# Welfare effects of common ownership in an international duopoly

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**Abstract**

We formulate an international oligopoly model in the presence of global common ownership. We theoretically investigate how common ownership affects the volume of international trade in an oligopoly market and global welfare. We find that welfare decreases (increases) with the degree of common ownership when the international transport costs are low (high), whereas common ownership reduces international trade. This conclusion remains valid in the presence of import tariffs and asymmetric common ownership share.

**JEL classification codes:** L13, F12, K21

**Keywords:** overlapping ownership, transport cost, welfare-improving production substitution

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# 1 Introduction

In this study, we investigate how common ownership by institutional investors and overlapping ownership in oligopoly markets affect international trade and economic welfare. We adopt a payoff interdependence approach discussed by López and Vives (2019) and formulate a simple two-country, duopoly model in which firms engage in Cournot competition in the two countries. Each firm is assumed to be concerned with its rival's as well as its own profits, and we discuss the relationship between the degree of concern about the rival's profits (i.e., the degree of common or overlapping ownership) and economic welfare. In contrast to the welfare-reducing effects of the significant common ownership emphasized in the previous literature, we present a counter-example and demonstrate that even a significant degree of common ownership may improve welfare when international transport costs are high. The concern about rival profit restricts competition and harms consumer welfare. However, it economizes international transport costs and increases industry profits. This effect may dominate the effect on consumer welfare, resulting in the improvement of welfare.

A distinct feature of financial markets in recent years is the high concentration in the investment industry (Backus et al., 2021). The big three index funds and passive management firms own more than 10% of the shares in listed firms globally, and they are the largest stockholders in many listed firms (Nikkei Market News, 2018/10/24). If listed firms are concerned about the interests of these common owners, then they are indirectly concerned about the profits of other firms (payoff interdependence). Moreover, these common owners may directly induce listed firms to pursue industry profits rather than individual profits through voting or communication with the management of these firms. Hence, the firms may account for industry profits rather than simply their own profits in the presence of common ownership (López and Vives, 2019, Vives, 2020). Such common ownership may reduce firms' incentive to compete (Azar et al., 2018, 2022; Moreno and Petrakis, 2022) and may harm consumer welfare.[1] Thus, common ownership has become a central issue in recent antitrust debates (Elhauge, 2016; Backus et al., 2021).

Cross-shareholding is another widely observed phenomenon, with several firms in the same industry holding stakes in each other.[2] In the presence of cross-shareholding, firms are concerned about the profits of other firms with ownership relationships and may generate an anti-competitive effect (Reynolds and Snapp, 1986; Farrell and Shapiro, 1990; Gilo et al., 2006). Both common ownership and cross-shareholding generate payoff interdependence among firms and may restrict competition similarly.

Although the literature emphasizes the anti-competitive and welfare-reducing effects of common or overlapping ownership, several studies also point out the possible welfare-improving effects of common ownership, especially when the degree of common ownership is insignificant. While common ownership reduces competition in product markets and raises prices, partial ownership by common owners in the same industry may lead firms to internalize industry-wide externalities and improve welfare. López and Vives (2019) assert that common ownership internalizes the positive externality of R&D, demonstrating that this welfare-improving effect may dominate the competition-reducing effects when the degree of common ownership is relatively low. In other words, they suggest a possible inverted U-shaped relationship between the degrees of common ownership and welfare. Sato and Matsumura (2020) investigate a free-entry market and find that common ownership internalizes the business-stealing effect and moderate common ownership may improve welfare.[3] They also demonstrate that significant common ownership always reduces welfare. Again, an inverted U-shaped relationship between the degree of common ownership and welfare is seen. Chen et al. (2021) investigate vertically related markets. They demonstrate that

[1] Azar et al. (2018) find that ticket prices are higher under common ownership in the US airline industry, and Azar et al. (2022) provide convincing evidence that common ownership and cross-ownership increase monopsony power in the banking industry.

[2] An example is the cross-shareholding among Toyota Motor Corporation, Suzuki Motor Corporation, and Isuzu Motors Limited (automobile makers).

[3] For a discussion on the business-stealing effect in free-entry markets, see Mankiw and Whinston (1986).

common ownership mitigates the problem of double marginalization, and that this welfare-improving effect dominates the competition-reducing effects on downstream markets, if the competition among downstream firms is weak. Hirose and Matsumura (2022) investigate the relationship between common ownership and firms' commitments to environmental corporate social responsibility. They demonstrate that common ownership may improve welfare, but it weakens firms' incentive for effective emission-reducing commitments. However, no study has analyzed how common ownership affects trading volumes and global welfare in the presence of international oligopolies.

This study considers the welfare effect of common ownership in an international duopoly. We formulate a symmetric two-country model with one firm in each country. Each duopolist chooses the quantities for the home-market and exports. The export incurs additional costs for international transport. We find that global welfare decreases (increases) with common ownership when transport costs are low (high). Our results suggest that even a high level of common ownership may improve welfare. We incorporate international transport costs into the model, and explain how it affects the relationship between common ownership and welfare. This constitutes the contribution of our study.

The remainder of this paper is organized as follows. Section 2 describes the model formulation. Section 3 analyzes equilibrium outcomes. Section 4 incorporates import tariffs (Section 4.1) and asymmetric overlapping ownership (Section 4.2). Finally, Section 5 concludes the paper.

# 2 The Model

We formulate a symmetric two-country, two-firm model with two countries, A and B, and two firms, 1 and 2. Firm 1 (2) is a home firm in country A (B) and also exports to country B (A). We assume that each firm has a marginal cost of $c$ for its domestic market and a marginal cost of $c + t$ for its foreign (export) market, where $c$ is the marginal cost of production, $t$

is the international transport cost, and both $c$ and $t$ are non-negative constants. Let $q_i^A$ and $q_i^B$ be the output of firm $i$ supplied for market A and B, respectively, where $i = 1, 2$. The common demand function for each country market is $p^k = p(Q^k)$ where $k = A, B$. We assume that $p' < 0$ and $p'' \leq 0$ as long as both $p$ and $Q$ are positive. This guarantees that the second-order conditions are satisfied, the strategies are strategic substitutes, and the stability condition is satisfied.[4] We adopted Dixit's (1984) segmented market setup. In other words, consumer and independent trader arbitrage is assumed to be prohibitively costly.[5]

The profits of firm 1 ($\pi_1$) and firm 2 ($\pi_2$) are respectively

$$\pi_1 = (p^A - c)q_1^A + (p^B - c - t)q_1^B, \tag{1}$$

$$\pi_2 = (p^A - c - t)q_2^A + (p^B - c)q_2^B. \tag{2}$$

Following recent theoretical literature on common ownership (López and Vives, 2019), we assume that each firm $i$ has the following objective function:

$$\psi_i = \pi_i + \lambda \pi_j, \tag{3}$$

where $\pi_i$ is firm $i$'s profit, $\pi_j$ is its rival's profit, and $\lambda$ is the degree of common ownership.[6] We restrict our attention to the case in which the solution is interior. In other words, we assume that both firms are active in both markets.

[4] The second-order conditions are satisfied, because $2p' + p''(q_i + \lambda q_j) < 0$ holds. The strategies are strategic substitutes, because $(1+\lambda)p' + p''(q_i + \lambda q_j) < 0$ holds. The stability condition is satisfied, because $\left|2p' + p''(q_i + \lambda q_j)\right| > \left|(1+\lambda)p' + p''(q_i + \lambda q_j)\right|$ holds, where $i,\ \ j = 1, 2,$ and $i \neq j$. Furthermore, we can replace the assumption $p'' \leq 0$ with $p' + p''Q < 0$ (the industry's marginal revenue is decreasing). Under this assumption, the second-order conditions are satisfied, and strategies are strategic substitutes. However, our main results do not hold when strategies are strategic complements. We discuss this point later.

[5] This assumption is not essential. Unless transport costs for consumers or independent traders are strictly lower than those of firms, arbitrage plays no role in our model.

[6] Prior studies have also investigated this type of payoff interdependence using a coefficient-of-cooperation model (Cyert and Degroot, 1973; Escrihuela-Villar, 2015) and a relative profit maximization model (Escrihuela-Villar and Gutiérrez-Hita, 2019; Hamamura, 2021; Matsumura and Matsushima, 2012; Matsumura et al., 2013).

