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Abstract

The existing literature does not provide a theoretical framework for conversion of remittances into investment. If a government introduces some measure for the earner abroad to save more, the earner's cost (to the extent of his/ her share of savings in the home country) jumps to the pre-policy cost minus the additional costs causing inefficiency prior to the remittance policy affecting the quantity of savings and hence pushing the market out of equilibrium. The supply and the demand of the savings, then adjust over time to bring the new post-policy market equilibrium. The interest rate adjustment mechanism is based on the fact that when the remittance-investment policy leads the market out of equilibrium, the buyers' and sellers' decisions are not coordinated at the current interest rate. It is essential to take into account the efficiency losses during the adjustment process while computing the benefits of remittance-investment policy. This paper develops a dynamic model and derives an optimal remittance-investment policy minimizing the efficiency losses (output and/ or consumption of funds lost) during the dynamic adjustment process taking into account the gains from the post-policy market equilibrium subject to a policy cost constraint. (JEL G20, G28, F2, F22, F24)

Keywords: Remittance-Investment Policy, Dynamic Efficiency, Interest Adjustment Path

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1 Introduction

Workers' remittances to developing countries have grown over time to be an important source of financing for household expenditures as well as investment. The funds are used for both consumption and investment in the home countries of the migrants. The increasing amount of inflows migrants remit to their home countries is of great importance. These inflows are increasingly substantial in terms of their stability, growth rates and as a share of GDP. In 2000, the world remittance inflow stood at \$131 billion. By 2005, this figure more than doubled to \$263 billion or 0.62% of the world's GDP. This is a substantial amount compared to the Official Development Assistance (ODA) to developing countries which constitutes less than 0.24% of the world GDP. The World Bank estimates indicate that in terms of external inflows, remittances are second only to Foreign Direct Investment (FDI). It is also important to point out that for the very poor nations, remittances have surpassed even FDI. Among the Least Developing Countries (LDCs), remittance inflows in 2005 are 5.4% of GDP whilst FDI stood at 2.7% of GDP in the same period[1].

The extent to which remittances will stimulate investment and thereby also growth in developing countries is dependent on a number of circumstances. These circumstances will influence the use of remittances for consumption, savings or investment. They will also have an effect on the choice of transferring funds through channels of a formal transparent character rather than channels of an informal character that is not recorded in official statistics.

To begin with, parts of remittances are not used for consumption by the receiver. Lucas and Stark (1985) present several hypotheses for motivations to remit, ranging from pure altruism to self-interest. Cox (1987) tests alternative hypotheses concerning motivation for inter vivos transfers. Adams Jr (1998) studies the marginal effects of remittances on consumption and investment. Arif et al. (1999) investigates the question whether return migrants and their families succeeded in directing remittances into investment. Ilahi and Jafarey (1999) model informal loans between migrant and extended family for financing the costs of international labor migration. In Glytsos (2002), various channels transmitting the impact of remittances on development are investigated based on the experience of countries from both sides of the Mediterranean basin. León-Ledesma and Piracha (2004) analyse the effect of remittances on employment performance for Central and East European (CEE) economies. Adams (2005) studies the relationship of remittances, household expenditure and investment in Guatemala. Chami, Fullenkamp and Jahjah (2005) develop a model of remittances based on the economics of the family that implies that remittances are not profit-driven, but are compensatory transfers, and should have a negative correlation with GDP growth. Mundaca (2005) finds in a sample of selected Central America countries that financial development tends to increase the responsiveness of growth to remittances. In a review of the global evidence, Adams (2007) documents that remittance receiving households invest more on the average than households without remittance receipts. Adams (2007) also shows that remittance receiving house-

[1] Bjuggren, Per-Olof, and James Dzansi. "Remittances and investment." In 55th Annual Meetings of the North American Regional Science Association International, New York, November, 2008.

holds tend to save more than the average households. The ability of financial intermediaries to expand credit to the private sector and thus give additional impetus to investment is thereby increased. Adams, Cuecuecha and Page (2008) use a nationally-representative household survey from Ghana to analyze within a rigorous econometric framework how the receipt of internal remittances (from within Ghana) and international remittances (from African or other countries) affects the marginal spending behavior of households on a broad range of consumption and investment goods, including food, education and housing. Fayissa and Nsiah (2008) suggest that remittances boost growth in countries with an underdeveloped financial sector. Ang, Jha and Sugiyarto (2009) studies the relationship of remittances and the household behavior in the Philippines. Giuliano and Ruiz-Arranz (2009) study how local financial sector development influences a country's capacity to take advantage of remittances. Mundaca (2009) analyzes the effects that both workers' remittances and financial intermediation have on economic growth. Finally, there are effects of remittances on investment from the multiplier effect of remittance-induced expenses. Several studies have found the multiplier effect to be quite large (see Lucas (2005)). Munyegera and Matsumoto (2016) find a positive and significant effect of mobile money access on household welfare, measured by real per capita consumption. The mechanism of this impact is the facilitation of remittances; user households are more likely to receive remittances, receive remittances more frequently, and the total value received is significantly higher than that of non-user households. According to Ratha (2016), remittances remain a key source of funds for developing countries, far exceeding official development assistance and even foreign direct investment.

It is important for developing countries to utilize the resources through remittances for investment purposes for the long term economic growth. In order to convert remittances into investment, the governments have to rely on some incentive mechanism for the senders of remittances to encourage them to send remittances as savings. If the governments want to spend some resources on encouraging the people to convert their remittances into savings and hence into investment, it is extremely important to spend those resources in an efficient manner so as to reap the maximum gains possible in terms of economic efficiency, and hence an optimal remittance-investment policy. However, following any remittance-investment measure, there is some transition period during which the market adjusts to arrive at the new equilibrium. This transition time is usually ignored when evaluating the effect of a policy, and the major reason being a lack of theoretical framework for such treatment. As governments continue their efforts to reduce investment costs by streamlining financial procedures and enhancing the quality of related services, it is important that officials in charge of developing future plans in this area be fully cognizant of the economic efficiency gains and losses during the adjustment process in order to devise an optimal policy and subsequent monitoring.

The existing literature does not provide a theoretical framework for conversion of remittances into investment through an incentive mechanism by the government by adopting an optimal policy with regard to minimizing the efficiency losses after the adoption of the policy. There have been some empirical studies regarding the impact of remittances on investment, however, they lack the theoretical support, and hence could not be used for prediction and hence the proposals for future policies. If a government introduces some measure in the form of an incentive for the earner abroad to save more, the earner's cost (to the extent of his/ her share of savings in the home country) jumps to the pre-policy cost minus the additional costs causing inefficiency prior to the remittance policy affecting the quantity of savings and hence pushing the domestic market out of equilibrium. The supply and the demand of the savings, then adjust over time to bring the new post-policy market equilibrium. The interest rate adjustment mechanism is based on the fact that when the remittance-investment policy leads the market out of equilibrium, the buyers' and sellers' decisions are not coordinated at the current interest rate. It is essential to take into account the efficiency losses during the adjustment process while computing the benefits of remittance-investment policy. This paper develops a dynamic model and derives an optimal remittance-investment policy minimizing the efficiency losses (output and/ or consumption of savings lost) during the dynamic adjustment process taking into account the gains from the post-policy market equilibrium subject to a policy cost constraint.

The remainder of this paper is organized as follows: Section 2 explains how the individual components of the market system are joined together to form a dynamic market model. Section 3 provides the solution of the model with a remittance-investment policy. Section 4 derives an optimal remittance-investment policy minimizing the efficiency losses subject to a policy cost constraint in a specific time period. Section 5 summarizes the findings and concludes.

2 The Model

Let us assume that there is a perfectly competitive market of savings in the domestic country in equilibrium (so our starting point is when the market is already in equilibrium). There are five market agents, i.e. the domestic producer of savings (the domestic household not receiving remittances), foreign producer of savings (the earner abroad), consumer of savings (the firm seeking funds for investment), a middleman (financial intermediary) and a government. The producer (of savings) is a price taker in a perfectly competitive market and the price (interest) cannot automatically jump to bring the new equilibrium after a shock, and rather the financial intermediary changes the interest rate depending on the supply and demand of funds which leads to the new equilibrium, where the financial intermediary finds it optimal not to change the interest rate further and the market stays in equilibrium until another shock hits the market. The producers produce the funds and supply those to the middleman (the financial intermediary), who keeps the funds and sells those to the consumer (borrower of funds) at the market interest rate. The producers of savings are households and their objective is to maximize their utility; the consumer is a firm seeking funds for investment, and has an objective of maximizing his/her profits and the middleman's objective is to maximize the difference between the revenue for providing funds to the borrowers and the costs of keeping the funds subject to the constraints.

