Adam Smith's Perfectly Competitive Market is Not Pareto Efficient: A Dynamic Perspective

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Adam Smith's Perfectly Competitive Market is Not Pareto Efficient: A Dynamic Perspective

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Abstract

The *invisible hand* of a perfectly competitive market refers to the self-regulating behavior of the market where if each consumer and producer is allowed to freely make their own choices, the market settles at an efficient outcome which is beneficial to all the individual members of the society and hence to the society as a whole. Two well-known facets of the *invisible hand* are generally mentioned in the economics literature - the first one is a static picture of a perfectly competitive market, i.e., a competitive market is efficient in an equilibrium; and the second one is that if the competitive market is disturbed from its equilibrium position, in the absence of a market failure and frictions, the market automatically settles at a new efficient equilibrium. **Existing literature does not consider the most important dynamic facet of the perfectly competitive market from perspective of Pareto efficiency, i.e., how efficient is a perfectly competitive market on the dynamic adjustment path after an economic shock in the absence of all kinds of frictions and price rigidities, and if all the ideal conditions are maintained. This research models the dynamic facet of the market and concludes that Adam Smith's perfectly competitive market is not Pareto efficient and coordinated actions of economic agents can result in a level of economic efficiency on the dynamic adjustment path which is not achievable by a free market mechanism.** (JEL D40, D41, D50, E32)

Keywords: Dynamic efficiency, Adjustment Path, Equilibrium, Coordination

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1 Introduction

Adam Smith's invisible hand was a clever mechanism for describing how information, idiosyncratic and dispersed among individuals, is accumulated and combined by the market mechanism such that the overall market equilibrium is the same as that would be obtained by an all-knowing social planner. This result is the core principle of all modern Samuelsonion welfare economics. However, the existing literature inadequately addresses the efficiency issue on the dynamic adjustment path of the market after an economic shock before it arrives at a new efficient equilibrium, and rather an adjustment mechanism has never been defined clearly for a perfectly competitive market. There is an inherent limitation in the definition of a perfectly competitive market, which precludes the explanation of movement of the market price after an economic shock. In a perfectly competitive market, all producers are price takers and the prices are perfectly flexible, however, a central question to the discussion is: who changes the price after an economic shock to the market to lead to a new equilibrium? If no one changes the price, then supply will never equalize the demand after an economic shock, resulting in an infinite level of inefficiency. If the producers have to be assumed of changing the price when the market is out of equilibrium, it comes in direct conflict with the assumption of the producers being "price takers". No other market agent (responsible for the movement of prices) has been defined under the theoretical concept of a perfectly competitive market. In this scenario, it is impossible to provide a theoretical framework for some drifting mechansim for a perfectly competitive market from one equilibrium to the other, as the "price taking" and the "price changing" assumptions contradict each other. A producer cannot be a "price taker" and a "price changer" at the same time, however, with the only assumption of "price taking behavior", the market will logically get stuck in an "out of equilibrium" state for an infinite period of time leading to an infinite level of inefficiency, whereas, the assumption of a "price changer" for a market agent (which is inevitable to explain the change in price from one equilibrium to the other) implies at least a minimal level of market power. This also implies that a perfectly competitive market has to incur some inefficiency after an economic shock, i.e., either it gets stuck in an "out of equilibrium" state (under the agents' price taking assumption) or a market agent changes the price to bring the new equilibrium, in which case, the transition cannot occur free of cost and has to come at the expense of some economic efficiency!

The only way a perfectly competitive market can be efficient during the transition is that the economic shock itself instantly settles the price and quantity from one equilibrium to the other, i.e., the market jumps from one equilibrium to the other as a result of a shock through some magic wand. However, the basis of market mechanism is self-interest by economic agents, which acts as a driving force to push the market toward an efficient equilibrium. When the market is in equilibrium, demand equals supply, however, after an economic shock the market does not instantaneously settle at a new equilibrium, the market forces once again become operative to drive the market and the market follows some adjustment process to move to the new equilibrium. The demand does not equal the supply throughout the adjustment process and there is some efficiency loss before the market settles at an efficient equilibrium. An important question to consider is: Can the market mechanism itself minimize this loss or is there some other mechanism which may improve the efficiency during the adjustment process? In order to address this question, we first need to quantify the efficiency on the dynamic adjustment path, and then compare it with an ideal situation.

Suppose there is only one producer in a market who produces a perishable good and sells it to the consumers living in a community. He sells a quantity exactly equal to the quantity he produces in each time period, and the market stays in equilibrium. If a demand shock decreases the demand of his product, some of his production will remain unsold and be wasted by the end of the time period in which the demand shock happened. Assuming that the producer can change the price and the production immediately, had he known the exact pattern of new demand, he would immediately pick the price and the quantity to maximize his profits and clear the market without wasting his production. However, he lacks this information, so he decreases the price based on his best guess about the new demand (based on the quantity of his unsold production), driving the market close to the new equilibrium. If in the next time period, his production is sold out, he will know that the new equilibrium has arrived, however, if a part of his production still remains unsold, he will reduce the price further (and production accordingly) to bring the market closer to the new equilibrium. The market will eventually settle at the new equilibrium after some efficiency loss. The resources wasted by the market mechanism are those which went into the unsold production in each time period during the adjustment process.

In contrast, suppose that after a demand shock, a community coordinator conveys the information about the new demand at various prices to the producer. The producer would immediately pick the profit maximizing, market clearing price and quantity without an efficiency loss. The new equilibrium will be identical in both scenarios, however, coordination seems to improve upon the market mechanism during the adjustment process, through revealing the information of new demand to the producer, which is missing in the market where consumers only care about their self-interest without being bothered about the resources the producer will have to waste as unsold production on account of lack of this information. Therefore, coordination can improve upon the market mechanism in terms of efficiency during the adjustment process, even though the new equilibrium is going to be the same as that would be achieved by the market mechanism.

In the above example, only one producer was assumed just to make intuition simple and clear, however, the same reasoning goes through for a perfectly competitive market with a large number of buyers and suppliers. In a perfectly competitive market, the exact magnitude of an economic shock is not known to suppliers, and they also lack information on how consumers are going to respond to a specific shock. They try to gauge the intensity of the shock through increasing or decreasing size of their inventories and adjust their economic actions accordingly. However, if there is coordination among producers and consumers, producers have more information about the new economic scenario and the adjustment to the new equilibrium is smoother and quicker as compared to the adjustment in a free market. It would be too imaginative to believe that there can be such a perfect coordination among all buyers and suppliers which may result in a zero efficiency loss after an economic shock, however, coordination can certainly improve economic efficiency during adjustment process through mutual sharing of information among buyers and suppliers. There might arise issues of coordination costs, however, if magnitude of shock is large enough and the adjustment process is associated with huge efficiency losses (which if saved could more than compensate coordination costs) then coordination is superior to a free market, whereas if the magnitude of shock is too small and coordination costs are comparatively too high, then free market might lead to a more efficient adjustment process. There could also be positive externalities of coordination. These issues need to be discussed separately in future research projects.

The issue of economic efficiency has much been discussed in the previous literature. The Fundamental Theorems of Welfare Economics are generally stated in a form attributed to Arrow et al. (1952), Arrow and Debreu (1954) and Debreu (1959). Interpreting the conditions under which the First Theorem is true provides the basis of the Market Failure approach. Bator (1958) focuses on externalities, natural monopolies, and public goods. Akerlof (1970) explains the quality uncertainty as another source of market failure. Kaldor (1972) argues that the general economic equilibrium originally formulated by Walras and mathematically developed by Gerard Debreu is too over simplistic to depict the real world phenomena. Green (1977) discusses the non-existence of informational equilibria. Grossman and Stiglitz (1980) also model situations where a competitive equilibrium would not exist. Greenwald and Stiglitz (1986) argue that in general when risk markets are incomplete and information is imperfect, markets are not constrained Pareto optimal: the Invisible Hand does not work. In Stiglitz's words (see Stiglitz (1991)): "The Welfare Theorems are just that: theorems, the conclusions of which follow inevitably from the assumptions. The research of the last two decades has not detected any major flaws of logic. The Theorems stand, as I have said, as one of the triumphs of modern mathematical economics. The question is not the logical status of these propositions, but their empirical relevance, the inferences which we make concerning how society should be organized and about the design of economic policy. These are, to be sure, matters of judgment. The earlier analyses of market failures basically agreed with the underlying conception of the market economy that was reflected in the assumptions of the Welfare Theorems. I am not so convinced." Ackerman, Nadal and Gallagher (2004) also call into question the assumptions in Arrow and Debreu's analysis which make the Theorems of limited relevance to modern industrial

economies.
There is also a huge literature discussing efficiency of markets in the context of a society, e.g., Arge and Hunt (1971) argue that for economic efficiency, what needs to be accomplished is a persuasion of the populace away from the doctrine of self-interest and toward a doctrine of cooperation and conservation. Sen (1993) argues that competitive market equilibria are weakly efficient in opportunity freedoms in terms of capabilities as well as commodity holdings. Oakley, Ashton et al. (1997) based on Titmus' famous study on paid and unpaid blood donations (1971) explains the social repercussions of creating a market for blood, inviting an un-ending debate on the issue. Frey and Oberholzer-Gee (1997) develop the argument further to show how the market mechanism for certain entities undermines an individual's sense of civic duty. Sunstein (1999) presents a new conception of the relationship between free markets and social justice. Folbre and Nelson (2000) elaborate that the market mechanism is unlikely to provide the quantum and quality of children, sick and elderly care. George (2004) challenges the view that free choice, as revealed by the preferences of individuals, maximizes the welfare of individuals given their income and market prices. Howell (2004) challenges the free market with regard to the labor markets around the world and persistent unemployment. Dolfsma, Finch and McMaster (2005) argue how the conceptualization of the market in relation to the society undermines the mainstream, Paretian perspective on welfare. Dolfsma (2005) highlights the importance of dynamic welfare. Satz (2010) delivers a comprehensive theory of the limits of the markets with regard to society. Hemsley-Brown (2011) portrays the challenges of a free market in higher education.
As the major focus of this paper is the efficiency on the dynamic adjustment path, it will not be out of place to acknowledge the previous literature highlighting the importance of adjustment path to equilibrium. Diamond (1971) develops a model of price adjustment assuming that firms know the demands they face. Eden (1981) explains a theory of competitive price adjustment. Hahn (1984) explains why a theory of the economy out of equilibrium is required. David (2007) emphasizes the importance of path dependence in economics. Nerlove (1958) discusses the Cobweb model which explains why prices might be subject to periodic fluctuations in certain types of markets. Cobweb model is based on adaptive expectations and subject to criticism by the advocates of rational expectations. Herings (1996) gives a price adjustment process for an exchange economy that converges generically to a Walrasian equilibrium. Tuinstra (2004) provides a price adjustment process in a model of monopolistic competition. There have been a number of ways of modeling paths to equilibrium under different set of frictions, such as incomplete markets (see Angeletos and Calvet (2005), Talman and Thijssen (2006)), adjustment costs (see Lucas (1967), Bertola and Caballero (1990), Danziger (1999) and Chen, Feng and Seshadri (2014)), rational inattention (see Chen, Levy, Ray and Bergen (2008), Maćkowiak and Wiederholt (2009) and Mackowiak and Wiederholt (2010)), and search costs (see Wright (1986)), etc. **Nawaz (2017), Nawaz (2019)a, Nawaz (2019)b,**

