No pain, no gain: implications in consumption and economic growth

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Abstract

Demand saturation occurs along with economic development, but the theoretical basis for demand saturation is lacking. This study adds to literature by proposing a novel concept named utilization cost, which denotes the physical or mental burden incurred to obtain utility. Correspondingly, we distinguish between quantity and quality of consumption and construct a general utility function. With a generative decision procedure, the analysis shows that utilization costs help to explain the economic dynamics across development stages in terms of demand saturation. And, the long-term state of demand is affected by the properties of utilization costs, determining development directions.

Keywords: Demand saturation; Consumption; Economic growth

JEL: D11, E10, O30, O40

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1 Introduction

The slowing down of real GDP growth rate has been a general trend and is widely observed around the world. This phenomenon has attracted extensive attention from researchers and policymakers. Conventional studies on the determinants of economic growth usually land on the supply capacity. In fact, economic growth can also be refined if demand is saturated, as further consumption would not make people better off. This point has been made in the pioneering work (Witt 2001) and following studies (Aoki and Yoshikawa 2002; Saint-Paul 2021). They highlight that moving away from demand satiation is crucial for releasing economic growth potential. To deviate from demand satiation, it is essential to understand what causes demand saturation. However, the literature offers fewer discussions in this aspect. This study attempts to fill this gap.

Our analysis roots in a novel concept called utilization cost, which represents the physical and mental burden one must bear in order to obtain utility. This concept allows us to construct a demand-driven theoretical framework for economic growth. The theoretical framework is based on two properties—quantity and quality of consumption—derived from the utilization cost. The quality and quantity are embedded in a utility function, which is generalized from Saint-Paul (2021). Based on the utility function, we define an indicator named demand saturation rate. With a generative decision procedure, the demand saturation rate is shown to intertwine with economic dynamics across development stages. An economy could prioritize quantity (resp. quality) of consumption initially due to shortage (resp. satiation), thereby increasing (resp. lowering) demand saturation rate. But eventually the economy would converge to a sustainable growth state to control the rise (resp. decline) in the demand saturation rate and avoid satiation (resp. shortage). A further analysis reveals that the long-term states of demand, obtained along with sustainable growth path, depend on properties of utilization cost.

The contributions of this study can be summarized in the following four aspects. First, it contributes to the small but growing literature on demand saturation by seeking answers to the question why demand is constrained. Although existing studies point out the importance of demand saturation in economic growth, they did not further investigate why demand saturation happens. For example, Witt (2001) and Aoki and Yoshikawa (2002) treat demand saturation as a stylized fact or a premise for further analysis. A more recent study by Saint-Paul (2021) proposes a specific utility function which embeds the idea that the utility gain of consumption is accompanied with utility loss, yet it does not reveal the fundamental reasons of the utility loss. In contrast, we devote efforts to explain what causes the utility loss from consumption and propose the concept of utilization cost, which sets the basis for the theoretical framework of demand saturation. Additionally, we extend the specific form of the utility function in Saint-Paul (2021) to a general form. The generalized utility function can be informative for any future research that involves utilization costs.

Second, this study helps to distinguish between quantity and quality of consumption in terms of utilization costs. The literature on creative destruction or the quality ladder assumes that both quality and quantity of consumption increase utility.[1] However, in the presence of demand satiation, an increase in the quantity of consumption should lower the utility, not raise it. To resolve this conflict, we propose that an increase in the quantity of consumption increases the positive utility as well as the utilization cost. So, the demand satiation happens when the marginal utilization cost equals the marginal positive utility with respect to the quantity of consumption. In contrast, an increase in the quality is assumed to be able to reduce the utilization cost due to a higher level of ease or harmlessness of consumption. Therefore, we make a clear distinction between the quantity and quality of consumption according to their contradictory effects on utilization costs.

Third, this study helps shed light on the mechanisms of economic growth. In addition to literature that emphasizes the essential role of innovations in supply side[2], we argue that the innovations can be driven by demand saturation to balance the quantity and quality

[1]See, for example, Aghion and Howitt (1992), Grossman and Helpman (1991), Jones (1995), Parello (2022), and Zheng, Huang, and Yang (2020).

[2]See, for example, Peretto and Connolly (2007), Chu, Cozzi, and Galli (2012), Foellmi, Wuergler, and Zweimüller (2014), and Akcigit and Kerr (2018).

of consumption. This demand-driven mechanism is inline with Jaimovich (2021), in which the demand is reflected by the market price. We move one step further to present a direct linkage between demand and economic growth. In particular, we propose an indicator named demand saturation rate to explain the economic dynamics in both developing and developed economies. In developing economies, numerous studies observed the decline of growth rates after high growth, which is widely known as the middle-income trap.[3] We argue that high growth in developing countries occurs when the demand saturation rate is low. Then, this high growth also leads to increased demand saturation rates and calls for higher quality to reduce utilization costs. The decline in the real GDP growth rate can thus be explained as a shift from prioritizing quantity to quality in development to avoid satiation.[4] In developed economies, economic fluctuations and business cycles draw attention of numerious studies.[5] We argue that an economic boom can take place if the quality of consumption leaps (demand saturation rate declines), while stagnation can happen after an overheat in the quantity of consumption (demand saturation rate rises).

This paper also contributes to the discussion on the long-term state of demand and its influencing factors. On this topic, Keynes (1930) points out that people's consumption will shift from material to spiritual, so the characteristics of consumption goods are essential. Saint-Paul (2021) shows that the satiation can be avoided in a social planner model. Our paper, on the other hand, proposes that the properties of utilization costs are key factors. Using the utility function from Saint-Paul (2021) as an example, if the utilization cost rises rapidly (resp. slowly) with the quantity consumed, then the long-term demand saturation rate will approach to unity (resp. zero). The long-term states of demand can be divergent even if the consumption is homogeneous and the economy is operated by a social planner. This is because properties of utilization costs can constrain the feasible region of the quantity

[3]See, for example, Aiyar et al. (2018), Furuoka et al. (2020), Lee (2020), and Glawe and Wagner (2020).

[4]This shift may fail in reality, creating economic crisis, as both the quantitative and qualitative developments are blocked. However, the discussion is beyond the scope of this paper and is left for future studies.

[5]See, for example, Foellmi and Zweimüller (2008), Lovcha and Perez-Laborda (2021), Ma and Samaniego (2022), Matsuyama (2002), and Galí (1999).

and quality of consumption of an economy. As the utilization cost becomes more sensitive to the quantity consumed, the surface of the utility function becomes more curved and the feasible region becomes smaller (see figure 3).

The rest of the paper is organized as follows. Section two introduces the core concepts. Section three presents a general utility function based on these core concepts. Section four introduces a complete economic model with the general utility function. Section five provides further discussion on economic dynamics and long run states. Section six concludes.

2 Concepts

This section introduces a set of interrelated concepts for the subsequent analysis. In particular, the utilization cost is proposed as the basis of demand saturation, and the quantity and quality of consumption are discussed accordingly to prepare for constructing the general utility function. The concepts of innovations are introduced in advance for the discussion on economic dynamics. We provide an essential illustration of these concepts in the main text as follows and leave a detailed interpretation in Appendix A.

Utilization cost We define the utilization cost as the physical and mental burden incurred in order to obtain "positive utility". Whether the utility comes from natural resources, durable goods, non-durable goods or intellectual products, there will be a utilization cost.[6] Moreover, utilization costs may occur before, during, or after consumption.[7] Consumption brings positive utility that people can enjoy, but also brings physical and mental burdens that must be taken into account, and these burden is the utilization cost. This relationship

[6] For example, the consumption of intelligent products requires thinking, the consumption of durable goods requires using, and the consumption of non-durable goods requires absorbing or destroying. That is, all forms of utility acquisition require physical and mental participation, and the cost of participation is the utilization cost. The case that utility comes from natural resources is likewise. For example, sun exposure damages the skin, and oxygen causes cellular aging.

[7] For example, we may prepare in advance for consumption or maintain afterwards. Besides, the fatigue or even illness, incurred to obtain utility, can last for some time.

can be represented by the following equation

$$\mathscr{U} = u - c, \tag{1}$$

which allows the utilization cost, c, to be larger than the positive utility, u, symbolizing excessive consumption harms.

Quantity and Quality Based on the aforementioned utilization costs, this study further distinguishes between quantity and quality of consumption. Conventional wisdom suggests that "positive utility" increases with the quantity consumed but often ignores the existence of utilization costs. In this paper, we argue that quantity and quality of consumption have contradictory effects on the utilization cost.

