How Does Competition Affect Incentives for Market Research?

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Abstract

We analyze firms' incentives to acquire information about market demand in a differentiated goods duopoly setting. We find two distinct benefits of having better information. Firstly, with better information, each firm can better match its price to demand. This benefit is decreasing in the level of market competition. Secondly, better information allows each firm to coordinate their prices with each other in different states, and each firm can make better use of its own information if the other firm acquires better information. This benefit is inverse u-shaped in the level of competition. Based on which effect dominates, each firm's total benefit from information can either be decreasing, or inverse u-shaped in the level of competition. Given endogenous information acquisition decisions by firms, the effect of competition on consumer welfare is ambiguous.

Keywords: Information acquisition, Bertrand duopoly, signals, competition.

1 Introduction

Firms acquire information about demand in order to optimally set prices. In competitive settings, market research not only directly informs a firm about demand for its own

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good, but indirectly informs the firm about how its competitor will price in the face of uncertain demand. Firms will only perform market research to the extent that the returns from doing so exceed the costs, and these returns may vary with the level of differentiation between one firm's product and its competitor's product.

We explore this phenomenon in the context of a standard differentiated duopoly Bertrand model with uncertain linear demand, in the style of Vives (1984). Rather than assuming exogenous signals of the demand intercept, we instead allow firms to covertly choose the accuracy of their signals at some cost. We compare the level of market research in (symmetric) equilibrium across different levels of competition, as measured by how differentiated the goods are. We give sufficient conditions such that endogenous market research monotonically decreases in the level of competition, as well as sufficient conditions such that endogenous market research is non-monotonic in the level of competition.

In this model, fixing some exogenous level of market research, a firm optimally chooses price by setting an average price plus a linear function of its signal. The more accurate a firm's signal, the more it will condition its price on its signal. Its average price will not change with signal accuracy, fixing the other firm's behavior. As the goods become less differentiated, competition sharpens: both firms' prices will go down for any given signal, which lowers overall profits.

Fixing the level of competition, as one firm's accuracy increases, its expected profits increase through two channels. First, it is better able to match its price to demand. Second, it is better able to coordinate its price with the other firm. Fixing average prices, one firm would rather price high when the other firm prices high, and low when the other firm prices low. A more accurate signal of demand is also a more accurate signal of the other firm's price. Because of this, if either firm's accuracy exogenously increases, both firms will condition their prices more on their signals. Otherwise, they will price conservatively in order to coordinate better. At any level of differentiation (other than perfectly homogenous goods), profits for both firms increase when either firm's accuracy increases.

The size of the marginal return to increasing accuracy varies with the amount of competition and can be broken down into two effects, which we call the *competitive profit effect* and the *coordination effect*. Both of these effects are weighted by the sensitivity of the firm's price to its signal; prices compress towards marginal cost as competition increases, so that the accuracy of a signal becomes less important fixing the other firm's behavior. The competitive profit effect is that as goods become less differentiated, so that both firms not only set prices lower on average but also condition prices less on the state, the firm cannot improve profits as much by setting high prices when the state is high and low prices when the state is low. If a firm is a monopoly, it can better align its prices with the state by increasing the accuracy of its signal. However, when the firm is forced to price conservatively because of increased competition, it cannot fully take advantage of a more accurate signal to match its price to the state.

The coordination effect has two components in addition to the sensitivity of the firm's price to its signal: the *substitution effect* and the *competitor pricing effect*. The substitution effect is that as goods become less differentiated, demand for one firm's good is more sensitive to the difference between the firms' prices. It becomes more important for a firm to coordinate its price with the other firm's price. The competitor pricing effect moves in the other direction. As competition intensifies, the firm's competitor not only lowers its price after any signal, but also compresses those prices towards marginal cost. This makes it easier to coordinate pricing, since the firms' prices are close even if their signals are very different. In the extreme case of homogenous goods, prices equal marginal cost and the competitor pricing effect is zero. At the other extreme, when goods are completely differentiated and firms function as monopolies, the substitution effect is zero. The total coordination effect is inverted U-shaped, so that it is highest at some intermediate level of competition.

We examine the competitive profit and coordination effects together. Marginally increasing accuracy always helps firms match the state better and coordinate better. However, the amount that it allows one firm to better coordinate with the other depends on the other's accuracy level. When both firms have very low accuracy, one firm marginally increasing its accuracy does not help it coordinate much with the other firm, whose price is not very correlated with demand. When both firms have high accuracy, one firm increasing its accuracy also allows it to better coordinate its price with the other firm. Thus, the relative importance of the competitive profit effect and the coordination effect depends on accuracy levels. We show that the competitive profit effect dominates when research costs are sufficiently high, so that equilibrium market research is monotonically decreasing in the level of competition. We also show that when research costs are sufficiently low, the coordination effect is large enough that equilibrium research is highest at an intermediate level of competition.

This paper is related to a wider literature on market research and competition. Building on the differentiated duopoly models of Singh & Vives (1984), Vives (1984) examines whether firms would prefer to commit to making their endogenous research public. He shows that firms prefer to pool their information in a Bertrand setting but not in a Cournot setting. Other models have endogenized market research, although they have tended to focus on Cournot rather than Bertrand competition, overt rather than covert research, and on different measures of competition than we do. For example, Hwang (1993) studies overt research in Cournot duopolies when goods are homogenous, but firms face different costs of acquiring information. Hwang (1995) also studies overt research in a Cournot setting with homogenous goods, but measures competition as the number of firms as well as a somewhat idiosyncratic "conjectural variation" model of competition. That paper finds a result qualitatively similar to ours: firms perform the least amount of research when competition is perfect, and perform the most amount of research either in an oligopoly or in a monopoly, depending on the parameters. Hauk & Hurkens (2001) study covert research in a Cournot setting, where competition is measured as the number of firms and goods are homogenous. Vives (1999) is an excellent overview of competition more broadly, and addresses some models of market research.

