The simple answer to the Social Discount Rate question

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THE SIMPLE ANSWER TO THE SOCIAL DISCOUNT RATE QUESTION

by Szabolcs Szekeres*

Abstract: The Social Time Preference Rate (STPR) correctly measures the rate of fall of the value of future benefits, while the Social Opportunity Cost Rate (SOCR) correctly measures the cost of capital of investment projects, but neither rate can correctly compute net present values (NPV) by itself. This paper shows that there is no choice, both must be used simultaneously, a method that is equivalent to shadow pricing capital. This reconciles the two approaches, as their joint use satisfies both of their requirements. Disagreements will remain, however, as reviewed in the paper, about the value of both rates.

Keywords: Social discount rate; Prescriptive discounting; Descriptive discounting; Two-rate discounting; Declining discount rates; Ramsey rule.

JEL classification: D61; H43

1. Introduction

Referring to the familiar descriptive and prescriptive classification of approaches to discounting in cost-benefit analysis (CBA), William Nordhaus (2019) observed that the debate about discounting is "just as unsettled as it was when first raised three decades ago."

According to Arrow *et al* (1995) "Two major approaches are used to determine the appropriate discount rate for climate change analysis. The normative or ethical perspective (called the prescriptive approach in this chapter) begins with the question, 'How (ethically) should impacts on future generations be valued?' The positive perspective, called here the descriptive approach, begins by asking, 'What choices involving trade-offs across time do people actually make?' and, 'To what extent will investments made to reduce greenhouse gas emissions displace investments elsewhere?' " The prescriptive approach usually defines a social time preference rate (STPR), while the descriptive approach generally seeks to measure the social opportunity cost rate (SOCR).

The above classification coined by Arrow *et al* (1995) is a common shorthand in the literature, even though the dichotomy of rates was already observed much earlier, see Baumol (1968). It is primarily caused by the effects of taxes on capital markets. In a simple undistorted market, the interest rate equals both the marginal rate of substitution (MRS) between present and future consumption of savers and the marginal rate of transformation (MRT) of producers. In a distorted market this equality no longer holds, as taxes introduce a wedge between interest paid and interest received. Because savers optimize their consumption path by reference to the interest rate available to them, the net interest received can be called the consumption rate of interest. This is the rate Arrow *et al* (1995) sought to define in discussing the STPR, so we can view it as a measure of how society values future consumption. On the other hand, entrepreneurs who borrow optimize by reference to the higher of the two rates, which leads to the SOCR. Those who advocate the prescriptive approach correctly argue that intertemporal welfare can only be gauged by the SPTR, whereas those who advocate the descriptive approach correctly argue that the SOCR is the true measure of capital costs and that it should be the feasibility hurdle rate.

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High stakes attach to the choice between these discounting approaches, as Nordhaus (2007) concludes in his "A Review of the Stern Review on the Economics of Climate Change." "The Review's unambiguous conclusions about the need for extreme immediate action will not survive the substitution of assumptions that are more consistent with today's marketplace real interest rates and savings rates. Hence, the central questions about global-warming policy—how much, how fast, and how costly—remain open." Advocates of both approaches often hold their views strongly. Freeman and Groom (2010) feel that these disagreements "raise the spectre of the near impossibility of reconciling" the prescriptive and the descriptive approaches to discounting. Spackman (2020) is blunter: "The divide between advocates of social opportunity cost and social time preference (STP) frameworks seems unbridgeable."

This paper proposes to bridge this divide by arguing that the only logical solution to the question of which discount rate to use is to recognize Baumol's (1968) admonition that no single rate can be used. It proposes, instead, to simultaneously use both the STPR and the SORC, but using each only in the role that it is well suited to play. The paper is organized as follows. Section 2 proposes an answer to the selection of social discount rate question. Section 3 presents an alternative path to reaching the same conclusions. Section 4 presents the corroboration of the assertions found in Section 2, by reference to a computable agent-based general equilibrium capital-market model. Section 5 discusses a commonly used way of measuring the SOCR and reviews related issues. Section 6 does the same with respect to the STPR. Finally, Section 7 presents conclusions.

2. The simple answer

The simple answer is that we must abandon the notion of using a single discount rate, because neither the STPR nor the SOCR can compute correct net present values (NPV) of investment projects. This was recognized by Baumol (1968) "We see now that no optimal rate exists. The rate that satisfies the one requirement cannot possibly meet the conditions of the other."

