Insider Trading with Semi-Informed Traders and Information Sharing: The Stackelberg Game

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Abstract

We study a generalization of the Kyle (1985) static model with two risk neutral insiders to the case where each insider is partially informed about the value of the stock and compete under Stackelberg setting. First, we characterize the linear Bayesian equilibrium. Then, we carry out a comparative statics analysis. Our findings reveal that partial information increases the insiders profits in a Stackelberg setting than in a Cournot setting. Finally we study the impact of the information sharing on equilibrium outcomes.

JEL classification: G14, D82

Keywords: Insider trading, Risk neutrality, Partial Information, Stackelberg structure, Kyle model

1 Introduction

Asymmetric information plays a crucial role in insider trading. Owing to their position within the firm, corporate insiders, such as corporate executives or board members, are more informed than outsiders about any event involving the firm they work for and whose stock is publicly traded. They can use this information to make profitable trades at the expense of other market participants who do not have access to the same information. Typically, this private, price-sensitive information is expected to alter the behavior of the company's stock and reduce market efficiency.

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With the recurrence of insider trading scandals, academic research has consistently renewed its interest in this topic. A large strand of the literature focused on the informational effect of insider trading following the pioneering work of Kyle (1985). In the Kyle model, there is an insider who knows the value of the stock and a market maker who only knows the distribution of the values of the stock, gets information from the total noisy stock order flow, and sets the stock price in a way that his expected profits are zero. The results show that the stock price reveals half of the inside information, regardless of the parameter values. Then, many other scholars investigated the impact of different ways of disclosing information on insider trading in the Kyle one-shot game model. Some papers extended the Kyle model to include multiple risk-averse traders (Subrahmanyam, 1991; Holden and Subrahmanyam, 1994; Vitale, 1995; Zhang, 2004 and Baruch, 2002). Recently, Daher et al. (2020) extended the Kyle model to the case of two insiders, one risk-neutral and one risk-averse, while Jiang and Liu (2022) studied the case of two insiders in which the first insider is risk-neutral while the second insider is overconfident. Another direction of the extension of the static Kyle model is in Jain and Mirman (1999) who allowed for the market maker to observe a second signal that is correlated to the order flow. Multiple papers have also investigated the real and financial effects of insider trading in a static Kyle model (Jain and Mirman, 2000 and 2002; Daher and Mirman, 2006 and 2007; Wang et al., 2009 and Wang and Wang; 2010). Carre et al. (2022) extend Kyle's model to the case where the insider may be subject to an additional trading penalty, increasing in the size of their trades, and characterize the set of insider trading penalties which are efficient from the point of view of a regulator who cares both about market liquidity and price informativeness.

A very interesting extension of the static Kyle model resides in the investigation of the effect of different market structures in the financial model on the dissemination of information. Indeed, there exists different types of insiders in the firm, some without any managerial responsibilities (the president and the members of the board of directors, for example), with the objective of maximizing their profits from trading the stock of the firm whose inside information they possess. Therefore, competition among insiders is another form of competition that influences the amount of information disseminated in the stock price. Daher and al. (2012) extend the Kyle-type model of Jain and Mirman (1999) with two signals to include Cournot duopoly among insiders in the financial market and find that each insider loses the market power and partially controls the stock price. Hence, the stock price reveals more information with respect to Jain and Mirman (1999). The unconditional profits of each insider also decrease. Daher and al. (2012) also extend the Kyle-type model of Jain and Mirman (1999) with two signals to investigate the effect Stackelberg competition in the financial market on information revelation. They assume that one of the insiders, the owner, is high in the organizational hierarchy and chooses the second insider, the manager, to serve his purpose. In other words, the owner is the leader and knows the reaction function of the manager. They show that with Stackelberg competition in the financial market, the manager trades less and hence earns less than in the Cournot case.

However, the owner, due to her role as leader, trades more than in the Cournot and earns more profits. In addition, the price reveals more information in the Stackelberg than in the Cournot structure. Gong et al. (2018) develops a sequential fair Stackelberg auction model in the spirit of Kyle (1985) in which each of the two risk-seeking insiders has an equal chance to be a leader or follower at each auction stage. The authors establish the existence, uniqueness of sequential fair Stackelberg equilibria when both insiders adopt linear strategies, and find that at the sequential equilibria such two insiders compete aggressively that cause the liquidity of market to drop, the information to be revealed and the profit to go down very rapidly while the trading intensity goes substantially high.

The paper adds to the literature on insider trading in the spirit of Kyle (1985) by modeling Stackelberg competition among the two partially-informed insiders in the financial market, where the owner acts as a Stackelberg leader in the financial market, and the manager acts as a Stackelberg-follower in the financial sector to the owner who knows his reaction function before making any decision. The Stackelberg structure in the financial market with the owner serving as the Stackelberg leader seems very realistic. Indeed, the owner designs the manager's compensation mechanism and chooses a capable manager to serve his purpose; therefore, he should have information on the manager's reaction. In addition, feedback from staff of various levels can tell the owner about any improper behavior of the manager, so the owner can frequently adjust his perception. Our results show that in the presence of partial information, the follower makes more profits than the leader.

Although insiders tend to compete in the financial markets, they also tend to share trading information. For instance, evidence suggests that mutual fund managers trade based on local word-of-mouth communication in the asset-management community. Sharing information reduces the informational advantage of the insider and therefore it is important to understand what benefit investors get from sharing information. Goldstein et al. (2023) study information sharing between strategic investors who are informed about asset fundamentals. They show that a coarsely informed investor optimally chooses to share information if his counterparty investor is well informed. By doing so, the coarsely informed investor invites the other investor to trade against his information, thereby reducing his price impact. Paradoxically, the well informed investor loses from receiving information because of the resulting worsened market liquidity and the more aggressive trading by the coarsely informed investor. Another contribution of the current paper is to investigate the effect of information sharing among partially informed insiders on the dissemination of information. Our results show that follower has no incentive to share information while the leader in indifferent between sharing and not sharing the information.