We utilize the central result of Persico (2000) in order to compare equilibrium market research at different levels of competition. That paper shows that when signals are ordered by accuracy, a concept first presented by Lehmann (1988), marginal returns to accuracy can be ranked according to a relatively straightforward single crossing condition. The paper then applies that ranking to compare information acquisition in first and second price auctions, building on the work of Milgrom and Weber (1982). To our knowledge, this is the first direct application of the theorem to a duopoly setting.

The paper shares some similarities to the literature on innovation, though in our setting market research hurts rather than helps consumers, since firms use the information to extract more surplus rather than to create better products.[1] Questions about the effects of competition on innovation have been raised and debated since seminal works by Schumpeter (1912, 1942). We do not address this debate, except to note that Aghion et al. (2005) find evidence of an inverted-U shape in equilibrium innovation that is qualitatively similar to our coordination effect. Goettler & Gordon (2014) also find an inverted-U shape between innovation and competition in their model of dynamic oligopoly with endogenous market structure.

The rest of the paper is organized as follows. Section 2 contains the model. Section 3 applies Persico's theorem to identify the two effects of competitiveness on returns to

[1] We address this further in Section 4.

market research and gives the main results. Section 4 concludes.

2 Model

We first describe the timing and payoffs and review the relevant results from Vives (1984). Two symmetric firms indexed by i each privately chooses a signal distribution indexed by $v_i \in [0, \infty)$ at differentiable cost $K(v_i)$. The state $\alpha \sim \mathcal{N}(\bar{\alpha}, V_\alpha)$ is realized. The cdf of this disribution, $G(\alpha)$, is commonly known to the firms when they choose v_i. Each firm receives a private signal realization $s_i = \alpha + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, v_i)$, and ϵ_1 and ϵ_2 are independent. Define $t_i = \frac{V_\alpha}{V_\alpha + v_i} \in (0, 1]$. Since for any V_α there is a one-to-one, continuous relationship between v_i and t_i, we consider firm i to be choosing t_i at cost $C(t_i)$. We assume that $C(t_i) \geq 0$ and $C'(t_i) \geq 0$.

We write the conditional distribution on α after seeing signal realization s_i as $G^{t_i}(\alpha|s_i)$. For a given α' and t_i we write the conditional distribution on all signals s_i as $F^{t_i}(s_i|\alpha')$. For a given t_i, we write the prior distribution on all signals s_i as $F^{t_i}(s_i)$.

After privately receiving signals, firms simultaneously set prices p_1 and p_2. Following Vives (1984), firm i faces the following linear inverse demand defined in the region of positive quantities:[2]

$$p_i = \alpha - q_i - \gamma q_{-i}.$$

Direct demand (for sufficiently low p_{-i}) is:[3]

$$q_i = \frac{\alpha}{1+\gamma} - \frac{1}{1-\gamma^2} p_i + \frac{\gamma}{1-\gamma^2} p_{-i}.$$

Goods are substitutes, i.e. $\gamma \in [0, 1)$.[4] The state α, the demand intercept, captures the level of demand, while increasing γ decreases the level of differentiation between firms. When $\gamma = 0$ the firms are monopolies, while as $\gamma \to 1$ demand approaches perfect competition. We normalize the cost of production to be 0 for simplicity. After privately observing a signal realization, each firm chooses its own price. Firm i earns

[2] This is a special case of Vives (1984) with β normalized to 1, so that $\gamma \in [0, 1]$ fully characterizes the level of substitutability across firms, and with independent signals to simplify the firm's choice of t.

[3] To avoid arbitrarily large demand, for values of $p_{-i} > \bar{p}_{-i}$, we set a (symmetric) upper bound on firm i's demand denoted by $q_\infty(p_i, \alpha, \gamma) = q_i(p_i, \bar{p}_{-i}, \alpha, \gamma)$, where $\bar{p}_{-i}$ is some suitably large value. This is merely for technical convenience and does not affect any of the results

[4] Direct demand is undefined at $\gamma = 1$, where profits are discontinuous in price.

profits $p_i q_i$.

We consider a Perfect Bayesian Equilibrium of this game, with firm i's equilibrium strategy written as $\{t_i^*, p_i^*(s_i|t_i)\}$ for any $s_i \in F^{t_i}(\cdot)$. In the Bertrand competition stage, firms maximize their expected profits given their conjecture of the other firm's pricing strategy as a function of their signal. Firm i's maximization problem after receiving signal s_i when their signal structure is indexed by t_i and the conjectured signal structure of firm $-i$ is indexed by t_i, is

$$\max_{p_i} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_i q_i(p_i, p_{-i}(s_{-i}), \alpha, \gamma) dF^{t_{-i}}(s_{-i}|\alpha) dG^{t_i}(\alpha|s_i).$$

Equilibrium prices are as in Vives (1984):

$$p_i^*(s_i|\gamma, t_i) = A + B_i t_i \left(\frac{s_i}{1+\gamma} - \frac{\bar{\alpha}}{1+\gamma} \right)$$

where

$$A = \frac{\bar{\alpha}(1-\gamma)}{2-\gamma}$$

$$B_i = \frac{(2+\gamma t_{-i})(1-\gamma^2)}{4-\gamma^2 t_1 t_2}.$$

Anticipating this, firm i chooses t_i to maximize $R(t_i) - C(t_i)$, with

$$R(t_i) \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_i^*(s_i|\gamma, t_i) q_i(p_i^*(s_i|\gamma, t_i), p_{-i}(s_{-i}|\gamma, t_{-i}), \alpha, \gamma) dF^{t_{-i}}(s_{-i}|\alpha) dF^{t_i}(s_i|\alpha) dG(\alpha).$$

We call this the *market research problem* and we call t_i firm i's *accuracy level*.