Marglin (1963) demonstrated that two rates are needed to correctly compute NPVs, but it remained unrecognized because this was not the focus of his argument. "The answer is remarkably straightforward. … we plan public projects to maximize their net present value at the marginal social rate of discount, but, in evaluating the social cost of public investment, an opportunity cost reflecting the social value of utilizing resources in private investment *replaces* the money cost of the portion of the resources that comes from the private investment sector." (Emphasis added.)

Assuming that the SOCR measures the opportunity cost of funds, the following simple example shows why both the STPR and the SOCR are needed to define a project's NPV. Take an infinitely lived project with capital costs K and yearly net operational benefits b accruing in perpetuity. In computing the present value of this project, Marglin's key insight was to "replace the money cost" K of the project by its future opportunity cost $K \times SOCR$. The project's NPV therefore is:

$$NPV = -\frac{K \times SOCR}{STPR} + \frac{b}{STPR} \tag{1}$$

The above expression shows that both STPR and SOCR are needed to calculate correctly the NPV of the project. Neither rate can do it by itself.

We can interpret the ratio *SOCR/STPR* as the shadow price of capital, and after having applied it by multiplication of K, it appears that the STPR is sufficient by itself, because it is the sole rate used for discounting benefits. This ignores an often-unrecognized fact, however, that the condition for NPV to be positive is:

$$\frac{b}{STPR} > \frac{K \times SOCR}{STPR} \qquad (2)$$

which reduces to $b > K \times SOCR$. This means that the feasibility hurdle rate of return for projects is the SOCR. This is logical, as projects that fail to cover the opportunity cost of their invested capital are welfare destroying, a fact that is unaffected by the value of STPR, because both benefits and the opportunity cost of capital lie in the future, so discounting them does not change their relative value. But hurdle rate is one thing, while NPV is another. It is not possible to choose between alternative projects of different useful lives without the use of the STPR. So, both rates are needed.

Could a weighted average of the two rates compute a correct conventional single discount rate NPV? The equivalent weighted average rate, called a, can be derived:

$$-K + \frac{b}{a} = -\frac{K \times SOCR}{STPR} + \frac{b}{STPR} \qquad (3)$$

It is possible to solve for a in (3), but computing it is not useful because it is not just a function of $STPR$ and $SOCR$, but also of K and b and is therefore only valid for a specific project and will not calculate the correct NPV for other projects.

Take as an example $K = 1$; $b = 0.7$; $STPR = 2\%$; $SOCR = 5\%$. Then $NPV = 1$, as computed by (1) and by the right-hand side of (3). The equivalent conventional weighted average discount rate obtained by solving for a is 3.5%, which when used in the left-hand side of (3) also results in $NPV = 1$. However, should $b = 0.6$, then $NPV = 0.5$ and the weighted average discount rate that would yield the same result is 4%. As the weighted average discount rate is only valid for a specific project, it is not useful for the evaluation of other projects, so there is no point in computing it.

A simple numerical example of a finitely lived project shows how to compute an NPV with two rates and that the two-rate calculation method is equivalent to conventionally discounting with the STPR after having shadow priced capital. Consider the following project flow.

Table 1
Project flow of a five-year project with constant benefits

Year	**0**	**1**	**2**	**3**	**4**	**5**
Project flow	-1	0.4	0.4	0.4	0.4	0.4

To compute the two-rate NPV we replace the capital cost by its yearly opportunity cost, which is the annuity necessary to repay $K = 1$ with a yield of $SOCR = 5\%$ over five years. This is equal to 0.23 per year. The modified project flow is the following (notice that there are no values in Year 0):

Table 2
Two rate net flow of the project in Table 1

Year	**0**	**1**	**2**	**3**	**4**	**5**
Benefits	0	0.4	0.4	0.4	0.4	0.4
Opportunity cost of capital	0	0.23	0.23	0.23	0.23	0.23
Net flow	0	0.17	0.17	0.17	0.17	0.17

The NPV of the above Net flow is 0.80, which is the difference between the present value of the benefits (1.89) and the present value of the opportunity costs of capital (1.09), all discounted at $STPR = 2\%$.

