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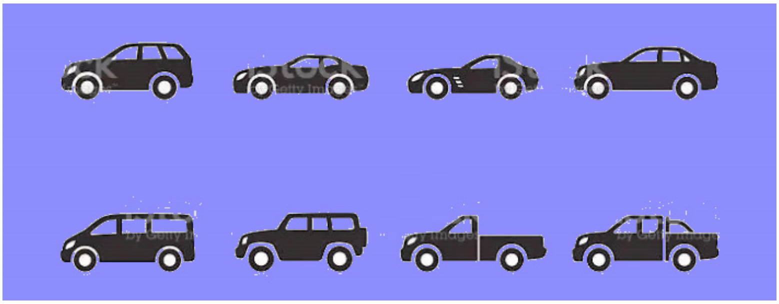

Validating the Postulates of rational Choice in the Context of economical Fuel Consumption of Vehicles

Joseph E. Mullat *

Abstract. We have introduced an innovative procedure called "blind statistical scoring" that simplifies the analysis of statistical indicators. This procedure aligns with the principle of parsimony, also known as Ockham's razor. By applying this procedure, we confirm several postulates within the concept of bounded rationality of choice. To illustrate this phenomenon, we conducted an experiment with the data provided by Spritmonitor.de website, which contains data, search results, texts, graphics, software and other information.

JEL: C25; G17

Keywords: car market; fuel impulses; monotonic; system; kernel

1. Introduction

It may seem to someone that the statements made in the article are so trivial that they do not require special experiments. However, even if a statement seems obvious, it is always best to verify it through rigorous scientific methods to ensure that it is indeed in accord with the reality. Regarding the car market, it is not always easy to predict car fuels consumption accurately. While it is true that well established car models tend to have more accurate forecasts, there are still many other factors that can affect car market, such as lifestyle choices, technological advancements and rising fuel costs. Therefore, it is also true that a simple statistical procedure could be useful (maybe not ideal) for studying such a phenomenon as hypothetical events at the car market masquerade, instead of a very complex probabilistic-statistical analysis of fuel indicators.

In probabilistic-statistical analysis, two opposing approaches can be distinguished: from subjective to objective knowledge and, in the opposite direction—from objective to subjective. In the first approach, such specialists as a physician, biologist, astronomer, practitioner or market analyzer... those who have knowledge in their field, use data statistical analysis for objective statements about the obtained estimates of experimental data, observations, etc.

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With this approach, from a subjective assessment to an objective assessment, probability research includes: Markov processes (Rogers and Williams, 2000), Lévy processes (Applebaum, 2004), Gaussian processes (Lifshits, 2012), random fields (Adler, 2010)… Statatistical analysis includes space state models (SSM, Koller and Friedman, 2009), parameter estimation (Walter and Pronzato, 1997), management and decision-making problems (Narula and Weistroffer, 1989), continuous modeling, multiple time series (Voelkl et al, 2012) and computational methods (Mirkin et al, 1995). In both areas, the knowledge of the distribution of judgments about the object under study is necessary. That is not always the case.

The objective of subjective assessment (Frey and Võhandu, 1966) both seem to be statistical and probabilistic indicators and, at first glance, seem to be contradictory. It seems that specialized knowledge is also required. Nevertheless, it is very possible to do without special knowledge, as well as knowledge about the distribution of numerical parameters—indicators.

The procedure of the objective to subjective approach considered below could be called the "blind glance of statistical scoring", which is what we need. The only thing the Data Explorer uses in blind scoring is that one number is greater/less than another. If common sense is achieved, then the well-known law of parsimony or "***Ockham's razor***" will come into force. A procedure that requires fewer assumptions about reality can be considered the most reliable.

2. Bounded Rationality Postulates

Rational choice theory is a framework that attempts to explain how individuals make decisions based on their preferences and constraints. There are several postulates of rational choice theory (e.g., Arrow 1948; Jamison 1973;…) that fall under the umbrella of so-called Bounded Rationality, including the assumption that individuals have well-defined preferences, make choices based on expected utility and make rational decisions based on the available information. It appears that we are discussing the dynamics of car evaluations in the car market and the introduction of the postulate of monotonicity to account for changing preferences and assessments.