Following Persico (2000), let *asymmetric marginal revenue* $AMR_\gamma(t, t')$ be firm i's marginal returns from increasing t_i from $t_i = t$ when the level of differentiation is γ and firm $-i$ plays pricing strategy $p_{-i}^*(s_i|\gamma, t_{-i} = t', t_i = t')$, i.e. when firm $-i$ has accuracy level t' and prices as if firm i also has accuracy level t'. Define marginal revenue of accuracy at level of differentiation γ as as $MR_\gamma(t) \equiv AMR_\gamma(t, t)$. Define the marginal cost of accuracy as $MC(t) \equiv C'(t)$.

We focus on symmetric equilibria in which $t_i^* = t_{-i}^* = t^*(\gamma)$ and $p_i^*(s_i|\gamma, t^*(\gamma)) = p_{-i}^*(s_{-i}|\gamma, t^*(\gamma))$, $\forall s_i = s_{-i}$. At such an equilibrium it must be that $MR_\gamma(t^*(\gamma)) = MC(t^*(\gamma))$.

3 Returns to Market Research

This section contains the main results of the paper. We state the relevant result from Persico (2000) in the framework of our model. Without directly solving for marginal returns to accuracy, we are able to apply the result in order to rank marginal returns to accuracy across different levels of differentiation. We decompose relative marginal revenue from accuracy into two components, the competitive profits component and the coordination component. We then give two main results: (1) when the cost of accuracy, $C(\cdot)$, is sufficiently high, market research in the unique symmetric equilibrium is decreasing in the level of competition, and (2) when $C(\cdot)$ is sufficiently low, equilibrium market research is higher at an intermediate level of competition than in either the monopoly or perfect competition setting. Finally, we show that the second result extends to a setting in which the both firms' choice of accuracy is publicly observed.[5]

Let $u_\gamma(\alpha, p_i^*(s_i|\gamma, t_i, t_{-i})) \equiv \int\limits_{s_{-i}=-\infty}^{\infty} p_i^*(s_i) q_i(p_i^*, p_{-i}^*, \alpha, \gamma) dF^{t_{-i}}(s_{-i}|\alpha)$. When $t_1 = t_2 = t$, denote this as $u_\gamma(\alpha, p^*(s,t))$. Given two payoff functions $u_{\gamma'}(\alpha, p_i^*(s,t))$ and $u_{\gamma''}(\alpha, p_i^*(s,t))$, we write $u_{\gamma'} \succeq u_{\gamma''}$ if $u_{\gamma'} - u_{\gamma''}$ has the single-crossing property in (α, p); i.e. if $\frac{\partial u_{\gamma'}(\alpha,p)}{\partial p}$ crosses $\frac{\partial u_{\gamma''}(\alpha,p)}{\partial p}$ at most once, and from below, as α increases. We write $u_{\gamma'} \succ u_{\gamma''}$ if $u_{\gamma'} \succeq u_{\gamma''}$ and $u_{\gamma''} \not\succeq u_{\gamma'}$.

Lemma 1. *For γ' and γ'', if $u_{\gamma'}(\alpha, p^*(s,t)) \succ u_{\gamma''}(\alpha, p^*(s,t))$, then $MR_{\gamma'}(t) > MR_{\gamma''}(t)$.*

Proof. See Appendix 5.1. □

The lemma states that in order to compare the marginal returns of accuracy at two different competition levels, it suffices to show that their difference satisfies single-crossing.[6]

Note that $p_i^*(s_i)$ is non-decreasing in s_i. In order to show for a given pair γ',γ'' that $MR_{\gamma''}(t_i) > MR_{\gamma'}(t_i)$, it suffices to show that

$$\frac{\partial}{\partial s_i}\left[u_{\gamma''}(\alpha, p_i^*(s_i|\gamma'', t)) - u_{\gamma'}(\alpha, p_i^*(s_i|\gamma', t))\right]$$

is increasing in α. To that end, we first examine $\frac{\partial^2}{\partial s_i \partial\alpha}\left[u_\gamma(\alpha, p_i^*(s_i|\gamma, t_i, t_{-i}))\right]$ for fixed $\gamma \in [0,1)$, which satisfies the following equation 1.[7]

[5] We do not extend the first result to the public setting.
[6] See Persico (2000) for a detailed discussion.
[7] See Appendix 5.2 for a derivation of this equation. Arguments are suppressed for neatness.

$$\frac{\partial^2}{\partial\alpha\partial s_i}\left[u_\gamma(\alpha, p_i^*(s_i|, t_i))\right] = \left(\frac{\partial q_\infty}{\partial\alpha}\frac{\partial p_i^*}{\partial s_i}\right) + \left(\frac{\partial q_i}{\partial p_{-i}}\frac{\partial p_{-i}}{\partial s_{-i}}\frac{\partial p_i^*}{\partial s_i}\right). \tag{1}$$

where q_∞ denotes $q_i(p_i^*, p_{-i}^*(\infty), \alpha, \gamma)$.

From the equation we see that firm i's marginal return to accuracy has two components. Both components are weighted by the sensitivity of the firm's optimal price to their signal, and they are smaller if the firm's optimal price is not very sensitive to the signal.