To compute the NPV using the shadow price of capital (SPC) method, we can use the calculation expression proposed by Cline (1992) and cited in the OECD CBA Manual (2018:207), which will calculate the present value of the opportunity cost of capital over 5 years.

$$SPC = \frac{SOCR}{STPR} \cdot \frac{1-(1+STPR)^{-5}}{1-(1+SOCR)^{-5}} = 1.09 \tag{4}$$

Notice that that the computed SPC is the same as the present value of the opportunity cost of capital shown in Table 2. The project flow using the SPC method is the following:

Table 3
Project flow of Table 1 with SPC adjustment

Year	**0**	**1**	**2**	**3**	**4**	**5**
Project flow	-1.09	0.4	0.4	0.4	0.4	0.4

The NPV of this flow discounted at the STPR is 0.80, the same value obtained using the two-rate method. A weighted average rate can be found that gives this same NPV when used to discount the flow shown in Table 1. It is 3.68% but adds no useful information.

Is the SPC computed in (4) useful for other projects as well? No, for it is also dependent on project-specific information: the useful life of the project, and the assumption that repayments of capital are constant in time, which is reasonable only if the benefits of the project are also constant. The following example shows how the two-rate NPV can be computed for any project.

Table 4
Simple generic project

Year	**0**	**1**	**2**	**3**	**4**	**5**
Project flow	-1	0	0	0.5	0.6	0.7

In this case calculating the net project flow after taking into account the opportunity cost of capital is more complex:

Table 5
Two rate net flow of the project in Table 4

Year	**0**	**1**	**2**	**3**	**4**	5
Benefits	0	0	0	0.5	0.6	0.7
Capital: previous balance	0	1.000	1.050	1.103	0.658	0.091
Capital: opportunity cost	0	0.050	0.053	0.055	0.033	0.005
Capital: amortization	0	0.000	0.000	0.500	0.600	0.095
Capital: current balance	0	1.050	1.103	0.658	0.091	0.000
Net flow after capital costs	0	0.000	0.000	0.000	0.000	0.605

Notice that project benefits are first devoted to amortizing capital and that net benefits only accrue after the capital investment has been fully serviced and repaid. Table 5 shows how the effective opportunity cost of capital, as well as the SPC factor derived from it, depend on the timing of project benefits. The longer it takes for the initial capital investment to be amortized, the longer capital is still committed to the project, and the higher its aggregate opportunity cost. The NPV of the above net flow after capital costs is 0.548, which is the difference between the present value of the benefits (1.659) and the present value of the opportunity costs of capital (1.112), all discounted at $STPR = 2\%$.

The project flow for the SPC adjustment method is:

Table 6
Project flow of Table 4 with SPC adjustment

Year	**0**	**1**	**2**	**3**	**4**	**5**
Project flow	-1.112	0.0	0.0	0.5	0.6	0.7

The NPV of this flow discounted at the STPR is 0.548, the same value obtained using the two-rate method. The NPV computation requires the calculation of how the project can meet its capital expenses and that information can then be used either to compute a specific SPC factor for the project, or a net flow after capital costs. The NPV value derived by either method will be the same. There is no escaping the conclusion that both the STPR and the SOCR are needed to compute project NPVs.

We can see that the SPC in this sample project is different from that of the previous ones, because the SPC adjustment is project-specific. Using expression (4) to compute the SPC would be wrong in this case, because its implicit assumption of even amortization of capital costs is not met and would consequently understate the capital needs of the project. Capital needed is not the same as the initial capital expenditure, but is defined, rather, by the amount and duration of the external financing required. Given equal initial outlays, the SPC of a project that amortizes its capital quickly is lower than that of another that takes longer to do so.

Again, a weighted average of the two rates can be found that gives this same NPV when used to conventionally discount the flow shown in Table 4. It is 3.75% but adds no useful information.

The preceding examples show that discounting a conventional project flow by the STPR without performing the SPC adjustment leads to an overstatement of the NPV. This is an all-too-common practice, as reported in the OECD CBA Manual (2018:221): "Using the Shadow Price of Capital approach (SPC) is advisable when using the [STPR], so that the opportunity cost of public capital can be reflected in the NPV calculation. This rarely happens in practice due to onerous informational requirements." This is regrettable, as it implies that because computing the right result is expensive, we'll settle for the wrong one. Spackman (2020) is again blunter: "One reason for this neglect is doubtless that bodies promoting a CBA can face an incentive to omit it, in searching for high NPVs or [benefit cost ratios]."

