A Dynamic Optimal Trade Facilitation Policy

Nawaz, Nasreen

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Nasreen Nawaz*

Federal Board of Revenue

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Abstract

The existing literature regarding the impact of trade facilitation in terms of export volume, per capita GDP, etc., only considers the market equilibria (before and after trade facilitation) to compare and account for the efficiency gains. However, when a trade facilitation measure is introduced by a government, the cost of the foreign producer gets a downward jump to the extent of costs reduced due to trade facilitation policy. This affects the quantity of imports and the market gets out of equilibrium. The market supply and demand of the commodity for which the trade facilitation measure is adopted, gets adjusted over time until the post-policy equilibrium arrives. The mechanism regarding the adjustment of price is based on the lack of coordination among buyers and sellers at the exisiting prices. Efficiency losses on the adjustment path are not taken into consideration when the efficiency gains of trade facilitation are computed in the exisiting literature. In this article, an optimal trade facilitation policy subject to a cost constraint has been derived, which minimizes the efficiency losses on the adjustment path, while the gains in the equilibrium, from the trade facilitation policy are accounted for. (JEL F10, F11, F13, H20, H21, H30, H540)

Keywords: Trade Facilitation Policy, Trade Facilitation Cost, Dynamic Efficiency, Price Adjustment Path, Equilibrium

*Email: nawaznas@msu.edu

1 Introduction

Trade facilitation refers to the improvement in efficiency of the customs administration and ports' logistics, and the simplification, harmonization, standardization, conformance to international regulations and modernization of trade procedures. A policy of trade facilitation is directed towards reduced complexity, time, and cost of the trade transactions and enhanced efficiency, transparency and predictability of the manner in which all the activities take place. It has an impact on the whole trade chain, i.e., exporter, importer, transportation and payment modes, with an emphasis on the crossing of border and the relevant agencies. There are some costs associated with trade facilitation. Costs incurred in introducing trade facilitation measures basically involve introducing new regulations; institutional changes; training; equipment; and infrastructure. It paves the way for the expansion of trade, which results in a wide variety of products for the consumers, a higher level of overall welfare, and a reduction in the market uncertainties and fluctuations. Measuring the efficiency gains as a result of trade facilitation is quite challenging.

The benefits of trade facilitation are hard to measure for business as well as for governments. According to a study by the United Nations Conference on Trade and Development (UNCTAD), the number of parties involved in a typical trade transaction is around 27 to 30, which includes banks, brokers, sureties, carriers, and freight forwarders. At the minimum, 40 documents are needed for government authorities as well as the related businesses. More than 200 data elements are normally asked for. Among those, around 60 to 70 per cent have to be re-keyed at least once, and 15 per cent have to be re-typed around 30 times.

According to a study by OECD, the estimated trade transaction costs fall in the range of 2 to 15 percent of the total value of a trade transaction, and the world trade is estimated to be 550 billion dollars per year. This is a huge waste of money and time, a hindrance in the smooth functioning of businesses, and hence economic growth and development, having a substantial impact in the developing economies.

All countries consider the simplified trade procedures based on the international standards and practices as extremely important. The landlocked countries have a lot of dependence on their neighbours' trade procedures, and hence are more vulnerable. The compliance cost of small and medium enterprises with complex trade procedures is higher, and hence their gains from trade facilitation are also proportionally higher.

The governments have to spend lots of resources on trade facilitation, therefore, in order to achieve the maximum economic gains, the resources must be spent efficiently, and that is an optimal trade facilitation policy. When a trade facilitation measure is implemented, the market adjusts during the transition period and arrives at a new equilibrium. The efficiency implications during the transition period are generally ignored, as no theoretical framework is available for this kind of treatment. The governments put in efforts for streamlined trade procedures, and enhanced quality of services and infrastructure, therefore, it is necessary to account for the economic efficiency losses during the transition before the new equilibrium, by the policy makers and administrators for policy formulation and monitoring respectively.

The existing literature regarding the impact of trade facilitation in terms of export volume, per capita GDP, etc., only considers the market equilibria (before and after trade facilitation) to compare and account for the efficiency gains. However, when a trade facilitation measure is introduced by a government, the cost of the foreign producer gets a downward jump to the extent of costs reduced due to trade facilitation policy. This affects the quantity of imports and the market gets out of equilibrium. The market supply and demand of the commodity for which the trade facilitation measure is adopted, gets adjusted over time until the post-policy equilibrium arrives. The mechanism regarding the adjustment of price is based on the lack of coordination among buyers and sellers at the exisiting prices. Efficiency losses on the adjustment path are not taken into consideration when the efficiency gains of trade facilitation are computed in the exisiting literature. In this article, an optimal trade facilitation policy subject to a cost constraint has been derived, which minimizes the efficiency losses on the adjustment path, while the gains in the equilibrium, from the trade facilitation policy are accounted for.

The remainder of this paper is organized as follows: Section 2 provides the literature review. Section 3 presents some stylized facts. Section 4 explains the theory: devises a dynamic model by introducing the individual problems of market agents; solves the model after introducing a trade facilitation measure, and an optimal trade facilitation policy has been derived which minimizes the efficiency losses subject to a trade facilitation cost constraint. In Section 5, a summary of findings and conclusion has been presented.

2 Literature Review

In Anderson (1979), a higher elasticity of substitution magnifies the effect of trade barriers on trade flows, even in the absence of increasing returns or monopolistic competition. Krugman (1980) predicts that a higher elasticity of substitution between goods magnifies the impact of trade barriers on trade flows. Rauch (1999) finds that trade barriers have a milder impact on trade volumes for goods that are more homogenous. Freund and Weinhold (2000) apply a gravity model to estimate the role of e-commerce in promoting bilateral trade. They find that a 10 percent increase in the relative number of web hosts in one country would have increased by one percent trade flows in 1998 and 1999. Obstfeld and Rogoff (2000) explain the six major puzzles in International Macroeconomics by the existence of trade barriers. Using a simple model, they illustrate how plausible values for trade barriers can have a large impact on trade flows depending on the magnification by the elasticity of substitution. Hertel, Walmsley and Itakura (2001) use computable general equilibrium analysis to quantify the impact on trade of greater standards harmonization for e-business and automating customs procedures between Japan and Singapore. They find these reforms will increase trade flows between these countries as well as their trade flows with the rest of the world.

Otsuki, Wilson and Sewadeh (2001)a and Otsuki, Wilson and Sewadeh (2001)b apply a gravity model to the case of food safety standards, finding that African export of cereals, nuts and dried fruits will decline by 4.3 (cereals) and 11 (nuts and dried fruits) percent with a 10 percent tighter EU standard on aflatoxin contamination levels of these products. Wilson, Mann and Otsuki (2003) analyze the relationship between trade facilitation and trade flows in the Asia-Pacific region. They estimate the causal correlation between trade facilitation and the per capita GDP. Wilson, Mann and Otsuki (2005) measure and estimate the relationship between trade facilitation and trade flows in manufactured goods in 2000-2001 in global trade, considering four important categories: port efficiency, customs environment, regulatory environment, and service sector infrastructure. Dennis and Shepherd (2007) show that lower entry costs for firms, and lower internal and external trade costs, are strongly and robustly associated with export diversification in developing countries. Sadikov (2007) shows how signatures required for exporting and business registration procedures affect the volume and composition of country's exports. Chaney (2008) finds that a higher elasticity makes the intensive margin more sensitive to changes in trade barriers, whereas it makes the extensive margin less sensitive. Martinez-Zarzoso and Márquez-Ramos (2008) analyse the effect of trade facilitation on sectoral trade flows. Persson (2013) tests whether trade facilitation affects the extensive margin by counting the number of 8-digit products that are exported from developing to EU countries, and using this as the dependent variable in an estimation. Moreover, she also tests whether the extensive margins in differentiated and homogeneous goods are affected in the same way by transaction costs. **Zaki (2015) evaluates the effect of different aspects of trade facilitation in developed and developing countries on bilateral trade through an augmented gravity model and uses the latter to estimate ad valorem equivalents (AVEs) of administrative barriers to trade. Zaki (2017) compares the cost and benefit of removing administrative barriers.**

3 Stylized Facts

From the previous empirical studies (Kleitz and Directorate (2002)), some interesting numbers are as follows:

- The Compliance costs have been estimated at 1.5%, in intra-EC trade (Cecchini);

- One day of shipping time saved can save a cost equivalent to a tariff reduction of 0.8% (Hummels);

- The price of the traded goods can be lowered by 0.2% through Customs automation (Mitsubishi Research Institute).

The overall estimates have been provided by a few studies in contrast, e.g.,

- **The transaction costs for the world trade have been estimated as falling in the rage**

of $7-10\%$ of the value of the world trade-the basis remains unclear though; customs compliance costs would be somewhat less.