To better understand this difference, we need to show what quantity and quality mean in this paper. In a nutshell, the quantity of consumption reflects the extent to which the consumption can fulfill our needs. In terms of production, the quantity of consumption is a condensation of the used labor, resources, and quantity technology (productivity), which follows the production functions widely used in literature. The quality, on the other hand, reflects the level of ease and harmlessness of consumption, which is assumed to be determined by quality technology level.[8]

Since we assume that utilization cost can be brought with consumption, the quantity consumed generates not only positive utility but also utilization costs. While, a higher quality of consumption lowers physical and mental burden, i.e. reduce utilization cost, due to a higher level of ease and harmlessness. Consequently, the utilization cost increases with the quantity of consumption while decreases with the quality of consumption, so the quantity and quality can be distinguished in terms of the utilization cost.

[8]We assume that the quality of consumption is a non-negative real number, so the quality can be improved by the qualitative innovation continuously. Further discussion is left in Appendix A.

Innovation Finally, this paper argues that both quantity and quality of consumption can be affected by innovation. We propose two types of innovation in an economy—quantitative and qualitative innovations. Quantitative innovation mainly improves quantity technology (productivity) and thus affects the quantity of consumption; while qualitative innovation mainly raises quality technology level and thus affects the quality of consumption. Minor spillover effects are allowed for generality.[9] Innovations can be made through research efforts from human capital.

3 Utility function

Based on the concepts above, this section provides a general utility function as well as its properties. This utility function is based on Saint-Paul (2021), in which the utility is allowed to be negative due to over consume. The utility loss from consumption in Saint-Paul (2021) is interpreted as utilization cost, and this interpretation allows for the generalization of the utility function.[10]

The general utility function is as follows

$$\mathscr{U}(m,q) = u(m) - c(m,q). \tag{2}$$

As described in Section 2, the positive utility increases with the quantity consumed, $u'(m) > 0$. And, the utilization cost rises with the quantity of consumption but decreases with the quality of consumption, $\partial c/\partial m > 0$ and $\partial c/\partial q < 0$. In addition, we assume that the positive utility concavely increase with the quantity of consumption, $u''(m) \leq 0$, and the utilization cost convexly increase (resp. decrease) with the quantity (resp. quality) of consumption,

[9] The spillover effects happen when the quantitative innovation also improves the quality technology level, or when the qualitative innovation raises the quantity technology level.

[10] In Saint-Paul (2021), the utility loss can be lowered by introducing new goods. In perspective of this paper, the introduced new goods can help to fulfill the same need with lower utilization cost due to more choices. Therefore, a greater variety of consumption is also a type of higher quality in terms of the utilization cost from the macro perspective.

$\partial^2 c/\partial m^2 > 0$ (resp. $\partial^2 c/\partial q^2 > 0$), and a higher quality reduces the marginal utilization cost, $\frac{\partial^2 c}{\partial m \partial q} < 0$. Besides, we assume that the utilization cost is zero when there is no quantity of consumption, $\lim_{m \to +0} \frac{\partial c}{\partial m} = 0$. Accordingly, derived properties of the general utility $\mathscr{U}$ are summarized in Lemma 1, in preparation for additional analysis on economic dynamics in Section 5.

Lemma 1. *The general utility function* $\mathscr{U} = u(m) - c(m, q)$ *satisfies that:*

1) Given $m > 0$*, we have* $\partial \mathscr{U}/\partial q > 0$*,* $\partial^2 \mathscr{U}/\partial q^2 < 0$ *and* $\frac{\partial^2 \mathscr{U}}{\partial m \partial q} > 0$*;*

2) Given $q > 0$*, we have* $\lim_{m \to +0} \frac{\partial \mathscr{U}}{\partial m} > 0$*,* $\frac{\partial^2 \mathscr{U}}{\partial m^2} < 0$ *and* $\lim_{m \to \infty} \frac{\partial \mathscr{U}}{\partial m} < 0$*;*

3) Given $q > 0$*,* $\exists! m > 0$ *that maximizes* $\mathscr{U}$ *and satisfies* $\frac{\partial c(m,q)/\partial m}{u'(m)} = 1$*.*

Based on this general utility function, we propose an indicator named demand saturation rate. The demand saturation rate represents the ratio of the marginal utilization cost to the marginal positive utility:

$$\phi(m, q) = \frac{\partial c(m, q)/\partial m}{u'(m)}. \tag{3}$$

Demand is satiated when its saturation rate is unity. When the demand saturation rate is less (resp. greater) than unity, a greater (resp. less) quantity of consumption increases the utility. The properties of the demand saturation rate are summarized in Lemma 2. We will show in Section 5 that the demand saturation rate can affect the trajectory of economic dynamics.

Lemma 2. *Given* $m, q > 0$*, the following statements are true: 1)* $\phi(m, q) > 0$*; 2)* $\partial \phi/\partial m > 0$*; 3)* $\partial \phi/\partial q < 0$*.*

Graphical representation The utility function defines a topological structure on the quantity and quality of consumption. If we assume that a rational one would not over consume, then this topological structure would be non-trivial because the feasible region of the quantity and quality of consumption is constrained. For example, we can draw a demand satiation path on the 3D surface of the utility function as shown in figure 1. This path represents the combination of quantity and quality of consumption for which demand saturation rate is unity. Given the utility function, it is easy to show that this path exists and is unique (see statement 3 of Lemma 1). Then, an economy can only choose the combination of quantity and quality to the left of this demand satiation path. Graphical representations can provide intuitive results and are therefore also shown in Sections 5 as a complement to the analysis.

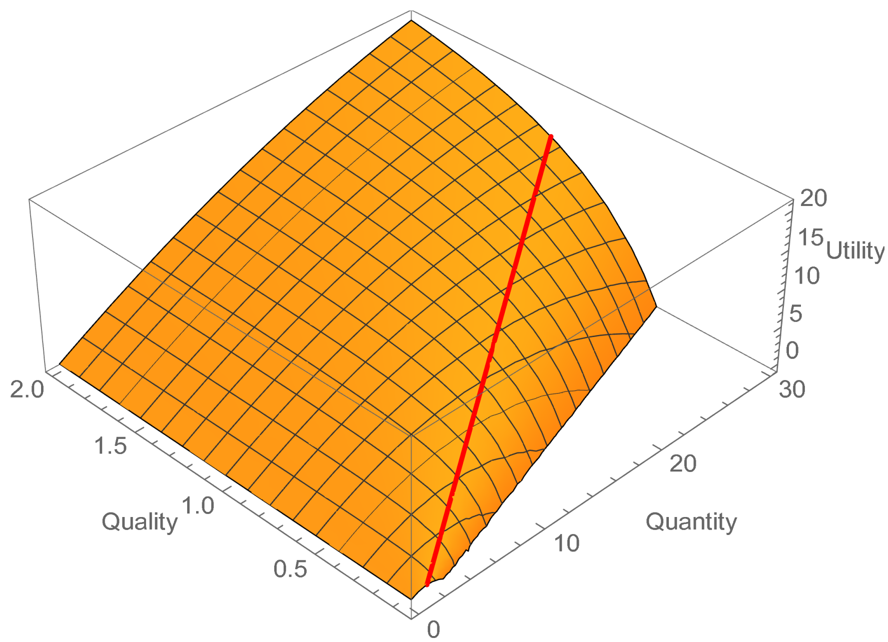


Figure 1: An illustrative 3D surface of the utility function
Note: the utility function takes the specification of $\mathscr{U} = m - 0.02\frac{m^2}{q}$. The quantity of consumption ranges from 0 to 30, and the quality of consumption ranges from 0 to 2. Only the utility above zero is reported to keep the figure clear.

4 Model

In this section, we construct a complete economic model to investigate the economic dynamics with utilization costs. For simplicity, we assume that the output is non-storable and can only be used for consumption, incurring utilization costs simultaneously.

As forementioned in Section 2, the quantity of output produced follows a Cobb-Douglas production function, which requires combinations of labor supply, natural resources, and quantity technology (productivity):

$$m_t = A_{m,t} L_t^{\alpha} X_t^{1-\alpha}, \tag{4}$$

We assume that labor supply and natural resources are given and normalized as one, i.e. $L_t \equiv 1$ and $X_t \equiv 1$, so the output is determined by the quantity technology level, $m_t = A_{m,t}$. The quality of output depends only on the quality technology level, $q_t = A_{q,t}$.

