Automated switching services

Garrod, Luke and Li, Ruochen and Wilson, Christopher

Loughborough Business School, Loughborough University, Centre for Economics Research, Shandong University, Loughborough Business School, Loughborough University

1 September 2023

Automated Switching Services

Luke Garrod, Ruochen Li and Chris M. Wilson*

September 1, 2023

Abstract

Automated switching services have recently emerged as online intermediaries that use algorithms to facilitate consumer switching. Unlike price comparison websites, these services i) act on behalf of consumers by actively switching them to the cheapest deals, ii) typically charge consumers directly, rather than charging suppliers commission, and iii) often survey across the entire market. We offer the first theoretical analysis of such services. In an oligopoly model with imperfect price information, we characterize an equilibrium with an auto-switching service, and analyze its impact on market outcomes and welfare.

Keywords: Consumer Switching; Consumer Search; Price Information; Intermediary; Automated; Competition.

JEL Codes: D43; L13; D83

*Garrod and Wilson: Loughborough Business School, Loughborough University, Loughborough, LE11 3TU, UK; l.garrod@lboro.ac.uk and c.m.wilson@lboro.ac.uk Li: Centre for Economics Research, Shandong University, Jinan, 250100, China; ruochen.li@sdu.edu.cn. The corresponding author is Ruochen Li. This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors. Declarations of interest: none. We thank Joe Harrington and an anonymous referee for their helpful suggestions.

1 Introduction

Automated switching services are online intermediaries that use algorithms to switch consumers to cheaper suppliers. They exist for a variety of products, such as energy, broadband and credit. Examples include Bill Hero (Australia), June (France), Switchd (UK), Utiliz (US), and Wechselpilot (Germany). Since emerging in 2016-17, they have become increasingly popular. For instance, in 2019, 300,000 consumers had used such services in the UK energy market, accounting for 2% of all switches (Citizens Advice 2020).

These auto-switching services (henceforth 'services') differ to price comparison websites (PCWs) in three main ways. First, rather than presenting competing offers, the services act on behalf of consumers by switching them to the cheapest deals. Second, rather than charging suppliers through commissions or advertising fees, most services charge consumers directly (typically with a monthly fixed fee, ranging from $2.50-7.50/€2.20-6.80). Third, rather than only comparing the suppliers that pay to list, they often survey all suppliers.

We provide the first theoretical analysis of auto-switching services. Within an oligopoly model with homogeneous products and imperfect price information (e.g. Varian 1980, Tappata 2009), we introduce a service that charges consumers a fee to switch them to the cheapest supplier. We evaluate the equilibrium impact of the service on market outcomes and welfare. Among other results, we show how i) the service earns higher profits when few consumers are otherwise informed and/or when the number of suppliers is large, ii) the introduction of a service intensifies price competition but price dispersion can increase or decrease depending on how many consumers are already informed, and iii) the service's existence always benefits all consumers, despite it only serving some consumers and charging them a fee.

The only existing paper on auto-switching services is Feldhaus et al. (2022) which runs a field experiment to empirically assess the willingness of consumers to use such services. The closest theoretical papers consider other forms of consumer information intermediaries, such as PCWs (e.g. Baye and Morgan 2001, Moraga-González and Wildenbeest 2012, Ronayne 2021, Ronayne and Taylor 2022). Within these models, intermediaries always charge suppliers to trade on their platform. In contrast, we study auto-switching services that only interact with the consumer-side of the market. This business model could be optimal for the service if transacting with suppliers is i) prohibitively costly, or ii) perceived negatively by consumers or authorities.

