Business cycle and factor income shares: a VAR sign restrictions approach

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Abstract

The paper first provides a comprehensive literature review of the theories explaining the cyclical interaction between factor shares and economic activity. Secondly, it assesses if empirical evidence supports those theories, overcoming the strong criticalities present in the current empirical literature. To this end, a Bayesian VAR identified with sign restrictions is set up. The results suggest that countercyclical fluctuations in the labor share are mainly driven by the pro-cyclicality of labor productivity - consistent with *overhead costs* and *risk distribution* theories - and by the Phillips Curve effects upheld by *Goodwin*. The model does not support the expansive effect of a capital share rise suggested by Goodwin. In contrast, there is partial evidence favoring the *biased technical change* theory.

Keywords: factor shares, business cycle, labor share, VAR, sign restrictions.

JEL Codes: C32, E25, E32.

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1 Introduction

The persistent cyclicality of factor shares of income is becoming increasingly recognized as a business cycle stylized fact (see, among others, Young, 2004 [26]; Rius- Rull and Llopis, 2010 [22]; Shaio and Silos, 2014 [23]; Boldrin, 2019 [7]; Barrales-Ruiz et al., 2022 [3]). This observed recurrent pattern takes the following form. The labor share falls during the early expansionary phase of the business cycle. However, the fall in labor share reaches its peak earlier than the output increase. Thus, in the late output expansionary phase, the labor share reverses its trend and starts rising along with output. Empirically, the diverging movement of labor share and output during the early output expansion phase results in a *simultaneous* negative correlation. In contrast, the subsequent increase in labor share in the late output expansion phase results in a *lagged* positive correlation. Figure (1) plots the leads and lags correlations between the quarterly US labor share and the real GDP from 1951 to 2019. The series are from the Bureau of Labor Statistics. After taking the natural logarithm, the data have been processed with the Hodrick-Prescott filter to obtain the cyclical component. The negative contemporaneous correlation (-0.32) confirms that the labor share is slightly counter-cyclical. As expected, the maximum correlation (about 0.47) is reached after four lags and is higher than the contemporaneous one. This pattern means that the labor share lags output, i.e. an increase in GDP is correlated with a rise in the labor share some quarters later.

Several theories have been proposed to explain this phenomenon. According to Goodwin (1967)[17], it is a fall in labor share that increases output by stimulating private investment, generating the simultaneous negative correlation. Then, as production and employment grow, increasing pressure on wages pushes up the labor share, causing the lagged positive correlation. According to Ambler and Cardia (1998)[2] and Hornstein (1993)[19], it is the presence of the so-called *overhead costs* to produce the simultaneous negative correlation. By acting as fixed costs, overhead costs cause unit costs to fall as output increases, resulting in higher profits per unit of output and a fall in the labor share. Gomme and Greenwood (1995)[16] and Boldrin and Horvath (1995)[6] see the *distribution of risk* between workers and entrepreneurs during the business cycle as the cause of the simultaneous negative correlation. Since workers want to insure themselves against fluctuations in their wages, they enter into contracts with entrepreneurs that prevent wages from matching movements of labor productivity. Consequently, the increase in labor productivity during an upturn results in a simultaneous GDP rise and labor share fall. Finally, there is the theory of *biased technical change* (Young, 2004[26]; Boldrin et al., 2019[7]). Rising labor costs and labor share induce firms to direct R&D toward technologies that allow substituting labor for

Figure 1: Correlation of Labor's share (t+i) and Real GDP (t)

capital. The resulting increase in labor productivity leads to a parallel rise in output and fall in labor share, generating the simultaneous negative correlation.

The soundness of these theories is difficult to assess because of the shortcomings characterizing the VAR literature on this topic. Authors rely on a bivariate GDP-labor share model identified with a recursive Cholesky's scheme, and the variables' ordering is justified based on the single theory they want to test. However, this method a priori discards the role that the other theories might play, forcing the results in a specific direction. Another critical issue is that the bivariate structure usually employed does not allow the different channels theorized in the literature to be properly disentangled and tested together. With the aim to overcome these criticalities, a Bayesian VAR identified with sign restrictions is set up. The variables entering the model are real GDP, hours worked, nominal hourly wage, Consumer Price Index and the number of workers who left their job voluntarily. To the best of my knowledge, this is the first paper to use the sign restrictions approach on this topic and to test the different theories in a single framework.

The results suggest that pro-cyclical labor productivity mainly drives the counter-cyclicality of the labor share. This phenomenon is consistent with the theories of *overhead costs* and *risk distribution*. In contrast, the evidence supporting the expansive effect of a capital share rise upheld by Goodwin is weak. Nevertheless, the results support the other pillar of Goodwin's model - *the*

Philips curve mechanism - which explains for the observed lagging behavior of the labor share to output. Finally, the results partially support the *biased technical change theory.*

The remainder of the paper is organized as follows. Section 2 reviews the theoretical channels the literature proposes to explain the cyclicality of factor shares of income. Section 3 summarizes and discusses the empirical literature, highlighting its main shortcomings. Section 4 builds a Bayesian VAR model identified with sign restrictions to test the theories reviewed in Section 2 and overcome the critical issues outlined in Section 3. Section 5 concludes.

2 Theoretical literature

The first theory on the cyclical interaction between functional income distribution and output can be attributed to Goodwin (1967) [17]. The model traces the predator-prey models used in biology (Lotka, 1925 [21]; Volterra, 1936 [25]), where labor and capital conflict with each other but exist symbiotically. Workers consume all of their income while entrepreneurs save it all. Say's law ensures that all entrepreneurs' savings are invested, causing an increase in the profit share to have an expansionary effect on output, producing the looked-for simultaneous negative correlation between labor share and GDP. A Phillips curve ensures that following the tightening of the labor market caused by the output expansion, wages begin to rise with some lags. The lagged wage increase reduces the profit share and output until the subsequent wage fall restarts the cycle. Phases of expansion of production and profit share are then followed by phases of contraction in a clockwise cycle in the profit share-activity plan. Note that all labor share movements are driven by changes in the real wage growth rate, as labor productivity growth is constant.

The second strain of theoretical literature focuses on factor substitution generated by 'biased technical change' as the driver of functional distribution fluctuations across the business cycle. Biased technical change (Blanchard et al., 1997 [5]; Caballero and Hammour, 1998 [8] and Acemoglu, 2002 [1]) refers to any introduction of new technologies, change in production methods, or change in the organization of work that increase labor productivity allowing to substitute labor with capital in response to an increase in labor cost. The rise in labor productivity and GDP reduces the original increase in the labor share, generating the looked-for negative correlation between labor share and GDP. Young (2004) [26] was the first to propose this mechanism as a possible source of the labor share share counter-cyclicality. This argument is used to justify stochastic factors' elasticities in the Cobb-Douglas production function. Since in a Cobb-Douglas production function, the output elasticity of a factor represents its income share, changes in the latter automatically cause variations in factor productivity, as predicted by the theory of Biased technical change. The cyclicality of factor shares is not self-sustaining as in Goodwin (1967) but stems from exogenous stochastic shocks as in all RBC models. A more structured model is that of Boldrin et al. (2019) [7], which splits the business cycle into two phases. During the *growth phase*, output growth is driven by the adoption of labor-saving technologies, i.e. by replacing a less advanced type of capital with a new vintage incorporating more advanced technology. In this phase, the profit share increases since labor productivity rises more than real wages due to the capital replacement process. During the *build-up phase* - i.e. when all the capital of the old vintage has been entirely replaced by the new one - firms keep widening the new capital, the employment increases, and so does the real wage. In this phase, a lower output growth, a declining capital price, and a rising wage reduce the profit share. When, during the build-up phase, the price of capital has declined enough, it becomes profitable to start replacing the old capital with a more advanced vintage of capital, and a new growth phase begins. In this paper, the fluctuations of the labor share during the business cycle are not occasional events arising as a response to exogenous shocks but are *"systematic and recurrent features of the economy"*. In other words, there is no balanced growth path; instead, the economy endogenously alternates between upturns accompanied by a rising profit share and downturns accompanied by a falling profit share. Furthermore, cyclical variations in factor prices are the engine of technological progress and long-run economic growth. This model and Goodwin (1967) [17] are the only ones generating a self-sustaining cycle in the factor shares of income. All other models described in the rest of this section only reproduce the observed correlations between variables following exogenous stochastic shocks. They do not generate endogenous cycles in the labor share-activity plane.