Nawaz (2020), Nawaz (2021), Nawaz (2022) model dynamic adjustment path for optimal policies.

The main contribution of this paper is that the commonly believed notion that if all the ideal conditions of a perfectly competitive market are maintained, the market is Pareto efficient has been challenged to the extent of dynamic adjustment path. It is shown that the market mechanism involves an efficiency loss during the adjustment process on account of agents' lack of perfect information about the new economic conditions after the shock. In the existing literature, the imperfect information generally refers to the asymmetric information among buyers and suppliers regarding the quality of a product or other factors leading either to an inefficient equilibrium or non-existence of an equilibrium. This paper shows that even though an equilibrium exists, the free market cannot get rid of inefficiency on the way to the equilibrium. This paper assumes perfect information regarding the quality of goods, just to distinguish and highlight the key point. The main source of inefficiency during the adjustment process is the agents' lack of perfect information about the exact magnitude of the shock and the new patterns of supply and demand. The only way a perfectly competitive market can be Pareto efficient during the adjustment process after an economic shock is that if all the ideal conditions of a perfectly competitive market are maintained and *also if all the economic agents have perfect information about the exact magnitude and direction of all future shocks and the exact patterns of supply and demand in an economy as a result of shocks for all times*, which would have been possible only if economic agents were Gods, hence making the study of economics needless!

When the term *market* is used in economics, it generally refers to the two main economic agents, i.e., the consumer and the producer. The producer produces a product and sells it to the consumer for profit who has some willingness to pay for the product for consumption. In this way, a mutual exchange takes place between the two agents which is beneficial for both of them. However, in the real world these two economic agents seldom directly meet each other for a transaction to take place. Consumers never go to the factory owners to buy products. Borrowers never directly meet lenders. Students do not visit their professors' houses to get education. Each transaction between a consumer and a producer takes place through the involvement of a *middleman*. In the real world, whole salers, retailers, financial institutions, educational institutions and hospitals, etc., reflect the very much presence of the *middleman* in the market. A market model in the absence of a *middleman* cannot capture the important aspects of reality. So, if the middleman is out there in the real world between a consumer and a producer, why not include it in the analysis and see how the static as well as the dynamic market behavior responds to the presence of a middleman?

In this paper, we develop a deterministic dynamic market model (for a single homogeneous commodity) in continuous time framework. The model takes into account a supply curve, a demand curve, an inventory curve for the middleman, and dead time to model sticky prices and production frictions. However, analytical solution to the model has been presented for the simple case when prices are perfectly flexible and there is no production friction. A more realistic case, which incorporates price rigidities and frictions will be presented in a future research project.

The model answers the following **research** questions: How do actions of three market agents, i.e., consumer, producer and middleman in their own interest lead to the market equilibrium? Which path does the market follow as a result of an endogenous or exogenous shock in the steady state equilibrium? How efficient is the market on that path? Can a free market on its own minimize efficiency losses during the adjustment process or is there any other mechanism which can help improve upon the market in terms of efficiency during the adjustment process? The model is based on mathematical formulations (in time domain) of practical behavior of key agents (without over-simplistic assumptions) endogenously capturing the feedback effects of price changes on market system. A mathematical innovation in the model is the introduction of deviation variables, i.e., deviation from an equilibrium (before shock) instead of absolute value of the variable. This makes the model free of initial conditions as the initial value of a deviation variable is always zero.

The mathematical picture provides concrete conditions for the existence of an equilibrium which is independent of middleman's response. This result is supported by the final steady state equilibrium (mathematical) expression which is dependent just on responses of consumer and producer. The parameter depicting the middleman's response drops out of the final steady state equilibrium expression. However, the mathematical analysis shows that the middleman has an important role in the dynamic adjustment process. The response of the middleman (along with that of consumer and producer) determines the adjustment path of the market after an economic shock. The middleman's response also determines the quantum of inefficiency during the adjustment process. The concept of dynamic efficiency has been discussed in the paper. When an economic shock hits the market, the market remains out of equilibrium during the adjustment period before it arrives at a new equilibrium. The longer the adjustment period, the greater are the efficiency losses. The market certainly cannot and does not jump to a new equilibrium (an ideal dynamic efficiency condition which is not achievable). A market is said to be closer to an ideal dynamically efficient state if it follows the smoothest and the shortest possible route between equilibriums after an economic shock, that is, if it minimizes the present value of output (and/or consumption, utility) lost. The model provides a basic framework for a dynamic welfare analysis and scope for extensions and applications in various goods, services, labor and financial markets.

The remainder of this paper is organized as follows: Section 2 explains how the individual components of the market system are joined together to form a dynamic market model. Section 3 provides a solution of the model in time domain when market has no production friction and price rigidities. Section 4 summarizes the findings and concludes. Section 5 explains the future research prospects.

2 The Model

The basic model has already been discussed in Nawaz (2017), Nawaz (2019)a, Nawaz (2019)b, Nawaz (2020), Nawaz (2021), and Nawaz (2022). Let's assume that there is a perfectly competitive market of a single homogeneous commodity in equilibrium (initial condition is when the market is already in equilibrium). There are three types of infinitely-lived agents: a representative -or a unit mass of- producer (that produces a good, and demand labor and capital), a middleman (who buys the good from firms to sell to consumers, and possibly accumulating inventories), and a representative –or a unit mass of– consumer (who buys the good, accumulates capital by investing and supplies labor inelastically). The role of middleman is motivated by real world scenario where producer and consumer seldom directly meet for a transaction to take place. The existence of retailers, wholesalers, financial institutions, educational institutions and the hospitals reflect the presence of middlemen between producers and consumers in most of the economic activity going on. The producer produces the goods and supplies those to the middleman, who keeps an inventory of goods and sells to the consumer at the market price. In the model, the middleman plays a key role, as he sets the selling price p by maximizing the difference between the revenue for selling goods to consumers and the costs of inventories. The buying price paid to the producer is αp with $\alpha < 1$, and the producer is a price taker.

The price adjustment mechanism is based on the fact that when a shock leads the market out of equilibrium, the buyers' and sellers' decisions are not coordinated at the current prices. An example can illustrate the working of this market. Consider that the market is initially in equilibrium. The middleman has an equilibrium stock of inventory. Then, an exogenous demand contraction will increase the stock of inventory, due to firms' output could not match with the –now lower– units demanded by the consumer at the current price. This excess of supply is accumulated in inventory held by the middleman. The middleman will decrease the price so that the producer will find optimal to produce a lower level of output. A new equilibrium with a lower price and a lower level of output is then reached. The equilibrium is defined as follows:

(i) The producer and the middleman maximize their profits and the consumer maximizes utility subject to the constraints they face (mentioned in their individual dynamic optimization problems in Section 2).

(ii) The quantity supplied by the producer equals the quantity consumed by the consumer (and hence the inventory does not change when the market is in equilibrium).

The conditions for the existence of the equilibrium (Routh–Hurwitz stability criterion, which provides a necessary and sufficient condition for the stability of a linear dynamical system) have been mentioned in Section 3.

As the set up is for a perfectly competitive market, therefore, the middleman who sells the goods to the consumer at the market price is a price taker when the market is in equilibrium. When the market is out of equilibrium, the middleman can change the price along the dynamic adjustment path until the new equilibrium arrives, where again the middleman becomes a price taker. If a demand or supply shock hits the market, the market does not suddenly jump to a new equilibrium, rather the price adjusts over time to bring the new equilibrium. This adjustment process involves endogenous decision making (in their own interest) by all agents in the market, i.e., consumer, producer and the middleman. For the mathematical treatment, the objective of each of the three market agents is maximized through the first order conditions of their objective functions and to capture the collective result of their individual actions, the equations representing their individual actions are solved simultaneously. For simplification, we assume that after the shock, the new equilibrium is not too far from the initial equilibrium. This assumption makes the linearization of supply and demand curves quite reasonable. Please look at figure 1 (the time axis is not shown). Linearization seems to be a good approximation when market moves from point a to b, whereas it is not a good approximation when it moves from point a to c. For modeling the movement of the market from point a to c, we need to model a non-linear dynamical system (which is not covered under the scope of this paper).

A small segment of the economy under study may be called as a *system* and everything else (which the system is to interact with) may be called as *surroundings*. Analysis of a system may be convenient if the system is sub-divided into smaller simpler components, which could be interconnected through their inputs/outputs. A mathematical equation correlating outputs with inputs may be written for each component giving a set of simultaneous equations. Solution of this set of equations may give an overall behavior of the system.