2 Model

Suppose a finite number of suppliers, $n \geq 2$, compete to sell a homogeneous product at zero cost. There is a unit mass of consumers with unit demand and common willingness-to-pay, $v > 0$. A proportion of consumers are 'shoppers', $\sigma_0 \in [0, 1)$, who know all prices and buy from the cheapest supplier. The remainder are 'non-shoppers', who do not know prices. They can buy from a random supplier or pay a subscription fee, κ, to use a monopoly auto-switching service. This 'service' arranges for users to trade with the cheapest supplier.[1] The service's costs are normalized to zero.[2]

In stage 1, the service selects $\kappa \geq 0$ and commits to the maximum number of consumers who can subscribe, $q_U \in [0, 1 - \sigma_0]$.[3] In stage 2, everyone observes κ and q_U, before i) non-shoppers decide whether to subscribe, and ii) each supplier sets its price, p_i. We denote the resulting proportion of users as σ_U, and the total proportion of 'active' and 'non-active' consumers as $\sigma = \sigma_U + \sigma_0 \in [\sigma_0, 1]$ and $1 - \sigma$, respectively. In stage 3, consumers' purchase decisions follow from the descriptions above. We study symmetric subgame perfect equilibria (SPE). To allow for mixed strategies, let $F(p)$ represent each supplier's equilibrium price distribution. Finally, as a tie-break rule, we assume that non-shoppers wish to subscribe whenever they are indifferent.[4]

3 Equilibrium

We proceed through backwards induction. Consumers' purchase decisions in stage 3 are trivial or automated.

3.1 Stage 2

3.1.1 Pricing Decisions

For any proportion of active and non-active consumers, suppliers' pricing decisions follow from Varian (1980).

[1] We assume it does this truthfully as consistent with its actions being verifiable or regulated.

[2] Positive marginal costs have no qualitative impact on the equilibrium but can affect total welfare.

[3] This is consistent with the service pre-committing to the scale of its operations through prior investments in IT systems and staff.

[4] Together with the use of q_U, this tie-break rule simplifies the equilibrium derivation. Without them, our final results remain robust; details available on request.

Lemma 1. *For any* $\sigma = \sigma_0 + \sigma_U \in [0,1]$*, the unique symmetric equilibrium has each supplier earning* $\pi(\sigma) = \frac{v(1-\sigma)}{n}$ *with*

$$F(p) = 1 - \left[\frac{(1-\sigma)(v-p)}{\sigma n p}\right]^{\frac{1}{n-1}} \text{ on } [\underline{p}, v] \tag{1}$$

where $\underline{p} = \frac{v(1-\sigma)}{n\sigma+(n-1)}$.

Intuitively, when setting $p \in [\underline{p}, v]$, supplier i's expected profits are

$$p\left[\frac{1-\sigma}{n} + \sigma(1-F(p))^{n-1}\right]$$

as it expects to supply $\frac{1-\sigma}{n}$ non-active consumers and σ active consumers with probability $(1-F(p))^{n-1}$. As standard, $F(p)$ in (1) ensures suppliers are indifferent over $[\underline{p}, v]$.

Lemma 2. *In equilibrium, for any* $\sigma = \sigma_0 + \sigma_U \in [0,1]$*, non-active and active consumers expect to pay the following prices, respectively:*

$$E(p|\sigma) = \int_{\underline{p}}^{v} pF'(p)dp = \int_0^1 \frac{v(1-\sigma)}{1-\sigma(1-nz^{n-1})}dz \in [0,v] \tag{2}$$

$$E(p_{min}|\sigma) = \int_{\underline{p}}^{v} p \cdot n(1-F(p))^{n-1} F'(p)dp = \int_0^1 \frac{v(1-\sigma)nz^{n-1}}{1-\sigma(1-nz^{n-1})}dz \in [0, E(p|\sigma)] \tag{3}$$

Each price is expressed in two forms. The first is standard. The second follows from Tappata (2009) by changing variables with $z = 1 - F(p)$.

3.1.2 Subscription Decisions

Non-active and active consumers' expected surpluses (gross of any fee) equal $V_{NA}(\sigma) = v - E(p|\sigma)$ and $V_A(\sigma) = v - E(p_{min}|\sigma)$, respectively.

