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Abstract

This study develops an open-economy Schumpeterian growth model with endogenous takeoff to explore the effects of exports on the transition of an economy from stagnation to innovation-driven growth. We find that a higher export demand raises the level of employment, which causes a larger market size and an earlier takeoff along with a higher transitional growth rate but has no effect on long-run economic growth. These theoretical results are consistent with empirical evidence that we document using cross-country panel data in which the positive effect of exports on economic growth becomes smaller, as countries become more developed, and eventually disappears. We also calibrate the model to data in China and find that its export share increasing from 4.6% in 1978 to 36% in 2006 causes a rapid growth acceleration, but the fall in exports after 2007 causes a growth deceleration that continues until recent times.

JEL classification: O30, O40, F43

Keywords: international trade, innovation, endogenous takeoff

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"Globalization brought much of Asia out of extreme poverty." The Economist (2009)¹

1 Introduction

In the 20th century, many Asian economies started to develop rapidly via export-led growth. It was first Japan in the early 20th century and then the so-called "tiger economies" (Hong Kong, Singapore, South Korea, and Taiwan) in the 1960s. At the end of the 1970s, China also opened up its economy and started to grow rapidly. More recently, the 21st century brought the rise of the "tiger cub economies" (Indonesia, Malaysia, the Philippines, Thailand, and Vietnam). Events of this magnitude naturally prompt the question: how does international trade affect the endogenous transition of an economy from stagnation to economic growth? To explore this question, we develop an open-economy Schumpeterian growth model with endogenous takeoff. We find that an increase in foreign demand for a country's exports gives rise to an earlier takeoff and a higher transitional growth rate of the country's output per capita; however, it does not affect steady-state economic growth.

The economic mechanism producing the results sketched above is that larger demand for a country's exports raises the country's employment and thereby expands the size of its internal market. This expansion gives rise to an earlier takeoff by activating innovation earlier and can even trigger an immediate takeoff.² The reason is that the larger market size increases firm size in the short run and thereby improves firms' incentive to invest in innovation. Post takeoff, as firms invest more in innovation, the transitional growth rate of the country's output per capita rises. In the long run, however, entry of new firms in response to the larger market size causes firm size to converge to a steady-state level that does not depend on the level of employment. As a result, export demand does not affect the steady-state growth rate. These theoretical results are consistent with empirical evidence that we document using cross-country panel data. In our empirical analysis, we indeed find that the positive effect of exports on economic growth becomes smaller as countries become more developed, and eventually disappears.

We also explore the quantitative implications of exports for the takeoff of the economy. We first derive a formula for the derivative of the takeoff time with respect to the export share and find that the magnitude of the effect of exports on takeoff is decreasing in the population growth rate and the degree of labor intensity in production, but increasing in the level of labor and the preference parameter for leisure. We then calibrate the model to data for China and find that an increase in the export share by 0.1 causes the transition to innovation-driven growth to happen over a decade earlier. Furthermore, the increase of China's export share from 4.6% in 1978 to 36% in 2006 causes a rapid growth acceleration, while the fall in exports after 2007 causes a growth deceleration that continues until recent times.

This study relates to the literature on innovation and economic growth. Romer (1990) is the seminal study that develops the R&D-based growth model with variety expansion. Another seminal study is Aghion and Howitt (1992), which develops the quality-ladder Schumpeterian growth model; see Grossman and Helpman (1991a) and Segerstrom *et al.* (1990) for other

¹<https://www.economist.com/asia/2009/03/25/the-export-trap>

²Examining data in the four tiger economies plus China and also India, Ang and Madsen (2011) find that innovation plays a key role for economic growth in these Asian economies.

early studies in this vein. Subsequent studies combine the two dimensions of innovation — variety and quality — to develop the Schumpeterian growth model with endogenous market structure;³ see Peretto (1998, 1999) and Smulders and van de Klundert (1995) for the variant with creative accumulation and Howitt (1999) for the variant with creative destruction.⁴ Our study contributes to this literature by developing an open-economy version of the Schumpeterian model with endogenous market structure and using it to explore the effects of international trade on the complete phase-transition dynamics of economic growth.

This study also relates to the literature on international trade and innovation-driven growth. Early studies by Grossman and Helpman (1990) and Rivera-Batiz and Romer (1991a, b) develop two-country versions of the Romer model, whereas Grossman and Helpman (1991b) develops a small-open-economy version; see Grossman and Helpman (1991c) for a textbook treatment of this literature. All these studies belong to the first generation of R&D-based growth models in which the long-run growth rate exhibits a counterfactual strong scale effect.⁵ Peretto (2003) develops a multi-country Schumpeterian growth model with endogenous market structure that removes the strong scale effect. Subsequent studies apply the open-economy Schumpeterian growth model to explore various issues, such as the cross-country effects of R&D subsidies in Impullitti (2010), the cross-country effects of changes in the resource endowment in Peretto and Valente (2011), and the interaction between comparative advantage in Ricardian trade and innovation-driven growth in Ji and Seater (2020). This study contributes to this literature by developing a small-open-economy version of the Schumpeterian growth model with endogenous market structure and endogenous takeoff to explore the effects of international trade on endogenous takeoff.

Finally, this study relates to the literature on endogenous takeoff and economic growth. The seminal study by Galor and Weil (2000) develops unified growth theory to explain the endogenous transition of an economy from stagnation to growth;⁶ see Galor and Moav (2002), Galor and Mountford (2008), Galor *et al.* (2009) and Ashraf and Galor (2011) for subsequent studies and empirical evidence that supports the theory and Galor (2005, 2011) for a comprehensive review of unified growth theory. A recent branch of this literature examines the transition from stagnation to innovation-driven growth. Peretto (2015) develops a closed-economy Schumpeterian growth model with endogenous takeoff. Subsequent studies by Iacopetta and Peretto (2021), Chu, Fan and Wang (2020), Chu, Kou and Wang (2020), Chu, Furukawa and Wang (2022) and Chu, Peretto and Wang (2022) explore different mechanisms, such as corporate governance, status-seeking culture, intellectual property rights, rent-seeking government and agricultural revolution, that affect endogenous takeoff in that Schumpeterian economy. This study contributes to this literature by developing an open-economy version of the Peretto model to explore the effects of international trade on the transition of the economy from pre-industrial stagnation to innovation-driven growth.

The rest of this study is organized as follows. Section 2 documents some stylized facts. Section 3 develops the model. Section 4 presents our theoretical and quantitative results.

³See Laincz and Peretto (2006), Ha and Howitt (2007) and Madsen (2008, 2010) for empirical evidence that supports this class of Schumpeterian growth models.

⁴A recent study by Garcia-Macia *et al.* (2019) shows that innovation is mostly driven by quality improvement by incumbents (i.e., creative accumulation).

⁵See Jones (1999) and Laincz and Peretto (2006) for a discussion of the strong scale effect.

⁶See also Hansen and Prescott (2002) and Kalemli-Ozcan (2002) for other early studies on endogenous takeoff.

Section 5 explores two extensions of the baseline model. Section 6 concludes.

2 Stylized facts

In this section we document an empirical relationship between exports and economic growth.⁷ We use the following regression specification:

$$g_{it} = \kappa_1 \text{Export}_{it} + \kappa_2 \text{Export}_{it} \times y_{it} + \kappa_3 y_{it} + \Phi_{it} + \zeta_i + \zeta_t + \epsilon_{it},$$

where g_{it} denotes the growth rate of real GDP, the growth rate of real GDP per capita or the growth rate of R&D expenditure in country i at time t . Export_{it} is the ratio of exports to GDP, whereas y_{it} is the initial level of income at time t measured by the log of real GDP per capita. Our theory predicts that $\kappa_1 > 0$ and $\kappa_2 < 0$. In other words, exports have a positive relationship with economic growth, but this positive relationship becomes weaker as the economy becomes more developed. Our theory also predicts that this positive relationship eventually disappears and becomes insignificant as y_{it} becomes large enough.

Φ_{it} denotes the following set of control variables: the log level of the capital stock, government spending as a share of GDP, the real interest rate, and the capital depreciation rate. The variables ζ_i and ζ_t denote country fixed effects and time fixed effects, respectively. Finally, ϵ_{it} is the error term.

Given that the cyclical fluctuations in annual data may bias our estimation, we consider five years as a period to remove these fluctuations.⁸ We thus have a sample of up to 1128 observations covering 205 countries for 1991-2020, after merging data from OECD Data, Penn World Table and World Bank Data. We provide the summary statistics of our data in Appendix A.

Table 1 features the following dependent variables: average annual growth rate of real GDP in columns (1)-(2); average annual growth rate of real GDP per capita in columns (3)-(4); average annual growth rate of R&D expenditure in columns (5)-(6). In all columns, the regression coefficient κ_1 on exports is significantly positive, whereas the regression coefficient κ_2 on the interaction term between exports and the income level is significantly negative.⁹

For example, in column (4) the estimated coefficient on exports is 0.2287, which is statistically significant at the 1% level, whereas the estimated coefficient on the interaction term is -0.0216, which is also statistically significant at the 1% level. These results suggest that exports have a positive relationship with economic growth. However, this positive relationship becomes weaker as the economy becomes more developed. Specifically, for a country with minimal GDP per capita, increasing exports by 1% is associated with an increase in the growth rate

⁷See Sachs and Warner (1995), Edwards (1998) and Frankel and Romer (1999) for empirical studies that also find a positive relationship between international trade and economic growth.

⁸We also consider ten years as a period. In this case, most of the estimated coefficients remain significant at least at the 5% level. Results are available upon request.

⁹We have also considered total factor productivity (TFP) growth as our dependent variable, but the coefficients are insignificant perhaps due to TFP data being too noisy. Interestingly, our results are robust when we use our own simple measures of TFP growth. First, we compute TFP growth as $g_{TFP} = g_y - 0.33g_k$, where g_y and g_k are the growth rate of GDP per capita and capital per capita, respectively. Second, we compute TFP growth as $g_{TFP} = g_y - 0.33g_k - 0.67g_h$, where g_h is human capital per capita; see for example, Klenow and Rodriguez-Clare (2005). Results are reported in Appendix A.

by 0.0953% ($= 0.2287 - 0.0216 \times 6.1777$), which is statistically significant at the 1% level. For a country with average GDP per capita, increasing exports by 1% is associated with an increase in the growth rate by 0.0313% ($= 0.2287 - 0.0216 \times 9.1372$), which is statistically significant at the 5% level. For a country with maximal GDP per capita, increasing exports by 1% is associated with a decrease in the growth rate by 0.0273% ($= 0.2287 - 0.0216 \times 11.8517$), but it is not statistically significant with a p -value of 0.14.¹⁰ Therefore, the positive effect of exports on economic growth becomes smaller, as the level of income rises, and eventually disappears.