It is worth recalling that the feasibility hurdle rate is the SOCR, as shown in (2) above. The example of Table 5 helps to visualize the reason. If the benefits of the project had not been sufficient to amortize the capital used, the project would have had no net benefits and a negative residual value (the unpaid balance on the capital account), which would then have resulted in a negative NPV, no matter what STPR was used for discounting.

The same conclusion holds if in computing the NPV the SPC calculation approach is followed instead of the two-rate method. If a project's IRR is below the SOCR, its benefits are insufficient to pay the opportunity cost of capital, the unpaid balance of which must then be charged to the last time-period of the project. In such cases the computed SPC will be greater than the present value of the benefits, regardless of the discount rate used.

To summarize, the simple answer to the choice of social discount rate question is that there is no choice to be made. Both the STPR and the SOCR are needed to compute NPVs. The conventional discounting equivalent weighted average of the two is unique to each project and is not useful for other projects.

This recognition reconciles the two approaches. Both the two-rate discounting method and the equivalent SPC method fully comply with the requirements of the prescriptive approach by only discounting with the STPR, thereby not undervaluing future benefits and also satisfy the requirements of the descriptive approach because they fully account for the opportunity cost of capital as quantified by the SOCR.

3. An alternative path to the same conclusions

There is an alternative path to the conclusions of the previous section. Baumol's admonition that no single rate can simultaneously provide the correct intertemporal weighting and calculate the correct cost of capital is true because the process of discounting accomplishes both tasks with whatever rate is being used. (This is why project flows never include interest, because including them would double count them, as discounting automatically imputes the interest costs implicit in the discount rate.) Consequently, the only possible solution to the problem is to calculate the NPV in two separate steps, using the correct rate in each.

This is commonly done in financial analysis to compute return to equity. The problem to be solved in that case is analogous to the one we face. A firm borrows to finance an investment, for which it pays interest. To calculate the return to the owners of the firm, we must convert the project flow into the flow to equity, namely the cash-flow of the owners. For simplicity, assume that the owners finance the entire capital expenditure. To convert the project flow, we *replace* the initial capital expenditure by the debt service flow of the loan that finances it. The result is the flow to equity, which contains no initial capital expenditure but reduces the benefits by the debt service of the loan. It is the project flow less the funding flow. This is the equity holders' cash flow, from which a present value can be computed using the equity holders' opportunity cost of funds, which is different from the interest rate on the loan.

The calculations presented in the previous section are analogous. The equity holder role is played by society, which finances its capital expenditures at the SOCR and discounts its cash flow at the STPR. The NPV so computed is correct from a welfare point of view, as the method uses the correct rate at each step: SOCR to account for the opportunity cost of capital and the STPR to discount. Table 5 in the previous Section is a practical example of how to do this.

4. Corroboration

Even though there is no theoretical reason to doubt the conclusions reached thus far, Szekeres (2022) built a computable capital-market general equilibrium model to experimentally corroborate them. It is a two-period agent-based model, with 58 agents. All have constant elasticity of substitution utility functions with coefficients of risk aversion equal to 1, and preset endowments (income) for Year 0 and Year 1. For half of the agents the Year 1 endowment is larger than that of Year 0, for the other half it is less. The agents optimize their consumption path by borrowing, lending, or investing in an equity with stochastic yield, the supply of which is infinitely elastic. Given the time profile of their endowments, half of the agents are inclined to borrow, and the other half are inclined to lend. The model finds the equilibrium market rate of interest at which lending equals borrowing and establishes the amount invested in equities. There is a tax paid on interest income and dividends.

The model calculates endogenously both STPR and SOCR, and it is possible to simulate the effect of undertaking a public sector project. The model assumes that the entire capital expenditure of the project is financed by issuing bonds that the agents subscribe, and that the net benefits of the project will be distributed equally among all agents (or if negative, then collected as a tax).