Let $W^A$, $W^B$, and $W$ denote country A's, country B's, and global welfare.

$$W^A = \left(\int_0^{Q^A} p(q)dq - p^A Q^A\right) + \pi_1,\ W^B = \left(\int_0^{Q^B} p(q)dq - p^B Q^B\right) + \pi_2,$$
$$W = W^A + W^B$$

# 3 Equilibrium

Given the symmetry between the two countries, we focus on the competition in market A. The first-order conditions of firms 1 and 2 are

$$p^{A'} q_1^A + (p^A - c) + \lambda p^{A'} q_2^A = 0, \tag{4}$$
$$p^{A'} q_2^A + (p^A - c - t) + \lambda p^{A'} q_1^A = 0. \tag{5}$$

Let $\bar{t}(\lambda)$ be the upper bound of $t$ that yields the interior solution. From (5), we obtain $\bar{t} = p^M - c + \lambda p^{M'} q^M$ where $p^M$ ($q^M$) is the price (output) when the home firm is the monopolist. As the monopoly price is independent of $\lambda$ and $t$, $\bar{t}$ decreases with $\lambda$.[7]

Because of the symmetry of the two countries, we focus on market A only and drop the superscript A until we discuss equilibrium outcomes. As (4) and (5) yield the equilibrium output, totally differentiating (4) and (5) generates,

$$\frac{\partial q_1}{\partial t} = -\frac{p'(1+\lambda) + p''(q_1 + \lambda q_2)}{p'\Omega_1},\ \frac{\partial q_2}{\partial t} = \frac{2p' + p''(q_1 + \lambda q_2)}{p'\Omega_1},\ \frac{\partial Q}{\partial t} = \frac{1-\lambda}{\Omega_1}, \tag{6}$$

$$\frac{\partial q_1}{\partial \lambda} = \frac{\Omega_2}{\Omega_1},\ \frac{\partial q_2}{\partial \lambda} = -\frac{[2p' + p''(q_1 + \lambda q_2)]\Omega_2 + p' q_2 \Omega_1}{\Omega_1 \Omega_3},\ \frac{\partial Q}{\partial \lambda} = \frac{-p'[(1-\lambda)\Omega_2 + q_2\Omega_1]}{\Omega_1\Omega_3}, \tag{7}$$

where

$$\Omega_1 = (1-\lambda)[(3+\lambda)p' + p''(1+\lambda)Q], \tag{8}$$
$$\Omega_2 = [p'(1+\lambda) + p'' q_1]q_1 - (2p' + p'' q_2)q_2, \tag{9}$$
$$\Omega_3 = p'(1+\lambda) + p''(q_1 + \lambda q_2). \tag{10}$$

[7] If $t \geq \bar{t}$, each firm becomes the monopolist in its home market; thus, a further increase in $\lambda$ does not affect the equilibrium outcomes.

The following Lemma 1 illustrates the properties of $\Omega_1$, $\Omega_2$, and $\Omega_3$, which are critical for determining the relationship between $\lambda$ and the equilibrium outcomes.

**Lemma 1** (i) $\Omega_1 < 0$. (ii) $\Omega_2 > 0$ if $t$ is sufficiently low (i.e., sufficiently close to 0). (iii) $\Omega_2 < 0$ if $t$ is sufficiently high (i.e., sufficiently close to $\bar{t}$). (iv) $\Omega_3 < 0$.

**Proof** See the Appendix.

We use superscript A* (B*) to denote equilibrium outcomes in market A (B). By symmetry of the two countries, $q_1^{A*} = q_2^{B*}$, $q_2^{A*} = q_1^{B*}$, and $Q^{A*} = Q^{B*}$. From (6),(7), and Lemma 1, we obtain the following lemma.

**Lemma 2** (i) $q_1^{A*}$ increases with $t$. (ii) $q_2^{A*}$ and $Q^{A*}$ decrease with $t$. (iii) $q_1^{A*}$ increases (decreases) with $\lambda$ if $t$ is sufficiently high (low). (iv) $q_2^{A*}$ and $Q^{A*}$ decrease with $\lambda$. (v) $\partial q_1^{A*}/\partial\lambda \geq \partial q_2^{A*}/\partial\lambda$, where the equality holds if and only if $t = 0$. (vi) $\left|\partial q_1^{A*}/\partial\lambda\right| \leq \left|\partial q_2^{A*}/\partial\lambda\right|$, if $t$ is sufficiently low. The equality holds if and only if $t = 0$.

**Proof** See the Appendix.

An increase in $t$ raises firm 2's marginal cost for market A, which reduces $q_2^{A*}$ (direct cost effect). As the strategies are strategic substitutes, $q_1^{A*}$ increases through strategic interactions between the two firms (indirect strategic effect). As the direct cost effect dominates the indirect strategic effect under the stability condition, an increase in $t$ decreases $Q^{A*}$.

When $\lambda$ is larger, each firm is more concerned with its rival's profit. Thus, an increase in $\lambda$ always reduces each firm's output to increase its rival's profit when firms have the same marginal costs for their home and export markets (i.e., $t = 0$). However, under cost differences for home and export markets (i.e., $t > 0$), an increase in $\lambda$ may stimulate the home firm's production, which seems to be counter-intuitive. This is because the home firm's production is more efficient than that of the foreign firm from the viewpoint of joint-profit-maximization. When $\lambda$ is larger, the equilibrium combination of outputs is close to the cooperative (joint-profit-maximizing) one. Thus, the foreign firm has a stronger incentive than the home firm to reduce its output. Because the strategies in the second stage are strategic substitutes, a reduction in the foreign firm's output naturally increases the home firm's output. This effect can be significant, especially when $t$ is high, and may dominate the standard output-reducing effect owing to common ownership. Consequently, the home firm's output may increase with $\lambda$.

Even when $q_1^{A*}$ decreases with $\lambda$ due to a sufficiently small $t$, the output-reducing effect of common ownership is greater for the foreign firm than the home firm. This leads to Lemma 2(v) and (vi).

Because of the symmetry between the two countries, the equilibrium global welfare $W^*$ can be re-written as follows.

$$W^* = 2[\int_0^{Q^{A*}} p(q)dq - cq_1^{A*} - (c+t)q_2^{A*}].$$

We thus obtain

$$\frac{\partial W^*}{\partial \lambda} \quad = \quad 2\left[p(Q^{A*})\frac{\partial Q^{A*}}{\partial \lambda} - c\frac{\partial q_1^{A*}}{\partial \lambda} - (c+t)\frac{\partial q_2^{A*}}{\partial \lambda}\right]. \tag{11}$$

We now present our main result.

**Proposition 1** (i) $\partial W^*/\partial\lambda < 0$ if $t$ is sufficiently low. (ii) $\partial W^*/\partial\lambda > 0$ if $t$ is sufficiently high.

**Proof** See the Appendix.

Proposition 1(i) (Proposition 1(ii)) characterizes the welfare consequence of common ownership when $t$ is low (high). We explain the intuition behind this result. Each firm's marginal cost for the home market is lower than that for the foreign market because of international transport costs. In other words, each firm's supply for the home market is more profitable than that for the foreign market. In the presence of common ownership, each firm is concerned with the rival's profit. Thus, each firm reduces its supply to the foreign market more significantly than it does to the home market. This reduces the (weighted) average of the two firms' costs and increases their joint profits. This welfare-improving effect is more pronounced when $t$ is higher, dominating the welfare-reducing effect owing to smaller total output (smaller consumer surplus). Therefore, common ownership improves welfare if $t$ is high.[8]

However, the assumption of strategic substitutes is crucial to our results. Under strategic complementarity, common ownership always reduces firms' outputs for both home and foreign markets. Thus, the welfare-improving production substitution from foreign to domestic firms becomes weak, and common ownership may harm welfare even when the international transport cost is high.

# 4 Extensions

In this section, we extend our analysis by introducing import tariffs (Section 4.1) and asymmetric ownership structure (Section 4.2). For this purpose, we adopt a linear demand function because of the tractability. The linear demand function is popular in literature (Moreno and Petrakis, 2022; Hirose and Matsumura, 2022). The inverse demand function in country $k$ is $p^k = a - Q^k$, where $Q^k := q_1^k + q_2^k$ and $k = A, B$.