Table 1: Effects of exports on economic growth

	GDP growth		per capita GDP growth		R&D growth	
	(1)	(2)	(3)	(4)	(5)	(6)
$Export_{it}$	0.1630*	0.2041**	0.1877**	0.2287***	1.2162***	1.1198**
	(0.0929)	(0.0787)	(0.0890)	(0.0801)	(0.4167)	(0.4367)
$Export_{it} \times y_{it}$	-0.0150*	-0.0188**	-0.0176**	-0.0216***	-0.1059***	-0.1005**
	(0.0083)	(0.0076)	(0.0079)	(0.0079)	(0.0341)	(0.0374)
y_{it}	-0.0483***	-0.0228**	-0.0494***	-0.0283***	0.0120	0.0282
	(0.0092)	(0.0087)	(0.0091)	(0.0077)	(0.0270)	(0.0989)
Control variables		✓		✓		✓
Country fixed effects	✓	✓	✓	✓	✓	✓
Time fixed effects	✓	✓	✓	✓	✓	✓
Observations	1128	529	1128	529	231	107
R^2	0.4111	0.6047	0.4249	0.6040	0.3892	0.5443

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Robust standard errors in parentheses. Standard errors are clustered at the country level.

To alleviate potential endogeneity, we follow Feyrer (2019) and Nigai (2022) and construct an instrument for exports using the gravity model. The intuition is that bilateral trade is heavily influenced by the distance between countries, which is not affected by economic growth. This makes both air distance and sea distance strong predictors of bilateral trade that can be used as instrumental variables for exports. Specifically, using annual bilateral trade data we estimate the following gravity regression:¹¹

$$\ln(Export_{ijk}) = \vartheta_{air,t} \times \ln(airdist_{ij}) + \vartheta_{sea,t} \times \ln(seadist_{ij}) + \gamma_{ij} + \gamma_t + \epsilon_{ijk},$$

where k denotes the year and t denotes the 5-year period. $\ln(Export_{ijk})$ denotes the export flow from country i to j in year k . $\ln(airdist_{ij})$ denotes the log of air distance and $\ln(seadist_{ij})$ denotes the log of sea distance between country i and j ,¹² whereas $\vartheta_{air,t}$ and $\vartheta_{sea,t}$ are their time-varying regression coefficients that change across periods. We also consider bilateral pair fixed effects γ_{ij} and period fixed effects γ_t to improve the predictive power of the gravity model.

¹⁰This insignificant effect at maximal GDP per capita also applies to other columns.

¹¹We follow Feyrer (2021) to exclude oil exporters due to their atypical trade patterns. Specifically, Egypt, Israel, Jordan, Syria, Sudan, Libya, Lebanon and Turkey are left out of the sample; however, our results (available upon request) are robust to including these countries.

¹²Data sources: air distance data is from *CEPII*, and sea distance data is from *CERDI*.

Table A3 in Appendix A reports the results. The relationship between the predicted value of exports and the actual value of exports are relatively strong with a F -statistic of 43.41, which is above the threshold of 10 for a weak instrument. We then follow Frankle and Romer (1999) and Feyrer (2019) to aggregate the export flows to obtain the predicted value of aggregate exports in each country for each 5-year wave as follows:

$$predicted_Export_{it} = \sum_{j \neq i} exp \left(\hat{\vartheta}_{air,t} \times \ln(airdist_{ij}) + \hat{\vartheta}_{sea,t} \times \ln(seadist_{it}) + \hat{\gamma}_{ij} + \hat{\gamma}_t \right).$$

We employ the identifying assumption that geographical bilateral distance across countries has no direct effect on economic growth; therefore, the use of the predicted value of exports alleviates any endogeneity concerns. Hence, we use the value of exports predicted by bilateral distance as an explanatory variable to explore the effects of exports on economic growth. Table 2 reports the results. In columns (1)-(6), κ_1 remains positive and statistically significant whereas κ_2 remains negative and statistically significant. Therefore, our IV regression results are consistent with our baseline regression results, implying that exports indeed have a positive effect on economic growth but this positive effect becomes smaller as the economy becomes more developed.

Table 2: Effects of exports on economic growth (IV)

	GDP growth		per capita GDP growth		R&D growth	
	(1)	(2)	(3)	(4)	(5)	(6)
$predicted_Export_{it}$	0.6699** (0.2838)	0.7684*** (0.2322)	0.6389** (0.2817)	0.6607*** (0.2235)	2.7289*** (0.9069)	2.7307** (1.0693)
$predicted_Export_{it} \times y_{it}$	-0.0674*** (0.0255)	-0.0772*** (0.0211)	-0.0640** (0.0253)	-0.0638*** (0.0201)	-0.2346*** (0.0768)	-0.2306** (0.0871)
y_{it}	-0.0435*** (0.0116)	-0.0145* (0.0080)	-0.0464*** (0.0114)	-0.0239*** (0.0071)	0.0384 (0.0376)	0.0540 (0.0994)
Control variables		✓		✓		✓
Country fixed effects	✓	✓	✓	✓	✓	✓
Time fixed effects	✓	✓	✓	✓	✓	✓
Observations	886	508	886	508	214	105
R^2	0.4195	0.6426	0.4317	0.6375	0.3650	0.5356

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Robust standard errors in parentheses. Standard errors are clustered at the country level.

3 An open-economy Schumpeterian growth model

The Schumpeterian growth model with endogenous takeoff is based on Peretto (2015). That model features both the development of new products and the improvement of the quality of existing products. The combination of the two dimensions of innovation gives rise to the endogenous market structure that removes the strong scale effect. We convert the closed-economy model into a small-open-economy version that preserves the tractability of the original

model and enables us to solve analytically for the transition dynamics of the economy from pre-industrial stagnation to modern growth.

3.1 Household

A representative household has the following utility function:

$$U = \int_0^{\infty} e^{-(\rho-\lambda)t} L_0 \left[\ln c_t + \delta \ln(1 - l_t) + \psi \frac{(l_t)^{1-\varepsilon}}{1-\varepsilon} \right] dt,$$

where $\varepsilon \in [0, 1)$. The parameter $\rho > 0$ is the subjective discount rate, $\psi > 0$ is the preference parameter for per capita consumption of an imported good l_t , and c_t is per capita consumption of a domestically produced final good, which is also the numeraire. $\delta > 0$ is the preference parameter for leisure $1 - l_t$, where l_t is the supply of labor per household member. Finally, the parameter $\lambda \in (0, \rho)$ is the growth rate of the population, which evolves according to $L_t = L_0 e^{\lambda t}$.

The asset-accumulation equation is

$$\dot{a}_t = (r_t - \lambda)a_t + w_t l_t - c_t - p_t l_t, \quad (1)$$

where a_t is the value of assets per household member and r_t is the domestic real interest rate.¹³ Each household member supplies l_t units of labor to earn the wage w_t . Finally, p_t is the price of the imported good relative to the domestic final good.

Dynamic optimization yields the familiar consumption Euler equation

$$\frac{\dot{c}_t}{c_t} = r_t - \rho, \quad (2)$$

the (inverse) demand for the foreign good

$$p_t = \frac{\psi c_t}{(l_t)^\varepsilon}, \quad (3)$$

and the supply of labor

$$l_t = 1 - \frac{\delta c_t}{w_t}. \quad (4)$$

3.2 Domestic final good

The final good Y_t is produced by competitive domestic firms. The production function is

$$Y_t = \int_0^{N_t} X_t^\theta(i) [Z_t^\alpha(i) Z_t^{1-\alpha} L_{y,t} / N_t^{1-\sigma}]^{1-\theta} di, \quad (5)$$

where $\{\theta, \alpha, \sigma\} \in (0, 1)$. There is a variety of N_t differentiated intermediate goods at time t . The quantity of each differentiated intermediate good $i \in [0, N_t]$ is denoted by $X_t(i)$, while

¹³We assume that the domestic financial market is not integrated in the global financial market; see Section 4.5 for a discussion of this assumption.

the good's quality level is denoted by $Z_t(i)$. The average quality across intermediate goods is $Z_t \equiv \frac{1}{N_t} \int_0^{N_t} Z_t(i) di$ and the parameter α determines the degree $1 - \alpha$ of technology spillovers. Production labor is denoted $L_{y,t}$ and the specification $L_{y,t}/N_t^{1-\sigma}$ captures a congestion effect $1 - \sigma$ of variety that removes the strong scale effect for $\sigma < 1$.

Profit maximization yields the conditional demand functions for $\{L_{y,t}, X_t(i)\}$:

$$L_{y,t} = (1 - \theta)Y_t/w_t, \quad (6)$$

$$X_t(i) = \left[\frac{\theta}{P_t(i)} \right]^{1/(1-\theta)} Z_t^\alpha(i) Z_t^{1-\alpha} L_{y,t}/N_t^{1-\sigma}, \quad (7)$$

where $P_t(i)$ denotes the price of $X_t(i)$. Competitive firms pay $(1 - \theta)Y_t = w_t L_{y,t}$ for production labor and $\theta Y_t = \int_0^{N_t} P_t(i) X_t(i) di$ for intermediate goods.

3.3 Intermediate goods and in-house R&D

Each differentiated intermediate good i is produced by a monopolistic firm, which uses a linear one-to-one production function. Specifically, the monopolistic firm employs $X_t(i)$ units of the final good to produce $X_t(i)$ units of intermediate good i . Moreover, it incurs a fixed operating cost $\phi Z_t^\alpha(i) Z_t^{1-\alpha}$ in units of the final good, where $\phi > 0$ is an operating cost parameter. To improve the quality $Z_t(i)$ of intermediate good i , the firm also invests $R_t(i)$ units of the final good; the in-house R&D process is

$$\dot{Z}_t(i) = R_t(i). \quad (8)$$

The profit flow (before R&D) of the firm at time t is

$$\Pi_t(i) = P_t(i) X_t(i) - X_t(i) - \phi Z_t^\alpha(i) Z_t^{1-\alpha}, \quad (9)$$

where $P_t(i) X_t(i)$ is the firm's revenue. The value of the monopolistic firm is

$$V_t(i) = \int_t^\infty \exp\left(-\int_t^s r_u du\right) [\Pi_s(i) - R_s(i)] ds. \quad (10)$$

The firm maximizes $V_t(i)$ subject to (7)-(9). We solve this optimization problem in Appendix B. The solution consists of the monopolistic price set by the firm and of an expression for the rate of return to quality innovation.

