	0.272	360	199201	202112
	Oat	-0.063 (-0.263)	0.311	0.014	0.128	0.853	0.611	0.150	0.062	0.195	360	199201	202112
	Orange juice	-0.115 (-1.300)	0.308	-0.130	0.004	0.288	0.361	0.050	0.052	0.125	360	199201	202112
	Rough rice	-0.138 (-2.085)	0.265	-0.042	0.345	2.448	0.404	0.009	-0.025	0.067	360	199201	202112
	Soyabean meal	0.064 (2.024)	0.274	-0.048	-0.013	1.379	0.680	0.116	-0.062	0.153	360	199201	202112
	Soyabean oil	-0.062 (-0.758)	0.240	-0.079	-0.321	2.187	0.674	0.129	0.078	0.281	360	199201	202112
	Soyabeans	0.008 (0.848)	0.239	-0.060	-0.509	1.442	0.795	0.138	0.000	0.251	360	199201	202112
	Sugar no. 11	-0.093 (-0.399)	0.339	0.143	-1.139	8.707	0.361	0.059	-0.005	0.180	360	199201	202112
	Wheat CBT	-0.142 (-2.268)	0.288	-0.068	0.174	1.125	0.641	0.114	0.045	0.221	360	199201	202112
	Lumber	-0.133 (-1.409)	0.325	0.029	0.155	0.602	0.331	0.027	0.093	0.165	360	199201	202112
Energy	WTI crude oil	-0.033 (-0.228)	0.311	0.173	-0.445	1.105	0.199	0.797	0.097	0.314	360	199201	202112
	PJM electricity	-0.213 (-2.006)	0.373	0.180	0.105	6.829	0.069	0.552	0.049	0.090	360	199201	202112
	Heating oil	0.006 (0.809)	0.320	0.098	-0.013	1.751	0.160	0.846	0.096	0.271	360	199201	202112
	Natural gas	-0.257 (-1.748)	0.480	0.052	-0.021	0.798	0.075	0.743	0.046	0.087	360	199201	202112
	RBOB gasoline	-0.006 (-0.545)	0.251	0.217	-1.297	10.622	0.286	0.492	0.054	0.374	141	200510	202112
	Unleaded gasoline	0.048 (1.701)	0.274	0.001	0.445	5.060	-0.006	0.661	0.066	0.085	180	199201	200701

(*cont.*)

Livestock	Feeder cattle	0.011 (0.734)	0.140	0.040	-0.354	0.927	-0.075	0.051	0.604	0.018	360	199201	202112
	Frozen pork bellies	-0.035 (0.246)	0.312	-0.173	0.172	2.873	0.116	0.057	0.724	0.072	234	199201	201107
	Lean hogs	-0.087 (-0.981)	0.288	-0.045	-0.191	1.520	-0.028	0.125	0.803	-0.033	360	199201	201507
	Live cattle	0.057 (2.296)	0.154	-0.020	-0.484	3.280	0.045	0.015	0.602	0.041	360	199201	202112
Metal	Copper	0.009 (0.723)	0.256	0.107	-0.543	4.841	0.335	0.259	0.056	0.661	360	199201	202112
	Gold 100 oz (CBT)	0.013 (0.941)	0.155	-0.110	-0.027	1.440	0.242	0.130	-0.024	0.698	360	199201	202112
	Palladium	0.040 (1.343)	0.328	-0.010	-0.302	2.444	0.241	0.186	0.047	0.742	360	199201	202112
	Platinum	0.010 (0.747)	0.211	0.076	-1.072	5.235	0.351	0.231	0.021	0.834	360	199201	202112
	Silver 1000 oz	-0.014 (0.550)	0.281	-0.075	-0.251	1.272	0.271	0.127	-0.023	0.796	360	199201	202112

Table 2. Performance of Individual Commodity Styles.

This table summarizes the annualized excess returns of styles or long-short portfolios formed according to different commodity futures return predictors (sorting signals). The carry style is based on the basis or the log futures price difference between the front- and second-nearest contract, the hedging pressure (HP) style is based on net hedgers' short positions over total positions, the momentum style is based on the past-year average return, the skewness style is based on the Pearson coefficient of skewness of the commodity futures return distribution estimated with past-year daily returns, and the basis-momentum style is based on the differential momentum between front- and second-nearest contracts. Panel A reports statistics over the full sample period January 1992 to December 2021. Panel B reports Sharpe ratios over 6-year non-overlapping subperiods and corresponding style ranks in parenthesis based on the Sharpe ratio with 1 (5) denoting top (bottom) performance.