The concept of monotonicity, as we described it, suggests that as the list of models for a proposed purchase or sale of cars is narrowed down, car owners' assessments or subjective utilities (referred to as impulses) will consistently and monotonously decrease. In other words, as the options become more limited, the car owners' preferences or perceived value of the remaining models will decrease in a predictable and continuous manner. This notion of monotonicity implies that as individuals eliminate certain car models from their consideration, their subjective evaluations or impulses associated with those models decline. It suggests that there is a decreasing trend in the desirability or attractiveness of the remaining options as the selection process progresses.

This concept could be relevant in various decision-making scenarios, such as when consumers are evaluating and comparing different products before making a purchase. It assumes that as individuals eliminate alternatives, their preferences and evaluations will consistently decline rather than fluctuating or exhibiting non-monotonic behavior.

However, it's important to note that the concept of monotonicity may not universally apply to all decision-making processes, as individual preferences and subjective assessments can vary significantly. Different people may have different valuation criteria or impulses and their evaluations might not always exhibit a monotonic decrease with narrowing options. Therefore, while monotonicity can provide a useful framework for understanding certain decision dynamics, it should be applied with caution and considered in conjunction with other factors that may influence individual choices. In this context, we have modified Arrow's, 1959, strict consistency postulate slightly to ensure the validity of the basic postulates of rational choice remains intact. This suggests that some modifications are being made to the standard rational choice framework to account for the dynamics of the exchange market and car owner behavior. The act of choice consists in selecting from X some variants $C(X)$ according to certain rules, Strzalecki (2011).

Let us recall in a Boolean, that is, in a more formal form, bounded rationality canonical postulates (cited by Aizerman and Malishevski, 1981, pp. 65-83, English version translated from Russian, p. 189), which we note in connection with the procedure supposedly rational choice of our fuel consumtion on the car exchange market:

- *Independence with respect to dropping rejected alternatives (or, for brevity, elimination of options), Postulate 5 (Chernoff, 1954, pp. 422-443) or Axiom 2 (Jamison and Lau, 1973, pp. 901-912):*
  From $C(Y) \subset X \subset Y$ *it follows that* $C(X) = C(Y)$*;*
- *Compatibility, the same as Postulate 10 of Chernoff and property* γ *of Sen:*
  From $X \cup Y$ *it follows that* $C(X) \cap C(Y) \subset C(X \cup Y)$
- *Succession, which is the same as Postulate 4 (Chernoff), or property* α *(Sen, 1971, pp. 307-317) or the axiom C2 of Arrow-Uzawa (Arrow, 1959, pp. 121-127):*
  From $X \subset Y$ *it follows that* $X \setminus C(X) \subset Y \setminus C(Y)$
  or equivalent to $X \cap C(Y) \subset C(X)$*;*
- *Strict Succession or constant residual choice (it is the same as postulate 6 (Chernoff, 1954) and one of the forms of the "weak axiom of revealed preference" of Samuelson, i.e., the axiom C4 (Arrow, 1959, pp. 121-127):*
  From $X \subset Y$ *and* $X \cap C(Y) \neq \varnothing$ *it follows that* $X \cap C(Y) = C(X)$.

We have modified the strict succession postulate in accord with the monotonicity postulate to take into account the changing dynamics of car owners' assessments of the value of fuel consumtion. The monotonicity postulate is a departure from the standard assumption of rational choice theory. This already mentioned postulate states that if the list of Y car models of different manufacturers for the proposed sale/purchase is narrowed down to X, $X \subset Y$, then the estimates of car owners or the elasticity of making fair deals or the corridor of satisfaction for purchasing the indicated model in the list monotonically decreases simultaneously with the narrowing of the considered list Y. Even if pair preferences remain in the narrowed choice set, the level of significance for a reasonable purchase or sale may turn out to be at a lower level satisfying reg. fe. fuel consumption in the narrowed set X than in the "covering" subset Y, which includes the narrowed subset. Indeed, we observed this phenomenon in a dataset downloaded from Spritmonitor.de. It was easy to slightly modify the strict sequence postulate C4 (Arrow) to ensure that the validity of the rational choice postulate C4 remained unchanged, cf., i.e.:

From $X \subset Y$ *and* $X \cap C(Y) \neq \varnothing$ *it follows that* $X \cap C(Y) = C(X) \cap C(Y)$.