• The benefits of the trade facilitation have been estimated at $1-5\%$ of the total world trade value (1994 Coumbus Ministerial Declaration on Trade Efficiency cited the figure of 2.5%).

Some research work done recently has raised doubts regarding these figures, and suggests that they might be on the higher side. The figures from various studies are based on which measures are taken into account, and the actual implementation practices. However, the overall conclusion about the importance of trade facilitation will not alter significantly irrespective of the estimates of the costs and the benefits of trade facilitation being on the higher or lower side. Even if the benefit of trade facilitation is estimated on the lower side, that is at or less than one percent of the world trade value, it could still have significant impacts on the world trade and the overall global welfare, due to the linkages of supply chain in the world economy.

4 Theory

4.1 Model Setup

The set up of the model is for a perfectly competitive market for a single good, which is initially in equilibrium. There are two types of producers, i.e., a domestic producer, and a foreign producer of the same identical good. There is a middleman in the market, who sells the product to the final consumer after buying it from the producers. The consumer consumes the good after buying it from the middleman, and the government formulates the trade facilitation policy. The producers are price takers, and when an exogenous shock hits the market, the middleman changes the price during the adjustment process drifting the market to the next equilibrium. As the demand and supply are not equal at all points in time, a stock or inventory of goods is held by the middleman, which he/ she purchases from the producers for subsequently selling those to the consumer. The producers have an objective of maximizing their profits; the middlemen also maximize their profits, whereas the consumer maximizes his/ her utility subject to their respective constraints.

The basis of price adjustment is a lack of coordination between buyer and seller at the exisiting prices. The working of this market can be illustrated by the following example: Consider a market in equilibrium. An equilibrium stock of inventory is held by the middleman. An exogenous supply shock increases the inventory stock, as the consumers' demand at the current price does not match with the supply by the producers. The middleman decreases the price in his/ her own benefit and the producers also find it optimal to produce a lower quantity than before. The next equilibrium with a higher quantity and a lower price is then reached. The equilibrium in this market is defined as follows:

(i) The middleman and both the producers maximize profits, and the consumer maximizes utility subject to their respective constraints.

(ii) When the market is in equilibrium, the inventory does not change as the quantity consumed by the consumer equals the quantity supplied by the producers.

Section 3 lists the conditions for the existence of an equilibrium in mathematical terms, i.e., the Routh–Hurwitz stability criterion as a necessary and sufficient condition for a stable dynamical linear system.

When the market is perfectly competitive, the middleman is also a price taker in equilibrium state of the market. When the market is adjusting, the middleman has an incentive to change the price. When the new equilibrium arrives, the middleman takes the price as given as far as the market remains in equilibrium. The government introduces a trade facilitation policy. When the policy gets implemented, the market does not instantaneously settle at the new equilibrium, and rather the price starts adjusting over time until the market is again in equilibrium. The adjustment of price is based on the endogenous decision making of all the market agents. Suppose a perishable good is produced by the producers in a market. They sell the product to the middleman, who subsequently sells it to the consumer in a community. The quantity bought by the consumer and the middleman equals the quantity produced by the producers, and the market is in equilibrium. If the cost of one of the producers decreases due to a favorable governmental policy, and enhances the supply in the market, some of the supply will not be sold out by the end of the time period, and hence will get wasted. We assume that there is no production friction, and the producers can change the production without delay. Similarly, there is no price rigidity in the market, and the middleman can change the price immediately after realization of the need to do so. If the middleman had perfect information about the new demand and supply pattern at various prices, he could easily have picked up the next equilibrium price, which would invite the equilibrium production by the producers, and hence the market would clear without any delay, however, this information is missing, and therefore, the middleman increases the price as per his/ her conjecture about the new supply and demand on the basis of the change in inventory. This drives the market toward the final equilibrium. The producers change their production according to their own objective. This phenomenon continues until the new equilibrium arrives after some efficiency loss, i.e., the output wasted during the adjustment of the market. There is some efficiency gain in the new equilibrium due to the trade facilitation measure. The net efficiency gain is the gain in the equilibrium minus the loss during the adjustment process.

For the illustration of the model in mathematical terms, the objective of each one of the market agents is maximized subject to the constraints, and the outcome equations are solved simultaneously in order to capture the collective impact of agents on the market. To justify the linearization of demand and supply curves, we assume that the post-policy equilibrium is not too off from the pre-policy equilibrium.

4.1.1 Middleman

The middleman sells the goods to the final consumer after purchasing those from the producers, and maintains an inventory as a difference of his purchases and sales. The difference of supply and demand in the market is reflected in the inventory, therefore the inventory is an important intermediary stage between supply and demand. When the supply and demand are the same, the inventory also remains the same in the market. However, when the inventory changes, it implies either supply, demand or both are changing at different rates.

The following explanation describes the connection between inventory, and the demand, supply and price. When supply in a market shifts to the right, while demand does not change, the inventory piles up due to excessive supply in the market, and the price goes down in the new equilibrium. Similarly, if the demand shifts to the right, while supply remains the same, the inventory level goes down in the market and the price goes up in the new equilibrium. Therefore, a change in price is inversely related to a change in inventory, ceteris peribus. If both demand and supply shocks shift the demand and supply curves in such a manner that there is no change in inventory, there will be no change in price either. The channel of both demand and supply shocks regarding influencing the market is the same, as they are both affecting inventory, and hence both kinds of shocks can be called as an, "inventory shock." Now let us discuss the mechanism of a price change with an inventory change, which are both inversely related. Let us assume a perfectly competitive market, where the inventories are held, and some costs incurred for mainitaining those by the middlemen. The cost and size of an inventory are positively related, i.e., it is more costly to hold a bigger inventory, and vice versa.

Now suppose that a positive supply shock happens in the market, which enhances the supply, while demand stays the same. There is a pile up in the market inventory due to excessive supply. The piled up inventories can be held either by the producers or the middlemen, however, the key point is that this scenario will not be sustainable in the long run, and hence there has to be some way out of this scenario. Assuming, that the piled up inventroies are with the middlemen, the middlemen will have to decrease the price to increase the demand along the demand curve in order to have a sustainable storage of inventories with them. The price will finally equal the new marginal cost, as the market is perfectly competitive, however, the adjustment path of the market will be determined by the response of the middleman regarding changing the price to bring the new equilibrium. Mathematically, the middleman's problem is described below:

Static/ Short Term Problem The middleman's short run (in discrete analog, one time period) problem is as follows:

$$\Pi = pq(p) - \varsigma(m(p,e)), \tag{1}$$

where
Π = profit,
p = market price,
$q(p)$ = quantity sold at price p,
m = inventory held by the *middleman*,
e = factors other than inventory which influence the market price,which also includes the purchase price of the middleman from the producer,
$\varsigma(m(p,e))$ = cost, increasing in inventory.
Taking the derivative of eq. (1), with respect to price gives:

$$pq'(p) + q(p) - \varsigma'(m(p,e))m_1'(p,e) = 0, \tag{2}$$

As the market is perfectly competitive, therefore, all the market agents have to be price takers when the market is in equilibrium, and the middleman has a benefit in changing the price only during the adjustment process. The middleman has no more incentive to change the price after the equilibrium is already arrived at, and rather will be losing business by deviating from the equilbrium price; whereas, when the market is adjusting, the demand and supply differ, and the price must be changed by an economic agent in his/ her own benefit to bridge the gap to bring the new equilibrium, therefore, a price change by the middleman on the adjustment path is in fact a market force, unlike when there is an equilibrium in a perfectly competitive market, and the demand is infinitely elastic as follows:

$$pq'(p) + q(p) = \varsigma'(m(p,e))m_1'(p,e),$$
$$p\left[1 + \frac{1}{demand\ elasticity}\right] = \varsigma'(m(p,e))\frac{m_1'(p,e)}{q'(p)}.$$

In the above expression, the price is equal to the marginal cost (the expression on the right hand side), when the elasticity of demand is infinite. Suppose, that a supply shock happens in the market as a result of which, the marginal cost of production decreases, and the supply shifts to the right. The supply will expand, no matter, whether the marginal cost reduction happens for the domestic supplier, the foreign supplier or both. As the supply and demand are no longer equal, the market is out of equilibrium. The supply and demand will adjust as a result of a price change in the market, however, price cannot jump on its own to bring the final equilibrium, and the middleman will realize about the supply shock after his/ her inventory begins piling up. Before that, he/ she will continue charging the previous price, which is higher than the exisiting marginal cost. After, the inventory starts building up, he/ she will reduce the price to maximize the profits. If the supply shock leads to a shrinkage in production, the price will go up in the new equilibrium. The middleman will not change the price, which is now lower than the new marginal cost until he/ she gets a signal of shrinkage in the production through his/ her depeleted inventory. In this scenario, the consumer will be a beneficiary regarding paying a price, which is now lower than the new marginal cost until the price increases in the final equilibrium. When the market arrives at the new equilibrium, no market agent is a beneficiary anymore, it is only during the adjustment process that the gains are reaped by some of them depending on a case by case basis.