	Carry	HP	Momentum	Skewness	Basis-Mom
Panel A: *Static portfolio evaluation*					
Mean	0.043	0.036	0.044	0.047	0.051
	(3.030)	(2.784)	(3.001)	(3.343)	(3.757)
StDev	0.081	0.078	0.088	0.072	0.077
Semi-deviation	0.221	0.224	0.247	0.192	0.216
Max Drawdown	-0.248	-0.140	-0.189	-0.209	-0.259
1% VaR	-0.051	-0.049	-0.055	-0.044	-0.047
Sharpe ratio	0.563	0.495	0.533	0.680	0.689
Sortino ratio	0.943	0.788	0.869	1.160	1.124
Omega ratio	1.533	1.440	1.488	1.663	1.696
Cert. equiv. return (CER)	0.0025	0.0021	0.0037	0.0038	0.0064
Panel B: *Dynamic porfolio evaluation: Sharpe ratio (style ranking)*					
Jan 1992 - Dec 1997	0.468(5)	0.476(4)	0.876(3)	1.178(1)	0.892(2)
Jan 1998 - Dec 2003	1.256(1)	0.544(4)	0.996(2)	0.291(5)	0.937(3)
Jan 2004 - Dec 2009	0.907(1)	0.837(3)	0.393(5)	0.902(2)	0.482(4)
Jan 2010 - Dec 2015	0.324(4)	0.279(5)	0.552(3)	0.664(2)	1.012(1)
Jan 2016 - Dec 2021	-0.129(5)	0.274(3)	-0.096(4)	0.440(1)	0.278(2)

Table 3. Standalone Styles Dependence Structure.

The table reports measures of dependence between the monthly excess returns of the individual long-short portfolios. Panel A reports the Pearson correlation (linear dependence). Panels B and C reports the non-parametric Spearman rank-order correlation and Kendall correlation, respectively, that capture linear and nonlinear dependence. The sample period is January 1992 to December 2021.

Panel A: *Pearson correlation*	Carry	HP	Momentum	Skewness	Basis-Mom
Basis	1.000	0.162	0.416	0.150	0.263
HP		1.000	0.240	0.071	0.124
Momentum			1.000	-0.046	0.322
Skewness				1.000	-0.146
Basis-Mom					1.000
Panel B: *Spearman rank-order corr.*	Carry	HP	Momentum	Skewness	Basis-Mom
Basis	1.000	0.179	0.343	0.101	0.317
HP		1.000	0.173	0.090	0.029
Momentum			1.000	-0.088	0.303
Skewness				1.000	-0.124
Basis-Mom					1.000
Panel C: *Kendall correlation*	Carry	HP	Momentum	Skewness	Basis-Mom
Basis	1.000	0.126	0.240	0.069	0.223
HP		1.000	0.120	0.062	0.021
Momentum			1.000	-0.061	0.217
Skewness				1.000	-0.086
Basis-Mom					1.000

Table 4. Performance of Commodity Style-Integrated Portfolios.

The table summarizes the annualized excess returns of the equal-weight style integrated (EWI) portfolio and eight optimized style-integrated (OI) portfolios, as outlined in Section 2 and Appendix C, including the Bayesian optimized integration (BOI). The style-weights estimation is based on L=60 month rolling windows. The commodity scores matrix $\mathbf{\Theta}_t$, Equation (1), contains standardized signals. The reported Ledoit and Wolf (2008) and Opdyke (2007) tests p-values are for the hypothesis $H_0: \Delta SR_j \leq 0$ vs $H_A: \Delta SR_j > 0$ (one-sided tests) where $\Delta SR_j = SR_j - SR_{EWI}$ with j denoting a (B)OI portfolio. The p-value of the Jobson and Korkie (1981) test (with the Memmel (2003) correction) is for the hypothesis $H_0: \Delta SR_j = 0$ vs $H_A: \Delta SR_j \neq 0$ (two-sided). The GMM test p-value is for the hypothesis $H_0: \Delta CER_j = 0$ vs $H_A: \Delta CER_j \neq 0$. Panel A reports statistics over the full-sample from January 1992 to December 2021. Panel B reports Sharpe ratios and ranking over 6-year non-overlapping periods.