Proposition 1. *For any* $\sigma = \sigma_0 + \sigma_U \in [0,1]$*, each non-shopper's willingness-to-pay for the service is*

$$\omega(\sigma) = V_A(\sigma) - V_{NA}(\sigma) = E(p|\sigma) - E(p_{min}|\sigma) = \int_0^1 \frac{v(1-\sigma)(1-nz^{n-1})}{1-\sigma(1-nz^{n-1})}dz \geq 0 \tag{4}$$

where $\omega(0) = \omega(1) = 0$*,* $\omega''(\sigma) < 0$ *and* $\widehat{\sigma} \in (0,1)$ *sets* $\omega'(\widehat{\sigma}) = 0$*.*

Consistent with Tappata (2009), $\omega(\sigma)$ has an inverted U-shape. As σ rises away from 0 (where all prices equal v) towards $\widehat{\sigma}$, $E(p|\sigma)$ and $E(p_{min}|\sigma)$ reduce while price dispersion increases (as captured by $\omega(\sigma) = E(p|\sigma) - E(p_{min}|\sigma)$); but as σ increases beyond $\widehat{\sigma}$, $E(p|\sigma)$ and $E(p_{min}|\sigma)$ tend towards 0 and price dispersion decreases.

3.1.3 Equilibria in Subgames Starting at Stage 2

We now state the equilibria in subgames starting at stage 2 for any $\kappa \geq 0$ and $q_U \in [0, 1-\sigma_0]$. We consider equilibria where all agents have correct beliefs over $\sigma_U \leq q_U$ and $\sigma = \sigma_0 + \sigma_U$. First, suppose $\kappa > \omega(\sigma)$ in equilibrium. Here, no non-shoppers would subscribe, so the only possible equilibrium has $\sigma_U = 0$ such that $\sigma = \sigma_0$. This equilibrium exists iff $\kappa > \omega(\sigma_0)$ but since it is uninteresting we discard it. Now suppose $\kappa \leq \omega(\sigma)$ in equilibrium. Here, given our tie-break rule, all non-shoppers wish to subscribe, so the only possible equilibrium has $\sigma_U = q_U$ and $\sigma = \sigma_0 + \sigma_U$. Such an equilibrium always exists for any $\sigma_U \in [0, 1-\sigma_0]$ iff $\kappa \leq \omega(\sigma_0 + \sigma_U)$.

3.2 Stage 1

Given $q_U = \sigma_U$ in equilibrium, the service's maximization problem is

$$\max_{\kappa,\sigma_U} \pi_S(\kappa, \sigma_U) = \kappa \cdot \sigma_U \text{ s.t. } \kappa \leq \omega(\sigma_0 + \sigma_U)$$

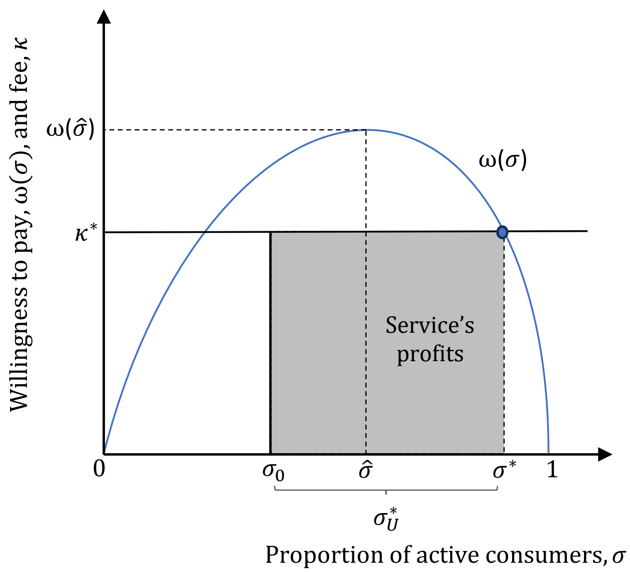
Or, since $\sigma_U = \sigma - \sigma_0 \geq 0$ and the constraint is clearly binding:

$$\max_\sigma \pi_S(\sigma) = \omega(\sigma) \cdot (\sigma - \sigma_0) \tag{5}$$

Proposition 2. *For any* $\sigma_0 \in [0,1)$*, there exists a unique symmetric SPE where* $\pi_S(\sigma^*) > 0$*. There are* $\sigma_U^* \in (0, 1-\sigma_0)$ *users who pay* $\kappa^* = \omega(\sigma^*) \in (0, \omega(\widehat{\sigma}))$*, where* $\sigma^* = \sigma_0 + \sigma_U^* \in (\widehat{\sigma}, 1)$*.*