In this slightly changed form it still operates in the same way that is presented above, as canonical Arrow's strict succession postulate. The founders of rational choice theory did not consider such dynamics, including Arrow in 1948 and 1959, Chernoff in 1954 and Sen in 1971.

The postulates of succession and compatibility are specific theoretical foundations used in economics to analyze behavior in decision-making. The postulates of succession and strict succession suggest that individuals make decisions based on a set of consistent preferences that do not change over time. As regards the postulate of strict succession C4, it is useful to paraphrase Arrow's intuitive interpretation. Indeed, the intuitive interpretation of the canonical postulate of Arrow, as well as the modified postulate of strict succession, is as the like: *"If some car models in the context of fuel consumtion are selected from the set of models* Y *available for sale and then the range for models available for sale is narrowed to* X*, but still contains some models previously selected for purchase, then previously unselected models do not become selected for purchase and previously selected models do not become unselected."*

3. Parsimonious approach

To validate the postulates of rational choice in the context of reasonable fuel consumption differences among car manufacturers, we can examine how consumers make decisions when selecting a car based on fuel efficiency and analyze whether their choices align with the assumptions of rational choice theory.

There are several statistical methods we could use to test the rational choice postulates. One approach could be to use econometric models to estimate the parameters of a utility function that describes how car owners make decisions. Another approach could be to use machine-learning algorithms to identify patterns in the data and test whether they are consistent with rational choice theory. It seems that the following discussion of the restaurant scenario is only an introduction to the main topic about the results and experiments carried out using the Excel spreadsheet of Information and interactive computer services, which are provided on https://www.spritmonitor.de/en/. The spreadsheet was subjected to a test of the truth of the independence postulates of the rejected alternative and the postulates of succession or strict succession with the established car models in the market. This suggests that car buyers may prioritize, such as engine power or fuel efficiency may be rational when postulating rejected alternatives. The presented postulates of succession and strict succession according to the procedure of Ockham's razor seem to confirm our experiments. A rigorous proof of these assertions is a good initiative for further research. However, the proof of independence from the rejected alternatives follows from the Proposition I in 3.3.

3.1. Wine Menu Exibit. Let's start the analysis with a "hypothetical" or "pedagogical exhibit". When accepting the order in a restaurant, the sommelier informs the guest that some of the preferred choices are unavailable, what can lead to irrational behavior on the part of the guest or the sommelier. From the guest's point of view, it may be wiser to try cheaper wines that were initially overlooked. On the other hand, the sommelier may suggest more expensive wines, even though there are cheaper and equally good options available. This behavior can be depending on the specific circumstances. Indeed, the absence on the list of the most expensive wines, for sure, will encourage guests to expand the list of cheap wines or at least keep the choice. On the contrary, the lack of approved, at first glance, cheap wines may induce the sommelier to suggest more expensive wines in favor of others available for order, also cheaper, but quite good and better wines. More often than not, guests agree with such a proposal. This behavior, sometimes rational or irrational, was our main motive for informing the reader about "these events." The phenomenon of such hypothetical events on the car market masquerade were discussed from the point of view of an innovative statistical procedure and illustrated on the basis of a probabilistic-statistical analysis of the numerical indicators of the exchange market.

The wine list is ordered in descending order of price and **1** multiplies the price of the most expensive wine, **2** multiplies the next local price, then **3** the next and so on. We call these numbers as price credentials or impulses. The local maximum of impulses and the price of wine are selected when this peak location from the top of the ordered list where the maximum is reached. The guest decides to accept the price of the wine at the local impulse maximum as an acceptable level of price of significance when choosing wines with a higher or equal price level, e.g., the list $\mathbf{10^2}$, $\mathbf{9^2}$, $\mathbf{8^2}$, $\mathbf{7^2}$, $\mathbf{6^2}$, $\mathbf{5^2}$, … suggests that the peak of this sequence is located at $\mathbf{7^2{=}49}$.