		Optimized Style-Integrations (OIs)							
	EWI	MV	MVshrinkage	MinVar	StyleVol	MaxDiv	PowerU	PowerDA	BOI
Panel A: *Static portfolio evaluation*									
Mean	0.080	0.054	0.051	0.075	0.082	0.083	0.052	0.052	0.092
StDev	0.101	0.094	0.094	0.084	0.102	0.096	0.093	0.094	0.087
Semi-deviation	0.272	0.258	0.258	0.209	0.275	0.248	0.258	0.262	0.212
Max Drawdown	-0.243	-0.297	-0.287	-0.158	-0.255	-0.219	-0.296	-0.296	-0.174
1% VaR	-0.061	-0.058	-0.058	-0.050	-0.062	-0.057	-0.058	-0.059	-0.051
Sharpe Ratio (SR)	0.815	0.606	0.577	0.904	0.823	0.886	0.588	0.587	1.060
Sortino ratio	1.393	1.012	0.960	1.677	1.400	1.566	0.976	0.970	1.987
Omega ratio	1.900	1.599	1.563	2.041	1.918	2.023	1.576	1.574	2.309
ΔSR (gain vs EWI)		-0.209	-0.239	0.089	0.008	0.071	-0.227	-0.229	0.245
Ledoit-Wolf test p-value		0.883	0.931	0.222	0.383	0.128	0.902	0.901	0.005
Opdyke test p-value		0.888	0.945	0.151	0.226	0.246	0.902	0.897	0.023
Jobson-Korkie test p-value		0.168	0.074	0.156	0.093	0.064	0.150	0.156	0.004
Cert. equiv. return (CER)	0.005	0.004	0.003	0.005	0.005	0.005	0.003	0.003	0.006
ΔCER (gain vs EWI)		-0.001	-0.002	0.000	0.000	0.000	-0.002	-0.002	0.001
GMM test p-value		0.115	0.066	0.470	0.354	0.250	0.105	0.103	0.034
Panel B: *Dynamic portfolio evaluation: Sharpe ratio (style ranking)*									
Jan 1992 - Dec 1997	1.108(7)	1.296(3)	1.107(8)	1.293(4)	1.103(9)	1.265(6)	1.278(5)	1.300(2)	1.373(1)
Jan 1998 - Dec 2003	0.999(4)	1.000(3)	0.860(8)	0.671(9)	1.002(2)	0.902(7)	0.923(6)	0.931(5)	1.005(1)
Jan 2004 - Dec 2009	1.115(2)	0.378(9)	0.464(6)	1.058(5)	1.113(3)	1.076(4)	0.411(7)	0.398(8)	1.314(1)
Jan 2010 - Dec 2015	0.979(4)	0.513(9)	0.604(6)	1.042(3)	0.977(5)	1.055(2)	0.547(8)	0.558(7)	1.180(1)
Jan 2016 - Dec 2021	0.193(5)	0.116(6)	0.089(7)	0.496(2)	0.194(4)	0.381(3)	0.081(8)	0.072(9)	0.583(1)

Table 5. Net Performance of Commodity Style-Integrated Portfolios (TC = 8.6 bp)

The table reports summary statistics for the out-of-sample annualized net excess returns of the style-integrated portfolios, as outlined in Section 2 and Appendix C, using the transaction cost estimate of 8.6 bp (Marshall et al., 2012). The style-weights estimation is based on L=60 month rolling windows. The commodity scores matrix $\mathbf{\Theta}_t$, Equation (1), contains standardized signals. The reported Ledoit and Wolf (2008) and Opdyke (2007) tests p-values are for the hypothesis $H_0: \Delta SR_j \leq 0$ vs $H_A: \Delta SR_j > 0$ (one-sided tests) where $\Delta SR_j = SR_j - SR_{EWI}$ with j denoting a (B)OI portfolio. The p-value of the Jobson and Korkie (1981) test with the Memmel (2003) correction is for the hypothesis $H_0: \Delta SR_j = 0$ vs $H_A: \Delta SR_j \neq 0$ (two-sided). The GMM test p-value is for the hypothesis $H_0: \Delta CER_j = 0$ vs $H_A: \Delta CER_j \neq 0$. Panel A reports statistics over the full sample from January 1992 to December 2021. Panel B reports Sharpe ratios and ranking over 6-year non-overlapping periods.