The first component is the *competitive profit effect*, $CMP(t, \gamma) \equiv \frac{\partial q_\infty}{\partial\alpha}\frac{\partial p_i^*}{\partial s_i}$. This depends on the change in expected profit as the state changes evaluated when firm $-i$ sets its price at ∞. When the state increases, the quantity demanded at any price also increases. As the firm's accuracy increases, it is better able to tailor its price to the state, α. However, if the firms are very insensitive to their signal due to either low accuracy or high competition, then they cannot benefit as much from a high state. Even though this effect is evaluated when the competing firm chooses a fixed high price, we call it the "competitive" profit effect because it is dependent on the firm's ability to price high and condition its price on the state, which is determined by the level of competition.

The second component is the *coordination effect*, $CRD(t, \gamma) \equiv \frac{\partial q_i}{\partial p_{-i}}\frac{\partial p_{-i}}{\partial s_{-i}}\frac{\partial p_i^*}{\partial s_i}$. As the firm's accuracy increases, it not only learns more about the state, but also learns more about the other firm's pricing. It is able to better coordinate its pricing with the competing firm. Fixing an average price, the firm is better off pricing high when its competitor prices high, and low when its competitor prices low. The coordination effect measures this benefit.

The coordination effect has two components in addition to the sensitivity of firm i's price to its signal: the sensitivity of firm i's demand to firm $-i$'s price, $\frac{\partial q_i}{\partial p_{-i}}$, i.e. the substitution effect, and the sensitivity of firm $-i$'s price to its signal, $\frac{\partial p_{-i}}{\partial s_{-i}}$, i.e. the competitor pricing effect. The substitution effect reflects that if demand is more sensitive to firm $-i$'s price, it is more important that firm i prices accordingly. As goods become less differentiated, the quantity a firm sells is highly dependent on the difference between the two firms' prices. This is magnified by the competitor pricing effect. If firm $-i$'s price is more sensitive to its signal, then it is more important for firm i to coordinate signals with firm $-i$. A small difference in signals leads to a large difference in prices when firm $-i$'s price is very sensitive to its signal.

We now plug in equilibrium prices to Equation 1. For given accuracy levels t_i, t_{-i}, the equation is equivalent to

$$\frac{\partial^2 u_\gamma(\alpha, p_i^*(s_i|\gamma, t_i))}{\partial\alpha\partial s_i} = \frac{1}{1+\gamma}B_i t_i + B_i t_i \frac{\gamma}{1-\gamma^2}B_{-i}t_{-i}. \tag{2}$$

Recall that $B_i = \frac{(2+\gamma t_{-i})(1-\gamma^2)}{4-\gamma^2 t_1 t_2}$. Since we are interested in symmetric equilibrium, suppose $t_i = t_{-i} = t$, in which case $B_i = B_{-i}$. Then Equation 2 is equivalent to

$$\frac{\partial^2 u_\gamma(\alpha, p_i^*(s_i|\gamma, t))}{\partial\alpha\partial s_i} = \frac{1-\gamma}{2-\gamma t}t + \frac{(1-\gamma^2)\gamma}{(2-\gamma t)^2}t^2 \tag{3}$$

We can now examine how both CMP and CRD depend on the level of competition γ.

Proposition 1. *For any $t \in (0,1]$, the competitive profit effect $CMP(t,\gamma)$ is strictly decreasing in γ.*

Proof. For $t \in (0,1]$, $\frac{\partial}{\partial\gamma}\left[\frac{1-\gamma}{2-\gamma t}t\right] = \frac{-t(2-t)}{(2-\gamma t)^2} < 0$. □

As the environment becomes more competitive and firms price more aggressively, not only does the size of the pie effectively shrink, but the firms are less able to maximize their profits by tailoring prices to demand. The more a firm is forced to compete, the less it is able to condition its price on its signal and better match its price to the state. Accuracy becomes marginally less valuable.

Proposition 2. *For any $t \in (0,1]$, the coordination effect $CRD(t,\gamma)$ is single-peaked in γ, and $CRD(t,0) = \lim_{\gamma\to 1} CRD(t,\gamma) = 0$.*

Proof. First note that at $\gamma = 0$ and at $\gamma = 1$ the coordination effect is $\frac{(1-\gamma^2)\gamma}{(2-\gamma t)^2}t^2 = 0$. The derivative of the coordination effect w.r.t. γ is

$$\frac{\partial}{\partial\gamma}\left[\frac{(1-\gamma^2)\,\gamma}{(2-\gamma t)^2}t^2\right] = \frac{-6\gamma^2 + (\gamma^3+\gamma)t + 2}{(2-\gamma t)^3}t^2.$$

This is continuous, positive at $\gamma = 0$, and negative at $\gamma = 1$. Setting it equal to zero, there is only one real-valued solution in γ, which must be interior by the intermediate value theorem. It must be the global maximum in γ on $\gamma \in [0,1)$. □

Changes in competition change the relative size of the coordination effect in two ways. First, as γ increases, firm i's profits are more dependent on firm $-i$'s price.

Thus, it becomes more important to learn the state in order to learn more about firm $-i$'s price. Second, the size of this effect depends on how sensitive firm $-i$'s price is to the signal s_{-i}. Since these effects are multiplicative, the coordination effect is highest at intermediate levels of competition, where prices are sensitive enough to signals that coordinating prices requires high accuracy, and goods are similar enough that price coordination is important.

Examples of the competitive profit effect and the coordination effect as a function of γ are shown in Figure 1 for $t = 0.5$.