3.2. Significance. We look at some of the details of our wine procedure for analyzing car market data. Let's define a set of fuel consumtion indicators $p_j \in W$, $|W| = n$ of n car models, $j = \overline{1,n}$. In particular, suppose that in the sample denoted by the letter H, some potential cars are collected as candidates according to the reasonable fuel consumption that car buyers might value. We can further define a totality of sets $\{H\}$ of all 2^n samples $H \subseteq W$. Impulses $\pi(p_j, H) = p_j \cdot |H|$ (in terms of Kempner et al., as monotone linkage functions) will evaluate so called credentials of fuel consupmtion. The procedure for finding the signifacance level of fuel consumtion is easy to set up. First, all the fuel consumtion indicators p_j, are sorted in descending order, constituting (like in wine list) the indicators order $\langle p_j \rangle$. Then a sequence $\overline{\pi}$ called impulses $\overline{\pi} = \langle \pi_j \rangle = \langle p_j \rangle \cdot j$, $j = \overline{1,n}$, is constructed. The list of fuel consumtion indicators $\langle p_j \rangle$, in contrast to original list p_j, is necessary descending. In 1971, we called such sequences $\overline{\pi}$ as defining.

3.3. The reasonable level. The impulses $\langle \pi_j \rangle$, are single peaked, where the peak denotes the kernel H^* (Mullat, 1971-1995) of a monotone system. The set H^* constitutes the rational, i.e., the monotone linkage choice implemented in our findings. At the location k^* from the top of the impulses $\overline{\pi} = \langle \pi_j \rangle$, i.e., from the top of defining sequence of models, $j = \overline{1,n}$, where the local maximum $u = \arg\max_{j=\overline{1,n}} \langle \pi_j \rangle$ is reached, the peak, denoted by u, will be called the level of Significance.

Proposition I. *Among the totality of all samples* $H \subseteq W$*, i.e., among all the sets* $\{H\}$ *of all* 2^n *samples, the kernel* H^* *guarantees reaching the global maximum of the impulse function* $F(H)$ *of samples* H *equal to* $u = \min_{p_j \in H} \pi(p_j, H)$: $H^* = \arg\max_{H \subseteq W} F(H)$.

The proposition I confirms the postulate of independence from rejected alternatives in two-person games, which was originally studied by John F. Nash in the 1950s, when he developed a solution to the bargaining problem. With regard to the market for the perchasing and production of cars, the proposition I states that any final decisions made or based on statistics should not be affected by the removal of any part of statistics that are not reliable or represent a very small number of cases in which, for example, statistics have been collected into a database and selected for review.

3.4. Threshold-based indicators. These types of indicators are commonly used in time series analysis, reasonable processing and other areas where we are interested in detecting deviations or changes in system behavior. In finance, a threshold-based indicator can be used to give a significance reasonable when a car's fuel rises above a certain positive or falls below a certain negative threshold—see Appendix for an illustration of the situation. In our case, indicators called fuel impulses create a dynamic system, since the previous state of the car market determines the subsequent state. It is worth noting that the postulates of strict or non-strict succession emphasize the rational behavior of car owners when some new models expand the list of available alternatives. In the event that car owners have chosen some of the best cars in the past, then these postulates state that car owners will still be inclined to consider old models—"old love does not rust."

