Economic Effects of Covid-19 and Non-Pharmaceutical Interventions: applying a SEIRD-Macro Model to Italy

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Abstract

We study the economic effects generated by the proliferation of the Covid-19 epidemic and the implementation of non-pharmaceutical interventions by developing a SEIRD-Macro model, where the outbreak and policy interventions shape the labour input dynamic. We microfound an Epidemic-Macro model grounded on the Neo-Classical tradition, useful for epidemic and economic analysis at business cycle frequency, which is able to reproduce the highly debated health-output trade-off. Assuming a positive approach, we show the potential of our model by matching the epidemic and macroeconomic empirical evidence of the Italian case.

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1 Introduction

In this work we propose an analysis on the dynamics of the Covid-19 outbreak along with its effects on the economy, developing an epidemiological-macroeconomic model, which also takes into account the implementation of Non-Pharmaceutical Interventions (NPIs).[1]

The simultaneous presence of an infectious disease and NPIs have pushed economists to analyse the generation and exacerbation of epidemic-driven economic shocks. As highlighted by Baldwin and di Mauro (2020), these shocks are triggered by three sources: Covid-19 generates a labour supply shock, as ill and deceased workers cannot supply labour. Further negative shocks are triggered by NPIs as these temporary shut down all activities that require a high level of physical contact, impose a lock-down regime on a portion of the population and suspend transport routes through air and sea. Thus, NPIs affect both the aggregate supply - via a disruption of industrial production - and the aggregate demand - through shrinkage of exports, imports and consumption. Households and Firms reaction to the epidemic evolution hit negatively aggregate demand. Accordingly, demand for goods and services drops as consumers follow saving-for-emergency, wait-and-see and hoarding strategies. Firms reduce investments due to closures of company plants, supply-chain contagions and negative expectations on future economic activity.

Part of the literature has theorised and modelled aggregate demand shortages triggered by aggregate supply plunges, envisaging the recurrence of a supply-demand vicious spiral (Fornaro and Wolf, 2020; Guerrieri et al., 2022). McKibbin and Fernando (2021) have considered multiple exogenous supply and demand shocks, while for Faria-e-Castro (2021), the Covid-19 outbreak generates a demand shock. Ciccarone et al. (2021) and Gagnon et al. (2020) set up an analysis on theintergenerational costs and benefits of pandemics and lockdowns, employing a life-cycle macroeconomic scheme.[2] Other researchers have focused the analysis on economic as well as health issues, developing a

[1]NPIs - also known as social-distancing measures - concern disruptions and closures of business activities and travel, testing and mask-wearing requirements as well as lock-downs on the population.

[2]These authors have modelled the epidemic shock in different ways. See Fornaro and Wolf (2020) for a permanent fall in the growth rate of labour productivity; see Guerrieri et al. (2022) for a one-period reduction in the labour supply; see Fernando and McKibbin (2021) for simultaneous reduction in labor supply, a rise in the cost of doing business, shifts in consumer preferences and increase in equity risk premia on companies and countries; see Faria-e-Castro (2021) for a negative shock to the marginal utility of consumption; see Ciccarone et al. (2021) and Gagnon et al. (2020) for an exogenous shock to the mortality rate of the elderly.

new category of models that can be defined as SIR-Macro/Health models, where epidemic dynamics are combined with an health-related macroeconomic framework (Alvarez et al., 2021; Eichenbaum et al., 2021; Gonzalez-Eiras and Nieptal, 2020; Piguillem and Shi, 2020). These studies have in common the formulation of a maximisation problem of a Social Planner who seeks to find the optimal social distancing policies taking into account a health-output trade-off.[3]

As for the epidemiological model, we employ a standard SIR model augmented with the inclusion of the exposed (E) and deceased (D) compartments, yielding the well-known SEIRD model.[4] Given the wide-ranging NPIs adopted by governments, epidemiologists have modified standard models to take into account their effects on transition probabilities, thus formulating models with time-varying parameters.[5] In this respect, we follow the idea of Romano et al. (2020), who define the transition probabilities as a direct function of a proxy of the NPIs.[6] The case fatality rate is another parameter that is highly affected by external factors.[7] We consider these externalities indirectly, setting the case fatality rate time-varying, described by a step-function.

As for the economic model, differently from the aforementioned literature, we set up a model grounded on the Neo-Classical tradition. To link the Macro framework with the SIR framework, we adapt the indivisible labour Hansen model (Hansen, 1985), which introduces a distinction between the intensive and extensive margin of labour, where their product defines the actual worked hours. Starting from this specification, we let the SEIRD block and NPIs determine the path of the extensive margin, while the intensive

[3]The Social Planner manages a trade-off as individuals, on one hand, benefit from the mitigation of the virus, and, on the other, worsen economically due to disruptions in activities.

[4]See Atkeson (2020), Cadwell et al. (2021) and Piguillem and Shi (2020) for studies on Covid-19 employing SIR models with exposed individuals, while see Loli Piccolomini et al. (2020) and Romano et. al (2020) for SEIRD models. See Romano et al. (2020) and Giordano et al. (2020) for more sophisticated models involving additional epidemic compartments.

[5]The effect of NPIs is captured by changing the infection rate at discretion (Giordano et al., 2020), by defining it as a step-function (Noll et al., 2020) or as a direct function of time (Loli Piccolomini et al., 2020).

[6]From the *Google Covid-19 Community Mobility Report*, they construct a mobility function as the weighted average on the mobility data in different places. This parameter gives an idea of how NPIs have modified community movements in specific locations. Differently, we utilise the *Containment and Health* Index (CH^{Index}) - computed by Hale et al. (2021) - and derive a continuous function describing how the CH^{Index} perturbs the infection rate.

[7]In particular, the inception of new variants, vaccines and deficiencies in the healthcare system can alter its value (Cadwell et al., 2021; Sadeghi et al., 2020)

margin is efficiently chosen by economic agents. In this respect, we are close to Goenka and Liu (2012) who also adopt a discrete time one sector growth model where the the extensive margin is fixed by the epidemic model while the intensive margin is determined by the supply decisions of healthy workers. Still, our analysis differs in many respects: first, we adopt a SEIRD model whereas Goenka and Liu (2012) employ a SIS specification; second, we do not follow a normative approach, but a positive one, since we aim to build a model that can replicate the empirical evidence for the Italian case. By contrast, the goal of Goenka and Lin (2012) is not to match any observed data, but rather to perform a theoretical analysis of the dynamic properties of their Macro-SIS model with a particular focus on possible emergence of periodic cycles and chaotic dynamics;[8] third, we are interested in analysing the epidemiological and economic effect of NPIs interventions while Goenka and Lin (2012) focus on vaccination and isolation of infective individuals as an instrument to stabilize the economy; finally, to match the drop in consumption observed in the data, we distinguish between *Optimising* and *Rule-of-Thumb* consumers as, for instance, in Galì et al. (2007);[9]

Three versions of the model are presented, each featuring a labour supply shock. In the simplest version, Model 0 (M_0), the possibility to work is restricted only by the harmfulness of the disease. In Model 1 (M_1), we show how the implementation and, successively, the removal of NPIs produce multiple epidemic waves. In Model 2 (M_2), we follow a quantitative approach, namely, the epidemic and NPIs shocks are modelled in order to replicate both the epidemiological and business cycle empirical evidence of Italy in the period 24/02/20-24/02/22.

The main results of this study can be summarised as follow. First of all, we developed a model for epidemic and economic analysis which is stunningly simple. Actually, M_0 can be seen as a toy model and a starting point for more complex analyses, which seek to capture further aspects. The possibility of reproducing epidemic waves - as done in M_1 - is an example. Secondly, comparing M_0 with M_1 shows that when the government has to decide whether to intervene with NPIs and choose their strictness, it bears a trade-off inasmuch as interventions reduce the number of infections and

[8]The SEIRD model that we employ is characterized by a unique and stable steady state.

[9]Our approach diverges also from the aforementioned step-up of the SIR-Macro/Health literature, inasmuch as our model is microfounded along the lines of the business cycle tradition. Basically, the SIR-Macro/Health literature employs health-related models, in which the relevant economic trade-offs are shaped by the health status of the economic agents and by the transition probabilities between epidemic compartments. This comes at the cost of giving up the canonical behavioural equations of economic agents, namely, the Euler equations are absent from the analysis.

save lives in the face of a more forceful and long-lasting economic recession. Finally, M_2, our most sophisticated version of the model, is able to match suitably the empirical evidence of Italy, reproducing the path of the five waves of infections and the quarterly conjunctural growth rate of the main macroeconomic variables.