The social planner can allocate human capital, $R_t > 0$, to quantitative or qualitative innovation. The allocated shares are $r_{m,t} \geq 0$ and $r_{q,t} \geq 0$, respectively, so we have

$$R_t = r_{m,t} R_t + r_{q,t} R_t = R_{m,t} + R_{q,t}, \tag{5}$$

where $R_{m,t} = r_{m,t} R_t$ (resp. $R_{q,t} = r_{q,t} R_t$) is the total human capital allocated to quantitative (resp. qualitative) innovation. Specifically, the innovation functions are assumed to be as follows

$$A'_{m,t} = g_m(R_{m,t}, R_{q,t}), \tag{6}$$

$$A'_{q,t} = g_q(R_{m,t}, R_{q,t}), \tag{7}$$

which satisfy that $\frac{\partial^2 g_i}{\partial R_{m,t}^2} \leq 0$, $\frac{\partial^2 g_i}{\partial R_{q,t}^2} \leq 0$ and $\frac{\partial^2 g_i}{\partial R_{m,t} \partial R_{q,t}} \geq 0$ to allow technologies to increase concavely with human capital. Minor spillover effects are allowed as foretold in Section 2, i.e. $\frac{\partial g_m}{\partial R_{m,t}} > \frac{\partial g_m}{\partial R_{q,t}} \geq 0$ and $\frac{\partial g_q}{\partial R_{q,t}} > \frac{\partial g_q}{\partial R_{m,t}} \geq 0$.

By allocating human capital, the social planner will maximize the discounted utility of a representative household in the long run:

$$\max_{\{r_{m,t}, r_{q,t}\}} \int_0^\infty e^{-\rho t} \mathscr{U}(m_t, q_t) dt, \tag{8}$$

where the parameter $\rho > 0$ is time preference.

Standard solution uses Hamiltonian (9) and the derived first order conditions (10)-(12):

$$\begin{aligned} H = & e^{-\rho t}\mathscr{U}\left(m_t, q_t\right) + e^{-\rho t}\lambda_{m,t} g_m(R_{m,t}, R_{q,t}) \\ & + e^{-\rho t}\lambda_{q,t} g_q(R_{m,t}, R_{q,t}) + \mu_{m,t} e^{-\rho t} r_{m,t} + \mu_{m,t} e^{-\rho t} r_{q,t}, \end{aligned} \tag{9}$$

$$\lambda_{m,t}\left(\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}\right) R_t - \lambda_{q,t}\left(\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}\right) R_t + \mu_{m,t} - \mu_{q,t} = 0, \tag{10}$$

$$\lambda'_{m,t} - \rho\lambda_{m,t} = -\left(u'(m_t) - \frac{\partial c(m_t, q_t)}{\partial m_t}\right), \tag{11}$$

$$\lambda'_{q,t} - \rho\lambda_{q,t} = \frac{\partial c(m_t, q_t)}{\partial q_t}, \tag{12}$$

where the variable $\lambda_{m,t}$ (resp. $\lambda_{q,t}$) denote the marginal value of quantitative (resp. qualitative) innovation, and the auxiliary variable $\mu_{m,t}$ (resp. $\mu_{q,t}$) satisfy that $\mu_{m,t} \geq 0$ (resp. $\mu_{q,t} \geq 0$) and $\mu_{m,t} r_{m,t} = 0$ (resp. $\mu_{q,t} r_{q,t} = 0$).

We can conclude an optimal decision rule (see Proposition 1) from equation (10), implying that the social planer weights the marginal values of human capital to make decisions.[11] If allocating human capital into qualitative (resp. quantitative) innovation has a higher marginal value, then the economy will prioritize qualitative (resp. quantitative) innovation. In a particular state, if there is an allocation that equates the marginal values of human capital, then the economy will choose this allocation. We will refer to this particular state as sustainable growth, since the human capital is allocated to both quantitative and qualitative innovations, and thus allows both the quantity and the quality of consumption to grow sustainably even if without the spillover effects.

[11] The marginal value of human capital in quantitative (resp. qualitative) innovation equates the marginal value of innovation to the objective function $\lambda_{m,t}$ (resp. $\lambda_{q,t}$) multiply the marginal value of human capital to innovation $\left(\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}\right)$ (resp. $\left(\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}\right)$).

Proposition 1. *We have the following statements:*

1) $\forall r_{m,t}, r_{q,t}$*, if* $\lambda_{m,t}\left(\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}\right) > \lambda_{q,t}\left(\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}\right)$*, then the social planner will choose* $(r_{m,t}, r_{q,t}) = (1, 0)$*;*

2) $\forall r_{m,t}, r_{q,t}$*, if* $\lambda_{m,t}\left(\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}\right) < \lambda_{q,t}\left(\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}\right)$*, then the social planner will choose* $(r_{m,t}, r_{q,t}) = (0, 1)$*;*

3) if $\exists r_{m,t} > 0, r_{q,t} > 0$*, s.t.* $\lambda_{m,t}\left(\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}\right) = \lambda_{q,t}\left(\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}\right)$*, then the social planner will choose this allocation.*

The quantity versus quality trade-off in consumption has realistic implications for analyzing economic growth. If an economy prioritizes the quantitative (resp. qualitative) innovation, then the real GDP (the quantity of output) will increase rapidly (resp. slowly). A sustainable growth state should present a modest real GDP growth rate in between.

5 Discussion

We have identified three states of an economy from the previous section. One concern is that if the above states can switch. For example, a strategy that prioritizes quantity may increase the demand saturation rate and increase the needs for a higher quality of consumption, which may drive the economy to allocate a higher share of human capital for qualitative innovations. However, the Hamiltonian and its first order conditions cannot reveal much further information.

5.1 Solution method

In this paper, we propose a generative decision procedure that allows informative analysis using the following two steps. First, we calculate the marginal value of innovations analytically by a simplifying assumption that the social planner will take the current state variables (m_t and q_t) as given. Second, the social planner allocates human capital according to a modified optimal decision rule, where the unsolvable true marginal value of innovation is replaced by the analytic ones derived in the first step.

In the first step, the modified marginal values of innovations are calculated as follows

$$\tilde{\lambda}_{m,t} = \int_t^{\infty} e^{-\rho s} \frac{\partial \mathscr{U}}{\partial m_t} ds = \frac{1}{\rho} \frac{\partial \mathscr{U}}{\partial m_t} = \frac{1}{\rho} \left(u'(m_t) - \frac{\partial c(m_t, q_t)}{\partial m_t} \right), \tag{13}$$

$$\tilde{\lambda}_{q,t} = \int_t^{\infty} e^{-\rho s} \left(\frac{\partial \mathscr{U}}{\partial q_t} \right) ds = \frac{1}{\rho} \frac{\partial \mathscr{U}}{\partial q_t} = -\frac{1}{\rho} \frac{\partial c(m_t, q_t)}{\partial q_t}. \tag{14}$$

Intuitively, a quantitative (resp. qualitative) innovation raises the utility for all times, and the marginal values of innovations, measure the discounted sum of the increments of utility. In the standard Hamiltonian approach, the marginal values of innovations, it is hard to reach analytical solutions of $\lambda_{m,t}$ and $\lambda_{q,t}$. If we modify the first order conditions (11) and (12) by taking the current state variables as given, the same analytical solutions as (13) and (14) can be derived. In this perspective, the generative decision procedure is computed in a similar way as the standard Hamiltonian for the equilibrium, except that the generative decision procedure is used to facilitate the social planner's decision making. The properties of the modified marginal values of innovations are summarized in Lemma 3.