		Optimized Style-Integrations (OIs)							
	EWI	MV	MVshrinkage	MinVar	StyleVol	MaxDiv	PowerU	PowerDA	BOI
Panel A: *Static portfolio evaluation*									
Mean	0.075	0.049	0.046	0.071	0.077	0.078	0.047	0.047	0.090
StDev	0.101	0.091	0.099	0.087	0.093	0.095	0.090	0.089	0.087
Semi-deviation	0.275	0.252	0.280	0.223	0.243	0.252	0.253	0.248	0.215
Max Drawdown	-0.245	-0.221	-0.296	-0.176	-0.202	-0.218	-0.223	-0.220	-0.173
1% VaR	-0.062	-0.056	-0.062	-0.052	-0.056	-0.057	-0.056	-0.055	-0.051
Sharpe Ratio (SR)	0.768	0.576	0.506	0.834	0.845	0.838	0.554	0.560	1.031
Sortino ratio	1.298	0.950	0.819	1.486	1.478	1.447	0.906	0.918	1.918
Omega ratio	1.830	1.578	1.480	1.924	1.944	1.938	1.549	1.556	2.252
ΔSR (gain vs EWI)		-0.192	-0.262	0.066	0.077	0.070	-0.214	-0.208	0.263
Ledoit-Wolf test p-value		0.883	0.931	0.222	0.383	0.128	0.902	0.901	0.005
Opdyke test p-value		0.900	0.949	0.140	0.214	0.230	0.914	0.910	0.019
Jobson-Korkie test p-value		0.152	0.070	0.148	0.291	0.258	0.134	0.139	0.003
Cert. equiv. return (CER)	0.004	0.004	0.003	0.005	0.005	0.005	0.003	0.003	0.006
ΔCER (gain vs EWI)		-0.001	-0.002	0.001	0.001	0.001	-0.001	-0.002	0.002
GMM test p-value		0.114	0.065	0.113	0.310	0.205	0.101	0.101	0.027
Panel B: *Dynamic portfolio evaluation: Sharpe ratio (style ranking)*									
Jan 1992 - Dec 1997	1.049(5)	0.982(7)	1.015(6)	1.112(3)	1.105(4)	1.131(2)	0.921(8)	0.897(9)	1.318(1)
Jan 1998 - Dec 2003	0.957(2)	0.814(7)	0.897(4)	0.833(6)	0.889(5)	0.936(3)	0.762(8)	0.742(9)	0.988(1)
Jan 2004 - Dec 2009	1.115(2)	0.378(9)	0.464(6)	1.058(5)	1.113(3)	1.076(4)	0.411(7)	0.398(8)	1.314(1)
Jan 2010 - Dec 2015	0.979(4)	0.513(9)	0.604(6)	1.042(3)	0.977(5)	1.055(2)	0.547(8)	0.558(7)	1.180(1)
Jan 2016 - Dec 2021	0.155(4)	0.076(6)	-0.007(9)	0.276(2)	0.114(5)	0.163(3)	0.021(8)	0.064(7)	0.579(1)

Table 6. Style-integrated portfolios with alternative scoring schemes.

The table summarizes the annualized excess returns of the EWI and OI portfolio strategies, as described in Section 2, based on three different commodity score schemes. The score matrix $\mathbf{\Theta}_t$, Equation (1), contains -1 or +1 (Panel A), standardized ranks (Panel B), and cross-sectionally winsorized signals (Panel C). The estimation is based on L=60 month rolling windows. The reported Ledoit and Wolf (2008) and Opdyke (2007) tests p-values are for the hypothesis $H_0: \Delta SR_j \leq 0$ vs $H_A: \Delta SR_j > 0$ (one-sided tests) where $\Delta SR_j = SR_j - SR_{EWI}$ with j denoting a (B)OI portfolio. The p-value of the Jobson and Korkie (1981) test with the Memmel (2003) correction is for the hypotheses $H_0: \Delta SR_j = 0$ vs $H_A: \Delta SR_j \neq 0$ (two-sided). The GMM test p-value is for $H_0: \Delta CER_j = 0$ vs $H_A: \Delta CER_j \neq 0$.The sample period is January 1992 to December 2021.