Given $\kappa = \omega(\sigma)$, the service chooses σ_U^* to balance the marginal revenue and marginal cost of serving another non-shopper. If it has too few users, its customer base is too small. If it has too many, price dispersion becomes limited and non-shoppers' willingness-to-pay is too low. This is illustrated in Figure 1 (for a given v, n and σ_0) where the service chooses $\sigma_U^* = \sigma^* - \sigma_0$ and $\kappa^* = \omega(\sigma^*)$ to maximize its profits (signified by the shaded rectangle).

Figure 1: An Example Equilibrium



3.3 Equilibrium Features

To help academics and policymakers understand how the service's behavior will vary across different types of markets, we now consider changes to two measures of market competitiveness: the proportion of shoppers, σ_0, and the number of suppliers, n.

Proposition 3. *As σ_0 rises, the service's users, fee and profits decrease,* $\frac{\partial \sigma_U^*}{\partial \sigma_0} = -(1 - \frac{\partial \sigma^*}{\partial \sigma_0}) \in (-1, 0)$, $\frac{\partial \kappa^*}{\partial \sigma_0} < 0$, and $\frac{d\pi_S(\sigma^*)}{d\sigma_0} < 0$.

The service earns its highest profits when $\sigma_0 = 0$. Intuitively, as σ_0 increases away from 0, price dispersion at σ^* falls, *ceteris paribus*. This lowers $\omega(\sigma^*)$ so the service decreases κ^* and σ_U^* reducing $\pi_S(\sigma^*)$. However, $\sigma^* = \sigma_0 + \sigma_U^*$ rises because σ_U^* decreases by less than σ_0 increases. As $\sigma_0 \to 1$, the service's users, fee and profits tend to zero.

Although fully analyzing how the service's behavior is affected by n is difficult, we can obtain the following.

Proposition 4. *As n rises, the service's fee and profits strictly increase, $\frac{\partial \kappa^*}{\partial n} > 0$ and $\frac{d\pi_S(\sigma^*)}{dn} > 0$. Further, when n is very large, the number of users is $\sigma_U^* \approx 1 - \sigma_0$, such that $\sigma^* \approx 1$, with $\kappa^* \approx v$ and $\pi_S(\sigma^*) \approx v(1 - \sigma_0)$.*

As n increases, equilibrium price dispersion rises. This enhances non-shoppers' willingness-to-pay, $\omega(\sigma)$, and allows the service to increase κ^* and $\pi_S(\sigma^*)$. Hence, the service benefits from switching consumers to a bigger set of suppliers. Indeed, when n is very large, the service extracts approximately the entire surplus of non-shoppers, $\pi_S(\sigma^*) \approx v(1 - \sigma_0)$. Intuitively, the service wishes to serve (almost) all non-shoppers, $\sigma_U^* \approx 1 - \sigma_0$, because it sets a fee close to the highest possible level, $\kappa^* = E(p|\sigma^*) - E(p_{min}|\sigma^*) \approx v$ (since $\lim_{n\to\infty} E(p|\sigma) = v$ and $\lim_{n\to\infty} E(p_{min}|\sigma) = 0$ as standard, e.g. Tappata 2009).

4 The Impact of the Service

Finally, we evaluate the impact of the service by comparing the equilibrium, $\sigma = \sigma^*$, versus $\sigma = \sigma_0 < \sigma^*$. We begin with prices.