2 The Epidemiological-Macroeconomic Model

We employed a discrete-time deterministic SEIRD model, where individuals are separated into groups - or compartments - depending on their disease status and can move from one compartment to another as the infectious virus spreads among the population.

The economy is featured by a Neoclassical Growth model, whose standard version is modified by two specifications. We revive the Hansen's version of indivisible labour model (Hansen, 1985) - making, however, some adjustments - and introduce one heterogeneity among households: they are distinguished between *Optimising* and *Rule-of-Thumb* as described in Galì et al. (2007).

2.1 The Epidemiological Framework

In a canonical discrete-time SEIRD model[10] the population is subdivided among five compartments: susceptible (S), where individuals have not been infected with the disease and are exposed to the risk of infection; exposed (E), composed of individuals who have been infected but cannot transmit the disease due to the incubation period;[11] infected (I), composed of individuals who have been infected with the disease and are capable of spreading it to those in the susceptible compartment; recovered (R), where individuals have been infected and then recovered from the disease through immunisation; deceased (D), composed of individuals who have deceased due to the infectious illness.

Since we are mainly interested in short-run analysis, we consider a closed population. Accordingly, this model does not take into account births, migration and deaths which are not related to the disease.[12]

[10] The model is deterministic, allowing us to exactly predict the path of epidemic variables throughout the progression of the infectious disease.

[11] i.e. the period between exposure and onset of clinical symptoms, in which the virus cannot be transmitted. In addition, in this model we assume the absence of asymptomatic individuals and, hence, exposed individuals can only be in the pre-symptomatic phase. For models with asymptomatic individuals, see Giordano et al. (2020) and Romano et al. (2020).

[12] The closed population hypothesis in a SIR model assures the presence of one stable

The rate at which susceptible individuals become exposed depends on the number of susceptible and infected individuals and their effective contact rate b. This parameter - also known in the literature as the infection rate - accounts for the transmissibility of the disease as well as the mean number of contacts per individual.[13] Consequently, the whole number of susceptible individuals contracting the disease from infectious individuals per unit of time is given by the product bS_tI_t. In other words, this term represents the number of newly exposed individuals in time t. Then, only a fraction ε of exposed individuals become infected in each period. The rate at which infected individuals move into the recovered compartment depends on the amount of time during which an individual is contagious, which is captured by the recovery rate γ.[14] Finally, the rate at which infected individuals decease is indicated with μ, known as the case fatality rate.

The infectiousness of a disease - namely its transmissibility within a population - is described by the *basic reproduction number*, R_0. This parameter characterises the initial propagation of an infectious virus and, in a SEIRD model without births and non-disease-related deaths, is computed as the ratio of the infection rate and the sum of the recovery and case fatality rate, $R_0 = b\,(\gamma + \mu)^{-1}$.[15] Hence, the infection rate can be written as:

$$b = R_0(\gamma + \mu) \tag{1}$$

Summing up, the epidemiological model is described by the following first

steady state for each parameter configuration. This is in contrast to what happen in SIS models where periodic fluctuations and chaotic dynamics may emerge (see Goenka and Liu, 2012; Fisman, 2007 and Allen, 1994)

[13] The model assumes homogeneous mixing of the population, meaning that all individuals in the population have an equal probability of making contact with one another. However, this does not reflect human social structures, where contact occurs within limited networks.

[14] ε^{-1} and γ^{-1} represent the incubation period and the average time of being infected, respectively.

[15] The basic reproduction number is the average number of secondary infections produced by a typical case of an infection in a population where everyone is susceptible. In general, an infectious disease spreads within a susceptible population if R_0 is greater than one. Under this condition, the number of infected individuals increases exponentially over time. Conversely, if $R_0 < 1$, the virus does not spread as the number of infected agents converges monotonically to zero, while with $R_0 = 1$ the infection remains constant.

order non-linear difference equations:

$$S_{t+1} - S_t = -b\frac{S_t I_t}{P} \tag{2}$$

$$E_{t+1} - E_t = b\frac{S_t I_t}{P} - \varepsilon E_t \tag{3}$$

$$I_{t+1} - I_t = \varepsilon E_t - \gamma I_t - \mu I_t \tag{4}$$

$$R_{t+1} - R_t = \gamma I_t \tag{5}$$

$$D_{t+1} - D_t = \mu I_t \tag{6}$$

where population is defined as $P = S_t + E_t + I_t + R_t + D_t$ and with the initial condition $S_0 > 0$ and $I_0 > 0$. Additionally, it is imposed that $S_t, E_t, I_t, R_t, D_t \geq 0$ and $S_t + E_t + I_t \leq P$.

2.2 Linking the SEIRD and RBC Blocks

An epidemic can be seen as an external real shock that perturbs the equilibrium of the economic system. In detail, it damages the health of individuals, compromising their ability and propensity to work and produce until recovery is achieved. After having established which epidemic compartments are still able to work, the SEIRD model deterministically derives the path of the available labour force over time. As result, macroeconomic variables display short-run variations until long-run equilibria are reached again.

We assume that the labour force is equal to the population net of infected and deceased individuals. As a matter of fact, we are supposing that infected individuals are too ill to work, while exposed individuals can still supply labour since they are in a pre-symptomatic phase. Hence, in the absence of NPIs, the labour force is given by:

$$N_t = S_t + E_t + R_t \tag{7}$$

From equation (7) it is possible to define the employment rate:

$$n_t = \frac{N_t}{P - D_t} \tag{8}$$

We stress that n_t is the variable that generates a bridge between the two frameworks. As long as the infectious disease is in circulation, the employment rate will change, modifying economic agents' behaviour.

2.3 The Macroeconomic Framework

In this study, we employ a simple neoclassical economic model where the economy is characterised by *indivisible labour* (Hansen, 1985). From Hansen (1985), we borrow the separation between extensive and intensive margin and the shape of the utility function,[16] making, however, some adjustments. In our model, the extensive margin, interpreted as the employment rate (n_t), is not a control variable for households and firms, but it is determined by the epidemiological model.[17] Regarding the intensive margin, represented by the worked hours (h_t^0), differently from the Hansen model, it is not constant but time-dependent. At each time, it adjusts to variations of the extensive margin and of the actual worked hours (h_t) in order to guarantee the following relation:

$$h_t = n_t h_t^0 \tag{9}$$

where variables are in per capita terms. Hence, the product of the two margins represents the actual worked hours in each time. As shown successively, h_t is set in privately efficient way through the simultaneous solution of households and firms' maximisation problems.

We assume that households' utility function is logarithmic in consumption and still linear in labour, even though the coefficient that multiplies actual worked hours, v_t, is no longer constant as in Hansen (1985). Accordingly, it is represented by the following equation:[18]

$$U(c_t, h_t; h_t^0) = \log(c_t) - v_t h_t \tag{10}$$

Hereafter, we refer to v_t as the Hansen's variable, which is defined by:

$$v_t = -\frac{\psi}{h_t^0}\left[\frac{(1-h_t^0)^{1-\theta} - 1}{1-\theta}\right] \tag{11}$$

where ψ and θ are the households' relative preference for labor and the inverse of the elasticity of labor, respectively.

To match the negative comovement between private consumption and virus spread waves (with related NPIs), we assume heterogeneous households,

[16] In Hansen (1985), households get to choose the probability of working, but once employed, they work for a fixed amount of hours. Thanks to this lotteries setup, the households' maximization problem returns to be convex, and the utility function is linear in labour irrespective of the individual elasticity of labour supply.

[17] This assumption is particularly suitable for the case of Italy, where the government has introduced a ban on dismissals on 17th March 2020 through the *Cura Italia* decree law, which has been gradually removed from the end of 2021.

[18] The derivation of the utility function is presented in appendix A.

namely, Optimising (Opt) and Rule-of-Thumb (RoT) households in the sense of Galì et al. (2007). In particular, RoT consumers are financially constrained so that they are assumed to consume their current income fully. Labour is supplied by both households, while capital is provided only by Opt households. Firms employ the two production factors in the production process, remunerating labor with the real wage (w_t) and capital (k_t) with the real interest rate (r_t). Markets are perfectly competitive and complete.