		Optimized Style-Integrations (OIs)							
	EWI	MV	MVshrinkage	MinVar	InvVar	MaxDiv	PowerU	PowerDA	BOI
Panel A: *Binary scores*									
Mean	0.079	0.055	0.052	0.076	0.078	0.078	0.055	0.054	0.094
StDev	0.076	0.072	0.071	0.075	0.076	0.076	0.072	0.072	0.074
semi StDev	0.186	0.195	0.195	0.187	0.187	0.189	0.194	0.196	0.175
Max Drawdown	-0.097	-0.159	-0.136	-0.115	-0.101	-0.105	-0.150	-0.151	-0.089
1% VaR	-0.044	-0.044	-0.044	-0.044	-0.044	-0.045	-0.044	-0.044	-0.042
Sharpe Ratio (SR)	1.040	0.786	0.744	1.012	1.034	1.028	0.787	0.770	1.243
Sortino ratio	1.938	1.327	1.245	1.862	1.914	1.904	1.340	1.299	2.420
Omega ratio	2.196	1.798	1.741	2.145	2.184	2.173	1.808	1.793	2.555
ΔSR (gain vs EWI)		-0.254	-0.296	-0.028	-0.005	-0.012	-0.253	-0.270	0.203
Ledoit-Wolf test *p*-value		0.961	0.982	0.630	0.570	0.577	0.963	0.972	0.000
Opdyke test *p*-value		0.998	0.969	0.642	0.450	0.426	0.998	0.998	0.045
Jobson-Korkie test *p*-value		0.045	0.003	0.507	0.534	0.546	0.025	0.027	0.017
Cert. equiv. return (CER)	0.006	0.004	0.003	0.006	0.007	0.006	0.004	0.004	0.007
ΔCER (gain vs EWI)		-0.002	-0.003	0.000	0.000	0.000	-0.002	-0.002	0.001
GMM test *p*-value		0.040	0.001	0.439	0.521	0.261	0.024	0.024	0.037
Panel B: *Standardized rankings*									
Mean	0.084	0.067	0.059	0.083	0.084	0.082	0.068	0.067	0.099
StDev	0.083	0.083	0.083	0.081	0.083	0.082	0.083	0.083	0.081
semi StDev	0.209	0.219	0.224	0.201	0.207	0.207	0.216	0.216	0.195
Max Drawdown	-0.129	-0.197	-0.258	-0.151	-0.130	-0.169	-0.171	-0.185	-0.140
1% VaR	-0.049	-0.050	-0.051	-0.047	-0.049	-0.048	-0.050	-0.050	-0.046
Sharpe Ratio (SR)	1.009	0.824	0.733	1.033	1.017	1.006	0.836	0.824	1.204
Sortino ratio	1.846	1.434	1.245	1.909	1.863	1.829	1.471	1.446	2.305
Omega ratio	2.168	1.889	1.748	2.263	2.189	2.205	1.907	1.891	2.565
ΔSR (gain vs EWI)		-0.185	-0.276	0.024	0.008	-0.003	-0.173	-0.185	0.196
Ledoit-Wolf test *p*-value		0.919	0.984	0.375	0.385	0.516	0.910	0.922	0.000
Opdyke test *p*-value		0.995	0.967	0.338	0.395	0.409	0.995	0.992	0.037
Jobson-Korkie test *p*-value		0.095	0.004	0.362	0.216	0.493	0.060	0.061	0.045
Cert. equiv. return (CER)	0.007	0.005	0.004	0.007	0.007	0.007	0.005	0.005	0.008
ΔCER (gain vs EWI)		-0.002	-0.003	0.000	0.000	0.000	-0.002	-0.002	0.001
GMM test *p*-value		0.070	0.003	0.409	0.245	0.511	0.045	0.041	0.038

