Dynamic Screening with Differentially Informed Principals

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Abstract

This paper studies a dynamic principal-agent model of adverse selection under competition among principals. Principals are ex-ante identical, but receive information about the agent independently which creates a setting of imperfect competition. I study how the agent's payoffs in this setting differ compared to the regular monopoly principal-agent case, and how that affects the agent's incentives to reveal information. The focus is on how the information structure affects the competition for the agent's services, and how the nature of competition in turn affects the agent's incentives. In a repeated setting with short term contracts and private observability of the agent's performance measure, the agent cannot be incentivized to fully reveal his private information as the familiar ratchet effect persists. Finally, I show that allowing voluntary information sharing among principals can benefit principals and improve welfare in general.

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1 Introduction

In an employment relationship, firms often acquire information over time about a worker's productivity, through various performance measures. This information is valuable for two reasons: first, it allows the firm to write more effective incentive contracts with their employees, and second, it gives them an informational advantage when competing with other firms to retain those employees. Nevertheless, many firms reveal at least some of this information to their potential rivals, either directly – through reference letters or outplacement services – or indirectly – through job titles or responsibilities.[1] Why might a firm reveal its private information to the rest of the labor market?

This paper argues that revealing information about past performance can effectively commit a firm to pay high wages to its high-ability workers. That is, without disclosing information, the firm would extract rents from its employees after they reveal that they are high ability. This manifestation of the classic ratchet effect means that firms would struggle to induce their high ability workers to stand out. By disclosing information and thereby inducing more severe competition for their employees, the firm can credibly promise to reward high-ability workers, which encourages those workers to reveal themselves.

To make this point, I develop tools for understanding imperfect competition among differentially-informed principals who seek to hire an agent. I show that this problem can be formulated as a first-price common-value auction, where each principal's bid is a contract that maps output to wages, which in turn generates a schedule of rents given to agents with different abilities. While identifying closed-form equilibrium contracts in this setting is not tractable, I establish comparative statics results on the agent's equilibrium rent as a function of the principals' information. These comparative statics allow me to analyze the costs and benefits to the incumbent principal of revealing her private information.

Formally, I consider a two-period model with two principals and one agent. In each period, the agent's ability is perfectly known only to the agent; however, both principals independently receive informative signals[2] about the agent's ability, after which they offer contracts simulataneously. Higher signals correspond to more favorable beliefs regarding the agent's ability. The agent can only choose to work with one principal in each period. In

[1]Top management consulting firms, for example, often have outplacement services that help their employees to find new jobs. They provide this help in the form of connecting the employee with prospective employers, and through providing credible information about the employee's productivity in the forms of references, evaluations, and performance reviews.

[2]This can be thought of as an interview of the agent conducted by the pricipal, or another type of information acquisition.

the two-period model, I consider both the setting where principals can credibly publicize information, and the one where they cannot.

I show that in each period, the rent offered to the agent by a principal is increasing in the principal's signal; that is, a more favorable belief induces the principal to pay the agent more. This is true both when principals receive signals of the same level of informativeness and when the level of informativeness is different. However, between these two cases, a meaningful comparion of the agent's payoffs can be made, and it can be shown that when one principal is more accurately informed than the other, this reduces the agent's ex ante expected payoff from the setting where both principals have the lower accuracy.

In the two-period model presented in this paper, if a principal learns something about the agent's productivity in the first period, it gives her an informational advantage over her opponent in the second period. However, this informational advantage is detrimental for the agent's payoff, which makes it harder to induce the agent to reveal information. For this reason, even though ex post the principal would benefit from having superior information, ex ante it is beneficial for her to commit to share any information about the agent's productivity she learns in the first period. Publicizing information works as an incentive tool because the incremental benefits of public information is higher for more productive agents, which means more productive agents will have a stronger incentive to exert effort in order to publicize their productivity. Availability of more public information makes the competition for the agent's services stronger in the second period; however, because we have two principals competing for the agent, the agent's second period expected payoff is the higher of the two payoffs offered to him by the two principals, whereas from the principal's perspective, the added payoff she must offer due to stronger competition is only the expected increase in her own bid in the second period. Because of this, from the point of view of the principal, the lowered cost of providing incentives dominates the effect of added competition, so publicizing information is ex ante beneficial for the principal's payoff.

The formal model is introduced in section 2. In section 3, I characterize necessary conditions for incentive compatible contracts and list the relevant results for the monopolist principal benchmark.

Results for the one-period model is presented in section 4. Because equilibria in the two-period game crucially depends on the information structure in the second period, it will be very useful to compare equilibrium payoffs for the agent under different information structures for the principals, and we build on these results from the static game in the two-period model. The analysis of the two-period game is in section 5. Section 6 discusses possible extensions. Section 7 concludes.

1.1 Related Literature

Monopolistic screening has been studied in its various forms as a partial solution to this problem in early works such as Mussa & Rosen (1978), Baron & Myerson (1982), and Laffont & Tirole (1986). However, much less emphasis has been given to forms of incentive contracts for screening in a setting with competition. Imperfect competition has been mostly studied within the domain of IO theory where the "imperfection" arises from some form of differentiation among the competitors. Most of this literature has been focused on intrinsic differences among the competitors themselves. However, even though less studied, interesting insights can be generated from models where competitors only differ in terms of the information they are endowed with. Spulber (1988) studies a simple model of Bertrand competition where the marginal costs of the producers are realized from a distribution at the beginning of the game, and becomes private information of each producer. In this setting, the Bertrand paradox is mitigated, and this private information generates enough differentiation among competitors that they each make positive expected profits in equilibrium. The game analyzed there is a specialized version of an independent private values auction setting.

The "symmetric", or "mineral-rights" model of common value auctions has been studied in early papers such as Wilson (1967) and Milgrom & Weber (1982). Common-value auctions with asymmetrically informed bidders have received much less attention, although this setting plays an important role in this paper. Such settings have been analyzed in Engelbrecht-Wiggans, Milgrom & Weber (1983) and Milgrom & Weber (1982b). Representing a screening contract as a form of auction has also been studied in Biglaiser & Mezzetti (1993, 2000) in the framework of the independent private values model.

The central theme of screening contracts with short-term commitment is the ratchet effect. Starting from Freixas, Guesnerie & Tirole (1985), the ratchet effect has been studied under a procurement setting in Laffont & Tirole (1988), as well as worker incentives in a firm in Gibbons (1987), Ickes & Samuelson (1987), and Carmichael & Macleod (2000), and in the economics of corruption (Choi & Thum 2003). Empirical analysis of the ratchet effect is not numerous, but recently there has been some work. For example, in Charness et al. (2011), they find that the ratchet effect is indeed a significant problem in the labor market when there is less competition between firms or between workers, however, competition in either side mediates the problem. They interpret this result as the parties' outside options playing an important role in solving the problem. Their work is partly inspired by the theoretical treatment of the ratchet effect in Kanemoto & Macleod (1992), which has a two period model of "second-hand workers" with short term commitment, but in the 2nd period the agent is free to choose offers from other competing principals, who crucially, does not observe the agent's performance in the 1st period. Competition among principals, therefore, mitigates the ratchet effect, and the first-best outcome is possible with perfect competition among principals.

2 The Model

There is one risk-neutral agent, A, and two risk-neutral principals P_1 and P_2, over two periods, $t = 1, 2$. The agent has private information about his type θ_t, which is drawn from a commonly known distribution $F(\cdot)$ at $t = 1$, and $F(\cdot|\theta_1)$ at $t = 2$, over the interval support $[\underline{\theta}, \bar{\theta}]$. We assume that $\underline{\theta} > 1$, and the associated density functions $f(\cdot)$ and $f(\cdot|\theta_1)$ are positive and atomless everywhere in the support. In each period, each principal P_i receives a private signal X_{it}, which is informative of the agent's type θ_t; the signals are distributed independently according to the signal-generating processes $S_{it}(\cdot|\theta_t)$ with the associated density functions $s_{it}(\cdot|\theta_t)$, which are positive and atomless everywhere in the support. This is also commonly known.

In each stage, if the agent chooses to work with one of the principals, he chooses a non-negative effort level $e_t \in [0, \bar{e}]$, and which produces output $y_t = e_t + \theta_t$. We assume that $\bar{e} > 1$. The agent's effort level is not observed by the principals. Output, however, is observed by the employing principal and it is contractible. Effort is costly for the agent, with $C(e_t) = e_t^2/2$. Each principal P_i can offer a contract $w_{it} : \mathbb{R}^+ \to \mathbb{R}$ which determines a payment $w_{it}(y_t)$ which the principal must pay the agent following the realization of output[3]. In the first period, as part of the contract, the principals can also commit to a public message $m_i(y_1)$ which the first period employing principal sends after realization of the output. We will look at two cases for the messages available to the principals. In the non-disclosure setting, the message space is $M_i^{ND} = \phi$, meaning principals cannot choose informative messages, whereas in the disclosure setting, the message space is $M_i^D = \{m_i : m_i \subseteq [\underline{\theta}, \bar{\theta}]\}$, so the principals can commit to reveal information about their beliefs regarding the agent's first-period type θ_1.