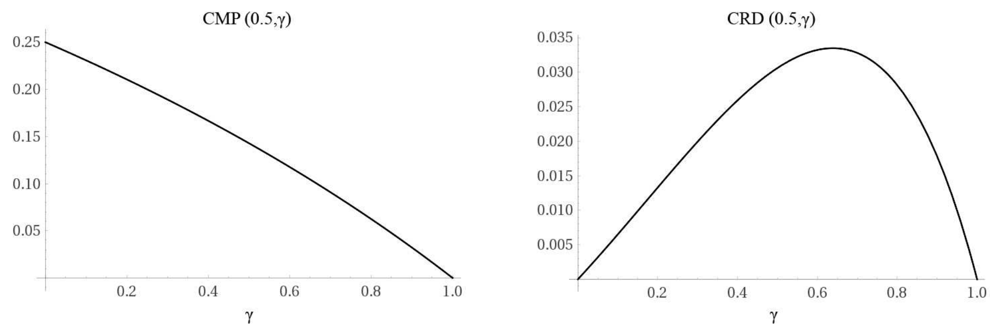

Figure 1: Competitive Profit Effect and Coordination Effect

Corollary 1. *For any* $t \in (0,1]$, $MR_0(t) > \lim_{\gamma \to 1} MR_\gamma(t)$.

The competitive profit effect is positive in the monopoly case, i.e. $\gamma = 0$, where firms' profits when they price optimally are very sensitive to the state. The coordination effect is 0 in the monopoly case, since one firm's price has no impact on the other firm's demand or optimal price. In the (almost) perfect competition case, i.e. as γ approaches 1, both the competitive profit effect and the coordination effect approach 0. Each firm's equilibrium price approaches marginal cost at all signals, so there are minimal returns to better information.

The change in the total effect across competition levels, $\frac{\partial}{\partial \gamma}\left[CMP(t,\gamma) + CRD(t,\gamma)\right]$, depends on the level of accuracy, t. If both firms' signals have low accuracy, then one firm getting better accuracy does not help coordination very much, but it does help that firm better match the state. Thus, when t is low enough, the competitive profit effect is relatively more important than the coordination effect. The marginal return to accuracy is monotonically decreasing in γ in that case. When t is high enough, the coordination effect dominates, so the marginal return to accuracy is no longer monotonically decreasing in the level of competition, but instead is highest at some intermediate level of competition. Examples of $CMP(t, \gamma) + CRD(t, \gamma)$ are shown in Figure 2 for $t = 0.5$ on the left and $t = 0.98$ on the right. In the right-hand graph, $CMP(t, \gamma) + CRD(t, \gamma)$ is maximized at an interior value of γ.

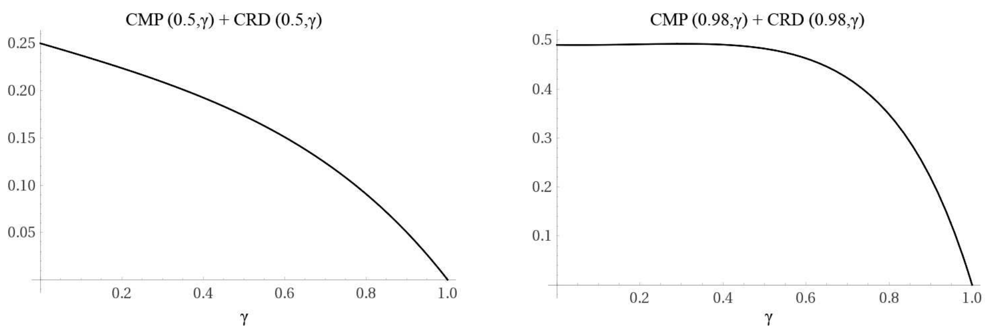

Figure 2: Total Effect at Low Accuracy and High Accuracy

The following lemmas formally state that the marginal return to accuracy is monotonically decreasing in γ when t is low, and that it is maximized at some interior γ when t is high.

Lemma 2. $\exists \bar{t}$ *such that for any* $t \in (0, \bar{t})$, $\frac{\partial}{\partial \gamma} [MR_\gamma(t)] < 0$.

Proof. See Appendix 5.3. □

Lemma 3. $\exists \underline{t}$ *such that for any* $t > \underline{t}$, $\exists \gamma' > 0$ *such that* $MR_{\gamma'}(t) > MR_0(t)$.[8]

Proof. See Appendix 5.4. □

We can compare equilibrium levels of market research across levels of competition as long as there exists a symmetric equilibrium in market research. For any pair (t, γ), both the competitive profit and coordination effects are weakly positive. This is true even if $t_i \neq t_{-i}$, as in Equation 2. It is immediate by inspection that for any tuple (γ, t_i, t_{-i}), $AMR_\gamma(t_i, t_{-t}) > 0$, i.e. firm i always benefits from more accuracy.[9]

This implies that we can find a cost function $C(t)$ such that when this is the cost of accuracy for both firms, at any γ there exists a unique symmetric equilibrium in accuracy $t^*(\gamma)$. Furthermore, we can find a cost function such that for some $\bar{t}$, $t^*(\gamma) \in (0, \bar{t})\ \forall \gamma \in [0, 1]$. Call such a cost function $C^{\bar{t}}(t)$. We can also find a cost function such that for some $\underline{t} > \bar{t}$, $t^*(\gamma) \in (\underline{t}, 1)\ \forall \gamma \in [0, 1]$. Call such a cost function $C_{\underline{t}}(t)$.

[8] The lowest such $\underline{t}$ is approximately 0.96778. Note that the notation is somewhat idiosyncratic in that the minimum $\underline{t}$ satisfying Lemma 3 is larger than the maximum $\bar{t}$ satisfying Lemma 2.

[9] $AMR_\gamma(t_i, t_{-t})$ approaches 0 as $\gamma \to 1$.