The monopolistic price is $P_t(i) = \min\{\mu, 1/\theta\} = \mu$, where $\mu \in (1, 1/\theta)$ is the unit production cost of competitive firms that can imitate the production of $X_t(i)$ with the same quality $Z_t(i)$ as the monopolistic firm. Bertrand competition implies that the monopolistic firm must charge the limit-price μ that is below the unconstrained profit-maximizing price $1/\theta$.

As shown in previous studies, the resulting equilibrium is symmetric with $Z_t(i) = Z_t$ and $X_t(i) = X_t$ for $i \in [0, N_t]$.¹⁴ Substituting $P_t(i) = \mu$ in (7), and using the labor market clearing condition $L_{y,t} = l_t L_t$, yields quality-adjusted firm size as

$$\frac{X_t}{Z_t} = \left(\frac{\theta}{\mu}\right)^{1/(1-\theta)} \frac{l_t L_t}{N_t^{1-\sigma}}, \quad (11)$$

¹⁴Symmetry also implies $\Pi_t(i) = \Pi_t$, $R_t(i) = R_t$ and $V_t(i) = V_t$.

where l_t is the employment ratio. This result suggests that we can express the dynamics of the economy in terms of the state variable

$$x_t \equiv \left(\frac{\theta}{\mu} \right)^{1/(1-\theta)} \frac{L_t}{N_t^{1-\sigma}}. \quad (12)$$

In this notation, the rate of return to quality-improving R&D is

$$r_t^q = \alpha [(\mu - 1)x_t l_t - \phi]; \quad (13)$$

see Appendix B for the details of the derivation.

3.4 Entrants

To support the symmetric equilibrium at any time t , we follow previous studies and assume that new firms enter with quality level Z_t . Developing a new intermediate good and setting up the physical structure (plant and equipment) to serve the market costs βX_t units of the final good, where $\beta > 0$ is an entry-cost parameter. The asset-pricing equation that determines the rate of return on assets is

$$r_t = \frac{\Pi_t - R_t}{V_t} + \frac{\dot{V}_t}{V_t}. \quad (14)$$

The free-entry condition is

$$V_t = \beta X_t. \quad (15)$$

We substitute (8), (9), (11), (12), (15) and $P_t(i) = \mu$ into (14) to derive the rate of return to entry as

$$r_t^e = \frac{1}{\beta} \left(\mu - 1 - \frac{\phi + z_t}{x_t l_t} \right) + \frac{\dot{l}_t}{l_t} + \frac{\dot{x}_t}{x_t} + z_t, \quad (16)$$

where $z_t \equiv \dot{Z}_t/Z_t$ is the quality growth rate.

3.5 International trade

We consider a small open economy that sells abroad a constant fraction of its final output because we want to explore the effects of exogenous changes in the export share. Specifically, the economy exports χY_t units of the final good, where $\chi > 0$ is an export demand parameter, and uses the export revenue to pay for the imported good. The balanced-trade condition is

$$p_t l_t = \chi y_t, \quad (17)$$

where $y_t \equiv Y_t/L_t$ is final output per capita.

3.6 Equilibrium

The equilibrium is a time path of allocations $\{a_t, \iota_t, c_t, Y_t, L_{y,t}, l_t, X_t(i), R_t(i)\}$ and a time path of prices $\{r_t, w_t, p_t, P_t(i), V_t(i)\}$ such that the following conditions are satisfied:

- the household chooses $\{\iota_t, c_t, l_t\}$ to maximize utility taking $\{p_t, r_t, w_t\}$ as given;

- competitive firms choose $\{L_{y,t}, X_t(i)\}$ to produce Y_t and maximize profit taking $\{w_t, P_t(i)\}$ as given;
- a monopolistic firm produces $X_t(i)$ and chooses $\{P_t(i), R_t(i)\}$ to maximize $V_t(i)$ taking r_t as given;
- entrants make entry decisions taking V_t as given;
- the value of monopolistic firms is equal to the total value of household assets such that $N_t V_t = a_t L_t$;
- the labor market clears such that $l_t L_t = L_{y,t}$;
- the balanced-trade condition holds such that $p_t l_t L_t = \chi Y_t$; and
- the domestic final-good market clears.

3.7 Aggregation

Substituting (7) and $P_t(i) = \mu$ into (5), imposing symmetry and using $L_{y,t} = l_t L_t$, yields the reduced-form production function for the final good

$$Y_t = \left(\frac{\theta}{\mu}\right)^{\theta/(1-\theta)} N_t^\sigma Z_t L_t l_t. \quad (18)$$

The resulting growth rate of output per capita is

$$g_t \equiv \frac{\dot{y}_t}{y_t} = \sigma n_t + z_t + \frac{\dot{l}_t}{l_t}, \quad (19)$$

where $n_t \equiv \dot{N}_t/N_t$ is the variety growth rate and $z_t \equiv \dot{Z}_t/Z_t$ is the quality growth rate.

3.8 Dynamics

As argued, we express the model's equilibrium dynamics in terms of the state variable x_t defined in (12). Its law of motion is given by

$$\frac{\dot{x}_t}{x_t} = \lambda - (1 - \sigma)n(x_t), \quad (20)$$

where the function $n(x_t)$ describes the variety growth rate n_t in the model's equilibrium. We construct the function in Appendix B. Here, we highlight the properties that we use to obtain our main results.

Investment in either variety expansion or quality improvement is irreversible and thus subject to a non-negativity constraint. Moreover, each form of investment is a sunk cost. Therefore, for each one, there exists a threshold of firm size below which the return that it promises falls below the reservation interest rate of savers who are then not willing to finance it. We denote these thresholds x_N and x_Z , respectively, and consider the case $x_N < x_Z$. Accounting for these thresholds and using a standard no-arbitrage argument to construct our equilibrium, we find

that $n(x_t)$ is zero for $x_t \leq x_N$ and positive for $x_t > x_N$. When positive, $n(x_t)$ is a decreasing function of x_t with two branches: one that applies for $x_t \leq x_Z$ and the other that applies for $x_t > x_Z$. This partition of the x axis allows us to represent the process described by (20) as consisting of the three phases discussed throughout the paper.

Appendix B shows that the dynamics of x_t are globally stable if the following parameter condition holds:

$$\beta\phi > \frac{1}{\alpha} \left[\mu - 1 - \beta \left(\rho + \frac{\sigma\lambda}{1-\sigma} \right) \right] > \mu - 1. \quad (21)$$

Specifically, given initial condition $x_0 < x_N$, the state variable x_t gradually increases towards its steady-state value

$$x^* \equiv \arg \underset{x_t}{\text{solve}} \left\{ \frac{\lambda}{1-\sigma} = n(x_t) \right\},$$

crossing first the threshold x_N and then the threshold x_Z . This process gives us our story: the economy begins in a pre-industrial era in which both the variety growth rate n_t and the quality growth rate z_t are zero because firm size $x_t l_t$ is not large enough to support innovation. When firm size becomes large enough, the economy enters the first phase of the industrial era in which monopolistic firms develop and bring to market new products ($n_t > 0$). As firm size continues to grow, the economy enters the second phase of the industrial era in which existing monopolistic firms improve the quality of their products ($z_t > 0$). In the long run, the economy converges to the balanced growth path with constant steady-state growth as x_t converges to its steady-state value x^* .

To flesh out the details of the story, we begin with the characterization of equilibrium employment and consumption. As in previous studies (such as Chu, Furukawa and Wang (2022) and Chu, Peretto and Wang (2022)), we assume that in the pre-industrial era monopolistic firms do not yet operate and competitive firms produce intermediate goods with the constant unit production cost μ , making zero profit.¹⁵ As the economy enters the industrial era, monopolistic firms take over the existing markets and innovation starts, first only variety expansion and then both variety expansion and quality improvement. The following proposition describes the behavior of consumption and employment throughout this process.

Proposition 1 (*Consumption and employment*) *Assume $1 - \theta > \chi$. At any time t , the consumption ratio c_t/y_t and the employment ratio l_t are, respectively:*

$$\frac{c_t}{y_t} = \begin{cases} 1 - \theta - \chi & 0 \leq x_t \leq x_N \\ \frac{(\rho-\lambda)\beta\theta}{\mu} + 1 - \theta - \chi & x_N < x_t < \infty \end{cases};$$

$$l_t = \left(1 + \frac{\delta}{1-\theta} \frac{c_t}{y_t} \right)^{-1} = l^* = \begin{cases} \left[1 + \frac{\delta}{1-\theta} (1 - \theta - \chi) \right]^{-1} & 0 \leq x_t \leq x_N \\ \left\{ 1 + \frac{\delta}{1-\theta} \left[\frac{(\rho-\lambda)\beta\theta}{\mu} + 1 - \theta - \chi \right] \right\}^{-1} & x_N < x_t < \infty \end{cases}.$$

Proof. See Appendix B. ■

These two results say that the employment and consumption ratios are always constant. This property is the reason why the model's equilibrium dynamics reduce to the single-variable differential equation (20).

¹⁵In Appendix C, we solve the model without this assumption to show that the dynamics becomes less realistic.

4 Export-led takeoff

In this section we explore how an expansion in export demand affects the transition of the economy from the pre-industrial era without innovation to the industrial era with innovation. After providing analytical results, we calibrate the model to data to perform a quantitative analysis in Section 4.5.

4.1 The pre-industrial era

In the pre-industrial era, firm size $x_t l^*$ is not large enough to support innovation. Consequently, the growth rate of output per capita is $g_t = \sigma n_t + z_t + \dot{l}_t/l_t = 0$. The state variable x_t follows

$$\frac{\dot{x}_t}{x_t} = \lambda - (1 - \sigma)n_t = \lambda > 0$$

and thus grows exponentially. This means that for initial condition $x_0 < x_N$, the state variable x_t crosses the threshold x_N in finite time.

4.2 The first phase of the industrial era

The first phase of the industrial era begins when the variety growth rate n_t becomes positive. In Appendix B, we show that the variety growth rate n_t is

$$n_t = \frac{1}{\beta} \left(\mu - 1 - \frac{\phi}{x_t l^*} \right) + \lambda - \rho, \quad (22)$$

where l^* is the constant value from Proposition 1. This expression says that variety growth n_t is positive if and only if

$$x_t > x_N(\chi) \equiv \frac{\phi}{\mu - 1 - \beta(\rho - \lambda)} \left\{ 1 + \frac{\delta}{1 - \theta} \left[\frac{(\rho - \lambda)\beta\theta}{\mu} + 1 - \theta - \chi \right] \right\}. \quad (23)$$

The threshold x_N is thus decreasing in the export share χ . Our mechanism, therefore, is that an expansion of export demand reduces the threshold x_N , and thereby gives rise to an earlier activation or even an immediate activation of innovation, because it raises the employment ratio l^* and thus firm size $x_t l^*$. The following Proposition summarizes this result.