Lemma 3. *When $0 \leq \phi_t \leq 1$ and $m_t > 0$, we have:*

1) $\tilde{\lambda}_{m,t} \geq 0$ and $\tilde{\lambda}_{q,t} > 0$;

2) $\partial \tilde{\lambda}_{m,t} / \partial m_t < 0$ and $\partial \tilde{\lambda}_{m,t} / \partial q_t > 0$;

3) $\partial \tilde{\lambda}_{q,t} / \partial m_t > 0$ and $\partial \tilde{\lambda}_{q,t} / \partial q_t < 0$;

4) $\frac{d}{dr_{m,t}} \left[\frac{d}{dt} \left(\frac{\lambda_{m,t}}{\lambda_{q,t}} \right) \right] < 0$ and $\frac{d}{dr_{q,t}} \left[\frac{d}{dt} \left(\frac{\lambda_{m,t}}{\lambda_{q,t}} \right) \right] > 0$;

5) $\frac{d\omega_t}{dr_{m,t}} \geq 0$ and $\frac{d\omega_t}{dr_{q,t}} \leq 0$, where

$$\omega_t = \frac{\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}}{\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}} > 0. \tag{15}$$

In the second step, the social planner allocates human capital according to the modified optimal decision rule in Remark 1. Since the Remark 1 preserves the conclusion of Proposition 1 and only substitutes using the modified marginal value, $\tilde{\lambda}_{m,t}$ and $\tilde{\lambda}_{q,t}$, we still follow the three states named in the discussion of Proposition 1, except that the three states will be based on the modified optimal decision rule.

Remark 1. The modified optimal decision rule is as follows:

1) $\forall r_{m,t}, r_{q,t}$, if $\tilde{\lambda}_{m,t}\left(\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}\right) > \tilde{\lambda}_{q,t}\left(\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}\right)$, then the social planner will choose $(r_{m,t}, r_{q,t}) = (1, 0)$;

2) $\forall r_{m,t}, r_{q,t}$, if $\tilde{\lambda}_{m,t}\left(\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}\right) < \tilde{\lambda}_{q,t}\left(\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}\right)$, then the social planner will choose $(r_{m,t}, r_{q,t}) = (0, 1)$;

3) if $\exists r_{m,t} > 0, r_{q,t} > 0$, s.t. $\tilde{\lambda}_{m,t}\left(\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}\right) = \tilde{\lambda}_{q,t}\left(\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}\right)$, then the social planner will choose this allocation.

Then, it can be shown that the generative decision procedure maximizes the general utility function at any given time point (Lemma 4) instead of the discounted sum of utilities in the long run (8). What drives us to use this approach is that we have to find a balance between perfection and feasibleness. On the one hand, we want to know how demand affects economic dynamics, but the laws of motions are difficult to parse. On the other hand, if the long-term optimization is based on the current optimum, then the properties of current optimum are bound to be important considerations as a baseline for the long-term optimization. Therefore, although our approach is second best, it still has important implications in presenting the direct linkage between demand and economic dynamics, which is obscure in the theoretical long-term optimization.

Lemma 4. *$\forall t > 0$, the modified optimal decision rule maximizes $d\mathscr{U}_t/dt$*, where $\mathscr{U}_t = \mathscr{U}(m_t, q_t)$.

5.2 Dynamics

Recall that we have proposed a question that whether the states of an economy can switch in the beginning of Section 5. With the generative decision procedure above, Proposition

2 responds to this concern and shows five properties of the dynamics.[12] First, the trajectory of economic growth would converge to sustainable growth state and remain thereafter. Second, the sustainable growth states can be expressed by equation (17), defining a sustainable growth path on the 3D surface of the utility function. And, in the sustainable growth path, the demand saturation rate must be between zero and one, implying that an economy would avoid shortage or satiation when the economy can trade off between quantitative and qualitative innovations. Third, if the demand saturation rate is lower (resp. higher) than that of the sustainable growth state, then the economy would prioritize quantitative (resp. qualitative) innovation. Thereby, the demand saturation rate intertwine with economic dynamics. Fourth, the generative decision procedure ensures that the trajectory of an economy is determined and unique by its initial quantity and quality of consumption. One caveat is that the sustainable growth path may not be the same for different economies. However, if the marginal values of human capital are constants, then trajectories of different economies will merge in the same sustainable growth path with unsatiated demand even if they are endowed with different initial quantity and quality of consumption (Statement five).

Proposition 2. *Under generative decision procedure, if we have*

$$\begin{cases} \frac{d}{dt}\left(\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}}\right) \ll 0, \quad r_{m,t} = 1, \\ \frac{d}{dt}\left(\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}}\right) \gg 0, \quad r_{m,t} = 0, \end{cases} \tag{16}$$

then the following statements are true:

1) $\exists T > 0$, *if* $t > T$, $\exists r_{m,t} > 0, r_{q,t} > 0$, *s.t.* $\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}} = \omega_t$;

[12] The assumption (16) suggests that an economy cannot rely on spillover effects to keep staying in states that prioritize quantity or quality forever. For example, when $r_{m,t} = 1$, the term $\frac{d}{dt}\left(\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}}\right) \ll 0$ suggests that $\exists \delta > 0$, s.t. $\frac{d}{dt}\left(\frac{\lambda_{m,t}}{\lambda_{q,t}}\right) < -\delta$. Thus, if an economy prioritizes quantity, then the marginal value of quantity would keep declining relatively, even if the possible spillover effect may also lower the marginal value of quality. Consequently, the economy would be urged to allocate research efforts to qualitative innovation at some time point.

2) $\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}} = \omega_t$ *can be rewritten as*

$$\phi(m_t, q_t) = 1 + \frac{\frac{\partial c(m_t,q_t)}{\partial q_t}}{u'(m_t)}\omega_t, \tag{17}$$

and we have $0 < \phi < 1$*;*

3) $\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}} > \omega_t \Longrightarrow \phi(m_t, q_t) < 1 + \frac{\frac{\partial c(m_t,q_t)}{\partial q_t}}{u'(m_t)}\omega_t$*, and* $\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}} < \omega_t \Longrightarrow \phi(m_t, q_t) > 1 + \frac{\frac{\partial c(m_t,q_t)}{\partial q_t}}{u'(m_t)}\omega_t$*;*

4) $\forall t > 0$*, given* m_t *and* q_t*,* $\exists! r_{m,t}$ *satisfy the modified optimal decision rule;*

5) if $\omega_t \equiv \omega > 0$*, then* $\forall q_t > 0$*,* $\exists! m_t > 0$*, s.t.* $\phi(m_t, q_t) = 1 + \frac{\frac{\partial c(m_t,q_t)}{\partial q_t}}{u'(m_t)}\omega$*.*

The above discussion on economic dynamics facilitate our analysis of different stages of economic development. In developing countries, we frequently see their high growth rates followed by a permanent decline, and this phenomenon is often referred to as the middle income trap. Our results suggest that this slowdown in output growth could imply a shift in the state of development: from prioritizing quantity to sustainable growth. Underlying this shift, the demand saturation rate rises, and the marginal value of qualitative innovation increases (see Lemma 2 and 3). Similarly, the low growth rates of developed countries can be explained by considering that they are in sustainable growth states. However, if a developed economy overheats, the increase in demand saturation rate may prompt the economy to adopt a strategy prioritizing quality, which in turn temporarily stagnates the growth of the quantity of total output (real GDP) but also sets the stage for the next boom. In this way, we show how changes in the demand saturation rate impact economic dynamics, including shifts in development stages and changes in economic growth.

Graphical representation We can use a graphical representation to illustrate the dynamics of an economy. Here, we assume that the utility function of the economy is the same as in figure 1 and that innovation functions are linear and produces no spillover effects. The dynamics of the economy depend on its initial state, which can be divided into the following three cases. The first (resp. second) case prioritizes the quantity (resp. quality). In this case, the economy will increase the quantity (resp. quality) of consumption along the surface until the sustainable growth state is reached. The third case is when the initial quantity and quality happen to be in the sustainable growth state. If so, the economy will not deviate from the sustainable growth path. Because linear innovation functions adopted in this example satisfy that the marginal values of human capital are constants, the dynamics from different initial positions will merge into the same sustainable growth path (statement 5 of Proposition 2). The confluence is shown clearly in this figure.