## 4.1 Import tariffs

In this subsection, we introduce import tariffs. We assume that $\lambda$ is common between firms 1 and 2, but the tariff rates can differ between the two countries. Because of the asymmetry between the two countries, we analyze both markets A and B. Each country's welfare is the sum of the local firm's profit, consumer surplus, and import tariff revenue:

$$W^A = \left(\int_0^{Q^A} p(q)dq - p^A Q^A\right) + \pi_1 + \tau^A q_2^A, \tag{12}$$

$$W^B = \left(\int_0^{Q^B} p(q)dq - p^B Q^B\right) + \pi_2 + \tau^B q_1^B, \tag{13}$$

[8]See Lahiri and Ono (1988) for discussions of welfare-improving production substitution.

where $\tau^A$ and $\tau^B$ are unit import tariffs of countries A and B, respectively. Global welfare $W$ is the sum of local welfare from countries A and B (i.e., $W^A + W^B$). The profit of firm 1 ($\pi_1$) and firm 2 ($\pi_2$) are

$$\pi_1 = (p^A - c)q_1^A + (p^B - c - t - \tau^B)q_1^B, \tag{14}$$

$$\pi_2 = (p^A - c - t - \tau^A)q_2^A + (p^B - c)q_2^B. \tag{15}$$

We restrict our attention to the case in which the solution is interior. In other words, we assume that both firms are active in both markets.

We now display the equilibrium outcomes in countries A and B. The first-order conditions for firms 1 and 2 in country A, which are derived from their respective objective functions presented in (3), are as follows.

$$p^{A'} q_1^A + (p^A - c) + \lambda p^{A'} q_2^A = 0, \tag{16}$$

$$p^{A'} q_2^A + (p^A - c - t - \tau^A) + \lambda p^{A'} q_1^A = 0. \tag{17}$$

In country B, the first-order conditions of firms 1 and 2 are respectively

$$p^{B'} q_1^B + (p^B - c - t - \tau^B) + \lambda p^{B'} q_2^B = 0. \tag{18}$$

$$p^{B'} q_2^B + (p^B - c) + \lambda p^{B'} q_1^B = 0. \tag{19}$$

The second-order conditions are satisfied.

The superscript † represents the equilibrium outcomes in this subsection. The first-order conditions yield the following equilibrium outputs in countries A and B:

$$q_1^{A\dagger} = \frac{(a-c)(1-\lambda)+t(1+\lambda)+\tau^A(1+\lambda)}{(3+\lambda)(1-\lambda)}, \tag{20}$$

$$q_2^{A\dagger} = \frac{(a-c)(1-\lambda)-2t-2\tau^A}{(3+\lambda)(1-\lambda)}, \tag{21}$$

$$Q^{A\dagger} = \frac{2(a-c)-t-\tau^A}{3+\lambda}, \tag{22}$$

$$q_1^{B\dagger} = \frac{(a-c)(1-\lambda)-2t-2\tau^B}{(3+\lambda)(1-\lambda)}, \tag{23}$$

$$q_2^{B\dagger} = \frac{(a-c)(1-\lambda)+t(1+\lambda)+\tau^B(1+\lambda)}{(3+\lambda)(1-\lambda)}, \tag{24}$$

$$Q^{B\dagger} = \frac{2(a-c)-t-\tau^B}{3+\lambda}. \tag{25}$$

Substituting these equilibrium outputs into the inverse demand, profit, and welfare functions yields

$$p^{A\dagger} = \frac{a(1+\lambda)+2c+t+\tau^A}{3+\lambda}, \tag{26}$$

$$p^{B\dagger} = \frac{a(1+\lambda)+2c+t+\tau^B}{3+\lambda}, \tag{27}$$

$$\begin{aligned}\pi_1^\dagger &= \frac{\Big[(a-c)(1+\lambda)+(t+\tau^A)\Big]\Big[(a-c)(1-\lambda)+(t+\tau^A)(1+\lambda)\Big]}{(3+\lambda)^2(1-\lambda)} \\ &\quad+\frac{\Big[(a-c)(1+\lambda)-(t+\tau^B)(\lambda+2)\Big]\Big[(a-c)(1-\lambda)-2(t+\tau^B)\Big]}{(3+\lambda)^2(1-\lambda)},\end{aligned} \tag{28}$$

$$\begin{aligned}\pi_2^\dagger &= \frac{\Big[(a-c)(1+\lambda)+(t+\tau^B)\Big]\Big[(a-c)(1-\lambda)+(t+\tau^B)(1+\lambda)\Big]}{(3+\lambda)^2(1-\lambda)} \\ &\quad+\frac{\Big[(a-c)(1+\lambda)-(t+\tau^A)(\lambda+2)\Big]\Big[(a-c)(1-\lambda)-2(t+\tau^A)\Big]}{(3+\lambda)^2(1-\lambda)},\end{aligned} \tag{29}$$

$$
\begin{aligned}
W^{A\dagger} &= \frac{\left[2(a-c)-(t+\tau^A)\right]^2}{2(\lambda+3)^2} + \pi_1^\dagger + \tau^A q_2^{A\dagger}, && (30)\\
W^{B\dagger} &= \frac{\left[2(a-c)-(t+\tau^B)\right]^2}{2(\lambda+3)^2} + \pi_2^\dagger + \tau^B q_1^{B\dagger}, && (31)\\
W^{\dagger} &= W^{A\dagger} + W^{B\dagger}. && (32)
\end{aligned}
$$

We investigate the relationship between the common ownership and global welfare.

**Proposition 2** $\partial W^\dagger/\partial\lambda < (=, >) 0$ if $t < (=, >) t^\dagger$, where

$$
t^\dagger = \frac{-2(a-c)(1-\lambda^2)(1-\lambda) - (7+6\lambda+3\lambda^2)(\tau^A+\tau^B) + \sqrt{\Delta_2}}{2(5\lambda^2+14\lambda+13)} > 0, \qquad (33)
$$

and

$$
\begin{aligned}
\Delta_2 = {} & 4a^2\lambda^6 + 32a^2\lambda^5 + 68a^2\lambda^4 - 32a^2\lambda^3 - 180a^2\lambda^2 + 108a^2 - 8ac\lambda^6 - 64ac\lambda^5 - 136ac\lambda^4 \\
& +64ac\lambda^3 + 360ac\lambda^2 - 216ac + 2a\lambda^5\tau^A + 2a\lambda^5\tau^B + 14a\lambda^4\tau^A + 14a\lambda^4\tau^B + 20a\lambda^3\tau^A \\
& +20a\lambda^3\tau^B - 36a\lambda^2\tau^A - 36a\lambda^2\tau^B - 54a\lambda\tau^A - 54a\lambda\tau^B + 54a\tau^A + 54a\tau^B + 4c^2\lambda^6 \\
& +32c^2\lambda^5 + 68c^2\lambda^4 - 32c^2\lambda^3 - 180c^2\lambda^2 + 108c^2 - 2c\lambda^5\tau^A - 2c\lambda^5\tau^B - 14c\lambda^4\tau^A \\
& -14c\lambda^4\tau^B - 20c\lambda^3\tau^A - 20c\lambda^3\tau^B + 36c\lambda^2\tau^A + 36c\lambda^2\tau^B + 54c\lambda\tau^A + 54c\lambda\tau^B \\
& -54c\tau^A - 54c\tau^B - \lambda^4(\tau^A)^2 + 18\lambda^4\tau^A\tau^B - \lambda^4(\tau^B)^2 + 28\lambda^3(\tau^A)^2 + 72\lambda^3\tau^A\tau^B \\
& +28\lambda^3(\tau^B)^2 + 98\lambda^2(\tau^A)^2 + 156\lambda^2\tau^A\tau^B + 98\lambda^2(\tau^B)^2 + 108\lambda(\tau^A)^2 + 168\lambda\tau^A\tau^B \\
& +108\lambda(\tau^B)^2 + 23(\tau^A)^2 + 98\tau^A\tau^B + 23(\tau^B)^2.
\end{aligned}
$$

**Proof** See the Appendix.

The effect of the degree of common ownership on global welfare is similar to that in Proposition 1. When $t$ is small (large), common ownership harms (improves) welfare.[9]

Moreover, even if we adopt a non-linear demand function discussed in the previous section, we obtain similar results. (i) If $t$ is sufficiently small, the global welfare decreases

[9] Solving the equation $\partial W^\dagger/\partial\lambda = 0$ gives us two solutions. The smaller solution $t$ is negative, and $t^\dagger$ we discussed in (33) is the larger solution of $t$. Thus, we obtain (i) when $0 < t < t^\dagger$, $\partial W^\dagger/\partial\lambda < 0$. (ii) when $t > t^\dagger$, $\partial W^\dagger/\partial\lambda > 0$.

in the degree of common ownership. (ii) If $t$ is sufficiently large, the global welfare increases in the degree of common ownership.