The paper is organized as follows: In Section 2, we present the model and provide the necessary and sufficient condition for the uniqueness of the linear Bayesian equilibrium. Moreover, we characterize the linear Bayesian equilibrium of the model. In Section 3, we conduct a comparative statics analysis of the equilibrium outcomes with respect to Tighe and Michener (1994) and Goldstein et al. (2023). Finally, in Section 4, we present the model with information sharing.

2 The Stackelberg Model

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space, all the random variables are defined with respect to this probability space. In this model we consider a modified version of the model of Kyle (1985) with two insiders, each of them, is partially informed about the underlying value of the stock. Consider an economy with one financial asset, the stock of the firm. There are three types of agents trading in the financial market. First, there are two risk-neutral rational traders (the insiders), each of them is partially informed about the the realization z of $\tilde{z}$, the value of the stock. Specifically, we assume that the partially informed trader $i, (i = 1, 2)$ observes the signal $\tilde{s}_i = \tilde{z} + \tilde{\varepsilon}_i$ correlated to $\tilde{z}$ where $\tilde{\varepsilon}_i$ is normally distributed with zero mean and variance σ_i^2. Second, there are (non-rational) noise traders, representing small investors with no information on z. The aggregate noise trade is assumed to be a random variable $\tilde{u}$, which is normally distributed with mean zero and variance σ_u^2. Finally, there are $K(K \geq 2)$ risk-neutral market makers who act like Bertrand competitors. We assume that each of the variables $\tilde{z}$ and $\tilde{\varepsilon}_i(i = 1, 2)$, is independent from $\tilde{u}$.[1]

Following Kyle (1985), the trading mechanism is organized in two steps. In step one, a linear pricing rule and optimal order rule are determined by the market makers and the insiders, respectively, as a Bayesian Nash equilibrium. The market makers determine a (linear) pricing rule p, based on their a priori beliefs, where p is a measurable function $p : \mathbb{R} \longrightarrow \mathbb{R}$. Each insider $i = 1, 2$ chooses a stock trade function $\tilde{x_i}$, where $x_i : \mathbb{R} \longrightarrow \mathbb{R}$ is a measurable function. In the second step, the insiders observe the realization of the signals, and submit their stock order to the market makers based on the equilibrium stock trade functions. The market makers also receive orders from the noise traders, all these orders arrive as a total order flow signal $\tilde{r} = \tilde{x}_1 + \tilde{x}_2 + \tilde{u}$. The order flow signal is used by the market makers to set the price $\tilde{p} = p(\tilde{r})$, based on the equilibrium price function, to clear the market. Each partially informed insider $i, i = 1, 2$ knows only the realization s_i of $\tilde{s}_i$ and does not know the values of $\tilde{u}, , \tilde{r}$, before the order flow decisions is made. Moreover, each market maker does not know neither the realization z of $\tilde{z}$ nor the realization s_i of $\tilde{s}_i$ but only knows their distribution. Finally, the market makers cannot observe either x_i or u.

In this model we introduce Stackelberg competition among the two insiders in the financial market. Specifically, we assume that one of insiders, the owner (insider 1), is high on the organizational hierarchy and acts as a Stackelberg leader in the financial market. The second insider (insider 2), the manager, is in the lower

[1]Random variables are denoted with a tilde. Realized values lack the tilde. The mean of the random variable is denoted with bar.

ladder of the organizational hierarchy and acts as a Stackelberg-follower in the financial sector to the owner who knows his reaction function before making any decision.

This is a game of incomplete information and we seek a Bayesian-Nash equilibrium. A Bayesian-Nash equilibrium is a vector of three functions $[x_1(.), x_2(.), p(.)]$ such that:

(a) Profit maximization of insider 1,

$$E[\tilde{z} - p(\tilde{x}_1 + \tilde{x}_2 + \tilde{u}))\tilde{x}_1|\tilde{s}_1] \geq E[\tilde{z} - p\tilde{x}_1' + \tilde{x}_2 + \tilde{u}))\tilde{x}_1'|\tilde{s}_1] \tag{1}$$

for any level of trading order $\tilde{x}_1'$ decided by the insider;

(b) Profit maximization of insider 2,

$$E[\tilde{z} - p(\tilde{x}_1 + \tilde{x}_2 + \tilde{u}))\tilde{x}_1|\tilde{x}_1, \tilde{s}_2] \geq E[\tilde{z} - p(\tilde{x}_1 + \tilde{x}_2' + \tilde{u}))\tilde{x}_2'|\tilde{x}_1, \tilde{s}_2] \tag{2}$$

for any level of trading order $\tilde{x}_2'$ decided by the insider;

(c) Semi-Strong Market Efficiency: The pricing rule $p(.)$ satisfies,

$$p(\tilde{r}) = E[\tilde{z}|\tilde{r}]. \tag{3}$$

An equilibrium is linear if the insiders' strategies are linear with respect to their observed signals and the pricing rule is linear with respect to the order flow signal. In other words, there exists constants $a_0, a_1, b_0, b_1, b_2, \mu, \lambda$ such that,

$$x_1(s_1) = a_0 + a_1 s_1 \qquad x_2(s_2, x_1) = b_0 + b_1 x_1 + b_2 s_2 \tag{4}$$

and

$$\forall r, \quad p(r) = \mu + \lambda r. \tag{5}$$

Note that conditions (1) and (2) define the optimal strategies of the insiders while condition (3) guarantees the zero expected profits for the market makers. The stock price, set by the market makers, is equal to the conditional expectation of the asset given the available information. We restrict our study to linear equilibrium. The normal distributions of the exogenous random variables, together with the particular expression of the demand, enable us to derive and to prove the existence of a unique linear equilibrium. In Proposition 1, we characterize the unique linear equilibrium outcomes of the model.