The third strain of literature focuses on the role played by increasing return to scale. The pro-cyclicality of profit share is generated by the presence of 'overhead costs', which in turn generate increasing returns to scale. Overhead costs are all those expenses associated with running a business that cannot be directly linked to producing a product or service (e.g., rent, facilities, management pay, insurance, accounting, or legal expenses). They act as fixed costs, generating a fall in unit labor costs and labor share as output increases. Ambler and Cardia (1998) [2] build an RBC model in which the production function differs from the conventional Cobb-Douglas in two aspects. Firstly, it shows decreasing average costs due to the overhead costs. Secondly, a return to scale parameter determines if the decreasing average costs are coupled with constant or decreasing marginal costs, amplifying the factor shares oscillations. On the other hand, the effect of overhead costs is counterbalanced by the possibility of firms entering (exiting) the market when the economy is in an upturn (downturn) phase. In the limiting case where the entry/exit is instantaneous, the distributive cycle disappears since the increase (decrease) in output is entirely borne by new firms' entry (exit) into the market. In this case, output per firm is constant, and so are the aggregate average costs. This is what happens in Devereux et al. (1996) [14], which therefore represents a limiting case of Ambler and Cardia (1998) [2] when the entry/exit of new firms into the market is instantaneous. On the other extreme, in Hornstein (1993) [19], as there is no possibility of entry/exit from the market, the fluctuation of factor shares is at its maximum.

Hansen and Prescott (2005) [18] shares with this strand the hypothesis of non-constant return to scale, although the returns are decreasing and generated by a different mechanism. They use a decreasing return to scale assumption at the plant level to guarantee that operating many small identical plants is optimal rather than one large one. In this economy, not all plants are always used in the production process, but there can be some idle capacity. When capacity constraints do not bind, plant capital is not a scarce factor and, consequently, does not earn income. Hence, in this case, the labor share is larger. When during periods of expansion, all capacity is used, the return on capital increases, generating a pro-cyclical profit share.

The fourth strand of literature focuses on the distribution of the risk between workers and entrepreneurs over the business cycle and, more generally, on the role played by a non-competitive wage setting and frictions in the labor market. The main idea is that workers enter into contracts with capitalists that prevent wages from falling during a recession (as if they were determined only by marginal labor productivity) and vice versa during an expansion. In other words, it is the will of workers to insure themselves against fluctuations of the business cycle that makes real wages acyclical, which in turn, combined with pro-cyclical labor productivity, generate the looked-for counter-cyclicality of the labor share. Gomme and Greenwood (1995) [16] assume that *"built into labour income is an insurance component designed to provide workers with some degree of protection against business cycle fluctuations. This insurance component inserts a wedge between marginal product of labour and measured wages"*. Boldrin and Horvath (1995) take a similar approach [6] which, differently from Gomme and Greenwood (1995), assumes that the desire of workers to hedge against the risk of fluctuations in the economic cycle stems from a double assumption. Firstly, since workers are not endowed with wealth, they cannot access (imperfect) financial markets like their employers to achieve intertemporal consumption smoothing. Secondly, employers are less risk-averse than workers and thus more willing to take on the business cycle risks by agreeing on partially fixed wages for a certain period before output is realized.

Closely to this strand of literature, Rios-rull and Choi (2009) [12] depart from the baseline RBC model by adding frictions in the labor market and a non-competitive wage setting. These two changes generate a countercyclical labor share. This occurs because as output increases, on the one hand, search and matching frictions cause employment to have a lagging behavior. On the other, Nash bargaining creates a wedge that prevents the real wage from adjusting immediately to productivity. However, the model fails in replicating the looked-for hump-shaped response of the labor share to an output shock found in Rios-Rull and Llopis (2010) [22]. Indeed - as we will see later - in a distribution-activity VAR identified with Cholesky where the activity variable is ordered first, the Impulse Response Function of the labor share to an output shock is negative on impact but overshoots after five quarters. Nash Bargaining is sufficient to replicate the countercyclicality but not the overshoot. Shao and Silos (2014) [23] introduce barriers to market entry to capture this effect. A productivity shock reduces the effective capital needed for a start-up to enter the market, reducing the demand for capital per entrant. The countercyclical interest rate dampens the initial fall in labor share, reducing the contemporaneous negative correlation with output and bringing it closer to that observed in the data. In addition, the costly entry delays the rise in wages and employment by producing the looked-for hump-shaped IRF of the labor share to a productivity shock. With the same aim of reproducing the observed overshooting, Colciago and Rossi (2015) [13] introduce a countercyclical mark-up generated by the entry of new firms into the market following an expansive shock. The greater competition resulting from the larger number of firms pushes down the mark-up resulting in an overshoot and a labor share higher than the initial one.

Finally, the fifth strain of literature relies on a standard Cobb-Douglas production function making the share parameter stochastic. Since in a Cobb-Douglas production function the factors' output elasticities represent factor shares, shocks to the latter result in changes in output. The aforementioned Young (2004) [26] relies on an RBC model with stochastic factors' elasticities in the Cobb-Douglas production function. Casteneda et al. (1998) [10] also propose a model along these lines, although their aim is not to analyze the relationship between functional income distribution and real activity. Rios-Rull and Llopis (2009) [22] firstly estimate a bivariate activity-labor share VAR identified with Cholesky with the activity variable ordered first. Then, the estimated relationship is plugged into the Cobb-Douglas production function of an RBC model. As the TFP increases, the labor share falls on impact because of their estimated negative relationship.

3 Empirical literature

The VAR literature on this topic can essentially be divided into two strands. Both are based on bivariate[1] distribution-activity models identified via Cholesky. The first strand (Carvalho and Rezai, 2016 [9]; Basu and Gautham, 2020 [4]; Barrales-Ruiz et al., 2022 [3]) orders the labor share

[1]Sometimes a third variable is added, but the results remain essentially the same.

before the real activity variable, thus assuming that distribution can have a contemporaneous impact on output but not vice versa. This assumption is justified based on Goodwin's (1967) [17] model. Accordingly, labor productivity growth is constant and movements in the labor share are entirely driven by real wage growth rate changes. Given the stickiness of the latter, it is reasonable to expect that the labor share does not respond immediately to changes in output. The results stemming from this identification scheme are that following a positive shock to the labor share, the GDP fall is instantaneous and long-lasting. Following a positive shock to GDP, the profit share decreases gradually from the second quarter onwards. These results, firmly in line with the predictions of Goodwin's model, are therefore brought to support this theory.