(*cont.*)

Panel C: *Winsorized signals*									
Mean	0.072	0.061	0.052	0.074	0.072	0.072	0.062	0.061	0.086
StDev	0.075	0.077	0.076	0.074	0.075	0.074	0.076	0.077	0.074
semi StDev	0.186	0.201	0.205	0.182	0.185	0.183	0.199	0.200	0.174
Max Drawdown	-0.167	-0.170	-0.171	-0.162	-0.162	-0.171	-0.173	-0.171	-0.151
1% VaR	-0.044	-0.046	-0.047	-0.044	-0.044	-0.044	-0.046	-0.046	-0.042
Sharpe Ratio (SR)	0.961	0.812	0.701	1.003	0.974	0.976	0.823	0.811	1.166
Sortino ratio	1.776	1.420	1.192	1.855	1.803	1.809	1.445	1.423	2.258
Omega ratio	2.078	1.854	1.694	2.178	2.109	2.130	1.862	1.846	2.439
∆SR (gain vs EWI)		-0.149	-0.260	0.042	0.013	0.015	-0.138	-0.150	0.205
Ledoit-Wolf test *p* -value		0.925	0.978	0.252	0.261	0.383	0.875	0.889	0.000
Opdyke test *p* -value		0.999	0.982	0.375	0.424	0.481	0.999	0.998	0.019
Jobson-Korkie test *p* -value		0.216	0.026	0.362	0.216	0.493	0.146	0.144	0.012
Cert. equiv. return (CER)	0.006	0.005	0.004	0.006	0.006	0.006	0.005	0.004	0.007
∆CER (gain vs EWI)		-0.001	-0.001	0.002	0.000	0.000	-0.001	0.000	0.002
GMM test *p* -value		0.191	0.014	0.607	0.562	0.824	0.138	0.126	0.012

Table 7. BOI portfolios with alternative priors.

The table summarizes the annualized excess returns of the EWI method alongside three Bayesian optimized style-integrated portfolio implementations based on different priors for the $K \times 1$ style-weights vector and $N \times N$ commodity excess return covariance matrix. BOI_1 is based on the style-weights prior $\boldsymbol{\omega}_t \sim N\left(\boldsymbol{\omega}_0, \frac{1}{\gamma}\boldsymbol{V}_{\boldsymbol{\Theta}}^{-1}\boldsymbol{V}_{\mu}\right)$ with $\boldsymbol{\omega}_0 = \left(\frac{1}{K}, \ldots, \frac{1}{K}\right)'$ and inverse-Wishart covariance prior $\boldsymbol{V}_t \sim IW(\boldsymbol{\Lambda}_0, \nu)$ with $\boldsymbol{\Lambda}_0 = \boldsymbol{I}_N$ (identity matrix) and $\nu = N + 1$ as described in Section 2.3. BOI_2 is based on $\boldsymbol{\omega}_0 = (\frac{1}{K}, \ldots, \frac{1}{K})'$ together with $\boldsymbol{\Lambda}_0 = \bar{\lambda}_t \boldsymbol{I}_N$ with $\bar{\lambda}_t$ is a scalar that represents the average variance across the N commodities, and BOI_3 is based on $\boldsymbol{\omega}_0 = (0.23, 0.23, 0.18, 0.18, 0.18)'$ reflecting stronger beliefs on the carry and hedging pressure styles rooted in fundamental commodity market theory as described in Section 4.3.

		Bayesian style-integration with different priors		
	EWI	BOI_1	BOI_2	BOI_3
Mean	0.080	0.092	0.094	0.090
StDev	0.101	0.087	0.086	0.090
Semi-deviation	0.272	0.212	0.210	0.225
Max Drawdown	-0.243	-0.174	-0.169	-0.186
1% VaR	-0.061	-0.051	-0.050	-0.053
Sharpe Ratio (SR)	0.815	1.060	1.085	1.004
Sortino ratio	1.393	1.987	2.055	1.836
Omega ratio	1.900	2.309	2.355	2.207
ΔSR (gain vs EWI)		0.105	0.131	1.004
Ledoit-Wolf test *p*-value		0.005	0.002	0.007
Opdyke test *p*-value		0.023	0.020	0.036
Jobson-Korkie test *p*-value		0.004	0.004	0.006
Cert. equiv. return (CER)	0.005	0.006	0.006	0.006
ΔCER (gain vs EWI)		0.006	0.006	0.006
GMM test *p*-value		0.034	0.031	0.080

Table 8. Style-integrated portfolio strategies with quarterly holding period.