A block diagram representation is a convenient tool for giving an overall picture of the relationships among variables by highlighting the function of each component and the flow of information around the system. A block indicates a mathematical operation or procedure applied on an input to generate an output. Let us consider the following function:

$$Y = f(X). \tag{1}$$

This function can be represented by the rectangular block shown in figure 2. The arrow entering the box is the forcing function or input variable, and the arrow leaving the box is the response or output variable. Inside the box is placed the function. We state that the function in the box operates on the input function X to produce an output function Y. The rectangular block can be used to represent a component of the system, which transforms its input to an output. X and Y may or may not be the physical input and output respectively.

Concept of dead time:

Sometimes, there is a time gap involved between receiving an information and making a decision or taking an action. This time gap may be called as *dead time* or *time of inaction*. Suppose some information or material is sent to us through a postal or courier service. We receive information or material without a change in it but after a delay in time. In Figure 3, $f(t)$ and $f(t-\tau d)$ are identical but $f(t-\tau d)$ is delayed by time τd. a and a' are two corresponding points on $f(t)$ and $f(t-\tau d)$. a' appears after a with a time delay of τd ($aa' = \tau d$). Dead time element may be represented by the rectangular block given in Figure 4. The involvement of dead time means that the basis of implementation is back dated. Sometimes information is prompt but its implementation is delayed due to practical reasons. At times information is delayed but the implementation is prompt. In certain other situations both information and its implementation are delayed. As the impact of each situation is the same (i.e., implementation is based on back dated information or happening), therefore the mathematical handling of each of these cases is the same.

A market system may be sub-divided into the following three physical entities: (1) The *producer*, (2) the *middleman* and (3) the *consumer*.

2.1 Middleman

The middleman purchases goods from producer and sells to consumer for profit. As happens in the real world, the middleman does not buy and sell exactly the same quantity at all points in time, thus he holds an inventory of the goods purchased to be sold subsequently. Inventory is an intermediary stage between supply and demand which reflects the quantum of difference between supply and demand of the goods in the market. If the inventory remains the same, it implies that demand and supply rates are the same. An increase or decrease in inventory implies a change in supply, demand or both at different rates.

Figure 2 illustrates the link among inventory, supply, demand and prices. When supply curve shifts to the right (while demand remains the same), inventory in the market increases at initial price, and new equilibrium brings price down. Similarly, when demand curve shifts to the right (while supply remains the same), inventory depletes from the market at previous price and new equilibrium brings price up. This shows that there is an inverse relationship between an inventory change and a price change (all else the same). If both supply and demand curves shift by the same magnitude such that the inventory does not change, then price will also remain the same. Inventory unifies supply and demand shocks in the sense that they are both affecting the same factor, i.e., inventory and are basically the faces of the same coin. Therefore, each kind of shock is in fact just an inventory shock. From above mentioned discussion, we have seen that there is an inverse relationship between an inventory change and a price change.

Now let's discuss the mechanism which brings about such a change. Consider a market of homogeneous goods where the middlemen, such as whole sellers, retailers, etc., hold inventories, incur some cost for holding those, and sell products to the consumers to make profits. Cost is a positive function of the size of an inventory, i.e., a larger inventory costs more to hold as compared to a smaller inventory. In absence of an exogenous shock, if supply and demand rates are equal then the system is in equilibrium and the price does not vary with time. Suppose that a technological advancement decreases the marginal cost of production and increases supply rate, whereas demand rate remains the same. As demand and supply rates are no longer equal, therefore the difference will appear somewhere in the economy in the form of piled up inventories. As the production flows from producer to consumer through the middleman, therefore it is reasonable to assume that the middleman will be holding the net difference (Explanation: The piled up inventories can also be in the form of producers' inventories of finished goods, which does not change the key point that a difference of supply and demand rates directly affect the inventories in the economy). The economy will not be able to sustain this situation for an indefinite time period, and the middlemen will have to think of some means of getting rid of piled up inventories. The only resort they have is to decrease the price which brings the demand up along the demand curve.

In a perfectly competitive market, price will eventually come down to equalize the new marginal cost, however the adjustment path depends on how the middlemen react to the change in their inventories. Notice that the marginal cost of production has decreased but the marginal cost of holding an extra unit of inventory for the middleman has increased. This is an intuitive explanation which is theoretically consistent with the demand, supply, utility and profit maximization by a consumer and a producer respectively. In the real world, we see examples of this behavior of middlemen, e.g., as consumers, we enjoy *end of year sales*, offers such as *buy one get one free*, *gift offers* if you buy above a certain quantity threshold, etc. For a mathematical treatment, we need to consider profit maximization problem of the *middleman* as follows:

2.1.1 Short-Term Problem

Let's first consider the short-term problem, i.e., the middleman's objective is myopic rather than doing dynamic optimization. In a discrete analog, this is a one period analysis, which is presented for an intuitive purpose in anticipation of the (more complicated) dynamic problem in section 2.1.2. Expression for middleman's profit is as follows:

$$\Pi = pq(p) - \varsigma(m(p, e)), \tag{2}$$

where

Π = profit,

p = market price,

$q(p)$ = quantity sold at price p,

m = inventory (total number of goods held by the *middleman*),

e = factors which influence inventory other than the market price including middleman's purchase price from producer,
$\varsigma(m(p,e))$ = cost as a function of inventory (increasing in inventory).
First order condition (with respect to price) is as follows:

$$pq'(p) + q(p) - \varsigma'(m(p,e))m_1'(p,e) = 0, \tag{3}$$

Middleman has an incentive to change price only during the adjustment process and will incur losses by deviating from the price (equal to marginal cost) when market is in equilibrium. During adjustment process, demand does not equal supply and the market drifts toward the new equilibrium (however, price cannot move automatically and it is reasonable to assume that some economic agent moves price in his own benefit), therefore a price change by middleman in the direction of bringing new equilibrium is not against market forces, so he does not lose business by changing price on the adjustment path unlike when the market is in equilibrium and where the middleman faces an infinitely elastic demand as follows:

$$pq'(p) + q(p) = \varsigma'(m(p,e))m_1'(p,e),$$
$$p\left[1 + \frac{1}{\textit{demand elasticity}}\right] = \varsigma'(m(p,e))\frac{m_1'(p,e)}{q'(p)}.$$

The right hand side of above expression is the marginal cost which equals price when the middleman faces an infinitely elastic demand. Suppose that as a result of a supply shock, the marginal cost of production decreases, and the supply curve shifts downwards. Now the competitive market is out of equilibrium as demand does not equal supply at the previous equilibrium price. The price must eventually decrease to bring the new equilibrium, however, the price will not jump to equalize demand and supply, and rather the middleman will continue charging a price higher than the new marginal cost until the market forces make him realize that the supply has increased and he needs to lower the price to satisfy profit maximization condition. The similar is the case of a reverse supply shock, where price must eventually increase to bring new equilibrium. In this case, the middleman will continue charging a price lower than the marginal cost until the market forces make him increase price, in which case it is the consumer who is the short term beneficiary. Again, the consumer will be paying a price less than the marginal cost only during the adjustment process and only until the middleman increases the price. The equilibrium price is equal to marginal cost of production plus the marginal cost of storage (i.e., total marginal cost) in absence of any kind of a tax, so neither does middleman earn any economic rent, nor does consumer benefit by paying a price less than marginal cost when competitive market is in equilibrium.
For mathematical treatment, suppose as a result of a supply shock (while demand remains the same) such as a technological advancement which reduces marginal cost of production and increases

supply by producers, if the middleman wants to hold an extra unit of inventory, his marginal cost of holding an extra unit i.e., $\varsigma'(m(p,e))\frac{m_1'(p,e)}{q'(p)}$ is higher at the previous price, because the term $\varsigma'(m(p,e))$ is higher at the previous price. This might be on account of higher storage charges because of increased demand of warehouses, godowns, etc. after increased supply in the market. The second term, i.e., $\frac{m_1'(p,e)}{q'(p)}$ is a function of price, and is the same as before as the price has not changed yet (we are assuming that middleman's purchase price is the same as before as producer is a price taker during the adjustment process as well and always charges a fixed fraction of market price to middleman). A discrete analog of this scenario is that middleman maximizes profits in each time period without considering future time periods, and in each time period he takes the purchase price from producer as given and only chooses the sale price. This implies that on the previous price, now middleman faces

$$\frac{\partial \Pi}{\partial p} = pq'(p) + q(p) - \varsigma'(m(p,e))m_1'(p,e) < 0, \tag{4}$$

which means that middleman must decrease price to hold an extra unit of inventory to satisfy the profit maximization condition after the supply shock. Note, that in this static scenario, the short term gains accrued from decreased marginal cost of production will be reaped by producer, as his marginal cost has decreased but he charges the same price to middleman until the middleman changes the price. If we plot together various profit maximizing combinations of inventories and respective prices chosen by middleman, a downward sloping *inventory curve* results with price on y-axis and inventory on x-axis. This is analogous to the concept of *supply* and *demand curves* for profit maximizing producers and utility maximizing consumers respectively.