Proposition 5. *The introduction of the service generates:*
i) strictly lower prices, $E(p|\sigma^*) < E(p|\sigma_0)$ and $E(p_{min}|\sigma^*) < E(p_{min}|\sigma_0)$;
ii) strictly less (more) price dispersion, $\omega(\sigma^*) < (>)\omega(\sigma_0)$, in markets with sufficiently many (few) shoppers, σ_0.

Intuitively, service users increase competition amongst suppliers which lowers prices. More surprisingly, depending upon σ_0, the service's existence can lead to increased or decreased price dispersion. For example, when $\sigma_0 = 0$, all prices equal v absent the service, so price dispersion increases under the service; but when $\sigma_0 = \widehat{\sigma}$, price dispersion is maximal absent the service, so it obviously decreases under the service.

Next, consider the service's welfare impact. Since total welfare is fixed due to unit demand, we analyze consumer surplus and industry profits. For $\sigma \in [\sigma_0, 1]$, these equal:

$$\begin{aligned} CS(\sigma) &= \sigma_0 V_A(\sigma) + (\sigma - \sigma_0)(V_A(\sigma) - \omega(\sigma)) + (1-\sigma) V_{NA}(\sigma) \\ \Pi(\sigma) &= n\pi(\sigma) + \pi_S(\sigma) \end{aligned} \tag{6}$$

Proposition 6. *The introduction of the service generates:*
i) strictly greater consumer surplus, $CS(\sigma^*) > CS(\sigma_0)$*;*
ii) strictly lower supplier profits, $n\pi(\sigma^*) < n\pi(\sigma_0)$*;*
iii) strictly lower industry profits, $\Pi(\sigma^*) < \Pi(\sigma_0)$*.*

By increasing the proportion of active consumers and thereby lowering prices, the introduction of the service decreases supplier profits. In contrast, despite charging users a fee and only serving some consumers, the service's effects are 'pro-consumer'. Indeed, *all* consumers are better off: some consumers become active which, in the language of Armstrong (2015), imposes a positive search externality on all other consumers. Specifically, each non-active consumer and shopper benefits from reduced prices, gaining respectively:

$$V_{NA}(\sigma^*) - V_{NA}(\sigma_0) = -[E(p|\sigma^*) - E(p|\sigma_0)] > 0$$

$$V_A(\sigma^*) - V_A(\sigma_0) = -[E(p_{min}|\sigma^*) - E(p_{min}|\sigma_0)] > 0.$$

Surprisingly, users only benefit from reduced prices too because κ^* exactly offsets the direct benefits of the service, $V_A(\sigma^*) - \kappa^* = V_{NA}(\sigma^*)$. Consequently, each user gains the same as a non-active consumer:

$$V_A(\sigma^*) - \kappa^* - V_{NA}(\sigma_0) = -[E(p|\sigma^*) - E(p|\sigma_0)] > 0.$$

Hence, while the service fully extracts users' direct benefits, it does not extract the wider benefits from lower prices. Thus, industry profits strictly fall because the service's profit gain only partially offsets the loss in supplier profits.

5 Conclusion

This paper has provided the first theoretical analysis of automated switching services. Our model could also help understand the effects of future policy interventions, such as the UK's plans to automatically switch some consumers onto cheaper energy tariffs.[5] For instance, if such a policy raises the proportion of shoppers, σ_0, our model suggests that this would crowd out a private auto-switching service by reducing its users, but still lower supplier prices and benefit consumers.[6]

Appendix

Proof of Proposition 1. From (4) note

$$\omega'(\sigma) = -v\int_0^1 \frac{nz^{n-1}\left(1-nz^{n-1}\right)}{\left[1-\sigma\left(1-nz^{n-1}\right)\right]^2}dz$$

$$\omega''(\sigma) = -2v\int_0^1 \frac{nz^{n-1}\left(1-nz^{n-1}\right)^2}{\left[1-\sigma\left(1-nz^{n-1}\right)\right]^3}dz$$