Proposition 1 *In the Stackelberg model with two partially informed traders, a linear equilibrium exists and it is unique. It is characterized by,*

$$\tilde{x}_1(\tilde{s}_1) = a_1(\tilde{s}_1 - \bar{s}_1) \quad \text{and} \quad \tilde{x}_2(\tilde{x}_1, \tilde{s}_2) = b_1\tilde{x}_1 + b_2(\tilde{s}_2 - \bar{s}_2) \tag{6}$$

$$a_1 = \frac{h_1 h_2}{2\lambda(1+h_1)(1+h_1+h_2)}, \; b_1 = -\frac{1}{2} + \frac{1+h_1}{h_2} \text{ and } b_2 = \frac{h_2}{2\lambda(1+h_1+h_2)} \quad (7)$$

$$\lambda = \frac{\sqrt{(1+h_1)[4h_1(1+h_1)^2 + 4(1+h_1)(1+2h_1)h_2 + (4+3h_1)h_2^2)\sigma_z^2]\sigma_u^2}}{4(1+h_1)(1+h_1+h_2)\sigma_u^2} \quad (8)$$

$$\mu = \bar{z}, \quad E[\pi_1] = a_1[\gamma(1-\lambda b_2) - \lambda a_1(1+b_1)](\sigma_z^2 + \sigma_1^2) \text{ and } E[\pi_2] = \lambda E[\tilde{x}_2^2] \quad (9)$$

Where $h_1 = \frac{\sigma_z^2}{\sigma_1^2}$, $h_2 = \frac{\sigma_z^2}{\sigma_2^2}$ *and* $\gamma = \frac{\sigma_z^2}{\sigma_z^2+\sigma_1^2}$,

Proof: See Appendix A.

Discussion of the equilibrium: Proposition 1 highlights the impact of both market structure (the Stackelberg setting) and information asymmetry on the equilibrium outcomes. Hence, the relationship between our model and existing literature papers should be clarified. First, our paper studies the impact of partial information structure on the equlibrium outcomes in alignment with Daher et al. (2012) and to Fuzhou and Yonghui (2018) which both consider a Stackelberg structure of the Kyle model with fully informed traders. Moreover, our model differs from Tighe and Michener (1994) and Goldstein et al. (2023) s which study the case of Cournot setting with partially informed traders, by studying the impact of Stackelberg structure in the financial sector. Consequently, in this paper, we will be able to show the effect of market structure and information asymmetry on the equilibrium outcomes.

The sequential structure of the players' information and moves in the Stackelberg game, has a direct effect on the equilibrium outcomes. Indeed, when the two insiders are fully informed, our paper reduces to the case studied by Daher et al. (2012). To achieve such result, we need to implement two steps procedure: in the first step, we characterize the equilibrium outcomes when we replace the follower's signal noise variance by zero (equivalently, we consider the limit when h_2 goes to infinity). In the second step, we replace the leader' signal noises variances (equivalently, we consider the limit when h_1 goes to infinity) of the resulting outcomes. Specifically, we have the following result.

Lemma 1 *When the two insiders are fully informed, our paper reduces to the case studied by Daher et al. (2012)*[2]*. Indeed*

$$\tilde{x}_1^D(\tilde{z}) = \lim_{h_1\to\infty}(\lim_{h_2\to\infty}(a_1(\tilde{s}_1 - \bar{s}_1))) = \frac{1}{2\lambda^D}(\tilde{z} - \bar{z}) \quad (10)$$

[2]The superscript "D" refers to Daher et al (2012)

$$\tilde{x}_2^D(\tilde{z}) = \lim_{h_1\to\infty}(\lim_{h_2\to\infty}(b_1\tilde{x}_1 + b_2(\tilde{s}_2 - \bar{s}_2))) = \frac{1}{2\lambda^D}(\tilde{z} - \bar{z}) \tag{11}$$

$$\mu = \bar{z} \quad \text{and } \lambda^D = \lim_{h_1\to\infty}(\lim_{h_2\to\infty}\lambda) = \frac{\sigma_z\sqrt{3}}{4\sigma_u} \tag{12}$$

$$E[\pi_1]^D = \lim_{h_1\to\infty}(\lim_{h_2\to\infty}(E[\pi_1]) = \frac{\sigma_z\sigma_u}{2\sqrt{3}} \tag{13}$$

$$E[\pi_2]^D = \lim_{h_1\to\infty}(\lim_{h_2\to\infty}(E[\pi_2]) = \frac{\sigma_z\sigma_u}{4\sqrt{3}} \tag{14}$$

Note that this procedure is restrictive to the Stackelberg structure. Indeed. in the Cournot model with partially informed traders studied in Tighe and Michener (1994) and Goldstein et al. (2023), the equilibrium outcomes converge to the full information case when both h_1 and h_2 go simultaneously to infinity. Moreover, it should be noted that the information structure in the Stackelberg game imposes the order of convergence. In other words, we do not obtain the full information case if we first let h_1 go to infinity and then h_2 which is not the case in the Cournot game.

Proposition 1 sheds light on the impact of partial information i on the equilibrium outcomes in a Stackelberg game strucutre. Indeed, Figure 1 shows that the profits of the follower (insider 2) are greater than the profits of the leader (insider 1).