The agent's outside option is zero. Let's denote principal P_i's payoff as π_{it}, and the agent's payoff as U_t. The

[3]Note that we restrict the contract to only be conditioned on the output produced. In particular, we do not allow one principal's contract to condition on another principal's contract

payoffs are therefore:

$$\pi_{it} = \begin{cases} y_t - w_{it}(y_t) & \text{if } A \text{ chooses } P_i\text{'s contract} \\ 0 & \text{otherwise} \end{cases}$$

$$U_t = \begin{cases} w_{it}(y_t) - e_t^2/2 & \text{when } w_{it} \text{ is the chosen contract} \\ 0 & \text{if no contract is chosen} \end{cases}$$

The timing in the first period is:

1. The agent's type θ_1 is realized and privately observed by the agent.

2. Each principal P_i observes a signal x_{i1}, realized according the distribution $S_{i1}(\cdot|\theta_1)$.

3. Each principal simultaneously and privately offers the agent a contract $w_{i1} : \mathbb{R}^+ \to \mathbb{R}$, which is a function that maps output to payment, and commits to a public message $m_i : \mathbb{R}^+ \to M_i$

4. The agent accepts at most one contract; the agent's decision is $d_1 \in \{\phi, P_1, P_2\}$

5. If no contract is accepted, the stage ends. If the agent accepts P_i's offer, he then exerts effort e_1, output y_1 is produced and only observed by P_i. The output accrues to P_i.

6. P_i pays the agent $w_{i1}(y_1)$, and sends public message $m_i(y_1)$.

The timing in the second period is similar to that of the first period; however, principals don't choose any messages, and both principals use all of their information at the time of offering contracts. We denote P_i's available information at the time of offering second-period contracts as $\mathcal{I}_i = \{x_{i1}, x_{i2}, d_1, \mathcal{C}_i, m\}$, where $\mathcal{C}_i$ contains any information about θ_1 that P_i gained through a contractual relationship with the agent in the first period, and m is the public message sent in the first period.

I use the following assumptions throughout the paper. The first characterizes the informative nature of the principals' signals. The second is a technical assumption commonly used in screening models.

Assumption 1. *The signals X_{it} are affiliated with the agent's realized type,θ_t.*

Assumption 1 implies, for any $\theta_t' > \theta_t$, the signal generating process $S_{it}(\cdot|\theta_t')$ stochastically dominates $S_{it}(\cdot|\theta_t)$ according to the likelihood ratio order.

Assumption 2. *For any possible pair of signal realizations (x, y) for the two principals, each principal's posterior belief about the agent's type, $F_{it}(\cdot|x, y)$ satisfies the monotone hazard rate condition.*

Assumption 2 implies, for any pair of signal realizations for the two principals (x, y), $\frac{f_{it}(\theta_t|x,y)}{1-F_{it}(\theta_t|x,y)}$ is non-decreasing in θ_t.

3 The Monopoly Benchmark

It is instructive to start with the one-period monopoly benchmark where one principal is inactive throughout. This case is a straightforward analog of the framework analyzed in Mussa & Rosen (1978). An important observation in this setting is that the agent's payoff depends on the principal's beliefs about his type. By the revelation principle, the principal's maximization problem after receiving realized signal x is:

$$\max_{e(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} [e(\theta) + \theta - w(\theta)] \, dF(\theta|x)$$

subject to:

$$\begin{aligned} U(\theta) &\geq U(\hat{\theta}|\theta), \forall \theta, \hat{\theta} \quad &\text{(IC)} \\ U(\theta) &\geq 0, \forall \theta \quad &\text{(IR)} \end{aligned}$$

Here, $U(\hat{\theta}|\theta)$ is the agent's payoff when his real type is θ and he chooses the allocation for type $\hat{\theta}$. $U(\theta) = U(\theta|\theta)$ is the agent's payoff under truth-telling.

Lemma 1. *For a smooth effort allocation $e(\cdot)$, The IC constraints are satisfied if and only if:*

1. $e(\theta) + \theta$ is non-decreasing
2. $U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} e(q)dq$

Proof. See appendix. □

Throughout the paper, the first condition is referred to as the monotonicity constraint, and the second one as the envelope condition.

By Lemma 1, we can see that because $e(q)$ is non-negative, setting $U(\underline{\theta}) = 0$ satisfies the IC constraints for all types. The principal's relaxed maximization problem can therefore be written as:

$$\max_{e(\cdot|x)} \int_{\underline{\theta}}^{\bar{\theta}} \left\{ e(\theta|x) + \theta - \frac{e(\theta|x)^2}{2} - \int_{\underline{\theta}}^{\theta} e(q|x)dq \right\} dF(\theta|x)$$

Let $e^M(\theta|x)$ be the solution to this problem, and $\pi^M(x)$ be the principal's expected payoff after receiving signal x.

Proposition 1 characterizes the e^M and π^M.

Proposition 1. *In the monopoly setting, the optimal contract in the stage game has the following properties:*

1. The optimal effort level is $e^M(\theta|x) = 1 - \frac{1-F(\theta|x)}{f(\theta|x)}$, which is strictly decreasing in x, for all $\theta < \bar{\theta}$.

2. $U(\theta|x)$ is strictly decreasing in x, for all θ.

3. The principal's expected payoff, $\pi^M(x)$, is strictly increasing in x.

Proof. See appendix. □

The main idea behind these results is the tradeoff between efficiency and rent extraction. For an agent of type θ, total surplus equals $e + \theta - e^2/2$, which is maximized by choosing $e(\theta) = 1$, that is to say, in our setting, $e^{FB}(\theta) = 1$ for any θ. As we see from Lemma 1, the θ-type agent's payoff, which consists entirely of his information rent in the monopoly setting, is increasing in the proposed effort level the principal chooses for all types lower than θ. This leads the principal to choose an effort level that is less than first-best for all types other than the highest type, $\bar{\theta}$.In other words, the principal distorts the effort level from the efficient level of 1 in order to reduce rent for higher types. The amount of distortion depends on the principal's beliefs about the agent's type. When higher types are more likely, reducing rent for higher types becomes more important compared to efficiency for lower types, which leads to more distortion for the low types. This is why after receiving a higher signal, which makes higher types of agents more likely according to the principal's updated beliefs, the principal chooses a lower (more distorted) effort level for all agent types other than $\bar{\theta}$. This leads to a lower payoff for all types of the agent. However, this increases the principal's expected payoff, which means the principal's value from working with the agent goes up following a high signal.

4 The Static Game under Competition

In this section the analysis of the one-period game with two principals is presented. Here I use a subclass of perfect Bayesian equilibrium which I call *regular equilibrium.* I put conditions on the principals' equilibrium strategies so that the equilibrium involves each principal using a continuous strategy as a function of her signal.

Let us then analyze the principals' maximization problem. Suppose P_{-i} is playing the strategy $e_{-i}(\cdot|\cdot)$ with the associated payoff schedule $U_{-i}(\cdot|\cdot)$. Because the space of allowed contracts here is the same as in the monopoly setting, lemma 1 still pins down the necessary conditions for the agent's IC constraints.[4] Therefore, after receiving signal x, principal P_i's maximization problem is:

$$\max_{e_i(\cdot|x),U_i(\underline{\theta}|x)} \int_{-\infty}^{\infty} \left[\int_{\underline{\theta}}^{\bar{\theta}} \left\{ \left(e_i(\theta|x) + \theta - \frac{e_i(\theta|x)^2}{2} - U_i(\underline{\theta}|x) - \int_{\underline{\theta}}^{\theta} e_i(t|x)dt \right) \mathbf{1}_{\{U_{-i}(\theta|y)<U_i(\theta|x)\}} \right\} dF_i(\theta|x,y) \right] dG_{-i}(y|x)$$

$$\text{Subject to: } \begin{cases} e_i^{'}(\theta|x) \geq -1, \forall\theta & \text{monotonicity} \\ U_i(\underline{\theta}|x) \geq 0 & \text{IR} \end{cases}$$

Here, $G_{-i}(y|x)$ is the distribution of the opponent's signal, which is updated using Bayes' rule after receiving signal x. In other words, this is the principal's posterior belief about the signal of her opponent.

Before proving existence of equilibrium, it is helpful to define a few terms.

Let the pair $(e_i^*(\cdot|x), U_i^*(\underline{\theta}|x))$ be an optimal strategy in the above maximization problem. Let

$$U_i^*(\theta|x) = U_i^*(\underline{\theta}|x) + \int_{\underline{\theta}}^{\theta} e_i^*(t|x)dt$$

In competing with P_{-i}, P_i decides, for every agent type θ, what level of rent $U_i^*(\theta|x)$ to offer to the agent of

[4]Here, the rent offered to agent type θ by P_{-i} with realized signal y, which is $U_{-i}(\theta|y)$, is the θ type agent's outside option for P_i's offered contract. This outside option may be a random variable if P_{-i} is playing a strategy conditional on y. The fact that lemma 1 still pins down the agent's IC constraints for P_i's contract is an implication of lemma 2 in Rochet & Stole (2002), which studies screening with random outside options.

type θ, as a function of P_i's realized signal x. This makes the game analogous to a first price common value auction, which we can then study as a bidding game, and analyze the two principals' bidding behaviors in terms of their information.

4.1 Regular Equilibrium

We will focus on a class of competitive equilibria where the principals' strategies are continuous functions of their signals. We will call an equilibrium in such strategies a *regular equilibrium*.

For any regular equilibrium, we define a "lowest contract" for that equilibrium, which is by construction the unique contract offered by both principals as their signal realizations approach the infimum of their signal supports. This lowest contract determines the initial value for the differential equations that govern how much rent P_i bids for each type of the agent, θ.

Definition. The lowest contract, $(\underline{\mathrm{e}}(\cdot), \underline{\mathrm{U}}(\underline{\theta}))$ is defined as:

$$\underline{\mathrm{e}}(\cdot) \coloneqq \lim_{x \to \underline{\mathrm{X}}_i} e_i^*(\cdot|x); \text{ for } i = 1, 2$$

$$\underline{\mathrm{U}}(\underline{\theta}) = \lim_{x \to \underline{\mathrm{X}}_i} U_i^*(\underline{\theta}|x); \text{ for } i = 1, 2$$

where $\underline{\mathrm{x}}_i$ is the infimum of P_i's signal space.