From the solution of the maximisation problem of RoT and Opt households and firms,[19] we obtain the following equations, i.e. the Euler equation of Opt households, the labor supply of RoT and Opt households and the capital and labor demand of firms:

$$\frac{1}{c_t^p} = \beta E\left[\frac{1+r_{t+1}}{c_{t+1}^p}\right] \tag{12}$$

$$w_t = v_t^p c_t^p \tag{13}$$

$$w_t = v_t^r c_t^r \tag{14}$$

$$r_t = (1-\phi)\frac{y_t}{k_t} - \delta \tag{15}$$

$$w_t = \phi\frac{y_t}{h_t} \tag{16}$$

where variables with the superscript p and r refer to Opt and RoT households, respectively. Households' time preference is indicated with β, while δ and ϕ are the deprecation rate of capital and the labor share of production, respectively. Aggregation is computed through a weighted average of the corresponding variables for each household type, i.e. $x_t = (1-\lambda)x_t^p + \lambda x_t^r$ with $x = \{h_t^0, c_t, i_t, k_t\}$ and where λ represents the share of rule-of-thumb households in the economy.

Combining the SEIRD with the Macro block, we derive the simplest version of our model - labeled as Model 0 (M_0) - where no NPIs are established.[20] M_0 may be seen as a counterfactual scenario, which illustrates epidemic and economic outcomes when the government decides not to intervene to hinder the outbreak from spreading.

2.4 Benchmark Calibration

As for the calibration of the SEIRD model, the basic reproduction number and the incubation period is derived from Romano et al. (2020), equal to

[19] See appendix B.

[20] The equilibrium equations and the steady state relations of the economic model are illustrated in appendix C and D, respectively.

3.8 and 3 days, respectively.[21] The number of days to recover is equal to 10, which is in line with values employed by Angeli et al. (2022),[22] and the case fatality rate is set equal to 2%, which is within the value range used by Loli Piccolomini et al. (2020) and Piguillem and Shi (2020). Lastly, from equation (1) we compute the value of the infection rate, equal to 0.456.[23] Table 1 summarises our calibration.

Macroeconomic parameters assume standard values, converted on a daily basis. The daily time preference β and the daily depreciation rate δ are set to match a yearly real interest rate equal to 4.04% and a yearly depreciation rate equal to 10%. In order to make comparisons across the three versions of the model, we maintain the same calibration for common parameters. Hence, θ and λ - given the wide range of values provided by the literature - are calibrated by adopting a sensitivity analysis to increase the matching with real data. The two households' relative preferences for labour (ψ^p and ψ^r) are determined from the steady state labour supply and budget constraint imposing that in steady state $\bar{h}^{0,p} = \bar{h}^{0,r} = 1/3$ and $\bar{n} = 1$.[24] The calibration of the economic model is displayed in table 2.

Table 1: SEIRD block Calibration

Definition	Parameter	Value	Reference
Population	P	$59,641,488$	[23]
Initial Infections	I_1	221	[27]
Initial Recoveries	R_1	1	[27]
Initial Deceases	D_1	7	[27]
Basic Reproduction Number	R_0	3.8	[15, 31]
Incubation Rate	ε	1/3	[19, 31]
Recovery Rate	γ	1/10	[3, 10, 29]
Infection Rate	b	0.456	[16, 25, 31]
Case Fatality Rate	μ	0.02	[25, 29]

[21]The adopted R_0 is in the range of values estimated by Gatto et al. (2020) and ε lies in the in the range of values obtained by Guan et al. (2020)

[22]and close to the value assumed by Piguillem and Shi (2020) and Ferguson et al. (2020).

[23]The obtained infection rate is in line with the values assigned by Giordano et al. (2020), Loli Piccolomini et al. (2020) and Romano et al., (2020).

[24]See Appendix D.

Table 2: RBC block Calibration

Definition	Parameter	Value
Daily Time Preference	β	0.9992
Daily Depreciation Rate	δ	$2.74e^{-4}$
Labor Share of Output	ϕ	0.64
Inverse of the Elasticity of Labor	θ	3
RoT Households' Relative Preference for Labor	ψ^r	1.6
Opt Households' Relative Preference for Labor	ψ^p	0.8843
Share of RoT Households in the economy	λ	0.8

Figures 1 displays the dynamics of the SEIRD and RBC blocks produced by M_0, where for each macroeconomic variable the relative deviation from the steady state ($\tilde{x_t}$) is conveyed, except for the real interest rate where we display the absolute deviation from the steady state.

As the virus kicks in, labour force reduces due to ill workers, namely, the outbreak generates a fall in the endowment of the labour input and, consequently, the extensive margin n_t reduces (-31.3%). Simultaneously, the intensive margin h_t^0 increases (18.8%). This occurs since disease-free members of the representative household are willing to work more hours in order to compensate for the lower income yielded by ill members. Likewise, firms want to compensate for the reduction of the extensive margin, which otherwise will generate a sharp plunge in production. However, the shrinkage of the extensive margin drowns out the rise of the intensive margin, resulting in a tumble of the actual worked hours h_t (-18.3%). Then, the plunge in actual worked hours undoubtedly generates a recessive effect on output (-12.2%) and since households' income resources are reduced, consumption and investment decline (-7.1% and -26.9%). Even though households are averse to deep consumption reductions as they prefer to smooth consumption over time, i.e. as income drops households are willing to reduce savings on behalf of consumption, the presence of RoT households (accounting for 80%) weakens the smoothing of consumption. As a result, the fall of consumption is significant.

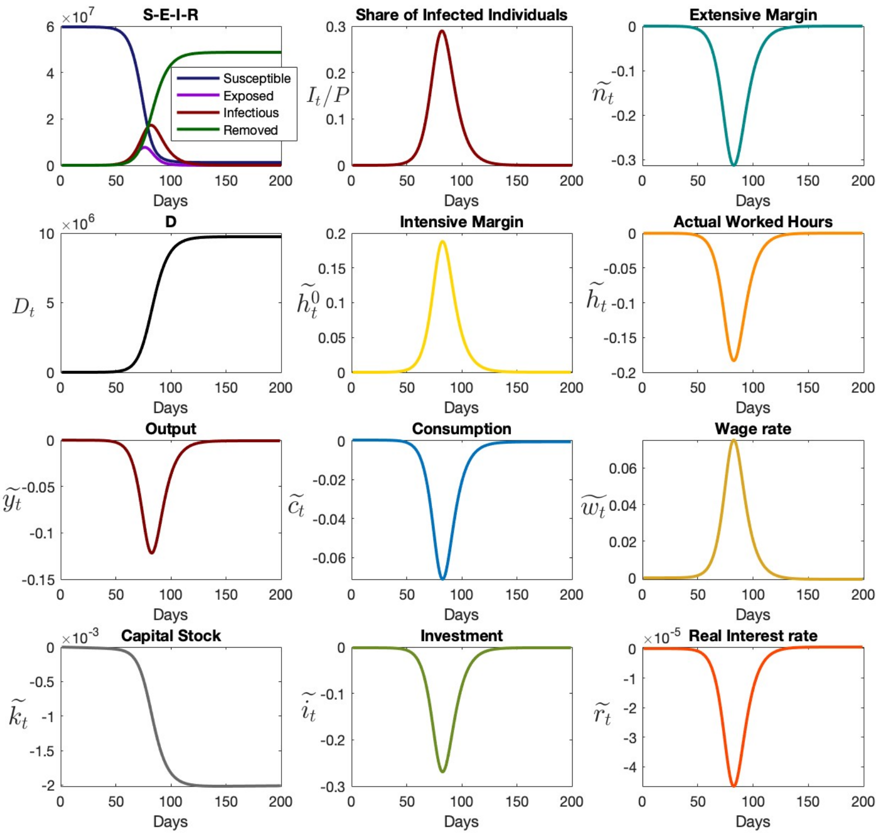
Successively, less investment today generates a lower capital accumulation tomorrow, determining a fall in the capital stock over time.[25] Considering

[25]From day 149, capital stock increases. However, due to the small value of investments, it augments slightly taking a long time before reaching the steady state again, which occurs after the time frame of our simulation.

the reduction in labour input, firms have to adjust the level of capital stock. Therefore, capital demand moves backward. This shift more than compensates for the reduction in savings supply as shown by the fall of the real interest rate. Conversely, the increase in the hourly wage rate w_t (7.5%) implicates that the fall in labour supply is higher than the drop in labour demand.