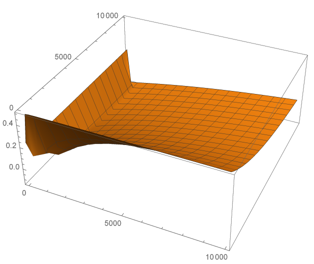

Figure 1: $\delta\Pi = \Pi_2 - \Pi_1$, The difference of two insiders' profits in the Stackelberg the same signal.

3 Comparative Statics

In this section, we carry out a comparative statics analysis of the results of our model with respect to the results of Tighe and Michener (1994) and Goldstein et al (2023). To shed light on the impact of the market structure on equilibrium outcomes, we restrict our analysis to the case when the two insiders observe the same signal. In this case, Proposition 1 becomes

Proposition 2 *When two signals' precisions are the same ($\sigma_1^2 = \sigma_2^2$), the equilibrium is characterized by,*

$$\tilde{x}_1(\tilde{s}_1) = a_1(\tilde{s}_1 - \bar{s}_1) \quad \text{and} \quad \tilde{x}_2(\tilde{x}_1, \tilde{s}_2) = b_1\tilde{x}_1 + b_2(\tilde{s}_2 - \bar{s}_2) \tag{15}$$

$$a_1 = \frac{h^2}{2\lambda(1+h)(1+2h)}, \quad b_1 = \frac{1}{2} + \frac{1}{h} \ \text{ and } b_2 = \frac{h}{2\lambda(1+2h)} \tag{16}$$

$$\mu = \bar{z} \ \text{ and } \lambda = \frac{\sigma_z\sqrt{h(1+h)(8+3h(8+5h))}}{4\sigma_u(1+h)(1+2h)} \tag{17}$$

$$E[\pi_1] = \frac{h^2(2+3h)\sigma_z\sigma_u}{2(1+2h)\sqrt{h(1+h)(8+3h(8+5h))}} \tag{18}$$

$$E[\pi_2] = \frac{h(4+3h)^2\sigma_z\sigma_u}{4(1+2h)\sqrt{h(1+h)(8+3h(8+5h))}} \tag{19}$$

Where $h = h_1 = h_2$

In Lemma 2, we show graphically the impact of the Stackelberg structure when compared to the Cournot setting studied in Tighe and Michener (1994) and Goldstein et al (2023). We obtain,

Lemma 2 *The relations between the Stackelberg and Cournot models when the signals errors have the same precisions, is given by*

$$\lambda^S > \lambda^C, \quad \pi_1^S > \pi_1^C \quad \text{and} \quad \pi_2^S > \pi_2^C \tag{20}$$

First note that the Stackelberg competition between the two insiders, increases their unconditional profits when compared to the Cournot competition between the insiders. Second, the market depth parameter λ is higher in our model than in the Cournot case. It should be pointed out that these results do not hold in the case when both insiders are fully informed about the stock value (see Daher et al. (2012)). Consequently, in the presence of incomplete information, the insiders benefit more in the Stackelberg setting than in Cournot setting.

To better understand the impact the information asymmetry between the insiders on the market structure, we consider the case when insider 2 is fully informed about the stock value ($\tilde{s}_2 = \tilde{z}$) while insider 1 is partially informed. In this case, Proposition 1 becomes

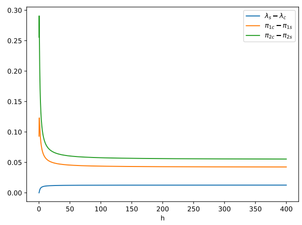

Figure 2: The market depth parameter λ and the two insiders' profits in the Stackelberg and Cournot settings in the case of observing the same signal.

Proposition 3 *When insider 2 is fully informed about the stock value ($\tilde{s}_2 = \tilde{z}$) and insider 1 is partially informed ($\tilde{s}_1 = \tilde{s}$ and $h_1 = h$), the equilibrium is characterized by,*

$$\tilde{x}_1(\tilde{s}) = a_1(\tilde{s}_1 - \bar{s}) \quad \text{and} \quad \tilde{x}_2(\tilde{x_1}, \tilde{z}) = b_1\tilde{x}_1 + b_2(\tilde{z} - \bar{z}) \tag{21}$$

$$a_1 = \frac{h}{2\lambda(1+h)}, \quad b_1 = \frac{-1}{2} \quad \text{and } b_2 = \frac{1}{2\lambda} \tag{22}$$

$$\mu = \bar{z} \quad \text{and } \lambda = \frac{\sigma_z\sqrt{(1+h)(3h+4)}}{4\sigma_u(1+h)} \tag{23}$$

$$E[\pi_1] = \frac{h\sigma_z\sigma_u}{2\sqrt{(1+h)(3h+4)}} \tag{24}$$

$$E[\pi_2] = \frac{(4+h)\sigma_z\sigma_u}{4\sqrt{(1+h)(3h+4)}} \tag{25}$$

In Lemma 3, we show graphically the impact of the Stackelberg structure when compared to the Cournot setting studied in Tighe and Michener (1994) and Goldstein et al (2023) when insider 2 is fully informed while insider 1 is partially informed. We obtain,

Lemma 3 *The relations between the Stackelberg and Cournot models when insider 2 is fully informed and insider 1 is partially informed, is given by*

$$\lambda^S < \lambda^C, \quad \pi_1^S < \pi_1^C \quad \text{and} \quad \pi_2^S < \pi_2^C \tag{26}$$

Lemma 3 reveals that Cournot structure is more beneficial more both insiders when they at one the two insiders in fully informed. Moreover, a quick comparison between the insiders' profits (see equations (24) and (25)), we can notice that the unconditional profits of the leader (insider 1) are greater than the profits of the follower (insider 2) when insider 1 signal is less noisy.