The table summarizes the out-of-sample annualized excess returns of the EWI portfolio, and optimized style-integrated portfolios, as outlined in Section 2 and Appendix C, rebalanced at the quarterly frequency. Each (B)OI style-integrated portfolio is formed at month-end *t* using a past window of 60-months to determine the style-weights, and the process is repeated at month-end *t*+3. Panel A is based on raw excess returns and Panel B is based on net excess returns using the conservative estimate TC = 17.2 bp. See note to Table 4.

		Optimized Style-Integrations (OIs)							
	EWI	MV	MVshrinkage	MinVar	StyleVol	MaxDiv	PowerU	PowerDA	BOI
Panel A: *excess returns* (TC=0)									
Mean	0.059	0.015	0.033	0.075	0.063	0.057	0.011	0.012	0.081
StDev	0.101	0.093	0.100	0.087	0.093	0.096	0.093	0.092	0.087
Semi-deviation	0.295	0.299	0.301	0.221	0.263	0.275	0.301	0.298	0.219
Max Drawdown	-0.304	-0.330	-0.324	-0.174	-0.236	-0.268	-0.379	-0.366	-0.175
1% VaR	-0.063	-0.061	-0.064	-0.052	-0.057	-0.059	-0.061	-0.061	-0.052
Sharpe Ratio (SR)	0.613	0.210	0.379	0.879	0.704	0.632	0.166	0.174	0.939
Sortino ratio	0.965	0.300	0.578	1.585	1.143	1.008	0.237	0.246	1.710
Omega ratio	1.625	1.180	1.357	1.995	1.730	1.641	1.140	1.147	2.091
ΔSR (gain vs EWI)		-0.402	-0.233	0.266	0.091	0.020	-0.446	-0.439	0.326
Ledoit-Wolf test *p*-value		1.000	0.952	0.299	0.121	0.276	1.000	1.000	0.010
Opdyke test *p*-value		0.999	0.948	0.337	0.131	0.394	1.000	1.000	0.036
Jobson-Korkie test *p*-value		0.001	0.080	0.055	0.033	0.564	0.000	0.000	0.022
Cert. equiv. return (CER)	0.003	0.000	0.001	0.005	0.004	0.003	-0.001	0.000	0.005
ΔCER (gain vs EWI)		-0.003	-0.002	0.002	0.001	0.000	-0.004	-0.003	0.002
GMM test *p*-value		0.001	0.080	0.103	0.091	0.740	0.000	0.001	0.043
Panel B: *net excess returns (TC = 17.2bps)*									
Mean	0.049	0.005	0.024	0.047	0.054	0.048	0.001	0.002	0.073
StDev	0.102	0.093	0.100	0.088	0.093	0.096	0.094	0.092	0.087
Semi-deviation	0.302	0.306	0.309	0.254	0.269	0.281	0.308	0.306	0.225
Max Drawdown	-0.309	-0.386	-0.346	-0.193	-0.241	-0.273	-0.442	-0.429	-0.178
1% VaR	-0.064	-0.062	-0.065	-0.055	-0.058	-0.060	-0.062	-0.061	-0.052
Sharpe Ratio (SR)	0.518	0.106	0.282	0.565	0.610	0.542	0.060	0.066	0.848
Sortino ratio	0.801	0.148	0.420	0.893	0.971	0.846	0.083	0.092	1.510
Omega ratio	1.508	1.087	1.254	1.541	1.607	1.527	1.048	1.054	1.945
ΔSR (gain vs EWI)		-0.413	-0.237	0.047	0.092	0.023	-0.459	-0.452	0.330
Ledoit-Wolf test *p*-value		1.000	0.957	0.278	0.120	0.239	1.000	1.000	0.009
Opdyke test *p*-value		1.000	0.956	0.310	0.104	0.361	1.000	1.000	0.014
Jobson-Korkie test *p*-value		0.000	0.072	0.544	0.030	0.495	0.000	0.000	0.019
Cert. equiv. return (CER)	0.002	-0.001	0.000	0.003	0.003	0.002	-0.001	-0.001	0.005
ΔCER (gain vs EWI)		-0.003	-0.002	0.000	0.001	0.000	-0.004	-0.004	0.002
GMM test *p*-value		0.001	0.078	0.658	0.060	0.562	0.000	0.000	0.032

Table 9. Additional optimized style-integration strategies.