Proposition 2 (*Export share in the 1st phase*) *A larger export share χ leads to an earlier transition of the economy from pre-industrial stagnation to innovation-driven growth and a higher transitional growth rate g_t in the first phase of the industrial era.*

Proof. Recall that x_t increases at the exogenous rate λ in the pre-industrial era. Then, use (23) to show that the threshold x_N is decreasing in χ . Finally, use (22) to show that $g = \sigma n_t$ is increasing in l^* , which is increasing in χ as derived in Proposition 1. ■

We now use (20) and (22) to obtain

$$\frac{\dot{x}_t}{x_t} = \lambda - (1 - \sigma) \left[\frac{1}{\beta} \left(\mu - 1 - \frac{\phi}{x_t l^*} \right) + \lambda - \rho \right]. \quad (24)$$

This is a stable process with its own steady state \bar{x} (shown in Appendix B). Under condition (21), however, the state variable x_t keeps rising because n_t is below its steady-state value $n^* = \lambda/(1 - \sigma)$ throughout the interval $x_N \leq x_t \leq x_Z$. This means that the economy enters the second phase of the industrial era in finite time.

4.3 The second phase of the industrial era

As stated above, the economy eventually enters the second phase of the industrial era in which both quality growth ($z_t > 0$) and variety growth ($n_t > 0$) take place. We combine (2) and (13) by setting $r_t = r_t^q$ to derive

$$g_t \equiv \frac{\dot{y}_t}{y_t} = \frac{\dot{c}_t}{c_t} = \alpha \left[(\mu - 1)x_t l^*(\chi) - \phi \right] - \rho, \quad (25)$$

where the first equality uses the result from Proposition 1 that the consumption ratio is constant. Equation (25) shows that for given x_t , the growth rate of output per capita g_t is once again increasing in the export share χ via firm size $x_t l^*$. In Appendix B, we derive separately the quality growth rate z_t and the variety growth rate n_t . We then show that $z_t = z(x_t)$ is positive if and only if the state variable x_t rises above the threshold

$$x_Z(\chi) \equiv \arg \underset{x}{\text{solve}} \left\{ [(\mu - 1)x l^*(\chi) - \phi] \left[\alpha - \frac{\sigma}{\beta x l^*(\chi)} \right] = (1 - \sigma)\rho + \sigma\lambda \right\}. \quad (26)$$

As said, we set our parameters to ensure $x_N < x_Z$. The derivation in Appendix B shows that the state variable x_t follows the law of motion

$$\dot{x}_t = \frac{1 - \sigma}{\beta} \left\{ \left[(1 - \alpha)\phi - \left(\rho + \frac{\sigma\lambda}{1 - \sigma} \right) \right] \frac{1}{l^*} - \left[(1 - \alpha)(\mu - 1) - \beta \left(\rho + \frac{\sigma\lambda}{1 - \sigma} \right) \right] x_t \right\} \quad (27)$$

and converges to the steady state

$$x^* = \frac{1}{l^*} \frac{(1 - \alpha)\phi - [\rho + \sigma\lambda/(1 - \sigma)]}{(1 - \alpha)(\mu - 1) - \beta [\rho + \sigma\lambda/(1 - \sigma)]} > x_Z. \quad (28)$$

Substituting (28) into (25) yields the steady-state growth rate

$$g^* = \alpha \left[(\mu - 1) \frac{(1 - \alpha)\phi - [\rho + \sigma\lambda/(1 - \sigma)]}{(1 - \alpha)(\mu - 1) - \beta [\rho + \sigma\lambda/(1 - \sigma)]} - \phi \right] - \rho > 0, \quad (29)$$

which shows that steady-state growth is independent of the employment ratio l^* and thus of the export share χ due to the scale-invariance property of the Schumpeterian growth model with endogenous market structure. Specifically, although steady-state economic growth depends on firm size $x^* l^*$, it is independent of the export share χ because a larger employment ratio l^* implies smaller x^* via a larger mass of firms N_t . One way to see this is to note that (28) yields that the product $x^* l^*$ is a constant independent of the export share χ .

The next Proposition summarizes the effects of the export share in the second phase of the industrial era.

Proposition 3 (*Export share in the 2nd phase*) A larger export share χ leads to an earlier activation of quality innovation and a higher transitional growth rate g_t in the second phase of the industrial era but does not affect the steady-state growth rate g^* .

Proof. Use (25) to show that for given x_t , growth g_t is increasing in l^* , which is increasing in χ by Proposition 1. Then, use (29) to show that g^* is independent of l^* and thus χ . ■

4.4 Exports and the timing of the key events

Before moving on to the quantitative exercise, we bring together all the results in the previous sections to highlight what novel aspects of the process of development our model can illuminate. Recall that we argued that the economy crosses the thresholds x_N and x_Z in finite time. We denote the dates of these two key events T_N and T_Z , respectively. The next proposition provides a formal result concerning these dates that illuminates the model's key mechanism.

Proposition 4 (*Export share and activation times*) For $x_0 < x_N$, the effects of the export share χ on the variety-growth activation time T_N and on the quality-growth activation time T_Z are, respectively:

$$\begin{aligned}\frac{\partial T_N}{\partial \chi} &= -\frac{1}{\lambda} \frac{\delta l^*}{1 - \theta} < 0; \\ \frac{\partial T_Z}{\partial \chi} &= \frac{\partial T_N}{\partial \chi}.\end{aligned}$$

For $x_0 > x_N$ the effect of the export share χ on the quality-growth activation time T_Z is

$$\frac{\partial T_Z}{\partial \chi} = -\frac{1}{\nu} \frac{x_0}{\bar{x} - x_0} \frac{\delta l^*}{1 - \theta} < 0,$$

where $\bar{x} \equiv \frac{\phi}{[\mu - 1 - \beta(\rho + \frac{\sigma\lambda}{1-\sigma})]l^*(\chi)}$ and $v \equiv \frac{1-\sigma}{\beta} [\mu - 1 - \beta(\rho + \frac{\sigma\lambda}{1-\sigma})] > 0$.

Proof. See Appendix B. ■

The magnitude of the negative effect of the export share on the activation time T_N is decreasing in the population growth rate λ and the degree of labor intensity $1 - \theta$ in production, but increasing in the employment ratio l^* and the leisure preference parameter δ . The population growth rate matters because it determines how fast x_t grows towards x_N , whereas $\delta l^*/(1 - \theta)$ matters because it determines the effect of the export share χ on the employment ratio l^* and firm size $x_t l^*$.

The model has the property that if the initial condition is $x_0 < x_N$, the process features a transition time from x_N to x_Z that does not depend on the export share χ (see the proof of the proposition for details). Consequently, the effect on the time of the first phase transition T_N is a summary statistic for the overall effect of the export share on the evolution of the economy. If, instead, the initial condition is $x_N < x_0 < x_Z$, the only possible effect is on the time of transition to the second phase and is governed by the parameters regulating the speed at which the state variable x_t moves in the first phase. These are the composite parameters ν and \bar{x} in the expression in the proposition (see the proof for details).

4.5 Quantitative application

We now calibrate the model to data for the Chinese economy. To set the stage for our analysis, it is useful to discuss briefly the applicability of our model to China.

4.5.1 Some considerations: why China, why this model?

China is a suitable laboratory for our quantitative illustration because its takeoff has been largely export-led.¹⁶ In the late 1970s, China implemented several market-oriented reforms designed to stimulate economic growth. The reforms were successful and China experienced a sharp growth acceleration. Naturally, economists took notice and studied the process in great detail; see, among others, Bai *et al.* (2006) and Song *et al.* (2011).

Many argue that the initial acceleration of Chinese growth was largely driven by capital accumulation, a perspective that might cast doubt on the relevance of our model since it does not feature the typical neoclassical rendition of capital accumulation. We think that our model is actually very relevant. First, our quantitative analysis focuses on TFP growth, which is the residual of economic growth after accounting for the accumulation of physical factors of production (capital and labor). To understand such residual, one needs a model of TFP growth, not of capital accumulation. Moreover, when a firm starts to produce a new good in our model, it needs to set up a production plant with its equipment; i.e., it invests in a lump of capital. In this sense, an expanding variety of products goes hand in hand with a specific form of capital accumulation that is not neoclassical but is capital accumulation nevertheless.¹⁷

A related potential criticism is that some commentators claim that Chinese growth is mostly driven by absorption of foreign technology, if not simple imitation or copying, while our model features innovation. In our judgment, Ang and Madsen (2011) dispose of such claims: the evidence is that in fact, China (and several other Asian economies) also innovate.¹⁸ Moreover, what is innovation in our model, especially variety expansion, can be interpreted as a mix of the transfer and adaptation of foreign technologies to the local context and of the domestic development of novel technologies. We do not model explicitly the transfer of technologies from abroad to keep things simple, but one can easily think of this process as being a part of variety growth while the improvement of the quality of these products represents the emergence of domestic innovation as has been witnessed in China since joining the World Trade Organization (WTO).¹⁹ For example, Figure 1 shows that there is a drastic surge in the number of patent applications and R&D share of GDP in China (relative to the US) around the time of China joining the WTO.²⁰

¹⁶See Wan, Lu and Chen (2007) and Yao (2014) for a discussion of Chinese growth and international trade.

¹⁷In addition, in our model, the cost of creating the marginal unit of N_t is βX_t and one can think of the traditional Tobin's q as this cost βX_t . Indeed, the economics of the accumulation of N_t in our model is formally identical to that of capital accumulation in the neoclassical model. The only difference of substance is that our model features irreversible investment, capturing the idea that the set-up process is the embodiment of product-specific knowledge in a physical structure owned and operated by a firm of non-negligible size.

¹⁸They also provide evidence in favor of the second-generation Schumpeterian growth model, for which the model in this paper belongs to.

¹⁹See Liu and Ma (2020) for empirical evidence.

²⁰See Hu *et al.* (2017) for a discussion on how big a share of these patent applications may reflect innovation.