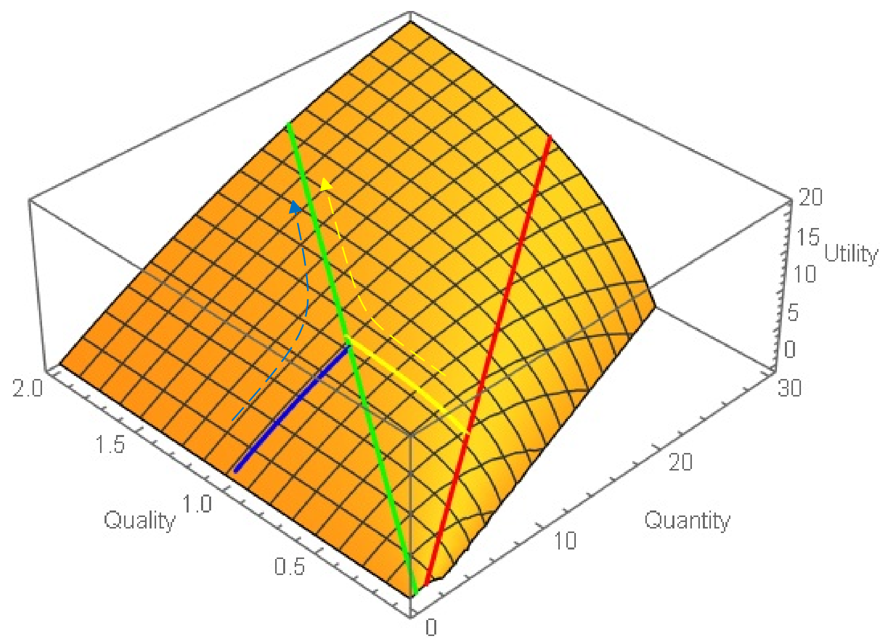


Figure 2: An illustrative example of the paths and dynamics

Note: The red line is the demand satiation path, and the green line is the sustainable growth path. Same as in figure 1, the utility function takes the specification of $\mathscr{U} = m - 0.02\frac{m^2}{q}$. The quantity of consumption ranges from 0 to 30, and the quality of consumption ranges from 0 to 2. Only the utility above zero is reported to keep the figure clear. The technology progresses take the specifications that $g_m(R_{m,t}, R_{q,t}) = 0.06R_{m,t}$ and $g_q(R_{m,t}, R_{q,t}) = 0.02R_{q,t}$. Then, the blue line initiates from $(m, q) = (0.5, 1)$, whose demend satiation rate is then 0.02 (shortage). The yellow line initiates from $(m, q) = (10, 0.4)$, whose demand satiation rate is one (satiated). The dashed curves with arrows show directions of the dynamics. Both dynamics converge to the sustainable growth path and remain thereafter.

5.3 Long-term states

From the previous subsection, we can see that an economy always converges to sustainable growth state, where the demand saturation rate must be between zero and one. However, demand saturation rate may still converge to zero or one in the long run following the sustainable growth path.

The long-term states of demand matter because it can intertwine with consumption patterns and thus development directions. To show this, we use the utility function specification from Saint-Paul (2021) as an example for illustration

$$\mathscr{U}\,(m_t, q_t) = u(m_t) - c(m_t, q_t) = m_t^{\alpha} - \theta \frac{m_t^{\gamma}}{q_t}. \tag{18}$$

And, we assume that the innovation functions are linear and have no spillover effects

$$g_m(R_{m,t}) = aR_{m,t}, \tag{19}$$

$$g_q(R_{q,t}) = bR_{q,t}, \tag{20}$$

where $a > 0$ and $b > 0$ are technology growths per unit of human capital.[13]

Then, the intertwine between demand and consumption in the long run are summarized in Lemma 5. When the demand saturation rate approaches unity (resp. zero) in the long run, $\lim_{t\to\infty} \phi_t = 1$ (resp. $\lim_{t\to\infty} \phi_t = 0$), quality (resp. quantity) is the dominated aspect of consumption, $\lim_{t\to\infty} \frac{m_t}{q_t} = 0$ (resp. $\lim_{t\to\infty} \frac{m_t}{q_t} = \infty$), which is abbreviated as pattern one (resp. two). When the demand saturation rate is between 0 and 1 in the long run, neither quantity nor quality would dominate, which is abbreviated as pattern three. Divergent consumption patterns suggest different development directions, foretelling the future of an economy.

Lemma 5. *Given the utility function (18), the sustainable growth path (17) can be written*

[13] The same innovation functions have been adopted in figure 2. The simplified innovation functions allow the variable to be a constant to deliver concise and intuitive results.

as $\frac{m_t}{q_t} = \frac{1-\phi_t}{\phi_t} \cdot \frac{a}{b}$*, Then, we have that:*

1) if $\lim_{t\to\infty} \phi_t = 1$ *, then* $\lim_{t\to\infty} \frac{m_t}{q_t} = 0$*;*

2) if $\lim_{t\to\infty} \phi_t = 0$*, then* $\lim_{t\to\infty} \frac{m_t}{q_t} = \infty$*;*

3) if $0 \ll \lim_{t\to\infty} \phi_t \ll 1$*, then* $0 \ll \lim_{t\to\infty} \frac{m_t}{q_t} \ll \infty$*.*

The long-term states of demand can vary with properties of utilization costs. For example, Proposition 3 states that whether utilization costs are sensitivity to the quantity of consumption is crucial in determining long-term states of demand. In particular, if the utilization cost is more (resp. less) sensitive to the quantity of consumption, $\gamma > \alpha + 1$ (resp. $\gamma < \alpha + 1$), that is, the utilization cost rises rapidly (resp. slowly) with the quantity consumed, pattern one (resp. two) emerges. In special cases, pattern three occurs when sensitivity of utilization costs to quantity of consumption is exactly at the threshold, $\gamma = \alpha + 1$.

Proposition 3. *Given utility function (18) and technology growth functions (19) and (20), then along the sustainable growth path:*

1) if $\gamma > \alpha + 1$*, then* $\lim_{t\to\infty} \phi_t = 1$ *and* $\lim_{t\to\infty} \frac{m_t}{q_t} = 0$*;*

2) if $\gamma < \alpha + 1$*, then* $\lim_{t\to\infty} \phi_t = 0$ *and* $\lim_{t\to\infty} \frac{m_t}{q_t} = \infty$*;*

3) if $\gamma = \alpha + 1$*, then* $0 < \lim_{t\to\infty} \phi_t = \phi^* < 1$ *and* $0 < \lim_{t\to\infty} \frac{m_t}{q_t} = s^* < \infty$*, where* ϕ^* *and* s^* *are positive constants.*

Is there a factor in reality that affects the sensitivity to utilization costs? Such factors, like aging and environmental pollution, have garnered significant attention from both academia and society. In case of aging, if the elderly are less able to bear the utilization cost than the young, then aging will mean a higher sensitivity of utilization costs to quantity of consumption and shift the economy toward the quality dominated pattern. Similarly, environmental pollution will result in illnesses and thus an escalation of utilization costs. Adding to the literature, our theory helps to explain the economic significance of these factors (aging and environmental pollution) in shaping the long-term states of demand and thus impacting the consumption patterns and development directions.

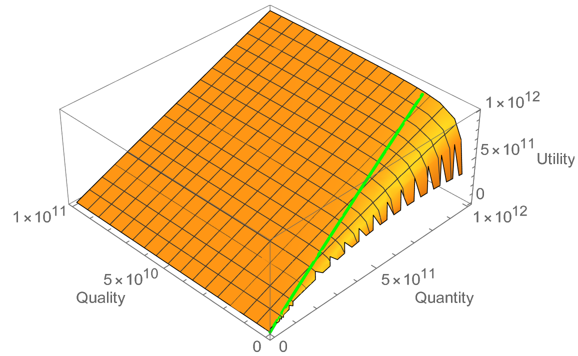
Graphical representation How do varying sensitivities of utilization costs to quantity of consumption lead to divergent long-term states of demand and development directions? The mechanism can be intuitively shown in figure. Due to demand satiation, the sensitivity constrains the feasible region of quantities and qualities of consumption. As can be seen, the feasible region in the economy is compressed as the sensitivity increases. In other words, the topological transformation induced by the change in the utility function is the cause underlying the shifts in long-term states of demand and consumption patterns.

6 Conclusion

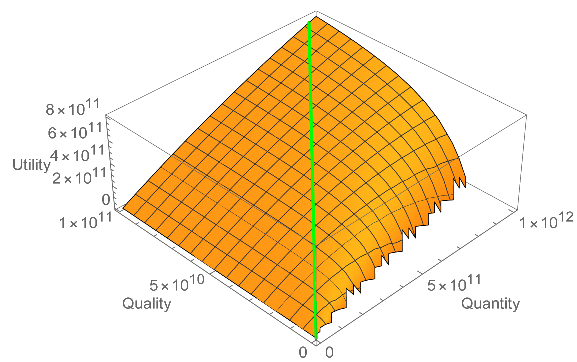
In this paper, a new concept is introduced to summarize the causes of demand saturation. This concept is referred to as utilization cost, which is the physical and mental burden that people bear in order to obtain utility. Based on this new concept, we define the quantity and quality of consumption and then provide a general utility function that allows for endogenous demand saturation. Moreover, we show that the economic dynamics can be driven by the changes in demand saturation rates, and the long-term states of demand rely on the sensitivity of utilization costs to quantity of consumption, which also affect consumption patterns in the long run. The topological transformation of the feasible region in the 3D surface of the utility function can explain the mechanism intuitively.