For a mathematical demonstration, suppose, as a result of a positive supply shock, such as a reduced marginal cost as a result of some technological innovation, the marginal cost for the middleman to hold another unit in his/ her inventory, i.e., the term $\varsigma'(m(p,e))\frac{m_1'(p,e)}{q'(p)}$ is bigger at the existing price, on account of the fact that the term $\varsigma'(m(p,e))$ is higher at the existing price. The reason is intuitive: as the storage capacity approaches its potential, storing the goods become more and more costly due to enhanced demand of the storage houses, godowns, warehouses, etc. The term, $\frac{m_1'(p,e)}{q'(p)}$ is a function of price, and has not changed as the price is the same as before. This is due to the reason that the purchase price for the middleman has not changed yet, on account of the fact that the producer is a price taker throughout, i.e., when the market is adjusting as well as in equilibrium; and the middleman is charged a fixed fraction of the market price by the producer. A discrete analog can help clarify the above scenario as follows: the middleman maximizes his/ her profits in each time period, where they take the purchase price as given and only chooses the sale/ market price. The middleman does not take into account the profits in furture time periods as his/ her problem is myopic. At the exisiting price, the middleman now faces the following inequality

$$\frac{\partial \Pi}{\partial p} = pq'(p) + q(p) - \varsigma'(m(p,e))m_1'(p,e) < 0. \tag{3}$$

The above inequality implies that the middleman must reduce the price to have another unit of inventory and maximize profits after the supply shock. In this example, the short term benefits accrue to the producer, as the producer's marginal cost has decreased but he/ she keeps receiving the same price from the middleman until the middleman reduces the price. A plot of various profit maximizing combinations of prices versus inventories is a downward sloping inventory curve with price on the y-axis and the inventory on the x-axis. The concept is analogous to the *demand* and *supply curves* for the utility maximizing consumers and profit maximizing producers respectively.

Dynamic/ Long Term Problem In this sub-section, a dynamic/ long term problem of the middleman has been considered. The present discounted value of future stream of profits is maximized by the middleman, and the following expression represents his/ her present value at rime zero:

$$V(0) = \int_0^\infty \left[pq(p) - \varsigma(m(p,e))\right] e^{-rt} dt, \tag{4}$$

r is the discount rate. $p(t)$ is the *control variable* and $m(t)$ the *state variable*. The middleman's maximization problem is framed below:

$$\underset{\{p(t)\}}{Max} V(0) = \int_0^\infty \left[pq(p) - \varsigma(m(p,e))\right] e^{-rt} dt,$$

subject to the constraints that
$\dot{m}(t) = m_1'(p(t), e(p(t), z))\dot{p}(t) + m_2'(p(t), e(p(t), z))e_1'(p(t), z)\dot{p}(t)$ (state equation, which describes how the state variable changes with time; z being the exogenous factors),
$m(0) = m_s$ (initial condition),
$m(t) \geq 0$ (non-negativity constraint on state variable),
$m(\infty)$ free (terminal condition).
The current-value Hamiltonian is as given below:

$$\widetilde{H} = p(t)q(p(t)) - \varsigma(m(p(t), e(p(t), z))) + \mu(t)\dot{p}(t) \begin{bmatrix} m_1'(p(t), e(p(t), z)) + m_2'(p(t), e(p(t), z))* \\ e_1'(p(t), z) \end{bmatrix}. \tag{5}$$

The maximizing conditions are
(i) $p^*(t)$ maximizes $\widetilde{H}$ for all t: $\frac{\partial \widetilde{H}}{\partial p} = 0$,
(ii) $\dot{\mu} - r\mu = -\frac{\partial \widetilde{H}}{\partial m}$,
(iii) $\dot{m}^* = \frac{\partial \widetilde{H}}{\partial \mu}$ (this just gives back the state equation),
(iv) $\lim_{t \to \infty} \mu(t)m(t)e^{-rt} = 0$ (the transversality condition).
The following are the first two conditions:

$$\frac{\partial \widetilde{H}}{\partial p} = 0, \tag{6}$$

and

$$\dot{\mu} - r\mu = -\frac{\partial \widetilde{H}}{\partial m} = \varsigma'(m(p(t), e(p(t), z))). \tag{7}$$

In equilibrium, $\dot{p}(t) = 0$, and the expression $\frac{\partial \widetilde{H}}{\partial p}$ boils down to the following:

$$p(t)\left[1 + \frac{1}{demand\ elasticity}\right] = \varsigma'(m(p(t), e(p(t), z))) \left\{ \frac{m_1'(p(t), e(p(t), z))}{q'(p(t))} + \frac{m_2'(p(t), e(p(t), z))e_1'(p(t), z)}{q'(p(t))} \right\},$$

that is, the price equals the marginal cost for an infinitely elastic demand. The right hand side of the above expression is the marginal cost, and not the same as in the static/ short term problem, due to the fact that in the long run, the middleman also considers the market price effect on his future purchase price from the producer.
Now, in case the middleman would like to add another unit to inventory, his/ her marginal cost

to have an extra unit will be higher, as the term $\varsigma'(m(p(t), e(p(t), z)))$ will be higher at the existing price at that time. In the above marginal cost expression, the term in parentheses, i.e., $\frac{m_1'(p(t),e(p(t),z))}{q'(p(t))} + \frac{m_2'(p(t),e(p(t),z))e_1'(p(t),z)}{q'(p(t))}$ depends on price, therefore has the same value at the exisiting price. Now, the middleman faces the following:

$$\frac{\partial \widetilde{H}}{\partial p} < 0.$$

Therefore, for maximization of profit in a dynamic set up after some positive supply shock, the middleman must reduce the price to have an extra unit of inventory, and hence an inverse relationship between inventory and price. An inventory in a goods market is a unifying factor between supply and demand. When the market is in equilibrium, the inventory stays the same as the supply and demand rates are the same. If either the supply or the demand rate changes due to some external shock, and no other counter shock happens, the inventory, and the price will continue changing until the saturation point of the market arrives. The following formulation is a mathematical depiction of the above explanation:

$$\begin{aligned}
&\textit{Price change} \propto \textit{change in market inventory.}\\
&P = \textit{price change.}\\
&M = m - m_s = \textit{change in inventory in the market,}\\
&m = \textit{inventory at time } t\textit{,}\\
&m_s = \textit{inventory in steady state equilibrium.}\\
&\textit{Input} - \textit{output} = \frac{dm}{dt} = \frac{d(m-m_s)}{dt} = \frac{dM}{dt},\\
&\text{or } M = \int (\textit{input} - \textit{output})\, dt.\\
&\textit{Price change} \propto \int (\textit{supply rate} - \textit{demand rate})\, dt, \text{ or}\\
&P = -K_m \int (\textit{supply rate} - \textit{demand rate})\, dt,
\end{aligned}$$

where K_m is a positive proportionality constant. The multiplying negative sign suggests that when the difference of *supply rate*, and the *demand rate* is positive, P is negative (i.e., the price decreases). Rearranging the above expression gives:

$$\int (\textit{supply rate} - \textit{demand rate})\, dt = -\frac{P}{K_m}, \text{ or}$$

$$\int (w_i - w_0)\, dt = -\frac{P}{K_m}, \tag{8}$$

$$w_i = \textit{supply rate},$$

$$w_0 = \textit{demand rate},$$

$$K_m = \textit{dimensional constant.}$$

Suppose that our initial condition is the initial market equilibrium, i.e., at time $t = 0$, *supply rate* = *demand rate*, and eq. (8) becomes

$$\int (w_{is} - w_{0s})\, dt = 0. \tag{9}$$

The subscript s denotes the initial equilibrium (steady state), and $P = 0$, when the market is in equilibrium. Subtracting eq. (9) from eq. (8) gives:

$$\int (w_i - w_{is})\, dt - \int (w_0 - w_{0s})\, dt = -\frac{P}{K_m}, \text{ or}$$

$$\int (W_i - W_0)\, dt = -\frac{P}{K_m}, \tag{10}$$

$$\text{where } w_i - w_{is} = W_i = \textit{change in supply rate},$$

$$w_0 - w_{0s} = W_0 = \textit{change in demand rate.}$$

P, W_i and W_0 are the variables of deviation from the initial equilibrium, with zero initial values. Rearranging eq. (10), we get:

$$P = -K_m \int W\, dt = -K_m M, \tag{11}$$

where $W = W_i - W_0$. If price changes due to some factor other than inventory, an input can be added which modifies eq. (11) as follows:

$$P = -K_m \int W\, dt + B = -K_m M + B. \tag{11a}$$

There could also be some input or external shock, other than the price feedback for inventory. **Eq.** (11*a*) **depicts how the price change in the market is correlated with an inventory**

change. Inventory could change either because of a supply shock, a demand shock, or both. Therefore, in a goods market, the price responds to an inventory change which is a unifying factor between supply and demand. When demand equals the supply, the market is in equilibrium, and none of the market supply, demand, inventory and the price are changing. As soon as a shock happens, the inventory gets affected and has an impact on the market price.