Finally, macroeconomic variables converge to their steady state value as soon as the epidemic is expelled from the population.

Figure 1: SEIRD Model and IRFs of Macroeconomic Variables (M_0)



Note: We report the first 200 days out of 732 of our simulation, corresponding to the period 24/02/20-24/02/22. In the top-left corner is shown the time path of susceptible, exposed, infected and removed individuals. The time path of deceased individuals and share of infected individuals is displayed, respectively, under and on the right of the latter panel. The rest of the panels exhibit the relative deviation from steady state of the intensive margin, extensive margin, actual worked hours, output, consumption, wage rate, capital stock, investment and the absolute deviation from steady state of the real interest rate. Source: Authors, developed with Dynare-4.6.1 and Matlab-R2018a.

3 Non-Pharmaceutical Interventions

In this section, we introduce the NPIs, where two extensions of the basic model are presented. In Model 1 (M_1), NPIs are introduced following a qualitative approach in order to show how they may generate epidemic waves and a forceful output downturn. In Model 2 (M_2), we assume a quantitative approach, modelling the effects of NPIs on transition probabilities more rigorously in order to match the epidemic and business cycle evidence of Italy.

As shown in Figure 1, without NPIs the outbreak dies out as soon as the first wave ends. To obtain multiple waves - which indicate repressions and revivals of the virus - the infection rate must change, reducing and augmenting over time (Atkeson, 2020). Actually, NPIs - such as school and workplace closures, lockdowns, face coverings, etc. - diminish contacts among individuals and thus lessen the infection rate. As these measures are relaxed, contacts rise again, which augments inevitably the infection rate. Consequently, to model the impact of NPIs, we take inspiration from Noll et al. (2020) and introduce a time-dependent infection rate in equations (2) and (3), defined by:

$$b_t = bf(\eta_t), \tag{17}$$

where b is the constant infection rate in the absence of NPIs[26] and $f(\eta_t)$, with $0 \leq \eta_t \leq 1$, describes a generic (decreasing) function of NPIs, labelled with η_t. This can be interpreted as the NPIs rate, which measures the strictness of interventions (1 = strictest), and it is described by a step-function, as shown in paragraph 3.1.

Furthermore, to model the negative supply shock generated by NPIs, we modify equation (7), assuming that the labour force is defined by:

$$N_t = (S_t + E_t + R_t)g(\chi_t), \tag{18}$$

where $g(\chi_t)$ is still a function of the NPIs, with $0 \leq \chi_t \leq 1$, representing the fraction of labour force that is effectively kept out from the production process due to Government measures. Hence, the labor force is not only reduced by infected and deceased individuals, but also by the share of individuals who are subjected to restrictions.

It is relevant to stress that we are distinguishing between the NPIs which affect the infection rate, η_t, and which affect the labour force, χ_t. This is since the non-pharmaceutical interventions consist of also those measures that do not impede workers from supplying labour, such as face-covering requirements, testing policies and other health measure. As shown in paragraph 3.2, we use

[26] See equation (1).

two different indexes as proxy of η_t and χ_t.

3.1 Model with multiple waves (M_1)

We assume $f(\eta_t)$ and $g(\chi_t)$ to be decreasing function of NPIs and represented by:

$$f(\eta_t) = (1 - \kappa\eta_t)$$
$$g(\chi_t) = (1 - \alpha\chi_t^2)$$

with $0 \leq \kappa, \alpha \leq 1$. Parameters κ and α represent efficacy coefficients of the interventions on reducing the infection rate and the labour forces, respectively.[27] Accordingly, it is possible to assume a limited effect of NPIs on the infection rate since restricted individuals might still be subject to some contact and thus infect and be infected. At the same time, even under restrictions, some labourers can still supply work.[28]

To show the potential of the model in producing multiple waves following the implementation and the relaxation of NPIs, we assume values for η_t according to a step-function and, for simplicity, we set $\eta_t = \chi_t$.[29] For $t \leq 60 \cup t > 160, \eta_t = 0$ and for $60 < t \leq 160, \eta_t = 0.8$. Thus, in the first 60 days, the government decides not to intervene, allowing the virus to spread across the population. Successively, strict NPIs are imposed for 100 days, which are removed from day 160 until the end of simulation. In addition, we conduct three simulations for M_1 in which α assumes different values, i.e. $\alpha = \{0.3, 0.5, 0.7\}$, while κ remains constant, equal to 0.8. Increasing values of α imply a higher degree of NPIs efficacy in reducing labour supply.

In figure 2, we report the outcome produced by Model 1 in which we allow for the presence of NPIs. It is possible to observe that by implementing measures that reduce the infection rate, the government is able to lessen the spread of the outbreak, which infects only 6.1% of the population (at the first peak) as compared to M_0 where nearly 30% of population is infected. As long as strict measures are maintained, the number of infections declines. However, the following removal of NPIs enables the virus to be transmitted more rapidly, producing a surge of infections and the manifestation of a second

[27] See Alavarez et al. (2020).

[28] It is easy to get this idea by thinking of the extraordinary implementation of remote - or smart - working displayed worldwide from March 2020, which allowed households to work even under lockdown requirements. Nonetheless, not all sectors of the economy can operate remotely, such as the manufacturing sector and, for this reason, NPIs still generate a significant negative shock on labor supply.

[29] In the next paragraph, we rule out this assumption, following the previous reasoning.

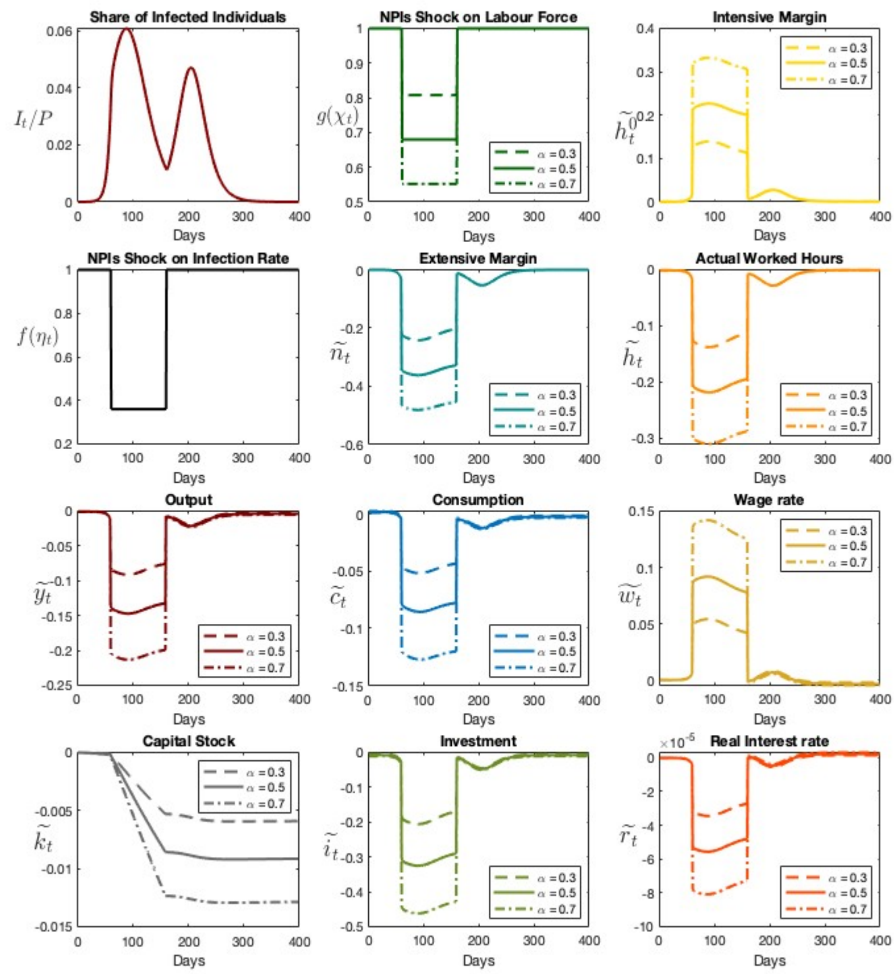
wave, which dies out after 488 days.