where $\omega''(\sigma) < 0\ \forall 2 \leq n < \infty$. Thus, to prove there exists a unique $\widehat{\sigma} \in (0,1)$ that sets $\omega'(\sigma) = 0$, we show $\omega'(0) > 0$ and $\omega'(1) < 0$. Manipulating $\omega'(0) = -v\int_0^1 nz^{n-1}\left(1-nz^{n-1}\right)dz$ and $\omega'(1) = -v\int_0^1 \frac{1-nz^{n-1}}{nz^{n-1}}dz$ by denoting $\overline{z} = \left(\frac{1}{n}\right)^{\frac{1}{n-1}}$ so $n\overline{z}^{n-1} = 1$, where nz^{n-1} is increasing in z, gives

$$\begin{aligned}
\omega'(0) &= v\left[-\int_0^{\overline{z}} nz^{n-1}\left(1-nz^{n-1}\right)dz + \int_{\overline{z}}^1 nz^{n-1}\left(nz^{n-1}-1\right)dz\right] \\
&> v\left[-n\overline{z}^{n-1}\int_0^{\overline{z}}\left(1-nz^{n-1}\right)dz + n\overline{z}^{n-1}\int_{\overline{z}}^1\left(nz^{n-1}-1\right)dz\right] \\
&= -vn\overline{z}^{n-1}\int_0^1\left(1-nz^{n-1}\right)dz = 0
\end{aligned}$$

[5] See https://www.reuters.com/world/uk/britain-plans-trial-automatic-switching-consumer-energy-bills-2021-07-23/, accessed 1st September 2023.

[6] Our main results remain robust if we replace our stage 2 pricing equilibrium with Myatt and Ronayne's (2023) asymmetric pure-strategy pricing equilibrium. The introduction of the service still lowers expected prices and benefits all consumers through a search externality. However, it now serves as many non-shoppers as possible and price dispersion always increases. Details available on request.

and

$$\begin{aligned}\omega'(1) &= v\left[-\int_0^{\overline{z}} \frac{1-nz^{n-1}}{nz^{n-1}}dz + \int_{\overline{z}}^1 \frac{nz^{n-1}-1}{nz^{n-1}}dz\right] \\ &< v\left[-\frac{1}{n\overline{z}^{n-1}}\int_0^{\overline{z}}\left(1-nz^{n-1}\right)dz + \frac{1}{n\overline{z}^{n-1}}\int_{\overline{z}}^1\left(nz^{n-1}-1\right)dz\right] \\ &= -\frac{v}{n\overline{z}^{n-1}}\int_0^1\left(1-nz^{n-1}\right)dz = 0,\end{aligned}$$

since $\int_0^1 (1-nz^{n-1})\,dz = 0$. Finally, note $\omega(0) = v\int_0^1 (1-nz^{n-1})\,dz = 0$ and $\omega(1) = 0$. □

Proof of Proposition 2. From (5),

$$\pi_S'(\sigma) = \omega(\sigma) + \omega'(\sigma)(\sigma - \sigma_0) \tag{7}$$

$$\pi_S''(\sigma) = 2\omega'(\sigma) + \omega''(\sigma)(\sigma - \sigma_0).$$

We show there exists a unique $\sigma^* \in (\sigma_0, 1)$ that sets $\pi_S'(\sigma^*) = 0$ such that $\pi_S(\sigma^*) > 0$.

First, we prove there exists a unique $\sigma^* \in (\widehat{\sigma}, 1)$ that sets $\pi_S'(\sigma^*) = 0$ where $\pi_S''(\sigma^*) < 0$. Note $\forall\sigma \geq \sigma_0$: i) if $\sigma \in (0, \widehat{\sigma})$, $\pi_S'(\sigma) > 0$ since $\omega(\sigma) > 0$ and $\omega'(\sigma) > 0$, and ii) if $\sigma \geq \widehat{\sigma}$, $\pi_S''(\sigma) < 0$ from $\omega'(\sigma) \leq 0$ and $\omega''(\sigma) < 0$, where $\pi_S'(\widehat{\sigma}) = \omega(\widehat{\sigma}) > 0$ and $\pi_S'(1) = \omega'(1)(1-\sigma_0) < 0$ so $\sigma^* \in (\widehat{\sigma}, 1)$. Given $\omega'(\sigma) < 0\ \forall\sigma > \widehat{\sigma}$, we know $\omega(\sigma^*) < \omega(\widehat{\sigma})$ and $\omega(\sigma^*) > \omega(1) = 0$.