Conversely, the second strand of literature orders the activity variable before the labor share, implicitly assuming that the former can impact the latter simultaneously but not vice versa. Following the empirical literature of Goodwinian inspiration, we will refer to the ordering of the variables of the first strand as 'standard ordering' and to that suggested by the second strand as 'reverse ordering'. Rius-Rull and Llopis (2010) [22], taking an RBC as their theoretical reference model, order productivity (and alternatively GNP) before labor share. Colciago and Rossi (2015) [13] and Shao and Silos (2014) [23] replicate this result. These papers find that an increase in real activity triggers an instantaneous fall in the labor share, which after about five quarters overshoots and remains at a higher level than it was at the beginning. The output response to a positive labor share shock is non-significant or positive. Cauvel (2019) [11] attempts to demonstrate the inconsistency of the standard ordering, which would ignore the strong procyclicality of labor productivity due to overhead costs. Firstly, he demonstrates that the results are profoundly different by reversing the variables ordering. Then, he shows that once the procyclicality of labor share is taken into account by cleaning up the labor share of its cyclical component, even applying the standard ordering, the expansionary effect of an increase of the capital share disappears.

3.1 A critique

The results obtained with the standard ordering have two shortcomings. First, imposing that economic activity cannot have any contemporaneous impact on the distribution a priori rules out the hypothesis that procyclical movements in labor productivity drive cyclical fluctuations in the labor share, as suggested by Cauvel (2019) [11] and Lavoie (2017) [20]. Therefore, the associated results could be nothing more than spurious associations from forcing the model in that direction. A similar criticism can be made against reverse ordering in limiting the theories of Goodwin and biased technical change. Indeed, in these theories the direction of causality runs from distribution to real activity and the reverse ordering imposes a zero restriction on the contemporaneous reaction of output in response to changes in functional distribution. However, there is a fundamental difference concerning the implicit assumptions in the two identification schemes. A reaction of labor productivity following an output shock could be automatic and instantaneous, as in the case of overhead costs or risk distribution theories. On the other hand, any direction of causality from distribution to economic activity requires a reaction on the part of economic agents, which can take more time to show significant effects. This is the case of the rise of firms' investment following an increase in the profit share predicted by the Goodwin model or the biased technical change triggered by the rise in labor cost. The slower response of agents makes the assumption embodied in the reverse ordering - no response of economic activity to changes in labor share for a quarter - less restrictive. The higher the frequency of the time series, the less restrictive is the reverse ordering.

We now come to the second criticism that can be made of the standard ordering. According to the models that apply this identification scheme, the expansionary effect caused by an increase in the profit share explains the negative correlation between labor share and economic activity. However, a shock to the labor share as such does not exist. Different shocks that give rise to an increase in the labor share can have an opposite effect on GDP. For example, the profit share could increase both following a decrease in wages and following an increase in the mark-up. However, in the first case, there would be a fall in prices and the real exchange rate and vice versa in the second, with potentially opposite effects on GDP.

This paper aims to overcome the critical issues associated with both Cholesky's orderings through a more sound identification scheme based on the sign restrictions approach. Through this new framework, this paper aims to determine the main theoretical mechanisms that drive the distributional cycle by examining whether there is empirical evidence supporting the theories outlined in Section 2.

4 Empirical strategy

4.1 Data

The data used in the following models refer to the U.S. economy from 2001q1 to 2019q4 and come from the Bureau of Labor Statistics. The reason for choosing this short period is the unavailability of earlier data regarding the number of workers who left their job voluntarily ($QUITS$) collected by the *Job Openings and Labor Turnover Survey* of the BLS. However, as shown in Section 4.2, the estimates with this sample for the bivariate model are in line with those in the literature for more extended time periods. In addition to *quits* the other variables used in this paper are the total number of hours worked in the economy ($HOURS$), real GDP (GDP)[2], nominal hourly wages ($WAGES$), the Consumer Price Index (CPI) and the labor share ($LABOR$ $SHARE$). Note that by combining the series of nominal wages, CPI, hours worked and real GDP, it is possible to reconstruct the labor share series provided by the Bureau of Labor Statistics. This is why - unlike the bivariate models of section 4.2 based on labor share and GDP - in section 4.3, the labor share does not enter directly into the model. All variables are in natural logarithms and enter the model in levels; the Bayesian specification of the model is, in fact, compatible with the use of non-stationary variables (Sims, 1990 [24]).

4.2 The bivariate model

The purpose of this section is twofold. First, to show how the adoption of the widely used Cholesky ordering, besides not being theoretically sound - as discussed in Section 3.1 - is not robust to variable inversion. Second, the results of the bivariate model are intended to show that even with this shorter sample period the findings are in line with those found in the literature using longer samples. This allows us to compare the results from the bivariate model identified via Cholesky with a sign restrictions identification scheme in the next section.

The reduced form of the estimated model reads:

$$Y_t = C + \sum_{i=1}^{n} A_i Y_{t-i} + u_t \tag{1}$$

Where Y_t is the vector of endogenous variables: $Y_t = [GDP_t;\ \text{labour share}\ _t]$, C is a vector of reduced-form constants, A_i is a matrix of reduced form parameters and u_t is a vector of reduced-form errors. The model is estimated with a Bayesian approach based on an Independent Normal-Wishart prior[3]. Hyperparameters are selected in such a way as to maximize the marginal likelihood through a grid search procedure. In section 4.3 a *dummy initial observation strategy* restricts the prior toward unit roots or cointegration to prevent draws obtained from the posterior from being characterized by explosive unit roots (Dieppe et al., 2018 [15]).

[2]The real GDP series is obtained by dividing the corresponding nominal series by the Consumer Price Index provided by the BLS.

[3]Since there is no closed-form solution for the posterior distribution, it is necessary to use the Gibbs sampler. This MCMC algorithm approximates the true distribution. The algorithm requires a certain number of iterations to approximate the posterior with sufficient accuracy; 100,000 draws are then performed, of which the first 80,000 are discarded. Hence, parameter estimates are based only on the last 20,000 draws.

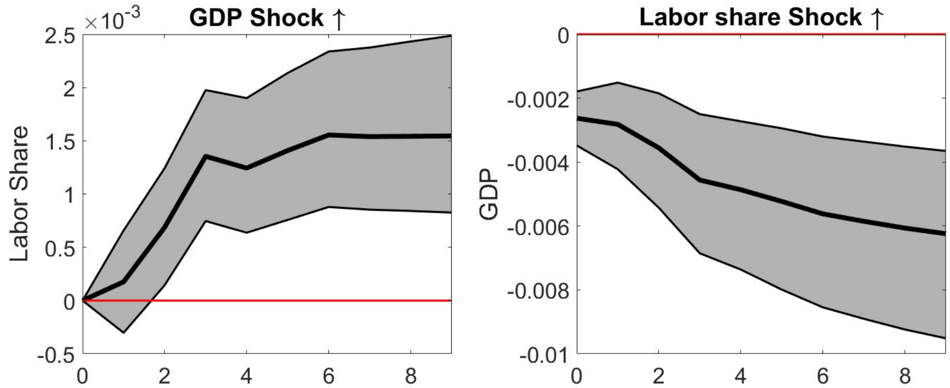

Figure 2: IRF with the standard ordering

As is well known, to perform structural analysis in a VAR framework, it is necessary to map the reduced-form errors to the structural ones by imposing restrictions on the variance-covariance matrix through the B matrix.

$$u_t = B\epsilon_t \tag{2}$$

The simplest method is to obtain B from a Cholesky decomposition of the variance-covariance matrix. This way, the variable ordered first in the vector Y will be exogenous on impact to the second, but not vice versa.