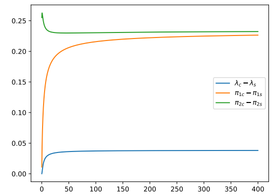

Figure 3: The market depth parameter λ and the two insiders' profits in the Stackelberg and Cournot settings in the case of fully informed insider 2 and partially informed insider 1.

$t = 0$

- Investors simultaneously make information-sharing decisions.

$t = 1$

- Investors observe their private information and, if any, the shared information;
- Investors and noise traders submit order flows, and the market maker sets the price.

$t = 2$

- The value of the assset is realized, and all agents consume.

4 Information Sharing

In this section we study the impact of information sharing on the insiders' decisions. For comparison purposes, we will adopt the same structure of Goldstein et al (2023) who study the impact of the information in the Cournot case. Specifically, we will assume that insider 2 is fully informed about the asset value, i.e. Proposition 30 holds. Moreover, we assume as in Goldstein et al. (2023), that $\bar{z} = 0$ and $\sigma_z^2 = 1$. Following Goldstein et al (2023), we denote the fully informed insider(insider 2 in our model) by H while the semi-informed insider (insider 1 in our model) by L. At $t = 0$, H and L simultaneously decide whether to share their private information to maximize their respective expected trading profits. For investor $i \in H, L$, we use $A_i \in \{S, \emptyset\}$ to denote information-sharing decisions, where $A_i = S$ means that the investor fully shares information with the other investor, whereas $A_i = \emptyset$ means that the investor keeps it secret. We assume that the date-0 information sharing decisions become common knowledge at the beginning of date 1, so that we can apply backward induction to compute the equilibrium of our economy.

Trading occurs on $t = 1$. Conditional on the endowed private information , as well as the shared information (if any), investor $i \in H, L$ places market order to maximize the expected trading profit. Let $\mathbb{F}_i$ indicates investor i's information set. For instance, if L shares information with H but H does not share information with L (i.e., $A_L = S$ and $A_H = \emptyset$), then the two investors' information sets are respectively $\mathbb{F}_L = \{\tilde{s}\}$ and $\mathbb{F}_H = \{\tilde{z}, \tilde{s}\}$ (see Figure 1 which describes the timeline of the economy).

Given investors' information-sharing decisions A_L and A_H on date 0, different trading subgames follow on date 1, depending on whether the investors share their respective private information. The investors' expected profits evaluated on the date-1 subgame equilibrium will serve as their payoffs of the date-0 information-sharing game. We next discuss each subgame separately.

Subgame 1: Neither Investor Shares Information ($A_L = A_H = \emptyset$).

When neither investor shares information, we obtain the expression of the expected profits stated in Proposition 30.

Subgame 2: L **Shares Information but** H **Does Not**($A_L = \{\tilde{s}\}$ **and** $A_H = \emptyset$).

Since H is the follower and plays at the second stage of the game, observing the signal of L, which is linearly dependent to his strategy, won't change the profits maximization problem of each of the two players. Hence, this case will coincide with subgame 1 and we obtain the expression of the expected profits stated in Proposition 30.

Subgame 3: H **Shares Information** ($A_H = S$).

There are two subgames when H shares information, depending on whether L shares his information. Nonetheless, it turns out that the equilibrium in these two subgames is the same because H owns perfect information about the asset fundamental, and once he shares it with L, L no longer uses his own noisy signal $\tilde{s}$ in predicting the asset fundamental. Thus, regardless of L's information-sharing decision, once H shares information, the trading game degenerates to the Daher el (2012) model with two perfectly-informed traders. Consequently, we obtain the expression of the expected profits stated in Lemma 1.

Now, we can summarize the insiders' payoffs of the date-0 information sharing game in Table 1. We can easily notice that this game has 2 equilibria with the same players' payoffs. Indeed, the strategy of player H , "Not share" is a dominant strategy [3]. Taking this information into account, player L is indifferent between "Share" or "Not share" since both strategies lead to the same payoff.

Finally, it should be noted that the result is quite different than the result found in Goldstein et al. (2023) in which the equilibrium consists of player L sharing his information whereas player H does not. This difference is due to the fact

[3] Consider the function $f(h) = \frac{(4+h)\sigma_u}{4\sqrt{(1+h)(3h+4)}}$. It easy to check that this function is decreasing on $(0, \infty)$ and having a horizontal asymptote with equation $y = \frac{\sigma_u}{4\sqrt{3}}$.

		H	
		$Notshare$	$Share$
L	$Notshare$	$\frac{h\sigma_u}{2\sqrt{(1+h)(3h+4)}}, \frac{(4+h)\sigma_u}{4\sqrt{(1+h)(3h+4)}}$	$\frac{\sigma_u}{2\sqrt{3}}, \frac{\sigma_u}{4\sqrt{3}}$
	$Share$	$\frac{h\sigma_u}{2\sqrt{(1+h)(3h+4)}}, \frac{(4+h)\sigma_u}{4\sqrt{(1+h)(3h+4)}}$	$\frac{\sigma_u}{2\sqrt{3}}, \frac{\sigma_u}{4\sqrt{3}}$

that in the Stackelberg game, on the one hand, the fully informed player is the follower and on the other hand, it is also due to the sequential structure of the game timing.