4.1.2 Producers

There are two types of producers, i.e., a domestic producer and a foreign producer. Their objective is to maximize the profits in a dynamic setting. Furthermore, their objective is identical, and hence not considered separately. They maximize the present discounted value of future stream of profits, and the following expression represents their present value at rime zero:

$$V(0) = \int_0^\infty \left[\alpha p(t)F\left(K\left(t\right), L\left(t\right)\right) - w(t)L\left(t\right) - \Re(t)I(t)\right]e^{-rt}dt. \tag{12}$$

The fraction of the market price charged by the producers to the middleman is α. r represents the discount rate. $L(t)$ (labor) and $I(t)$ (level of investment) are the *control variables* and $K(t)$ the *state variable*. The producer's maximization problem is framed below:

$$\underset{\{L(t),I(t)\}}{Max} V(0) = \int_0^\infty \left[\alpha p(t)F\left(K\left(t\right), L\left(t\right)\right) - w(t)L\left(t\right) - \Re(t)I(t)\right]e^{-rt}dt,$$

subject to the constraints that
$\dot{K}(t) = I(t) - \delta K(t)$ (state equation, describing how the state variable changes with time),
$K(0) = K_0$ (initial condition),
$K(t) \geq 0$ (non-negativity constraint on state variable),
$K(\infty)$ free (terminal condition).
The current-value Hamiltonian for this case is

$$\widetilde{H} = \alpha p(t)F\left(K\left(t\right), L\left(t\right)\right) - w(t)L\left(t\right) - \Re(t)I(t) + \mu(t)\left[I(t) - \delta K(t)\right]. \tag{13}$$

The maximizing conditions are as follows:
(i) $L^*(t)$ and $I^*(t)$ maximize $\widetilde{H}$ for all t: $\frac{\partial \widetilde{H}}{\partial L} = 0$ and $\frac{\partial \widetilde{H}}{\partial I} = 0$,
(ii) $\dot{\mu} - r\mu = -\frac{\partial \widetilde{H}}{\partial K}$,
(iii) $\dot{K}^* = \frac{\partial \widetilde{H}}{\partial \mu}$ (this just gives back the state equation),
(iv) $\lim\limits_{t\to\infty} \mu(t)K(t)e^{-rt} = 0$ (the transversality condition).
The first two conditions are given below:

$$\frac{\partial \widetilde{H}}{\partial L} = \alpha p(t)F_2'\left(K\left(t\right), L\left(t\right)\right) - w(t) = 0, \tag{14}$$

$$\frac{\partial \widetilde{H}}{\partial I} = -\Re(t) + \mu(t) = 0, \tag{15}$$

and

$$\dot{\mu} - r\mu = -\frac{\partial \widetilde{H}}{\partial K} = -\left[\alpha p(t) F_1' \left(K\left(t\right), L\left(t\right)\right) - \delta\mu(t)\right]. \tag{16}$$

If we plug the value of $\dot{\mu}$ and μ from eq. (15) into eq. (16), we obtain:

$$\alpha p(t) F_1' \left(K\left(t\right), L\left(t\right)\right) - (r+\delta)\Re(t) + \dot{\Re}(t) = 0.$$

If there is a price increase, i.e., the value of $p(t)$ goes up, the producer faces the following inequalities at the exisiting level of investment and labor

$$\alpha p(t) F_2' \left(K\left(t\right), L\left(t\right)\right) - w(t) > 0,$$
$$\alpha p(t) F_1' \left(K\left(t\right), L\left(t\right)\right) - (r+\delta)\Re(t) + \dot{\Re}(t) > 0.$$

Therefore, for maximization of profit in a dynamic set up, the producer must increase the production after an increase in the price. Let p be the market price; c, a reference price, including the production cost, and the profits of the producers and the middlemen, determining the feasibility of the business, and with respect to which the variation in the market price is considered by the producers for enhancing or reducing the production levels.

$$W_m = \textit{Change in production as a result of a price change.}$$

A higher value of $(p-c)$ provides an incentive for a higher level of production to the producer, i.e.,

$$W_m \propto \alpha(p-c), \text{ or}$$

$$W_m = K_s(p-c). \tag{17}$$

In equilibrium, $W_m = 0$, or

$$0 = K_s(p_s - c_s). \tag{18}$$

K_s is a positive proportionality constant. p_s and c_s denote the initial equilibrium values. Subtracting eq. (18) from eq. (17) gives:

$$W_m = K_s\left[(p-p_s)-(c-c_s)\right] = -K_s\left(C-P\right) = -K_s\varepsilon, \tag{19}$$

where W_m, C and P are the variables of deviation from the initial steady state equilibrium.
Eq. (19) **depicts that the change in production by the producer is a positive function of the price change, i.e., as the price increases, the production by the producer increases. We are already well aware of this well known phenomenon in the form of a positively sloped supply curve in the existing literature. The only difference here is that this response has been derived through dynamic optimization by the producer.**

4.1.3 Consumer

The present discounted value of future stream of utilities is maximized by the consumer, and the following expression represents his/ her present value at rime zero:

$$V(0) = \int_0^\infty U(x(t))e^{-\rho t}dt, \tag{20}$$

ρ is the discount rate, and $x(t)$, the *control variable*. The consumer's maximization problem is framed below:

$$\underset{\{x(t)\}}{Max} V(0) = \int_0^\infty U(x(t))e^{-\rho t}dt,$$

subject to the constraints that
$\dot{a}(t) = R(t)a(t) + w(t) - p(t)x(t)$ (state equation, describing how the state variable changes with time). $a(t)$ is asset holdings (a *state variable*) and $w(t)$ and $R(t)$ are exogenous time path of wages and return on assets.
$a(0) = a_s$ (initial condition),
$a(t) \geq 0$ (non-negativity constraint on state variable),
$a(\infty)$ free (terminal condition).
The current-value Hamiltonian is

$$\widetilde{H} = U(x(t)) + \mu(t)\left[R\left(t\right)a\left(t\right) + w\left(t\right) - p\left(t\right)x\left(t\right)\right]. \tag{21}$$

The maximizing conditions are as given below:
(i) $x^*(t)$ maximizes $\widetilde{H}$ for all t: $\frac{\partial \widetilde{H}}{\partial x} = 0$,
(ii) $\dot{\mu} - \rho\mu = -\frac{\partial \widetilde{H}}{\partial a}$,
(iii) $\dot{a}^* = \frac{\partial \widetilde{H}}{\partial \mu}$ (this just gives back the state equation),
(iv) $\lim_{t\to\infty} \mu(t)a(t)e^{-\rho t} = 0$ (the transversality condition).
The first two conditions are as follows:

$$\frac{\partial \widetilde{H}}{\partial x} = U'(x(t)) - \mu(t)p(t) = 0, \tag{22}$$

and

$$\dot{\mu} - \rho\mu = -\frac{\partial \widetilde{H}}{\partial a} = -\mu(t)R(t). \tag{23}$$

If the price of good x increases, then the consumer faces the following inequality at the existing level of consumption:

$$\frac{\partial \widetilde{H}}{\partial x} = U'(x(t)) - \mu(t)p(t) < 0.$$

Therefore, for maximization of utility in a dynamic set up, the consumer must decrease the consumption after an increase in the price. Let the change in demand as a result of a price change, is proportional to P, i.e., a price change with respect to the initial equilibrium, i.e.,

$$\textit{Change in demand} \propto P, \text{ or}$$

$$W_d = -K_d P. \tag{24}$$

W_d and P are variables of deviation from the initial equilibrium, and when P is positive, W_d is negative.

Eq. (24) **depicts that the change in demand by the consumer is a negative function of the price change, i.e., as the price increases, the demand by the consumer decreases. We are already well aware of this well known phenomenon in the form of a negatively sloped demand curve in the existing literature. The only difference here is that this response has been derived through dynamic optimization by the consumer.**

To summarize section 3, the middleman, producers, and the consumer make their choices independently, and in their own self interest. Eqs. ($11a$), (19), **and** (24) **depict the behaviors of the market agents in mathematical terms. This system of equations represent the market system, and when solved simultaneously depicts the market response. The impact of the market response has been fedback into the responses of the market agents, which further impact the market, thus capturing the market's dynamic response at each point in time.**

4.2 Solving the Model

4.2.1 Solution of the Model with Trade Facilitation Policy

In the previous section, it has been explained how the market agents make their choices. This section provides the behavior of the market in the presence of a trade facilitation policy reflecting the collective outcome of the individual behaviors of the market agents.