The interest rate adjustment mechanism is based on the fact that when a shock leads the domestic market (of savings) out of equilibrium, the buyers' and sellers' decisions are not coordinated at the current interest rate. An example can illustrate the working of this market. Consider that the market is initially in equilibrium. The middleman has an equilibrium stock of funds. Then, an exogenous supply expansion will increase the stock of funds, due to consumers' demand could not match with the –now higher– funds supplied by the producers at the current interest rate. This higher supply is reflected in the piled up funds held by the middleman. The middleman will decrease the interest rate so that the producers will find optimal to produce a lower level of funds. A new equilibrium with a lower interest rate and a higher level of investment is then reached. The equilibrium is defined as follows:
(i) The producers (households) maximize their utility; the consumer (firm) maximizes his/her profits and the middleman maximizes the difference between the revenue for providing funds to the borrowers and the costs of keeping the funds subject to the constraints they face (mentioned in their individual dynamic optimization problems in Section 2).
(ii) The quantity of funds supplied by the producers equals the quantity consumed by the consumer (and hence the inventory of funds held by the middleman does not change when the market is in equilibrium).
The conditions for the existence of the equilibrium (Routh–Hurwitz stability criterion, which provides a necessary and sufficient condition for the stability of a linear dynamical system) have been mentioned in Section 3.
As the set-up is for a perfectly competitive market, therefore, the middleman who sells the funds to the consumer at the market interest rate is a price taker when the market is in equilibrium. When the market is out of equilibrium, the middleman can change the interest rate along the dynamic adjustment path until the new equilibrium arrives, where again the middleman becomes a price taker. The government adopts a remittance-investment policy through an incentive mechanism for the earner abroad to convert remittances into investment and when the policy is adopted, the market does not suddenly jump to the post-policy market equilibrium, rather the interest rate adjusts over time to bring the new equilibrium. This adjustment process involves endogenous decision making (in their own interest) by all the agents in the market, i.e. consumer, producers and the middleman as follows: Suppose there are two producers in a market who save funds and sell those to a middleman who further sells those to a consumer. The consumer and the middleman buy a quantity exactly equal to the quantity the producers produce in each time period, and the market stays in equilibrium. If the government adopts a favorable policy for one of the producers which decreases his/her cost of saving and hence increases the supply of the funds, some of the funds will remain unsold by the end of the time period in which the policy was adopted. Assuming that the producers and the middleman can change the production of funds and the interest rate respectively, immediately, had the middleman known the exact pattern of new supply and demand (after he would change the interest rate), he would immediately pick the interest rate to maximize

the profits and clear the market without wasting his profits through unsold funds. However, he lacks this information, so the middleman decreases the interest rate based on his best guess about the new supply at the new interest rate (based on the quantity of the increased funds by the producers), driving the market close to the new equilibrium. At the lower interest rate, the producers produce a lower quantity of funds than before. If in the following time period, their production of funds sold to the middleman is fully sold out to the consumer, the middleman will know that the new equilibrium has arrived, however, if there is still some unsold supply of funds, the middleman will decrease the interest rate further (and the producers, the production of funds accordingly) to bring the market closer to the new equilibrium. The market will eventually settle at the new equilibrium after some efficiency loss. The efficiency loss by the remittance-investment policy is the output of unutilized funds in each time period during the adjustment process. A new equilibrium with an efficiency gain due to remittance-investment policy is finally arrived at. The total efficiency loss as a result of policy is the loss during the adjustment process minus the gain in the final equilibrium. For the mathematical treatment, the objective of each of the four market agents is maximized through the first order conditions of their objective functions and to capture the collective result of their individual actions, the equations representing their individual actions are solved simultaneously. For simplification, we assume that after the adoption of the remittance-investment policy, the new equilibrium is not too far from the initial equilibrium. This assumption makes the linearization of supply and demand curves quite reasonable. Please look at figure 1 (the time axis is not shown). Linearization seems to be a good approximation when we move from point a to b, whereas it is not a good approximation when we move from point a to c. For modeling the movement of the market from point a to c, we need to model a non-linear dynamical system (which is not covered under the scope of this paper).

2.1 Middleman (Financial Intermediary)

The middleman purchases funds from the producers and sells those to the consumer for profit. As happens in the real world, the middleman does not buy and sell exactly the same quantity of funds at all points in time, thus he holds an inventory of the funds purchased to be sold subsequently. Inventory is an intermediary stage between supply and demand which reflects the quantum of difference between supply and demand of the funds in the financial market. If the inventory remains the same, it implies that demand and supply rates are the same. An increase or decrease in inventory implies a change in supply, demand or both at different rates.

Please look at figure 2 to understand the link between inventory of funds (savings), supply, demand and price (interest rate). When the supply curve shifts to the right (while demand remains the same), the inventory in the market increases at the initial price (interest rate), and the new equilibrium brings the price (interest rate) down. Similarly, when the demand curve shifts to the right (while supply remains constant), the inventory of funds (savings) depletes from the market at the previous interest rate and the new equilibrium brings the interest rate up. This shows that there is an inverse relationship between an inventory change and an interest rate change (all else the same). If both the supply and demand curves shift by the same magnitude such that the inventory does not change, then interest rate will also remain the same. Inventory unifies the supply and demand shocks in the sense that they are both affecting the same factor, i.e. inventory and are basically the faces of the same coin. Therefore, each kind of shock is in fact just an inventory shock. From the above mentioned discussion, we have seen that there is an inverse relationship between an inventory change and an interest rate change. Now let's discuss the mechanism which brings about such a change. Consider a market of funds where the middlemen (financial intermediaries) hold inventories of funds, incur some cost for holding those, and lend funds to the consumers to make profits. The cost is a positive function of the size of an inventory of funds, i.e. a larger inventory costs more to hold as compared to a smaller inventory. In the absence of an exogenous shock, if the supply and demand rates are equal then the system is in equilibrium and the interest rate does not vary with time.

Suppose that an intervention decreases the marginal cost of production of funds (savings) and increases the supply rate, whereas the demand rate remains the same. As the demand and supply rates are no longer equal, therefore the difference will appear in the form of piled up funds by the financial institutions. The middlemen will have to think of some means of lending the additional funds to the borrowers (consumers). The only resort they have is to decrease the interest rate which brings the demand up along the demand curve. In a perfectly competitive market, the interest rate will eventually come down to equalize the new marginal cost, however the adjustment path depends on how the middlemen react to the change in their inventories of funds. Notice that the marginal cost of production has decreased but the marginal cost of holding an extra unit of funds for the middleman has increased. For a mathematical treatment, we need to consider the profit maximization problem of the *middleman* as follows:

2.1.1 Static Problem

Let us first consider the static problem (Explanation: The static problem means that the middleman's objective is myopic rather than doing dynamic optimization) of the middleman as follows:

$$\Pi = rq(r) - \varsigma(m(r, e)), \tag{1}$$

where

Π = profit,

r = market interest rate,

$q(r)$ = quantity of funds sold (selling means lending) at interest rate r,

m = inventory (quantity of funds held by the *middleman*),

e = other factors which influence inventory of funds other than the market interest rate including the middleman's purchase interest rate from the producer,

$\varsigma(m(r,e))$ = cost as a function of inventory (increasing in inventory).
The first order condition (with respect to interest rate) is as follows:

$$rq'(r) + q(r) - \varsigma'(m(r,e))m_1'(r,e) = 0, \tag{2}$$

The middleman has an incentive to change the interest rate only during the adjustment process and will incur losses by deviating from the interest rate (equal to the marginal cost) when the market is in equilibrium. During the adjustment process, the demand does not equal the supply and the market drifts toward the new equilibrium, therefore an interest rate change by the middleman in the direction of bringing the new equilibrium is not against the market forces, so he does not lose business by changing interest rate on the adjustment path unlike when the market is in equilibrium and where the middleman faces an infinitely elastic demand as follows:

$$\begin{aligned} rq'(r) + q(r) &= \varsigma'(m(r,e))m_1'(r,e), \\ r\left[1 + \frac{1}{demand\ elasticity}\right] &= \varsigma'(m(r,e))\frac{m_1'(r,e)}{q'(r)}. \end{aligned}$$

The right hand side of the above expression is the marginal cost which equals the interest rate when the middleman faces an infinitely elastic demand. Suppose that as a result of a supply shock, the marginal cost of production of funds decreases, and the supply curve shifts downwards (if the marginal cost of production decreases either for domestic producer, foreign producer or both, the total domestic supply curve will shift downwards in all the three cases however by different magnitudes). Now the competitive market is out of equilibrium as the demand does not equal the supply at the previous equilibrium interest rate. The interest rate must eventually decrease to bring the new equilibrium, however, the interest rate will not jump to equalize the demand and supply, and rather the middleman will continue charging an interest rate higher than the new marginal cost until the market forces make him realize that the supply has increased and he needs to lower the interest rate to satisfy the profit maximizing condition. The similar is the case of a reverse supply shock, where the interest rate must eventually increase to bring the new equilibrium. In this case, the middleman will continue charging an interest rate lower than the marginal cost until the market forces make him increase the interest rate, in which case it is the consumer who is the short term beneficiary. Again, the consumer will be paying an interest rate less than the marginal cost only during the adjustment process and only until the middleman increases the interest rate. The equilibrium interest rate is equal to the marginal cost of production plus the marginal cost of holding funds by the financial intermediary (i.e. the total marginal cost) in the absence of any kind of a policy intervention, so neither does the middleman earn any economic rent, nor does the consumer benefit by paying an interest rate less than the marginal cost when the competitive market is in equilibrium.