Throughout the rest of the paper, I put conditions that the lowest contract satisfy the monotonicity constraint, it allocates an inefficient effort level for all interior types of the agent, and the efficient level for the lowest type.

Condition 1. $\underline{\mathrm{e}}(\cdot)$ is continuous and differentiable over $[\underline{\theta}, \bar{\theta}]$, and $\underline{\mathrm{e}}^{'}(\theta) > -1$, for all $\theta \in [\underline{\theta}, \bar{\theta}]$.

Condition 2. In any regular equilibrium, $\underline{\mathrm{e}}(\underline{\theta}) = e^{FB}$, $\underline{\mathrm{e}}(\theta) < e^{FB}$ for all $\theta \in (\underline{\theta}, \bar{\theta})$, and $\underline{\mathrm{U}}(\underline{\theta}) = \frac{1}{2} + \underline{\theta}$.

We will look at regular equilibria where in all equilibrium contracts, the effort allocation is efficient for the lowest type who gets the total surplus generated, and the effort level is inefficient for all interior types. If we start from a lowest contract where principals offer efficient effort allocations for all types, then the only equilibrium is where they both offer the same contract regardless of their realized signals. Condition 2 is required to have an equilibrium where principals choose non-constant strategies based on their signals.

Next, it is useful to formalize some tools that allow the contracting problem to be discussed in terms of a common values auction. I make use of these terms in proving results for the competitive screening game, through using results from auction theory.

Definition. The *actual value* to principal P_i of obtaining agent of type θ, given P_i's realized signal x and $P'_{-i}s$ realized signal y, is

$$\hat{R}_i(\theta|x,y) := \int_{\theta}^{\bar{\theta}} \left[\left\{ e_i^*(q|x) + q - \frac{e_i^*(q|x)^2}{2} - \int_{\theta}^{q} e_i^*(t|x)dt \right\} \mathbf{1}_{\left\{U_{-i}^*(q|y) < U_i^*(q|x)\right\}} \right] dF_i(q|x,y)$$

This term is the expected payoff to the principal generated from all types of the agent above θ, under the optimal contract when the principal's own signal is x and the opponent's signal is y.

It will also be useful sometimes to define this function without the indicator. So let's define:

$$R_i(\theta|x,y) := \int_{\theta}^{\bar{\theta}} \left\{ e_i^*(q|x) + q - \frac{e_i^*(q|x)^2}{2} - \int_{\theta}^{q} e_i^*(t|x)dt \right\} dF_i(q|x,y)$$

Definition. The *interim value* to principal P_i, conditional on winning, of obtaining agent of type θ, given her realized signal x, is

$$V_i(\theta|x) := \int_{-\infty}^{\infty} \left(\left\{ \hat{R}_i(\theta|x,y) \right\} \mathbf{1}_{\left\{U_{-i}^*(\theta|y) < U_i^*(\theta|x)\right\}} \right) dG_{-i}(y|x)$$

The interim value is P_i's expected payoff given her realized signal in the equilibrium from all agent types above θ, where the expectation is taken over the agent's types and the opponent's signal realizations, conditional on P_i winning.

Definition. The *ex ante value* to principal P_i, conditional on winning, of obtaining agent of type θ, is

$$V_i(\theta) := \int_{-\infty}^{\infty} V_i(\theta|x)dS_i(x|\theta)$$

The ex ante value is the expected interim value for agent type θ, where the expectation is taken over all possible signal realizations for P_i given the agent's type is θ.

Now I present the main result for the static game with competition, that highlights the role of competition on the agent's equilibrium payoff. Unlike in the monopoly benchmark, in a regular equilibrium with competition, the agent's payoff is increasing in the principals' signals, which mean favorable beliefs are beneficial for the agent's payoff.

Proposition 2. *In any regular equilibrium of the static game, for any agent type $\theta \in (\underline{\theta}, \bar{\theta})$, $U_i^*(\theta|x)$ is strictly increasing in x, for $i = 1, 2$.*

Proof. First of all, suppose that in the equilibrium the opponent principal P_{-i} is playing a strictly increasing strategy in her signal for all $\theta \in (\underline{\theta}, \bar{\theta})$, that is, $U_{-i}^*(\theta|y)$ is strictly increasing in y for all $\theta \in (\underline{\theta}, \bar{\theta})$.

We consider principal P_i's equilibrium strategy after receiving signal x. Now pick an arbitrary $\theta \in (\underline{\theta}, \bar{\theta})$. Let $u = U_i^*(\theta|x)$.

We can separate out the principal's payoff coming from agent types below θ, from the ones above θ. This is useful because this allows us to focus on the effect of changing u only on the associated change in winning probability for types above θ. That is,

$$\pi_i(x) = \int_{-\infty}^{\infty} \int_{\underline{\theta}}^{\theta} \left(\left\{ e_i^*(q|x) + q - \frac{e_i^*(q|x)^2}{2} \right\} \mathbf{1}_{\left\{U_{-i}^*(q|y) < U_i^*(q|x)\right\}} \right) dF_i(q|x,y) dG_{-i}(y|x) + \pi_i(\theta|x;u)$$
$$+ \int_{-\infty}^{\infty} \left(\left\{ \hat{R}_i(\theta|x,y) \right\} \mathbf{1}_{\left\{U_{-i}^*(\theta|y) > U_i^*(\theta|x)\right\}} \right) dG_{-i}(y|x)$$

where

$$\pi_i(\theta|x;u) = \int_{-\infty}^{\infty} \left(\left\{ \hat{R}_i(\theta|x,y) \right\} \mathbf{1}_{\left\{U_{-i}^*(\theta|y) < U_i^*(\theta|x)\right\}} - u \right) dG_{-i}(y|x)$$

Notice that the benefit of marginally increasing u in the form of increasing the probability of winning all agents of type θ or above, can be captured by the term $\pi_i(\theta|x;u)$, because as $u = U_i^*(\theta|x)$, and for any $\theta^{'} \in [\theta, \bar{\theta}]$, $U_i^*(\theta^{'}|x) = U_i^*(\theta|x) + \int_{\theta}^{\theta^{'}} e(q)dq$, increasing u increases the rent offered to all types between θ and $\bar{\theta}$, and given P_{-i} is playing a strictly increasing strategy, increasing the rent for all types between θ and $\bar{\theta}$ increases the probability of winning all types between θ and $\bar{\theta}$.

Because P_{-i} is playing a strategy that is strictly increasing in her signal, we can write[5]

$$\pi_i(\theta|x;u) = \int_{-\infty}^{U^{*\,-1}_{-i}(u)} \left\{\hat{R}_i(\theta|x,y) - u\right\} dG_{-i}(y|x)$$

It is easy to see that in any equilibrium, no principal would ever offer a contract that specifies $e_i^*(\theta|x) > e^{FB} = 1$, because there are always profitable deviations that offer $e_i^*(\theta|x) = e^{FB}$, and adjusts the payment accordingly so that $U_i^*(\theta|x)$ remain unchanged and total surplus goes up, thereby increasing the principal's payoff without affecting the agent's incentive constraints or choice of contract. Now, for any $\theta > \underline{\theta}$, as long as $e_i^*(\theta|x) < e^{FB}$, because signals are affiliated, for any pair of signals (x,y), $R_i(\theta|x,y)$ is strictly increasing in x,by the same proof as that of part 3 in proposition 1. Because P_{-i} is playing a strictly increasing strategy, there are signals $y \in (-\infty, U^{*\,-1}_{-i}(u)]$ of P_{-i} such that for $q \in (\theta, \bar{\theta})$ the indicator $\mathbf{1}_{\{U_{-i}(q|y)<U_i(q|x)\}}$ equals 1. Hence, $V_i(\theta|x) = \int_{-\infty}^{U^{*\,-1}_{-i}(u)} \hat{R}_i(\theta|x,y) dG_{-i}(y|x)$ is strictly increasing in x.

Now, we can rewrite $\pi_i(\theta|x;u) = G_{-i}\left[\left(U^{*\,-1}_{-i}(u)\right)|x\right](V_i(\theta|x) - u)$.

By taking the cross-partial derivative in P_i's own signal x and bid u, we can see that

$$\frac{\partial^2 \pi_i}{\partial u \partial x} = g_{-i}\left[\left(U^{*\,-1}_{-i}(u)\right)|x\right] \frac{1}{U^{*'}_{-i}\left(U^{*\,-1}_{-i}(u)\right)} V_x$$

We assumed that the distribution of signals has strictly positive density everywhere within the domain, we assumed the opponent is playing a strictly increasing strategy in her signal, and we showed $V_i(\theta|x)$ is strictly increasing in x. All of these together imply that that π_i is strictly supermodular in $(x;u)$. By Theorem 2.1 in Edlin & Shannon (1998) and the monotonicity theorem in Milgrom & Shannon (1994), we can say that $U_i^*(\theta|x)$ is strictly increasing in x. Because θ was arbitrarily chosen from $(\underline{\theta}, \bar{\theta})$, this holds for all $\theta \in (\underline{\theta}, \bar{\theta})$.