The model computed the NPV of several sample projects using the two-rate discounting method described in Section 2. The corroboration of the results obtained derives from the gold standard of welfare analysis: the computation of an equivalent compensating variation. The model records the level of expected utility attained by each agent after consumption path optimization for both the with-project and the without-project situations. After this, taking the without-project expected utilities as a benchmark, a calculation was made to determine how much additional Year 0 income would make each agent's expected utility equal to that of the with-project situation. The aggregate welfare improvement attributable to the project is the aggregation of these compensating variations. For all tested sample projects, the aggregate welfare gain (or loss) coincided with the NPV's computed using the two-rate NPV calculation method, which corroborates that the method computes welfare correctly.

The model offers interesting insights into the computation of both the STPR and the SOCR, which will be mentioned as relevant in the following Sections. References there to "the model" are to the model mentioned in this Section.

5. The SOCR

Harberger (1972), in writing on measuring the social opportunity cost of public funds, defines what we call the SOCR as the weighted average of the net-of-tax yield of savings and the gross-of-tax cost of borrowing paid by investors, with the weights being proportional to their respective elasticities of supply and demand. This is consistent with how CBA calculates shadow prices. Raising capital for a public sector project will cause a slight increase in the market rate structure, resulting (1) in a small increase in savings that is valued at the rate that savers require to part from their money and (2) in a small decrease of borrowing by investors, valued at what they would have been willing to pay for it. The aggregate welfare cost of funding a public sector project can therefore be calculated as a weighted average of the two cited rates and constitutes the SOCR.

This weighted average computes the SOCR, which is not the ephemeral project-specific average of the STPR and the SOCR that was found not to be very useful in Section 2 above. The Harberger weighted average is relatively stable, changing only with market conditions, but it computes only the SOCR. Using the SOCR in conventional discounting by itself will yield the correct feasibility hurdle rate, as we have seen, but it will not yield a welfare correct NPV because it will undervalue future benefits. For accurate welfare measurement, the Harberger SOCR must be used in conjunction with the STPR, following either the two-rate discounting method or the equivalent SPC method.

Burgess and Zerbe (2011) expanded on Harberger's formulation by adding foreign borrowing as a third source of funds, thereby adding a third rate and a third weight to the formula. The model described in Section 4 contains further possible elements. As in the model some agents borrow for consumption smoothing, funding a public sector project also displaces some consumer borrowing, the opportunity cost of which is the market interest rate, which is lower than the opportunity cost of reducing investment in higher yield equities. The model adds two more elements to arrive at the endogenous SOCR. First, the transaction cost of intermediation between retail savings and the market in equities. This recognizes that the capital market is segmented. When public funding displaces productive investments, less intermediation is needed, which frees up resources and therefore reduces the SOCR. Second, a compensating variation value is computed that leaves all agents with the same expected utility that they would have had absent public sector borrowing.

It is remarkable how small this compensation variation is, which shows the effective adaptations that agents make to the new conditions. Because the model has an infinitely elastic supply of equities[1] (which is demand for loanable funds for investment purposes) the weighted average SOCR is pulled strongly upwards, but this is partly offset by the inclusion of the intermediation cost.

[1] This is not a realistic assumption but serves to reduce the dimensionality of the optimization problem and reduces the computational burden of solving the model. As the model only serves didactic purposes, it is not a problem.

The model corroborates a statement by Burgess and Zerbe (2013): "the [SOCR] criterion measures the impact of the project on the government's budget when the private sector is kept at pre-project utility." In the model, the sum of the effects of the classical components of the SOCR calculation (displaced productive investments, displaced consumer borrowing and induced savings) nearly exactly corresponds to the sum of two financial items: the interest paid by the public sector on the amount it borrowed, and the amount of taxes forgone because of its intervention in the capital market. Thus, the result of a welfare impact calculation nearly equals the computed out-of-pocket expense of the public sector, making the opportunity cost of capital very tangible.

The Harberger definition of SOCR does not consider any secondary effects from the reinvestment of project benefits, either of the public sector projects or the private ones displaced, to which Marglin (1963) devoted considerable attention. There are two good reasons for this. First, because it is standard CBA practice to ignore most secondary effects, as they might be impossible to accurately trace, and it would be impossible to ascertain whether an imperfect quantification is better than the omission. Second, because alternative projects might have similar effects, so in comparing one to the other, secondary effects would not be important as long as both projects are treated equally. If secondary effects are deemed important, it is better to assess them at the level of project benefits and costs and not make global assumptions by incorporating them in the SOCR, given that projects may differ significantly in this regard. For instance, projects that generate non-pecuniary benefits will have no monetary benefits to reinvest.