For the mathematical treatment, suppose that as a result of a supply shock (while demand remains the same) which reduces the marginal cost of production of funds and increases the supply by the producers (either by the domestic producer, foreign producer or both), if the middleman wants to hold an extra unit of inventory of funds, his marginal cost of holding an extra unit i.e. $\varsigma'(m(r,e))\frac{m_1'(r,e)}{q'(r)}$ is higher at the previous interest rate, because the term $\varsigma'(m(r,e))$ is higher at the previous interest rate. This might be on account of higher cost of holding unutilized funds by the financial intermediary after increased supply in the market. The second term, i.e. $\frac{m_1'(r,e)}{q'(r)}$ is a function of the interest rate, and is the same as before as the interest rate has not changed yet (we are assuming that the middleman's purchase interest rate is the same as before as the producer is a price taker during the adjustment process as well and always charges a fixed fraction of the market interest rate to the middleman). A discrete analog of this scenario is that the middleman maximizes profits in each time period without considering the future time periods, and in each time period he takes the purchase interest rate from the producer as given and only chooses the sale (lending) interest rate. This implies that on the previous interest rate, now the middleman faces

$$\frac{\partial \Pi}{\partial r} = rq'(r) + q(r) - \varsigma'(m(r,e))m_1'(r,e) < 0, \tag{3}$$

which means that the middleman must decrease the interest rate to hold an extra unit of inventory of funds to satisfy the profit maximizing condition after the supply shock. Please notice that in this static scenario, the short term gains accrued from the decreased marginal cost of production will be reaped by the producer, as his marginal cost has decreased but he charges the same interest rate to the middleman until the middleman changes the interest rate. If we plot together various profit maximizing combinations of inventories of funds and the respective interest rates chosen by a middleman, we will get a downward sloping *inventory curve* with the interest rate on the y-axis and the inventory of funds on the x-axis. This is analogous to the concept of *supply* and *demand curves* for the profit maximizing producers and the consumers of funds respectively.

2.1.2 Dynamic Problem

Now let us consider the dynamic problem of the financial intermediary. In a dynamic setting, the financial intermediary maximizes the present discounted value of the future stream of profits, and his present value at time zero is as follows:

$$V(0) = \int_0^\infty [rq(r) - \varsigma(m(r,e))]\, e^{-\sigma t} dt, \tag{4}$$

σ denotes the discount rate. $r(t)$ is the *control variable* and $m(t)$ the *state variable*. The maximization problem can be written as

$$\underset{\{r(t)\}}{Max} V(0) = \int_0^\infty [rq(r) - \varsigma(m(r,e))]\, e^{-\sigma t} dt,$$

subject to the constraints that

$\dot{m}(t) = m_1'(r(t), e(r(t), z))\dot{r}(t) + m_2'(r(t), e(r(t), z))e_1'(r(t), z)\dot{r}(t)$ (state equation, describing how the state variable changes with time; z are exogenous factors),

$m(0) = m_s$ (initial condition),

$m(t) \geq 0$ (non-negativity constraint on state variable),

$m(\infty)$ free (terminal condition).

The current-value Hamiltonian for this case is

$$\widetilde{H} = r(t)q(r(t)) - \varsigma(m(r(t), e(r(t), z))) + \mu(t)\dot{r}(t) \begin{bmatrix} m_1'(r(t), e(r(t), z)) + m_2'(r(t), e(r(t), z)) * \\ e_1'(r(t), z) \end{bmatrix}. \tag{5}$$

Now the maximizing conditions are as follows:

(i) $r^*(t)$ maximizes $\widetilde{H}$ for all t: $\frac{\partial \widetilde{H}}{\partial r} = 0$,

(ii) $\dot{\mu} - \sigma\mu = -\frac{\partial \widetilde{H}}{\partial m}$,

(iii) $\dot{m}^* = \frac{\partial \widetilde{H}}{\partial \mu}$ (this just gives back the state equation),

(iv) $\lim\limits_{t \to \infty} \mu(t)m(t)e^{-\sigma t} = 0$ (the transversality condition).

The first two conditions are as follows:

$$\frac{\partial \widetilde{H}}{\partial r} = 0, \tag{6}$$

and

$$\dot{\mu} - \sigma\mu = -\frac{\partial \widetilde{H}}{\partial m} = \varsigma'(m(r(t), e(r(t), z))). \tag{7}$$

When the market is in equilibrium, $\dot{r}(t) = 0$, and the expression $\frac{\partial \widetilde{H}}{\partial r}$ boils down to the following (see appendix):

$$r(t)\left[1 + \frac{1}{demand\ elasticity}\right] = \varsigma'(m(r(t), e(r(t), z)))\left\{\frac{m_1'(r(t), e(r(t), z))}{q'(r(t))} + \frac{m_2'(r(t), e(r(t), z))e_1'(r(t), z)}{q'(r(t))}\right\},$$

suggesting that the interest rate equals the marginal cost (the right hand side of the above expression is the marginal cost in a dynamic setting, which is different from that in a static problem on account of the fact that in a dynamic setting the middleman also takes into account the impact of interest rate chosen on his purchase interest rate from the producer) when the demand is infinitely elastic. Now suppose that as a result of a supply shock, if the middleman wants to hold an extra unit of inventory of funds, then the marginal cost of holding an extra unit is higher because the term $\varsigma'(m(r(t), e(r(t), z)))$ is higher at the previous interest rate at that point in time. The term in parentheses in the expression of the marginal cost, i.e. $\frac{m_1'(r(t),e(r(t),z))}{q'(r(t))} + \frac{m_2'(r(t),e(r(t),z))e_1'(r(t),z)}{q'(r(t))}$ is a function of interest rate and is the same at the previous interest rate. This implies that on the previous interest rate, now the middleman faces

$$\frac{\partial \widetilde{H}}{\partial r} < 0.$$

Therefore in order to satisfy the condition of dynamic optimization, the middleman must decrease the interest rate for an increase in inventory of funds. This implies a negative relationship between interest rate and the inventory of funds. The concept of inventory unifies the market supply and demand. If the supply and demand rates are equal, the market is in a steady state equilibrium. If a difference of finite magnitude is created between the supply and demand rates and the consumer and the producer do not react to an interest rate change induced by a difference in the supply and demand rates, the interest rate will continue changing until the system saturates. This behavior can be depicted by the following formulation:

$$\begin{aligned}
&\textit{Interest rate change} \propto \textit{change in market inventory of funds.}\\
&\qquad R = \textit{interest rate change.}\\
&\qquad M = m - m_s = \textit{change in inventory of funds in the market,}\\
&\qquad m = \textit{inventory of funds at time } t,\\
&\qquad m_s = \textit{inventory of funds in steady state equilibrium.}\\
&\textit{Input} - \textit{output} = \frac{dm}{dt} = \frac{d(m-m_s)}{dt} = \frac{dM}{dt},\\
&\qquad \text{or } M = \int (\textit{input} - \textit{output})\, dt.\\
&\textit{Interest rate change} \propto \int (\textit{supply rate} - \textit{demand rate})\, dt, \text{ or}\\
&\qquad R = -K_m \int (\textit{supply rate} - \textit{demand rate})\, dt,
\end{aligned}$$

where K_m is the proportionality constant. A negative sign indicates that when ($\textit{supply rate} - \textit{demand rate}$) is positive, then R is negative (i.e. the interest rate decreases). The above equation can be re-arranged as follows:

$$\int (\textit{supply rate} - \textit{demand rate})\, dt = -\frac{R}{K_m}, \text{ or}$$

$$\int (w_i - w_0)\, dt = -\frac{R}{K_m}, \tag{8}$$

$$w_i = \textit{supply rate},$$

$$w_0 = \textit{demand rate},$$

$$K_m = \textit{dimensional constant.}$$

Let at time $t = 0$, *supply rate* = *demand rate* (market is in a steady state equilibrium), then eq. (8) can be written as

$$\int (w_{is} - w_{0s})\, dt = 0. \tag{9}$$

The subscript s indicates the steady state equilibrium and $R = 0$ in steady state. Subtracting eq. (9) from eq. (8), we get:

$$\int (w_i - w_{is})\, dt - \int (w_0 - w_{0s})\, dt = -\frac{R}{K_m}, \text{ or}$$

$$\int (W_i - W_0)\, dt = -\frac{R}{K_m}, \tag{10}$$

$$\text{where } w_i - w_{is} = W_i = \textit{change in supply rate},$$

$$w_0 - w_{0s} = W_0 = \textit{change in demand rate.}$$

R, W_i and W_0 are deviation variables, which indicate deviation from the steady state equilibrium. The initial values of the deviation variables are zero. Eq. (10) may also be written as follows:

$$R = -K_m \int W dt = -K_m M, \tag{11}$$

where $W = W_i - W_0$. If R gets a jump as a result of some factor other than an inventory of funds change, that is considered as a separate input and can be added to eq. (11) as follows:

$$R = -K_m \int W dt + B = -K_m M + B. \tag{11a}$$

Similarly, there can be an exogenous shock in inventory of funds other than the interest rate feedback.