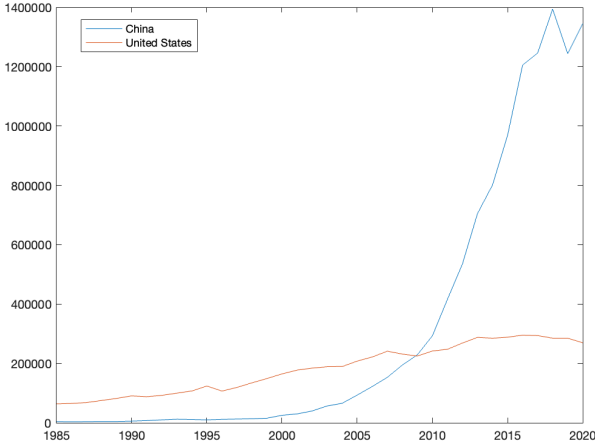


Figure 1a: Number of patent applications

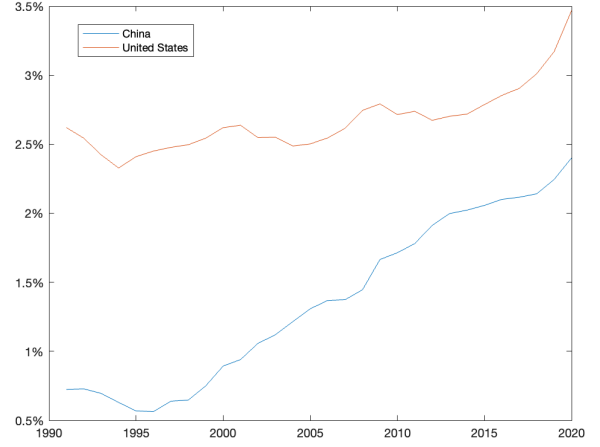


Figure 1b: R&D share of GDP

A final consideration concerns the relevance of our small-open-economy specification. Before China joined the WTO in 2001, the size of its economy (US\$1.21 trillion in 2000) was smaller than that of the UK (US\$1.66 trillion in 2000),²¹ which is often viewed as a small open economy. It wasn't until 2006 that the size of the Chinese economy caught up with that of the UK economy. Furthermore, we show that extending the model to a two-country setting leaves our story the same (see Subsection 5.2). Finally, China's financial market is not integrated to the global financial market, making our assumption of a locally determined interest rate r_t valid. With these considerations in mind, we now turn to our quantitative exercise.

4.5.2 Calibration and counterfactuals

As mentioned, China started opening its economy at the end of the 1970s. In 1978, its export share of GDP was 4.6%. It then rose to roughly 20% in 2000 and reached a peak of 36% in 2006 before falling below 20% in recent times.²² The average population growth rate in China from 1980 to 2020 is 0.9%,²³ and the labor share of output $1 - \theta$ in China is about 0.5,²⁴ which is lower than a typical Western economy. Furthermore, we set the share of time devoted to employment l^* to 0.4, which is higher than a typical Western economy due to the longer working hours in China. As for the leisure preference parameter δ , it is inversely related to the equilibrium level of labor l^* . From Proposition 1, $l^* \approx 1/(1 + \delta)$ if $c/y \approx 1 - \theta$, which are both roughly 0.5 in China. So, setting $\delta = 1.5$, which corresponds to $l^* \approx 1/(1 + \delta) = 0.40$, is a reasonable back-of-the-envelope value. Then, we have

$$\frac{\partial T_N}{\partial \chi} = -\frac{1}{\lambda} \frac{\delta l^*}{1 - \theta} \approx 133,$$

²¹The comparison here is based on the market exchange rate as we are comparing the market size of two economies for international trade, rather than domestic purchasing power.

²²Data source: World Bank Data.

²³Data source: World Bank Data.

²⁴See Bai, Hsieh and Qian (2006).

which implies that an increase in the export share χ by 0.1 triggers an earlier transition to innovation-driven growth in China by 13.3 years.

In the rest of this section, we calibrate the entire model to the Chinese economy to perform a more complete quantitative analysis. The model features the following set of parameters: $\{\lambda, \theta, \rho, \alpha, \sigma, \mu, \beta, \phi, \delta, \chi\}$.²⁵ As before, we set the population growth rate λ to 0.9% and the labor share $1 - \theta$ of output to 0.5. We set the discount rate ρ to a conventional value of 0.03. We follow Iacopetta *et al.* (2019) to set the degree of technology spillovers $1 - \alpha$ to 0.833 and the social return of variety σ to 0.25. We set the markup ratio μ to 1.3 according to the empirical estimates in Lu and Yu (2015), Fan *et al.* (2018) and Wen (2022). Then, we calibrate $\{\beta, \phi, \delta\}$ by matching the following moments of the Chinese economy: 49.5% for the consumption share,²⁶ 1% for the average TFP growth rate,²⁷ and 0.40 for the share of time devoted to work. Finally, the export share in China was 4.6% in 1978. Table 3 summarizes the parameter values.

λ	θ	ρ	α	σ	μ	β	ϕ	δ	χ_{1978}
0.009	0.500	0.030	0.167	0.250	1.300	5.076	0.177	1.515	0.046

Figure 2 plots the path of the export share in China and shows that it rises from 4.6% in 1978 to 36.0% in 2006 before falling to 18.5% in 2020. Using this data, we compute a time path of χ and model the changes in χ as a sequence of unanticipated and permanent changes (i.e., MIT shocks). Given this calibrated path of χ , Figure 3 presents the simulated path of the technology growth rate $\sigma n_t + z_t$ and shows that it matches the HP-filter trend of the TFP growth rate in China reasonably well. Specifically, the technology growth rate gradually increased and was below 1% before China joined the WTO. After that, it accelerated sharply to above 2.5% in 2006, and then it kept falling as in the data.²⁸

Figure 4 presents again the simulated path of the technology growth rate, along with a simulated path of the growth rate without changes in the export share (i.e., χ remains at its initial value of 0.046) and a simulated path of the growth rate in which the export share χ hypothetically remains at its peak value of 0.36 after 2006. This figure shows that the rapid rise in the export share since 1978 (and especially since joining the WTO) has caused a rapid growth acceleration in the Chinese economy. However, the fall in exports since 2007 has also caused a growth deceleration that continues until recent times.

²⁵The import preference parameter ψ affects the equilibrium level of import ι but not the rest of the economy.

²⁶Data source: CEIC Data.

²⁷Data source: Federal Reserve Bank of St. Louis.

²⁸In the last two years of the data, there were the China-US trade war and the COVID-19 pandemic, which are likely responsible for the strongly negative TFP growth.

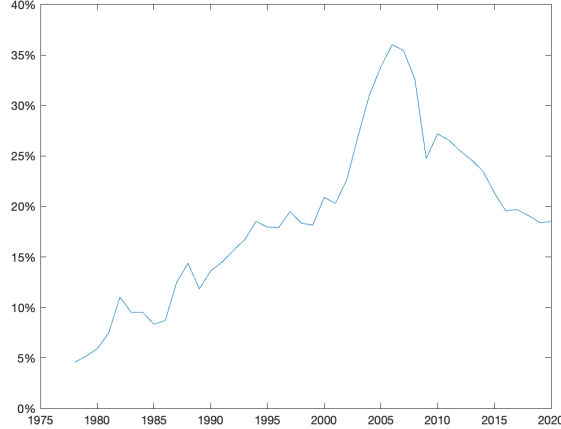


Figure 2: Export share in China

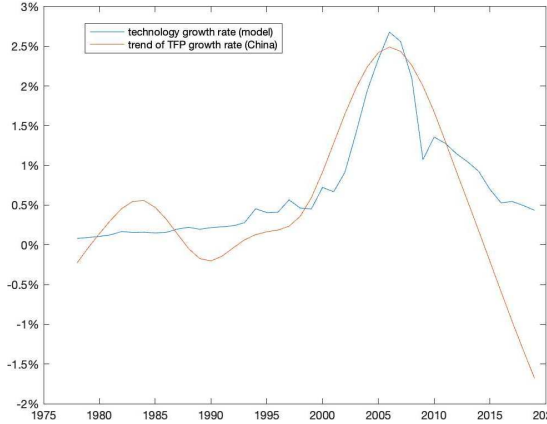


Figure 3: Simulation and data

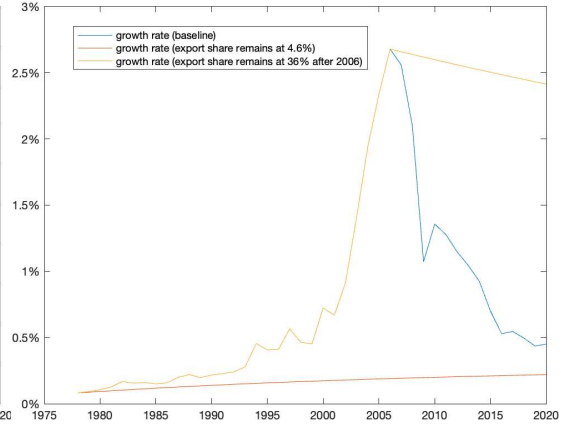


Figure 4: Simulation and counterfactual

Finally, Figure 5 presents the decomposition of the simulated technology growth rate $\sigma n_t + z_t$ in Figure 3 into variety-driven growth σn_t and quality-driven growth z_t . Our quantitative analysis shows that the Chinese economy did not feature quality-driven growth until the mid-1990s. Before that, variety-driven growth was the driver of its slow technological progress. From our simulation results, we also find that without the rise in the export share since 1978 (i.e., χ remains at its initial value of 0.046 as in Figure 4), quality-driven growth would have remained inactive (i.e., the state variable x_t remains below the threshold x_Z) until 2040. Figure 5 also shows that after the mid-1990s, quality growth remained relatively unimportant to technological progress in China (despite the rise in exports) until its entry to the WTO. After joining the WTO, quality-driven growth became the main driver of its rapid technological progress, which coincided with the start of the Chinese government intervention in stimulating innovation in 1999.²⁹ However, the fall in exports since 2007 has led to a rapid decline in quality-driven growth, which even becomes lower than variety-driven growth during the recent decoupling of the China-US economies.

²⁹See Ding and Li (2015) for a detailed discussion.