Finally, in order to state the main result we must formally define "higher costs" and "lower costs" of accuracy. For a given cost function $\hat{C}(t)$, let $\{\hat{C}(t)\}_L$ be the set of all cost functions $C(t)$ such that $\forall\gamma$ there exists a symmetric equilibrium, and $\forall t' \in [0,1]$, $C(t') \leq \hat{C}(t')$ and $C'(t') \leq \hat{C}'(t')$. Similarly, let $\{\hat{C}(t)\}_H$ be the set of all cost functions $C(t)$ such that $\forall\gamma$ there exists a symmetric equilibrium, and $\forall t' \in [0,1]$, $C(t') \geq \hat{C}(t')$ and $C'(t') \geq \hat{C}'(t')$.

Theorem 1. *There exist* $\{\bar{t}, \underline{t}\}$ *with* $1 > \underline{t} > \bar{t} > 0$ *such that:*
(1) $\exists C^{\bar{t}}(t)$ *such that for any cost function* $C(t) \in \{C^{\bar{t}}(t)\}_H$*, at every* $\gamma \in [0,1)$ *there exists a unique symmetric equilibrium with market research* $t^*(\gamma)$ *s.t.* $\frac{\partial}{\partial\gamma}[t^*(\gamma)] < 0$*, and*
(2) $\exists C_{\underline{t}}(t)$ *such that for any cost function* $C(t) \in \{C_{\underline{t}}(t)\}_L$*, at every* $\gamma \in [0,1)$ *there exists a unique symmetric equilibrium with market research* $t^*(\gamma)$ *s.t.* $t^*(\gamma') > t^*(0) > \lim_{\gamma\to 1} t^*(\gamma)$ *for some* $\gamma' \in (0,1)$*.*

Proof. By Lemma 1, single crossing is sufficient for ranking marginal returns to accuracy. Existence of symmetric equilibrium is immediate from Lemmas 2 and 3. $t^*(\gamma)$ is continuous in γ by the continuity of equilibrium prices and equilibrium payoffs in all arguments. For (1), by Lemma 2 there exist some $\bar{t}$ and $C^{\bar{t}}(t)$ such that $\frac{\partial}{\partial\gamma}[MR_\gamma(t)] \leq 0$, $\forall t \in [0,\bar{t}]$, $\forall\gamma$; and $t^*(\gamma) < \bar{t}$, $\forall\gamma$. This is true for all higher cost functions such that there exists a unique equilibrium at every γ. For (2), by Lemma 3 there exist some $\underline{t}$ and $C_{\underline{t}}(t)$ such that $\forall t > \underline{t}$, $\exists\gamma' \in (0,1)$ s.t. $MR_{\gamma'}(t) > MR_0(t)$ and $t^*(\gamma) > \underline{t}$,$\forall\gamma$. In particular, for $t^*(0)$, $\exists\gamma'$ s.t. $MR_{\gamma'}(t^*(0)) > MR_0(t^*(0))$. Therefore it must be that $t^*(\gamma') > t^*(0)$. This is true for all lower cost functions such that there exists a unique equilibrium at every γ. $\square$

The theorem states that, for cost functions such that there exists a unique equilibrium at all levels of competition, equilibrium private market research is decreasing in competition when accuracy costs are sufficiently high, and is maximized at some intermediate level of competition when accuracy costs are sufficiently low.

The second part of the result readily extends to the case of public market research. Suppose that after firms choose accuracy levels v_i and v_{-i}, both firms observe v_i and v_{-i} prior to choosing prices. The game is otherwise as in Section 2. Call this the overt game. In this setting, both accuracy and prices are strategic complements.[10] Thus, firms have weakly higher marginal returns to accuracy compared to the private

[10] See Chapter 8 in Vives (2000) for a more thorough discussion.

market research game. However, in the monopoly case there is no strategic effect from increasing accuracy, so marginal returns are the same in both settings. Let $t^*_O(\gamma)$ denote market research in a symmetric equilibrium of the overt market research game. For any cost function such that there exists a unique symmetric equilibrium in both the private research game and the overt game at some γ, it must be that $t^*_O(\gamma) \geq t^*(\gamma)$. In the monopoly case ($\gamma = 0$), $t^*_O(0) = t^*(0)$. Furthermore, returns to market research approach zero in both settings as competition approaches perfect competition: $\lim\limits_{\gamma \to 1} t^*_O(\gamma) = \lim\limits_{\gamma \to 1} t^*(\gamma) = 0$.

As in the private market research setting, in the overt game for any $\underline{t}$ one can find a cost function $C_{\underline{t}}(t)$ such that at every γ there exists a symmetric equilibrium in the overt game with $1 > t^*_O(\gamma) > \underline{t}$. Define $\{\hat{C}(t)\}^O_L$ in the overt game analogously to $\{\hat{C}(t)\}_L$ in the private market research game. Corollary 2 immediately follows.

Corollary 2. $\exists \underline{t}, C_{\underline{t}}(t)$ *such that for any cost function* $C(t) \in \{C_{\underline{t}}(t)\}^O_L$*, at every* $\gamma \in [0, 1)$ *there exists a unique symmetric equilibrium with market research* $t^*_O(\gamma)$ *s.t.* $t^*_O(\gamma') > t^*_O(0) > \lim\limits_{\gamma \to 1} t^*_O(\gamma)$ *for some* $\gamma' \in (0, 1)$*.*

As in the private market research game, in the overt game when accuracy costs are sufficiently low, firms facing some intermediate level of competition invest more in market research than monopolistic firms.