2.2 Consumer

The consumer of funds (such as a firm seeking funds for investment) maximizes the present discounted value of the future stream of profits, and his present value at time zero is as follows:

$$V(0) = \int_0^\infty \left[p(t)F\left(K\left(t\right), L\left(t\right)\right) - w(t)L\left(t\right) - r(t)I(t)\right] e^{-\sigma t} dt, \tag{12}$$

$p(t)$ is the price of the output of the firm. σ denotes the discount rate. $L(t)$ (labor) and $I(t)$ (level of investment) are the *control variables* and $K(t)$ the *state variable*. The maximization problem can be written as

$$\underset{\{L(t), I(t)\}}{Max} V(0) = \int_0^\infty \left[p(t)F\left(K\left(t\right), L\left(t\right)\right) - w(t)L\left(t\right) - r(t)I(t)\right] e^{-\sigma t} dt,$$

subject to the constraints that
$\dot{K}(t) = I(t) - \delta K(t)$ (state equation, describing how the state variable changes with time),
$K(0) = K_0$ (initial condition),
$K(t) \geq 0$ (non-negativity constraint on state variable),
$K(\infty)$ free (terminal condition).
The current-value Hamiltonian for this case is

$$\widetilde{H} = p(t)F\left(K\left(t\right), L\left(t\right)\right) - w(t)L\left(t\right) - r(t)I(t) + \mu(t)\left[I(t) - \delta K(t)\right]. \tag{13}$$

Now the maximizing conditions are as follows:
(*i*) $L^*(t)$ and $I^*(t)$ maximize $\widetilde{H}$ for all t: $\frac{\partial \widetilde{H}}{\partial L} = 0$ and $\frac{\partial \widetilde{H}}{\partial I} = 0$,
(*ii*) $\dot{\mu} - \sigma\mu = -\frac{\partial \widetilde{H}}{\partial K}$,
(*iii*) $\dot{K}^* = \frac{\partial \widetilde{H}}{\partial \mu}$ (this just gives back the state equation),
(*iv*) $\lim_{t\to\infty} \mu(t)K(t)e^{-\sigma t} = 0$ (the transversality condition).
The first two conditions are as follows:

$$\frac{\partial \widetilde{H}}{\partial L} = p(t)F_2'\left(K\left(t\right), L\left(t\right)\right) - w(t) = 0, \tag{14}$$

$$\frac{\partial \widetilde{H}}{\partial I} = -r(t) + \mu(t) = 0, \tag{15}$$

and

$$\dot{\mu} - \sigma\mu = -\frac{\partial \widetilde{H}}{\partial K} = -\left[p(t)F_1'\left(K\left(t\right), L\left(t\right)\right) - \delta\mu(t)\right]. \tag{16}$$

If the interest rate, i.e. $r(t)$ goes up, (at the previous level of investment) the firm faces

$$-r(t) + \mu(t) < 0.$$

Therefore in order to satisfy the condition of dynamic optimization after the interest rate increase, the firm (consumer of funds) must decrease the investment. Let the change in demand be proportional to the change in interest rate, i.e. R. Then we can write:

$$\textit{Change in demand of funds} \propto R, \text{ or}$$

$$W_d = -K_d R. \tag{17}$$

W_d is the change in demand due to R; when R is positive W_d is negative.

2.3 Producers

The foreign and the domestic producers' objective is identical, therefore the problem of only one of the producers has been considered and the results extended to both of them. The producer of funds (the earner abroad or the foreign household) maximizes the present discounted value of the future stream of utilities, and his present value at time zero is as follows:

$$V(0) = \int_0^\infty U(x(t)) e^{-\rho t} dt, \tag{18}$$

ρ denotes the discount rate and $x(t)$ is the *control variable*. The maximization problem can be written as

$$\underset{\{x(t)\}}{Max} V(0) = \int_0^\infty U(x(t)) e^{-\rho t} dt,$$

subject to the constraints that
$\dot{a}(t) = r(t)a(t) + w(t) - p(t)x(t)$ (state equation, describing how the state variable changes with time). $a(t)$ is asset holdings (a *state variable*) in the domestic country, and $w(t)$ and $p(t)$ are exogenous time path of wages and price of consumption $x(t)$.
$a(0) = a_s$ (initial condition),
$a(t) \geq 0$ (non-negativity constraint on state variable),
$a(\infty)$ free (terminal condition).
The current-value Hamiltonian for this case is

$$\widetilde{H} = U(x(t)) + \mu(t) \left[r(t) a(t) + w(t) - p(t) x(t) \right]. \tag{19}$$

Now the maximizing conditions are as follows:

(i) $x^*(t)$ maximizes $\widetilde{H}$ for all t: $\frac{\partial \widetilde{H}}{\partial x} = 0$,
(ii) $\dot{\mu} - \rho\mu = -\frac{\partial \widetilde{H}}{\partial a}$,
(iii) $\dot{a}^* = \frac{\partial \widetilde{H}}{\partial \mu}$ (this just gives back the state equation),
(iv) $\lim_{t\to\infty} \mu(t)a(t)e^{-\rho t} = 0$ (the transversality condition).
The first two conditions are as follows:

$$\frac{\partial \widetilde{H}}{\partial x} = U'(x(t)) - \mu(t)p(t) = 0, \tag{20}$$

and

$$\dot{\mu} - \rho\mu = -\frac{\partial \widetilde{H}}{\partial a} = -\mu(t)r(t). \tag{21}$$

If the interest rate $r(t)$ goes up, the above maximization condition becomes as follows:

$$\dot{\mu} - \rho\mu + \mu(t)r(t) > 0.$$

Therefore in order to satisfy the conditions of dynamic optimization after the interest rate increase, the asset holdings in the domestic country must increase and hence the supply of remittances. Let $r =$ market interest rate, $c =$ a reference price of assets (such as the yield on an asset which includes the cost of savings, profit of producer and profit of the financial intermediary). c is a parameter which may vary with time or be kept fixed for a limited time period, e.g. the cost of savings may vary over time or can also remain constant for a while. It is the reference point with respect to which the variation in r is considered by the producer of funds for decision making.

$$W_m = \textit{Change in production due to change in the interest rate},$$

$(r - c)$ acts as an incentive for the producer to provide more funds. We can write:

$$W_m \propto \alpha(r - c), \text{ or}$$

αr is the buying interest rate paid to the producer of funds by the financial intermediary, where $\alpha < 1$. The above expression can also be written as:

$$W_m = K_s(r - c). \tag{22}$$

When the market is in equilibrium, then $W_m = 0$, or

$$0 = K_s(r_s - c_s). \tag{23}$$

K_s is the proportionality constant. r_s and c_s are the steady state equilibrium values. Subtracting eq. (23) from eq. (22), we get:

$$W_m = K_s \left[(r - r_s) - (c - c_s) \right] = -K_s \left(C - R \right) = -K_s \varepsilon, \tag{24}$$

where W_m, C and R are deviation variables.

3 Solution of the Model with Remittance-Investment Policy

As $W_m(t)$ is the change in the total supply in the domestic market, it includes both the domestic supply change as well the change in foreign funds, and thus can be bifurcated as follows:

$$W_m(t) = -K_{sd} \left[C_d(t) - R(t) \right] - K_{se} \left[C_e(t) - R(t) \right], \tag{25}$$

where the subscripts d and e denote the domestic producer (of investment funds) and the exporter (foreign producer, i.e. the domestic earner in the foreign counrty who transfers funds for investment in the domestic country) in the foreign country respectively. The solution of the model can be written as follows:

$$\frac{dR(t)}{dt} + K_m(K_{sd} + K_{se} + K_d)R(t) = K_m \left[K_{sd}C_d(t) + K_{se}C_e(t) \right]. \tag{26}$$

If $C_e(t) = f(T)$, i.e. a decreasing function of T (incentive for converting remittances into investment), and as a simple example, suppose that $C_e(t) = -T$, and $C_d(t) = 0$, i.e. the government's remittance-investment policy reduces the per unit cost on the flow of foreign funds by T at $t = 0$, then the above differential equation becomes as follows:

$$\frac{dR(t)}{dt} + K_m(K_{sd} + K_{se} + K_d)R(t) = -K_m K_{se} T. \tag{27}$$

The Routh–Hurwitz stability criterion (which provides a necessary and sufficient condition for stability of a linear dynamical system) for the stability of the above differential equation is $K_m(K_{sd}+K_{se}+K_d) > 0$, which holds as K_m, K_{sd}, K_{se} and K_d are all defined to be positive. This ensures that, away from a given initial equilibrium, every adjustment mechanism will lead to another equilibrium. The solution has the form

$$R(t) = C_1 + C_2 e^{-[K_m(K_{sd}+K_{se}+K_d)]t}. \tag{28}$$

Substituting the values of C_1 and C_2 in eq. (28), we get:

$$R(t) = -\frac{K_{se}T}{(K_{sd} + K_{se} + K_d)} + \frac{K_{se}T}{(K_{sd} + K_{se} + K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}. \tag{29}$$

When $t = 0$, $R(0) = 0$ (the initial condition), and when $t = \infty$, $R(\infty) = \frac{-K_{se}T}{(K_{sd}+K_{se}+K_d)}$ (the final steady state equilibrium value). The interest rate dynamics as a result of remittance-investment policy depends on the parameters K_{sd}, K_{se}, K_d, K_m and T. In the final equilibrium, the quantity demanded must equal the quantity supplied, which holds.

4 A Dynamic Optimal Remittance-Investment Policy

When a remittance-investment policy is adopted, there is some efficiency gain as a result of a comparison of the pre and post policy market equilibrium. However, the dynamic picture shows that there is some efficiency loss on the dynamic adjustment path to the new equilibrium after the introduction of the remittance-investment policy before the market arrives at the new efficient equilibrium (the equilibrium before the policy was inefficient because of additional costs to the exporter of funds, and which the government eliminates as a result of remittance-investment policy). After the policy gets implemented, the cost of the savings part of the remittances jumps to the previous cost minus the additional costs causing inefficiency. This affects the foreign savings quantity. The demand of funds does not equal the supply any longer and the interest rate adjusts over time to bring the new equilibrium interest rate which is lower than the previous equilibrium rate by a magnitude depending on the elasticity of demand and supply schedules. By eq. (25), the change in total domestic supply as a result of remittance-investment policy is as follows:

$$W_m(0) = -K_{sd}\left[C_d(0) - R(0)\right] - K_{se}\left[C_e(0) - R(0)\right] = K_{se}T, \tag{30}$$

as $R(0) = 0$.