The results have following implications. First, real GDP growth rate will slow down with the stage of development. As an economy trades off quantity and quality of consumption, the greater the demand saturation rate, the higher the emphasis on quality. Therefore, managing utilization cost should emerge as a critical concern in development. Today's challenges, such as aging and environmental pollution, may increase sensitivity of utilization costs to quantity of consumption and further push consumption patterns toward quality, slowing down the real GDP growth rate.

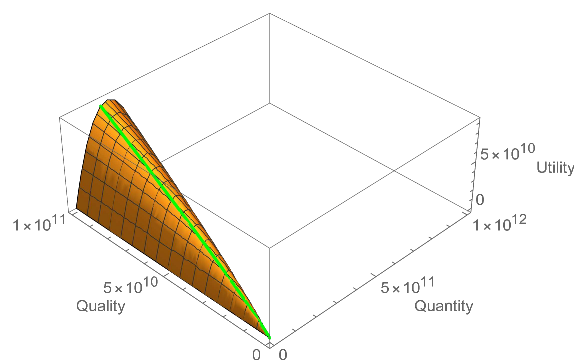
One caveat is that we have used a generative decision procedure to obtain analytical



(a) Case one: $\gamma = 1.9$



(b) Case two: $\gamma = 2$



(c) Case three: $\gamma = 2.1$

Figure 3: Sustainable growth paths under different utilization costs

Note: The utility function and innovation functions are the same as in figure 2. The value of γ varies in three cases to represent different sensitivities of utilization costs to the quantity of consumption. The green lines are corresponding sustainable growth paths.

solutions for the model. We chose this approach because of a dilemma faced by this study on presenting the economic consequences of abstract concepts. On the one hand, if abstract concepts and general functional forms are discarded, no general conclusions can be drawn, because simulations of particular examples are always not universally representative. On the other hand, if abstract concepts and general functional forms are used, the laws of motion are ambiguous to show the impact of demand on economic dynamics due to long-term optimization. Therefore, this study opts for the latter and uses a generative decision procedure to perform a general analysis on how demand affects the current decisions of an economy. Subsequent studies can choose the former and develop numerical simulations based on utility functions that include utilization cost for specific economies. With the theoretical foundation of this paper in hand, it is believed that related work can be carried out more easily.

Appendix

A Further discussion on concepts

This section will add some details to the concepts introduced in the main text by discussing the following issues, including: 1) What are positive utility and utilization cost? 2) What are quantity and quality? 3) What are quantity and quality technologies? 4) What are quantitative and qualitative innovations?

To further investigate positive utility and utilization costs, we need to make a distinction between *ends* and *means*. If we use different means to reach the same desirable *end*, then we consider the positive utility obtained to be equal. However, different *means* may have different side effects and hence their utilization costs are not the same. In short, positive utility measures the extent to which desirable *ends* are achieved, while utilization costs measure the side effects of *means*. For example, when we treat a disease by taking a medicine, curing the disease is our desirable *end* and taking the medicine is the *means*. The curative effect is a positive utility, while the drug side effect is an utilization cost.

The above discussion will lead to a further analysis of quantity and quality. The quantity in this paper is not the number of a specific commodity, but the quantity of an abstract basic consumption good (the *means*). The same amount of basic consumption goods brings the same positive utility, or can achieve the same desirable *end*. Similarly, the quality in this paper is the quality of this abstract basic consumption good, implying that this abstract basic consumption good is upgradable, generating smaller side effect for the same desirable *end*.

We proceed to analyze what technologies and innovations are. Firstly, quantity technology in this paper refers to the efficiency of using labor and resources to achieve desirable *ends*, and a quantitative innovation means that the same labor and resources can achieve better *ends*. Secondly, the quality technology in this paper is the ability to control the utilization cost. A qualitative innovation implies that the utilization cost is smaller to achieve the same desirable *end*. When the quality is zero, the utilization cost is infinite, meaning that the desirable *end* is not feasible. A qualitative innovation then can make it feasible by providing a positive quality technology level.

Lastly, we turn to the macroeconomic implications. This paper adopts the common practice of viewing the output as real GDP. From the perspective of this paper, the higher the real GDP is, the more desirable *ends* can be achieved. In reality, an upgrade in product quality may create demand and thus raise output thereafter. Correspondingly, this paper argues that a qualitative innovation will reduce utilization cost and demand saturation rate, providing increment spaces for the quantity of consumption. From this perspective, the theory in this paper is consistent with stylized facts in reality.

B Proofs

Lemma 1

Proof. Statement 1. From properties of the utilization cost $c(m,q)$, we have $\frac{\partial \mathscr{U}}{\partial q} = -\frac{\partial c(m,q)}{\partial q} > 0$, $\frac{\partial^2 \mathscr{U}}{\partial q^2} = -\frac{\partial^2 c(m,q)}{\partial q^2} < 0$, and $\frac{\partial^2 \mathscr{U}}{\partial m \partial q} = -\frac{\partial^2 c(m,q)}{\partial m \partial q} > 0$.

Statement 2. From $u'(m) > 0$ and $\lim_{m \to +0} \frac{\partial c}{\partial m} = 0$, we have $\lim_{m \to +0} \frac{\partial \mathscr{U}}{\partial m} = \lim_{m \to +0} \left(u'(m) - \frac{\partial c}{\partial m} \right) > 0$. From $u''(m) \leq 0$ and $\frac{\partial^2 c}{\partial m^2} > 0$, we have $\frac{\partial^2 \mathscr{U}}{\partial m^2} = \frac{\partial^2 u}{\partial m^2} - \frac{\partial^2 c}{\partial m^2} < 0$. Also, from $u'(m) > 0$,

$\frac{\partial^2 u}{\partial m^2} \leq 0$ and $\frac{\partial c}{\partial m} > 0$, $\frac{\partial^2 c}{\partial m^2} > 0$, we can conclude that $\lim_{m\to\infty} \frac{\partial \mathscr{U}}{\partial m} = \lim_{m\to\infty} \left(u'(m) - \frac{\partial c}{\partial m}\right) < 0$.

Statement 3. From the statement 2, we know that $\lim_{m\to+0} \frac{\partial \mathscr{U}}{\partial m} > 0$, $\frac{\partial^2 \mathscr{U}}{\partial m^2} < 0$ and $\lim_{m\to\infty} \frac{\partial \mathscr{U}}{\partial m} < 0$. Consequently, there is a unique $m > 0$ s.t. $\frac{\partial \mathscr{U}}{\partial m} = u'(m) - \partial c/\partial m = 0$, maximizing $\mathscr{U}$ and satisfies $\frac{\partial c(m,q)/\partial m}{u'(m)} = 1$. □

Lemma 2

Proof. Statement 1. From $\partial c(m,q)/\partial m \geq 0$ and $u'(m) > 0$, we have $\phi(m,q) = \frac{\partial c(m,q)/\partial m}{u'(m)} > 0$.

Statement 2. We have

$$\partial\phi/\partial m = \frac{\partial^2 c/\partial m^2}{u'(m)} - \frac{u''(m)}{u'(m)^2}\frac{\partial c}{\partial m}.$$

Because $\partial^2 c/\partial m^2 > 0$, $u'(m) > 0$, $u''(m) \leq 0$, $\partial c/\partial m > 0$, we have $\partial\phi/\partial m > 0$.

Statement 3. From $\frac{\partial^2 c}{\partial m \partial q} < 0$ and $u'(m) > 0$, we have $\partial\phi/\partial q = \frac{\frac{\partial^2 c}{\partial m\partial q}}{u'(m)} < 0$. □

Proposition 1

Proof. Statement 1. From the first order condition (10) and $\mu_{i,t} \geq 0$, $\mu_{i,t} r_{i,t} = 0$, $i = m, q$, we know that, if $\lambda_{m,t}\left(\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}\right) R_t - \lambda_{q,t}\left(\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}\right) R_t > 0$, then $\mu_{m,t} = 0$ and $\mu_{q,t} > 0$. Thus, we have $r_{q,t} = 0$ and $r_{m,t} = 1 - r_{q,t} = 1$.