Because $V_i(\theta|x)$ is positive whenever $e_i^*(q|x) < e^{FB}$ for some $q \in [\theta, \bar{\theta}]$, there cannot be a regular equilibrium in which P_i plays constant bidding strategy $U_i^*(\theta|x)$ for some realizations x, because if the constant bidding is for some $e_i^*(\theta|x) < e^{FB}$, there is an atom at that effort level and P_{-i} can get a positive payoff by placing an atom at $e_i^*(\theta|x) + \epsilon$ and by making ϵ small enough, which makes P_i's strategy suboptimal. If there is constant bidding at some signal realization x with $e_i^*(q|x) = e^{FB}$ for all $q \in [\theta, \bar{\theta}]$, then a profitable deviation exists by choosing

[5] Here $U^{*\,-1}_{-i}(u)$ refers to the signal of P_{-i} that induces her to offer rent u, for agent type θ.

a smaller $e_i^*(\theta|x)$ because by condition 2, in a regular equilibrium $\underline{e}(\theta) < e^{FB}$, so for a positive measure of signal realizations, P_{-i} offers a less than first-best contract, therefore reducing $e_i^*(\theta|x)$ increases payoff because P_i can still win for some signal realizations of P_{-i}, which makes it a profitable deviation. Finally, because $V_i(\theta|x)$ is strictly increasing in x whenever $e_i^*(q|x) < e^{FB}$ for some $q \in [\theta, \bar{\theta}]$, both principals cannot play a decreasing strategy for any signal realizations, because P_i can increase her payoff by placing an atom at some $e_{-i}^*(\theta|y)$ where P_{-i} plays a decreasing strategy for y.

□

Unlike in the monopoly setting, where the principal has no benefit from offering rent to the agent, under competition the principals can increase the probability of winning the agent by offering more rent to the agent. Two factors pin down the increase in rents offered to the agent in terms of the principal's signal. First, just as in the monopolist principal benchmark, for any given interior agent type, a higher signal increases the principal's expected payoff from hiring the agent, so the agent becomes more valuable for the principal to hire. This leads to the principal wanting to increase the probability of winning the agent, which is done by increasing the rent offered to the agent. Second, because both principals' signals are affiliated with the agent's type, they are affiliated with each other, so a higher signal makes it more likely from the principal's perspective that the other principal also received a higher signal, which in this equilibrium means that the other principal is more likely to bid a higher rent. Therefore, in order to win the agent, the first principal must also offer higher rent to the agent. Both of these forces work in the same direction, therefore with a higher signal realization, principals offer more rent to all interior agent types.

Following is a couple of useful results that follow from proposition 2.

Corollary 1. *In any regular equilibrium, $U_i^*(\underline{\theta}|x)$ is nondecreasing in x, for $i = 1, 2$.*

Proof. Take any $x < x'$. Suppose towards a contradiction that $U_i^*(\underline{\theta}|x) > U_i^*(\underline{\theta}|x')$, and let the difference be $\delta = U_i^*(\underline{\theta}|x) - U_i^*(\underline{\theta}|x') > 0$. By proposition 2, for any $\theta > \underline{\theta}$, we must have $U_i^*(\theta|x) < U_i^*(\theta|x')$, which means $U_i^*(\underline{\theta}|x) + \int_{\underline{\theta}}^{\theta} e_i^*(q|x)dq < U_i^*(\underline{\theta}|x') + \int_{\underline{\theta}}^{\theta} e_i^*(q|x')dq$. So $\delta < \int_{\underline{\theta}}^{\theta} e_i^*(q|x')dq - \int_{\underline{\theta}}^{\theta} e_i^*(q|x)dq$. But as $\theta \to \underline{\theta}$, this inequality cannot be satisfied for $\delta > 0$ because by condition 1, in a regular equilibrium contracts are continuous functions of θ and hence their integrals cannot have a discrete jump in value. Therefore $U_i^*(\underline{\theta}|x)$ must be nondecreasing in x. □

Corollary 2. *In any regular equilibrium, for all $\theta \in (\underline{\theta}, \bar{\theta})$, $e_i^*(\theta|x)$ is strictly increasing in x, for $i = 1, 2$.*

Proof. Notice that in any regular equilibrium we must have $U_i^*(\underline{\theta}|x) = \frac{1}{2} + \underline{\theta}$ for all x. Towards a contradiction, suppose not. By proposition 2, $U_i^*(\theta|x)$ is strictly increasing in x, and by condition 2 and corollary 1, $U_i^*(\underline{\theta}|x) \geq \frac{1}{2} + \underline{\theta}$. If $U_i^*(\underline{\theta}|x) > \frac{1}{2} + \underline{\theta}$, and $e_i^*(\theta|x) = e^{FB}$ for all θ, then P_i is making a negative payoff so can benefit by decreasing $U_i^*(\underline{\theta}|x)$. If $e_i^*(\theta|x) < e^{FB}$ for some θ, then it is profitable for P_i to decrease $U_i^*(\underline{\theta}|x)$ and increase $e_i^*(\theta|x)$ as to keep $U_i^*(\theta|x)$ unchanged, but this increases total surplus so increases P_i's payoff. So we must have $U_i^*(\underline{\theta}|x) = \frac{1}{2} + \underline{\theta}$ for all x. We know that for any $\theta > \underline{\theta}$, $U_i^*(\theta|x) = U_i^*(\underline{\theta}|x) + \int_{\underline{\theta}}^{\theta} e_i^*(q|x)dq$. By proposition 2, $\frac{\partial}{\partial x}\left(U_i^*(\theta|x)\right) > 0$, and $U_i^*(\underline{\theta}|x)$ is constant in x, so $\frac{\partial}{\partial x}\left(U_i^*(\theta|x)\right) = \int_{\underline{\theta}}^{\theta} \frac{\partial}{\partial x}\left(e_i^*(q|x)\right) dq > 0$, and because this holds for all $\theta > \underline{\theta}$, it follows that in a regular equilibrium the allocated effort level, $e_i^*(\theta|x)$ is strictly increasing in x. □

As proposition 2 shows, a higher signal induces principals to offer more rent to the agent. The way principals offer higher rent in a regular equilibrium is through offering higher effort allocations to the agent. The alternative is paying higher wages without increasing efficiency, which is less profitable, because increasing effort to a more efficient level increases total surplus, so for the same increase in rent for the agent, the principal can benefit more in terms of capturing the added surplus.

Having proven in proposition 2 that in any regular equilibrium the bids for all interior types of the agent are strictly increasing in the principal's signal, and because both principals have a common lowest contract, it is now possible to define a correspondence between realized signals of the two principals, based on their optimal bidding strategies. For some given $\theta > \underline{\theta}$, and for any realized signal x of P_i, we can find the corresponding realization y of P_{-i} that makes her bid the same amount. We will call this the *tying function*. The tying function $Q_i(\cdot)$ maps P_i's realized signals to P_{-i}'s corresponding realized signal that induces P_{-i} to bid the same amount. We also define the inverse bidding functions. Both of these are well-defined by proposition 2. The following definitions are used to formalize this.

Definition. For an arbitrary $\theta > \underline{\theta}$, the inverse bidding function $\phi_i(\cdot)$ is defined as

$$\phi_i(u) := U_i^{*^{-1}}(u)$$

That is, if $U_i^*(\theta|x) = u$, then $\phi_i(u) = x$.

Definition. For an arbitrary $\theta > \underline{\theta}$, for any signal x of P_i, define the *tying function* $Q_i(x)$ as

$$Q_i(x) := \phi_{-i}\left(U_i^*(\theta|x)\right)$$

Definition. For an arbitrary $\theta > \underline{\theta}$, for any signal x of P_i and signal y of P_{-i}, define the *total value* of obtaining agent of type θ as

$$\tilde{R}_i(\theta|x,y) := \int_{\underline{\theta}}^{\theta} \left(\left\{ e_i^*(q|x) + q - \frac{e_i^*(q|x)^2}{2} \right\} \mathbf{1}_{\left\{U_{-i}^*(q|y) < U_i^*(q|x)\right\}} \right) dF_i(q|x,y) + \hat{R}_i(\theta|x,y)$$

The following result formalizes how each principal's bidding strategy is determined based on her signals as well as the other principal's bidding strategy. It is also useful in establishing the existence of equilibrium in the static game.

Proposition 3. *Given a common lowest contract $\underline{e}(\cdot)$ for both principals, the tying function for an arbitrary $\theta > \underline{\theta}$ is the solution to the following differential equation:*[6]

$$\frac{dQ_i(x)}{dx} = \left\{ \frac{\tilde{R}_{-i}\left(\theta|Q_i(x),x\right) - U_{-i}^*(\theta|Q_i(x))}{\tilde{R}_i\left(\theta|x,Q_i(x)\right) - U_i^*(\theta|x)} \right\} \frac{s_i(x)}{s_{-i}(Q(x))} \frac{G_{-i}(Q_i(x)|x)}{G_i(x|Q_i(x))}$$

With the associated initial condition $Q_i(\underline{x}_i) = \underline{x}_{-i}$

And the associated equilibrium bid profile is characterized by:

$$U_i^*(\theta|x) = \underline{U}(\theta) + \int_{-\infty}^{x} \tilde{R}_i(\theta|t,Q_i(t))dL(t|x)$$

$$U_{-i}^*(\theta|y) = U_i^*\left(\theta|Q_i^{-1}(y)\right)$$

where $L(t|x) := \exp\left(-\int_t^x \frac{g_i(s|Q_i(s))}{G_i(s|Q_i(s))}ds\right)$, and $\underline{U}(\theta)$ is the type-θ agent's payoff under a lowest contract satisfying conditionss 1 and 2.