Regarding macroeconomic variables, we see that as soon as $g(\chi_t)$ varies the extensive margin bounces, while, when it is constant, it is possible to appreciate the gradual effect of the epidemic shock on the economy. Given $0.3 \leq \alpha \leq 0.7$, the extensive margin reduces by $24.3 - 48.3\%$ at the first peak, and, in the second wave, it falls by 5.3%. As already highlighted in M_0, other macroeconomic variables are affected by variations in the extensive margin exhibiting two minima as well.[30]

From the comparison of the first two models a trade-off between health and output arises. Its severity is conditional on the value of the efficacy coefficient α. According to our simulations, when NPIs are not implemented, the epidemic runs out in 238 days, hitting the population strenuously, namely, infecting 17,3 million individuals at the peak and taking 9.7 million lives overall. The epidemic shock knocks off output by 12.2% at the minimum. Conversely, in the scenario in which the government intervenes with NPIs, the outbreak lasts for a longer period, conveying multiple waves. However, it infects a lower portion of the population (3.6 million at the first peak and 2.8 million at the second peak) and causes the death of fewer individuals (8.8 million). Notwithstanding this, when $\alpha > 0.415$, the NPIs shock, which adds to the remaining epidemic shock, generates a more forceful and long-lasting recession.

[30] At the first minimum, actual worked hours, output, consumption and investments tumble, respectively, by 13.8-31.1%, 9.1-21.3%, 5.2-12.8% and 20.6-46.2%, while, at the second minimum, decline by 2.9%, 2.3%, 1.3% and 5.0%.

Figure 2: NPIs and IRFs of Macroeconomic Variables (M_1)



Note: We report the first 400 days out of 732 of our simulation, corresponding to the period 24/02/20-24/02/22. In the top-left corner is shown the time path of the share of infected individuals. The time path of the NPIs shock on the infection rate and the labour force is displayed, respectively, under and on the right of the latter panel. The rest of the panels exhibit the relative deviation from steady state of the intensive margin, extensive margin, actual worked hours, output, consumption, wage rate, capital stock, investment and the absolute deviation from steady state of the real interest rate. Source: Authors, developed with Dynare-4.6.1 and Matlab-R2018a.

3.2 The Quantitative Model (M_2)

In order to replicate the empirical evidence of Italy, we need to take into account further aspects. When the analysis is focused on an extended period, external factors may change the properties of the spreading epidemic, i.e. transition probabilities. In this circumstance, it is necessary to adopt time-varying parameters to include the effects of these external factors.[31]

To this aim, we impose a time-dependent infection rate (b_t) and case fatality rate (μ_t), which modify equations (2)-(3) and (6) of the canonical SEIRD model. As for $f(\eta_t)$, we borrow from reality a proxy of NPIs, namely, a measure that assigns a numerical value to restrictions adopted in Italy. We employ the *Containment and Health* Index (CH^{Index}) - which is one of the efforts of the OxCGRT[32] project - elaborated by Hale et al. (2021). More specifically, this index is a composite measure based on thirteen policy response indicators including school closures, workplace closures, travel bans, testing policy, contact tracing, face coverings, and vaccine policy re-scaled to a value from 0 to 100 (100 = strictest). The functional form of $f(\eta_t)$ is given by:

$$f(\eta_t) = \Big(1 - \kappa_{1,t}\eta_t - \kappa_{2,t}\eta_t^2\Big) \tag{19}$$

where $\eta_t = (CH_t^{Index}/100)$, with $0 \leq \kappa_{i,t} \leq 1$, $i = 1, 2$. Differently from M_1, the two coefficients - which multiply, respectively, η_t and its square - are time-varying, reflecting the fact that the effectiveness of NPIs in reducing the infection rate changes over time.[33]

Moreover, to model the impact of restrictions on the labour force we do not use the CH^{Index}, as some of the interventions that it contains do not affect labour, but we build our own index, labelled as *Modified Stringency* Index (MS^{Index}). This is obtained starting from the dataset collected by the

[31] In particular, the infection rate is remarkably reduced by self-protective behaviours and government actions - i.e. NPIs - which aim at diminishing contacts among individuals (Giordano et al., 2020; Loli Piccolomini et al., 2020; Noll et al., 2020; Romano et al., 2020; Sadeghi et al., 2020; Zhang et al., 2021). In addition, the infection rate can be scaled down through the production and distribution of vaccines that immunise the susceptible population and can be moved up by more transmissible virus' variants (Angeli et al., 2022; Caldwell et al., 2021 and Zhang et al., 2021). The incubation period is disease-specific and thus virus variants may present slightly different incubation rates. The recovery rate depends not only on the nature of the epidemic disease but also on the available medical resources and the efficiency of the healthcare system (Giordano et al., 2020; Romano et al., 2020 and Sadeghi et al., 2020). Finally, the case fatality rate can be pushed up by the inception of new variants and deficiencies in the healthcare system (Caldwell et al., 2021 and Sadeghi et al., 2020).

[32] The Oxford Coronavirus Government Response Tracker.

[33] When considering an extended period, it is difficult to assume that NPIs maintain the same effect on the infection rate, as other factors perturb their impact. Thus, by imposing time-varying efficacy parameters we are assuming that equal policy interventions, implemented at different times, can have differential effects on reducing the infection rate.

OxCGRT project and following their methodology for calculating indices.[34] In detail, the new index is constituted by five indicators, i.e. workplaces closures, public transport disruptions, stay-at-home requirements, restrictions on internal movements and international travel.[35] As in the previous paragraph, we set $g(\chi_t) = (1 - \alpha_t \chi_t^2)$, with $\chi_t = (MS_t^{Index}/100)$. Contrary to M_1, the efficacy of NPIs on reducing N_t is not constant but it varies over time (α_t) according to a step-function.[36]

We calibrate the model on Italian data. Our analysis covers a time period that goes from 24/02/2020 to 24/02/2022 (732 days), in which Italy faced a sequence of five waves of Covid-19. Since epidemiological models usually present a daily frequency, we decided to maintain it, calibrating macroeconomic parameters on a daily basis.[37] Epidemic data is taken from the dataset of the Italian Civil Protection Department, available in the GitHub repository.[38] To estimate time-varying efficacy coefficients $\kappa_{i,t}$ $(i = 1, 2)$ and the case fatality rate, μ_t, we minimise the simple mean of the *sum of squared errors* (SSE) of the share of infections and deceases, where I_t^{Real} and D_t^{Real} refers to data from the ICPD, subject to the SEIRD model with time-variant parameters. Additionally, we highlight two more feature of our problem. Firstly, in our model NPIs reduces promptly b_t. Nevertheless, it has been experienced in reality that their effect is displayed with a certain temporal lag. Thus, we assume a delay of 33 days in the impact of NPIs on the infection rate,[39] which is determined in order to optimise the matching between real and artificial data on infections and deceased.[40] Secondly, the total period (T) consists of 732 days and the problem is solved dividing T into five sub-intervals, imposing each parameters to estimate to be constant in the different intervals.

[34] A full description is available at `https://github.com/OxCGRT/covid-policy-tracker/blob/master/documentation/index_methodology.md`.

[35] Referring to the code-book for OxCGRT, our index is computed using only indicators C2, C5, C6, C7 and C8. Conversely, the *Stringency* Index - computed by the Oxford team - contains also C1, C3, C4 and H1. It is for this reason that we rename our index as *Modified Stringency* Index.

[36] This assumption reflects the fact that in the first phase of the Covid-19 outbreak, policy interventions were more disruptive than in the following phases, since production processes were not designed to work properly under restrictions.

[37] This is also the choice of Alvarez et al (2021), Gonzalez-Eiras and Niepelt (2020) and Piguillem and Shi (2020).

[38] Find data at `https://github.com/pcm-dpc/COVID-19`.

[39] Conversely, effects on the labour force are observed instantly, since, for instance, the decision to restrict travel and/or to lock-down households has an immediate impact on the possibility to supply labour.

[40] In practice, we impose that on 24^{th} February 2020 the CH_t^{Index} assumes the value exhibited on 22^{nd} January 2020, that is equal to 0.