Appendix

Proof of Proposition 1

Applying the backward induction approach to solve for the equilibrium, insider 2 solves,

$$\begin{aligned}\underset{\tilde{x}_2}{\text{Max}}\, E[(\tilde{z}-p(\tilde{r}))\tilde{x}_2|\tilde{s}_2,\tilde{x}_1] &= \underset{\tilde{x}_2}{\text{Max}}\, E[(\tilde{z}-\mu-\lambda(\tilde{x}_1+\tilde{x}_2+\tilde{u}))\tilde{x}_2|\tilde{s}_2,\tilde{x}_1] \\ &= \underset{\tilde{x}_2}{\text{Max}}(E[\tilde{z}|\tilde{s}_2,\tilde{x}_1]-\mu-\lambda\tilde{x}_1-\lambda\tilde{x}_2)\tilde{x}_2\end{aligned}$$

The F.O.C implies that

$$\tilde{x}_2 = \frac{E[\tilde{z}|\tilde{s}_2,\tilde{x}_1]-\mu-\lambda\tilde{x}_1}{2\lambda} \tag{27}$$

and the S.O.C. requires $\lambda > 0$. Since all the variables $\tilde{z}, \tilde{s}_2$ and $\tilde{x}_1$ are all normally distributed, then by the projection Theorem, we have $E[\tilde{z}|\tilde{s}_2,\tilde{x}_1] = \alpha_0+\alpha_1\tilde{x}_1+\alpha_2\tilde{s}_2$ where α_0, α_1 and α_2 will be expressed later in the paper. Hence, equation (27) becomes,

$$\tilde{x}_2(\tilde{s}_2,\tilde{x}_1) = \frac{\alpha_0+\alpha_1\tilde{x}_1+\alpha_2\tilde{s}_2-\mu-\lambda\tilde{x}_1}{2\lambda} = \frac{\alpha_0-\mu}{2\lambda}+\frac{(\alpha_1-\lambda)}{2\lambda}\tilde{x}_1+\frac{\alpha_2}{2\lambda}\tilde{s}_2 \tag{28}$$

Thus we get,

$$b_0 = \frac{\alpha_0-\mu}{2\lambda}, \qquad b_1 = \frac{(\alpha_1-\lambda)}{2\lambda}, \qquad b_2 = \frac{\alpha_2}{2\lambda} \tag{29}$$

We move now to the insider 1's problems. He solves

$$\underset{\tilde{x}_1}{\text{Max}}\, E[(\tilde{z}-p(\tilde{r}))\tilde{x}_1|\tilde{s}_1] = \underset{\tilde{x}_1}{\text{Max}}\, E[(\tilde{z}-\mu-\lambda(\tilde{x}_1+\tilde{x}_2+\tilde{u}))\tilde{x}_1|\tilde{s}_1]$$

Plugging equation (28) into the above equation insider 1 solves,

$$\underset{\tilde{x}_1}{\text{Max}}\, E[(\tilde{z} - \mu - \lambda\tilde{x}_1 - \lambda[\frac{\alpha_0 - \mu}{2\lambda} + \frac{(\alpha_1 - \lambda)}{2\lambda}\tilde{x}_1 + \frac{\alpha_2}{2\lambda}\tilde{s}_2])\tilde{x}_1|\tilde{s}_1]$$
$$= \underset{\tilde{x}_1}{\text{Max}} \left(E[\tilde{z}|\tilde{s}_1] + \frac{\mu - \alpha_0}{2} - \frac{(\alpha_1 + \lambda)}{2}\tilde{x}_1 - \frac{\alpha_2}{2}E[\tilde{s}_2|\tilde{s}_1] \right) \tilde{x}_1$$

The F.O.C implies that

$$\tilde{x}_1(\tilde{s}_1) = \frac{E[\tilde{z}|\tilde{s}_1] + \frac{\mu - \alpha_0}{2} - \frac{\alpha_2}{2}E[\tilde{s}_2|\tilde{s}_1]}{\alpha_1 + \lambda} \tag{30}$$

Recall that $E[\tilde{s}_2|\tilde{s}_1] = E[\tilde{z} + \tilde{\varepsilon}_2|\tilde{s}_1] = E[\tilde{z}|\tilde{s}_1]$ since $\tilde{\varepsilon}_2$ is independent of $\tilde{s}_1$. Hence, equation (30) becomes

$$\tilde{x}_1(\tilde{s}_1) = \frac{(\mu - \alpha_0) + (2 - \alpha_2)E[\tilde{z}|\tilde{s}_1]}{2(\alpha_1 + \lambda)} \tag{31}$$

Since $\tilde{z}$ and $\tilde{s}_1$ are jointly distributed, we have,

$$E[\tilde{z}|\tilde{s}_1] = \gamma\tilde{s}_1 + \eta \quad \text{where} \quad \gamma = \frac{\sigma_z^2}{\sigma_z^2 + \sigma_1^2} \quad \text{and} \quad \eta = (1 - \gamma)\bar{z} \tag{32}$$

Thus, equation (31) becomes

$$\tilde{x}_1(\tilde{s}_1) = \frac{(\mu - \alpha_0) + (2 - \alpha_2)(\gamma\tilde{s}_1 + \eta)}{2(\alpha_1 + \lambda)} \tag{33}$$

Consequently, we get,

$$a_0 = \frac{(\mu - \alpha_0) + (2 - \alpha_2)\eta}{2(\alpha_1 + \lambda)}, \qquad a_1 = \frac{(2 - \alpha_2)\gamma}{2(\alpha_1 + \lambda)} \tag{34}$$

In order to express the coefficients of the insiders, we need to find the values of α_0, α_1 and α_2. We begin to recall the Theorem that we use to find these values.