The domestic market's total supply, i.e., imports as well as the domestic production is $W_m(t)$. We can bifurcate the domestic supply and the imports as follows:

$$W_m(t) = -K_{sd}\left[C_d(t) - P(t)\right] - K_{se}\left[C_e(t) - P(t)\right]. \tag{25}$$

The subscript d represents the domestic producer, and e denotes the exporter, i.e., the foreign producer. After solving the model, we get the following expression:

$$\frac{dP(t)}{dt} + K_m(K_{sd} + K_{se} + K_d)P(t) = K_m\left[K_{sd}C_d(t) + K_{se}C_e(t)\right]. \tag{26}$$

Let $C_e(t) = f(T)$**, i.e., a decreasing function of trade facilitation measure,** T**. As a specific example, let** $C_e(t) = -T$, **and** $C_d(t) = 0$. **This implies that the government's trade facilitation measure reduces the per unit cost on the imports by** T **at** $t = 0$**. A trade facilitation policy decreases the per unit cost of import. The quantum of a decrease in the per unit import cost does not have to be the same for all the units, and it can be a non-linear function of the trade facilitation policy. However, as the simplest case, a uniform per unit decrease in the import cost has been considered which is equivalent to a per unit import subsidy.**

Substituting the values of $C_e(t)$, and $C_d(t)$, the above differential equation can be written as:

$$\frac{dP(t)}{dt} + K_m(K_{sd} + K_{se} + K_d)P(t) = -K_m K_{se} T. \tag{27}$$

A necessary and sufficient stability condition for a dynamical (linear) system is given by Routh–Hurwitz, which is the following in this case: $K_m(K_{sd} + K_{se} + K_d) > 0$, which holds on account of the fact that K_m, K_{sd}, K_{se} and K_d all have been defined to be positive. This guarantees the existence of an equilibrium the domestic market settles at, after an economic shock. The above differential equation's solution can be written as:

$$P(t) = C_1 + C_2 e^{-[K_m(K_{sd}+K_{se}+K_d)]t}. \tag{28}$$

After we plug the values of C_1 and C_2 in eq. (28):

$$P(t) = -\frac{K_{se}T}{(K_{sd} + K_{se} + K_d)} + \frac{K_{se}T}{(K_{sd} + K_{se} + K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}. \tag{29}$$

The initial condition is that $P(0) = 0$, when $t = 0$. The final condition or the final equilibrium value is $P(\infty) = \frac{-K_{se}T}{(K_{sd}+K_{se}+K_d)}$, when $t = \infty$. The dynamics of price after a trade facilitation measure is adopted by the government depends on the values of the parameters K_{sd}, K_{se}, K_d, K_m and T. The demand must equal the supply in the final equilibrium, which holds. **In qualitative terms, eq. (29) refelcts the movement of price from initial equilibrium before the policy intervention to the new equilibrium after the policy intervention, provided that no other shock hits the market.**

4.2.2 A Dynamic Optimal Trade Facilitation Policy

Generally, when the efficiency gains are quantified in the economics literature, only the equilibrium states before and after a specific measure are compared. However, there are some efficiency losses during the adjustment of the market to the new equilibrium. When a trade facilitation measure is adopted by a government, the import cost decreases, and hence affects the quantity of imports. The demand and the supply in the domestic market adjust over time as a feedback of price adjustment, and the market settles at a new equilibrium. The total domestic supply in the market varies as a result of a trade facilitation measure, and the change in domestic supply by eq. (25) is given below:

$$W_m(0) = -K_{sd}\left[C_d(0) - P(0)\right] - K_{se}\left[C_e(0) - P(0)\right] = K_{se}T, \tag{30}$$

as $P(0) = 0$.

The supply in the market increases right after the trade facilitation measure, whereas the demand has not yet changed, therefore, the inventory gets a positive increase by $K_{se}T$ at time zero. Now the market is already out of equilibrium, and the adjustment process starts to drift the market toward a new equilibrium. The price adjusts and the inventory changes as a feedback of price change. The quantum of supply and demand determine the quantum of inventory. The inventory piles up if the supply is higher than the demand and vice versa. When the market is in equilibrium, the supply and demand are the same, and there are no efficiency losses. If the market is out of equilibrium, and the supply is different from demand, either the output and/ or consumption gets wasted. This waste is the efficiency loss. If this waste is summed up, the total efficiency loss during the adjustment process can be quantified. **The total efficiency losses, including the losses before trade facilitation and those during the adjustment process are given below:**

$$EL = \int_{-\infty}^{0} W_m(\infty)dt + \int_{0}^{\infty} [W_m(t) - W_d(t)]\, dt.$$
$$= \int_{-\infty}^{0} W_m(\infty)dt + M(t). \tag{31}$$

In terms of forgone surplus, the efficiency loss is given below:

$$EL\ (Surplus) = \frac{1}{2}\left[\int_{-\infty}^{0} \{W_d(\infty) - W_d(0)\}\{P(\infty) + 2p_s\}\, dt - \int_{0}^{\infty} \{W_d(t) - W_d(\infty)\}\{p(t) - c(t)\}\, dt\right]$$

plus the sum of the consumer and the producer surplus in each time period wasted (which could have been earned by diverting resources to some other market) due to over production during the adjustment process. (32)

Minimizing the efficiency loss either in terms of quantity or surplus leads to the same result in terms of an optimal trade facilitation measure. However, the solution to the problem of finding an optimal trade facilitation policy is more tractable when the efficiency losses in terms of quantity rather than the surplus are considered.

With Trade Facilitation Cost Constraint:

The trade facilitation cost (TFC) for the government is given below:

$$TFC = g\left[T, \{w_{ime}(0) + K_{se}\{T + P(t)\}\}\right].$$

It is a function of the trade facilitation measure, and the import quantity after the measure is implemented. In order to obtain an analytic/ closed form solution, the following expression as an example of the trade facilitation cost is considered:

$$TFC = T\left[w_{ime}(0) + K_{se}\{T + P(t)\}\right]. \tag{33}$$

A practical example is an import subsidy to counter the inefficiencies at the import stage by reducing the cost of import. Our problem of minimizing the efficiency losses subject to the trade facilitation cost constraint, i.e., the cost does not exceed a certain amount, G, in a specific time period is given below:

$$\min_T EL \quad \text{s.t.} \quad TFC \leq G.$$

G is the import cost to the importer in addition to his/ her due import costs as a source of inefficiency. The trade facilitation measure is the choice variable. When TFC is given by eq. (33), **the constraint is binding.** We can express the Lagrangian for this problem as follows:

$$\mathcal{L} = \int_{-\infty}^{0} \frac{K_{se}K_d T}{(K_{sd}+K_{se}+K_d)} dt - \frac{1}{K_m}\left[-\frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t} - K_m K_{se} T\right]$$
$$+ \lambda\left[G - T\left[w_{ime}(0) + K_{se}\left\{T - \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}\right]\right].$$

The first order condition with respect to T, is given below:

$$T = -\frac{\lambda w_{ime}(0) - \left[\int_{-\infty}^{0} \frac{K_{se}K_d}{(K_{sd}+K_{se}+K_d)} dt - \frac{1}{K_m}\left[-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t} - K_m K_{se}\right]\right]}{2\lambda K_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right]}. \tag{34}$$

The first order condition with respect to λ, is as follows:

$$G - T\left[w_{ime}(0) + K_{se}\left\{T - \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}\right] = 0. \tag{35}$$

After substituting the value of T in eq. (35) with the expression given by eq. (34), the following value for λ is obtained:

$$\lambda = \frac{J}{\sqrt{w_{ime}^2(0) + 4QG}}.$$

λ has to be positive, since the minimum efficiency loss increases with an increase in G.

$$Q = K_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right],$$
$$J = \int_{-\infty}^{0} \frac{K_{se}K_d}{(K_{sd}+K_{se}+K_d)} dt - \frac{1}{K_m}\left[-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t} - K_m K_{se}\right].$$

Eq. (34) can also be expressed as:

$$T = -\frac{\lambda w_{ime}(0) - J}{2\lambda Q}. \tag{36}$$

If we plug in the value of λ in the above expression, we obtain:

$$T = -\frac{w_{ime}(0) - \sqrt{w_{ime}^2(0) + 4QG}}{2Q}. \tag{37}$$

For minimization of the objective function, the second order condition is satisfied. Now let us consider a numerical example as follows: a government has resources to the amount of $1000 in terms of trade facilitation cost. The initial value for the quantity of the imported good is 100. The value of each parameter, i.e., K_m, K_{sd}, K_{se} and K_d is one. Plugging in these values in eq. (37), we obtain:

$$T = -\frac{100 - \sqrt{10000 + 4000}}{2} = 9.161,$$

where $Q = 1 - 0.333 + 0.333e^{-3t}$, and at $t = 0$, $Q = 1$. The trade facilitation cost is $TFC = T\left[w_{ime}(0) + QT\right] = 1000$. Therefore, subject to the cost constraint, the optimal trade facilitation policy for the government must be to provide a subsidy to the amount of $9.161 per unit. In contrast to this, in a static model which ignores the dynamic dimension of the problem, the government would only be taking into consideration the efficiency in the equilibrium. Based upon that, the government would decide to provide a sub-optimal subsidy.