The minimisation problem is described by:

$$\min_{\{\kappa_{1,t},\kappa_{2,t},\mu_t\}_{t=1}^{T}} \quad \frac{1}{2}\left[\sum_{t=1}^{T}\left(\frac{I_t^{Real}-I_t}{P}\right)^2+\sum_{t=1}^{T}\left(\frac{D_t^{Real}-D_t}{P}\right)^2\right]$$
$$\text{s.t.} \quad \begin{aligned} & b_t = b\big(1-\kappa_{1,t}\eta_t-\kappa_{2,t}\eta_t^2\big) \\ & S_{t+1}-S_t = -b_t\frac{S_tI_t}{P} \\ & E_{t+1}-E_t = b_t\frac{S_tI_t}{P}-\varepsilon E_t \\ & I_{t+1}-I_t = \varepsilon E_t-\gamma I_t-\mu_t I_t \\ & R_{t+1}-R_t = \gamma I_t \\ & D_{t+1}-D_t = \mu_t I_t \end{aligned} \quad , \tag{20}$$

and table 3 summarizes our results.[41]

Table 3: Calibration of μ_t and $\kappa_{i,t}$ $(i = 1, 2)$

Period (Date)	Interval (Day)	μ_t (%)	$\kappa_{1,t}$	$\kappa_{2,t}$
25/02/20 – 21/07/20	2 – 149	0.812	0.3114	0.9855
22/07/20 – 19/02/21	150 – 362	0.091	0.9978	0.0423
20/02/21 – 12/07/21	363 – 505	0.053	0.1441	0.9945
13/07/21 – 22/10/21	506 – 607	0.026	0.4152	0.7544
23/10/21 – 24/02/22	608 – 732	0.038	0.0005	0.8744

Finally, parameter α_t is calibrated in order to match the conjunctural quarterly growth rate of actual worked hours in Italy. Accordingly, its values are depicted in table 4. The fact that α_t assumes a value of 0.65 in the first part of simulation and then declines progressively implies that, in the first months of the Covid-19 epidemic, policy interventions are more effective in reducing labour supply, while, successively, their negative effects weaken.

[41] The length of intervals is related to the time period of the five waves exhibited in Italy. On day one (24/02/20), we set $\mu_t = \mu$ and $\kappa_{i,t} = 0$, i.e. we are imposing the absence of NPIs in the first period. We checked the robustness of our calibration to changes in the algorithm's initialisation. Numerical results are available upon request from the authors.

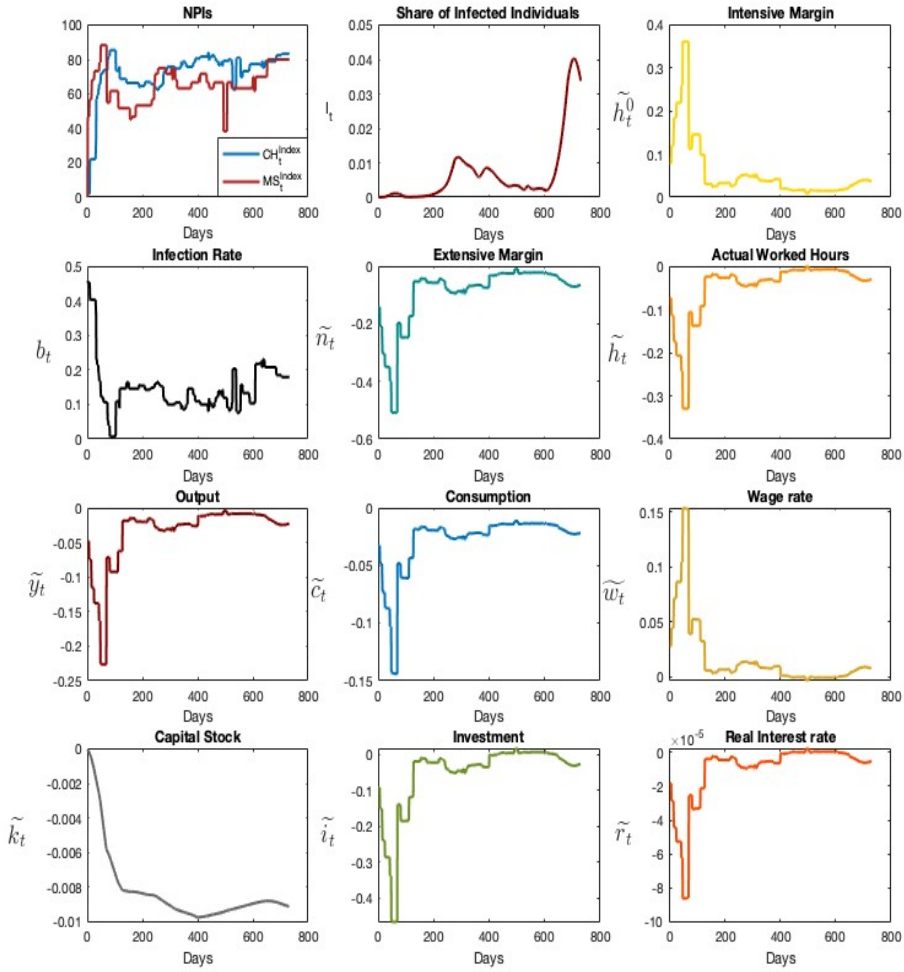
Table 4: Calibration of α_t

Period	Value
1	0
$2-128$	0.65
$129-220$	0.20
$221-402$	0.15
$403-732$	0.05

In figure 3, it is possible to observe that as the CH_t^{Index} increases (stricter NPIs), the infection rate (b_t) reduces slackening the propagation of the virus among individuals, whereas, when the strictness of interventions softens, the infection rate rises. The presence of rises and declines in the index replicates the interchange of suppression phases and weakening periods adopted by the Italian government. In addition, we can appreciate the effect of the MS_t^{Index} on the extensive margin which results as being the main shock on n_t,[42] reducing it by 50.8% at the lowest peak. The presence of several bottom points - exhibited by macroeconomic variables - highlights the presence of multiple epidemic waves, where infections rise, reach a maximum and, then, decline. Given that on day 732, i.e. the end of our simulation, the number of infections is still considerable, in M_2 macroeconomic variables do not come back to the initial steady state.

[42] Accordingly, NPIs almost neutralise the epidemic shock.

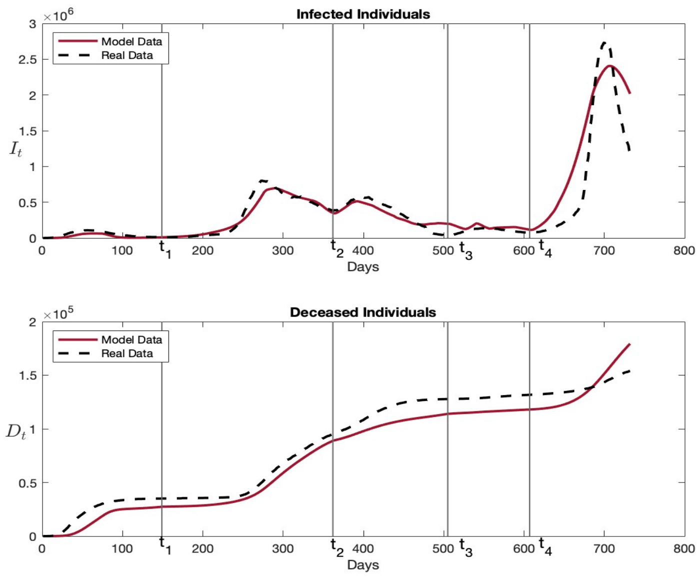
Figure 3: NPIs and IRFs of Macroeconomic Variables (M_2)



Note: Our simulation consists of 732 days, corresponding to the period 24/02/20-24/02/22. In the top-left corner is shown the time path of the CH_t^{Index} (blue line) and the MS_t^{Index} (red line). The time path of the share of infected individuals and the infection rate are displayed, respectively, under and on the right of the latter panel. The rest of the panels exhibit the relative deviation from steady state of the intensive margin, extensive margin, actual worked hours, output, consumption, wage rate, capital stock, investment and the absolute deviation from steady state of the real interest rate. Source: Authors, developed with Dynare-4.6.1 and Matlab-R2018a.