I first run the model with the labor share ordered first and GDP as the second variable (*standard ordering*). Figure (2) shows the resulting Impulse Response Functions.

As can be seen, GDP reacts negatively to an increase in the labor share, while the latter responds positively from the third quarter onward to an increase in GDP. These results perfectly align with the literature that adopts this Cholesky ordering (Carvalho and Rezai, 2016 [9]; Basu and Gautham, 2020 [4]; Barrales-Ruiz et al., 2022 [3]).

Figure (3) shows the results from the same model with the variables ordering reversed, i.e. with the *GDP* exogenous on impact to the *labor share*, but not vice versa (*reverse ordering*).

The response of the labor share to an increase in GDP is very different from the previous one; in fact, it is negative on impact and overshoots becoming positive after seven quarters. These results are in line with those found by Rios-Rull and Llopis (2009) [22] and Cauvel (2019) [11] in bivariate models identified with the reverse ordering. Notice that the response of GDP to an increase in the labor share is non-significant, which is firmly at odds with the results obtained

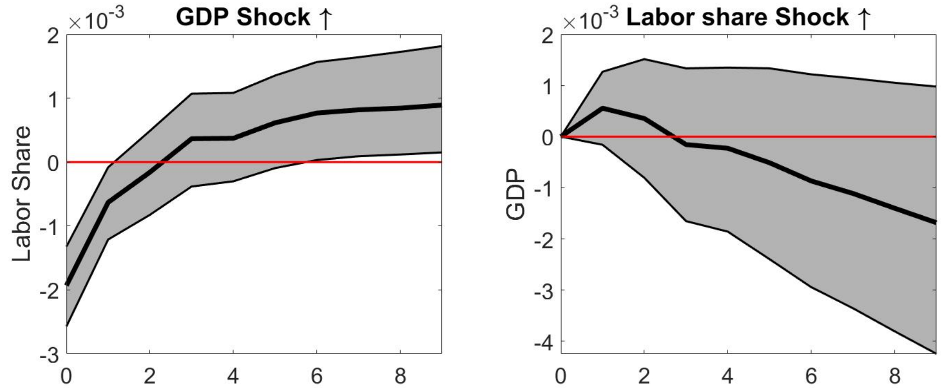

Figure 3: IRF with the reverse ordering

with the previous ordering.

Given the theoretical criticalities of both Cholesky orderings and the non-robustness of the results to their inversion, I rely on sign restrictions as an alternative identification scheme. Through this method - more theoretically robust and less restrictive than Cholesky - we can test the predictions of the theories exposed in Section 2 and check if the results of one of the two (if at all) strands of empirical literature are supported.

4.3 A sign restrictions approach

In this section, in order to overcome the theoretical and empirical criticalities exposed above, I estimate a five variables model identified with the sign restrictions approach. This approach fixes the fundamental structural identification problem - each VAR in reduced form corresponds to infinite VARs in structural form - retaining all possible structural models associated with a single reduced form that satisfies the imposed sign restrictions. The consequence is that there is no longer a mapping between shocks and variables as in Cholesky. The median is generally taken of all the IRFs generated in this way, and credibility intervals are constructed based on their distribution.

The benefit of this new framework is twofold. On the one hand, it allows us to avoid imposing hardly justifiable zero restrictions and, on the other, to decompose what in the bivariate model were the shocks to GDP and labor share into the shocks that theoretically drive these two variables. The applied identification scheme is reported in Table table (1) and table (2).

The five variables that enter the model are: hours worked, nominal hourly wage, consumer

	Demand ↑	Workers Bargaining Power ↑	Firm Bargaining Power ↓	Automation ↑
Hours	+	/	/	-
Wages	/	/	/	/
CPI	+	/	-	-
GDP	+	/	/	+
Quits	+	/	/	-

Table 1: Sign Restrictions on periods 1 and 2

	Demand ↑	Workers Bargaining Power ↑	Firm Bargaining Power ↓	Automation ↑
Hours	+	/	/	-
Wages	+	+	+	-
CPI	+	+	-	-
GDP	+	/	/	+
Quits	+	-	/	-

Table 2: Sign Restrictions on period 3

price index (CPI), real GDP and the number of voluntary quits from the labor market. The structural shocks are: aggregate demand, automation, labor bargaining power and firm bargaining power. Given the small sample, the following model is estimated with two lags to preserve degrees of freedom[4].

All the restrictions are applied on the first three quarters except for the nominal hourly wage, which is restricted just on the third quarter. This choice is made not to force wages to vary immediately if they were sticky. As we will see, only for the shock to workers' bargaining power, restrictions on prices and quits are imposed just on the third quarter after the shock.

In detail, a positive *aggregate demand* shock is the shock that generates an increase in hours

[4]Nevertheless, Section 4.5 estimates different lag specifications as a robustness check.

worked, GDP, price level and the number of workers voluntarily quitting their jobs for the first three quarters. The nominal wage is restricted to rise only in the third quarter, allowing for wage stickiness. The rationale behind the job quits restriction comes from the observation that this variable generally increases in periods of expansion in line with the number of new people hired (Appendix **??**). This phenomenon is probably due to the labor market's greater job opportunities and dynamism when aggregate demand increases. The probability of quickly getting a new job is higher in periods of expansion, making it less risky to quit your current job voluntarily.

Since we have restricted all variables in this shock, we are not really interested in the response of the variables entering the model but rather in the response of labor share, labor productivity and real wages. These variables can easily be derived by combining the responses of the nominal wage, prices, hours worked, and GDP. The Impulse Response Functions of the labor share, labor productivity and real wages are computed as follows:

$$ln(LS_t) = ln(Hours_t) + ln(NominalWage_t) - ln(GDP_t) - ln(CPI_t) \quad (3)$$

$$ln(PROD_t) = ln(GDP_t) - ln(Hours_t) \quad (4)$$

$$ln(RealWage_t) = ln(NominalWage_t) - ln(CPI_t) \quad (5)$$

The response of these three variables to this shock allows us to evaluate the predictions of the theories discussed in Section 2. Any fall in the labor share on impact caused by an increase in labor productivity would support the *overhead costs* theory. It would also be compatible with the *risk distribution* theory predictions if coupled with a real wage that does not immediately respond to the shock. This result would also invalidate the soundness of *standard ordering* since the latter imposes no contemporaneous changes in the labor share following an output shock. On the contrary, a labor share that does not respond significantly to the shock, coupled with a lagged real wage and labor share increase, would confirm what is predicted by the *Goodwin cycle*. This pattern would also support the soundness of the *standard ordering* identification scheme.

A positive shock to the *bargaining power of workers* is assumed to generate an increase in the real hourly wage and the price level (because firms face higher production costs) after two quarters and a fall in the number of workers quitting their jobs. This last assumption is fundamental to disentangle this shock from the demand and automation shocks. It is motivated by the fact that fewer workers may decide to leave their job as the working conditions improve. Or, vice versa, that more workers choose to leave their jobs when working conditions deteriorate. The variables 'hours' and 'GDP' are left unrestricted. Furthermore, only the posterior draws producing a rise in nominal wages larger than prices are retained to ensure that this shock generates an increase in real wages.