Our Ockham's razor procedure can be applied to adjust some "induced" $\pm\Delta$ indicators by fixing an interval $[-\Delta_1 + u, u + \Delta_2]$, which represents the range of values within which the rational choice postulates, according to the proposion, fluctuate around the threshold u (calculations in the **A2**). The interval can be seen as analogous to a confidence interval used in statistics. By considering this interval and observing whether the dynamic indicators cross the threshold u, one can make significance decisions. Indeed, in the context of economy cars, if the indicators consistently cross below $-\Delta_1 + u$, it indicates a reasonable potential car purchase. On the contrary, if the dynamic indicators intersect higher than $u + \Delta_2$, this indicates an unreasonable purchase of a car. By utilizing this interval approach, we can incorporate $\pm\Delta$ limits into more significance strategy, allowing for more nuanced decision-making based on the behavior of the dynamic indicators relative to the threshold u.

4. Car Market Data

We have taken advantage of the standard mechanisms and techniques of the Windows platform to view data in Excel spreadsheets for thousands of car users. We have looked at the list of cars that are not only economical, but also reasonably inexpensive or even expensive luxury cars of all available models. This information has been extracted and recompiled from the interactive computer services provided on the Spritmonitor.de website. This includes vehicle fuel data, significant volumes and other relevant variables.

Some comments are needed to clarify the implementation of our Ockham's razor "procedure" for analyzing cars fuel consumption dynamics. The reliability of data on leasing or purchase cars with regardd to fuel consumtion, where all fuel consumption data have been available to everyone, is given by the fact that the MPG (mileage or mile per gallon) data is guaranteed by Cost Calculator and Tracker at the date to date basic activity at Spritmonitor.de database. The spreadsheet was compiled using domain https://www.spritmonitor.de/en/search.html (Accessed: Monday, July 10, 2023).

Here is an overview of the different fuel types commonly found in the car models in our dataset:

- ***Gasoline***/Petrol: Gasoline or petrol is the most widely used fuel type for cars. It is a fossil fuel derived from crude oil and used in internal combustion engines.
- ***Diesel***: Diesel fuel is another common fuel type for cars. It has a higher energy density than gasoline and is often used in larger vehicles or those requiring more torque, such as trucks and SUVs.
- ***Electric***: Electric cars run entirely on electricity stored in rechargeable batteries. They have electric motors instead of traditional internal combustion engines and produce zero tailpipe emissions.
- ***Hybrid***: Hybrid cars combine an internal combustion engine with an electric motor and battery. They can run on both gasoline/diesel and electric power, with the ability to switch between the two depending on driving conditions.
- ***Plug-in*** Hybrid Electric Vehicle (PHEV): PHEVs are hybrid cars that can be charged by plugging them into an external electric power source. They have larger batteries than regular hybrids and can run for longer distances on electric power alone.

- ***Natural Gas***: Some car models can run on compressed natural gas (CNG) or liquefied natural gas (LNG), which are cleaner-burning fuels compared to gasoline or diesel.
- ***Hydrogen*** Fuel Cell: Hydrogen fuel cell vehicles use hydrogen gas to generate electricity, which powers an electric motor. They produce zero emissions, with the only byproduct being water vapor. We did not put this type of fuel into consideration.
- ***Ethanol***: Ethanol, also known as bioethanol, is an alcohol-based fuel derived from plant sources such as corn or sugarcane. Flex-fuel vehicles can run on gasoline or a blend of gasoline and ethanol.
- ***LPG*** (Liquefied Petroleum Gas): LPG is a mixture of propane and butane and is commonly used as an alternative fuel in some car models. It burns cleaner than gasoline or diesel.

These are the main fuel types you might encounter in our dataset (download accessed from http://www.datalaundering.com/download/MPG-MileAge-Data.xls, 11-01-2023). Each fuel type has its advantages and considerations in terms of efficiency, emissions, availability and infrastructure. Analyzing fuel consumption across these categories can provide valuable insights into the efficiency and environmental impact of different car models. The car market monitoring input data consists of 374 car models—the shortened version below.