Statement 2 can be proved likewise. If $\lambda_{m,t}\left(\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}\right) R_t - \lambda_{q,t}\left(\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}\right) R_t < 0$, then $\mu_{m,t} > 0$ and $\mu_{q,t} = 0$, so we have $r_{m,t} = 0$ and $r_{q,t} = 1 - r_{m,t} = 1$.

Statement 3 is the case when $\lambda_{m,t}\left(\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}\right) R_t - \lambda_{q,t}\left(\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}\right) R_t = 0$, then $\mu_{m,t} = \mu_{q,t} = 0$, and thus $r_{m,t} > 0$ and $r_{q,t} > 0$. □

Lemma 3

Proof. Statement 1. From (13), we have $\tilde{\lambda}_{m,t} = \frac{1-\phi_t}{\rho} u'(m_t)$. Since $u'(m_t) > 0$, we have $\tilde{\lambda}_{m,t} \geq 0$ when $0 \leq \phi_t \leq 1$. From (14), we have $\lambda_{q,t} = -\frac{\partial c(m_t,q_t)/\partial q_t}{\rho} > 0$ since $\partial c(m_t, q_t)/\partial q_t < 0$ and $\rho > 0$.

Statement 2. We have $\frac{\partial \tilde{\lambda}_{m,t}}{\partial m_t} = \frac{u''(m_t) - \partial^2 c(m_t,q_t)/\partial m_t^2}{\rho} < 0$ since $u''(m_t) \leq 0$ and $\partial^2 c(m_t, q_t)/\partial m_t^2 > 0$. We have $\frac{\partial \tilde{\lambda}_{m,t}}{\partial q_t} = -\frac{\partial^2 c(m_t,q_t)/\partial q_t^2}{\rho} > 0$ since $\partial^2 c(m_t, q_t)/\partial q_t^2 < 0$.

Statement 3. We have $\frac{\partial \tilde{\lambda}_{q,t}}{\partial m_t} = -\frac{1}{\rho}\frac{\partial^2 c(m_t,q_t)}{\partial m_t \partial q_t} > 0$ since $\frac{\partial^2 c(m_t,q_t)}{\partial m_t \partial q_t} < 0$. We have $\frac{\partial \tilde{\lambda}_{q,t}}{\partial q_t} = -\frac{1}{\rho}\frac{\partial^2 c(m_t,q_t)}{\partial q_t^2} < 0$ since $\frac{\partial^2 c(m_t,q_t)}{\partial q_t^2} > 0$.

Statement 4. We have

$$\frac{d}{dr_{m,t}}\left[\frac{d}{dt}\left(\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}}\right)\right] = R_t \frac{1}{\tilde{\lambda}_{q,t}^2}\left(\tilde{\lambda}_{q,t}\Gamma_1 + \tilde{\lambda}_{m,t}\Gamma_2\right),$$

where $\Gamma_1 = \frac{\partial \tilde{\lambda}_{m,t}}{\partial m_t}\left(\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}\right) + \frac{\partial \tilde{\lambda}_{m,t}}{\partial q_t}\left(\frac{\partial g_q}{\partial R_{m,t}} - \frac{\partial g_q}{\partial R_{q,t}}\right)$ and $\Gamma_2 = \frac{\partial \tilde{\lambda}_{q,t}}{\partial m_t}\left(\frac{\partial g_m}{\partial R_{q,t}} - \frac{\partial g_m}{\partial R_{m,t}}\right) + \frac{\partial \tilde{\lambda}_{q,t}}{\partial q_t}\left(\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}\right)$. Because $\frac{\partial \tilde{\lambda}_{m,t}}{\partial m_t} < 0$, $\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}} > 0$, $\frac{\partial \tilde{\lambda}_{m,t}}{\partial q_t} > 0$ and $\frac{\partial g_q}{\partial R_{m,t}} - \frac{\partial g_q}{\partial R_{q,t}} < 0$, the term $\Gamma_1 < 0$. Likewise, because $\frac{\partial \tilde{\lambda}_{q,t}}{\partial m_t} > 0$, $\frac{\partial g_m}{\partial R_{q,t}} - \frac{\partial g_m}{\partial R_{m,t}} < 0$, $\frac{\partial \tilde{\lambda}_{q,t}}{\partial q_t} < 0$ and $\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}} > 0$, the term $\Gamma_2 < 0$. Consequently, because $R_t > 0$, $\tilde{\lambda}_{m,t} > 0$ and $\tilde{\lambda}_{q,t} > 0$, we have $\frac{d}{dr_{m,t}}\left[\frac{d}{dt}\left(\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}}\right)\right] < 0$. Because $r_{m,t} + r_{q,t} = 1$, $\frac{d}{dr_{q,t}}\left[\frac{d}{dt}\left(\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}}\right)\right] = -\frac{d}{dr_{m,t}}\left[\frac{d}{dt}\left(\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}}\right)\right] > 0$.

Statement 5. Denote that $\omega_t = \frac{\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}}{\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}}$. Because we have assumed that the spillover effects are minor, i.e. $\frac{\partial g_q}{\partial R_{q,t}} > \frac{\partial g_q}{\partial R_{m,t}} \geq 0$ and $\frac{\partial g_m}{\partial R_{m,t}} > \frac{\partial g_m}{\partial R_{q,t}} \geq 0$, we have $\omega_t > 0$.

Then, we can calculate that

$$\begin{aligned} \frac{d\omega_t}{dr_{m,t}} &= \frac{d}{dr_{m,t}}\left(-\frac{\frac{\partial g_q(r_{m,t}R_t,(1-r_{m,t})R_t)}{\partial r_{m,t}}}{\frac{\partial g_m(r_{m,t}R_t,(1-r_{m,t})R_t)}{\partial r_{m,t}}}\right) \\ &= \frac{\Xi_1\left(\frac{\partial g_q}{\partial R_m} - \frac{\partial g_q}{\partial R_q}\right) + \Xi_2\left(\frac{\partial g_m}{\partial R_q} - \frac{\partial g_m}{\partial R_m}\right)}{\left(\frac{\partial g_m}{\partial R_q} - \frac{\partial g_m}{\partial R_m}\right)^2} R_t, \end{aligned}$$

where $\Xi_1 = \frac{\partial^2 g_m}{\partial R_{q,t}^2} - 2\frac{\partial^2 g_m}{\partial R_{m,t}\partial R_{q,t}} + \frac{\partial^2 g_m}{\partial R_{m,t}^2}$ and $\Xi_2 = \frac{\partial^2 g_q}{\partial R_{q,t}^2} - 2\frac{\partial^2 g_q}{\partial R_{m,t}\partial R_{q,t}} + \frac{\partial^2 g_q}{\partial R_{m,t}^2}$. Because $\frac{\partial^2 g_m}{\partial R_{q,t}^2} \leq 0$, $\frac{\partial^2 g_m}{\partial R_{m,t}^2} \leq 0$ and $\frac{\partial^2 g_m}{\partial R_{m,t}\partial R_{q,t}} \geq 0$, we have $\Xi_1 \leq 0$. Because $\frac{\partial^2 g_q}{\partial R_{q,t}^2} \leq 0$, $\frac{\partial^2 g_q}{\partial R_{m,t}^2} \leq 0$ and $\frac{\partial^2 g_q}{\partial R_{m,t}\partial R_{q,t}} \geq 0$, we have $\Xi_2 \leq 0$. From $\frac{\partial g_q}{\partial R_{m,t}} - \frac{\partial g_q}{\partial R_{q,t}} < 0$, $\frac{\partial g_m}{\partial R_{q,t}} - \frac{\partial g_m}{\partial R_{m,t}} < 0$ and $R_t > 0$, we have $\frac{d\omega_t}{dr_{m,t}} \geq 0$. Because $r_{m,t} + r_{q,t} = 1$, $\frac{d\omega_t}{dr_{q,t}} = -\frac{d\omega_t}{dr_{m,t}} \leq 0$. □

Lemma 4

Proof. We have

$$\frac{d\mathscr{U}_t}{dt} = \frac{\partial \mathscr{U}_t}{\partial m_t}\frac{dm_t}{dt} + \frac{\partial \mathscr{U}_t}{\partial q_t}\frac{dq_t}{dt}. \tag{21}$$

Given $r_{q,t} = 1 - r_{m,t}$ and innovation functions (6) and (7), we can find the condition to maximize $d\mathscr{U}_t/dt$ by calculating

$$\begin{aligned}\frac{d}{dr_{m,t}}\left(\frac{d\mathscr{U}_t}{dt}\right) &= \frac{d}{dr_{m,t}}\left(\frac{\partial \mathscr{U}_t}{\partial m_t}\frac{dm_t}{dt} + \frac{\partial \mathscr{U}_t}{\partial q_t}\frac{dq_t}{dt}\right)\\ &= \frac{\partial \mathscr{U}_t}{\partial m_t}\left(\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}\right) - \frac{\partial \mathscr{U}_t}{\partial q_t}\left(\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}\right).\end{aligned} \tag{22}$$

Given $\rho > 0$, $\tilde{\lambda}_{m,t} = \frac{1}{\rho}\frac{\partial \mathscr{U}}{\partial m_t}$, and $\tilde{\lambda}_{q,t} = \frac{1}{\rho}\frac{\partial \mathscr{U}}{\partial q_t}$ from (13) and (14), it can easily be shown that the condition to maximize $d\mathscr{U}_t/dt$ is equivalent to the modified optimal decision rule in Remark 1. □

Proposition 2

Proof. Statement 1. We first show that the states prioritizing quantity and quality would switch to sustainable growth, and then show that the economy would stay in sustainable growth thereafter.