We then discuss the relationship between $\lambda$ and each country's welfare. Henceforth, we postulate $\tau^A \leq 2\tau^B$ to simplify the computation.

**Proposition 3** (i) $\partial W^{A\dagger}/\partial\lambda < (=, >)0$ if $t < (=, >)t^{A\dagger}$ where

$$t^{A\dagger} = \frac{-(a-c)(1-\lambda)^2(1+\lambda) + \tau^A(3+4\lambda+\lambda^2) - 2\tau^B(5+5\lambda+2\lambda^2) + \sqrt{\Delta_2^A}}{5\lambda^2+14\lambda+13}$$

and

$$\begin{aligned}
\Delta_2^A \;=\; & -(\lambda+3)\Big[-a^2\lambda^5 - 5a^2\lambda^4 - 2a^2\lambda^3 + 14a^2\lambda^2 + 3a^2\lambda - 9a^2 + 2ac\lambda^5 + 10ac\lambda^4 + 4ac\lambda^3 \\
& -28ac\lambda^2 - 6ac\lambda + 18ac + 2a\lambda^4\tau^A - 3a\lambda^4\tau^B + 10a\lambda^3\tau^A - 14a\lambda^3\tau^B + 34a\lambda^2\tau^A \\
& -32a\lambda^2\tau^B + 54a\lambda\tau^A - 42a\lambda\tau^B + 28a\tau^A - 37a\tau^B - c^2\lambda^5 - 5c^2\lambda^4 - 2c^2\lambda^3 + 14c^2\lambda^2 \\
& +3c^2\lambda - 9c^2 - 2c\lambda^4\tau^A + 3c\lambda^4\tau^B - 10c\lambda^3\tau^A + 14c\lambda^3\tau^B - 34c\lambda^2\tau^A + 32c\lambda^2\tau^B - 54c\lambda\tau^A \\
& +42c\lambda\tau^B - 28c\tau^A + 37c\tau^B - 16\lambda^3(\tau^A)^2 + 8\lambda^3\tau^A\tau^B + 4\lambda^3(\tau^B)^2 - 62\lambda^2(\tau^A)^2 + 28\lambda^2\tau^A\tau^B \\
& +14\lambda^2(\tau^B)^2 - 88\lambda(\tau^A)^2 + 40\lambda\tau^A\tau^B + 20\lambda(\tau^B)^2 - 42(\tau^A)^2 + 20\tau^A\tau^B + 10(\tau^B)^2\Big].
\end{aligned}$$

(ii) $t^{A\dagger} > 0$ if $\tau^A = \tau^B$.

**Proof** See the Appendix.

Again, we obtain a similar result to our main result. Local welfare also increases with common ownership if the international transport cost is large. However, one important difference exists. $t^{A\dagger}$ can be negative when $\tau^A \neq \tau^B$. In this case, an increase in the degree of common ownership increases local welfare ($W^A$) for any $t \geq 0$. This is because the asymmetry of import tariffs induces the redistribution of tariff revenue between two countries in response to the increase of common ownership, and only one country may benefit from the increase of common ownership.

As we stated, we obtain a similar result on global welfare under non-linear demand functions. This implies that a similar result on local welfare if $\tau^A = \tau^B$. However, when

$\tau^A \neq \tau^B$, a non-linear demand yields ambiguous results on local welfare. In country A, if $t$ is sufficiently high, an increase in common ownership ($\lambda$) may encourage local firm 1 to increase production, owing to the absence of international transport costs. Consequently, the consumer surplus in country A and the profit of firm 1 both rise. Conversely, the tariff revenue in country A may decline as firm 2 exports fewer goods to country A in terms of the high value of $t$, the increase in common ownership, and the asymmetric tariff rates ($\tau^A \neq \tau^B$). Thus, the impact of common ownership on country A's welfare hinges on these two effects.

## 4.2 Asymmetric $\lambda$

In this subsection, we allow firms to have different ownership structures and thus have different objective functions. A typical example is asymmetric cross-ownership in which firm 1 (2) owns firm 2's (1's) share, and the cross-ownership share is different. Another example is where the degree of ownership by the institutional investors is the same, but the ownership structure of other owners differs between these two firms.[10] Put differently, we allow different degree of concerns with the rival's profits. We assume that

$$\psi_1 = \pi_1 + \lambda_{21}\pi_2, \quad \psi_2 = \pi_2 + \lambda_{12}\pi_1. \tag{34}$$

Without loss of generality, we assume $\lambda_{21} \leq \lambda_{12}$. For analytical simplicity, we ignore the import tariffs.

We now display the equilibrium outcomes in countries A and B, respectively. The first-order conditions of firm 1 and firm 2 in country A, which are derived from their respective objective functions presented in (34), are

$$p^{A'} q_1^A + (p^A - c) + \lambda_{21} p^{A'} q_2^A = 0, \tag{35}$$

$$p^{A'} q_2^A + (p^A - c - t) + \lambda_{12} p^{A'} q_1^A = 0. \tag{36}$$

[10]For the discussion on the latter case, see López and Vives (2019) and Vives (2020).

Similarly in country B, the first-order conditions of firm 1 and firm 2 are

$$p^{B'} q_1^B + (p^B - c - t) + \lambda_{21} p^{B'} q_2^B = 0, \tag{37}$$

$$p^{B'} q_2^B + (p^B - c) + \lambda_{12} p^{B'} q_1^B = 0. \tag{38}$$

The second-order conditions are satisfied.

The superscript ‡ represents the equilibrium outcomes in this subsection. The first-order conditions yield

$$q_1^{A\ddagger} = \frac{(a-c)(1-\lambda_{21}) + t(1+\lambda_{21})}{3-\lambda_{12}-\lambda_{21}-\lambda_{12}\lambda_{21}}, \tag{39}$$

$$q_2^{A\ddagger} = \frac{(a-c)(1-\lambda_{12}) - 2t}{3-\lambda_{12}-\lambda_{21}-\lambda_{12}\lambda_{21}}, \tag{40}$$

$$Q^{A\ddagger} = \frac{(2-\lambda_{12}-\lambda_{21})(a-c) - t(1-\lambda_{21})}{3-\lambda_{12}-\lambda_{21}-\lambda_{12}\lambda_{21}}, \tag{41}$$

$$q_1^{B\ddagger} = \frac{(a-c)(1-\lambda_{21}) - 2t}{3-\lambda_{12}-\lambda_{21}-\lambda_{12}\lambda_{21}}, \tag{42}$$

$$q_2^{B\ddagger} = \frac{(a-c)(1-\lambda_{12}) + t(1+\lambda_{12})}{3-\lambda_{12}-\lambda_{21}-\lambda_{12}\lambda_{21}}, \tag{43}$$

$$Q^{B\ddagger} = \frac{(2-\lambda_{12}-\lambda_{21})(a-c) - t(1-\lambda_{12})}{3-\lambda_{12}-\lambda_{21}-\lambda_{12}\lambda_{21}}. \tag{44}$$