A gradual increase in labor productivity that generates a parallel increase in output such that an initial increase in labor share dies out would confirm what the *biased technical change* predicts. A GDP fall would support both *Goodwin's model* and the soundness of the *standard ordering*.

A positive shock to *firms' bargaining power* is identified as an increase in the mark-up. Hence, it is specified imposing the price level to rise for the first three quarters and the nominal wage to fall only in the third quarter. The other variables are left unrestricted.

If the bivariate model - which does not distinguish between sources of variation in labor share - is robust, we would expect the GDP to respond in the same direction following both an increase in workers' bargaining power and a fall in firms' bargaining power. Conversely, an output response of the opposite sign would indicate that the results coming from a shock to the labor share are misleading.

Finally, an *automation shock* is identified imposing an increase in GDP and a fall in hours worked, nominal wages, prices and quits. The latter is motivated in a similar way to the demand shock. The labor demand lowers following a positive automation shock, resulting in fewer workers leaving their jobs because they fear not finding another one.

Figure (4) shows the theoretical differences compared with the bivariate model. In the latter, there are only two types of shocks, one distributive (Labor Share) and one to GDP (last line of Figure 4). In the sign restrictions model, the labor share shock is decomposed into two sub-shocks (bargaining power of firms and workers), allowing for a possible different response of the GDP (second line, Figure 4). What was the GDP shock in the bivariate model has also been partitioned into two sub-shocks: an aggregate demand shock and an automation shock. Finally, the first row of Figure 4 divides these shocks into demand shocks (aggregate demand shock) and supply shocks (automation, firms and labor bargaining power shocks).

4.4 Results

Figure (5) shows the structural Impulse Response Functions of model variables, while Figure (6) shows the implied IRFs of labor share, labor productivity and real wages.

After a positive *demand shock*, the response of the labor share (Figure 6) is negative on impact to become non-significant from the third quarter onwards, eventually overshooting the initial level, although not significantly. At the same time, the impact response of labor productivity is

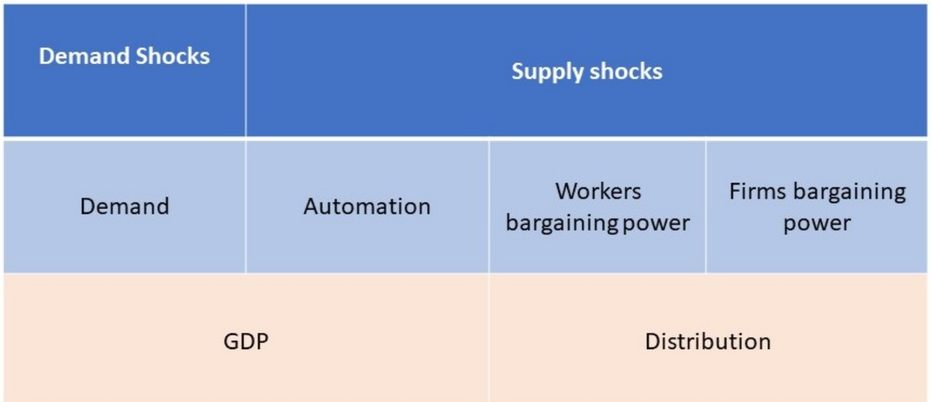

Figure 4: Structural shocks

positive on impact to become non-significant from the second quarter. In contrast, the response of real wages is non-significant on impact and then becomes increasingly positive (and statistically significant) from the fifth quarter onward.

This leads us to conclude that fluctuations in demand produce a countercyclical labor share driven by procyclical movements in labor productivity. This result confirms what is stated by the theory of *overhead costs*. It is also in line with what is expected from *risk distribution* theory. Real wages do not react immediately to changes in labor productivity but begin to increase significantly only after five quarters. This lagged increase in real wages contributes to the overshooting of the labor share even though it is not statistically significant. This behavior of real wages is also in line with the Phillips Curve mechanism in *Goodwin*'s model. Overall, this shock suggests that cyclical fluctuations in the labor share are driven by procyclical labor productivity - as theorized by *overhead cost* and *risk distribution* theories - coupled with a real wage that lags outputs - as in the *Goodwin cycle*. While the former effect generates the observed countercyclicality, the induced lagged increase in real wages causes the labor share to return to its initial level, possibly overshooting.

Moreover, the labor share reaction to the demand shock is similar to that of the labor share to a GDP shock in the bivariate model identified with the *reverse ordering*. In both cases the initial decline of the labor share is followed by an overshooting, although the latter is non-significant in the demand shock. Instead, it is very different from the response in the bivariate VAR model identified with the *standard ordering*, in which the response of the labor share is never negative.

An increase in *Labor bargaining power* raises hours worked given the same nominal wage and output (Figure 5). This results in a gradual but steady increase in labor productivity, such that it