Theorem 1 *If the $p \times 1$ vector Y is normally distributed with mean U and covariance V and if the vector Y is partitioned into two subvectors such that* $Y = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$ *and if* $Y^* = \begin{pmatrix} Y_1 \\ Y_2^* \end{pmatrix}$

$$U = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} \text{ and } V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$$

are the corresponding partitions of Y^*, U and V, then the conditional distribution of the $m \times 1$ ($m < p$) vector Y_1 given the vector $Y_2 = Y_2^*$ is the multivariate normal distribution with mean $U_1 + V_{12}V_{22}^{-1}(Y_2^*$ - $U_2)$ and covariance matrix $V_{11} - V_{12}V_{22}^{-1}V_{21}$.

Proof: See Graybill, Theorem 3.10 pp 63.

Now, applying Theorem 1 to the normal random vector $B = (\tilde{z}, \tilde{x}_1, \tilde{s}_2)$ with $p = 3$ and $m = 1$. By identification, we have $Y_1 = \tilde{z}$ and $Y_2 = \begin{pmatrix} \tilde{x}_1 \\ \tilde{s}_2 \end{pmatrix}$. $U_1 = \bar{z}$, $U_2 = \begin{pmatrix} \bar{x}_1 \\ \bar{s}_2 \end{pmatrix}$ and

$$V = \begin{pmatrix} \sigma_z^2 & \sigma_{zx_1} & \sigma_{zs_2} \\ \sigma_{zx_1} & \sigma_{x_1}^2 & \sigma_{x_1 s_2} \\ \sigma_{zs_2} & \sigma_{x_1 s_2} & \sigma_{s_2}^2 \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$$

Where $V_{11} = \sigma_z^2, V_{12} = (\sigma_{zx_1}, \sigma_{zs_2}), V_{21} = \begin{pmatrix} \sigma_{zx_1} \\ \sigma_{zs_2} \end{pmatrix}$ and $V_{22} = \begin{pmatrix} \sigma_{x_1}^2 & \sigma_{x_1 s_2} \\ \sigma_{x_1 s_2} & \sigma_{s_2}^2 \end{pmatrix}$.

Note that

$$V_{22}^{-1} = \frac{1}{D} \begin{pmatrix} \sigma_{s_1}^2 & -\sigma_{x_1 s_2} \\ -\sigma_{x_1 s_2} & \sigma_{s_2}^2 \end{pmatrix},$$

where D is the determinant of V_{22}, that is $D = \sigma_{x_1}^2 \sigma_{s_2}^2 - \sigma_{x_1 s_2}^2$.

Since we assume that $x_1(\tilde{s}_1) = a_0 + a_1 \tilde{s}_1$, we have

$$\sigma_{zs_2} = \sigma_z^2,\ \sigma_{zx_1} = a_1 \sigma_z^2,\ \sigma_{x_1 s_2} = a_1 \sigma_z^2,\ \sigma_{s_2}^2 = \sigma_z^2 + \sigma_2^2,\ \ \sigma_{x_1}^2 = a_1^2(\sigma_z^2 + \sigma_1^2).$$

Since $E[\tilde{z}|\tilde{s}_2, \tilde{x}_1] = \alpha_0 + \alpha_1 \tilde{x}_1 + \alpha_2 \tilde{s}_2$, we can now express the values of α_0, α_1 and α_2.

$$\alpha_0 = \bar{z} - \alpha_1 \bar{x}_1 - \alpha_2 \bar{s}_2 \tag{35}$$

$$\alpha_1 = \frac{\sigma_{zx_1} \sigma_{s_2}^2 - \sigma_{zs_2} \sigma_{x_1 s_2}}{D} \tag{36}$$

$$\alpha_2 = \frac{\sigma_{zx_1} \sigma_{s_2}^2 - \sigma_{zs_2} \sigma_{x_1 s_2}}{D} \tag{37}$$

First note that $D = \sigma_{x_1}^2 \sigma_{s_2}^2 - \sigma_{x_1 s_2}^2 = a_1^2[\sigma_1^2(\sigma_z^2 + \sigma_2^2) + \sigma_z^2 \sigma_2^2]$. Second, equation (36) becomes

$$\alpha_1 = \frac{a_1 \sigma_z^2 \sigma_2^2}{D} \tag{38}$$

Similarly, equation (37) becomes after simplification

$$\alpha_2 = \frac{a_1^2 \sigma_z^2 \sigma_1^2}{D} \tag{39}$$

Combining equations (2), (32), (34), (38) and (39), we obtain

$$b_1 = \frac{2\sigma_z^2\sigma_2^2 + 2\sigma_1^2\sigma_2^2 - \sigma_z^2\sigma_1^2}{2\sigma_z^2\sigma_1^2} \text{ and } b_2 = \frac{\sigma_z^2\sigma_1^2}{2\lambda(\sigma_z^2\sigma_1^2 + \sigma_z^2\sigma_2^2 + \sigma_1^2\sigma_2^2)} \tag{40}$$

$$b_0 = -\mu b_2 \text{ and } a_1 = \frac{\sigma_z^4\sigma_1^2}{2\lambda(\sigma_z^2 + \sigma_1^2)(\sigma_z^2\sigma_1^2 + \sigma_z^2\sigma_2^2 + \sigma_1^2\sigma_2^2)}, \tag{41}$$

$$a_0 = \frac{-\sigma_z^2\sigma_1^2(\mu\sigma_1^2 - \bar{z}\sigma_1^2 + \mu\sigma_z^2)}{2\lambda(\sigma_z^2 + \sigma_1^2)(\sigma_z^2\sigma_1^2 + \sigma_z^2\sigma_2^2 + \sigma_1^2\sigma_2^2)}, \ \alpha_0 = \frac{\mu\sigma_2^2(\sigma_z^2 + \sigma_1^2)}{\sigma_z^2\sigma_1^2 + \sigma_z^2\sigma_2^2 + \sigma_1^2\sigma_2^2} \tag{42}$$

$$\alpha_1 = \frac{2\lambda(\sigma_z^2 + \sigma_1^2)\sigma_2^2}{\sigma_z^2\sigma_1^2} \text{ and } \alpha_2 = \frac{\sigma_z^2\sigma_1^2}{\sigma_z^2\sigma_1^2 + \sigma_z^2\sigma_2^2 + \sigma_1^2\sigma_2^2} \tag{43}$$