Therefore, at time zero (when the policy is just adopted), the inventory of funds with the financial intermediary goes up by $K_{se}T$ (as the demand remains the same). As the market gets out of equilibrium, the market forces come into play and the market interest rate starts changing, affecting the inventory of funds through the feedback. A pile up of inventory of funds indicates a higher supply than demand, and a depletion of inventory of funds occurs when demand is higher than the supply in a given time period. When the demand and supply are the same, there is no efficiency loss. If the demand and supply are different, the output and/ or consumption of funds is being lost at that point in time. Therefore if we sum up the supply and demand difference at all points in time, we get the total efficiency loss during the adjustment process, As we also need to take into account the efficiency losses before the remittance-investment policy took place, which is the difference of the pre-policy and the post-policy equilibrium quantity of funds, the total efficiency loss is as follows:

$$\begin{aligned} EL &= \int_{-\infty}^{0} W_m(\infty)dt + \int_{0}^{\infty} \left[W_m(t) - W_d(t)\right] dt. \\ &= \int_{-\infty}^{0} W_m(\infty)dt + M(t). \end{aligned} \tag{31}$$

In traditional analysis, the measure of lost efficiency is the total surplus foregone which can be expressed as follows:

$$EL\ (Surplus) = \frac{1}{2}\left[\int_{-\infty}^{0} \{W_d(\infty) - W_d(0)\}\{R(\infty) + 2r_s\}\, dt - \int_{0}^{\infty} \{W_d(t) - W_d(\infty)\}\{r(t) - c(t)\}\, dt\right]$$

plus the sum of the consumer and the producer surplus in each time period wasted (which could have been earned by diverting resources to some other market) due to over production of funds during the adjustment process. (32)

In a dynamic setting, minimizing the efficiency loss in terms of quantity is more tractable as compared to that in terms of surplus. However, minimizing the efficiency loss in terms of quantity is equivalent to that in terms of the surplus.

With Remittance-Investment Policy Cost Constraint:

The expression for the remittance-investment policy cost ($RIPC$) can be written as follows:

$$RIPC = g\left[T, \{w_{ime}(0) + K_{se}\{T + R(t)\}\}\right],$$

i.e. the remittance-investment policy cost is a function of the policy as well as the foreign funds quantity as a result of policy. As a simple example in order to ensure an analytic/ closed form solution, the remittance-investment policy cost may be represented as

$$RIPC = T\left[w_{ime}(0) + K_{se}\{T + R(t)\}\right]. \tag{33}$$

This could be a kind of a specific remittance subsidy as a remittance-investment policy in order to counter the additional costs casuing inefficiency during the flow of foreign funds. If we want to minimize the efficiency loss subject to the constraint that remittance-investment policy cost is less than or equal to G in a given time period, our problem is as follows:

$$\min_T EL \quad \text{s.t.} \quad RIPC \le G.$$

G is the additional cost during the flow of funds causing inefficiency prior to the policy. The choice variable is the remittance-investment policy, and the constraint is binding in this particular example where $RIPC$ is given by eq. (33). The Lagrangian for the above problem is as follows:

$$\mathcal{L} = \int_{-\infty}^{0} \frac{K_{se}K_d T}{(K_{sd}+K_{se}+K_d)} dt - \frac{1}{K_m}\left[-\frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t} - K_m K_{se} T\right]$$
$$+\lambda\left[G - T\left[w_{ime}(0) + K_{se}\left\{T - \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}\right]\right].$$

Taking the first order condition with respect to T, we get:

$$T = -\frac{\lambda w_{ime}(0) - \left[\int_{-\infty}^{0} \frac{K_{se}K_d}{(K_{sd}+K_{se}+K_d)} dt - \frac{1}{K_m}\left[-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t} - K_m K_{se}\right]\right]}{2\lambda K_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right]}. \tag{34}$$

Taking the first order condition with respect to λ, we get:

$$G - T\left[w_{ime}(0) + K_{se}\left\{T - \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}\right] = 0. \tag{35}$$

Substituting the value of T from eq. (34) into (35), we get:

$$\lambda = \frac{J}{\sqrt{w_{ime}^2(0) + 4QG}}.$$

λ has to be positive (since an increase in G causes the minimum efficiency loss to increase as well).

$$Q = K_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right],$$
$$J = \int_{-\infty}^{0} \frac{K_{se}K_d}{(K_{sd}+K_{se}+K_d)} dt - \frac{1}{K_m}\left[-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t} - K_m K_{se}\right].$$

Eq. (34) can also be written as

$$T = -\frac{\lambda w_{ime}(0) - J}{2\lambda Q}. \tag{36}$$

Substituting the value of λ into eq. (36), we get:

$$T(t) = -\frac{w_{ime}(0) - \sqrt{w_{ime}^2(0) + 4QG}}{2Q}. \tag{37}$$

The second order condition for minimization has been checked (see appendix). Suppose that the government can spend \$1000 in terms of remittance-investment policy cost. The initial foreign funds quantity is purchase of 100 bonds, and the value of each one of K_m, K_{sd}, K_{se} and K_d is equal to one. Substituting these values in eq. (37) yields

$$T(0) = -\frac{100 - \sqrt{10000 + 4000}}{2} = 9.161,$$

where $Q = 1 - 0.333 + 0.333e^{-3t}$, and at $t = 0$, $Q = 1$. The remittance-investment policy cost is $RIPC = T\left[w_{ime}(0) + QT\right] = 1000$. Now when $t = \infty$, $Q = 0.667$. This implies that

$$T(\infty) = -\frac{100 - \sqrt{10000 + 2668}}{1.334} = 9.409.$$

The remittance-investment policy cost is again \$1000 as desired. Therefore the optimal remittance-investment policy is that the government initially provides a savings subsidy of \$9.161 per unit bond and then gradually increase it over time up to a final rate of \$9.4077 per unit bond.

5 Conclusion

It is important for the developing countries receiving remittances to spend those resources for the long term economic growth of the country. Therefore, a policy for converting remittances into investment might be beneficial for the developing countries. However, the exisiting literature does not provide a theoretical framework for the design of such policy. This paper fills this gap. In order to convert remittances into investment, governments must have to rely upon some incentive mechanism for the senders of remittances to send those as savings rather than for consumption purposes. If a government introduces some measure in the form of an incentive for the earner abroad to save more, the earner's cost (to the extent of his/ her share of savings in the home country) jumps to the pre-policy cost minus the additional costs causing inefficiency prior to the remittance policy affecting the quantity of savings and hence pushing the domestic market out of equilibrium. The supply and the demand of the savings, then adjust over time to bring the new post-policy market equilibrium. The interest rate adjustment mechanism is based on the fact that when the remittance-investment policy leads the market out of equilibrium, the buyers' and sellers' decisions are not coordinated at the current interest rate. It is essential to take into account the efficiency losses during the adjustment process while computing the benefits of remittance-investment policy. Eq. (37) gives an optimal remittance-investment policy over time which requires the same cost at any given point in time considering the adjustment of demand and supply of funds over time. The expression is a function of the slopes of the demand, supply (domestic as well as foreign) and the inventory curves, and the initial pre-policy equilibrium quantity of funds. In a static model, the government would only care about the efficiency in the equilibrium and hence choose a higher value of T, which would be a burden on the government exchequer.

6 Appendix:

6.1 Dynamic Problem of the Middleman/ Financial Intermediary

In a dynamic setting, the financial intermediary maximizes the present discounted value of the future stream of profits, and his present value at time zero is as follows:

$$V(0) = \int_0^{\infty} \left[rq(r) - \varsigma(m(r,e)) \right] e^{-\sigma t} dt, \tag{38}$$

σ denotes the discount rate. $r(t)$ is the control variable and $m(t)$ the state variable. The maximization problem can be written as

$$\underset{\{r(t)\}}{Max} V(0) = \int_0^{\infty} \left[rq(r) - \varsigma(m(r,e)) \right] e^{-\sigma t} dt,$$

subject to the constraints that
$\dot{m}(t) = m_1'(r(t), e(r(t),z))\dot{r}(t) + m_2'(r(t), e(r(t),z))e_1'(r(t),z)\dot{r}(t)$ (state equation, describing how the state variable changes with time; z are exogenous factors),
$m(0) = m_s$ (initial condition),
$m(t) \geq 0$ (non-negativity constraint on state variable),
$m(\infty)$ free (terminal condition).
The current-value Hamiltonian for this case is

$$\widetilde{H} = r(t)q(r(t)) - \varsigma(m(r(t), e(r(t),z))) + \mu(t)\dot{r}(t) \begin{bmatrix} m_1'(r(t), e(r(t),z)) + m_2'(r(t), e(r(t),z)) * \\ e_1'(r(t),z) \end{bmatrix}. \tag{39}$$