[6] where $G_{-i}(\cdot|x)$ is the cumulative distribution of P_{-i}'s signal given P_i's signal x, and $s_i(\cdot)$ is the prior unconditional density function of P_i's signal, that is, $s_i(x) = \int_{\underline{\theta}}^{\bar{\theta}} s_i(x|\theta)dF(\theta)$

Proof. See appendix. □

4.2 Existence of Regular Equilibria

Taking any lowest contract that satisfies conditions 1 and 2 as the boundary condition, we can write down a differential equation for each principal using the tying function from proposition 3, which pins down, for any given θ, how $U_i(\theta|x)$ must be increasing in x. This differential equation takes $\tilde{V}_i(\theta|x) := \int_{-\infty}^{Q_i(x)} \tilde{R}_i(\theta|x,y)dG_{-i}(y|x)$ as given, and using it we can write down an expression for $U_i(\theta|x)$ in terms of $\tilde{V}_i(\theta|x)$. For some given signal x, we can then write down the optimal bid for two interior agent types $\hat{\hat{\theta}}$ and $\hat{\theta}$, then by taking the limit as $\hat{\hat{\theta}} \to \hat{\theta}$, we can set it up as a calculus of variations problem using the observation that $U_i^{'}(\theta|x) = e_i(\theta|x)$ where the derivative is taken with respect to θ, along with the boundary conditions $e_i(\underline{\theta}) = e_i(\bar{\theta}) = e^{FB}$. Because the integrand in the principal's maximization problem is concave in $e_i(\cdot|x)$ and $U_i(\cdot|x)$, a solution exists to the maximization problem.

4.3 Other Equilibria

Apart from regular equilibria, where principals offer contracts based on their signals, there is an equilibrium in constant strategies. In this equilibrium, both principals offer the efficient effort allocation to all types of the agent, and all of the surplus generated to the agent as rent. However, unlike the regular equilibria, this equilibrium is in weakly dominated strategies, because both principals receive a payoff of zero, principal P_i can deviate by choosing a distorted contract and still receive the same payoff. However, in that case, it is no longer a best response for P_{-i} to offer the first-best contract. Therefore, this equilibrium is not robust to perturbations in principals' strategies.

Proposition 4. *(Price war equilibrium) Both principals offering the first-best contract to all agent types, and offering all the surplus to the agent is an equilibrium in the static game.*

Proof. See appendix. □

4.4 Accuracy of Principals' Information

We now look at how the informativeness of the principals' signals affects the payoff of the agent. In the terminology used in auction theory, this is analyzing the expected revenue under different signal structures. Note that we are still in a pure common value environment, which keeps the analysis more tractable than with a general interdependent values setting.

As shown previously, a principal's expected payoff is supermodular in her signal and bid, and higher signals result in higher optimal bids. The principal's decision problem is therefore a monotone decision problem. The most common approach to modeling quality of information is Blackwell's "sufficiency" criterion, whereby one signal is more informative than another if the less informative signal is constructed by "garbling" the more informative one, meaning the better informed principal cannot learn anything from the less informative signal. Not only is this a very restrictive setting which does not allow ranking a wide range of signal structures where intuitively some signals seem more informative than others, it is also not very tractable in an affiliated information setting. A more general (and more convenient) notion of informativeness is what's called "accuracy" in Persico (2000), which first appeared in Lehmann (1988). In terms of notation of the signal structures, here we will drop the subscripts for the principals and replace them with superscripts as accuracy levels $\{\alpha_i\}_{i=1.2}$

Definition. Given two signal structures $S^{\alpha_1}(\cdot|\theta)$ and $S^{\alpha_2}(\cdot|\theta)$, both of which are *affiliated* with the parameter θ, we say that $S^{\alpha_1}(\cdot|\theta)$ is *more accurate* than $S^{\alpha_2}(\cdot|\theta)$ if

$$T_{\alpha_1,\alpha_2,\theta}(x) \coloneqq S^{\alpha_1^{-1}}(S^{\alpha_2}(x|\theta)|\theta)$$

is strictly increasing in θ, for all signals x.[7]

Here, $T_{\alpha_1,\alpha_2,\theta}(x)$ is the signal y with accuracy α_1 which is the corresponding signal to signal x with accuracy α_2 under the parameter θ in the sense that the probability of getting a signal no higher than y under structure S^{α_1} is the same as the probability of getting a signal no higher than x under structure S^{α_2}. Suppose we take any signal x from S^{α_2}, find the corresponding signal y from S^{α_1} when the underlying parameter is θ. For a higher parameter $\theta' > \theta$, for the same signal x from S^{α_2}, the new corresponding y' from S^{α_1} will be to the right of y if S^{α_1} is more accurate than S^{α_2}. One way to understand this notion of informativeness is that given both signals are affiliated with θ, both of them will respond to an increase in θ by redistributing probabilities and by putting more probability mass to the right. The more accurate signal structure responds "more", in the sense that for an equal increase in the

[7]Here we impose "strictly increasing" as opposed to "non-decreasing", this is without loss because we have an unbounded signal space in our setup.

parameter θ, the more accurate signal structure shifts more probability to the right compared to the less accurate one. As explained in Persico (2000), this can also be understood by noticing that the transformation $T_{\alpha_1,\alpha_2,\theta}(x)$ varies together with θ, meaning by plugging in the same signal x, for a low θ, the transformation gives us a lower signal y compared to a higher signal y' when θ is high. In this way, the transformed signal is more correlated with the parameter, hence "more accurate".

Now consider a symmetric setting with $\alpha_1 = \alpha_2 = \alpha_S$. Contrast that with an asymmetric setting where $\alpha_1 > \alpha_2 = \alpha_S$. For an arbitrary type of the agent $\theta > \underline{\theta}$, let $\left\{U_i^S(\theta|\cdot)\right\}_{i=1,2}$ be the payoffs offered to the agent under equilibrium contracts in the symmetric setting, and $\left\{U_i^D(\theta|\cdot)\right\}_{i=1,2}$ be the payoffs offered in the setting with different accuracy levels. Let $U_i^S(\theta)$ be the expected payoff offered to agent type θ in the symmetric equilibrium by P_i, where the expectation is taken over all realizations of the signal. That is,

$$U_i^S(\theta) = \int_{-\infty}^{\infty} U_i^S(\theta|x) dS_i(x|\theta)$$

And similarly define $U_i^D(\theta)$ for the asymmetric case.

Just as in section 4, for an arbitrary $\theta > \underline{\theta}$, when P_{-i} is using an increasing strategy $U_{-i}(\cdot)$, we can write down P_i's payoff from bidding u after receiving signal x as:

$$\pi_i(x, u) = \int_{-\infty}^{U_{-i}^{-1}(u)} \left\{\hat{R}_i(\theta|x, y) - u\right\} dG(y|x)$$

Let's define $\Pi_i(\alpha) := \max_u \int_{-\infty}^{\infty} \pi_i(x, u) dS_i(x|\theta)$ under the signal X^α where α denotes the accuracy level.

By proposition 2, in a regular equilibrium P_{-i} indeed does use an increasing strategy, and as shown in the proof of proposition 2, this implies that this payoff is supermodular in (x, u), which implies that it has the single-crossing property in (x, u). We now restate an important result from Lehmann (1988) that links the accuracy of signals with payoffs having the single-crossing property.

Lemma 2. *Suppose signals X^{α_1}, X^{α_2} are affiliated with θ. Then, X^{α_1} is more accurate than X^{α_2} if and only if for all payoffs $\pi(x, u)$ having the single-crossing property,* $\Pi(\alpha_1) > \Pi(\alpha_2)$

Proof. See Lehmann (1988). □

Proposition 5. *In any regular equilibrium, $U_i^D(\theta) < U_i^S(\theta)$ for any $\theta > \underline{\theta}$, for i=1,2.*

Proof. See appendix. □

The intuition here is the following. In both settings, we are looking at a competition analogous to a common-value auction. Therefore there will be a winner's curse effect active in either situation. However, in the symmetric information setting, after winning, the winning principal will only know that her opponent's signal was lower than that of hers. This could be because she herself received an unusually high signal and the agent's type is actually quite low (winner's curse), or it could be that her opponent got an unlikely low signal and the agent's type is actually quite high. Because both possibilities exist in the symmetric information setting, the winner's curse is weaker compared to the setting with asymmetrically informed principals. In the asymmetric setting, after winning, the less informed principal will induce that it's more likely that her signal was unusually high and the agent's realized type is more likely to be low, because the other principal received a more accurate signal. Therefore the less informed principal must bid pessimistically enough to account for this stronger winner's curse, and knowing this, the informed principal will also lower her bid. This leads to overall lower utility (revenue) for the agent regardless of his type.

5 The Two-period Game

Suppose now that the game is repeated in a second period, where in period 1, the agent's type, θ_1, is realized from the distribution $F(\cdot)$, whereas in period 2, his type, θ_2, is realized from the distribution $F(\cdot|\theta_1)$. In the two-period setting with only short-term contracts, we will assume that all players maximize the undiscounted sum of their payoffs over the two periods.

Assumption 3. *θ_1 and θ_2 are affiliated.*

Assumption 3 says that for any $\theta_1^{'} > \theta_1$, $F(\cdot|\theta_1^{'})$ stochastically dominates $F(\cdot|\theta_1)$ in the likelihood ratio sense. This.captures the connection between the agent's productivity in period 1 and his productivity in period 2. This assumption means that a higher ability agent in period 1 is also more likely to be higher ability in period 2, and thus any information learned by the principals in period 1 about θ_1 is useful in period 2 as well.

In the repeated game, payoffs are the same for each period as in the static game. For the two-period setting, as described in section 2, we denote P_i's information structure at the end of period 1 as $\mathcal{I}_i$.

Let $U_{i2}^*(\theta_2|m)$ be the expected equilibrium payoff offered by P_i in the second period to the agent of realized type θ_2, when the public message m was sent in the first period. That is,

$$U_{i2}^*(\theta_2|m) = \int\limits_{-\infty}^{\infty} U_{i2}^*(\theta_2|m,x)dS_i(x|\theta_2)$$

Let $U_2(\theta_2|m)$ denote the expected payoff of type θ_2 when message m was sent; that is, $U_2(\theta_2|m)$ is the expected value of the higher of the two payoffs offered by the principals.