We observe in figure 4 that the employment of a time-varying infection rate allows us to replicate approximately the five different waves of Covid-19 infections, which characterised the evolution of the epidemic in Italy in the period 24/02/20 - 24/02/22. In the same way, the path of deceased is closely matched thanks to the variation of the case fatality rate.[43]

Figure 4: The Epidemiological Empirical Evidence of Italy vs Model 2



The two panels report, respectively, the time path of infected and deceased individuals derived from simulation (red lines) and real data (dashed black lines). Vertical grey lines are drawn in correspondence of t_i, with $i = \{1, 2, 3, 4\}$, which highlight intervals of the five epidemic waves. Source: Authors, developed with with Dynare-4.6.1 and Matlab-R2018a.

We conclude this paragraph by reporting in figure 5 a comparison between the quarterly conjunctural growth rate of actual worked hours, output, consumption and investment computed from M_2 and those obtained from real data,[44] covering a time period that goes from Q1-2020 to Q4-2021.[45] It is possible to observe that our model is able to match quantitatively - with a suitable degree of accuracy - the quarterly conjunctural growth rates of Italian economic aggregates. Even though aggregate consumption exhibits a smaller volatility, its path replicates the oscillations reported by the empirical evidence. As explained before, this is due to the consumption smoothing of *Opt* households. The goodness-of-fit of the model relies also on the implementation of the time-varying coefficient α_t. This is necessary inasmuch as

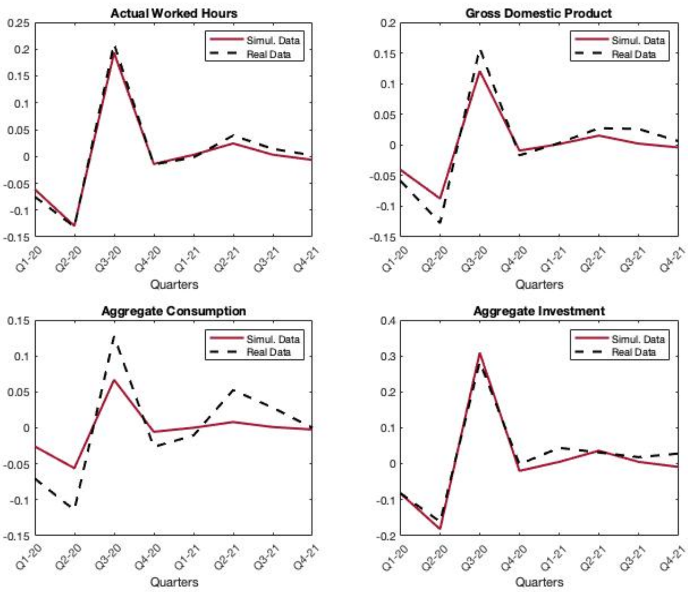
[43]In appendix E, table E.1 illustrates the goodness-of-fit of data on infections and deceased.

[44]Real data is taken from Istat, where aggregate consumption is compared with the final household's spending and aggregate investment with gross fixed investment.

[45]Since our simulation starts from 24/02/20, to compute growth rates in Q1-2020 we assume that each variable is in steady steady state from 01/01/2020 until 24/02/2020.

the reintroduction of NPIs to hinder new waves of Covid-19 did not have the same negative impact on the economy as the implementation of first interventions. Actually, in the first months of 2020, the outbreak along with policy interventions sneaked up on economic agents, yielding severe recessive effects. However, as the government lingered on the implementation of NPIs, firms and households adapted, weakening the negative impact of interventions. This may explain why a lower fall of economic aggregates is associated with equal values of MS_t^{Index}, starting from the third quarter of 2020.

Figure 5: The Macroeconomic Empirical Evidence of Italy vs Model 2



Note: the four panels report, respectively, the quarterly conjunctural growth rate of actual worked hours, output, consumption and investment computed from simulation (red lines) and real data (dashed black lines). To conduct a fair comparison, we have converted the frequency of model data from daily to quarterly and, since variables in our model are in per capita terms, each measure was transformed in absolute terms, multiplying it by $(P - D_t)$. Source: Authors, developed with Dynare-4.6.1 and Matlab-R2018a.

4 Conclusion

This paper proposes a unified framework for analysing the economic effects generated by the proliferation of an epidemic and the implementation of non-pharmaceutical interventions. Such framework is based on direct integration of a Neoclassical Growth Model (accounting for extensive margin of labour and financially constrained households) with a SEIRD epidemiological model. We depart from the standard SIR-Macro set-up that typically employs health-related models in which the health status and the transition probabilities between epidemic compartments shape the relevant agents' decisions; conversely, we propose a simplified and direct integration in which the SEIRD block together with the NPIs determine exogenously the path of the extensive margin of the labour input, while economic agents derive optimally the actual worked hours and the intertemporal allocations of consumption and asset accumulation.

We present three versions of our model in order to highlight different aspects. M_0 points out how the epidemic shock alters the equilibrium of macroeconomic variables when the government decides not to interfere with the spread of an infectious virus. In M_1, NPIs are introduced; since these interventions not only reduce the infection rate but also have a negative impact on the labour force, NPIs shock adds to the remaining epidemic shock producing a forceful economic recession and the occurrence of new waves of infections and deceased when NPIs are relaxed. This eventuality lays the foundation for a debate on the presence of a health-output trade-off faced by the government when choosing whether to intervene to hinder the spread of the outbreak and save lives at the cost of a more forceful and long-lasting economic recession. In M_2, we bring our model to data; we calibrate the NPIs on the basis of proxies such as the *Containment and Health* Index and the *Modified Stringency* Index. Thanks to time-varying infection and case fatality rates and efficacy coefficients the model is able to replicate the time path of infections and deceased as well as quarterly conjunctural variations of the main macroeconomic variables exhibited in Italy between 24/02/20 and 24/02/22.

We believe that the strength of our analysis lies in its simplicity and capacity to replicate the epidemic and business cycle evidence of the Italian economy, in which employment variations have been strictly related to the harmfulness of the epidemic and the implementation of non-pharmaceutical interventions and not to the decisions of agents. This is particularly true for the Italian case where the government has introduced a long-lasting ban on dismissals. Further, because of its parsimony, it is straightforward to integrate our setup into more structured business-cycle models in order to study a much broader set of issues.

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Appendix

A Derivation of the Utility Function

We consider a utility function logarithmic in consumption (c_t) and CRRA in leisure ($1 - h_t^0$). Following Hansen (1985), we can write:

$$U = \log(c_t) + \left[n_t \psi \frac{(1-h_t^0)^{1-\theta} - 1}{1-\theta} + (1-n_t)\psi \frac{1^{1-\theta} - 1}{1-\theta} \right]$$

Hence, utility from leisure is multiplied by the respective probability of being employed and unemployed. Rearranging the equation and substituting $n_t = h_t (h_t^0)^{-1}$, we obtain:

$$U = \log(c_t) + \frac{h_t}{h_t^0} \psi \left[\frac{(1-h_t^0)^{1-\theta} - 1}{1-\theta} - \frac{1^{1-\theta} - 1}{1-\theta} \right] + \psi \frac{1^{1-\theta} - 1}{1-\theta}$$

Omitting constant terms - as they do not affect the household's optimal choice - we get equation (10):

$$U = \log(c_t) - v_t h_t$$

where

$$v_t = -\frac{\psi}{h_t^0} \left[\frac{(1-h_t^0)^{1-\theta} - 1}{1-\theta} \right] \geq 0$$

B Maximisation Problems

***Opt* households' maximisation problem**. As in the basic model, households maximise their expected utility - defined, here, by equation (10) - subject to the budget constraint and the low of motion of capital. Thus, the problem is described by:

$$\max_{\left\{c_t^p, h_t^p, k_{t+1}^p\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta_t \left[\log c_t^p - v_t^p h_t^p\right]$$
$$\text{s.t.} \quad c_t^p + i_t^p \leq w_t h_t^p + (\delta + r_t) k_t^p$$
$$k_{t+1}^p = (1-\delta) k_t^p + i_t^p$$

with

$$v_t^p = -\frac{\psi^p}{h_t^{0,p}} \left[\frac{(1 - h_t^{0,p})^{1-\theta} - 1}{1-\theta}\right]$$

By applying the Lagrange multiplayer approach, we derive the first-order conditions (FOCs). Combining them, we obtain equations (12) and (13).