Negative significance level →				-0,61
Positive significance level →			6,97	3,19
	Count	Fuel type	l/100km	
Alfa Romeo	2053	Gasoline	9,28	2,31
Aston Martin	24	Gasoline	13,22	6,25
Bentley	12	Gasoline	15,56	8,59
BMW	29508	Gasoline	8,86	1,89
Bugatti	2	Gasoline	12,38	5,41
Chevrolet	1677	Gasoline	9,76	2,79
Cadillac	135	Gasoline	13,66	6,69
Chrysler	811	Gasoline	10,84	3,87
Daewoo	366	Gasoline	7,53	0,56
Citroen	5793	Gasoline	7,14	0,17
Daihatsu	1200	Gasoline	5,86	-1,11
Datsun	4	Gasoline	10,89	3,92
Ford	20799	Gasoline	7,99	1,02

Negative significance level →				-5,21
Positive significance level →			15,39	3,87
	Count	Fuel type	kWh/100km	
BMW	315	Electricity	16,55	1,16
Bugatti	1	Electricity	10,18	-5,21
Citroen	72	Electricity	15,94	0,55
Ford	25	Electricity	21,65	6,26
Ferrari	2	Electricity	41,45	26,06
Fiat	131	Electricity	17,11	1,72
Honda	13	Electricity	19,65	4,26
Hyundai	532	Electricity	15,93	0,54
Jaguar	5	Electricity	20,90	5,51
Kia	249	Electricity	17,34	1,95
Mazda	40	Electricity	19,43	4,04
Mercedes-Benz	90	Electricity	24,26	8,87
Mitsubishi	31	Electricity	14,23	-1,16

As you can see, some cells differ from others in certain patterns and frames. These highlighted patterns and frames are the result of using the macro—Ctrl+s. In accord with the proposition I above, an analysis of the significance levels of the negative/positive values of the car indicators dynamics has been conducted. Using the macro in columns, (selected or *"pasted"*) areas X of the spreadsheet A in their entirety may consist of negative/positive numbers distributed throughout the areas without any special order for negative or positive numbers. However, the standard EXCEL data sorting options allow you to sort selected areas in ascending or descending order depending on the specified columns or rows. Thus, having, for example, negative values scattered across a spreadsheet in different cells, these cells can be redistributed together into "contiguous areas" of negative or positive values in the columns or row patterns to satisfy the necessary conditions. Such contiguous areas can help performing experiments with the analysis results. The C(X) operator is compiled into the Ctrl+s macro, using car market share fuels X as initial data table below in column format of alternative X.

5. IRRATIONAL BEHAVIOR, FINDINGS AND EXPERIMENTS

Clients may fixate on the initial price presented by the salesperson or listed on the sticker. They may struggle to negotiate or deviate from this anchor point, even if it's not the best deal available. Some clients may prioritize the social status associated with owning a particular brand or model of a car over its practicality or affordability. They may be willing to spend more than they can afford just to maintain or enhance their social image.

Impulsive behavior is prevalent in car buying, where clients make quick decisions without conducting thorough research or considering long-term consequences. They may fall in love with a particular car at first sight and rush into the purchase without evaluating alternatives. Clients may be influenced by the opinions and actions of others, leading to herd mentality. They may purchase a car simply because their friends, family, or colleagues have it, without adequately assessing their own needs or preferences. Emotional attachments to a specific brand, model, or even the color of a car can cloud judgment.

Clients may overlook practical aspects such as fuel efficiency, maintenance costs, or resale value, prioritizing their emotional connection instead. Some clients may exhibit overconfidence in their negotiation skills or knowledge about cars, leading them to make irrational decisions. They may refuse to seek expert advice, rely solely on their own judgment and end up paying more or making suboptimal choices. Clients may have a strong bias towards purchasing brand-new cars, believing that newer models are inherently superior, even if a used car with similar features could meet their needs at a lower cost. This bias can lead to overspending and financial strain.

Clients may be overly concerned about the fear of missing out or losing a perceived opportunity. This fear can lead them to make impulsive decisions or agree to unfavorable terms, driven by the desire to secure a deal quickly, even if it's not the best option available. It's important to note that while these behaviors may be irrational from a purely logical perspective, they often stem from human psychology and the complex interplay of emotions, biases and social factors.