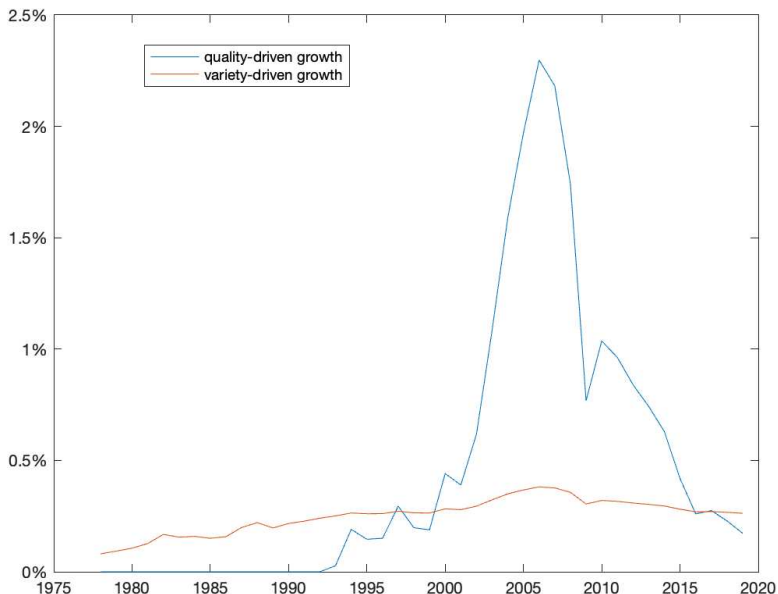


Figure 5: Decomposition into variety and quality growth

We conclude with an observation on the robustness of our main result. In our counterfactual "the export share remaining at 4.6%", the specific channel for the effect of the export share differs depending on whether we assume that China is not yet or is already in the first phase of the industrial era. In the first case, the entire sequence is delayed with the effect on the dates due to T_N going up and $T_Z - T_N$ remaining the same. In the second case, the delay concerns only quality innovation and the effect is due to T_Z going up. Since the qualitative result is the same (i.e., China grows slowly for much longer), our qualitative story is robust to the choice of the initial condition x_0 . In the calibration, we choose $x_0 > x_N$, but we could also choose $x_0 < x_N$ without changing the substance of our results. The choice of scenario boils down to an argument about when, exactly, the export share starts rising and whether at that date China is not yet or is already in the first phase of the industrial era. The conclusion of this brief analysis is that the story remains the same; namely, the path of the export share caused the transition to quality innovation to occur earlier than otherwise and drove the subsequent hump-shaped evolution of TFP growth in China.

5 Extensions

In this section, we consider two extensions of our baseline model. In Section 5.1, we introduce an agricultural sector. In Section 5.2, we convert our baseline small-open-economy model into a two-country model.

5.1 An agricultural sector

Like any typical macroeconomic model, our baseline model features a single final good. A model with a more realistic structure of the economy would feature multiple final goods, such as an

industrial final good and an agricultural final good. In this section, we explore an extension of our model with an agricultural sector to explore the robustness of our results.³⁰

Specifically, we modify the utility function of the household as follows:

$$U = \int_0^{\infty} e^{-(\rho-\lambda)t} \left[\ln c_t + \delta \ln q_t + \psi \frac{(l_t)^{1-\varepsilon}}{1-\varepsilon} \right] dt,$$

where $\delta > 0$ is now a preference parameter for the per capita consumption of an agricultural good q_t .³¹ For simplicity, we assume perfectly inelastic labor supply. The asset-accumulation equation becomes

$$\dot{a}_t = (r_t - \lambda)a_t + w_t - c_t - p_t l_t - p_{q,t} q_t,$$

where $p_{q,t}$ is the price of the agricultural good. The optimality condition for agricultural consumption per capita is

$$q_t = \frac{\delta c_t}{p_{q,t}}. \quad (30)$$

We follow Lagakos and Waugh (2013) and model the agricultural sector as competitive with production technology

$$Q_t = AL_{q,t}, \quad (31)$$

where Q_t is the aggregate output of the agricultural good and the parameter $A > 0$ determines the productivity of agricultural labor $L_{q,t}$. Profit maximization yields

$$w_t = p_{q,t} A, \quad (32)$$

which equates the wage rate to the value of the marginal product of agricultural labor.

The rest of the model is the same except for the labor market clearing condition, which now reads

$$L_{q,t} + L_{y,t} = L_t. \quad (33)$$

With these modifications, quality-adjusted firm size is

$$\frac{X_t}{Z_t} = \left(\frac{\theta}{\mu} \right)^{1/(1-\theta)} \frac{L_{y,t}}{N_t^{1-\sigma}} = x_t l_{y,t}, \quad (34)$$

which uses the definition of x_t in (12) and the newly defined industrial labor share $l_{y,t} \equiv L_{y,t}/L_t$. To solve the extended model, we simply replace the employment ratio l_t with the industrial labor share $l_{y,t}$ in all of the other equations. Combining (6) and (30)-(33) yields

$$l_{y,t} = \left(1 + \frac{\delta}{1-\theta} \frac{c_t}{y_t} \right)^{-1},$$

where the industrial consumption ratio c_t/y_t is determined as before in Proposition 1. Finally, we replace l_t with $l_{y,t}$ in (23) to derive the condition for the activation of innovation,

$$x_t > x_N(\chi) \equiv \frac{\phi}{\mu - 1 - \beta(\rho - \lambda)} \left\{ 1 + \frac{\delta}{1-\theta} \left[\frac{(\rho - \lambda)\beta\theta}{\mu} + 1 - \theta - \chi \right] \right\},$$

³⁰The extension is essentially an open-economy version of the model in Chu, Peretto and Wang (2022).

³¹Our results are robust to a subsistence parameter φ in $\delta \ln(q_t - \varphi)$ as in Chu, Peretto and Wang (2022).

which is identical to (23). Therefore, the addition of the agricultural sector does not change the structure of model's phase-transition dynamics.

The last result seems to suggest that the mechanism in this extension is the same as in the baseline model; however, this is not entirely true. Here, the higher export share χ of the industrial good causes a reallocation of labor from the agricultural sector to the industrial sector. Despite the different microeconomic mechanisms, however, in both models the macroeconomic trigger of the takeoff is the expansion in industrial production labor $L_{y,t}$.

This analysis assumes that the agricultural good is not exported. Allowing exports of the agricultural good does not change our results as long as the expansion of the demand for industrial exports is not accompanied by a higher demand for agricultural exports. In this case, we obtain

$$l_{y,t}(\chi_q) = \left(1 + \frac{\delta}{1-\theta} \frac{c_t/y_t}{1-\chi_q}\right)^{-1},$$

where χ_q is the share of Q_t exported and c_t/y_t is given by Proposition 1. This equation shows that an expansion in the export demand χ_q for the agricultural good (holding constant the export demand χ for industrial goods) would lead to a reallocation of labor from the industrial sector to the agricultural sector, which then yields a smaller market size for the industrial sector and thus delays the activation of innovation.

5.2 A two-country model

In this section, we convert our baseline small-open-economy model into a simple two-country setting. The utility function of the representative household in country $j \in \{h, f\}$ is given by

$$U^j = \int_0^\infty e^{-(\rho-\lambda^j)t} [\ln c_t^j + \delta^j \ln(1-l_t^j) + \chi^j \ln \iota_t^j] dt,$$

where we have converted the quasi-linear utility function into log utility in ι_t^j to allow the balanced growth path to feature different growth rates in the two countries. In this case, the demand for import consumption is

$$\iota_t^j = \frac{\chi^j c_t^j}{p_t^j}. \quad (35)$$

The rest of the economy in each country j is the same as before. In other words, the two economies only engage in trade in final output $\{Y_t^h, Y_t^f\}$. Country h imports some units of Y_t^f as import consumption ι_t^h and exports some units of Y_t^h to country f as its import consumption ι_t^f . The balanced-trade condition is

$$p_t^h \iota_t^h L_t^h = \iota_t^f L_t^f, \quad (36)$$

where $p_t^h \iota_t^h L_t^h$ is the value of country h 's imports and $\iota_t^f L_t^f$ is the value of its exports.

This modification replaces the values of the consumption ratio in Proposition 1 with

$$\frac{c_t^h}{y_t^h} = \begin{cases} \frac{1-\theta^h}{1+\chi^h} & 0 \leq x_t^h \leq x_N^h \\ \frac{1-\theta^h+(\rho-\lambda^h)\beta^h\theta^h/\mu^h}{1+\chi^h} & x_N^h < x_t^h < \infty \end{cases}. \quad (37)$$

The consumption ratio is thus decreasing in the preference parameter χ^h . The reason is that stronger preference for the imported good implies higher expenditure on imports, which must be matched by higher exports. Therefore, in equilibrium country h exports a larger share of its output Y_t^h to country f and experiences a decrease in its consumption ratio c_t^h/y_t^h . The reduction in the consumption ratio c_t^h/y_t^h in turn raises the equilibrium employment ratio

$$l_t^h = \left(1 + \frac{\delta^h c_t^h}{1 - \theta^h y_t^h}\right)^{-1}.$$

Therefore, in our simple two-country extension, a stronger preference for the foreign consumption good χ^h gives rise to a larger market size for the domestic industrial sector and thus to an earlier activation of innovation. The channel is the higher employment ratio exactly as in our baseline small-open-economy model.

Finally, we calibrate the two-country model to data on the economies of China and US to examine its quantitative implications. We use the same moments from the Chinese economy to calibrate the set of country h 's parameters: $\{\rho, \lambda^h, \theta^h, \alpha^h, \sigma^h, \mu^h, \beta^h, \phi^h, \delta^h, \chi^h\}$. Then, we use the corresponding moments from the US economy to calibrate the set of country f 's parameters: $\{\lambda^f, \theta^f, \alpha^f, \sigma^f, \mu^f, \beta^f, \phi^f, \delta^f, \chi^f\}$. The US population growth rate λ^f is also 0.9%.³² We set the labor share $1 - \theta^f$ of output to a conventional value of 2/3. We set the degree of technology spillovers $1 - \alpha^f$ to 0.833 and the social return of variety σ^f to 0.25 as before. We also set the markup ratio μ^f to 1.3, which is within the range of empirical estimates reported in Jones and Williams (2000). Then, we calibrate $\{\beta^f, \phi^f, \delta^f, \chi^f\}$ by matching the following moments of the US economy: 60% for the consumption share, 1% for the average TFP growth rate, 0.33 for the share of time devoted to work, and 10% for the average export share in the US. Table 4 summarizes the calibrated parameter values.