Eq. (37) **provides an expression for an optimal trade facilitation policy. This expression has been derived by minimizing the efficiency losses on account of an inefficient equilibrium before policy, and on the dynamic adjustment path, after the implementation of the optimal trade facilitation policy. The numerical example suggests how an optimal trade facilitation policy could be chosen, given the parameter values in expression** (37).

5 Conclusion

When a trade facilitation measure is adopted by a government, the cost of the foreign producer gets a downward jump to the extent of costs reduced due to trade facilitation policy. This affects the quantity of imports and the market gets out of equilibrium. The market supply and demand of the commodity for which the trade facilitation measure is adopted, gets adjusted over time until the post-policy equilibrium arrives. In the existing literature, the efficiency losses during the time the market is adjusting are not accounted for, when the costs and benefits of a trade facilitation measure are computed. As during the adjustment process, the market is out of equilibrium and there are some efficiency losses, it is worthwhile to consider those losses while designing an optimal trade facilitation policy. An expression for an optimal trade facilitation policy, i.e., eq. (37) has been derived, which satisfies the cost constraint at all points in time while taking into consideration the adjustment of the supply and demand over time. An optimal trade facilitation policy is a function of the demand, supply and the inventory curves' slopes as well as the initial equilibrium quantity. In contrast to this, in a static model which ignores the dynamic dimension of the problem, the government would only be taking into consideration the efficiency in the equilibrium. Based upon that, the government would decide to provide an inefficient level of subsidy.

The main lesson, we derive from the model is that the trade facilitation policy has to be cost effective, i.e., in order to derive the maximum efficiency gain, with a minimal cost, we must account for the efficiency losses during the adjustment process of the market after the implementation of the policy. If the efficiency losses during the adjustment process are not accounted for, the policy cannot be an optimal trade facilitation policy.

6 Online Appendix:

6.1 Dynamic Problem of the Middleman

The present discounted value of future stream of profits is maximized by the middleman, and the following expression represents his/ her present value at rime zero:

$$V(0) = \int_0^\infty \left[pq(p) - \varsigma(m(p,e))\right] e^{-rt} dt, \tag{38}$$

r is the discount rate. $p(t)$ is the *control variable* and $m(t)$ the *state variable*. The middleman's maximization problem is framed below:

$$\underset{\{p(t)\}}{Max} V(0) = \int_0^\infty \left[pq(p) - \varsigma(m(p,e))\right] e^{-rt} dt,$$

subject to the constraints that
$\dot{m}(t) = m_1'(p(t), e(p(t), z))\dot{p}(t) + m_2'(p(t), e(p(t), z))e_1'(p(t), z)\dot{p}(t)$ (state equation, describing how the state variable changes with time; z are exogenous factors),
$m(0) = m_s$ (initial condition),
$m(t) \geq 0$ (non-negativity constraint on state variable),
$m(\infty)$ free (terminal condition).
The current-value Hamiltonian for this case is

$$\widetilde{H} = p(t)q(p(t)) - \varsigma(m(p(t), e(p(t), z))) + \mu(t)\dot{p}(t) \begin{bmatrix} m_1'(p(t), e(p(t), z)) + m_2'(p(t), e(p(t), z)) * \\ e_1'(p(t), z) \end{bmatrix}. \tag{39}$$

The maximizing conditions are
(i) $p^*(t)$ maximizes $\widetilde{H}$ for all t: $\frac{\partial \widetilde{H}}{\partial p} = 0$,

(ii) $\dot{\mu} - r\mu = -\frac{\partial \widetilde{H}}{\partial m}$,
(iii) $\dot{m}^* = \frac{\partial \widetilde{H}}{\partial \mu}$ (this just gives back the state equation),
(iv) $\lim_{t\to\infty} \mu(t)m(t)e^{-rt} = 0$ (the transversality condition).
The first two conditions are as follows:

$$\frac{\partial \widetilde{H}}{\partial p} = q(p(t)) + p(t)q'(p(t)) - \varsigma'(m(p(t), e(p(t), z))) \left\{ \begin{array}{c} m_1'(p(t), e(p(t), z)) + m_2'(p(t), e(p(t), z))* \\ e_1'(p(t), z) \end{array} \right\}$$
$$+ \mu(t)\dot{p}(t) * \left[\begin{array}{c} m_{11}''(p(t), e(p(t), z)) + m_{12}''(p(t), e(p(t), z))e_1'(p(t), z)+ \\ m_{21}''(p(t), e(p(t), z))e_1'(p(t), z) + m_{22}''(p(t), e(p(t), z))e_1'^2(p(t), z)+ \\ m_2'(p(t), e(p(t), z))e_{11}''(p(t), z) \end{array} \right]$$
$$= 0, \tag{40}$$

and

$$\dot{\mu} - r\mu = -\frac{\partial \widetilde{H}}{\partial m} = \varsigma'(m(p(t), e(p(t), z))). \tag{41}$$

In equilibrium, $\dot{p}(t) = 0$, and the expression $\frac{\partial \widetilde{H}}{\partial p}$ boils down to the following:

$$q(p(t)) + p(t)q'(p(t)) - \varsigma'(m(p(t), e(p(t), z))) \left\{ \begin{array}{c} m_1'(p(t), e(p(t), z)) + m_2'(p(t), e(p(t), z))* \\ e_1'(p(t), z) \end{array} \right\}$$
$$= 0,$$

$$p(t)q'(p(t)) + q(p(t)) = \varsigma'(m(p(t), e(p(t), z))) \left\{ \begin{array}{c} m_1'(p(t), e(p(t), z)) + m_2'(p(t), e(p(t), z))* \\ e_1'(p(t), z) \end{array} \right\},$$
$$p(t)\left[1 + \frac{1}{demand\ elasticity}\right] = \varsigma'(m(p(t), e(p(t), z))) \left\{ \frac{m_1'(p(t), e(p(t), z))}{q'(p(t))} + \frac{m_2'(p(t), e(p(t), z))e_1'(p(t), z)}{q'(p(t))} \right\},$$

that is, the price equals the marginal cost for an infinitely elastic demand. The right hand side of the above expression is the marginal cost, and not the same as in the static/ short term problem, due to the fact that in the long run, the middleman also considers the market price effect on his future purchase price from the producer.
Now, in case the middleman would like to add another unit to inventory, his/ her marginal cost to have an extra unit will be higher, as the term $\varsigma'(m(p(t), e(p(t), z)))$ will be higher at the ex-

isting price at that time. In the above marginal cost expression, the term in parentheses, i.e., $\frac{m_1'(p(t),e(p(t),z))}{q'(p(t))} + \frac{m_2'(p(t),e(p(t),z))e_1'(p(t),z)}{q'(p(t))}$ depends on price, therefore has the same value at the exisiting price. Now, the middleman faces the following:

$$\frac{\partial \widetilde{H}}{\partial p} = q(p(t)) + p(t)q'(p(t)) - \varsigma'(m(p(t),e(p(t),z))) \left\{ \begin{matrix} m_1'(p(t),e(p(t),z)) + m_2'(p(t),e(p(t),z)) * \\ e_1'(p(t),z) \end{matrix} \right\}$$

$$+ \mu(t)\dot{p}(t) * \left[\begin{matrix} m_{11}''(p(t),e(p(t),z)) + m_{12}''(p(t),e(p(t),z))e_1'(p(t),z) + \\ m_{21}''(p(t),e(p(t),z))e_1'(p(t),z) + m_{22}''(p(t),e(p(t),z))e_1'^2(p(t),z) + \\ m_2'(p(t),e(p(t),z))e_{11}''(p(t),z) \end{matrix} \right]$$

$$< 0.$$

Therefore, for maximization of profit in a dynamic set up after some positive supply shock, the middleman must reduce the price to have an extra unit of inventory, and hence an inverse relationship between inventory and price. An inventory in a goods market is a unifying factor between supply and demand. When the market is in equilibrium, the inventory stays the same as the supply and demand rates are the same. If either the supply or the demand rate changes due to some external shock, and no other counter shock happens, the inventory, and the price will continue changing until the saturation point of the market arrives. The following formulation is a mathematical depiction of the above explanation:

$$\textit{Price change} \propto \textit{change in market inventory.}$$

$$P = \textit{price change.}$$

$$M = m - m_s = \textit{change in inventory in the market,}$$

$$m = \textit{inventory at time } t,$$

$$m_s = \textit{inventory in steady state equilibrium.}$$

$$\textit{Input} - \textit{output} = \frac{dm}{dt} = \frac{d(m - m_s)}{dt} = \frac{dM}{dt},$$

$$\text{or } M = \int (\textit{input} - \textit{output})\, dt.$$

$$\textit{Price change} \propto \int (\textit{supply rate} - \textit{demand rate})\, dt, \text{ or}$$

$$P = -K_m \int (\textit{supply rate} - \textit{demand rate})\, dt,$$

where K_m is a positive proportionality constant. The multiplying negative sign suggests that when the difference of *supply rate*, and the *demand rate* is positive, P is negative (i.e., the price decreases).