Nonetheless, some authors, such as Broughel (2020), for example, do take reinvestment effects into account in defining the SPC. In such cases the equivalence between two-rate discounting and computation via SPC will only hold if reinvestment effects are incorporated into the SOCR as well.

Both the SOCR and the SPC are estimated assuming that public sector projects are funded by borrowing and derive their value from capital market conditions. There is a large body of literature devoted to the opportunity cost of public funds (OCPF) that devotes attention instead to the costs of taxation. This cost is also called marginal cost of public funds (MCF). This literature largely ignores future opportunity costs. In a survey article Massiani and Picco (2013) listed twenty articles on the subject, of which only five mentioned the effect of crowding out of private investments. In commenting on this they added "Eventually, this implies that the question about crowding out is not to know how much it counts but whether it should be taken into account. … Generally, the actual relevance of the crowding out in the OCPF's analysis has to be carefully questioned." Following this line of reasoning, some state that the role of SPC should be played by MCF or OCPF and conclude that the opportunity cost of public funds lies in the present and cannot be expressed as a rate.

In this regard Burgess and Zerbe (2011) stated that "according to the [social opportunity cost of capital] approach the marginal source of funding for all projects is the capital market, thus keeping the issue of tax reform separate from project evaluation. If a particular tax is being proposed to finance a particular project, the revenue from the tax could be used to pay down the debt instead of funding the project, so an alternative use of funds for any project is to pay down the debt."

Using the model, we simulated the effects of debt retirement in the same amount that was borrowed in the debt financing case. The result was a welfare gain virtually identical to the welfare cost of the financing alternative. This is not surprising because the two cases are mirror images. When the public sector raises financing, consumer borrowing and investment in equities decline, whereas when the public sector retires debt nearly the same changes take place with the opposite sign.

We also simulated funding the project solely by a lump sum tax, payable equally by all agents. In the model the SOCR is not much lower than when funding is achieved by borrowing (ignoring the administrative cost of raising the tax). The computed consequence of the lump sum tax is the reduction of investment in equities by about half as much as when the project is funded by borrowing, due to the decline in Year 0 disposable income. However, the compensation variation item grows from being almost negligible to the amount of the lump sum tax times 1+STPR. This stands to reason: to restore agents' utilities to their pretax level, the required Year 1 compensating variation is the future value of the tax. So, to obtain the SOCR we must add the product of the capital cost of the project by the STPR to the opportunity cost of the forgone investments and treat the original invested amount as reimbursable. This is the return that savers require on their forced investment in the project to restore their utility.

The combination of a time zero cost of raising taxes and a lower SOCR for tax funded projects could be easily accommodated by the two-rate calculation method if it was empirically warranted. See Spackman (2020) for a fuller description of the SOCR vs MCF points of view.

The paper has not yet considered the possibility of the SOCR changing over time. Weitzman (1998) asserted that when market interest rates show positive autocorrelation over time, their certainty equivalent rate should be a declining function of time. He stated that the certainty equivalent discount factor should be derived from the expected value of all possible discount factors, which results in declining discount rates (DDR). Gollier (2004) came to the opposite conclusion by treating compound factors likewise, and this became the Weitzman-Gollier Puzzle. It turns out that Weitzman's assertion is based on a fallacy that statisticians warn against: the inverse of a stochastic expected value is not the expected value of the possible inverses[2]. Weitzman discounting violates the definition of present value. Correcting this error results in increasing certainty equivalent discount rates, rather than declining ones. See Szekeres (2020).

The foregoing describes some of the open questions related to the estimation of the SOCR. The best way to elucidate these doubts is through actual empirical studies. These studies do exist (and are not onerously expensive, as they serve all projects in the country for which results are calculated). An estimation for the US was presented in Burgess and Zerbe (2011). The World Bank prepared one for Mexico in 2014, also following Harberger's methodology. See Coppola *et al* (2014).