ρ	λ^h	θ^h	α^h	σ^h	μ^h	β^h	ϕ^h	δ^h	χ_{1978}^h
0.030	0.009	0.500	0.167	0.250	1.300	5.076	0.177	1.515	0.093
ρ	λ^f	θ^f	α^f	σ^f	μ^f	β^f	ϕ^f	δ^f	χ^f
0.030	0.009	0.333	0.167	0.250	1.300	6.135	0.105	2.257	0.167

Figure 6 presents the calibrated path of χ^h using the time path of export share in China. The calibrated value of χ^h is different from before for the following reason. In the baseline model, the export share of output is χ . In the two-country model, the export share of output in country h is $\chi^h c^h/y^h$.³³ Although the calibrated path of χ^h that we input into the model is different from before, the simulated path of the technology growth rate $\sigma n_t^h + z_t^h$ turns out to be the same as before. Specifically, the technology growth rate gradually increased prior to China joining the WTO and then accelerated sharply until 2006 before falling to pre-WTO level recently. From (37), we can rewrite the consumption-output ratio in country h as

$$\frac{c_t^h}{y_t^h} = \frac{(\rho - \lambda^h)\beta^h\theta^h}{\mu^h} + 1 - \theta^h - \chi^h \frac{c_t^h}{y_t^h} \quad (38)$$

³²Data source: World Bank Data.

³³Recall from (35) and (36) that $l_t^f L_t^f/Y_t^h = p_t^h l_t^h L_t^h/Y_t^h = \chi^h c_t^h/y_t^h$.

for $x_N^h < x_t^h < \infty$. Given that the calibrated path of χ in the baseline model is the same as the calibrated path of $\chi^h c^h / y^h$ in the two-country model, the simulated path of c_t / y_t in the baseline model (see Proposition 1) is also the same as the simulated path of c_t^h / y_t^h in the two-country model in (38), which in turn implies that the simulated employment ratio l_t^h and technology growth rate $\sigma n_t^h + z_t^h$ would also be the same as before.

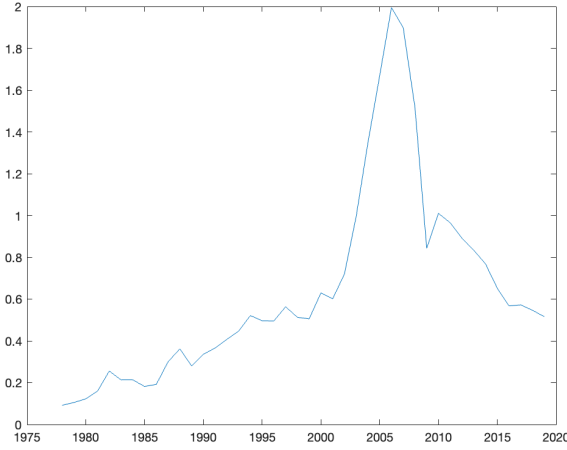


Figure 6: Calibrated path of χ^h in China

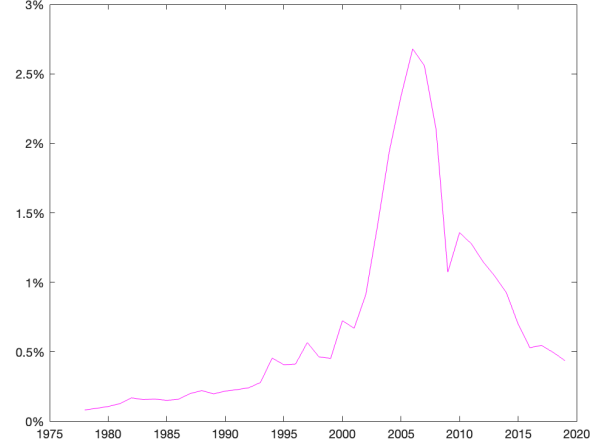


Figure 7: Simulated path of $\sigma n_t^h + z_t^h$ in China

6 Conclusion

In this study, we have developed a small-open-economy Schumpeterian growth model to explore the effects of exports on endogenous takeoff and economic growth. We find that higher demand for a country's exports expands the size of the market for the country's own production and thus causes an earlier takeoff and faster transitional growth. It does not affect, however, steady-state growth due to the scale-invariance property of the Schumpeterian growth model with endogenous market structure. Using cross-country panel data, we find supportive evidence for a positive effect of exports on economic growth that becomes smaller as the level of income rises, until it eventually disappears as our theory predicts. Despite this neutral effect of exports on long-run economic growth, we find a quantitatively significant effect of exports on the endogenous takeoff of the Chinese economy. This finding suggests that the opening up of the Chinese economy at the end of the 1970s has been crucial for the transition to innovation-driven growth; however, the fall in exports since 2007 has also caused a deceleration of such growth that continues until recent times. Finally, we stress that while we have focused on export-led growth as one of the mechanisms driving economic development, we most surely do not rule out other important mechanisms, such as mass education, political institutions, and investment in capital and infrastructure. We simply consider their effects as independent from the effects of exports on economic growth.

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Appendix A: Data

Table A1: Summary statistics

Variable	Observations	Mean	Sd	Min	Max
Growth of real GDP	1128	0.033	0.035	-0.206	0.323
Growth of real GDP per capita	1128	0.018	0.033	-0.199	0.330
Growth of R&D expenditure	231	0.052	0.065	-0.152	0.296
Export share of GDP	1128	0.395	0.278	0.004	2.119
Log real GDP per capita	1128	9.137	1.167	6.178	11.852
Log capital stock	806	12.731	2.135	7.715	18.312
Government expenditure share of GDP	811	0.188	0.081	0.007	0.608
Depreciation rate	806	0.042	0.012	0.013	0.098
Real interest rate	650	0.063	0.083	-0.415	0.410

Data sources: Penn World Table for the capital stock and the depreciation rate. OECD Data for R&D expenditure. World Bank for others.

Table A2: Using TFP as dependent variable

	TFP1 growth		TFP2 growth	
	(1)	(2)	(3)	(4)
$Export_{it}$	0.9035** (0.4006)	1.3242*** (0.4974)	0.7769* (0.4414)	1.3351*** (0.4666)
$Export_{it} \times y_{it}$	-0.1036** (0.0438)	-0.1493*** (0.0535)	-0.0943* (0.0483)	-0.1614*** (0.0493)
y_{it}	-0.0359 (0.0253)	-0.0003 (0.0256)	-0.0323 (0.0302)	-0.0144 (0.0281)
Control variables		✓		✓
Country fixed effects	✓	✓	✓	✓
Time fixed effects	✓	✓	✓	✓
Observations	794	523	634	406
R^2	0.2198	0.3859	0.2395	0.4571

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Robust standard errors in parentheses. Standard errors are clustered at the country level. TFP1 growth: $g_{TFP} = g_y - 0.33g_k$. TFP2 growth: $g_{TFP} = g_y - 0.33g_k - 0.67g_h$

Table A3: Gravity model estimation

	$\ln(Export_{ijk})$
$\ln(airdist_{ij}) \times I(1996 \geq year \geq 2000)$	-0.1012*** (0.0188)
$\ln(airdist_{ij}) \times I(2001 \geq year \geq 2005)$	-0.2107*** (0.0239)
$\ln(airdist_{ij}) \times I(2006 \geq year \geq 2010)$	-0.2165*** (0.0267)
$\ln(airdist_{ij}) \times I(2011 \geq year \geq 2015)$	-0.2465*** (0.0281)
$\ln(airdist_{ij}) \times I(2016 \geq year \geq 2020)$	-0.2212*** (0.0298)
$\ln(seadist_{ij}) \times I(1996 \geq year \geq 2000)$	0.0411** (0.0137)
$\ln(seadist_{ij}) \times I(2001 \geq year \geq 2005)$	0.0683*** (0.0176)
$\ln(seadist_{ij}) \times I(2006 \geq year \geq 2010)$	0.0595** (0.0201)
$\ln(seadist_{ij}) \times I(2011 \geq year \geq 2015)$	0.0901** (0.0214)
$\ln(seadist_{ij}) \times I(2016 \geq year \geq 2020)$	0.1009** (0.0228)
Pair fixed effects	✓
Wave fixed effects	✓
F -statistic	43.408
Observations	541946
R^2	0.8109

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Robust standard errors in parentheses. Standard errors are clustered at the country-pair level.

Appendix B: Proofs

The firm's maximization problem. The current-value Hamiltonian of firm i is

$$H_t(i) = \Pi_t(i) - R_t(i) + \eta_t(i) \dot{Z}_t(i) + \xi_t(i) [\mu - P_t(i)], \quad (\text{B1})$$

where $Z_t(i)$ is the state variable, $\eta_t(i)$ is the costate variable on $\dot{Z}_t(i)$ and $\xi_t(i)$ is the multiplier on $P_t(i) \leq \mu$. We substitute (7)-(9) into (B1) and derive

$$\frac{\partial H_t(i)}{\partial P_t(i)} = 0 \Rightarrow \frac{\partial \Pi_t(i)}{\partial P_t(i)} = \xi_t(i), \quad (\text{B2})$$

$$\frac{\partial H_t(i)}{\partial R(i)} = 0 \Rightarrow \eta_t(i) = 1, \quad (\text{B3})$$

$$\frac{\partial H_t(i)}{\partial Z_t(i)} = \alpha \left\{ [P_t(i) - 1] \left[\frac{\theta}{P_t(i)} \right]^{1/(1-\theta)} \frac{L_{y,t}}{N^{1-\sigma}} - \phi \right\} \frac{Z_t^{1-\alpha}}{Z_t^{1-\alpha}(i)} = r_t \eta_t(i) - \dot{\eta}_t(i), \quad (\text{B4})$$

If $P_t(i) < \mu$, then $\xi_t(i) = 0$. In this case, $\partial \Pi_t(i) / \partial P_t(i) = 0$ yields $P_t(i) = 1/\theta$. If the constraint on $P_t(i)$ is binding, then $\xi_t(i) > 0$. In this case, we have $P_t(i) = \mu$. Then, the assumption $\mu < 1/\theta$ implies $P_t(i) = \mu$. Substituting (B3), (12) and $P_t(i) = \mu$ into (B4) and imposing symmetry yield (13). ■