To show $\pi_S(\sigma^*) > 0$, it suffices to demonstrate $\sigma^* \in (\sigma_0, 1)\ \forall\sigma_0 \in [0, 1)$. Note a) when $\sigma_0 = 0$, then $\sigma^* > \widehat{\sigma} > \sigma_0 = 0$; b) $\lim_{\sigma_0 \to 1}\sigma^* = \sigma_0$ from $\pi_S'(1) = \omega'(1)(1-\sigma_0)$; and c) applying the implicit function theorem to (7) yields

$$\frac{\partial\sigma^*}{\partial\sigma_0} = -\frac{1}{\pi_S''(\sigma)}\frac{\partial\pi_S'(\sigma)}{\partial\sigma_0} = \frac{\omega'(\sigma)}{2\omega'(\sigma) + \omega''(\sigma)(\sigma - \sigma_0)} \in (0, 1).$$

□

Proof of Proposition 3. Note $\frac{d\sigma_U^*}{d\sigma_0} = \frac{\partial\sigma^*}{\partial\sigma_0} - 1 < 0$ since $\sigma_U^* = \sigma^* - \sigma_0$ and $\frac{\partial\sigma^*}{\partial\sigma_0} \in (0, 1)$. Further, $\frac{d\omega(\sigma^*)}{d\sigma_0} = \frac{\partial\sigma^*}{\partial\sigma_0}\omega'(\sigma^*) < 0$ since $\sigma^* \in (\widehat{\sigma}, 1)$ and $\omega'(\sigma^*) < 0$. Finally, using (5), $\frac{d\pi_S(\sigma^*)}{d\sigma_0} = \left.\frac{\partial\pi_S(\sigma)}{\partial\sigma_0}\right|_{\sigma=\sigma^*} = -\omega(\sigma^*) < 0\ \forall\sigma^* \in (0, 1)$. □

Proof of Proposition 4. Using (4) and (5) with the envelope theorem yields $\frac{d\kappa^*}{dn} = \frac{d\omega(\sigma^*)}{dn} = \frac{\partial(E(p)-E(p_{min}))}{\partial n}$ and $\frac{d\pi_S(\sigma^*)}{dn} = \frac{\partial(E(p)-E(p_{min}))}{\partial n}(\sigma^* - \sigma_0)$. Given $\sigma^* > \sigma_0$, these are both strictly positive because $\frac{\partial(E(p)-E(p_{min}))}{\partial n} > 0$ follows from Morgan et al. (2006, Proposition 3) (while using (2) and (3) to allow for any $v > 0$). Next, using (2) and (3), $\lim_{n\to\infty} E(p|\sigma) = v$ and $\lim_{n\to\infty} E(p_{min}|\sigma) = 0 \ \forall\ \sigma \in (0,1)$ so $\lim_{n\to\infty}\omega(\sigma) = v$ and $\lim_{n\to\infty}\pi_S(\sigma) = v(\sigma - \sigma_0)$. Hence, when n is very large, $\sigma^* \approx 1$ and $\sigma_U^* \approx 1 - \sigma_0$, $\kappa^* = \omega(\sigma^*) = v$ and $\pi_S(\sigma^*) \approx v(1-\sigma_0)$. □