Now the maximizing conditions are as follows:
(i) $r^*(t)$ maximizes $\widetilde{H}$ for all t: $\frac{\partial \widetilde{H}}{\partial r} = 0$,
(ii) $\dot{\mu} - \sigma\mu = -\frac{\partial \widetilde{H}}{\partial m}$,
(iii) $\dot{m}^* = \frac{\partial \widetilde{H}}{\partial \mu}$ (this just gives back the state equation),
(iv) $\lim_{t \to \infty} \mu(t)m(t)e^{-\sigma t} = 0$ (the transversality condition).
The first two conditions are as follows:

$$
\begin{aligned}
\frac{\partial \widetilde{H}}{\partial r} &= q(r(t)) + r(t)q'(r(t)) - \varsigma'(m(r(t), e(r(t), z))) \left\{ \begin{array}{c} m_1'(r(t), e(r(t), z)) + m_2'(r(t), e(r(t), z)) * \\ e_1'(r(t), z) \end{array} \right\} \\
&+ \mu(t)\dot{r}(t) * \left[\begin{array}{c} m_{11}''(r(t), e(r(t), z)) + m_{12}''(r(t), e(r(t), z))e_1'(r(t), z) + \\ m_{21}''(r(t), e(r(t), z))e_1'(r(t), z) + m_{22}''(r(t), e(r(t), z))e_1'^2(r(t), z) + \\ m_2'(r(t), e(r(t), z))e_{11}''(r(t), z) \end{array} \right] \\
&= 0,
\end{aligned} \tag{40}
$$

and

$$
\dot{\mu} - \sigma\mu = -\frac{\partial \widetilde{H}}{\partial m} = \varsigma'(m(r(t), e(r(t), z))). \tag{41}
$$

When the market is in equilibrium, $\dot{r}(t) = 0$, and the expression $\frac{\partial \widetilde{H}}{\partial r}$ boils down to the following:

$$
\begin{aligned}
& q(r(t)) + r(t)q'(r(t)) - \varsigma'(m(r(t), e(r(t), z))) \left\{ \begin{array}{c} m_1'(r(t), e(r(t), z)) + m_2'(r(t), e(r(t), z)) * \\ e_1'(r(t), z) \end{array} \right\} \\
& = 0,
\end{aligned}
$$

$$
\begin{aligned}
r(t)q'(r(t)) + q(r(t)) &= \varsigma'(m(r(t), e(r(t), z))) \left\{ \begin{array}{c} m_1'(r(t), e(r(t), z)) + m_2'(r(t), e(r(t), z)) * \\ e_1'(r(t), z) \end{array} \right\}, \\
r(t)\left[1 + \frac{1}{\mathit{demand\ elasticity}}\right] &= \varsigma'(m(r(t), e(r(t), z))) \left\{ \frac{m_1'(r(t), e(r(t), z))}{q'(r(t))} + \frac{m_2'(r(t), e(r(t), z))e_1'(r(t), z)}{q'(r(t))} \right\},
\end{aligned}
$$

suggesting that the interest rate equals the marginal cost (the right hand side of the above expression is the marginal cost in a dynamic setting, which is different from that in a static problem on account of the fact that in a dynamic setting the middleman also takes into account the impact of interest rate chosen on his purchase interest rate from the producer) when the demand is infinitely elastic. Now suppose that as a result of a supply shock, if the middleman wants to hold an extra unit of inventory of funds, then the marginal cost of holding an extra unit of funds is higher because the term $\varsigma'(m(r(t), e(r(t), z)))$ is higher at the previous interest rate at that point in time. The term in parentheses in the expression of the marginal cost, i.e. $\frac{m_1'(r(t), e(r(t), z))}{q'(r(t))} + \frac{m_2'(r(t), e(r(t), z))e_1'(r(t), z)}{q'(r(t))}$ is a function of interest rate and is the same at the previous interest rate. This implies that on the previous interest rate, now the middleman faces

$$\frac{\partial \widetilde{H}}{\partial r} = q(r(t)) + r(t)q'(r(t)) - \varsigma'(m(r(t), e(r(t), z))) \left\{ \begin{array}{c} m'_1(r(t), e(r(t), z)) + m'_2(r(t), e(r(t), z))* \\ e'_1(r(t), z) \end{array} \right\}$$

$$+ \mu(t)\dot{r}(t) * \left[\begin{array}{c} m''_{11}(r(t), e(r(t), z)) + m''_{12}(r(t), e(r(t), z))e'_1(r(t), z) + \\ m''_{21}(r(t), e(r(t), z))e'_1(r(t), z) + m''_{22}(r(t), e(r(t), z))e'^2_1(r(t), z) + \\ m'_2(r(t), e(r(t), z))e''_{11}(r(t), z) \end{array} \right]$$

$$< 0.$$

Therefore in order to satisfy the condition of dynamic optimization, the middleman must decrease the interest rate for an increase in inventory of funds. This implies a negative relationship between interest rate and the inventory of funds. The concept of inventory unifies the market supply and demand. If the supply and demand rates are equal, the market is in a steady state equilibrium. If a difference of finite magnitude is created between the supply and demand rates and the consumer and the producer do not react to an interest rate change induced by a difference in the supply and demand rates, the interest rate will continue changing until the system saturates. This behavior can be depicted by the following formulation:

$$\textit{Interest rate change} \propto \textit{change in market inventory of funds.}$$

$$R = \textit{interst rate change.}$$

$$M = m - m_s = \textit{change in inventory of funds in the market,}$$

$$m = \textit{inventory at time } t,$$

$$m_s = \textit{inventory in steady state equilibrium.}$$

$$\textit{Input} - \textit{output} = \frac{dm}{dt} = \frac{d(m - m_s)}{dt} = \frac{dM}{dt},$$

$$\text{or } M = \int (\textit{input} - \textit{output})\, dt.$$

$$\textit{Interest rate change} \propto \int (\textit{supply rate} - \textit{demand rate})\, dt, \text{ or}$$

$$R = -K_m \int (\textit{supply rate} - \textit{demand rate})\, dt,$$

where K_m is the proportionality constant. A negative sign indicates that when ($\textit{supply rate} - \textit{demand rate}$) is positive, then R is negative (i.e. the interest rate decreases). The above equation can be rearranged as follows:

$$\int (\textit{supply rate} - \textit{demand rate})\, dt = -\frac{R}{K_m}, \text{ or}$$

$$\int (w_i - w_0)\, dt = -\frac{R}{K_m}, \tag{42}$$

$$w_i = \textit{supply rate},$$

$$w_0 = \textit{demand rate},$$

$$K_m = \textit{dimensional constant.}$$

Let at time $t = 0$, supply rate = demand rate (market is in a steady state equilibrium), then eq. (42) can be written as

$$\int (w_{is} - w_{0s})\, dt = 0. \tag{43}$$

The subscript s indicates the steady state equilibrium and $R = 0$ in steady state. Subtracting eq. (43) from eq. (42), we get:

$$\int (w_i - w_{is})\, dt - \int (w_0 - w_{0s})\, dt = -\frac{R}{K_m}, \text{ or}$$

$$\int (W_i - W_0)\, dt = -\frac{R}{K_m}, \tag{44}$$

$$\text{where } w_i - w_{is} = W_i = \textit{change in supply rate},$$

$$w_0 - w_{0s} = W_0 = \textit{change in demand rate.}$$

R, W_i and W_0 are deviation variables, which indicate deviation from the steady state equilibrium. The initial values of the deviation variables are zero. Eq. (44) may also be written as follows:

$$R = -K_m \int W dt = -K_m M, \tag{45}$$

where $W = W_i - W_0$. If R gets a jump as a result of some factor other than an inventory of funds change, that is considered as a separate input and can be added to eq. (45) as follows:

$$R = -K_m \int W dt + B = -K_m M + B. \tag{44a}$$

Similarly, there can be an exogenous shock in inventory of funds other than the interest rate feedback.

6.2 Solution of the Model with Remittance-Investment Policy

From eqs. ($11a$), (17) and (24) we have the following expressions:

$$
\begin{aligned}
\frac{dR(t)}{dt} &= -K_m W(t), \\
W_d(t) &= -K_d R(t), \\
W_m(t) &= -K_s \varepsilon(t), \\
\varepsilon(t) &= C(t) - R(t),
\end{aligned}
$$

and

$$W(t) = W_m(t) - W_d(t),$$

if there is no exogenous change in supply and demand. As $W_m(t)$ is the total supply in the domestic market, it includes both the domestic supply as well the foreign funds, and thus can be bifurcated as follows:

$$W_m(t) = -K_{sd}\left[C_d(t) - R(t)\right] - K_{se}\left[C_e(t) - R(t)\right], \tag{46}$$

where the subscripts d and e denote the domestic producer (of investment funds) and the exporter (foreign producer, i.e. the domestic earner in the foreign counrty who transfers funds for investment in the domestic country) in the foreign country respectively. From the above equations, we can write

$$
\begin{aligned}
\frac{dR(t)}{dt} &= -K_m\left[W_m(t) - W_d(t)\right] \\
&= -K_m\left[-K_{sd}\left\{C_d(t) - R(t)\right\} - K_{se}\left\{C_e(t) - R(t)\right\} + K_d R(t)\right] \\
&= -K_m\left[-K_{sd}C_d(t) - K_{se}C_e(t) + (K_{sd} + K_{se} + K_d)R(t)\right].
\end{aligned}
$$

The above expression can be rearranged as follows:

$$\frac{dR(t)}{dt} + K_m(K_{sd} + K_{se} + K_d)R(t) = K_m\left[K_{sd}C_d(t) + K_{se}C_e(t)\right]. \tag{47}$$