2.1.2 Dynamic Problem

Now let's consider the dynamic problem of middleman. In a dynamic setting, middleman maximizes present discounted value of future stream of profits, and his present value at time zero is as follows:

$$V(0) = \int_0^\infty \left[pq(p) - \varsigma(m(p,e))\right] e^{-rt} dt, \tag{5}$$

r denotes the discount rate. $p(t)$ is *control variable* and $m(t)$ the *state variable.* The maximization problem can be written as

$$\underset{\{p(t)\}}{Max} V(0) = \int_0^\infty \left[pq(p) - \varsigma(m(p,e))\right] e^{-rt} dt,$$

subject to the constraints that

$\dot{m}(t) = m_1'(p(t), e(p(t), z))\dot{p}(t) + m_2'(p(t), e(p(t), z))e_1'(p(t), z)\dot{p}(t)$ (state equation, describing how state variable changes with time; z are exogenous factors),

$m(0) = m_s$ (initial condition),
$m(t) \geq 0$ (non-negativity constraint on state variable),
$m(\infty)$ free (terminal condition).
The current-value Hamiltonian for this case is

$$\widetilde{H} = p(t)q(p(t)) - \varsigma(m(p(t), e(p(t), z))) + \mu(t)\dot{p}(t) \left[\begin{array}{c} m_1'(p(t), e(p(t), z)) + m_2'(p(t), e(p(t), z))* \\ e_1'(p(t), z) \end{array} \right]. \tag{6}$$

Now the maximizing conditions are as follows:
(i) $p^*(t)$ maximizes $\widetilde{H}$ for all t: $\frac{\partial \widetilde{H}}{\partial p} = 0$,
(ii) $\dot{\mu} - r\mu = -\frac{\partial \widetilde{H}}{\partial m}$,
(iii) $\dot{m}^* = \frac{\partial H}{\partial \mu}$ (this just gives back the state equation),
(iv) $\lim_{t \to \infty} \mu(t)m(t)e^{-rt} = 0$ (the transversality condition).
The first two conditions are as follows:

$$\begin{aligned} \frac{\partial \widetilde{H}}{\partial p} &= q(p(t)) + p(t)q'(p(t)) - \varsigma'(m(p(t), e(p(t), z))) \left\{ \begin{array}{c} m_1'(p(t), e(p(t), z)) + m_2'(p(t), e(p(t), z))* \\ e_1'(p(t), z) \end{array} \right\} \\ &+ \mu(t)\dot{p}(t) * \left[\begin{array}{c} m_{11}''(p(t), e(p(t), z)) + m_{12}''(p(t), e(p(t), z))e_1'(p(t), z)+ \\ m_{21}''(p(t), e(p(t), z))e_1'(p(t), z) + m_{22}''(p(t), e(p(t), z))e_1'^2(p(t), z)+ \\ m_2'(p(t), e(p(t), z))e_{11}''(p(t), z) \end{array} \right] \\ &= 0, \end{aligned} \tag{7}$$

and

$$\dot{\mu} - r\mu = -\frac{\partial \widetilde{H}}{\partial m} = \varsigma'(m(p(t), e(p(t), z))). \tag{8}$$

When the market is in equilibrium, $\dot{p}(t) = 0$, and the expression $\frac{\partial \widetilde{H}}{\partial p}$ boils down to the following:

$$\begin{aligned} &q(p(t)) + p(t)q'(p(t)) - \varsigma'(m(p(t), e(p(t), z))) \left\{ \begin{array}{c} m_1'(p(t), e(p(t), z)) + m_2'(p(t), e(p(t), z))* \\ e_1'(p(t), z) \end{array} \right\} \\ &= 0, \end{aligned}$$

$$p(t)q'(p(t)) + q(p(t)) = \varsigma'(m(p(t), e(p(t), z))) \left\{ \begin{array}{c} m_1'(p(t), e(p(t), z)) + m_2'(p(t), e(p(t), z))* \\ e_1'(p(t), z) \end{array} \right\},$$

$$p(t) \left[1 + \frac{1}{\textit{demand elasticity}} \right] = \varsigma'(m(p(t), e(p(t), z))) \left\{ \frac{m_1'(p(t), e(p(t), z))}{q'(p(t))} + \frac{m_2'(p(t), e(p(t), z))e_1'(p(t), z)}{q'(p(t))} \right\},$$

suggesting that price equals marginal cost (the right hand side of above expression is marginal cost in a dynamic setting, which is different from that in a short-term problem due to the fact that in a dynamic setting, middleman also considers the impact of price chosen on his purchase price from producer) when demand is infinitely elastic. Now suppose that as a result of a supply shock, if middleman wants to hold an extra unit of inventory, then the marginal cost of holding an extra unit is higher because the term $\varsigma'(m(p(t), e(p(t), z)))$ is higher at the previous price at that point in time. The term in parentheses in the expression for marginal cost, i.e., $\frac{m_1'(p(t),e(p(t),z))}{q'(p(t))} + \frac{m_2'(p(t),e(p(t),z))e_1'(p(t),z)}{q'(p(t))}$ is a function of price and is the same at the previous price. This implies that on the previous price, now the middleman faces

$$\begin{aligned}\frac{\partial \widetilde{H}}{\partial p} &= q(p(t)) + p(t)q'(p(t)) - \varsigma'(m(p(t), e(p(t), z))) \left\{ \begin{array}{c} m_1'(p(t), e(p(t), z)) + m_2'(p(t), e(p(t), z))* \\ e_1'(p(t), z) \end{array} \right\} \\ &+ \mu(t)\dot{p}(t) * \left[\begin{array}{c} m_{11}''(p(t), e(p(t), z)) + m_{12}''(p(t), e(p(t), z))e_1'(p(t), z)+ \\ m_{21}''(p(t), e(p(t), z))e_1'(p(t), z) + m_{22}''(p(t), e(p(t), z))e_1'^2(p(t), z)+ \\ m_2'(p(t), e(p(t), z))e_{11}''(p(t), z) \end{array} \right] \\ &< 0.\end{aligned}$$

Therefore in order to satisfy the condition of dynamic optimization, middleman must decrease price for an increase in inventory. This implies there is a negative relationship between price and inventory. The concept of inventory unifies market supply and demand. If supply and demand rates are equal, market is in a steady state equilibrium. If a difference of finite magnitude is created between supply and demand rates and consumer and producer do not react to a price change induced by a difference in supply and demand rates, price will continue changing until the system saturates. This behavior can be depicted by the following formulation:

$$\begin{aligned} \textit{Price change} &\propto \textit{change in market inventory.} \\ P &= \textit{price change.} \\ M &= m - m_s = \textit{change in inventory in the market,} \\ m &= \textit{inventory at time } t, \\ m_s &= \textit{inventory in steady state equilibrium.} \\ \textit{Input} - \textit{output} &= \frac{dm}{dt} = \frac{d(m - m_s)}{dt} = \frac{dM}{dt}, \\ \text{or } M &= \int (\textit{input} - \textit{output})\, dt. \\ \textit{Price change} &\propto \int (\textit{supply rate} - \textit{demand rate})\, dt, \text{ or} \\ P &= -K_m \int (\textit{supply rate} - \textit{demand rate})\, dt, \end{aligned}$$

where K_m is the proportionality constant; *supply* and *demand rate* is the supply and demand per unit time respectively. A negative sign indicates that when (*supply rate* $-$ *demand rate*) is positive, P is negative (i.e., price decreases). The above equation can be re-arranged as follows:

$$\int (\textit{supply rate} - \textit{demand rate})\, dt = -\frac{P}{K_m}, \text{ or}$$

$$\int (w_i - w_0)\, dt = -\frac{P}{K_m}, \tag{9}$$

$$w_i = \textit{supply rate},$$
$$w_0 = \textit{demand rate},$$
$$K_m = \textit{dimensional constant.}$$

Let at time $t = 0$, *supply rate* $=$ *demand rate* (market is in a steady state equilibrium), then eq. (9) can be written as

$$\int (w_{is} - w_{0s})\, dt = 0. \tag{10}$$

The subscript s indicates the steady state equilibrium and $P = 0$ in steady state. Subtracting eq. (10) from eq. (9), we get:

$$\int (w_i - w_{is})\, dt - \int (w_0 - w_{0s})\, dt = -\frac{P}{K_m}, \text{ or}$$

$$\int (W_i - W_0)\, dt = -\frac{P}{K_m}, \tag{11}$$

$$\text{where } w_i - w_{is} = W_i = \textit{change in supply rate},$$
$$w_0 - w_{0s} = W_0 = \textit{change in demand rate.}$$

P, W_i and W_0 are deviation variables, which indicate deviation from the steady state equilibrium. The initial values of the deviation variables are zero. Eq. (11) may also be written as follows:

$$P = -K_m \int W\, dt = -K_m M, \tag{12}$$

where $W = W_i - W_0$. In the above analysis, we have assumed that P instantly responds to M. In reality, P persists for a while say τ_{d1} until the middleman realizes that there is a shortage or surplus of a commodity. The middleman is reluctant to change the price during this time without confirming the fact whether trend is transitory or permanent. Thus a dead time element is needed to be introduced after the rectangular block in figure 6 which modifies it to figure 7.

2.2 Producer

Producer maximizes present discounted value of future stream of profits, and his present value at time zero is as follows:

$$V(0) = \int_0^\infty \left[\alpha p(t) F\left(K\left(t\right), L\left(t\right)\right) - w(t) L\left(t\right) - \Re(t) I(t)\right] e^{-rt} dt, \tag{13}$$

α is fraction of market price the producer charges to middleman. r denotes the discount rate. $L(t)$ (labor) and $I(t)$ (level of investment) are *control variables* and $K(t)$ the *state variable*. The maximization problem can be written as follows:

$$\underset{\{L(t),I(t)\}}{Max} V(0) = \int_0^\infty \left[\alpha p(t) F\left(K\left(t\right), L\left(t\right)\right) - w(t) L\left(t\right) - \Re(t) I(t)\right] e^{-rt} dt,$$

subject to the constraints that
$\dot{K}(t) = I(t) - \delta K(t)$ (state equation, describing how state variable changes with time),
$K(0) = K_0$ (initial condition),
$K(t) \geq 0$ (non-negativity constraint on state variable),
$K(\infty)$ free (terminal condition).
The current-value Hamiltonian for this case is as given below:

$$\widetilde{H} = \alpha p(t) F\left(K\left(t\right), L\left(t\right)\right) - w(t) L\left(t\right) - \Re(t) I(t) + \mu(t)\left[I(t) - \delta K(t)\right]. \tag{14}$$