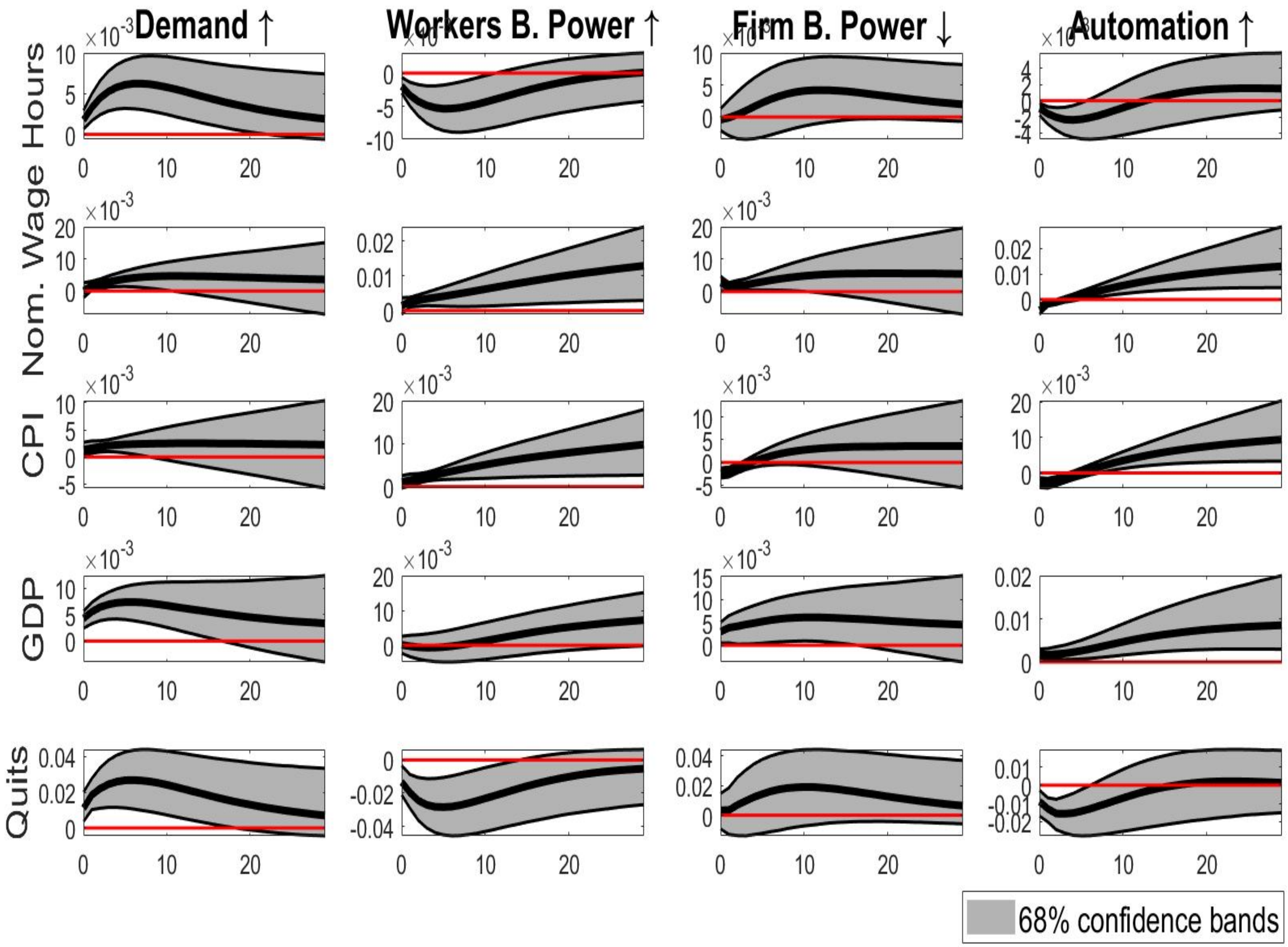

Figure 5: IRFs to structural changes

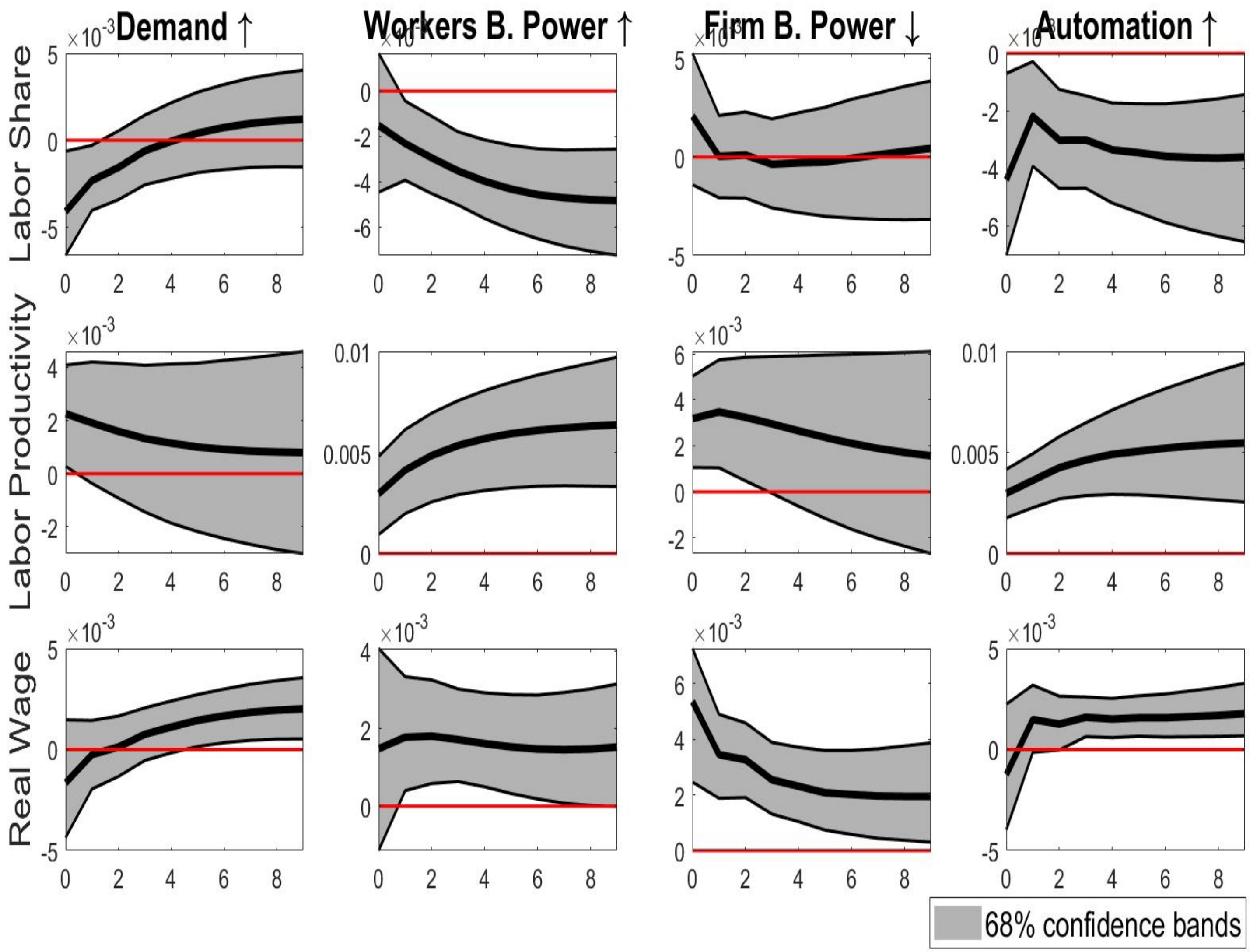

Figure 6: Implied IRFs to structural changes

more than offsets the rise in real wages. Consequently, the labor share declines rather than rise as expected (Figure 6).

This outcome is partly consistent with what is predicted by the theory of *biased technical change*. Indeed, the increase in productivity can be interpreted as the result of firms' efforts to replace the labor factor, which has become more expensive. A missing piece is an initial increase in the labor share associated with the increase in wages. In addition, GDP increases with a much longer lag than expected (roughly 30 quarters). A possible interpretation is that the initial increase in productivity comes at the expense of hours worked, which fall in the attempt to replace the labor factor. Gradually the hours worked return to the initial level and, after about 30 quarters, the higher productivity is passed on from lower hours to a higher GDP.

The lack of an initial increase in the labor share makes it difficult to judge the supposed contractionary effect of a labor share increase predicted by *Goodwin*'s model. Nevertheless, this shock actually produces a positive association between profit share and output, as in Goodwin (1967)[17], although it is an increase in wages and bargaining power of workers to trigger the increase in profit share and output paradoxically. Thus, the mechanism must be different from the one intended in Goodwin's model.

A reduction in *Firms bargaining power* generates an increase in GDP (Figure 5), labor productivity and real wages (Figure 6). This is what economic theory predicts following a reduction of the degree of monopoly and the competitiveness increase, indicating that the shock is well captured. However, although the median labor share reaction is positive on impact as expected, it is not significant. This happens because the simultaneous increase in labor productivity offsets a large part of the increase in the real wage.

Overall, we can observe that the two distributional shocks (workers' and firms' bargaining power) do not generate reactions in labor share and GDP comparable to those generated with the standard ordering identification. This weakens the clear-cut expansionary effect of a labor share fall found by the strand of literature relying on the standard ordering.

The *Automation shock* produces - inevitably because of the imposed identification - a fall in labor share and an increase in labor productivity on impact (Figure 6). Unlike the demand shock, the labor share after the initial fall does not return to the original level but remains at a lower point.

4.5 Sensitivity Analysis

As a first robustness check, I estimate the same model with three and four lags to check whether the number of lags used could affect the results. The results - not reported for convenience - are almost identical to those shown in Figures (5) and (6).