Substituting these equilibrium outputs into the inverse demand, profit, and welfare functions generates

$$p^{A\ddagger} = \frac{a(1-\lambda_{12}\lambda_{21}) + c(2-\lambda_{12}-\lambda_{21}) + t(1-\lambda_{21})}{3-\lambda_{12}-\lambda_{21}-\lambda_{12}\lambda_{21}}, \tag{45}$$

$$p^{B\ddagger} = \frac{a(1-\lambda_{12}\lambda_{21}) + c(2-\lambda_{12}-\lambda_{21}) + t(1-\lambda_{12})}{3-\lambda_{12}-\lambda_{21}-\lambda_{12}\lambda_{21}}, \tag{46}$$

$$\begin{aligned} \pi_1^\ddagger = {} & \frac{\Big[(a-c)(1-\lambda_{12}\lambda_{21}) + t(1-\lambda_{21})\Big]\Big[(a-c)(1-\lambda_{21}) + t(1+\lambda_{21})\Big]}{(3-\lambda_{12}-\lambda_{21}-\lambda_{12}\lambda_{21})^2} \\ & + \frac{\Big[(a-c)(1-\lambda_{21}) - 2t\Big]\Big[(a-c-t)(1-\lambda_{12}\lambda_{21}) - t(1-\lambda_{21})\Big]}{(3-\lambda_{12}-\lambda_{21}-\lambda_{12}\lambda_{21})^2}, \end{aligned} \tag{47}$$

$$\pi_2^\ddagger = \frac{\Big[(a-c)(1-\lambda_{12})-2t\Big]\Big[(a-c-t)(1-\lambda_{12}\lambda_{21})-t(1-\lambda_{12})\Big]}{(3-\lambda_{12}-\lambda_{21}-\lambda_{12}\lambda_{21})^2} + \frac{\Big[(a-c)(1-\lambda_{12}\lambda_{21})+t(1-\lambda_{12})\Big]\Big[(a-c)(1-\lambda_{12})+t(1+\lambda_{12})\Big]}{(3-\lambda_{12}-\lambda_{21}-\lambda_{12}\lambda_{21})^2}, \quad (48)$$

$$W^{A\ddagger} = \frac{\Big[(2-\lambda_{21}-\lambda_{12})(a-c)-t(1-\lambda_{21})\Big]^2}{2(3-\lambda_{12}-\lambda_{21}-\lambda_{12}\lambda_{21})^2} + \pi_1^\ddagger, \quad (49)$$

$$W^{B\ddagger} = \frac{\Big[(2-\lambda_{12}-\lambda_{21})(a-c)-t(1-\lambda_{12})\Big]^2}{2(3-\lambda_{12}-\lambda_{21}-\lambda_{12}\lambda_{21})^2} + \pi_2^\ddagger, \quad (50)$$

$$W^\ddagger = W^{A\ddagger} + W^{B\ddagger}. \quad (51)$$

We now discuss the relationship between $\lambda_{21}$ and welfare.

**Proposition 4** (i) $\partial W^\ddagger/\partial\lambda_{21} > 0$ if $t > t_{21}^\ddagger$, where

$$t_{21}^\ddagger = \frac{(a-c)(1-\lambda_{12})\Big[(1-\lambda_{12})(1-\lambda_{12}\lambda_{21})-\sqrt{\Delta_{21}}\Big]}{6\lambda_{21}-7\lambda_{12}+6\lambda_{12}\lambda_{21}+4\lambda_{12}^2\lambda_{21}+3\lambda_{12}^2+\lambda_{12}^3-13},$$

and

$$\Delta_{21} = (\lambda_{12}\lambda_{21}-1)(\lambda_{12}+3)(4\lambda_{21}-\lambda_{12}+3\lambda_{12}\lambda_{21}+\lambda_{12}^2\lambda_{21}+2\lambda_{12}^2-9).$$

(ii) $\partial W^{A\ddagger}/\partial\lambda_{21} > 0$ if $t > t^{A\ddagger}$ where

$$t^{A\ddagger} = -\frac{(a-c)(1-\lambda_{12})\Big(7-\lambda_{12}-5\lambda_{21}\lambda_{12}-\lambda_{21}-\sqrt{\Delta_3^A}\Big)}{4\Big[(1+\lambda_{12})^2(1-\lambda_{21})+2(\lambda_{12}-\lambda_{21})\Big]},$$

and

$$\Delta_3^A = -8\lambda_{12}^4\lambda_{21}+8\lambda_{12}^4+15\lambda_{12}^3\lambda_{21}^2-58\lambda_{12}^3\lambda_{21}+23\lambda_{12}^3+87\lambda_{12}^2\lambda_{21}^2-114\lambda_{12}^2\lambda_{21}+23\lambda_{12}^2 +113\lambda_{12}\lambda_{21}^2-102\lambda_{12}\lambda_{21}+57\lambda_{12}-23\lambda_{21}^2-102\lambda_{21}+81.$$

(iii) $\partial W^{B\ddagger}/\partial\lambda_{21} > 0$ if $t > t^{B\ddagger}$, where

$$t^{B\ddagger} = \frac{(1-\lambda_{21})(1-\lambda_{12}^2)\left[4-2(1+\lambda_{12})\lambda_{12}\lambda_{21}\right]+\sqrt{\Delta_3^B}}{2(11-\lambda_{12}-2\lambda_{12}\lambda_{21}-2\lambda_{12}^2\lambda_{21}-5\lambda_{12}^2-\lambda_{12}^3)}$$

and

$$\begin{aligned}\Delta_3^B \;=\;& (\lambda_{12}-1)\Big(4\lambda_{12}^5\lambda_{21}^2+8\lambda_{12}^5\lambda_{21}-4\lambda_{12}^5+24\lambda_{12}^4\lambda_{21}^2+40\lambda_{12}^4\lambda_{21}-24\lambda_{12}^4+41\lambda_{12}^3\lambda_{21}^2\\ &+26\lambda_{12}^3\lambda_{21}-31\lambda_{12}^3+9\lambda_{12}^2\lambda_{21}^2-126\lambda_{12}^2\lambda_{21}+9\lambda_{12}^2-13\lambda_{12}\lambda_{21}^2-130\lambda_{12}\lambda_{21}\\ &+51\lambda_{12}-\lambda_{21}^2+54\lambda_{21}+63\Big).\end{aligned}$$

**Proof** See the Appendix.

An increase in $\lambda_{21}$ reduces firm 1's incentive to expand its outputs for both markets because firm 1 is more concerned with firm 2's profits. This effect for market B is stronger than that for market A because firm 1's marginal cost for market B (foreign market) is higher than that for market A (home market) owing to the international transport cost. Because the strategies are strategic substitutes, firm 2 increases its output for both markets, and this effect is stronger in market B than in market A. The production substitution in market B economizes firms' total costs and may increase both firms' profits. This positive effect may dominate the negative effect on consumer surplus, especially when $t$ is large. As such, global and local welfare increase with $\lambda_{21}$.

However, we fail to derive this result under a general non-linear demand function. We can demonstrate that global welfare is decreasing in $\lambda_{21}$ when $t$ is sufficiently small, but we fail to demonstrate that welfare is increasing in $\lambda_{21}$ when $t$ is sufficiently large. Thus, a possible welfare-improving effect discussed in this study might depend on the assumption of the linearity of the demand function.

# 5 Concluding remarks

In this study, we investigate how common ownership affects the volume of international trade in an oligopoly market and global welfare. We find that common ownership reduces international trade. Welfare decreases (increases) with the degree of common ownership when the international transport cost is low (high). Therefore, common ownership can improve welfare, especially when the international transport cost is high. Moreover, this result holds in the presence of import tariffs or asymmetric ownership structures in a linear demand model. Common ownership reduces the volume of export levels more significantly than the production levels, and economizes international transport costs. This effect may dominate the consumer-welfare reducing effect and thus improve welfare.

In this study, we consider Cournot competition in homogeneous product markets. If we consider Bertrand competition in differentiated product markets, the welfare-improving effect may become weak because the reduction of exports does not stimulate the rival's supply for the domestic market. Therefore, our result may not hold in such circumstances.

Furthermore, we examine common ownership that induces non-profit-maximizing behavior. Other aspects, such as corporate social responsibility, also affect firms' objective functions. Another interesting extension of this study could incorporate a different type of non-profit maximizing behavior into our analysis and investigate how the interaction between different types of non-profit-maximizing objectives affects global welfare.[11]

[11] See Corneo and Jeanne (1994), Fjell and Pal (1996), Pal and White (1998), Bárcena-Ruiz and Garzón (2005a, b) for a discussion on the relationship between the welfare-maximizing objective of a public firm and international trade. See Bárcena-Ruiz and Sagasta (2022) for a discussion on the relationship between corporate social responsibility and international trade.