To avoid these pitfalls when buying a car, the potential owners should focus on fundamental analysis, which involves studying a model's financial statements and business operations. It's always important to carefully consider all available data and information when making the purchas decisions. It's possible that this new information could be factored into an decisions's analysis of a model's financial health and potential for growth. Regarding our specific findings, this sounds like including higher dynamic fuel consumtion for consideration may have an impact on buyer behavior, even when stable fuel consumtion are still available for sale.

It also appears that our rational choice postulates have yielded some interesting and nuanced results. Our findings suggest that decision-making behavior is complex and influenced by multiple factors. Indeed, while the postulate of independence of rejected alternatives may explain why car owners may choose dynamic options over stable fuel consumtion, the postulate of succession or strict succession for the stable fuel consumtion may also be a factor in this decision-making behavior. The specific options available, individual preferences and situational factors may all play a role in determining which option a car owner chooses. Understanding buyer behavior and user context is crucial for making accurate and reliable decisions, especially in the automotive industry. By considering these factors, car manufacturers can align their strategies and offerings with the needs and preferences of their target customers. Recognizing and addressing motorist biases is also essential. People may have preconceived notions or preferences based on their past experiences, cultural influences, or personal beliefs. Car manufacturers need to be aware of these biases and strive to base their decisions on objective data and analysis, rather than relying solely on subjective opinions or assumptions. By doing so, they can develop products and services that cater to a broader audience and maximize customer satisfaction. In addition, diversifining by incorporating a data analysis strategy through "blind statistical scoring" can be a useful tool for customers in the automotive market.

5.1. Ockham's Razor procedure analyzing statistical indicators. The procedure as said above is called "blind statistical scoring" and it involves finding the simplest explanation or model that fits the data. This approach is based on the principle of parsimony, which suggests that simpler explanations are more likely to be true than complex ones. It's important to note that this procedure is not necessarily equivalent to other well-known statistical methods like the 0-hypothesis. It is a separate approach that can be useful in certain contexts, particularly when dealing with complex data sets. The guide we are presenting here could be useful for analysts who are interested in applying the principle of parsimony to their statistical analysis. However, it's important to keep in mind that this approach is just one tool in a larger toolkit of statistical methods and may not always be appropriate for every situation.

5.2. Activating the Ctrl+s Macro. Any standard Microsoft Excel spreadsheet has a section in which so-called macros, written in the Visual Basic programming language, are available. In our German Exchange Market spreadsheet you can find our Ockham's Razor Visual Basic macro. Copy the text of this macro into your spreadsheet. In the properties of this macro, specify that the new macro can be executed using the Ctrl+s command. Your data, whether data in a row, column or table in the form of numerical data, can now be used by running the Ctrl+s macro to determine whether it is reasonable to purchase the desired or reject the uneconomical models available in the automotive market. Remember that the first two rows of the spreadsheet must be free —insert at least two free rows at the top of the spreadsheet.

5.3. Succession Postulates Validation. From the information provided in the main body of the article, it appears that we are discussing an "impulse indicator" that was used as a scalar to make decisions based on fuel consumption in the context of selecting an optimal alternative for vehicles. The impulse indicator was calculated as the product of the position number and fuel consumption of an option in a descending linear order list of indicators. It serves as a scalar to measure each option's desirability based on its fuel consumption. Such a measurement involves applying Ockham's Razor procedure to select the optimal option using the scalar criterion (fuel impulse). Ockham's Razor favors simpler explanations or models when choosing between competing options.

The "strict succession postulate" in terms of "monotonicity postulate" has been modified to fit this decision-making process, and we are suggesting that the impulse indicator, along with the modified postulates, provides a reliable and reasonable way to make rational decisions. It will be challenging to provide a thorough analysis or evaluation of the statement. However, the theorem of Aizerman and Malishevski (Teorem I, 1981) states that the scalar condition is necessary and sufficient for the truth of strick succession postulate.