Rearranging the above expression gives:

$$\int (\textit{supply rate} - \textit{demand rate})\, dt = -\frac{P}{K_m}, \text{ or}$$

$$\int (w_i - w_0)\, dt = -\frac{P}{K_m}, \tag{42}$$

$$w_i = \textit{supply rate},$$

$$w_0 = \textit{demand rate},$$

$$K_m = \textit{dimensional constant}.$$

Suppose that our initial condition is the initial market equilibrium, i.e., at time $t = 0$, *supply rate* = *demand rate*, and eq. (42) becomes

$$\int (w_{is} - w_{0s})\, dt = 0. \tag{43}$$

The subscript s denotes the initial equilibrium (steady state), and $P = 0$, when the market is in equilibrium. Subtracting eq. (43) from eq. (42) gives:

$$\int (w_i - w_{is})\, dt - \int (w_0 - w_{0s})\, dt = -\frac{P}{K_m}, \text{ or}$$

$$\int (W_i - W_0)\, dt = -\frac{P}{K_m}, \tag{44}$$

$$\text{where } w_i - w_{is} = W_i = \textit{change in supply rate},$$

$$w_0 - w_{0s} = W_0 = \textit{change in demand rate}.$$

P, W_i and W_0 are the variables of deviation from the initial equilibrium, with zero initial values. Eq. (44) can also be written as:

$$P = -K_m \int W\, dt = -K_m M, \tag{45}$$

where $W = W_i - W_0$. If price changes due to some factor other than inventory, an input can be added which modifes eq. (45) as follows:

$$P = -K_m \int W\, dt + B = -K_m M + B. \tag{44a}$$

There could also be some input or external shock, other than the price feedback for inventory.

6.2 Solution of the Model with Trade Facilitation Policy

From eqs. (11a), (19) and (24):

$$\frac{dP(t)}{dt} = -K_m W(t),$$
$$W_m(t) = -K_s \varepsilon(t),$$
$$\varepsilon(t) = C(t) - P(t),$$
$$W_d(t) = -K_d P(t),$$

and

$$W(t) = W_m(t) - W_d(t),$$

when there is no exogenous demand or supply shock. The domestic market's total supply, i.e., imports as well as the domestic production is $W_m(t)$. We can bifurcate the domestic supply and the imports as follows:

$$W_m(t) = -K_{sd}\left[C_d(t) - P(t)\right] - K_{se}\left[C_e(t) - P(t)\right], \tag{46}$$

The subscript d represents the domestic producer, and e denotes the exporter, i.e., the foreign producer. After solving the model, we get the following expression:

$$\begin{aligned}\frac{dP(t)}{dt} &= -K_m\left[W_m(t) - W_d(t)\right]\\ &= -K_m\left[-K_{sd}\left\{C_d(t) - P(t)\right\} - K_{se}\left\{C_e(t) - P(t)\right\} + K_d P(t)\right]\\ &= -K_m\left[-K_{sd}C_d(t) - K_{se}C_e(t) + (K_{sd} + K_{se} + K_d)P(t)\right].\end{aligned}$$

Rearranging the above expression gives:

$$\frac{dP(t)}{dt} + K_m(K_{sd} + K_{se} + K_d)P(t) = K_m\left[K_{sd}C_d(t) + K_{se}C_e(t)\right]. \tag{47}$$

If $C_e(t) = f(T)$, i.e. a decreasing function of T (trade facilitation measure), and as a simple example, suppose that $C_e(t) = -T$, and $C_d(t) = 0$, i.e. the government's trade facilitation measure reduces the per unit cost on the imports by T at $t = 0$, then the above differential equation can be written as:

$$\frac{dP(t)}{dt} + K_m(K_{sd} + K_{se} + K_d)P(t) = -K_m K_{se} T. \tag{48}$$

The characteristic function of the differential equation is as follows:

$$x + K_m(K_{sd} + K_{se} + K_d) = 0.$$

The single root is given by:

$$x = -K_m(K_{sd} + K_{se} + K_d).$$

The complementary solution is given by:

$$P_c(t) = C_2 e^{-[K_m(K_{sd}+K_{se}+K_d)]t}.$$

The form of the particular solution is as follows:

$$P_p(t) = C_1.$$

The solution, therefore, has the following form:

$$P(t) = C_1 + C_2 e^{-[K_m(K_{sd}+K_{se}+K_d)]t}. \tag{49}$$

By substituting the above expression into the differential equation, the value of the constant C_1 could be found as follows:

$$-K_m(K_{sd} + K_{se} + K_d)C_2 e^{-[K_m(K_{sd}+K_{se}+K_d)]t} + K_m(K_{sd} + K_{se} + K_d)C_1$$

$$+K_m(K_{sd} + K_{se} + K_d)C_2 e^{-[K_m(K_{sd}+K_{se}+K_d)]t} = -K_m K_{se} T,$$

$$C_1 = \frac{-K_{se}T}{(K_{sd} + K_{se} + K_d)}.$$

The initial condition, i.e., $P(0) = 0$, can help determine the value of C_2 as follows:

$$P(0) = \frac{-K_{se}T}{(K_{sd} + K_{se} + K_d)} + C_2 = 0,$$
$$C_2 = \frac{K_{se}T}{(K_{sd} + K_{se} + K_d)}.$$

After we plug the values of C_1 and C_2 in eq. (49):

$$P(t) = -\frac{K_{se}T}{(K_{sd} + K_{se} + K_d)} + \frac{K_{se}T}{(K_{sd} + K_{se} + K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t}. \tag{50}$$

The initial condition is that $P(0) = 0$, when $t = 0$. The final condition or the final equilibrium value is $P(\infty) = \frac{-K_{se}T}{(K_{sd}+K_{se}+K_d)}$, when $t = \infty$. The dynamics of price after a trade facilitation measure is adopted by the government depends on the values of the parameters K_{sd}, K_{se}, K_d, K_m and T.

6.3 A Dynamic Optimal Trade Facilitation Policy

Generally, when the efficiency gains are quantified in the economics literature, only the equilibrium states before and after a specific measure are compared. However, there are some efficiency losses when the market is adjusting and leading to the new equilibrium. When a trade facilitation measure is adopted by a government, the import cost decreases, and hence affects the quantity of imports. The demand and the supply in the domestic market adjust over time as a feedback of price adjustment, and the market settles at a new equilibrium. The total domestic supply in the market varies as a result of a trade facilitation measure, and the change in domestic supply by eq. (46) is given below:

$$W_m(0) = -K_{sd}\left[C_d(0) - P(0)\right] - K_{se}\left[C_e(0) - P(0)\right] = K_{se}T, \tag{51}$$

as $P(0) = 0$.

The supply in the market increases right after the trade facilitation measure, whereas the demand has not yet changed, therefore, the inventory gets a positive increase by $K_{se}T$ at time zero. Now the market is already out of equilibrium, and the adjustment process starts to drift the market toward a new equilibrium. The price adjusts and the inventory changes as a feedback of price change. The quantum of supply and demand determine the quantum of inventory. The inventory piles up if the supply is higher than the demand and vice versa. When the market is in equilibrium, the supply and demand are the same, and there are no efficiency losses. If the market is out of equilibrium, and the supply is different from demand, either the output and/ or consumption gets wasted. This waste is the efficiency loss. If this waste is summed up, the total efficiency loss during the adjustment process can be quantified. The total efficiency losses, including the losses before trade facilitation and those during the adjustment process are given below:

$$\begin{aligned} EL &= \int_{-\infty}^{0} W_m(\infty)dt + \int_{0}^{\infty} \left[W_m(t) - W_d(t)\right]dt \\ &= \int_{-\infty}^{0} W_m(\infty)dt + M(t). \end{aligned} \tag{52}$$

In terms of forgone surplus, the efficiency loss is given below:

$$EL\ (Surplus) = \frac{1}{2}\left[\int_{-\infty}^{0}\{W_d(\infty)-W_d(0)\}\{P(\infty)+2p_s\}\,dt-\int_{0}^{\infty}\{W_d(t)-W_d(\infty)\}\{p(t)-c(t)\}\,dt\right]$$

plus the sum of the consumer and the producer surplus in each time period wasted (which could have been earned by diverting resources to some other market) due to over production during the adjustment process. (53)