# Appendix

## Proof of Lemma 1

$\Omega_1$ can be re-written as follows:

$$\Omega_1 = (1-\lambda)[\overbrace{(p'+p''q_1)+(p'+\lambda p''q_2)}^{(-)}+\overbrace{(p'+p''q_2)+\lambda(p'+p''q_1)}^{(-)}] < 0.$$

This implies Lemma 1(i).

The sign of $\Omega_2$ depends on $q_1$ and $q_2$.

$$\begin{aligned}\Omega_2 &= [p'(1+\lambda)+p''q_1]q_1-(2p'+p''q_2)q_2\\ &= [p'(1+\lambda)+p''q_1+\lambda p''q_2]q_1-(2p'+p''q_2+\lambda p''q_1)q_2\\ &= \overbrace{[(1+\lambda)p'+p''(q_1+\lambda q_2)]}^{(-)}q_1-\overbrace{[2p'+p''(q_2+\lambda q_1)]}^{(-)}q_2.\end{aligned}$$

Therefore, $\Omega_2$ is more likely to be positive when $q_2^{A*}/q_1^{A*}$ is smaller.

If $t$ is sufficiently low (i.e., sufficiently close to zero), then $q_2 \to q_1$. Hence, $\Omega_2|_{t\to 0} = -(1-\lambda)p'q > 0$. This implies Lemma 1(ii). If $t$ is sufficiently high (i.e., sufficiently close to $\bar{t}$), $q_2 \to 0$. Hence, $\Omega_2|_{t\to\bar{t}} = [(1+\lambda)p'+p''q_1]q_1 < 0$. This implies Lemma 1(iii).

$\Omega_3 < 0$ is obtained because $p' < 0$ and $p'' < 0$. Q.E.D.

## Proof of Lemma 2

From (6) and (8), we obtain Lemma 2(i,ii). Note that $(p'+p''q_1)+\lambda(p'+p''q_2) < 0$ and $(p'+p''q_2)+\lambda(p'+p''q_1) < 0$.

Lemma 2(iii) is derived from (7) and Lemma 1(i,ii,iii).

From (7), we have

$$\begin{aligned}
\frac{\partial q_2^{A*}}{\partial \lambda} &= -\frac{[2p' + p''(q_1 + \lambda q_2)]\left\{[p'(1+\lambda) + p''q_1]q_1 - [p'(1+\lambda) + p''q_2]q_2\right\}}{\Omega_1\Omega_3} \\
&\quad -\frac{p'q_2(1-\lambda)[(p' + p''q_2) + \lambda(p' + p''q_1)]}{\Omega_1\Omega_3} \\
&= -\frac{[2p' + p''(q_1 + \lambda q_2)](q_1 - q_2)[p'(1+\lambda) + p''(q_1 + q_2)]}{\Omega_1\Omega_3} \\
&\quad -\frac{p'q_2(1-\lambda)[(p' + p''q_2) + \lambda(p' + p''q_1)]}{\Omega_1\Omega_3} < 0,
\end{aligned}$$

where we use $q_1 \geq q_2$.

Similarly, we have

$$\frac{\partial Q^{A*}}{\partial \lambda} = \frac{-p'(1-\lambda)Q[(p' + p''q_1) + \lambda(p' + p''q_2)]}{\Omega_1\Omega_3} < 0.$$

Thus, Lemma 2(iv) is obtained.

$\Omega_2$ can be re-written as $\Omega_2 = (q_1 - q_2)(p' + p''Q) + p'(\lambda q_1 - q_2)$. Thus, from (7), we present $\partial q_1^{A*}/\partial\lambda - \partial q_2^{A*}/\partial\lambda$ as follows:

$$\begin{aligned}
&\frac{\partial q_1^{A*}}{\partial \lambda} - \frac{\partial q_2^{A*}}{\partial \lambda} \qquad (52) \\
= &\frac{\Omega_2[p'(3+\lambda) + 2p''(q_1 + \lambda q_2)] + p'q_1\Omega_1}{\Omega_1\Omega_3} \\
= &\frac{p'(3+\lambda)(q_1 - q_2)[p'(1+\lambda) + p''Q] + p''[2(q_1 + \lambda q_2)\Omega_2 + p'q_2(1-\lambda^2)Q]}{\Omega_1\Omega_3} \\
= &\frac{q_1 - q_2}{\Omega_1\Omega_3}\Delta,
\end{aligned}$$

where

$$\Delta = p'(3+\lambda)[p'(1+\lambda) + p''Q] + p''\left\{2(q_1 + \lambda q_2)p''Q + p'[2(1+\lambda)q_1 - q_2 + \lambda(4+\lambda)q_2]\right\}.$$

Because we assume $p' < 0$ and $p'' \leq 0$, we find that $\Delta > 0$. Since $q_1 \geq q_2$ and equality holds only if $t = 0$, we obtain Lemma 2(v). From Lemma 2(iii,iv), we obtain $\partial q_1^{A*}/\partial\lambda < 0$ and $\partial q_2^{A*}/\partial\lambda < 0$ when $t$ is sufficiently low. We thus obtain

$$\frac{\partial q_1^{A*}}{\partial \lambda} - \frac{\partial q_2^{A*}}{\partial \lambda} \geq 0$$

from (52), which is equivalent to

$$-\left|\frac{\partial q_1^{A*}}{\partial \lambda}\right| + \left|\frac{\partial q_2^{A*}}{\partial \lambda}\right| \geq 0.$$

Thus, Lemma 2(vi) is obtained. Q.E.D.

**Proof of Proposition 1**

From (7) and (11), we obtain

$$\frac{\partial W^*}{\partial \lambda}|_{t\to 0} = \frac{2\Omega_2(1+\lambda)}{\Omega_1\Omega_3}(p' + p''q_1)\left[2(p-c) - t\right] < 0,$$

where we use Lemma 1 and the fact that $q_2 \to q_1$ as $t \to 0$. This implies Proposition 1(i).

Similarly, we obtain

$$\begin{aligned}
\frac{\partial W^*}{\partial \lambda}|_{t\to \bar{t}} &= \frac{2\Omega_2}{\Omega_1\Omega_3}\left\{-(p-c)p'(1-\lambda) + [(p-c) + \lambda p'q^M](2p' + p''q^M)\right\} \\
&= \frac{2\Omega_2}{\Omega_1\Omega_3}\left\{(p-c)[(1+\lambda)p' + p''q^M] + \lambda p'q^M(2p' + p''q^M)\right\}, \\
&= \frac{2\Omega_2}{\Omega_1\Omega_3}(1-\lambda)(p-c)(p' + p''q^M) > 0,
\end{aligned}$$

where we use Lemma 1 and the facts $\bar{t} = p - c + \lambda p'q^M$ and $p - c + p'q^M = 0$. Note that if $t \to \bar{t}$, then $q_1^{A*} \to q^M$. This implies Proposition 1(ii). Q.E.D.

**Proof of Proposition 2**

Given the equilibrium global welfare presented in (32), we found $\partial W^\dagger/\partial \lambda$ written as follows.

$$\begin{aligned}
&\left[(\lambda-1)^2(\lambda+3)^3\right]\cdot\frac{\partial W^\dagger}{\partial \lambda} = \\
&2(5\lambda^2 + 14\lambda + 13)t^2 + \Big[4(1-\lambda)(a-c) + (14+12\lambda)(\tau^A + \tau^B) - 4(a-c)\lambda^2(1-\lambda) \\
&+6\lambda^2(\tau^A+\tau^B)\Big]t - 4(a-c)^2(1-\lambda)(1-\lambda^2) - \lambda^2(a-c)(3-\lambda)(\tau^A+\tau^B) \\
&+(a-c)(\tau^A+\tau^B)(3\lambda-1) + (\lambda-1)^2\left[(\tau^A)^2 + (\tau^B)^2\right], \qquad (53)
\end{aligned}$$

where $\partial W^\dagger/\partial\lambda$ in (53) is a convex function of $t$. $\partial W^*/\partial\lambda$ is positive (zero, negative) if $t > \ (=, <) t^\dagger$, where

$$t^\dagger = \frac{-2(a-c)(1-\lambda^2)(1-\lambda) - (7+6\lambda+3\lambda^2)(\tau^A+\tau^B) + \sqrt{\Delta_2}}{2(5\lambda^2+14\lambda+13)} > 0$$