RoT households' maximisation problem. In this case, the optimisation problem is different from that of *Opt* households, as the low of motion of capital does not enter as a constraint and the budget constraint is modified. Hence, the problem is described by:

$$\max_{\left\{c_t^r, h_t^r\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta_t \left[\log c_t^r - v_t^r h_t^r\right]$$
$$\text{s.t.} \quad c_t^r \leq w_t h_t^r$$

with

$$v_t^r = -\frac{\psi^r}{h_t^{0,r}} \left[\frac{(1 - h_t^{0,r})^{1-\theta} - 1}{1-\theta}\right]$$

As done before, we compute the FOC of the problem, yielding equation (14).

Firms' maximisation problem. Firms maximise profits by combining capital stock and labor through a Cobb-Douglas production function. Accordingly, the problem is defined as:

$$\max_{h_t, k_t} \quad y_t - w_t h_t - (r_t + \delta) k_t$$
$$\text{s.t.} \quad y_t = k_t^{1-\phi} h_t^{\phi}$$

From the FOCs of the problem, we determine equations (15) and (16).

C Equilibrium of the RBC Block

The equilibrium of the RBC model is described by the following system of 17 equations:

$$
\begin{aligned}
\frac{1}{c_t^p} &= \beta E\left[\frac{1+r_{t+1}}{c_{t+1}^p}\right] \\
w_t &= v_t^p c_t^p \\
c_t^p + i_t^p &= w_t h_t^p + (\delta + r_t) k_t^p \\
k_{t+1}^p &= (1-\delta) k_t^p + i_t^p \\
v_t^p &= -\frac{\psi^p}{h_t^{0,p}}\left[\frac{(1-h_t^{0,p})^{1-\theta} - 1}{1-\theta}\right] \\
w_t &= v_t^r c_t^r \\
c_t^r &= w_t h_t^r \\
v_t^r &= -\frac{\psi^r}{h_t^{0,r}}\left[\frac{(1-h_t^{0,r})^{1-\theta} - 1}{1-\theta}\right] \\
h_t^0 &= (1-\lambda) h_t^{0,p} + \lambda h_t^{0,r} \\
c_t &= (1-\lambda) c_t^p + \lambda c_t^r \\
i_t &= (1-\lambda) i_t^p \\
k_t &= (1-\lambda) k_t^p \\
n_t &= \frac{N_t}{P - D_t} \\
h_t &= h_t^0 n_t \\
y_t &= k_t^{1-\phi} h_t^{\phi} \\
r_t &= (1-\phi)\frac{y_t}{k_t} - \delta \\
w_t &= \phi \frac{y_t}{h_t}
\end{aligned}
$$

D Steady State of the Model

Differently from a short-run macroeconomic model, in an epidemic model the initial steady state is different from the final steady state inasmuch as individuals move permanently from the susceptible compartment to the recovered and deceased compartments. Therefore, in the first period:

$$S_1 = P - (E_1 + I_1 + R_1 + D_1),$$

while with $t \to \infty$, $S_t, E_t, I_t \to 0$ and $(R_t + D_t) \to P$.

In the macroeconomic model initial and final steady states (s.s.) coincide as time converges to infinity. From equation (7), we derive the s.s. labour force:[46]

$$\bar{N} = \bar{S} + \bar{E} + \bar{R}.$$

From equation (8), we derive the s.s. extensive margin:[47]

$$\bar{n} = \frac{\bar{N}}{P - \bar{D}}.$$

Combining the budget constraint of *Opt* Households with equation (13) and making some substitutions, we derive the s.s. intensive margin of *Opt* Households:

$$\bar{h}^{0,p} = 1 - \left(1 - \frac{1-\theta}{\psi^p(1+z)}\frac{1}{\bar{n}}\right)^{\frac{1}{1-\theta}},$$

where

$$z = \frac{f}{\phi(1-\lambda)}\left(\frac{1-\phi}{f+\delta}\right) \quad \text{and} \quad f = \frac{1}{\beta} - 1.$$

Combining the budget constraint of *RoT* Households with equation (14), we derive the s.s. intensive margin of *RoT* Households:

$$\bar{h}^{0,r} = 1 - \left(1 - \frac{1-\theta}{\psi^r}\frac{1}{\bar{n}}\right)^{\frac{1}{1-\theta}}.$$

From the definition of Hansen's variable for *Opt* Households, we derive its steady state value:

$$\bar{v}^p = -\frac{\psi^p}{\bar{h}^{0,p}}\left[\frac{(1-\bar{h}^{0,p})^{1-\theta} - 1}{1-\theta}\right].$$

From the definition of the Hansen's variable for *RoT* Households, we derive its steady state value:

$$\bar{v}^r = -\frac{\psi^r}{\bar{h}^{0,r}}\left[\frac{(1-\bar{h}^{0,r})^{1-\theta} - 1}{1-\theta}\right].$$

From the aggregation of h_t^0, we derive the s.s. aggregate intensive margin:

$$\bar{h}^0 = (1-\lambda)\bar{h}^{0,p} + \lambda\bar{h}^{0,r}.$$

[46] $N_1 = P - (I_1 + D_1)$ and $\lim_{t\to\infty} N_t = P$.

[47] $n_1 = 1 - \frac{I_1}{P - D_1} \approx 1$ and $\lim_{t\to\infty} n_t = 1$.

From equation (9), we derive the s.s. aggregate actual worked hours:

$$\bar{h} = \bar{n}\bar{h}^0.$$

Combining the production function with the s.s. output-capital ratio,[48] we derive the s.s. aggregate capital stock:

$$\bar{k} = \left(\frac{1-\phi}{f+\delta}\right)^{\frac{1}{\phi}} \bar{h}.$$

From the aggregation of k_t, we derive the s.s. capital stock of *Opt* Households:

$$\bar{k}^p = \frac{\bar{k}}{1-\lambda}.$$

Combining the law of motion of capital stock for *Opt* Households with the s.s. aggregate capital stock, we derive the s.s. aggregate investment level:

$$\bar{\imath} = \delta \left(\frac{1-\phi}{f+\delta}\right)^{\frac{1}{\phi}} \bar{h}.$$

From the aggregation of i_t, we derive the s.s. investment level of *Opt* Households:

$$\bar{\imath}^p = \frac{\bar{\imath}}{1-\lambda}.$$

Combining the production function with the s.s. aggregate capital stock, we derive the s.s. aggregate output:

$$\bar{y} = \left(\frac{1-\phi}{f+\delta}\right)^{\frac{1-\phi}{\phi}} \bar{h}.$$

Combining equation (16) with the s.s. aggregate output, we derive the s.s. wage rate:

$$\bar{w} = \left(\frac{1-\phi}{f+\delta}\right)^{\frac{1-\phi}{\phi}}.$$

From equation (12), we derive the s.s. real interest rate:

$$\bar{r} = \frac{1}{\beta} - 1.$$

[48] It is obtained combining the s.s. Euler equation with the s.s. capital demand and it is equal to $\frac{\bar{y}}{\bar{k}} = \frac{f+\delta}{1-\phi}$.

From equation (13), we derive the s.s. consumption of *Opt* Households:

$$\bar{c}^p = \frac{\bar{w}}{\bar{\upsilon}^p}.$$

From equation (14), we derive the s.s. consumption of *RoT* Households:

$$\bar{c}^r = \frac{\bar{w}}{\bar{\upsilon}^r}.$$

From the aggregation of c_t, we derive the s.s. aggregate consumption:

$$\bar{c} = (1 - \lambda)\bar{c}^p + \lambda\bar{c}^r.$$

E Goodness-of-Fit of Data

To evaluate the goodness-of-fit of simulated data (infections and deceased), we compute the *Root Mean Squared Error* (RMSE), expressed by:

$$RMSE = \sqrt{\frac{\sum_t^T \left(X_t^{Real} - X_t\right)^2}{T - t + 1}}$$

where X_t^{Real} and X_t refer, respectively, to real and simulated data of a single epidemic compartment among infected and deceased.
Results are reported in table E.1, where goodness-of-fit measures are computed for each epidemic wave interval.

Table E.1: Goodness-of-Fit of Data on Infections and Deceases

Period	RMSE		RMSE/P	
$t - T$	Infections (e^{+5})	Deceased (e^{+4})	Infections (%)	Deceased (%)
$1 - 149$	0.308	0.941	0.05	0.016
$150 - 362$	0.777	0.710	0.13	0.012
$363 - 505$	0.870	1.372	0.15	0.023
$506 - 607$	0.634	1.338	0.11	0.022
$608 - 732$	4.689	1.248	0.79	0.021