The modified decision rule can be rewritten to depend on the following relationship:

$$\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}} \gtreqless \frac{\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}}{\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}}. \tag{23}$$

Suppose an economy starts from prioritizing quantity, $\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}} > \frac{\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}}{\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}}$, and thus $(r_{m,t}, r_{q,t}) = (1, 0)$. According to assumption (16), the value of the LHS of (23) would decline, and the value of the RHS of (23) would remain the same. Because the speed of decline has a lower bound, i.e. $\exists \delta > 0$, s.t. $\frac{d}{dt}\left(\frac{\lambda_{m,t}}{\lambda_{q,t}}\right) < -\delta$, there must be a time T, s.t. $\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}} = \frac{\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}}{\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}}$. The case that an economy initiates from prioritizing quality can be proved likewise.

Then, we show that there is a contradiction if an economy can switch back from sustainable growth to prioritizing quantity or quality. Suppose the economy swtich from sustainable growth to prioritizing quantity of consumption, then $r_{m,t}$ rises. According to assumption (16), the LHS of (23) declines, while the RHS of (23) would not decline from Lemma 3. Then, we have $\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}} < \frac{\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}}{\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}}$, which is the condition of prioritizing quality and thus contradicts with the assumption. The case that an economy cannot switch back from sustainable growth to prioritizing quality can be proved likewise.

Statement 2. By plugging (13) and (14) in $\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}} = \frac{\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}}{\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}}$, we have that

$$\frac{\frac{1}{\rho}\left(u'(m_t) - \frac{\partial c(m_t,q_t)}{\partial m_t}\right)}{-\frac{1}{\rho}\frac{\partial c(m_t,q_t)}{\partial q_t}} = \omega_t, \tag{24}$$

where $\omega_t = \frac{\frac{\partial g_q}{\partial R_{q,t}} - \frac{\partial g_q}{\partial R_{m,t}}}{\frac{\partial g_m}{\partial R_{m,t}} - \frac{\partial g_m}{\partial R_{q,t}}}$. Given that $\phi(m_t, q_t) = \frac{\partial c(m_t,q_t)/\partial m_t}{u'(m_t)}$ as in equation (3), we can simplify (24) to be $\phi(m_t, q_t) = 1 + \frac{\frac{\partial c(m_t,q_t)}{\partial q_t}}{u'(m_t)}\omega_t$. Then, because $\frac{\partial c(m_t,q_t)}{\partial q_t} < 0$, $u'(m_t) > 0$ and $\omega_t > 0$, we have $\phi(m_t, q_t) < 1$. Given $\phi(m_t, q_t) > 0$ from Lemma (2), we have that $0 < \phi(m_t, q_t) < 1$.

Statement 3. We have $\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}} > \omega_t \Longrightarrow \frac{\frac{1}{\rho}\left(u'(m_t) - \frac{\partial c(m_t,q_t)}{\partial m_t}\right)}{-\frac{1}{\rho}\frac{\partial c(m_t,q_t)}{\partial q_t}} > \omega_t \Longrightarrow \phi(m_t, q_t) < 1 + \frac{\frac{\partial c(m_t,q_t)}{\partial q_t}}{u'(m_t)}\omega_t$, and the case that $\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}} < \omega_t$ can be proved likewise.

Statement 4. We first prove that the statement is true in sustainable growth state, and then extend to the states that prioritize quantity and quality. In sustainable growth state, the choice of $r_{m,t}$ exists from statement one. Then, without loss of generality, suppose there is a $r^*_{m,t} > r_{m,t}$, subject to $\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}} = \omega_t$. Then, since $\frac{d}{dr_{m,t}}\left(\frac{\tilde{\lambda}_{m,t}}{\tilde{\lambda}_{q,t}}\right) < 0$ and $\frac{d\omega_t}{dr_{m,t}} \geq 0$ from Lemma 3, the economy would turn to the state that prioritizes quality. This result contradicts with statement one that suggests that the economy should remain in sustainable growth. Thus, the choice of $r_{m,t}$ is unique in sustainable growth. When prioritizing quantity (resp. quality), we have that $r_{m,t} = 1$ (resp. $r_{m,t} = 0$), so the choice of $r_{m,t}$ exists and is unique.

Statement 5. Consider the function

$$f(m, q) = \phi(m, q) - \frac{\frac{\partial c(m,q)}{\partial q}}{u'(m)}\omega - 1,$$

and sustainable growth path can be described by $f(m,q)=0$. For this function, we have

$$\frac{\partial f}{\partial m}=\frac{\omega u''(m)\frac{\partial c(m,q)}{\partial q}}{u'(m)^2}-\frac{\omega\frac{\partial^2 c(m,q)}{\partial m\partial q}}{u'(m)}+\frac{\partial\phi(m,q)}{\partial m}>0,$$

because $\omega>0$, $u''(m)<0$, $\frac{\partial c(m,q)}{\partial q}<0$, $u'(m)>0$, $\frac{\partial^2 c(m,q)}{\partial m\partial q}<0$ and $\frac{\partial\phi(m,q)}{\partial m}>0$. Also, when $q>0$, we can derive that $\lim_{m\to 0} f(m,q)=-1<0$ and $\lim_{m\to\infty} f(m,q)=\infty$. Consequently, for any $q>0$, there is a unique $m>0$ satisfying that $f(m,q)=0$, i.e. the sustainable growth path is unique. □

Lemma 5

Proof. From Proposition 2, we know that

$$\phi_t=1+\frac{\frac{\partial c(m_t,q_t)}{\partial q_t}}{u'(m_t)}\omega_t.$$

Applying utility function (18) and innovation functions, (19) and (20), yields:

$$\frac{m_t}{q_t}=\frac{1-\phi_t}{\phi_t}\cdot\frac{a}{b}. \tag{25}$$

Given $\frac{a}{b}>0$, we can check the three statements are true. □

Proof of Proposition 3

Proof. From (3) and model specifications (18), (19) and (20), we can derive that

$$\phi_t=\frac{\theta\gamma m_t^{\gamma-\alpha}}{\alpha q_t}. \tag{26}$$

Plugging in (25), we have $\frac{m}{q} = \frac{\alpha a q m^{\alpha-\gamma}\left(1 - \frac{\theta\gamma m^{\gamma-\alpha}}{\alpha q}\right)}{\theta\gamma b}$, from which we can solving q:

$$q_t = \frac{\left(\sqrt{a^2\gamma^2\theta^2 + 4a\alpha b\theta m_t^{\alpha+1-\gamma}} + a\gamma\theta\right)}{2a\alpha} m_t^{\gamma-\alpha}, \tag{27}$$

Plugging (27) in (26), yields:

$$\phi_t = \frac{2a\gamma\theta}{\sqrt{a^2\gamma^2\theta^2 + 4ab\alpha\theta m_t^{\alpha+1-\gamma}} + a\gamma\theta}. \tag{28}$$

When $m_t \to \infty$, if $\gamma < \alpha + 1$, then we can derive that $\phi_t \to 0$ from equation (28) and $\frac{m_t}{q_t} \to \infty$ from Lemma 5. So the statement 1 is proved. The proofs of statement 2 and 3 are likewise according to (28) and Lemma 5. □

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