Now the maximizing conditions are as follows:
(i) $L^*(t)$ and $I^*(t)$ maximize $\widetilde{H}$ for all t: $\frac{\partial \widetilde{H}}{\partial L} = 0$ and $\frac{\partial \widetilde{H}}{\partial I} = 0$,
(ii) $\dot{\mu} - r\mu = -\frac{\partial \widetilde{H}}{\partial K}$,
(iii) $\dot{K}^* = \frac{\partial H}{\partial \mu}$ (this just gives back the state equation),
(iv) $\lim_{t\to\infty} \mu(t) K(t) e^{-rt} = 0$ (the transversality condition).
The first two conditions are as follows:

$$\frac{\partial \widetilde{H}}{\partial L} = \alpha p(t) F_2'\left(K\left(t\right), L\left(t\right)\right) - w(t) = 0, \tag{15}$$

$$\frac{\partial \widetilde{H}}{\partial I} = -\Re(t) + \mu(t) = 0, \tag{16}$$

and

$$\dot{\mu} - r\mu = -\frac{\partial \widetilde{H}}{\partial K} = -\left[\alpha p(t) F_1'\left(K\left(t\right), L\left(t\right)\right) - \delta\mu(t)\right]. \tag{17}$$

Substituting the value of $\dot{\mu}$ and μ from eq. (16) into eq. (17) yields

$$\alpha p(t) F_1'(K(t), L(t)) - (r+\delta)\Re(t) + \dot{\Re}(t) = 0.$$

If price, $p(t)$ goes up, (at the previous level of investment and labor) producer faces following inequalities:

$$\alpha p(t) F_2'(K(t), L(t)) - w(t) > 0,$$
$$\alpha p(t) F_1'(K(t), L(t)) - (r+\delta)\Re(t) + \dot{\Re}(t) > 0.$$

Therefore in order to satisfy condition of dynamic optimization after price increase, producer must increase production level. Let $p =$ market price, $c =$ a reference price (such as retail price which includes production cost, profit of producer and profit of middleman). c is a parameter which may vary with time or be kept fixed for a limited time period, e.g., cost of a product may vary over time or can also remain constant for a while. It is the reference point with respect to which variation in p is considered by producer for decision making.

$$W_m = \textit{Change in production due to change in price},$$

$(p-c)$ acts as an incentive for producer to produce more. We can write:

$$W_m = F_s(\alpha(p-c)),$$

F_s is a function of $\alpha(p-c)$, which is assumed to be linear (or linearization of production function of price around steady state equilibrium could lead to the similar expression) and leads to the following expression:

$$W_m = K_s(p-c), \tag{18}$$

When market is in equilibrium, then $W_m = 0$, or

$$0 = K_s(p_s - c_s). \tag{19}$$

K_s is being endogenously determined, and turned out to be a constant by virtue of linearization of F_s around steady state equilibrium (and whether F_s is practically linear or non-linear is an empirical question). p_s and c_s are steady state equilibrium values. Subtracting eq. (19) from eq. (18), we get:

$$W_m = K_s\left[(p-p_s) - (c-c_s)\right] = -K_s\left(C-P\right) = -K_s\varepsilon, \tag{20}$$

where W_m, C and P are deviation variables. Eq. (20) can be represented by the block diagram in figure 8 where it is assumed that producer instantly responds to market price; however, in reality extra time for planning and preparation to overcome technological and credit constraints is needed to change production, e.g., purchase or import of raw materials, enhancement of production capacity or waiting until the next season for seasonal agricultural products, etc., all necessitate dead time, i.e., τ_{d2} (by definition). Therefore, figure 8 should get amended to figure 9.

2.3 Consumer

Consumer maximizes present discounted value of future stream of utilities, and his present value at time zero is as follows:

$$V(0) = \int_0^{\infty} U(x(t))e^{-\rho t}dt, \tag{21}$$

ρ denotes the discount rate and $x(t)$ is *control variable*. The maximization problem can be written as

$$\underset{\{x(t)\}}{Max} V(0) = \int_0^{\infty} U(x(t))e^{-\rho t}dt,$$

subject to the constraints that
$\dot{a}(t) = R(t)a(t) + w(t) - p(t)x(t)$ (state equation, describing how state variable changes with time).
$a(t)$ is asset holdings (a *state variable*) and $w(t)$ and $R(t)$ are exogenous time path of wages and return on assets.
$a(0) = a_s$ (initial condition),
$a(t) \geq 0$ (non-negativity constraint on state variable),
$a(\infty)$ free (terminal condition).
The current-value Hamiltonian for this case is

$$\widetilde{H} = U(x(t)) + \mu(t)\left[R\left(t\right)a\left(t\right) + w\left(t\right) - p\left(t\right)x\left(t\right)\right]. \tag{22}$$

Now the maximizing conditions are as follows:
(i) $x^*(t)$ maximizes $\widetilde{H}$ for all t: $\frac{\partial \widetilde{H}}{\partial x} = 0$,
(ii) $\dot{\mu} - \rho\mu = -\frac{\partial \widetilde{H}}{\partial a}$,
(iii) $\dot{a}^* = \frac{\partial H}{\partial \mu}$ (this just gives back the state equation),
(iv) $\lim_{t\to\infty} \mu(t)a(t)e^{-\rho t} = 0$ (the transversality condition).
The first two conditions are as follows:

$$\frac{\partial \widetilde{H}}{\partial x} = U'\left(x\left(t\right)\right) - \mu(t)p(t) = 0, \tag{23}$$

and

$$\dot{\mu} - \rho\mu = -\frac{\partial \widetilde{H}}{\partial a} = -\mu(t)R(t). \tag{24}$$

If price of good x goes up, consumer faces (at the previous level of consumption) following inequality:

$$\frac{\partial \widetilde{H}}{\partial x} = U'(x\,(t)) - \mu(t)p(t) < 0.$$

Therefore in order to satisfy the condition of dynamic optimization after price increase, consumer must decrease consumption of good x.

$$\textit{Change in demand} = F_d(P).$$

Let change in demand be proportional to change in price, i.e., P, or equivalently linearization of demand function of price around the steady state equilibrium leads to the following expression:

$$\textit{Change in demand} \propto P, \text{ or}$$

$$W_d = -K_d P. \tag{25}$$

W_d is change in demand due to P; when P is positive W_d is negative. Change in demand is endogenously determined as a result of a feedback of the price to consumer, and whether or not demand curve is linear is an empirical question. This equation can be represented by the block diagram in figure 10. We combine figures 7, 9 and 10 to get figure 11 (our desired dynamic market model) where all variables are functions of time.

3 Time Domain Solution with Flexible Prices and No Production Friction

The ideal scenario is when prices are perfectly flexible and there is no production friction, i.e., $\tau_{d1} = \tau_{d2} = 0$. If τ_{d1} is zero, price is adjustable without delay, and $\tau_{d2} = 0$, implies that producer can adjust production without delay, which is possible if producer does not face any production friction. Let's solve the model for this case as follows:
From eqs. (12) and (25), we have the following expressions:

$$\frac{dP(t)}{dt} = -K_m W(t), \tag{26}$$

$$W_d(t) = -K_d P(t), \tag{27}$$

$$W(t) = W_i(t) - W_0(t) + W_m(t) - W_d(t) \equiv W_1(t) - W_d(t). \tag{28}$$

$W_i(t)$ and $W_0(t)$ are exogenous *supply* and *demand* shocks respectively. We can solve the above equations simultaneously for $P(t)$ in terms of $W_1(t)$. Substituting $W(t)$ from eq. (28) into eq. (26), we get:

$$\frac{dP(t)}{dt} = -K_m \left[W_1(t) - W_d(t)\right].$$

Now plugging eq. (27) into the above expression, we get:

$$\frac{dP(t)}{dt} = -K_m \left[W_1(t) + K_d P(t)\right], \text{ or}$$

$$\frac{dP(t)}{dt} + K_m K_d P(t) = -K_m W_1(t), \tag{29}$$

where $W_1(t) = W_i(t) - W_0(t) + W_m(t) \equiv D(t) + W_m(t)$.

$$D(t) = W_i(t) - W_0(t) = \textit{supply rate change} - \textit{demand rate change}.$$

$D(t)$ captures the exogenous shocks. It will change either as a result of a demand shock, a supply shock or both. Now including eq. (20) and assuming $C(t) = 0$ (i.e., production cost does not change), we have the following equations:

$$\frac{dP(t)}{dt} + K_m K_d P(t) = -K_m \left[D(t) + W_m(t)\right], \tag{30}$$

$$W_m(t) = -K_s \varepsilon(t), \tag{31}$$

$$\varepsilon(t) = -P(t). \tag{32}$$

Plugging eq. (32) into (31) and then substituting the value of $W_m(t)$ into eq. (30), we get:

$$\frac{dP(t)}{dt} + K_m K_d P(t) = -K_m \left[D(t) + K_s P(t)\right], \text{ or}$$

$$\frac{dP(t)}{dt} + K_m(K_d + K_s)P(t) + K_m D(t) = 0. \tag{33}$$

The Routh-Hurwitz stability criterion (which provides a necessary and sufficient condition for stability of a linear dynamical system) for stability of above differential equation is $K_m(K_d + K_s) > 0$, which holds as K_m, K_d and K_s are all defined to be positive. This ensures that, away from a given initial equilibrium, every adjustment mechanism will lead to another equilibrium. Laplace transform of the above expression gives:

$$\frac{P(s)}{D(s)} = \frac{-K_m}{s + K_m(K_s + K_d)}. \tag{34}$$

Suppose that a permanent supply (or demand) shock hits the market, i.e., $D(t) = A$. Substituting the transformed expression into the above equation, we get:

$$P(s) = \frac{-AK_m}{s\,[s + K_m(K_s + K_d)]}. \tag{35}$$

Using the Final Value Theorem of Laplace transform, we get:

$$P(\infty) = \frac{-A}{K_s + K_d}. \tag{36}$$

It is evident from the above expression that the final steady state equilibrium is dependent just on the responses of consumer and producer. The parameter, depicting the response of middleman, i.e., K_m dropped out of the final steady state equilibrium expression. Similarly using the Initial Value Theorem of Laplace transform on eq. (35), we get:

$$P(0) = 0. \tag{37}$$

Now let's see how the response of the middleman affects the adjustment path when the prices are flexible and there is no production friction, that is, there is no *dead time* for middleman as well as for producer. Both these assumptions do not seem to be a depiction of the real world as prices are sticky in the short term in reality and the producers cannot instantaneously change production, however, the main purpose is to prove thesis of this research when all ideal conditions of a perfectly competitive market are met. Middleman can influence efficiency on dynamic adjustment path even if a perfectly competitive market is free of production frictions and short term price rigidities. Let's consider the following three cases:

3.1 Case *a*

Let $K_m = 1, K_s = 1, K_d = 1$ and $A = 1$, then by eq. (33), we have

$$\frac{dP(t)}{dt} + 2P(t) = -1. \tag{38}$$

The characteristic function for this differential equation is as follows:

$$x + 2 = 0, \text{ or}$$
$$x = -2.$$

Thus the complementary solution is

$$P_c(t) = C_2 e^{-2t}.$$

The particular solution has the form

$$P_p(t) = C_1.$$

Thus the solution has the form

$$P(t) = C_1 + C_2 e^{-2t}. \tag{39}$$

The constant C_1 is determined by substitution into the differential equation as follows:

$$\begin{aligned} -2C_2 e^{-2t} + 2C_1 + 2C_2 e^{-2t} &= -1, \\ C_1 &= -0.5. \end{aligned}$$

C_2 is determined by the initial condition as follows:

$$\begin{aligned} P(0) = -0.5 + C_2 &= 0, \\ C_2 &= 0.5. \end{aligned}$$

Substituting the values of C_1 and C_2 in eq. (39), we get:

$$P(t) = -0.5 + 0.5e^{-2t}. \tag{40}$$

When $t = 0$, $P(0) = 0$ (the initial condition), and when $t = \infty$, $P(\infty) = -0.5$ (the final steady state equilibrium value). Both these values are consistent with eq. (36) and eq. (37).

3.2 Case *b*

Let $K_m = 2, K_s = 1, K_d = 1$ and $A = 1$, (only the value of K_m has been changed from 1 to 2) then by eq. (33), we have

$$\frac{dP(t)}{dt} + 4P(t) = -2. \tag{41}$$

The characteristic function for this differential equation is as follows:

$$\begin{aligned} x + 4 = 0, &\text{ or} \\ x = -4. \end{aligned}$$

Thus the complementary solution is

$$P_c(t) = C_2 e^{-4t}.$$

The particular solution has the form

$$P_p(t) = C_1.$$

Thus the solution has the form

$$P(t) = C_1 + C_2 e^{-4t}. \tag{42}$$

The constant C_1 is determined by substitution into the differential equation as follows:

$$\begin{aligned} -4C_2 e^{-4t} + 4C_1 + 4C_2 e^{-4t} &= -2, \\ C_1 &= -0.5. \end{aligned}$$

C_2 is determined by the initial condition as follows:

$$\begin{aligned} P(0) = -0.5 + C_2 &= 0, \\ C_2 &= 0.5. \end{aligned}$$

Substituting the values of C_1 and C_2 in eq. (42), we get:

$$P(t) = -0.5 + 0.5e^{-4t}. \tag{43}$$

When $t = 0$, $P(0) = 0$ (the initial condition), and when $t = \infty$, $P(\infty) = -0.5$ (the final steady state equilibrium value). Both these values are consistent with eq. (36) and eq. (37).

3.3 Case c

Let $K_m = 4, K_s = 1, K_d = 1$ and $A = 1$, (only the value of K_m has been changed to 4) then by eq. (33), we have

$$\frac{dP(t)}{dt} + 8P(t) = -4. \tag{44}$$

The characteristic function for this differential equation is as follows:

$$\begin{aligned} x + 8 = 0, &\text{ or} \\ x = -8. \end{aligned}$$

Thus the complementary solution is

$$P_c(t) = C_2 e^{-8t}.$$

The particular solution has the form

$$P_p(t) = C_1.$$

Thus the solution has the form

$$P(t) = C_1 + C_2 e^{-8t}. \tag{45}$$

The constant C_1 is determined by substitution into the differential equation as follows:

$$-8C_2 e^{-8t} + 8C_1 + 8C_2 e^{-8t} = -4,$$
$$C_1 = -0.5.$$

C_2 is determined by the initial condition as follows:

$$P(0) = -0.5 + C_2 = 0,$$
$$C_2 = 0.5.$$

Substituting the values of C_1 and C_2 in eq. (45), we get:

$$P(t) = -0.5 + 0.5e^{-8t}. \tag{46}$$

When $t = 0$, $P(0) = 0$ (the initial condition), and when $t = \infty$, $P(\infty) = -0.5$ (the final steady state equilibrium value). Both these values are consistent with eq. (36) and eq. (37). Figure 12 depicts the dynamics of the price of the market for all above three cases from one equilibrium to the other after an economic shock. In all above cases, we got the same steady state equilibrium change, i.e., $P(\infty) = -0.5$. This is due to the fact that the responses of producer and consumer were kept fixed and the final steady state equilibrium depends only on their responses as shown in eq. (36). However, as we increased the value of K_m, which means changing the response of middleman (K_m is the slope of the inventory curve, a large value of K_m in absolute sense, indicates that middleman changes price by a large magnitude for a small change in inventory), the system's adjustment path to the new steady state equilibrium changed. The vertical line in figure 12 is for ideal dynamic efficiency, where the market jumps from one equilibrium to the next. This ideal dynamic efficiency is not achievable through any practical means, however, some options are better than others, e.g., case c is the closest to the ideal dynamic efficiency as compared to the other two cases, which shows that if middleman reacts strongly to the shock and changes price by a large magnitude for a small change in inventory (which is possible if three economic agents have better coordination with each other), the market will quickly adjust to the new equilibrium with smaller efficiency loss.

A pile up of inventory after an economic shock indicates a higher supply than demand, and a depletion of inventory occurs when demand is higher than supply in a given time period. When demand and supply are same, there is no efficiency loss. If demand and supply are different, output and/or consumption is being lost at that point in time. Therefore if we sum up the inventory change at all points in time, we get total efficiency loss, which is as follows:

$$EL = \int_{0}^{t} [W_m(t) - W_d(t)]\, dt = M(t). \tag{47}$$

The area under the curves in figure 13 is the dynamic efficiency loss for all three cases, which is the smallest in case c (strong coordination) as compared to those for cases a (free market) and b (weak coordination). The characteristic function of eq. (33) has a single root given by:

$$x = -K_m(K_s + K_d) = -K.$$

If any of the $K's$, i.e., K_s, K_d or K_m (the slope of supply, demand or inventory curve respectively) increase, the root moves on negative real axis and away from the origin, so market response to the new equilibrium becomes faster. However, it is important to notice that K_m is being multiplied by the sum of K_s and K_d, therefore the middleman has a stronger role on adjustment path than that of producer or consumer.

4 Conclusion

When an economic shock hits a perfectly competitive market, the market remains out of equilibrium during the adjustment period before it arrives at a new steady state equilibrium. The longer the adjustment period, the greater are the efficiency losses. An ideal dynamically efficient market would jump from one equilibrium to the other after an economic shock, and the market would always stay in equilibrium without an efficiency loss, however, this ideal situation is not practically achievable. An achievable dynamic efficiency is the smoothest and the shortest possible route which the market may adopt to move to the new steady state equilibrium after an economic shock. This path is determined by the collective action of all agents in the market, i.e., producer, middleman and consumer. A free market on its own cannot minimize efficiency loss during adjustment to the final equilibrium due to lack of perfect information of magnitude of shock and new patterns of supply and demand by economic agents. However, if the agents coordinate among themselves for their economic activity, they can minimize efficiency loss during adjustment period.

Most of transactions between producer and consumer take place through middleman, therefore middleman can play a significant role during adjustment process. As, the middleman acts like a post office, the final equilibrium is determined only by responses of producer and consumer as shown in eq. (36). However, middleman has an important role during adjustment of market. Suppose that after a demand shock (while supply remains the same), a consumers' representative inform middleman about new demand pattern at various prices, middleman will be able to change price much more quickly and efficiently to bring final equilibrium as compared to that in a free market where middleman will get a signal of a demand shock only through a variation in quantity of his inventory (because consumers are not obliged to inform middleman about their changed preferences), which will involve more time and efficiency loss before adjustment to new equilibrium takes place. Similarly if a shock happens on supply side (while demand remains the same), such as a marginal cost change due to some technological innovation, producers can lead to lower efficiency losses if they coordinate with middleman through provision of information about new marginal cost. The incentives for coordination may vary and there can be some short term winners at the expense of others in a free market, however, coordination can improve overall economic efficiency on adjustment path of market to equilibrium. As equilibrium is a short lived condition in a dynamic world where shocks happen quite frequently, efficiency during adjustment of market cannot be ignored.

A well informed middleman can take a stronger action regarding change in price as a result of a change in inventory or even before an actual inventory change takes place. In all three cases mentioned in previous section, only response of middleman has been varied through the value of K_m while responses of producer and consumer have been kept fixed. The dead times for producer and middleman have been assigned zero values which implies flexible prices and no production friction (i.e., ideal conditions of a perfectly competitive market are maintained). The plot for cases a (free market), b (weak coordination) and c (strong coordination), i.e., figure 12 shows that market arrives at the same new steady state equilibrium in all three cases. However, increasing the value of K_m (the slope of the inventory curve), which implies improved coordination among three agents, makes transition to the new equilibrium as short as possible, thus bringing market closer to the ideal dynamically efficient system. Like all natural systems, market has a self-regulation property, however it has certain limitations. In spite of an optimal behavior of economic agents, market can never achieve an ideal dynamic efficiency as depicted in figure 12. However, a transition path closer to the ideal situation (and practically achievable) is more desirable than others which is attainable through some coordinated actions by economic agents.

5 Future Research Prospects

Some potential future research areas are as follows:

Dynamic welfare analysis: A complete dynamic welfare analysis against various governmental policies is important and this research paper provides a foundation for that.