If $C_e(t) = f(T)$, i.e. a decreasing function of T (incentive for converting remittances to investment), and as a simple example, suppose that $C_e(t) = -T$, and $C_d(t) = 0$, i.e. the government's remittance-investment policy reduces the per unit cost on the flow of foreign funds by T at $t = 0$, then we can solve the above differential equation as follows:

$$\frac{dR(t)}{dt} + K_m(K_{sd} + K_{se} + K_d)R(t) = -K_m K_{se} T. \tag{48}$$

The characteristic function of the differential equation is as follows:

$$x + K_m(K_{sd} + K_{se} + K_d) = 0.$$

The characteristic function has a single root given by:

$$x = -K_m(K_{sd} + K_{se} + K_d).$$

Thus the complementary solution is

$$R_c(t) = C_2 e^{-[K_m(K_{sd}+K_{se}+K_d)]t}.$$

The particular solution has the form

$$R_p(t) = C_1.$$

Thus the solution has the form

$$R(t) = C_1 + C_2 e^{-[K_m(K_{sd}+K_{se}+K_d)]t}. \tag{49}$$

The constant C_1 is determined by substitution into the differential equation as follows:

$$-K_m(K_{sd} + K_{se} + K_d)C_2 e^{-[K_m(K_{sd}+K_{se}+K_d)]t} + K_m(K_{sd} + K_{se} + K_d)C_1$$

$$+K_m(K_{sd} + K_{se} + K_d)C_2 e^{-[K_m(K_{sd}+K_{se}+K_d)]t} = -K_m K_{se} T,$$

$$C_1 = \frac{-K_{se}T}{(K_{sd} + K_{se} + K_d)}.$$

C_2 is determined by the initial condition as follows:

$$R(0) = \frac{-K_{se}T}{(K_{sd} + K_{se} + K_d)} + C_2 = 0,$$
$$C_2 = \frac{K_{se}T}{(K_{sd} + K_{se} + K_d)}.$$

Substituting the values of C_1 and C_2 in eq. (49), we get:

$$R(t) = -\frac{K_{se}T}{(K_{sd} + K_{se} + K_d)} + \frac{K_{se}T}{(K_{sd} + K_{se} + K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}. \tag{50}$$

When $t = 0$, $R(0) = 0$ (the initial condition), and when $t = \infty$, $R(\infty) = \frac{-K_{se}T}{(K_{sd}+K_{se}+K_d)}$ (the final steady state equilibrium value). The interest rate dynamics as a result of remittance-investment policy depends on the parameters K_{sd}, K_{se}, K_d, K_m and T.

6.3 A Dynamic Optimal Remittance-Investment Policy

The efficiency gain as a result of a remittance-investment policy is not just the gain as a result of comparisons of the pre and post policy market equilibriums. The dynamic picture shows that there is some efficiency loss on the dynamic adjustment path to the new equilibrium after the introduction of the remittance-investment policy before the market arrives at the new efficient equilibrium (the equilibrium before the policy was inefficient because of additional costs to the exporter of funds, and which the government eliminates as a result of remittance-investment policy). After the policy gets implemented, the cost of the savings part of the remittances jumps to the previous cost minus the additional costs causing inefficiency. This affects the foreign savings quantity. The demand does not equal the supply any longer and the interest rate adjusts over time to bring the new equilibrium interest rate which is lower than the previous equilibrium rate by a magnitude depending on the elasticity of demand and supply schedules. By eq. (46), the change in total domestic supply as a result of remittance-investment policy is as follows:

$$W_m(0) = -K_{sd}\left[C_d(0) - R(0)\right] - K_{se}\left[C_e(0) - R(0)\right] = K_{se}T, \tag{51}$$

as $R(0) = 0$.

Therefore, at time zero, the inventory of funds with the financial intermediary goes up by $K_{se}T$ (as the demand remains the same). As the market gets out of equilibrium, the market forces come into play and the market interest rate starts changing, affecting the inventory of funds through the feedback. A pile up of inventory of funds indicates a higher supply than demand, and a depletion of inventory of funds occurs when demand is higher than the supply in a given time period. When the demand and supply are the same, there is no efficiency loss. If the demand and supply are different, the output and/ or consumption of funds is being lost at that point in time. Therefore if we sum up the supply and demand difference at all points in time, we get the total efficiency loss during the adjustment process, As we also need to take into account the efficiency losses before the remittance-investment policy took place, which is the difference of the pre-policy and the post-policy equilibrium quantity of funds, the total efficiency loss is as follows:

$$\begin{aligned} EL &= \int_{-\infty}^{0} W_m(\infty)dt + \int_{0}^{\infty} \left[W_m(t) - W_d(t)\right]dt \\ &= \int_{-\infty}^{0} W_m(\infty)dt + M(t). \end{aligned} \tag{52}$$

In traditional analysis, the measure of lost efficiency is the total surplus foregone which can be expressed as follows:

$$EL\ (Surplus) = \frac{1}{2}\left[\int_{-\infty}^{0} \{W_d(\infty) - W_d(0)\}\{R(\infty) + 2r_s\}\,dt - \int_{0}^{\infty} \{W_d(t) - W_d(\infty)\}\{r(t) - c(t)\}\,dt\right]$$

plus the sum of the consumer and the producer surplus in each time period wasted (which could have been earned by diverting resources to some other market) due to over production of funds during the adjustment process. (53)

In a dynamic setting, minimizing the efficiency loss in terms of quantity of funds is more tractable as compared to that in terms of surplus. However, minimizing the efficiency loss in terms of quantity is equivalent to that in terms of the surplus.

The increase in the final equilibrium quantity as compared to that in the initial equilibrium is the efficiency gain as a result of remittance-investment policy. From eq. (44a), we have

$$R(t) = -K_m M(t) + B.$$

The value of B can be found through the initial conditions as follows:

$$\begin{aligned} R(0) &= -K_m M(0) + B, \\ 0 &= -K_m K_{se} T + B, \\ B &= K_m K_{se} T. \end{aligned}$$

Substituting the value of B in eq. (44a), we get

$$\begin{aligned} R(t) &= -K_m M(t) + K_m K_{se} T, \text{ or} \\ M(t) &= -\frac{1}{K_m}\left[R(t) - K_m K_{se} T\right]. \end{aligned}$$

With Remittance-Investment Policy Cost Constraint:

From eq. (46), the change in supply due to a change in interest rate is as follows:

$$W_m(t) = -K_{sd}\left[C_d(t) - R(t)\right] - K_{se}\left[C_e(t) - R(t)\right].$$

The component of supply from the exporter in the foreign country for which remittance-investment policy is adopted is $-K_{se}\left[C_e(t) - R(t)\right]$, i.e.

$$W_{me}(t) = -K_{se}\left[C_e(t) - R(t)\right],$$

$$w_{nme}(t) - w_{ime}(0) = -K_{se}\left[C_e(t) - R(t)\right],$$

where $w_{ime}(0)$ is the initial foreign funds quantity and $w_{nme}(t)$ is the new foreign funds quantity after remittance-investment policy, because $W_{me}(t)$ is a deviation variable, i.e. deviation from the initial equilibrium value. Therefore the expression for the remittance-investment policy cost ($RIPC$) can be written as follows:

$$RIPC = g\left[T, \{w_{ime}(0) + K_{se}\{T + R(t)\}\}\right],$$

i.e. the remittance-investment policy cost is a function of the policy as well as the foreign funds quantity as a result of policy. As a simple example in order to ensure an analytic/ closed form solution, the remittance-investment policy cost may be represented as

$$RIPC = T\left[w_{ime}(0) + K_{se}\{T + R(t)\}\right]. \tag{54}$$

This could be a kind of a specific remittance subsidy as a remittance-investment policy in order to counter the additional costs casuing inefficiency during the flow of foreign funds. If we want to minimize the efficiency loss subject to the constraint that remittance-investment policy cost is less than or equal to G in a given time period, our problem is as follows:

$$\min_T EL \quad \text{s.t.} \quad RIPC \leq G.$$

G is the additional cost during the flow of funds causing inefficiency prior to the policy. The choice variable is the remittance-investment policy, and the constraint is binding in this particular example where $RIPC$ is given by eq. (54). The Lagrangian for the above problem is as follows:

$$\begin{aligned}
\mathcal{L} &= \int_{-\infty}^{0} W_m(\infty)dt + M(t) + \lambda\left[G - T\left[w_{ime}(0) + K_{se}\{T + R(t)\}\right]\right] \\
&= \int_{-\infty}^{0}\left[K_{se}T - \frac{K_{se}(K_{sd}+K_{se})T}{(K_{sd}+K_{se}+K_d)}\right]dt - \frac{1}{K_m}\left[\begin{array}{c} -\frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t} \\ -K_mK_{se}T \end{array}\right] \\
&\quad + \lambda\left[G - T\left[w_{ime}(0) + K_{se}\left\{T - \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}\right]\right] \\
&= \int_{-\infty}^{0}\frac{K_{se}K_dT}{(K_{sd}+K_{se}+K_d)}dt - \frac{1}{K_m}\left[-\frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t} - K_mK_{se}T\right] \\
&\quad + \lambda\left[G - T\left[w_{ime}(0) + K_{se}\left\{T - \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}\right]\right].
\end{aligned}$$