6. The STPR

Harberger (1972) assumed that "the 'social rate of time preference' refers to an appropriately weighted average of the different marginal rates of time preference applicable to the individuals who compose the society." The model referred to in this paper computes the STPR by finding the compensating variation payment that must be given to agents in Year 0 so that their utilities are the same as when they receive a very small payment in Year 1. The aggregation of these payments indicates the aggregate value that agents place in Year 0 on a small sum received in Year 1. From this the STPR can be computed. This rate turns out to be equal to the weighted average of the rates implicit in the agents' marginal rates of substitution between consumption in Year 0 and Year 1. For those that borrow it is the market interest rate, whereas for those that lend it is the market interest rate net of taxes. The STPR so calculated, which corresponds to Harberger's (1972) description, could be called the revealed preference STPR and is the only one that will compute welfare correct NPV's. Any other STPR value would either underestimate or overestimate future benefits from the point of view of aggregate welfare.

This is not how Arrow *et al* (1995) defined the prescriptive approach to finding the STPR, however. Their stance was rather based on normative or ethical considerations. Perhaps all definitions that depart from Harberger's (1972) description could be called authoritarian, because they replace the revealed preferences of individuals by the preference of someone with authority to

[2] As shown in Szekeres (2020) a Weitzman present value is equal to the correct present value times one minus the covariance between the compound and discount factors. As this covariance is always negative, because the factors are each other's reciprocals, the Weitzman present value is always larger than the correct present value.

know better, be they politicians, social planners, or anyone else invested with the authority to decide. Two surveys of the field of social discounting provide a comprehensive overview: Greaves (2017) and Groom *et al* (2022).

The Ramsey Rule is one method of defining the STPR that is often used according to both surveys. Groom *et al* (2022) characterizes the simple Ramsey Rule as a "workhorse" model for social discounting. Greaves (2017) states that "The standard approach to determining the discount rate is via *the Ramsey equation*." Because of its prominence in the literature, the Ramsey equation merits a closer look. It is derived from a Constant Elasticity of Substitution utility function of the following form:

$$U = \frac{C^{1-\eta} - 1}{1-\eta} \tag{4}$$

where U is utility, C is consumption and η is the constant elasticity of marginal utility of consumption, which can also be interpreted as the coefficient of risk aversion. The first derivative of (4) with respect to C is:

$$U' = \frac{1}{C^{\eta}} \tag{5}$$

The first order condition for consumption path optimization is:

$$\frac{1}{C_0^{\eta}} = exp\ (r - \delta)\frac{1}{C_1^{\eta}} \tag{6}$$

The left-hand side is the marginal utility of consumption in time-period 0, the right-hand side is the marginal utility of consumption in time-period 1. Interest rate *r* for one period reflects the rate at which consumption can be transferred between periods and the pure rate of time preference δ indicates the loss of value of future consumption. The agent achieves equality in (6) by transferring money between the two time-periods until the two sides of the equation become equal, while respecting the agent's budget constraint. Rearranging (6) we obtain:

$$\frac{C_1^{\eta}}{C_0^{\eta}} = exp\ (r - \delta) \tag{7}$$

and taking natural logarithms of both sides and rearranging we reach the Ramsey formula, in which *g* is the growth rate of consumption implicit in C_0 and C_1:

$$r = \delta + \eta g \tag{8}$$

Economists have used this expression to estimate the STPR. Greaves (2017) reports that Stern assumed δ = 0.1%, η =1 and g = 1.3% to arrive at r = 1.4%, whereas Nordhaus (2019) states that "I adopt a time discount rate of 1.5 percent per year with a consumption elasticity of 2. These yield an equilibrium real interest rate of 5.5 percent per year with the consumption growth that is projected over the next century by the DICE-2007 model."

The Ramsey formula may have become the "workhorse" model for social discounting, but the problem is that it is a circular definition. Notice that expression (8) is derived from the basic assumption underlying expression (6), namely that the agent has optimized his consumption path by reference to a *known r*. Expression (8) should not be used to find *r*, therefore.

The Ramsey formula is a tautology that is true for any point along the agent's indifference curve, provided it is tangent to an exogenously determined *r*, for only then will expression (6) be true. Therefore, the Ramsey formula is not a behavioral function that can be used to determine an unknown *r*, because if *r* is not known, then expression (6) cannot be assumed to be true. This is confirmed by the model. Reducing the pure rate of time preference of all agents by half a percentage point does not reduce the endogenous STPR of the model by a like amount, as it should if the Ramsey formula were a behavioral function. The STPR does fall, but by much less. The modification causes a change in the equilibrium interest rate, and new optimal consumption values are chosen, for which the Ramsey expression will be true, as it would be for any possible equilibrium point. Obviously, the Ramsey formula cannot deal with a change in taxes payable on interest revenues that will directly affect *r*. As expression (8) will hold for any conceivable market equilibrium, it cannot be used to find the actual market equilibrium *r*.