When in the first period, P_i offers a contract that specifies for a given θ_1 its allocated effort level $e_{i1}(\theta_1)$ together with $w_{i1}(\theta_1)$ and $m_i(y_1(\theta_1))$, let $U_i^{TP}(\hat{\theta}_1|\theta_1)$ denote the agent's two-period expected payoff when he chooses to work with P_i in the first period, and mimics type $\hat{\theta}_1$. Therefore,

$$U_i^{TP}(\hat{\theta}_1|\theta_1) = w_{i1}(\hat{\theta}_1) - C\left(e_{i1}(\hat{\theta}_1|\theta_1)\right) + \int\limits_{\underline{\theta}}^{\bar{\theta}} \left\{U_2\left(\theta_2|m_i(y_1(\hat{\theta}_1))\right)\right\} dF(\theta_2|\theta_1)$$

$$= w_{i1}(\hat{\theta}_1) + \int\limits_{\underline{\theta}}^{\bar{\theta}} \left\{U_2\left(\theta_2|m_i(y_1(\hat{\theta}_1))\right)\right\} dF(\theta_2|\theta_1) - C\left(e_{i1}(\hat{\theta}_1|\theta_1)\right)$$

$$\frac{d}{d\hat{\theta}_1}\left(U_i^{TP}(\hat{\theta}_1|\theta_1)\right) = w_{i1}^{'}(\hat{\theta}_1) - \left(C^{'}\left(e_{i1}(\hat{\theta}_1|\theta_1)\right)\right)\left(e_{i1}(\hat{\theta}_1) + 1\right) + \int\limits_{\underline{\theta}}^{\bar{\theta}} \left[\frac{d}{d\hat{\theta}_1}\left\{U_2\left(\theta_2|m_i(y_1(\hat{\theta}_1))\right)\right\}\right] dF(\theta_2|\theta_1)$$

Similar to Lemma 1, by applying the requirement for first period local incentive compatibility, we get

$$w_{i1}^{'}(\theta_1) = \left(C^{'}\left(e_{i1}^*(\theta_1)\right)\right) e_{i1}^{*'}(\theta_1) + C^{'}\left(e_{i1}^*(\theta_1)\right) - \int\limits_{\underline{\theta}}^{\bar{\theta}} \left[\frac{d}{d\theta_1}\left\{U_2\left(\theta_2|m_i(y_1(\theta_1))\right)\right\}\right] dF(\theta_2|\theta_1)$$

Which means,

$$U_{i1}^{'}(\theta_1) = e_{i1}^{'}(\theta_1) - \int\limits_{\underline{\theta}}^{\bar{\theta}} \left\{\frac{d}{d\theta_1}\left(U_2\left(\theta_2|m_i(y_1(\theta_1))\right)\right)\right\} dF(\theta_2|\theta_1)$$

This leads to the necessary envelope condition for first-period incentive compatibility:

$$U_{i1}(\theta_1) = \int_{\underline{\theta}}^{\theta_1} \left\{ e_{i1}(q) - \int_{\underline{\theta}}^{\bar{\theta}} (U_2(\theta_2|m_i(y_1(q)))\, dF(\theta_2|\theta_1) \right\} dq$$

However, without knowing the shape of $m_i(\cdot)$, we cannot say whether this envelope condition is sufficient for incentive compatibility.

5.1 Non-Disclosure Policy

Now consider a second-period situation where P_i employed the agent in the first period. In the second period, at the time of offering contracts, P_i may have learned some information about θ_1 through her contractual relationship with the agent in the first period. Because θ_1 and θ_2 are affiliated, any such information is also informative of the agent's second period type θ_2. This is an informational advantage that the first period employing principal may have over the outsider principal, in case the informational learned through the first period contractual relationship, $\mathcal{C}_i$, is nonempty. We model this by assuming that under a non-disclosure policy (where $M_i = \phi$), P_i's second-period information has accuracy level $\alpha_{i_2} > \alpha_S$, where α_S is the accuracy level of the outsider principal's information, and the accuracy level of both principals' information in period 1.

Proposition 5 illustrates that we cannot have a first period separating equilibrium in the setting with non-disclosure, because the required high-powered incentive in the first-period will attract lower types to mimic as higher types, and to "take the money and run". This is the Ratchet effect as described in Laffont & Tirole (1988).

Assumption 4. *Under a non-disclosure policy, if $\mathcal{C}_i$ is nonempty, then in the second period, the accuracy levels of signals satisfy $\alpha_{i_2} > \alpha_{-i_2} = \alpha_S$.*

We now focus on what this implies about possible equlibria in the two-period game.

Proposition 6. *In the two-period game under non-disclosure, there does not exist an equilibrium where the agent fully reveals his type to the employing principal in the first period.*

Proof. See appendix. □

This is simply an instance of the ratchet effect as in Laffont & Tirole (1988). Even though unlike that paper (which has a monopolist principal offering a spot contract to the agent), we have competition in our setting, as long as we do not have second period competition between symmetrically informed principals, the ratchet effect persists. Because as we see in proposition 6, the agent's second-period payoff will be lower if he completely reveals his type to the employing principal in the first period, so he has an extra incentive to not reveal his type. Only monetary incentives are ineffective in overcoming this ratchet effect problem, because without long-term commitment, nothing stops lower types to mimic higher types and take the extra money in the first period.

5.2 Disclosure Policy

Under the disclosure policy, the employing principal in the first period can commit to sending a public message containing any information learned in period 1. We will assume that the principals' second period signals are of symmetric accuracy, therefore using a public message, the first period incumbent principal can give away any informational advantage.

Assumption 5. *Under the disclosure policy, principals in the second period receive symmetric signals.*

Proposition 7. *In the repeated game under disclosure, there exists a regular equilibrium where the agent fully reveals his type to the employing principal in the first period, and the employing principal chooses to publicly reveal the agent's first-period type.*

Proof. When both principals and the agent play the strategies under this separating equilibrium, in the first period, P_i would choose $m_i(y) = \theta_1$ such that $y = e^*_{i1}(\theta_1) + \theta_1$. In this case, $m_i(y)$ is a sufficient statistic for $\mathcal{I}_i^D$ because in the second period, the realization of the agent's first-period type is the only piece of information from $\mathcal{I}_i^D$ which is payoff-relevant. When choosing P_i's contract in the first period and exerting effort $e^*_{i1}(\theta_1)$, an agent of type θ_1 can get a two-period expected payoff of

$$U_i^{TP}(\theta_1) = U^*_{i1}(\theta_1) + \int_{\underline{\theta}}^{\bar{\theta}} U^S(\theta_2|\theta_1) dF(\theta_2|\theta_1)$$

where $U^S(\theta_2|\theta_1)$ is the θ_2 type agent's expected payoff in the second period when both principals receive symmetric signals and update their beliefs to $F_i(\cdot|\theta_1, x_i)$, where x_i is the realization of P_i's signal in the second period. Because principals update their beliefs using Bayes' rule, and by assumption 3, for any $\theta_1^{'} > \theta_1$, $F(\cdot|\theta_1^{'})$ stochastically dominates $F(\cdot|\theta_1)$ according to the monotone likelihood ratio, applying proposition 2, we get that $U^S(\theta_2|\theta_1^{'}) > U^S(\theta_2|\theta_1)$ for all $\theta_2 \in (\underline{\theta}, \bar{\theta}]$. Moreover, for any $\theta_2^{'} > \theta_2$, because

$$\frac{d}{d\theta_1}U^S(\theta_2^{'}|\theta_1) = \int_{\underline{\theta}}^{\theta_2^{'}} \frac{d}{d\theta_1}e^S(q|\theta_1)dq > \int_{\underline{\theta}}^{\theta_2} \frac{d}{d\theta_1}e^S(q|\theta_1)dq = \frac{d}{d\theta_1}U^S(\theta_2|\theta_1)$$

and because $F(\cdot|\theta_1^{'})$ first order stochastically dominates $F(\cdot|\theta_1)$, we have that

$$\int_{\underline{\theta}}^{\bar{\theta}} \frac{d}{d\theta_1^{'}}U^S(\theta_2|\theta_1^{'})dF(\theta_2|\theta_1^{'}) > \int_{\underline{\theta}}^{\bar{\theta}} \frac{d}{d\theta_1}U^S(\theta_2|\theta_1)dF(\theta_2|\theta_1)$$

which means that the marginal benefit of disclosure is higher for higher first period types.

We can thus say that incentive-compatibility in the two-period game requires that for any $\theta_1^{'} > \theta_1$,

$$U_{i1}(\theta_1^{'}) = U_{i1}(\theta_1) + \int_{\theta_1}^{\theta_1^{'}} \left\{ e_{i1}(q) - \int_{\underline{\theta}}^{\bar{\theta}} U^S(\theta_2|q)dF(\theta_2|\theta_1^{'}) \right\} dq$$

In particular, we can thus write down the first period rent that needs to be paid for any type θ_1 as

$$U_i(\theta_1) = U_i(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_1} \left\{ e_{i1}(q) - \int_{\underline{\theta}}^{\bar{\theta}} U^S(\theta_2|q)dF(\theta_2|\theta_1) \right\} dq$$

Notice that this is smaller than the required rent in the static game. For any choice of first-period effort allocation $e_{i1}(\cdot)$, the principal can adjust the payment accordingly so that this IC requirement is satisfied.