Taking the first order condition with respect to T, we get:

$$\int_{-\infty}^{0} \frac{K_{se}K_d}{(K_{sd}+K_{se}+K_d)}dt - \frac{1}{K_m}\left[-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t} - K_mK_{se}\right]$$
$$-\lambda\left[w_{ime}(0) + K_{se}\left\{T - \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}\right]$$

$$-\lambda TK_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right] = 0.$$

This implies that

$$\int_{-\infty}^{0} \frac{K_{se}K_d}{(K_{sd}+K_{se}+K_d)}dt - \frac{1}{K_m}\left[-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t} - K_mK_{se}\right]$$
$$-2\lambda TK_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right]$$

$$= \lambda w_{ime}(0),$$

or

$$T = -\frac{\lambda w_{ime}(0) - \left[\int_{-\infty}^{0} \frac{K_{se}K_d}{(K_{sd}+K_{se}+K_d)}dt - \frac{1}{K_m}\left[-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t} - K_mK_{se}\right]\right]}{2\lambda K_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right]}. \tag{55}$$

Taking the first order condition with respect to λ, we get:

$$G - T\left[w_{ime}(0) + K_{se}\left\{T - \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}\right] = 0. \tag{56}$$

Substituting the value of T from eq. (55) into (56), we get:

$$G =$$

$$
-w_{ime}(0).\frac{\lambda w_{ime}(0)-\left[\int\limits_{-\infty}^{0}\frac{K_{se}K_d}{(K_{sd}+K_{se}+K_d)}dt-\frac{1}{K_m}\left[-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)}+\frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}-K_mK_{se}\right]\right]}{2\lambda K_{se}\left[1-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)}+\frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right]}
$$

$$
+K_{se}\left\{1-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)}+\frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}
$$

$$
*\left[-\frac{\lambda w_{ime}(0)-\left[\int\limits_{-\infty}^{0}\frac{K_{se}K_d}{(K_{sd}+K_{se}+K_d)}dt-\frac{1}{K_m}\left[-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)}+\frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}-K_mK_{se}\right]\right]}{2\lambda K_{se}\left[1-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)}+\frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right]}\right]^2,
$$

$$
\text{or } 4\lambda^2QG=-2\lambda^2w_{ime}^2(0)+2\lambda w_{ime}(0)J+\lambda^2w_{ime}^2(0)+J^2-2\lambda w_{ime}(0)J,
$$

$$
\text{where } Q=K_{se}\left[1-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)}+\frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right],
$$

$$
J=\int\limits_{-\infty}^{0}\frac{K_{se}K_d}{(K_{sd}+K_{se}+K_d)}dt-\frac{1}{K_m}\left[-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)}+\frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}-K_mK_{se}\right].
$$

This implies that

$$
\left\{w_{ime}^2(0)+4QG\right\}\lambda^2-J^2=0.
$$

$$
\lambda=\frac{J}{\sqrt{w_{ime}^2(0)+4QG}}.
$$

λ has to be positive (since an increase in G causes the minimum efficiency loss to increase as well). Eq. (55) can also be written as

$$
T=-\frac{\lambda w_{ime}(0)-J}{2\lambda Q}. \tag{57}
$$

Substituting the value of λ into eq. (57), we get:

$$T = -\frac{\frac{w_{ime}(0)J}{\sqrt{w_{ime}^2(0)+4QG}} - J}{\frac{2QJ}{\sqrt{w_{ime}^2(0)+4QG}}},$$

$$T = -\frac{w_{ime}(0) - \sqrt{w_{ime}^2(0) + 4QG}}{2Q}. \tag{58}$$

In order to check the second order condition for minimization, we proceed as follows: The Lagrangian can be written as

$$\mathcal{L} = JT + \lambda \left[G - T\left(w_{ime}(0) + QT\right)\right].$$

The Bordered Hessian matrix of the Lagrange function is as follows:

$$BH = \begin{bmatrix} 0 & w_{ime}(0) + 2QT \\ w_{ime}(0) + 2QT & \frac{-2QJ}{\sqrt{w_{ime}^2(0)+4QG}} \end{bmatrix},$$

the determinant of which is negative as $-\left(w_{ime}(0) + 2QT\right)^2 < 0$, which implies that the efficiency loss is minimized.

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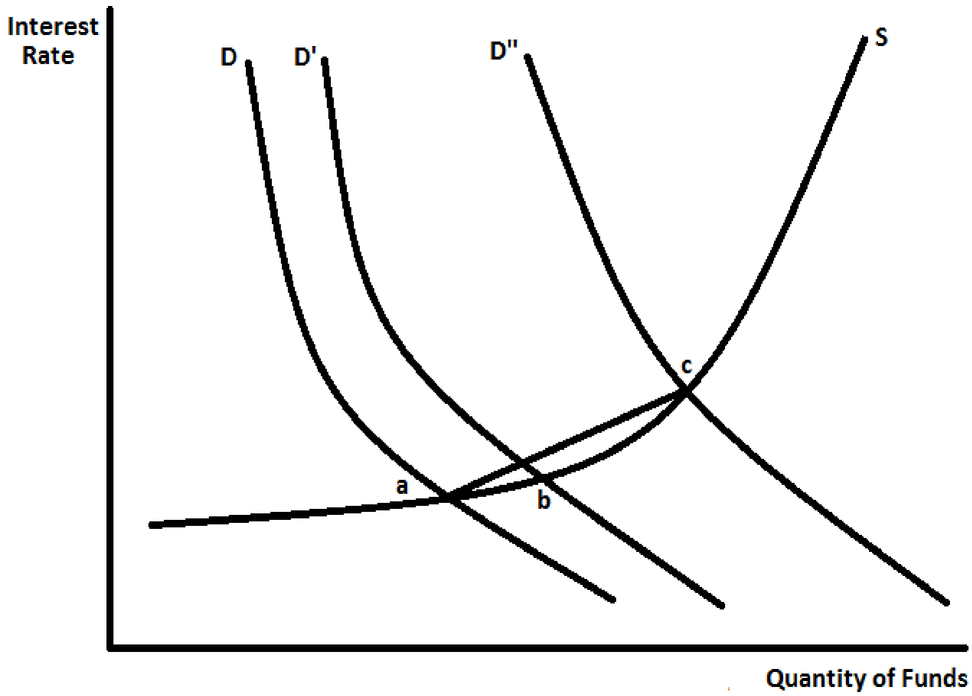

Figure 1: When is Linearity a Reasonable Assumption?

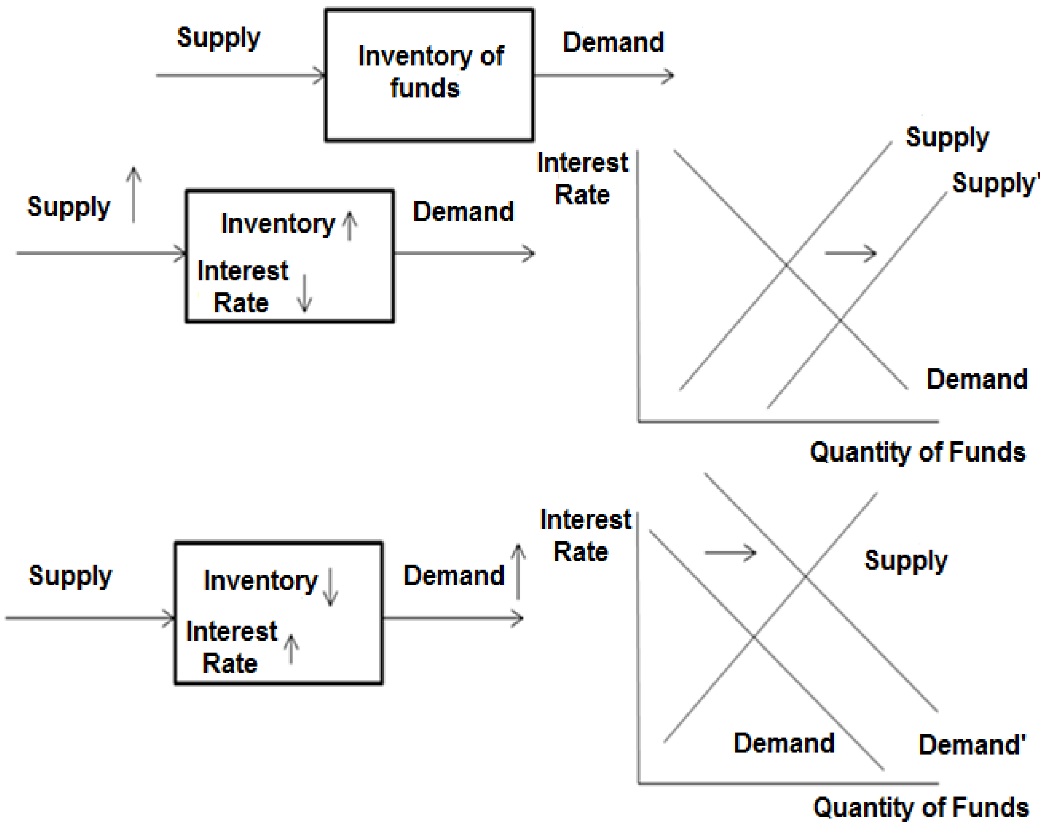

Figure 2: Movement of Interest Rate with Inventory of Funds.