We turn now to find the expressions of μ and λ. First, recall that the market efficiency condition together with the price linearity imply that

$$p(\tilde{r}) = E[\tilde{z}|\tilde{r}] = \mu + \lambda\tilde{r}$$

taking the expectation on both sides of the above equation we obtain,

$$\begin{aligned} \bar{z} &= \mu + \lambda\bar{r} = \mu + \lambda[\bar{x}_1 + \bar{x}_2] \\ &= \mu + \lambda[a_0 + b_0 + b_1 a_0 + a_1(1 + b_1)\bar{s}_1 + b_2\bar{s}_2] \end{aligned}$$

Using the expressions of $a_0, b_0, a_1, b_1, b_2, \alpha_0, \alpha_1$ and α_2 written above, we find that $\mu = \bar{z}$. We move now to find the value of λ. Since the orders of the insiders and the liquidity traders are normally distributed, applying the projection theorem for normal random variables, we have

$$\lambda = \frac{Cov(\tilde{z}, \tilde{r})}{Var(\tilde{r})}$$

where $\tilde{r} = \tilde{x}_1 + \tilde{x}_2 + \tilde{u} = a_0 + b_0 + b_1 a_0 + a_1(1 + b_1)\tilde{s}_1 + b_2\tilde{s}_2 + \tilde{u}$. Hence,

$$\lambda = \frac{Cov(\tilde{z}, \tilde{r})}{Var(\tilde{r})} = \frac{(a_1(1 + b_1) + b_2)\sigma_z^2}{a_1^2(1 + b_1)^2(\sigma_z^2 + \sigma_1^2) + b_2^2(\sigma_z^2 + \sigma_2^2) + 2a_1(1 + b_1)b_2\sigma_z^2 + \sigma_u^2} \tag{44}$$

Substituting equations (40),(41), (42) and (43) into (44) and using the following change of variable $h_1 = \frac{\sigma_z^2}{\sigma_1^2}$, $h_2 = \frac{\sigma_z^2}{\sigma_2^2}$ and $\gamma = \frac{\sigma_z^2}{\sigma_z^2 + \sigma_1^2}$ to solve for the positive root λ, we find expressions (7) and (8) of the equilibrium outcomes as stated in Proposition 1. It remains to find the expressions of the unconditional profits. Indeed, we have

$$\begin{aligned} E[(\tilde{z} - p(\tilde{r}))\tilde{x}_1|\tilde{s}_1] &= E[(\tilde{z} - \mu - \lambda(\tilde{x}_1 + \tilde{x}_2 + \tilde{u}))\tilde{x}_1|\tilde{s}_1] \\ &= (E[(\tilde{z}|\tilde{s}_1] - \mu - \lambda\tilde{x}_1 - \lambda E[\tilde{x}_2|\tilde{s}_1])\tilde{x}_1 \\ &= (E[(\tilde{z}|\tilde{s}_1] - \bar{z} - \lambda a_1(\tilde{s}_1 - \bar{s}_1) - \lambda E[b_1 a_1(\tilde{s}_1 - \bar{s}_1) + b_2(\tilde{s}_2 - \bar{s}_2)|\tilde{s}_1])(a_1(\tilde{s}_1 - \bar{s}_1)) \\ &= (\bar{z} + \gamma) - \bar{z} - \lambda a_1(\tilde{s}_1 - \bar{s}_1) - \lambda b_1 a_1(\tilde{s}_1 - \bar{s}_1) - \lambda b_2\gamma(\tilde{s}_1 - \bar{s}_1))(a_1(\tilde{s}_1 - \bar{s}_1)) \\ &= a_1(\tilde{s}_1 - \bar{s}_1)^2[\gamma(1 - \lambda b_2) - \lambda a_1(1 + b_1)] \end{aligned}$$

Taking the expectation on both sides of the above equation, we obtain the expression in (9). We turn now to compute the conditional profits of insider 2. We have,

$$
\begin{aligned}
E[(\tilde{z}-p(\tilde{r}))\tilde{x}_2|\tilde{x}_1,\tilde{s}_2] &= E[(\tilde{z}-\mu-\lambda(\tilde{x}_1+\tilde{x}_2+\tilde{u}))\tilde{x}_2|\tilde{x}_1,\tilde{s}_2] \\
= E[(\tilde{z}-\mu-\lambda\tilde{x}_1-\lambda\tilde{x}_2)\tilde{x}_2|\tilde{x}_1,\tilde{s}_2] &= \left(E[\tilde{z}|\tilde{x}_1,\tilde{s}_2]-\mu-\lambda\tilde{x}_1-\lambda\tilde{x}_2\right)\tilde{x}_2 \\
&= \left(\alpha_0+\alpha_1\tilde{x}_1+\alpha_2\tilde{s}_2-\mu-\lambda\tilde{x}_1-\lambda\tilde{x}_2\right)\tilde{x}_2 \\
&= \lambda\tilde{x}_2^2
\end{aligned}
$$

where the last equality is based on equations (29) and (41).

■

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