We still need to show that the principal cannot do better by choosing a first-period contract that involves pooling, and a message rule that does not completely reveal the agent's type. Based on the linkage principle in Milgrom & Weber (1982)[8] we know that the agent's second period rent will be highest when the maximum possible information is publicly available, and as can be seen from the IC requirement above, the first period principal can extract the incremental rent the agent can get in period 2 from improving public information about his type in the

[8]Especially Theorem 17 in Milgrom &Weber (1982) and Theorem 7 in Milgrom &Weber (1982b).

first period. In the first period, suppose $U_{i1}^*(\theta_1|x)$ is the two-period payoff-maximizing bid offered by P_i. Because

$$U^S(\theta_2|\theta_1) = \mathbb{E}_{X_j}\left[\max_{j=1,2}\left\{U_{j2}^*(\theta_2|\theta_1)\right\}\right] \geq \mathbb{E}_{X_i}\left[U_{i2}^*(\theta_2|\theta_1)\right]$$

Lastly, given this is a symmetric contract, if P_{-i} offers a separating contract, and by condition 2, the lowest contract satifies the monotonicity constraint, it is optimal for P_i to also offer a separating contract.

P_i can maximize her two-period payoff by choosing a separating equilibrium and revealing all information learned in the first period. □

Committing to disclosure of information has two effects on both total surplus and the incumbent principal's payoff. First, by committing to disclose information to her opponent, the principal implicitly commits to bid aggressively for the agent in period 2. That is, the principal commits to pay the agent more rent in period 2, relative to the case with no disclosure. But by corollary 2 of proposition 2, the principal optimally promises rent to the agent by asking that agent to exert more effort. Higher effort increases total surplus, so more aggressive bidding in period 2 implies higher total surplus. The incumbent principal can extract some of this future surplus by paying lower wages in period 1. Consequently, both principals earn higher expected ex ante payoffs in an equilibrium with disclosure. Note that principals cannot fully extract the additional surplus created from disclosure, since they compete with one another. However, they earn at least part of the additional surplus whenever that competition is imperfect, as it is when each of them has private information about the agent's ability.

Second, committing to disclosure increases the slope of the agent's expected second period rent as a function of his first period type. Consequently, with disclosure, it is cheaper to incentivise higher ability agents to separate from the lower ability agents, this second effect mitigates the ratchet effect problem, and allows principals to offer screening contracts in the first period.

6 Extensions

6.1 More Than Two Principals

It is natural to think about implications of having stronger competition for the agent when there are $N > 2$ principals. As in auction theory, the analysis of this situation is very similar to the two principals case. From each principal's perspective, the relevant belief about the opponents' bidding behavior is only the distribution of the highest of the $N-1$ other principals' bid. The inverse bidding function $\phi_{-i}(u)$ that is used to map the opponent's bid to her signal needs to be modified to be $\phi_{Y_1}(u)$, which is the inverse bidding function that maps the highest of the $N-1$ bids to a random variable Y_1 which is the signal associated the highest bid. That is, $U^*_{Y_1}(\cdot|\cdot)$ is the bidding strategy P_i bids against, where for a given agent type θ, and for signal realization y,

$$U^*_{Y_1}(\theta|y) = \max_{j\neq i} U^*_j(\theta|y)$$

The effect of increased competition in this way is straightforward. As seen earlier, with 2 principals, the best response to a more aggressive bidding strategy from the opponent is to become more aggressive. When there are more than two principals, the highest of the other principal's bids is higher as we are now considering the first order statistic of $N-1$ other bids. The outcome is that each principal bids more aggressively, and as $N \to \infty$ the contract offered by P_i approaches the first-best contract given any realization of her signal, and the agent gets all of the surplus he generates.

For this part we will again assume that all signals are affiliated with the agent's type θ_t.

Assumption 6. *X_{it} is affiliated with θ_t, for $i = 1, ..., N$*

Here we will show that the analog of proposition 2 for $N > 2$ principals holds; that is, with more than 2 principals, the utility offered to the agent by each principal is strictly increasing in the principal's signal realization. Before establishing this result, we state a useful lemma which is part of theorem 2 in Milgrom & Weber (1982).

Lemma 3. *X_{it} and Y_{1t} are affiliated.*

Proof. See Milgrom & Weber (1982). □

Because in the one-period game with $N > 2$ principals, from P_i's perspective, the maximization problem is the same as in the two principal case with Y_1 being the relevant signal, and because X_i and Y_1 are affiliated, the following analog for proposition 2 holds.

Proposition 8. *In the one-period game with competition between N principals, for any agent type $\theta \in (\underline{\theta}, \bar{\theta})$, $U^*_i(\theta|x)$ is strictly increasing in x, for $i = 1, ..., N$.*

6.2 Short-lived Principals

Consider a two-period setup where there are $N_1 \geq 1$ principals active in the first period, and $N_2 \geq 2$ principals active in the second period. This means there is competition among principals in the second period. We can have a set of active principals in period 1, $\mathcal{P}_1$, and a set of principals active in period 2, $\mathcal{P}_2$. It may be that some of the principals belonging to $\mathcal{P}_1$ are also in $\mathcal{P}_2$, while some are not, and $\mathcal{P}_2$ can have principals that are not in $\mathcal{P}_1$. So some principals may be short-lived, while others may be long-lived, and there may be some who are only active in period 2. However, the set of active principals in each period is fixed at the beginning of the game, so there are no entry or exit decisions made by principals. In this setup, in a disclosure setting, that is, in the first period, letting the message space be M_i^D, there is a fully separating equillibrium in the two-period game.

Proposition 9. *In the repeated game under disclosure with sets of principals $\mathcal{P}_1$ and $\mathcal{P}_2$, there exists a regular equilibrium where the agent fully reveals his type to the employing principal in the first period, and the employing principal chooses to publicly reveal the agent's first-period type.*

Proof. Under the strategies described, if $P_i \in \mathcal{P}_1$ employs the agent of type θ_1 in the first period and chooses message $m_i(y) = \theta_1$ such that $y = e^*_{i1}(\theta_1) + \theta_1$. Because θ_1 and θ_2 are affiliated, and $N_2 \geq 2$, $U^S(\theta_2|\theta_1)$ is strictly increasing in θ_1, by propositions 2 and 8. Similar to proposition 7, under the revealing public message, the agent's first period IC constraints under P_i's contract can be written as

$$U_i(\theta_1) = U_i(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_1} \left\{ e_{i1}(q) - \int_{\underline{\theta}}^{\bar{\theta}} U^S(\theta_2|q) dF(\theta_2|\theta_1) \right\} dq$$

This means P_i can extract the incremental rent the agent gets in the second period by adjusting the payment accordingly in the first period. If $P_i \notin \mathcal{P}_2$, then it is clearly beneficial for P_i to maximize the agent's second period payoff by revealing all information and extracting it in the first period. If $P_i \in \mathcal{P}_2$, because $U^S(\theta_2|\theta_1) = \mathbb{E}_{X_j}\left[\max_{j \in \mathcal{P}_2} \left\{U^*_{j2}(\theta_2|\theta_1)\right\}\right] \geq \mathbb{E}_{X_i}\left[U^*_{i2}(\theta_2|\theta_1)\right]$, it is still profitable for P_i to reveal the agent's type. □

6.3 Public Output

When the agent works with P_i in the first period, generates output y which is publicly observable, but the contract he was offered is not publicly observed, this may lead to a setting where P_i still has an informational advantage over the other principal because she may infer the agent's first-period type more accurately as she knows what contract the agent chose, while P_{-i} can only use a probability distribution over P_i's signal and the subsequent contract offered by P_i. As we saw in proposition 4, such a setting does not allow for separating contracts, as the ratchet effect is still present. When both output and contract offers are publicly observed, this makes information structure the same as under the disclosure policy. So similar to the disclosure setting, the first period employing principal can still capture some of the incremental rent the agent receives in the second period due to improved public information, because it is still the first period employing principal who is generating this value by giving the agent to credibly signal his type.

7 Conclusion

The implications of public disclosure of some performance measure can be seen through an increase in the degree of competition for the agent's services in the future. This creates value for the agent in the future through a reduction in the firms' uncertainty regarding the agent's worth (winner's curse), which leads to firms offering more efficient contracts, generating more surplus. However, this value is being created by the principals through their ability to credibly reveal information about the agent's performance, and as such, the principals will appropriate this additional surplus upfront by offering lower payment to the agent in the first period, utilizing the agent's incentive to work hard in the first period for the rent in the future. As the incremental rent from better public information is higher for higher types, this incentive is also stronger for higher types, which makes screening higher types from lower types easier for the principal. However, because both principals can generate this value, they compete away some of these rents to the agent, and how much of these rents the agent gets depends on the initial level of competition.