and

$$\begin{aligned}
\Delta_2 \;=\; & 4a^2\lambda^6 + 32a^2\lambda^5 + 68a^2\lambda^4 - 32a^2\lambda^3 - 180a^2\lambda^2 + 108a^2 - 8ac\lambda^6 - 64ac\lambda^5 - 136ac\lambda^4 \\
& +64ac\lambda^3 + 360ac\lambda^2 - 216ac + 2a\lambda^5\tau^A + 2a\lambda^5\tau^B + 14a\lambda^4\tau^A + 14a\lambda^4\tau^B + 20a\lambda^3\tau^A \\
& +20a\lambda^3\tau^B - 36a\lambda^2\tau^A - 36a\lambda^2\tau^B - 54a\lambda\tau^A - 54a\lambda\tau^B + 54a\tau^A + 54a\tau^B + 4c^2\lambda^6 \\
& +32c^2\lambda^5 + 68c^2\lambda^4 - 32c^2\lambda^3 - 180c^2\lambda^2 + 108c^2 - 2c\lambda^5\tau^A - 2c\lambda^5\tau^B - 14c\lambda^4\tau^A \\
& -14c\lambda^4\tau^B - 20c\lambda^3\tau^A - 20c\lambda^3\tau^B + 36c\lambda^2\tau^A + 36c\lambda^2\tau^B + 54c\lambda\tau^A + 54c\lambda\tau^B \\
& -54c\tau^A - 54c\tau^B - \lambda^4(\tau^A)^2 + 18\lambda^4\tau^A\tau^B - \lambda^4(\tau^B)^2 + 28\lambda^3(\tau^A)^2 + 72\lambda^3\tau^A\tau^B \\
& +28\lambda^3(\tau^B)^2 + 98\lambda^2(\tau^A)^2 + 156\lambda^2\tau^A\tau^B + 98\lambda^2(\tau^B)^2 + 108\lambda(\tau^A)^2 + 168\lambda\tau^A\tau^B \\
& +108\lambda(\tau^B)^2 + 23(\tau^A)^2 + 98\tau^A\tau^B + 23(\tau^B)^2.
\end{aligned}$$

We prove $t^\dagger > 0$ by showing

$$\left[2(1+\lambda)(a-c)^2 - (\tau^A)^2\right] + \left[2(1+\lambda)(a-c)^2 - (\tau^B)^2\right] + (a-c)(\tau^A+\tau^B)(1-\lambda) > 0.$$

Because $q_2^A > 0$, we obtain the first term in the above inequality is positive. The second term in the above inequality is positive because $q_1^B > 0$ in (23).

Note that the equation $\partial W^\dagger/\partial\lambda = 0$ has two solutions; the smaller solution of $t$ is negative and the larger solution is $t^\dagger$. Thus, we obtain (i) when $0 < t < t^\dagger$, $\partial W^\dagger/\partial\lambda < 0$, and (ii) when $t > t^\dagger$, $\partial W^\dagger/\partial\lambda > 0$. This implies Proposition 2. Q.E.D.

**Proof of Proposition 3**

From (30), we obtain

$$
\begin{aligned}
&\Bigg[(\lambda-1)^2(\lambda+3)^3\Bigg]\cdot\frac{\partial W^{A\dagger}}{\partial\lambda}=\\
&(5\lambda^2+14\lambda+13)t^2+\Bigg[2(a-c)(1-\lambda-\lambda^2+\lambda^3)-2(3+4\lambda+\lambda^2)\tau^A+4(5+5\lambda+2\lambda^2)\tau^B\Bigg]t\\
&+\Bigg[-2(a-c)^2(1-\lambda^2-\lambda+\lambda^3)+2(a-c)(\lambda^2+4\lambda+3)\tau^A+(a-c)(\lambda^3-5\lambda^2-5\lambda-7)\tau^B\\
&-3(\lambda^2+4\lambda+3)(\tau^A)^2+2(2\lambda^2+5\lambda+5)(\tau^B)^2\Bigg], \qquad (54)
\end{aligned}
$$

where $\partial W^{A\dagger}/\partial\lambda$ in (54) is a convex function of $t$. Solving $\partial W^{A\dagger}/\partial\lambda=0$, we find two solutions:

$$
\begin{aligned}
t^{A\dagger} &= \frac{-(a-c)(1-\lambda)^2(1+\lambda)+\tau^A(3+4\lambda+\lambda^2)-2\tau^B(5+5\lambda+2\lambda^2)+\sqrt{\Delta_2^A}}{5\lambda^2+14\lambda+13}, \quad (55)\\
t_2^{A\dagger} &= \frac{-(a-c)(1-\lambda)^2(1+\lambda)+\tau^A(3+4\lambda+\lambda^2)-2\tau^B(5+5\lambda+2\lambda^2)-\sqrt{\Delta_2^A}}{5\lambda^2+14\lambda+13}
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta_2^A &= -(\lambda+3)\Bigg[-a^2\lambda^5-5a^2\lambda^4-2a^2\lambda^3+14a^2\lambda^2+3a^2\lambda-9a^2+2ac\lambda^5+10ac\lambda^4+4ac\lambda^3\\
&-28ac\lambda^2-6ac\lambda+18ac+2a\lambda^4\tau^A-3a\lambda^4\tau^B+10a\lambda^3\tau^A-14a\lambda^3\tau^B+34a\lambda^2\tau^A\\
&-32a\lambda^2\tau^B+54a\lambda\tau^A-42a\lambda\tau^B+28a\tau^A-37a\tau^B-c^2\lambda^5-5c^2\lambda^4-2c^2\lambda^3+14c^2\lambda^2\\
&+3c^2\lambda-9c^2-2c\lambda^4\tau^A+3c\lambda^4\tau^B-10c\lambda^3\tau^A+14c\lambda^3\tau^B-34c\lambda^2\tau^A+32c\lambda^2\tau^B-54c\lambda\tau^A\\
&+42c\lambda\tau^B-28c\tau^A+37c\tau^B-16\lambda^3(\tau^A)^2+8\lambda^3\tau^A\tau^B+4\lambda^3(\tau^B)^2-62\lambda^2(\tau^A)^2+28\lambda^2\tau^A\tau^B\\
&+14\lambda^2(\tau^B)^2-88\lambda(\tau^A)^2+40\lambda\tau^A\tau^B+20\lambda(\tau^B)^2-42(\tau^A)^2+20\tau^A\tau^B+10(\tau^B)^2\Bigg].
\end{aligned}
$$

The smaller solution $t_2^{A\dagger}$ is always negative. These imply Proposition 3(i).

When $\tau^A=\tau^B$, $t^{A\dagger}$ in (55) can be re-written as

$$
t^{A\dagger}=\frac{-(a-c)(1-\lambda)^2(1+\lambda)-\tau(7+6\lambda+3\lambda^2)+\sqrt{\Delta}}{5\lambda^2+14\lambda+13},
$$

where

$$\Delta = (\lambda+3)(a^2\lambda^5+5a^2\lambda^4+2a^2\lambda^3-14a^2\lambda^2-3a^2\lambda+9a^2-2ac\lambda^5-10ac\lambda^4-4ac\lambda^3+28ac\lambda^2$$
$$+6ac\lambda-18ac+a\lambda^4\tau+4a\lambda^3\tau-2a\lambda^2\tau-12a\lambda\tau+9a\tau+c^2\lambda^5+5c^2\lambda^4+2c^2\lambda^3-14c^2\lambda^2-3c^2\lambda$$
$$+9c^2-c\lambda^4\tau-4c\lambda^3\tau+2c\lambda^2\tau+12c\lambda\tau-9c\tau+4\lambda^3\tau^2+20\lambda^2\tau^2+28\lambda\tau^2+12\tau^2).$$

Because

$$\Delta-\left[(a-c)(1-\lambda)^2(1+\lambda)+\tau(7+6\lambda+3\lambda^2)\right]^2$$
$$=(\lambda-1)^2(5\lambda^2+14\lambda+13)(2a-2c-\tau)[(a-c)(1+\lambda)+\tau])>0,$$

we obtain $\sqrt{\Delta}>(a-c)(1-\lambda)^2(1+\lambda)+\tau(7+6\lambda+3\lambda^2)$. Thus, from Proposition 3(i) we obtain $t^{A\dagger}>0$ if $\tau^A=\tau^B$, which implies Proposition 3(ii). Q.E.D.