In Section 4.3, only the posterior draws producing a rise in nominal wages greater than prices after the workers' bargaining power shock were retained to ensure that this shock generated a rise in real wages. Now, to check that this method does not affect our results, I replace it with the direct restriction that the real wage increases, which enters as the sixth variable in the model. The new identification scheme is shown in Table 3. This check aims only to confirm the direction of the IRFs, while undue weight should not be given to their significance. The reason is that with an additional variable the available degrees of freedom are largely exceeded and the confidence intervals are no longer reliable.

	Demand ↑	Workers Bargaining Power ↑	Firm Bargaining Power ↓	Automation ↑
Hours	+	/	/	-
Nom. Wage	+	+	+	-
Prices	+	+	-	-
GDP	+	/	/	+
Quits	+	-	/	-
Real Wage	/	+	/	-

Table 3: Sign Restrictions on period 3

There is only one difference to the previous model. It is further imposed that the real wage increases in the third quarter following both a positive shock to workers' bargaining power and a negative shock to firms' bargaining power. In addition, the real wage is restricted to decrease in response to a positive automation shock. The resulting structural IRFs are shown in Figure (7). Figure (8) shows the implied IRFs of labor share, labor productivity and real wage. The real wage IRF no longer comes from the combination of nominal wages and prices but comes directly from the new real wage variable entering the model. The latter is also used to calculate the labor share as follows:

$$LS_t = Hours_t + RealWage_t - GDP_t \quad (6)$$

Labor productivity is calculated as in Eq. (4).

The results obtained are in line with those found in the previous section. The main differences concern the response of labor productivity and real wages to the demand shock. The IRFs of these two variables maintain the same sign. However, the lower bound is on the zero line for labor productivity, and the real wage response is non-significant. Nevertheless, their combined effect does not alter the negative impact response of labor share. In any case, given the large number of degrees of freedom consumed by adding the sixth variable, the results obtained from the previous section's model remain more reliable.

5 Conclusion

The first part of the paper provided a comprehensive review of the theoretical and empirical literature explaining the observed cyclicality of factor shares of income. The empirical literature on this topic relies on VAR models identified with Cholesky schemes, whose restrictions skew results in favor of specific theories while ruling out others. Moreover, the associated results are not robust to the inversion of the variables' ordering. In the second part of the paper, a Bayesian VAR model identified with sign restrictions was set up to overcome these criticalities. The results suggest that the pro-cyclicality of labor productivity mainly drives countercyclical fluctuations in the labor share, consistent with *overhead costs* and *risk distribution* theories. Indeed, the instantaneous fall of the labor share following a demand shock is compatible with the increasing returns to scale generated by overhead costs. The instantaneous rise in productivity coupled with sticky real wages is also consistent with the workers' will to set acyclical wages to hedge against business cycle risk, as the risk distribution theory predicts. Also, the lagged response of the real wage to demand shocks contributes to the retraction of the fall in the labor share to the initial level, supporting the Phillips curve mechanism proposed by *Goodwin*. In contrast, it is difficult to judge the other prediction of Goodwin's model - the contractionary effect of an increase in the labor share. Indeed, following a rise in workers' bargaining power, the growth of real wages is more than offset by an increase in labor productivity, yielding - paradoxically - a fall in the labor share. Nevertheless, as in Goodwin's model, this shock produces a negative association between labor share and output, although the underlying mechanism generating it is necessarily different. The rise in labor productivity following an increase in workers' bargaining power can