Proof of Proposition 1. We use the labor demand (6) and the balanced trade condition (17) to reduce the pre-industrial-era household budget constraint (1) to

$$\frac{c_t}{y_t} = \frac{w_t l_t - p_t \iota_t}{y_t} = 1 - \theta - \chi.$$

This value is positive if and only if $1 - \theta > \chi$. In the industrial era, we use the entry condition $V_t = \beta X_t$ to derive

$$a_t = \frac{V_t N_t}{L_t} = \frac{\beta X_t N_t}{L_t} = \frac{\beta \theta}{\mu} y_t, \quad (\text{B5})$$

which also uses $\theta Y_t = \mu X_t N_t$. Differentiating (B5) with respect to t yields

$$\frac{\beta \theta}{\mu} \dot{y}_t = \dot{a}_t = (r_t - \lambda) a_t + w_t l_t - c_t - p_t \iota_t = (r_t - \lambda) a_t + (1 - \theta - \chi) y_t - c_t, \quad (\text{B6})$$

which uses (1) and the last equality uses (6) and (17). Then, we use (2) and (B5) to rearrange (B6) as

$$\frac{\dot{c}_t}{c_t} - \frac{\dot{y}_t}{y_t} = \frac{\mu}{\beta \theta} \frac{c_t}{y_t} - \left[\frac{\mu(1 - \theta - \chi)}{\beta \theta} + \rho - \lambda \right],$$

which implies that the consumption-output ratio jumps to the steady-state value c/y for $x_t > x_N$ in Proposition 1. Next, we substitute $L_{y,t} = l_t L_t$ and the labor demand (6) into (4) to obtain the employment ratio

$$l_t = \left(1 + \frac{\delta}{1 - \theta} \frac{c_t}{y_t} \right)^{-1}. \quad (\text{B7})$$

Substituting the two stationary values of the consumption ratio into this expression yields the two stationary values of the employment ratio in Proposition 1. ■

Derivation of the growth rates n and z . We use (19), the fact that l^* is constant and $g_t = \sigma n_t + z_t$ to write the quality growth rate as

$$z_t = \alpha [(\mu - 1)x_t l^* - \phi] - \rho - \sigma n_t. \quad (\text{B8})$$

We then combine (2) with (16) and use Proposition 1 to write

$$g_t = \frac{1}{\beta} \left(\mu - 1 - \frac{\phi + z_t}{x_t l^*} \right) + \frac{\dot{x}_t}{x_t} + z_t - \rho. \quad (\text{B9})$$

We use $g_t = \sigma n_t + z_t$ and (20) to rewrite this expression as

$$n_t = \frac{1}{\beta} \left(\mu - 1 - \frac{\phi + z_t}{x_t l^*} \right) + \lambda - \rho. \quad (\text{B10})$$

Substituting (B10) into (B8) yields $z_t = z(x_t)$, which is positive if and only if $x_t > x_Z$, where

$$x_Z(\chi) \equiv \underset{x}{\arg \text{solve}} \left\{ [(\mu - 1)x l^*(\chi) - \phi] \left[\alpha - \frac{\sigma}{\beta x l^*(\chi)} \right] = (1 - \sigma)\rho + \sigma\lambda \right\}. \quad (\text{B11})$$

As said, we assume $x_Z > x_N$. Substituting (B8) into (B10), we obtain $n_t = n(x_t)$. We then substitute it into (20) to derive the *non-linear* dynamics of x_t as

$$\dot{x}_t = \frac{1 - \sigma}{\beta - \sigma/(x_t l^*)} \left\{ \left[(1 - \alpha)\phi - \left(\rho + \frac{\sigma\lambda}{1 - \sigma} \right) \right] \frac{1}{l^*} - \left[(1 - \alpha)(\mu - 1) - \beta \left(\rho + \frac{\sigma\lambda}{1 - \sigma} \right) \right] x_t \right\}, \quad (\text{B12})$$

where $x_t l^* > \alpha x_Z l^* > \sigma/\beta$ is ensured by (B11). Letting $\sigma/(x_t l^*) \rightarrow 0$ yields the linearized dynamics of x_t in (27). Finally, x_t converges to x^* given (21). ■

Proof of Proposition 4. In the pre-industrial era x_t follows the exponential process $\dot{x}_t/x_t = \lambda$. We solve this differential equation for initial condition $x_0 < x_N$ to obtain $x_t = x_0 e^{\lambda t}$. Setting the right-hand side at x_N , we obtain the activation date

$$T_N = \frac{1}{\lambda} \ln \left(\frac{x_N}{x_0} \right).$$

Using (23) and Proposition 1 we then have

$$\frac{\partial T_N}{\partial \chi} = -\frac{1}{\lambda} \frac{1}{x_N} \frac{\partial x_N}{\partial \chi} = -\frac{1}{\lambda} \frac{\delta l^*}{1 - \theta}.$$

In the first phase of the industrial era, x_t follows the *linear* differential equation (24), reproduced here for convenience:

$$\dot{x}_t = \frac{1 - \sigma}{\beta} \frac{\phi}{l^*} - \frac{1 - \sigma}{\beta} \left[\mu - 1 - \beta \left(\rho + \frac{\sigma\lambda}{1 - \sigma} \right) \right] x_t > 0. \quad (\text{B13})$$

We denote the steady state of (B13)

$$\bar{x} \equiv \frac{\phi}{\left[\mu - 1 - \beta \left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)\right] l^*(\chi)} \quad (\text{B14})$$

and define the composite parameter

$$v \equiv \frac{1-\sigma}{\beta} \left[\mu - 1 - \beta \left(\rho + \frac{\sigma\lambda}{1-\sigma} \right) \right] > 0.$$

We then solve (B13) with initial condition x_0 , obtaining

$$x_t = x_0 + (\bar{x} - x_0) (1 - e^{-vt}).$$

Setting the right-hand side equal to x_Z give us the activation date of quality innovation.

We then have two cases.

1. The economy starts from initial condition $x_0 < x_N$. Accounting for the pre-industrial era dynamics, we have

$$T_Z = T_N + \frac{1}{v} \ln \left(\frac{\bar{x} - x_N}{\bar{x} - x_Z} \right).$$

From (23), (B11), (B14) and Proposition 1, we see that x_N , x_Z and \bar{x} are all proportional to $1/l^*$; therefore, the ratio $(\bar{x} - x_N)/(\bar{x} - x_Z)$ is independent of l^* and thus of χ . As a result, the effect of a change in χ in the pre-industrial era is $\partial T_Z/\partial\chi = \partial T_N/\partial\chi$.

2. The economy starts from initial condition $x_0 > x_N$. In this case T_Z is

$$T_Z = \frac{1}{v} \ln \left(\frac{\bar{x} - x_0}{\bar{x} - x_Z} \right) = \frac{1}{v} \ln \left(\frac{1 - x_0/\bar{x}}{1 - x_Z/\bar{x}} \right),$$

where x_Z/\bar{x} is independent of l^* and x_0/\bar{x} is increasing in l^* as shown in (B14). Therefore, T_Z is again decreasing in l^* and χ . Specifically, using Proposition 1 we have

$$\frac{\partial T_Z}{\partial\chi} = \frac{1}{v} \frac{x_0/\bar{x}^2}{1 - x_0/\bar{x}} \frac{\partial\bar{x}}{\partial\chi} = -\frac{1}{v} \frac{x_0/\bar{x}}{1 - x_0/\bar{x}} \frac{1}{l^*} \frac{\partial l^*}{\partial\chi} = -\frac{1}{v} \frac{x_0}{\bar{x} - x_0} \frac{\delta l^*}{1 - \theta}.$$

■

Appendix C: Monopolistic firms in the pre-industrial era

In this appendix, we consider an alternative assumption in the model in which monopolistic firms operate even in the pre-industrial era. In this case, we need to assume that the initial value of x_t is sufficiently large (despite being lower than x_N). Specifically,

$$x_0 > \frac{\phi}{(\mu - 1)l_0}, \quad (\text{C1})$$

where l_0 is determined below in (C7). Equation (C1) is equivalent to $\Pi_0 > 0$. Therefore, it is possible for monopolistic profits to be positive in the pre-industrial era before the takeoff occurs. When $n_t = 0$, the entry condition in (15) does not hold. However, the asset-pricing equation in (14) still holds and becomes

$$r_t = \frac{\Pi_t}{V_t} + \frac{\dot{V}_t}{V_t}, \quad (\text{C2})$$

where $R_t = z_t = 0$. We use $a_t = V_t N_t / L_t$ and $n_t = 0$ to derive $\dot{a}_t / a_t = \dot{V}_t / V_t - \lambda$ and then substitute this equation into (1) to obtain

$$\frac{\dot{V}_t}{V_t} - \lambda = \frac{\dot{a}_t}{a_t} = r_t - \lambda + \frac{w_t l_t - p_t l_t - c_t}{a_t}. \quad (\text{C3})$$

Substituting (C2) into (C3) yields

$$c_t = \frac{\Pi_t}{V_t} a_t + w_t l_t - p_t l_t = \frac{N_t}{L_t} \Pi_t + (1 - \theta - \chi) y_t, \quad (\text{C4})$$

where we have used $a_t = V_t N_t / L_t$, $p_t l_t = \chi y_t$ and $w_t l_t = (1 - \theta) y_t$. Then, substituting (9) and $P_t = \mu$ into (C4) yields

$$c_t = \frac{N_t X_t (\mu - 1 - \phi Z_t / X_t)}{L_t} + (1 - \theta - \chi) y_t = \frac{\theta}{\mu} \left(\mu - 1 - \frac{\phi}{x_t l_t} \right) y_t + (1 - \theta - \chi) y_t, \quad (\text{C5})$$

which uses $\theta Y_t = \mu N_t X_t$ and (11)-(12). Then, the consumption-output ratio from (C5) is

$$\frac{c_t}{y_t} = \frac{\theta}{\mu} \left(\mu - 1 - \frac{\phi}{x_t l_t} \right) + 1 - \theta - \chi, \quad (\text{C6})$$

which would increase from $1 - \theta - \chi$ to $1 - \theta - \chi + (\rho - \lambda)\beta\theta/\mu$ if firm size $x_t l_t$ were to start from $\phi/(\mu - 1)$ and increases towards $\phi/[\mu - 1 - \beta(\rho - \lambda)]$. Finally, we substitute (C6) into (B7) and manipulate the resulting equation to obtain the equilibrium firm size as follows:

$$x_t l_t = \frac{x_t + \frac{\delta\phi}{\mu} \frac{\theta}{1-\theta}}{1 + \delta \left(1 + \frac{\theta}{1-\theta} \frac{\mu-1}{\mu} - \frac{\chi}{1-\theta} \right)}, \quad (\text{C7})$$

which is increasing in export demand χ for a given x_t .

Given that the dynamics of x_t is given by (20) and $n_t = 0$ in the pre-industrial era, firm size $x_t l_t$ gradually increases towards the threshold $\phi/[\mu - 1 - \beta(\rho - \lambda)]$ to trigger the takeoff as before. The only difference is that as x_t increases over time, l_t in (C7) gradually decreases (instead of jumping at the time of the takeoff). This additional dynamics of l_t in the pre-industrial era gives rise to negative growth in domestic output per capita before the takeoff, which is not as realistic as the dynamics in the baseline model.