**Proof of Proposition 4**

Partially differentiating the equilibrium global welfare in (51) w.r.t. $\lambda_{21}$, we obtain,

$$\begin{aligned}&\left(\lambda_{12}+\lambda_{21}+\lambda_{12}\lambda_{21}-3\right)^3\cdot\frac{\partial W^{\ddagger}}{\partial\lambda_{21}}=\\&(6\lambda_{21}-7\lambda_{12}+6\lambda_{12}\lambda_{21}+4\lambda_{12}^2\lambda_{21}+3\lambda_{12}^2+\lambda_{12}^3-13)t^2\\&+2(a-c)(1-\lambda_{12})^2(1-\lambda_{12}\lambda_{21})(a-c-t).\end{aligned}\tag{56}$$

Because $\lambda_{12}+\lambda_{21}+\lambda_{12}\lambda_{21}-3<0$ and $6\lambda_{21}-7\lambda_{12}+6\lambda_{12}\lambda_{21}+4\lambda_{12}^2\lambda_{21}+3\lambda_{12}^2+\lambda_{12}^3-13<0$, thus $\partial W^{\dagger}/\partial\lambda_{21}$ in (56) is a convex function of $t$. We obtain $\partial W^{\ddagger}/\partial\lambda_{21}$ is positive if $t>t_{21}^{\ddagger}$, where

$$t_{21}^{\ddagger}=\frac{(a-c)(1-\lambda_{12})\left[(1-\lambda_{12})(1-\lambda_{12}\lambda_{21})-\sqrt{\Delta_{21}}\right]}{\underbrace{6\lambda_{21}-7\lambda_{12}+6\lambda_{12}\lambda_{21}+4\lambda_{12}^2\lambda_{21}+3\lambda_{12}^2+\lambda_{12}^3-13}_{(-)}},\tag{57}$$

and

$$\Delta_{21}=(\lambda_{12}\lambda_{21}-1)(\lambda_{12}+3)(4\lambda_{21}-\lambda_{12}+3\lambda_{12}\lambda_{21}+\lambda_{12}^2\lambda_{21}+2\lambda_{12}^2-9).$$

Note that the equation $\partial W^{\ddagger}/\partial\lambda_{21} = 0$ has two solutions of $t$ and $t_{21}^{\ddagger}$ in (57) is the larger solution of $t$. These imply Proposition 4(i).

Substituting the equilibrium outcomes into $W^{A\ddagger}$ in (49) yields,

$$\begin{aligned}
&\left[(\lambda_{12}+\lambda_{21}+\lambda_{12}\lambda_{21}-3)^3\right]\cdot\frac{\partial W^{A\ddagger}}{\partial\lambda_{21}} = \\
&\left(6\lambda_{21}-8\lambda_{12}+4\lambda_{12}\lambda_{21}+2\lambda_{12}^2\lambda_{21}-2\lambda_{12}^2-2\right)t^2 \\
&+\left(-7+8\lambda_{12}+\lambda_{21}-\lambda_{12}^2+4\lambda_{12}\lambda_{21}-5\lambda_{12}^2\lambda_{21}\right)(a-c)t \\
&+\left(4+\lambda_{21}-5\lambda_{12}+2\lambda_{12}^2-\lambda_{12}^3+5\lambda_{12}^2\lambda_{21}-6\lambda_{12}\lambda_{21}\right)(a-c)^2, \qquad (58)
\end{aligned}$$

where $\partial W^{A\ddagger}/\partial\lambda_{21}$ is a convex function of $t$, by assuming $\lambda_{21} \leq \lambda_{12}$ because $\lambda_{12}+\lambda_{21}+\lambda_{12}\lambda_{21}-3) < 0$ and $(6\lambda_{21}-8\lambda_{12}+4\lambda_{12}\lambda_{21}+2\lambda_{12}^2\lambda_{21}-2\lambda_{12}^2-2) < 0$ holds. We find that $\partial W^{A\ddagger}/\partial\lambda_{21}$ in (58) is positive if $t > t^{A\ddagger}$ where

$$t^{A\ddagger} = -\frac{(a-c)(1-\lambda_{12})\left[7-\lambda_{12}-5\lambda_{21}\lambda_{12}-\lambda_{21}-\sqrt{\Delta_3^A}\right]}{4[(1+\lambda_{12})^2(1-\lambda_{21})+2(\lambda_{12}-\lambda_{21})]}, \qquad (59)$$

and

$$\begin{aligned}
\Delta_3^A = {} & -8\lambda_{12}^4\lambda_{21}+8\lambda_{12}^4+15\lambda_{12}^3\lambda_{21}^2-58\lambda_{12}^3\lambda_{21}+23\lambda_{12}^3+87\lambda_{12}^2\lambda_{21}^2-114\lambda_{12}^2\lambda_{21}+23\lambda_{12}^2 \\
& +113\lambda_{12}\lambda_{21}^2-102\lambda_{12}\lambda_{21}+57\lambda_{12}-23\lambda_{21}^2-102\lambda_{21}+81.
\end{aligned}$$

Note that the equation $\partial W^{A\ddagger}/\partial\lambda_{21} = 0$ has two solutions for $t$ and $t^{A\ddagger}$ in (59) is the larger solution of $t$. These imply Proposition 4(ii) .

Substituting the equilibrium outcomes into $W^{B\ddagger}$ in (50) generates,

$$\begin{aligned}
&\left[(\lambda_{12}+\lambda_{21}+\lambda_{12}\lambda_{21}-3)^3\right]\cdot\frac{\partial W^{B\ddagger}}{\partial\lambda_{21}} = \\
&\left(\lambda_{12}+2\lambda_{12}\lambda_{21}+2\lambda_{12}^2\lambda_{21}+5\lambda_{12}^2+\lambda_{12}^3-11\right)t^2 \\
&+\left(5-4\lambda_{12}-\lambda_{21}-\lambda_{12}^2-2\lambda_{12}\lambda_{21}+\lambda_{12}^2\lambda_{21}+2\lambda_{12}^3\lambda_{21}\right)(a-c)t \\
&+\left(\lambda_{12}-\lambda_{21}-2+\lambda_{12}^3-\lambda_{12}^2\lambda_{21}-2\lambda_{12}^3\lambda_{21}+4\lambda_{12}\lambda_{21}\right)(a-c)^2. \qquad (60)
\end{aligned}$$

Note that both the coefficient (denominator) in front of $\partial W^{B\ddagger}/\partial\lambda_{21}$, and the coefficient of $t^2$ are negative. Therefore $\partial W^{B\ddagger}/\partial\lambda_{21}$ in (60) is a convex function of $t$.

The equation $\partial W^{B\ddagger}/\partial\lambda_{21} = 0$ yields two solutions of $t$ and $t^{B\ddagger}$ in (61) is the larger one.

$$t^{B\ddagger} = \frac{\overbrace{(1-\lambda_{21})(1-\lambda_{12}^2)[4-2(1+\lambda_{12})\lambda_{12}\lambda_{21}]}^{(+)} + \sqrt{\Delta_3^B}}{\underbrace{2(11-\lambda_{12}-2\lambda_{12}\lambda_{21}-2\lambda_{12}^2\lambda_{21}-5\lambda_{12}^2-\lambda_{12}^3)}_{(+)}} > 0, \tag{61}$$

and

$$\begin{aligned}\Delta_3^B = {} & (\lambda_{12}-1)\Big(4\lambda_{12}^5\lambda_{21}^2 + 8\lambda_{12}^5\lambda_{21} - 4\lambda_{12}^5 + 24\lambda_{12}^4\lambda_{21}^2 + 40\lambda_{12}^4\lambda_{21} - 24\lambda_{12}^4 + 41\lambda_{12}^3\lambda_{21}^2 \\ & +26\lambda_{12}^3\lambda_{21} - 31\lambda_{12}^3 + 9\lambda_{12}^2\lambda_{21}^2 - 126\lambda_{12}^2\lambda_{21} + 9\lambda_{12}^2 - 13\lambda_{12}\lambda_{21}^2 - 130\lambda_{12}\lambda_{21} \\ & +51\lambda_{12} - \lambda_{21}^2 + 54\lambda_{21} + 63\Big).\end{aligned}$$

These imply Proposition 4(iii). Q.E.D.

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