4 Conclusion

This paper examines how differentiation affects equilibrium market research in a Bertrand duopoly. We conjecture that in symmetric Bertrand oligopolies with $n > 2$ firms, the results hold qualitatively, meaning there exist parameters such that firms with partially differentated goods invest more in market research than firms with completely differentiated goods.

We do not explicitly analyze consumer welfare across differentiation levels, as to do so would require finding equilibrium market research levels in closed form, but we can say something about it. Increased accuracy has competing effects on consumer welfare. When firms increase their accuracy, they condition their prices more on their signals and thus better align their prices with the state. This is partially beneficial for consumers, since fixing the average price, they would prefer to pay a high price when the state is high and a low price when the state is low, rather than the same price in all states.

However, consumers also prefer for firms to have different prices from each other, as it allows them to substitute the cheaper good for the more expensive good. When firms increase their accuracy, their prices tend to be closer. This harms consumers. The net effect in our model is that consumer surplus decreases in the firms' accuracy.[11]

Fixing the accuracy of both firms, consumer welfare increases as goods become closer substitutes. However, as we have shown accuracy is sometimes non-monotonic in the level of differentiation. This highlights a challenge in regulating either market research or pricing behavior when market research is endogenous. For a given market research cost function, it may be that consumer welfare is sometimes higher when goods are less differentiated than when goods are more differentiated.

5 Appendix

5.1 Proof of Lemma 1

The result is a special case of Theorem 2 in Persico (2000). It suffices to show that the market research problem satisfies the assumptions of that Theorem.

First we show that for each firm i, signal s_i is *affiliated* with α. Two random variables S and A with joint density $f(s,\alpha)$ are affiliated if for any realizations $s^{'} > s$ and $\alpha^{'} > \alpha$, $f(s^{'},\alpha^{'})f(s,\alpha) \geq f(s,\alpha^{'})f(s^{'},\alpha)$.

Using the probability density functions of normal distributions with equal variance, for any two states $\alpha^{'} > \alpha$ and any two signal realizations $s^{'} > s$, we can see that

$$\frac{f(s^{'},\alpha^{'})}{f(s,\alpha^{'})} = \frac{\exp\left(-\frac{(s^{'}-\alpha^{'})^2}{2v}\right)}{\exp\left(-\frac{(s-\alpha^{'})^2}{2v}\right)} = \exp\left(\frac{(2\alpha^{'}-s^{'}-s)(s^{'}-s)}{2v}\right)$$
$$> \exp\left(\frac{(2\alpha-s^{'}-s)(s^{'}-s)}{2v}\right) = \frac{\exp\left(-\frac{(s^{'}-\alpha)^2}{2v}\right)}{\exp\left(-\frac{(s-\alpha)^2}{2v}\right)} = \frac{f(s^{'},\alpha)}{f(s,\alpha)}.$$

So by definition of affiliation, s_i is affiliated with α.

Given two signals S^{t_1} and S^{t_2}, we say that S^{t_1} is more *accurate* than S^{t_2} if $F^{t_1^{-1}}(F^{t_2}(s|\alpha)|\alpha)$ is nondecreasing in α, for every s; where $F^{t_1}(\cdot|\cdot)$ and $F^{t_2}(\cdot|\cdot)$ are cumulative distibution functions for S^{t_1} and S^{t_2}, respectively. (See Lehmann (1988).) For each firm i, the accuracy of its signal s_i is increasing in t_i. (See example 4 in Section 3.2 of Persico

[11]See Proposition 6 in Vives (1984).

(1996).)

By inspection, for each firm i, $u_i(\alpha, p_i) \equiv \int_{-\infty}^{\infty} p_i q_i(p_i, p_{-i}(s_{-i}), \alpha) dF^{t_{-i}}(s_{-i}|\alpha)$ is differentiable in p_i, and the optimal action $p_i^*(s_i, t_i)$ is differentiable in s_i and t_i.

Finally, the cdf of the normal distribution with variance v_i and state α is

$$F(x|\alpha, v_i) = \int_{-\infty}^{x} \left(\frac{1}{\sqrt{2\pi v_i}} \exp\left(-\frac{(z-\alpha)^2}{2v_i} \right) \right) dz,$$

which is differentiable with respect to v_i and continuous in α. Now, because $v_i = \frac{V_\alpha}{t_i} - V_\alpha$ is differentiable in t_i, it follows that $F(x|\alpha, v_i)$ is differentiable in t_i.

Thus, the conditions of Theorem 2 in Persico (2000) are satisfied in the market research problem.

5.2 Derivation of Equation 1

At a given signal s_i and with accuracy t_i, denote firm i's optimal price p_i^* as in Vives (1984). Define p_{-i}^* similarly.

$$u_\gamma(\alpha, p_i^*) = \int_{s_{-i}=-\infty}^{\infty} p_i^* q_i(p_i^*, p_{-i}^*, \alpha, \gamma) dF(s_{-i}|\alpha)$$

Integrating by parts:

$$\begin{aligned}
u_\gamma(\alpha, p_i^*) &= p_i^* \left\{ \Big[q_i(p_i^*, p_{-i}^*, \alpha, \gamma) F(s_{-i}|\alpha) \Big]_{s_{-i}=-\infty}^{\infty} - \int_{-\infty}^{\infty} \left(F(s_{-i}|\alpha) \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}^*}{\partial s_{-i}} \right) ds_{-i} \right\} \\
&= p_i^* \Bigg\{ \Big(q_i(p_i^*, p_{-i}^*(\infty), \alpha, \gamma) F(\infty|\alpha) - q_i(p_i^*, p_{-i}^*(-\infty), \alpha, \gamma) F(-\infty|\alpha) \Big) \\
&\quad - \int_{-\infty}^{\infty} \left(F(s_{-i}|\alpha) \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}^*}{\partial s_{-i}} \right) ds_{-i} \Bigg\} \\
&= p_i^* q_\infty - p_i^* \int_{-\infty}^{\infty} \left(F(s_{-i}|\alpha) \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}^*}{\partial s_{-i}} \right) ds_{-i}
\end{aligned}$$