Proof of Proposition 5. i) Using (2) and (3),

$$\frac{\partial E(p|\sigma)}{\partial \sigma} = -v \int_0^1 \frac{nz^{n-1}}{\left[1 - \sigma\left(1 - nz^{n-1}\right)\right]^2} dz < 0$$

and

$$\frac{\partial E(p_{min}|\sigma)}{\partial \sigma} = -v \int_0^1 \frac{\left(nz^{n-1}\right)^2}{\left[1 - \sigma\left(1 - nz^{n-1}\right)\right]^2} dz < 0.$$

ii) If $\sigma_0 \geq \widehat{\sigma}$, then $\omega(\sigma^*) < \omega(\sigma_0)$ as $\omega'(\sigma) < 0\ \forall \sigma > \widehat{\sigma}$ and $\sigma^* > \sigma_0\ \forall \sigma_0 < 1$. However, if $\sigma_0 \leq \widehat{\sigma}$, there exists a unique $\sigma_0' \in (0, \widehat{\sigma})$ that sets $\omega(\sigma^*) - \omega(\sigma_0') = 0$ so $\omega(\sigma^*) - \omega(\sigma_0') < (>) 0\ \forall \sigma_o > (<) \sigma_o'$. This follows because a) $\omega(\sigma^*) > \omega(\sigma_0)$ when $\sigma_0 = 0$ as $\omega(0) = 0$; b) $\omega(\sigma^*) < \omega(\sigma_0)$ when $\sigma_0 = \widehat{\sigma}$ as $\widehat{\sigma} < \sigma^*$ and $\omega''(\sigma) < 0$; and c) $\frac{\partial(\omega(\sigma^*)-\omega(\sigma_0))}{\partial \sigma_0} = \omega'(\sigma^*)\frac{\partial \sigma^*}{\partial \sigma_0} - \omega'(\sigma_0) < 0$ since $\omega'(\sigma^*) < 0 < \omega'(\sigma_0)$ and $\frac{\partial \sigma^*}{\partial \sigma_0} \in (0,1)$. □

Proof of Proposition 6. As $\pi(\sigma) = \frac{v(1-\sigma)}{n}$, $n(\pi(\sigma^*) - \pi(\sigma_0)) = -(\sigma^* - \sigma_0)v$. Given (6), $\Pi(\sigma^*) - \Pi(\sigma_0) = -(\sigma^* - \sigma_0)(v - \omega(\sigma^*))$. Given fixed welfare, $CS(\sigma^*) - CS(\sigma_0) = -(\Pi(\sigma^*) - \Pi(\sigma_0))$. Their signs follow as $\sigma^* > \sigma_0$ and $v > \omega(\sigma^*)$ from (4) $\forall\ 2 \leq n < \infty$. □

References

[1] Armstrong, M. (2015). Search and Ripoff Externalities. Review of Industrial Organization, 47, 273-302.

[2] Baye, M., and Morgan, J. (2001). Information Gatekeepers on the Internet and the Competitiveness of Homogeneous Product Markets. American Economic Review, 91(3), 454-474.

[3] Citizens Advice (2020). Stuck in the Middle. Available at https://www.citizensadvice.org.uk/Global/CitizensAdvice/Energy/TPIs%20report%20-%20FINAL%20(1).pdf.

[4] Feldhaus, C., Lingens, J., Löschel, A., and Zunker, G. (2022). Encouraging Consumer Activity Through Automatic Switching of the Electricity Contract - A Field Experiment. Energy Policy, 164, 112855.

[5] Moraga-González, J-L. and Wildenbeest, M. (2012). Comparison Sites. Handbook of the Digital Economy, M. Peitz and J. Waldfogel (eds), p.224-53, Oxford University Press.

[6] Morgan, J., Orzen, H., and Sefton, M. (2006). A Laboratory Study of Advertising and Price Competition. European Economic Review, 50(2), 323-347.

[7] Myatt, D. and Ronayne, D. (2023). A Theory of Stable Price Dispersion. Working paper

[8] Ronayne, D. (2021). Price Comparison Websites. International Economic Review, 62(3), 1081-1110.

[9] Ronayne, D., and Taylor, G. (2022). Competing Sales Channels with Captive Consumers. Economic Journal, 132(642), 741-766.

[10] Tappata, M. (2009). Rockets and Feathers: Understanding Asymmetric Pricing. RAND Journal of Economics, 40(4), 673-687.

[11] Varian, H. (1980). A Model of Sales. American Economic Review, 70(4), 651-659.