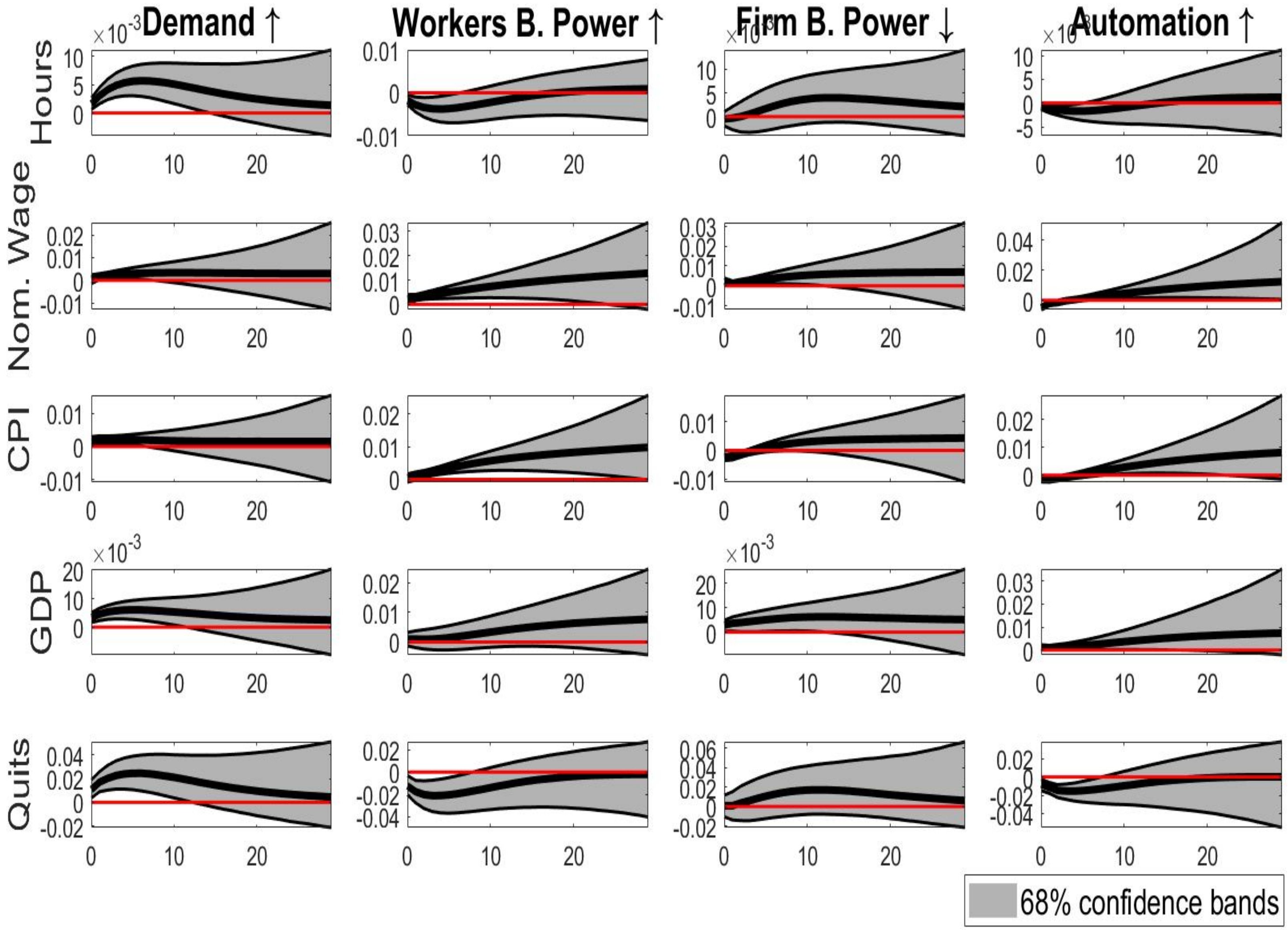

Figure 7: Structural IRFs

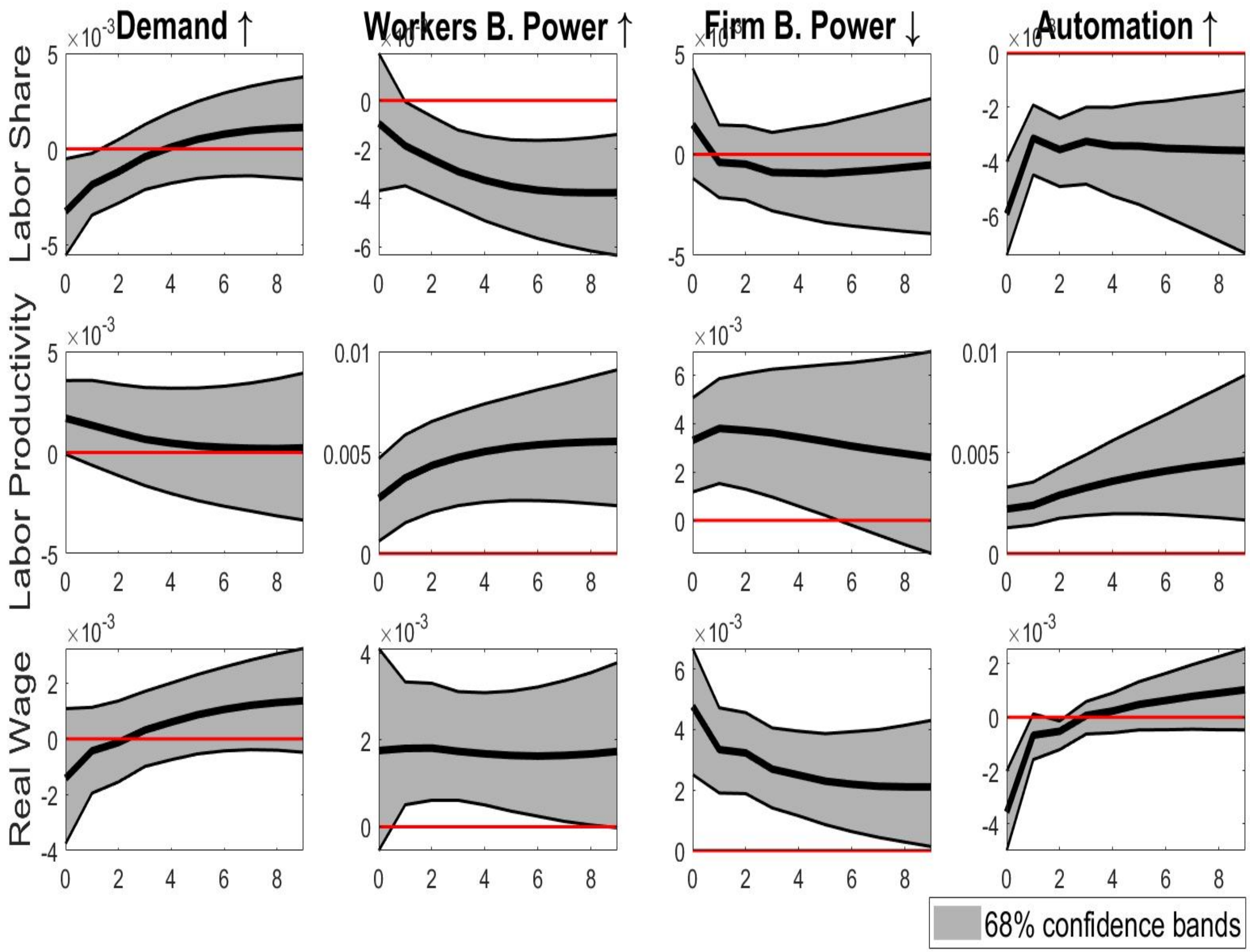

Figure 8: Implied structural IRFs

be interpreted as the result of firms' efforts to replace the labor factor, which has become more expensive, consistently with the *biased technical change theory*. However, a similar observation as for Goodwin's model applies here. Namely, the comovement between labor productivity and real wages does not correspond to a positive correlation between labor productivity and labor share as predicted by the theory. Indeed, the labor share falls because labor productivity grows more than real wages.

Finally, the results are at odds with those found by the literature relying on bivariate activity-distribution models identified with Cholesky, where the distribution variable is ordered first. This identification scheme a priori rules out the theories of overhead costs and risk distribution and the hypothesis that pro-cyclical movements in labor productivity drive counter-cyclical fluctuations in the labor share. My model supports this hypothesis instead. In contrast, the results are compatible with those found by the strand of literature ordering the activity variable before the labor share in a Cholesky scheme.

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Appendices

A

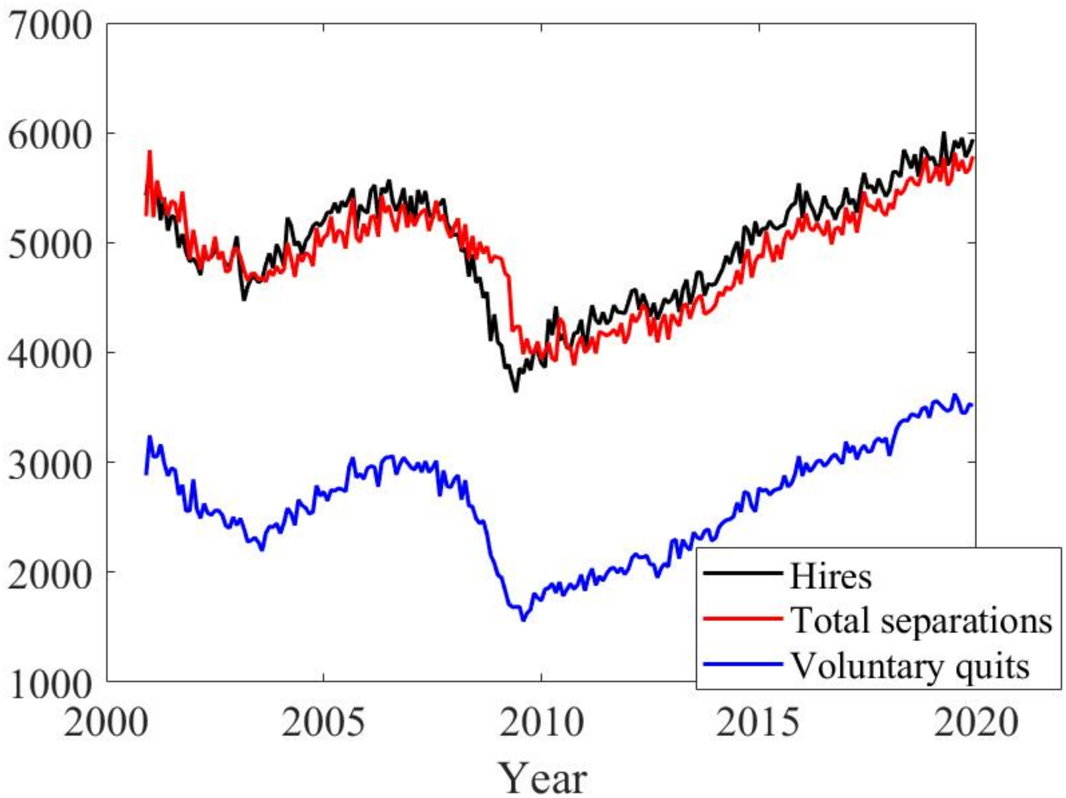

Figure 9: Labor turnover over the business cycle