Extensions to various kinds of markets in economy: This paper serves as a fundamental unit for applications and extensions to various markets, such as goods, services, financial and labor markets, etc., in economy.

Macroeconomic Model: This paper is a foundation stone for developing a macroeconomic model for business cycles by interconnection of n number of markets for n commodities in economy.

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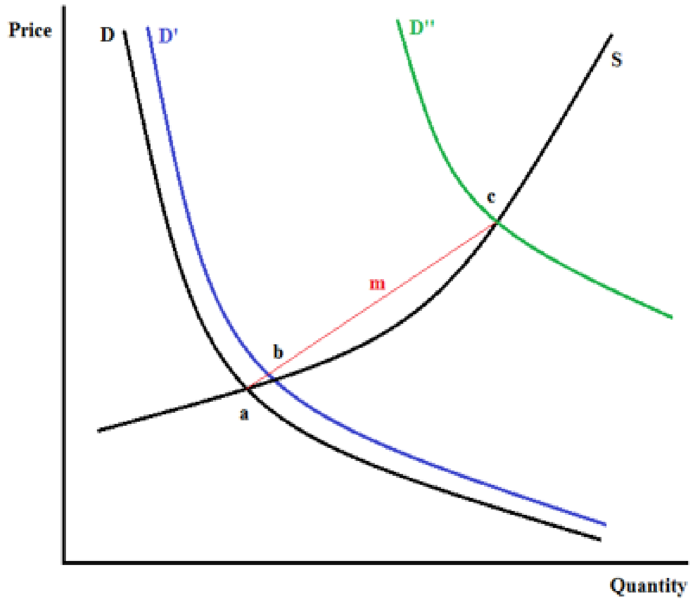

Figure 1: When is Linearity a Reasonable Assumption?

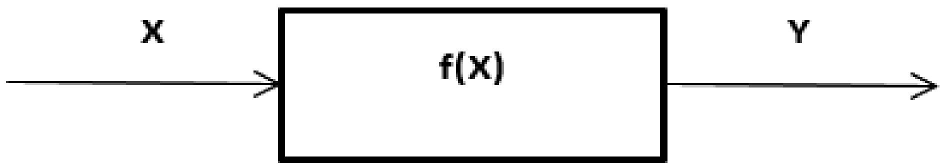

Figure 2: Block Diagram Representation of Input and Output.

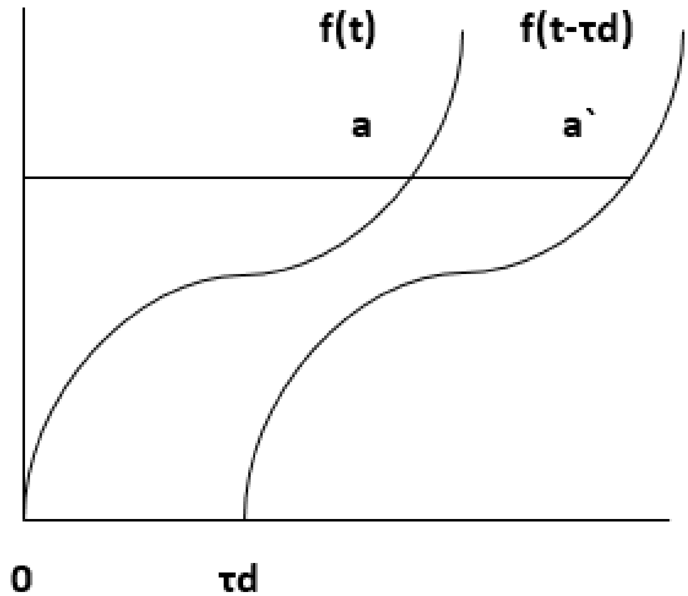

Figure 3: Concept of *Dead Time.*

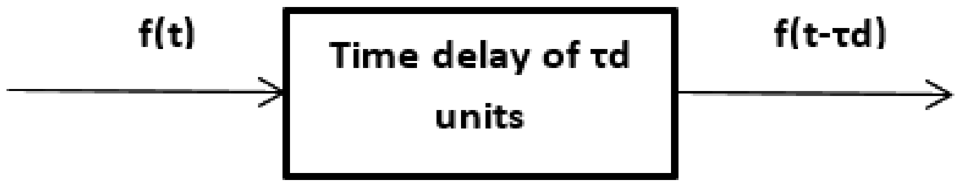

Figure 4: *Dead Time* between Input and Output.

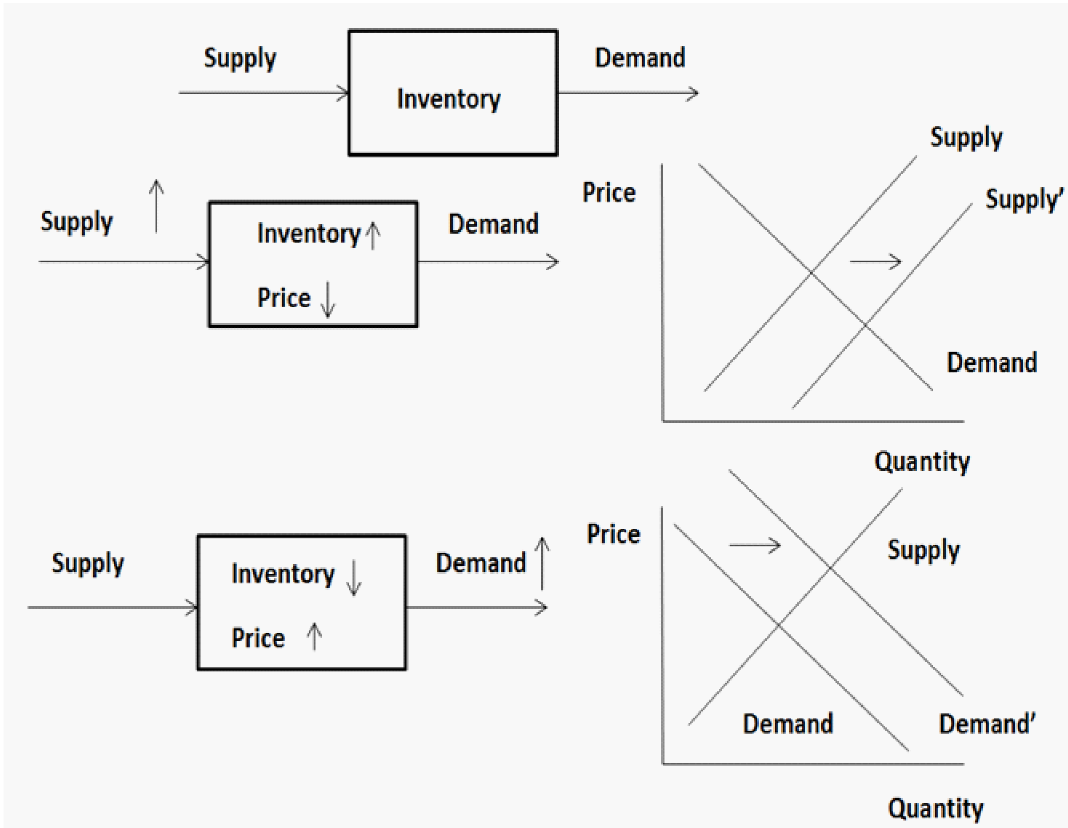

Figure 5: Movement of Price with Inventory.

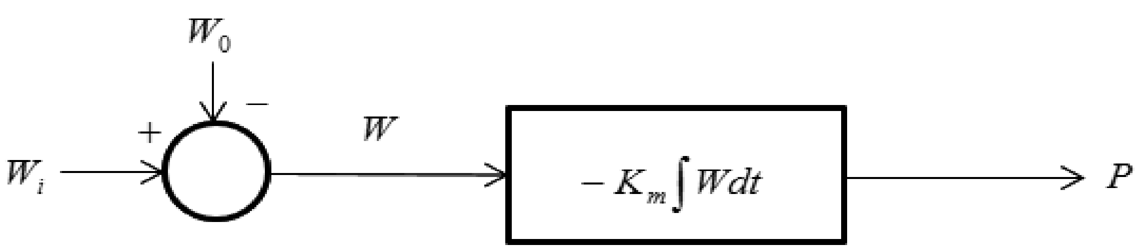

Figure 6: Block of the Middleman without *Dead Time*.