Further, the *g* variable of the Ramsey equation is not an exogenous variable because it is the result of optimization. The incomes of agents in Year 0 and Year 1 can be set exogenously, but *g*, computed from C_0 and C_1, can only be calculated once the model has found the market clearing equilibrium *r*, by which time all agents will have reached their optimal consumption paths at the equilibrium rate found.

A moment of reflection should suffice to realize that no expression containing data only about one side of a market, the supply of savings, carries enough information to determine the market interest rate (or set of distorted market rates). For that, information about the demand for loanable funds is also necessary, which in turn is a function of many other things, not the least of which is the technological progress that dominates the productivity of investments, that are entirely exogenous to social time preferences.

Groom *et al* (2022) reported that there is an extended Ramsey rule that adds an additional term to expression (8) because “the social planner is concerned that future consumption levels may be below their expected value and therefore is prepared to save out of precaution in projects that pay off in the future.” The extended formulation would not be able to predict market rates either, not just for the reason of circularity, but also because adding a term to expression (8) implies a violation of expression (6). The reference to the social planner, however, seems to indicate that the Ramsey rule is viewed not as a method of estimating a real-world discount rate, but rather as an intellectual construct useful in devising an authoritarian STPR.

Whether risks should affect the value of the STPR has not been addressed in this paper. For a description of this topic see Spackman (2020).

Unfortunately, empirical estimations of revealed preference STPRs do not seem to exist, even though there is no reason to think that they should be difficult to make. Having such estimates would be useful to cost-benefit analysts interested in calculating the welfare impact of projects and to social planners to measure costs imposed on the present generation by their choice of authoritarian STPRs.

7. Conclusions

Neither the STPR nor the SOCR can compute the correct NPV of investment projects by itself. Both are needed to find the right answer, there is no need to choose between them. Advocates of the prescriptive approach are right: only the STPR can be used for discounting. Advocates of the descriptive approach are also right: the feasibility hurdle rate of return is the SOCR. If a project fails to cover its opportunity cost of capital, it will have a negative residual value, the NPV of which will be negative, no matter what the value of the STPR is. This answer reconciles the two approaches.

To borrow a phrase from Marglin (1963), “the answer is remarkably straightforward.” Even though this conclusion is a direct consequence of elementary equation (1), some will not agree the SOCR is the hurdle rate. Such disagreement is only tenable, however, by taking issue with the equation’s first term.

Some will argue that the first term should be interpreted as the multiplication of the capital expense by the SPC factor (SOCR/STPR), which is not a rate, but a factor, and therefore the SOCR cannot be the hurdle rate. Those holding this view should realize that a properly estimated SPC will assume a value such that no project yielding less than the SOCR will have a positive NPV, as shown in Section 2 above.

Some will take issue with the first term of equation (1) for a different reason, by claiming that the ratio SOCR/STPR should be replaced by a factor such as the OCPF or the MCF. Both generally restrict the cost of raising public funds to the present, which implies that SOCR = 0. Some of those holding this view are likely to persist in their disagreement, despite the two arguments raised in Section 5: (1) that any tax revenue could be used to retire debt instead of funding a project, in which case the opportunity cost of the funds used is again the SOCR, and (2) that even raising taxes has an effect in the future, as tax increases induce taxpayers to reduce their savings. The formal answer to these doubters must be that this is an empirical question to resolve.

The method to calculate NPVs correctly using both the SPTR and the SOCR is clear, but disputes remain regarding the values of these rates. Because public resource allocation is inherently a political process, the setting of the STPR by fiat seems unavoidable. It would be useful, nevertheless, to have empirically measured revealed preference STPR values so that policymakers could assess the cost they impose on the current generation when raising funds through taxation. In contrast, the SOCR is not a discretionary policy variable and should be empirically measured as accurately as possible. This is important to avoid undertaking projects that destroy welfare for future generations. As Becker, Murphy and Topel (2011) stated: “Future generations would not thank us for investing in a low-return project.”

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