The table reports summary statistics for the annualized excess returns of the optimized style-integrations (OI) outlined in Section 4.5, alongside the EWI benchmark and the proposed BOI. RSI is Rotation-of-Styles Integration. CSI is Cross-Sectional Pricing Integration. PCI is Principal Components Integration. SMI is Style-Momentum Integration. The style-weights estimation is based on L=60 month rolling windows. The commodity scores matrix $\mathbf{\Theta}_t$, Equation (1), contains standardized signals. The reported Ledoit and Wolf (2008) and Opdyke (2007) tests p-values are for the hypothesis $H_0: \Delta SR_j \leq 0$ vs $H_A: \Delta SR_j > 0$ (one-sided tests) where $\Delta SR_j = SR_j - SR_{EWI}$ with j denoting a (B)OI portfolio. The p-value of the Jobson and Korkie (1981) test with the Memmel (2003) correction is for the hypothesis $H_0: \Delta SR_j = 0$ vs $H_A: \Delta SR_j \neq 0$ (two-sided). The GMM test p-value is for the hypothesis $H_0: \Delta CER_j = 0$ vs $H_A: \Delta CER_j \neq 0$. Panel A reports statistics over the full sample from January 1992 to December 2021. Panel B reports Sharpe ratios (and corresponding style-integrated portfolio rankings) over 6-year non-overlapping periods.

		Optimized Style-Integrations (OIs)				
	EWI	RSI	CSI	PCI	SMI	BOI
Panel A: *Static portfolio evaluation*						
Mean	0.080	0.062	0.078	0.001	0.034	0.092
StDev	0.101	0.117	0.099	0.114	0.101	0.087
Semi-deviation	0.272	0.333	0.264	0.356	0.302	0.212
Max Drawdown	-0.243	-0.378	-0.217	-0.076	-0.281	-0.174
1% VaR	-0.061	-0.073	-0.060	-0.566	-0.065	-0.051
Sharpe Ratio (SR)	0.815	0.569	0.809	0.063	0.382	1.060
Sortino ratio	1.393	0.920	1.393	0.092	0.585	1.987
Omega ratio	1.900	1.563	1.901	1.051	1.332	2.309
ΔSR (gain vs EWI)		-0.246	-0.006	-0.753	-0.433	0.245
Ledoit-Wolf test p -value		0.926	0.572	1.000	0.991	0.005
Opdyke test p -value		0.923	0.526	1.000	0.990	0.023
Jobson-Korkie test p -value		0.119	0.845	0.001	0.014	0.004
Cert. equiv. return (CER)	0.005	0.003	0.005	-0.002	0.001	0.006
ΔCER (gain vs EWI)		-0.002	0.000	-0.007	-0.004	0.001
GMM test p -value		0.156	0.681	0.000	0.013	0.034
Panel B: *Dynamic portfolio evaluation: Sharpe ratio (style ranking)*						
Jan 1992 - Dec 1997	1.108(2)	0.512(5)	1.095(3)	0.309(6)	0.681(4)	1.373(1)
Jan 1998 - Dec 2003	0.999(2)	0.973(3)	0.936(4)	-0.669(6)	0.658(5)	1.005(1)
Jan 2004 - Dec 2009	1.115(2)	0.401(6)	0.960(3)	0.699(4)	0.565(5)	1.314(1)
Jan 2010 - Dec 2015	0.979(2)	0.643(4)	0.923(3)	0.626(5)	-0.076(6)	1.180(1)
Jan 2016 - Dec 2021	0.193(4)	0.374(2)	0.257(3)	-0.226(6)	0.086(5)	0.583(1)