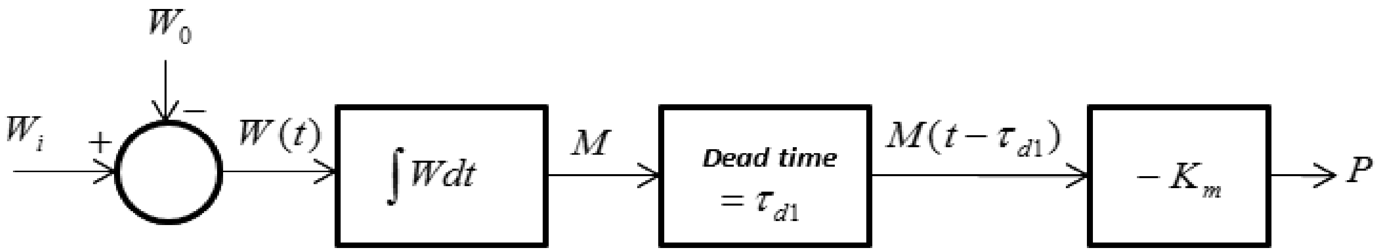

Figure 7: Block of the Middleman with *Dead Time.*

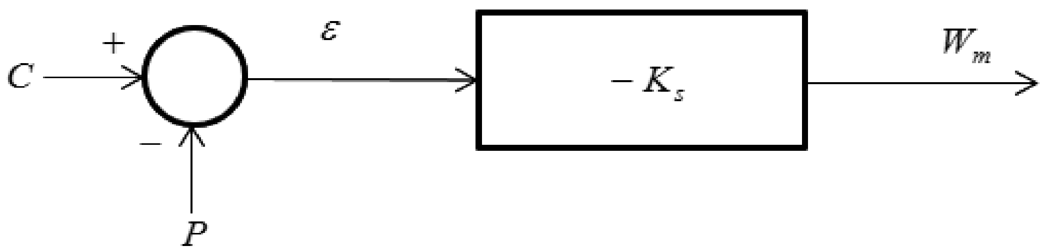

Figure 8: Block of the Producer without *Dead Time.*

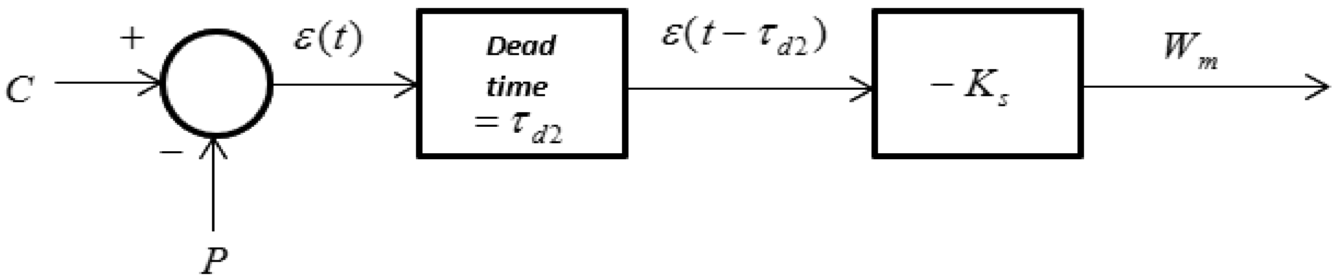

Figure 9: Block of the Producer with *Dead Time.*

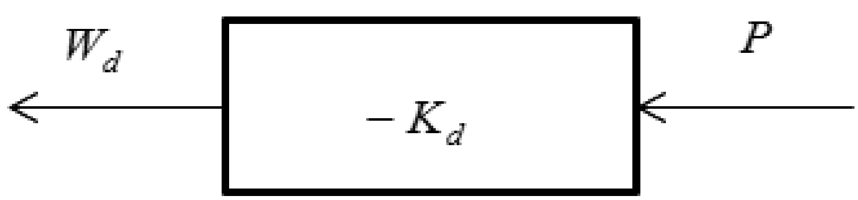

Figure 10: Block of the Consumer.

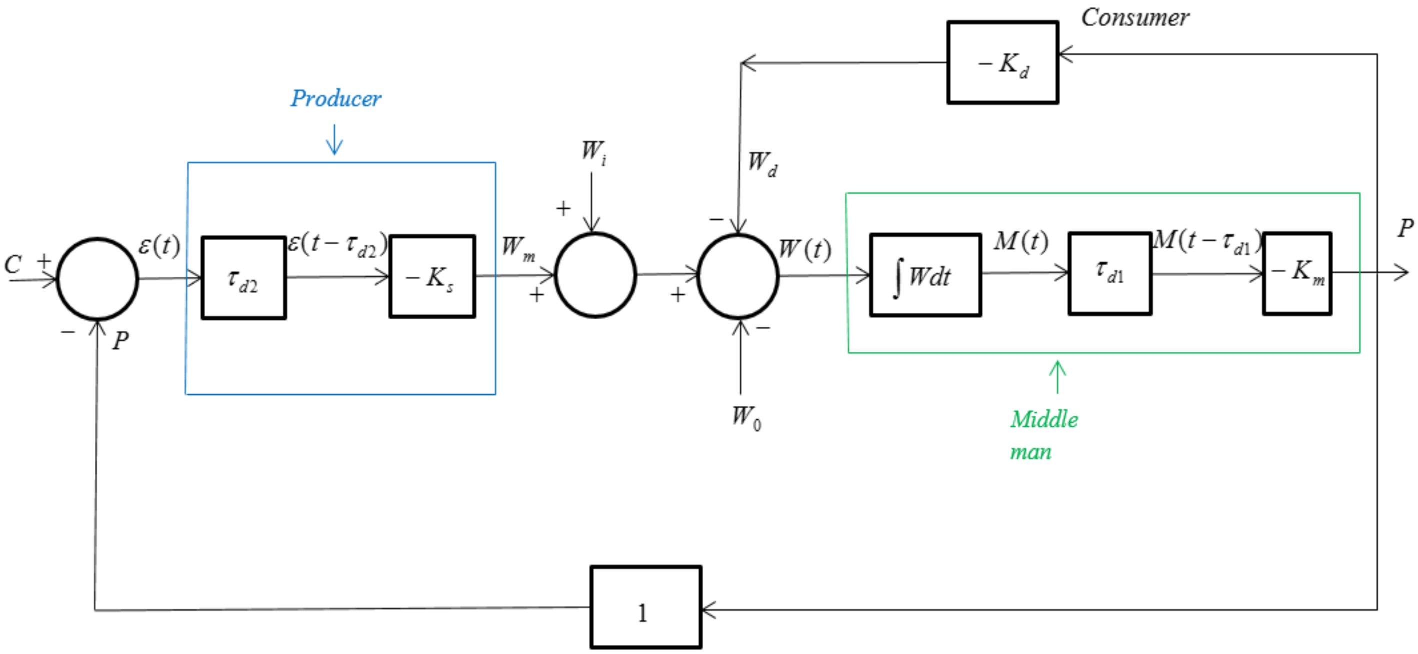

Figure 11: Dynamic Market Model in Block Diagram Representation.

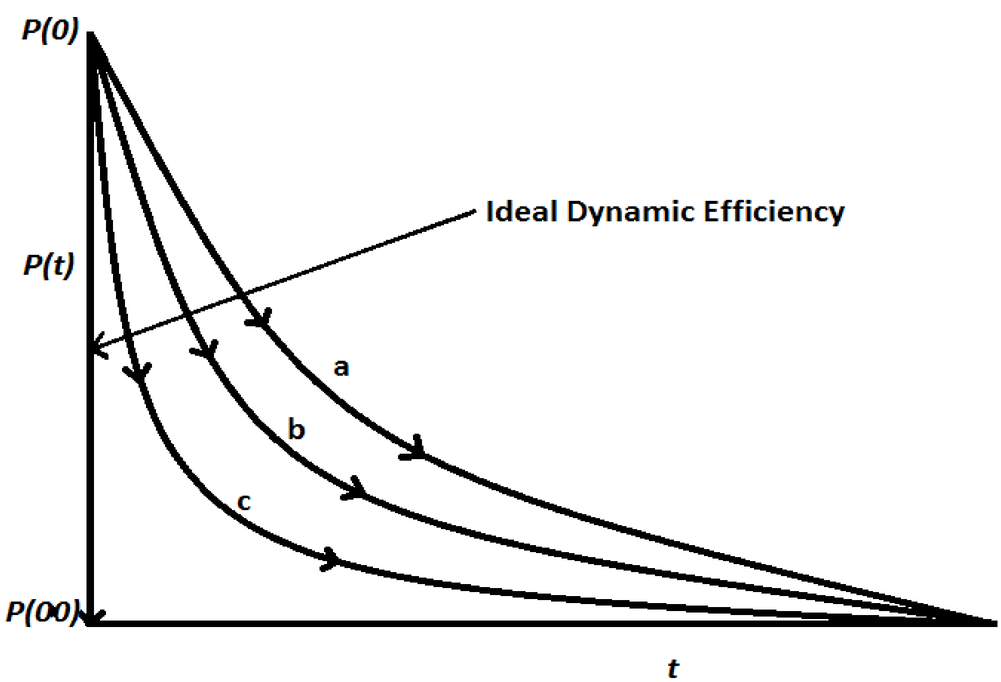

Figure 12: Plot of $P(t)$ versus t for Cases a, b and c.

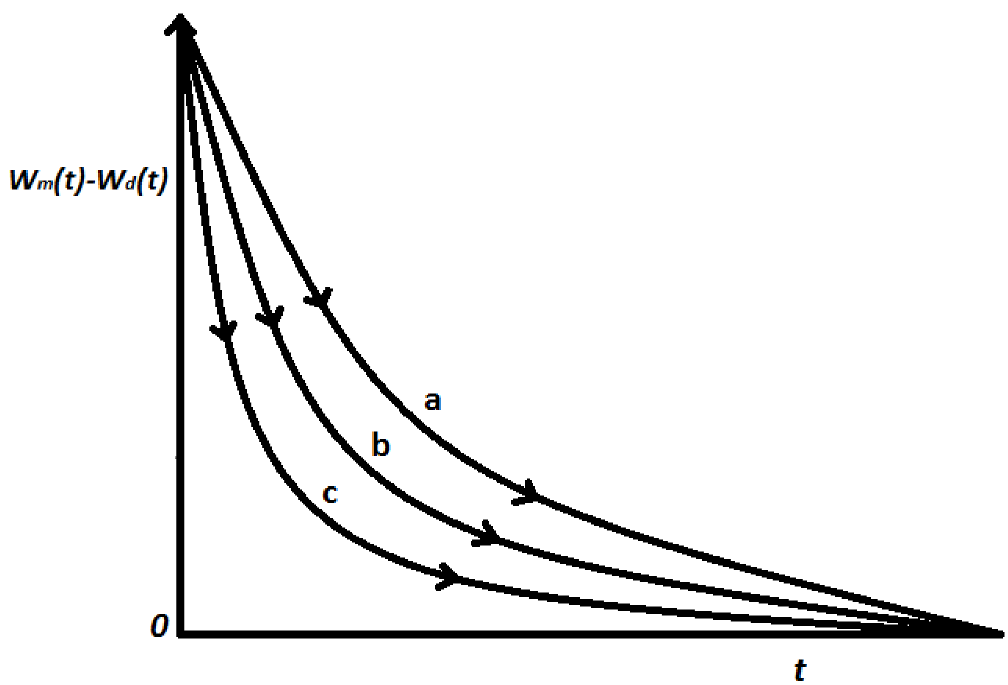

Figure 13: Plot of $[W_m(t) - W_d(t)]$ versus t for Cases a, b and c.