Wherever it was necessary, the set of indicators was presented in the form of a linear order. This means that the choice operator $C(X)$ on the subset $X \subseteq A$ of the set A of alternatives/indicators acts on a certain set of segments $S(A)$ or intervals (open closed, doesn’t matter) in contrast to the set of all subsets 2^A. The set A can be identified by all sets, now named as segments $X = [x\_left,\ x\_right]$ (already named as intervals) of the indicators under consideration. Now narrowing a segment Y to a segment X is an action of narrowing the interval $Y = [y\_left,\ y\_right]$ to $X = [x\_left,\ x\_right]$. In view of this understanding, the situation with intervals can preserve our model of choice operators $C(X)$ nomenclature. We are ready to state and prove the following proposition II.

Define first a set function (further on it is the function $f(X)$ of the interval X) in the notations just introduced: $f(X) = x\_left$.

Proposition II.

When narrowing the interval $Y \in S(A)$ to the interval $X \in S(A)$ as part of the intervals of indicators of the common supersegment A, the condition $f(C(X)) \le f(C(Y))$ for the fulfillment of the postulates of the succession (inclusive strict succession) is necessary and sufficient.

Proof.

Necessity. Assume that the condition $f(C(X) < f(C(Y)))$ is satisfied. This means that segments $[c(x)\_left,\ c(x)\_right]$, $[c(y)\_left,\ c(y)\_right]$ satisfy the inequality $c(x)\_left < c(y)\_left$, which leads to the inequality $x \setminus c(x)\_right < y \setminus c(y)\_right$. We can rewrite the last inequality in a set theoretical notations as $X \setminus C(X) \subset Y \setminus C(Y)$ what actually indicates at the statement of validity of the succession postulate.

Sufficiency. Suppose that the succession postulate is not satisfied for some intervals $C(X), C(Y) \in S(A)$. Contra assumption to the succession postulate means that some indicator p can be found that lies inside interval $X \setminus C(X)$ but does not lie within $Y \setminus C(Y)$, or, what is equivalent to $p \in C(Y)$. Now a new equivalent statement to the latter would be the validity of the inequality $p > f(C(Y))$ and the next inequality $p \le f(C(X))$, or at last the validity of the inequality $f(C(Y)) < p \le f(C(X))$ what contradicts the supposed validity of the condition of the **Proposition II** validity, ie., $X \setminus C(X) \not\subset Y \setminus C(Y)$.

APPENDIX

A1. We discuss an integrated approach that uses machine learning algorithms and rational choice theory to identify patterns in data related to fuel consumption of vehicles. The method involves searching for subsets X of alternatives A that meet certain criteria. A well-known approach in this direction is a closer system $C(X)$ of subsets $X \in 2^A$ of alternatives. Equivalent to a more precise definition, cf. Seiffarth et al., 2021, our nomenclature will include a set $X' \subseteq A$ such that it will not be possible to find a proper subset $X \subset X'$, for which, in Proposition I, our impulse function $F(X) = F(X')$. Indeed, in the database https://www.spritmonitor.de/en/ from the set A of all gasoline cars, including almost all Audi models and Mazda models, it can be concluded that the subset $X' \subseteq A$ of gasoline cars with fuel consumption over **6.80 l/100km** represents a closer set because $X' = C(X')$ and also a subset $Y' \subseteq A$ of cars with a consumption of less than **6.80 l/100km** but with a fuel consumption of more than **4.45 l/100km** is also a closer set since $Y' = C(Y')$. Hereby, the sets X' and Y' represent an intersection of all gasoline cars.

A2. Based on the information provided in the database, it appears that the interval **[5.36-9.56]** liters per **100 km** has been determined as a "reliable" range for gasoline consumption for all gasoline car models. Within this range, models to the left are considered more fuel-efficient, while those outside the range to the right are deemed to consume fuel more excessively. Particularly, we noticed that based on the experiment conducted using the models **A3** and **A4**, these two models fall within a range of no more than **2.76 l/100 km** to the right of the significance value u **= 6.80 l/100 km**. On the other hand, the **A5**, **A6**, **A7** and **A8** models exceed the significant level u by more than **2.76 l/100 km**, indicating a noticeable increase in fuel consumption. We also noticed that the most fuel efficient Audi models are the **A1** and **A2**, which are highlighted in green, see attached file.

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