Where q_∞ denotes $q_i(p_i^*, p_{-i}^*(\infty), \alpha, \gamma)$. We take the derivative with respect to s_i. Note that when pricing functions are as in the equilibrium of Vives (1984), both $\frac{\partial q_i}{\partial p_{-i}}$ and

$\frac{\partial p_{-i}^*}{\partial s_{-i}}$ are independent of s_{-i}.

$$\frac{\partial u_\gamma(\alpha, p_i^*)}{\partial s_i} = \left(q_\infty + p_i^* \frac{\partial q_\infty}{\partial p_i}\right) \frac{\partial p_i^*}{\partial s_i} - \left\{ \frac{\partial p_i^*}{\partial s_i} \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}^*}{\partial s_{-i}} \int\limits_{-\infty}^{\infty} (F(s_{-i}|\alpha))\, ds_{-i} \right\}$$

We take the derivative with respect to α. Note that $\frac{\partial q_i(\infty)}{\partial p_i}$, p_i^*, and $\frac{\partial p_i^*}{\partial s_i}$ are independent of α, and that conditional on some realization α' of the state signals are normally distributed with mean α' and some variance that is independent of α. The derivative is

$$\begin{aligned}\frac{\partial^2 u_\gamma(\alpha, p_i^*)}{\partial\alpha\partial s_i} =& \frac{\partial q_\infty}{\partial\alpha} \frac{\partial p_i^*}{\partial s_i} - \left\{ \frac{\partial p_i^*}{\partial s_i} \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}}{\partial s_{-i}} \int\limits_{-\infty}^{\infty} (F_\alpha(s_{-i}|\alpha))\, ds_{-i} \right\} \\ =& \frac{\partial q_\infty}{\partial\alpha} \frac{\partial p_i^*}{\partial s_i} - \left\{ \frac{\partial p_i^*}{\partial s_i} \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}}{\partial s_{-i}} \int\limits_{-\infty}^{\infty} (-f(s_{-i}|\alpha))\, ds_{-i} \right\} \\ =& \frac{\partial q_\infty}{\partial\alpha} \frac{\partial p_i^*}{\partial s_i} + \left(\frac{\partial p_i^*}{\partial s_i} \frac{\partial q_i}{\partial p_{-i}} \frac{\partial p_{-i}}{\partial s_{-i}} \right).\end{aligned}$$

5.3 Proof of Lemma 2

By Lemma 1, it suffices to show that $\exists t'$ s.t. $\frac{\partial}{\partial\gamma}\left[CMP(t,\gamma) + CRD(t,\gamma)\right] < 0\ \forall t < t'$.

$$\begin{aligned}CMP(t,\gamma) + CRD(t,\gamma) &= \frac{1-\gamma}{2-\gamma t} t + \frac{(1-\gamma^2)\gamma}{(2-\gamma t)^2} t^2 \\ \frac{\partial}{\partial\gamma}\left[CMP(t,\gamma) + CRD(t,\gamma)\right] &= \frac{t\left(t^2\gamma^3 + t\left(4 + 2\gamma - 6\gamma^2\right) - 4\right)}{(2-t\gamma)^3} \\ \therefore \frac{\partial}{\partial\gamma}\left[CMP(t,\gamma) + CRD(t,\gamma)\right] < 0 &\Leftrightarrow \left(t^2\gamma^3 + t\left(4 + 2\gamma - 6\gamma^2\right) - 4\right) < 0.\end{aligned}$$

Suppose $t \leq \frac{1}{2}$. Then $t^2\gamma^3 + t\left(4 + 2\gamma - 6\gamma^2\right) - 4$ is maximized on the domain $0 \leq \gamma < 1$ at $\gamma = 0$. At $\gamma = 0$

$$t^2\gamma^3 + t\left(4 + 2\gamma - 6\gamma^2\right) - 4 = 4t - 4 < 0.$$

The result follows.

5.4 Proof of Lemma 3

$$CMP(t,\gamma) + CRD(t,\gamma) = \frac{1-\gamma}{2-\gamma t}t + \frac{(1-\gamma^2)\gamma}{(2-\gamma t)^2}t^2$$

At $\gamma = 0$, $CMP(t,0) + CRD(t,0) = \frac{t}{2}$. Since $\lim_{\gamma\to 1}[CMP(t,\gamma) + CRD(t,0)] = 0$ and both $CMP(t,\gamma)$ and $CRD(t,\gamma)$ are continuous, it must be that if there exists some γ s.t. $CMP(t,\gamma) + CRD(t,\gamma) > \frac{t}{2}$, then there exist two values of γ such that $CMP(t,\gamma)+CRD(t,\gamma) = \frac{t}{2}$. There are two solutions γ^* for $CMP(t,\gamma^*)+CRD(t,\gamma^*) = \frac{t}{2}$:

$$\gamma^* = \frac{1}{2} - \frac{t}{4} \pm \frac{\sqrt{t^3 - 4t^2 + 36t - 32}}{4\sqrt{t}}$$

If $t < 1$, the solutions are real-valued and interior exactly when

$$t^3 - 4t^2 + 36t - 32 \geq 0.$$

When $t = 1$, the smaller of the two solutions is not interior, but the higher solution is interior. The left hand side of this expression is increasing in t, strictly negative at $t = 0$ and strictly positive at $t = 1$. The results follow.

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