Minimizing the efficiency loss either in terms of quantity or surplus leads to the same result in terms of an optimal trade facilitation measure. However, the solution to the problem of finding an optimal trade facilitation policy is more tractable when the efficiency losses in terms of quantity rather than the surplus are considered.
The increase in the final equilibrium quantity as compared to that in the initial equilibrium is the efficiency gain as a result of trade facilitation measure. From eq. ($44a$), we have

$$P(t) = -K_m M(t) + B.$$

By imposing the initial conditions, the value of B can be found as follows:

$$\begin{aligned} P(0) &= -K_m M(0) + B, \\ 0 &= -K_m K_{se} T + B, \\ B &= K_m K_{se} T. \end{aligned}$$

Plugging the value of B in eq. ($44a$):

$$\begin{aligned} P(t) &= -K_m M(t) + K_m K_{se} T, \text{ or} \\ M(t) &= -\frac{1}{K_m}\left[P(t) - K_m K_{se} T\right]. \end{aligned}$$

With Trade Facilitation Cost Constraint:
The domestic supply change by eq. (46), is as follows:

$$W_m(t) = -K_{sd}\left[C_d(t) - P(t)\right] - K_{se}\left[C_e(t) - P(t)\right].$$

The component of supply from the exporter in the foreign country for which trade facilitation measure is adopted is $-K_{se}\left[C_e(t) - P(t)\right]$, i.e.,

$$W_{me}(t) = -K_{se}\left[C_e(t) - P(t)\right],$$

$$w_{nme}(t) - w_{ime}(0) = -K_{se}\left[C_e(t) - P(t)\right],$$

where $w_{ime}(0)$ is the initial import quantity and $w_{nme}(t)$ is the new import quantity after trade facilitation policy. The trade facilitation cost (TFC) for the government is given below:

$$TFC = g\left[T, \{w_{ime}(0) - K_{se}\{T - P(t)\}\}\right],$$

It is a function of the trade facilitation measure, and the import quantity after the measure is implemented. In order to obtain an analytic/ closed form solution, the following expression as an example of the trade facilitation cost is considered:

$$TFC = T\left[w_{ime}(0) - K_{se}\{-T - P(t)\}\right],$$

which implies that

$$TFC = T\left[w_{ime}(0) + K_{se}\{T + P(t)\}\right]. \tag{54}$$

A practical example is an import subsidy to counter the inefficiencies at the import stage by reducing the cost of import. Our problem of minimizing the efficiency losses subject to the trade facilitation cost constraint, i.e., the cost does not exceed a certain amount, G, in a specific time period is given below:

$$\min_{T} EL \quad \text{s.t.} \quad TFC \leq G.$$

G is the import cost to the importer in addition to his/ her due import costs as a source of inefficiency. The trade facilitation measure is the choice variable. When TFC is given by eq. (54), the constraint is binding. We can express the Lagrangian for this problem as follows:

$$\mathcal{L} = \int_{-\infty}^{0} W_m(\infty)dt + M(t) + \lambda\left[G - T\left[w_{ime}(0) + K_{se}\left\{T + P(t)\right\}\right]\right]$$

$$= \int_{-\infty}^{0} \left[K_{se}T - \frac{K_{se}\left(K_{sd}+K_{se}\right)T}{\left(K_{sd}+K_{se}+K_d\right)}\right]dt - \frac{1}{K_m}\left[\begin{array}{c} -\frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t} \\ -K_mK_{se}T \end{array}\right]$$

$$+ \lambda\left[G - T\left[w_{ime}(0) + K_{se}\left\{T - \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}\right]\right]$$

$$= \int_{-\infty}^{0} \frac{K_{se}K_dT}{(K_{sd}+K_{se}+K_d)}dt - \frac{1}{K_m}\left[-\frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t} - K_mK_{se}T\right]$$

$$+ \lambda\left[G - T\left[w_{ime}(0) + K_{se}\left\{T - \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}\right]\right].$$

The first order condition with respect to T, is given below:

$$\int_{-\infty}^{0} \frac{K_{se}K_d}{(K_{sd}+K_{se}+K_d)}dt - \frac{1}{K_m}\left[-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t} - K_mK_{se}\right]$$

$$-\lambda\left[w_{ime}(0) + K_{se}\left\{T - \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}\right]$$

$$-\lambda TK_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right] = 0.$$

This implies that

$$\int_{-\infty}^{0} \frac{K_{se}K_d}{(K_{sd}+K_{se}+K_d)}dt - \frac{1}{K_m}\left[-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t} - K_mK_{se}\right]$$

$$-2\lambda TK_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right]$$

$$= \lambda w_{ime}(0),$$

or

$$T = -\frac{\lambda w_{ime}(0) - \left[\int\limits_{-\infty}^{0} \frac{K_{se}K_d}{(K_{sd}+K_{se}+K_d)}dt - \frac{1}{K_m}\left[-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t} - K_m K_{se}\right]\right]}{2\lambda K_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right]}. \tag{55}$$

The first order condition with respect to λ, is as follows:

$$G - T\left[w_{ime}(0) + K_{se}\left\{T - \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}T}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}\right] = 0. \tag{56}$$

If we plug the value of T from eq. (55) to (56), we obtain:

$$G =$$

$$-w_{ime}(0).\frac{\lambda w_{ime}(0) - \left[\int\limits_{-\infty}^{0} \frac{K_{se}K_d}{(K_{sd}+K_{se}+K_d)}dt - \frac{1}{K_m}\left[-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t} - K_m K_{se}\right]\right]}{2\lambda K_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right]}$$

$$+K_{se}\left\{1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right\}$$

$$* \left[-\frac{\lambda w_{ime}(0) - \left[\int\limits_{-\infty}^{0} \frac{K_{se}K_d}{(K_{sd}+K_{se}+K_d)}dt - \frac{1}{K_m}\left[-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t} - K_m K_{se}\right]\right]}{2\lambda K_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right]}\right]^2,$$

$$\text{or } 4\lambda^2 QG = -2\lambda^2 w_{ime}^2(0) + 2\lambda w_{ime}(0)J + \lambda^2 w_{ime}^2(0) + J^2 - 2\lambda w_{ime}(0)J,$$

$$\text{where } Q = K_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)}e^{-[K_m(K_{sd}+K_{se}+K_d)]t}\right],$$

$$J = \int_{-\infty}^{0} \frac{K_{se}K_d}{(K_{sd}+K_{se}+K_d)} dt - \frac{1}{K_m}\left[-\frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t} - K_m K_{se} \right].$$

This implies that

$$\left\{ w_{ime}^2(0) + 4QG \right\} \lambda^2 - J^2 = 0.$$

$$\lambda = \frac{J}{\sqrt{w_{ime}^2(0) + 4QG}}.$$

λ has to be positive, since the minimum efficiency loss increases with an increase in G. Eq. (34) can also be expressed as:

$$T = -\frac{\lambda w_{ime}(0) - J}{2\lambda Q}. \tag{57}$$

If we plug the value of λ in the above expression, we obtain:

$$T = -\frac{\frac{w_{ime}(0)J}{\sqrt{w_{ime}^2(0)+4QG}} - J}{\frac{2QJ}{\sqrt{w_{ime}^2(0)+4QG}}},$$

$$T = -\frac{w_{ime}(0) - \sqrt{w_{ime}^2(0) + 4QG}}{2Q}. \tag{58}$$

In order to estimate the optimal trade facilitation policy empirically, we need to estimate Q using the following formula:

$$\mathbf{Q} = \mathbf{K}_{se}\left[1 - \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} + \frac{K_{se}}{(K_{sd}+K_{se}+K_d)} e^{-[K_m(K_{sd}+K_{se}+K_d)]t} \right],$$

The empirical literature provides us various methods for the estimation of demand and supply elasticities, using which we can easily estimate the value of Q. Plugging in the estimated value of Q at $t = 0$, in expression (58), we can estimate the optimal trade facilitation policy. Delta method can be used for the confidence interval.

Now, let us check the second order condition for minimization. The Lagrangian can be written as

$$\mathcal{L} = JT + \lambda \left[G - T\left(w_{ime}(0) + QT \right) \right].$$

The Bordered Hessian matrix of the Lagrange function is as follows:

$$BH = \begin{bmatrix} 0 & w_{ime}(0) + 2QT \\ w_{ime}(0) + 2QT & \frac{-2QJ}{\sqrt{w_{ime}^2(0)+4QG}} \end{bmatrix}.$$

As the determinant of the Bordered Hessian matrix is negative, i.e., $-\left(w_{ime}(0) + 2QT\right)^2 < 0$, it implies that the efficiency loss is minimized.

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