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Appendix

Proof of Lemma 1

Given an allocation rule $e(\theta)$, define $e(\hat{\theta}|\theta)$ by the equation $e(\hat{\theta}|\theta) + \theta = e(\hat{\theta}) + \hat{\theta}$; that is, it is the level of effort an agent of type θ has to exert in order to mimic type $\hat{\theta}$. Therefore, $e^{'}(\hat{\theta}|\theta) = e^{'}(\hat{\theta}) + 1$, where the derivative on LHS is taken with respect to $\hat{\theta}$. For an agent of type θ, his payoff if he mimics type $\hat{\theta}$ is:

$$U(\hat{\theta}|\theta) = w(\hat{\theta}) - C\left(e(\hat{\theta}|\theta)\right);$$

where $C(\cdot)$ is the cost of effort function. Therefore,

$$U^{'}(\hat{\theta}|\theta) = w^{'}(\hat{\theta}) - C^{'}\left(e(\hat{\theta}|\theta)\right)e^{'}(\hat{\theta}|\theta) = w^{'}(\hat{\theta}) - C^{'}\left(e(\hat{\theta}|\theta)\right)\left(e^{'}(\hat{\theta}) + 1\right)$$

Local incentive compatibility[9] requires that

$$U^{'}(\hat{\theta}|\theta)\Big|_{\hat{\theta}=\theta} = w^{'}(\theta) - C^{'}(e(\theta))\,e^{'}(\theta) - C^{'}(e(\theta)) = 0$$

So, $w^{'}(\theta) = C^{'}(e(\theta))e^{'}(\theta) + C^{'}(e(\theta))$

Now, as $U(\theta) = w(\theta) - C(e(\theta))$, $U^{'}(\theta) = w^{'}(\theta) - C^{'}(e(\theta))e^{'}(\theta) = C^{'}(e(\theta))$

Because we are using the quadratic cost of effort function $C(e) = {}^{e^2}/_2$, $C^{'}(e(\theta)) = e(\theta)$, therefore $U^{'}(\theta) = e(\theta)$, which gives us the envelope condition

$$U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} e(t)dt$$

For the monotonicity part, let $\theta_1 > \theta_2$ be arbitrary. IC requires

$$U(\theta_2|\theta_1) \leq U(\theta_1)$$

Now, when IC is satisfied,

[9]Because the agent's utility function $U(e, \theta)$ satisfies the single-crossing condition, local IC implies global IC.

$$\begin{aligned}
U(\theta_2|\theta_1) &= w(\theta_2) - \frac{e(\theta_2|\theta_1)^2}{2} \\
&= w(\theta_2) - \frac{1}{2}\left[e(\theta_2) + (\theta_2 - \theta_1)\right]^2 \\
&= w(\theta_2) - \frac{1}{2}e(\theta_2)^2 + e(\theta_2)(\theta_1 - \theta_2) - \frac{1}{2}(\theta_1 - \theta_2)^2 \\
&= U(\theta_2) + (\theta_1 - \theta_2)\left[e(\theta_2) - \frac{1}{2}(\theta_1 - \theta_2)\right] \leq U(\theta_1) \\
\therefore U(\theta_1) - U(\theta_2) &\geq (\theta_1 - \theta_2)\left[e(\theta_2) - \frac{1}{2}(\theta_1 - \theta_2)\right]
\end{aligned}$$

Similarly, we need $U(\theta_1|\theta_2) \leq U(\theta_2)$. Now,

$$\begin{aligned}
U(\theta_1|\theta_2) &= w(\theta_1) - \frac{e(\theta_1|\theta_2)^2}{2} \\
&= w(\theta_1) - \frac{1}{2}\left[e(\theta_1) + (\theta_1 - \theta_2)\right]^2 \\
&= w(\theta_1) - \frac{1}{2}e(\theta_1)^2 - e(\theta_1)(\theta_1 - \theta_2) - \frac{1}{2}(\theta_1 - \theta_2)^2 \\
&= U(\theta_1) - (\theta_1 - \theta_2)\left[e(\theta_1) + \frac{1}{2}(\theta_1 - \theta_2)\right] \leq U(\theta_2) \\
\therefore U(\theta_1) - U(\theta_2) &\leq (\theta_1 - \theta_2)\left[e(\theta_1) + \frac{1}{2}(\theta_1 - \theta_2)\right]
\end{aligned}$$

Combining the two, we get:

$$(\theta_1 - \theta_2)\left[e(\theta_2) - \frac{1}{2}(\theta_1 - \theta_2)\right] \leq U(\theta_1) - U(\theta_2) \leq (\theta_1 - \theta_2)\left[e(\theta_1) + \frac{1}{2}(\theta_1 - \theta_2)\right]$$

Dividing by $\theta_1 - \theta_2$ and rearranging gives us $e(\theta_2) + \theta_2 \leq e(\theta_1) + \theta_1$, which establishes the monotonicity requirement.

Proof of Proposition 1

Using integration by parts, the monopolist principal's maximization problem can be written as

$$\max_{e(\cdot|x)} \int_{\underline{\theta}}^{\bar{\theta}} \left\{ e(\theta|x) + \theta - \frac{e(\theta|x)^2}{2} - \frac{1-F(\theta|x)}{f(\theta|x)} e(\theta|x) dq \right\} dF(\theta|x)$$

Let $H(\theta|x) = \frac{f(\theta|x)}{1-F(\theta|x)}$, then by taking the first order condition, the principals maximization problem can be pointwise solved and the optimal effort allocation $e^M(\theta|x)$ can be written out as

$$e^M(\theta|x) = 1 - \frac{1}{H(\theta|x)}$$

By assumption 1, for any pair of signals $x_1 > x_2$, $F(\cdot|x_1)$ stochastically dominates $F(\cdot|x_2)$ in the likelihood-ratio sense (LRD). LRD implies hazard-rate dominance (HRD), therefore $F(\cdot|x_1)$ hazard-rate dominates $F(\cdot|x_2)$, that is, $H(\theta|x_1) < H(\theta|x_2)$ for any θ. Therefore, $1 - \frac{1}{H(\theta|x_1)} < 1 - \frac{1}{H(\theta|x_2)}$, that is, $e^M(\theta|x)$ is decreasing in x for any θ.

The type-θ agent's payoff is simply

$$U(\theta|x_1) = \int_{\underline{\theta}}^{\theta} e^M(t|x_1)dt < \int_{\underline{\theta}}^{\theta} e^M(t|x_2)dt = U(\theta|x_2)$$

This proves the second part of proposition 1.

Finally, consider the principal's expected payoffs under the signals x_1 and x_2. LRD implies first-order stochastic dominance (FOSD), therefore $F(\cdot|x_1)$ FOSD $F(\cdot|x_2)$.

$$\pi(\theta|x_1) = \max_{e(\theta|x_1)} \int_{\underline{\theta}}^{\bar{\theta}} \left\{ e(\theta|x_1) + \theta - \frac{e(\theta|x_1)^2}{2} - \frac{1}{H(\theta|x_1)} e(\theta|x_1) \right\} dF(\theta|x_1)$$

Consider the Principal's payoff even if she (suboptimally) choses the schedule $e^M(\theta|x_2)$ after receiving signal x_1. Her payoff in this case is:

$$\int_{\underline{\theta}}^{\bar{\theta}} \left\{ e^M(\theta|x_2) + \theta - \frac{e^M(\theta|x_2)^2}{2} - \frac{1}{H(\theta|x_1)} e^M(\theta|x_2) \right\} dF(\theta|x_1)$$

Now, because $e^M(\theta|x_2)$ is increasing in θ, and $e^M(\theta|x_2) < 1$ for all θ other than $\bar{\theta}$,

$$\frac{d}{d\theta}\left(e^M(\theta|x_2) - \frac{e^M(\theta|x_2)^2}{2}\right) = \left(1 - e^M(\theta|x_2)\right)\frac{d}{d\theta}\left(e^M(\theta|x_2)\right) > 0$$

Hence, $e^M(\theta|x_2) - \frac{e^M(\theta|x_2)^2}{2}$ is an increasing function of θ. Therefore,

$$\int_{\underline{\theta}}^{\bar{\theta}}\left\{e^M(\theta|x_2) - \frac{e^M(\theta|x_2)^2}{2}\right\}dF(\theta|x_1) > \int_{\underline{\theta}}^{\bar{\theta}}\left\{e^M(\theta|x_2) - \frac{e^M(\theta|x_2)^2}{2}\right\}dF(\theta|x_2)$$

It can be showed using integration by parts that

$$E\left[\theta|x_1\right] = \int_{\underline{\theta}}^{\bar{\theta}}\theta f(\theta|x_1)d\theta = \theta F(\theta|x_1)|_{\underline{\theta}}^{\bar{\theta}} = \bar{\theta} - \int_{\underline{\theta}}^{\bar{\theta}}F(\theta|x_1)d\theta$$

And similarly, $E\left[\theta|x_2\right] = \bar{\theta} - \int_{\underline{\theta}}^{\bar{\theta}}F(\theta|x_2)d\theta$

So, $E\left[\theta|x_1\right] - E\left[\theta|x_2\right] = \left(\bar{\theta} - \int_{\underline{\theta}}^{\bar{\theta}}F(\theta|x_1)d\theta\right) - \left(\bar{\theta} - \int_{\underline{\theta}}^{\bar{\theta}}F(\theta|x_2)d\theta\right) = \left(\int_{\underline{\theta}}^{\bar{\theta}}F(\theta|x_2)d\theta\right) - \left(\int_{\underline{\theta}}^{\bar{\theta}}F(\theta|x_1)d\theta\right) = \int_{\underline{\theta}}^{\bar{\theta}}\left\{F(\theta|x_2) - F(\theta|x_1)\right\}d\theta = \int_{\underline{\theta}}^{\bar{\theta}}\left\{(1 - F(\theta|x_1)) - (1 - F(\theta|x_2))\right\}d\theta$

Because x_1 FOSD x_2,

$$\begin{aligned}
E[\theta|x_1] &> E[\theta|x_2] \\
\text{So, } \int_{\underline{\theta}}^{\bar{\theta}} \{(1-F(\theta|x_1)) - (1-F(\theta|x_2))\} d\theta &> 0 \\
\int_{\underline{\theta}}^{\bar{\theta}} \left\{((1-F(\theta|x_1)) - (1-F(\theta|x_2)))\left(1-e^M(\theta|x_2)\right)\right\} d\theta &> 0 \\
\int_{\underline{\theta}}^{\bar{\theta}} \left\{(F(\theta|x_2) - F(\theta|x_1)) - ((1-F(\theta|x_1)) - (1-F(\theta|x_2)))\, e^M(\theta|x_2)\right\} d\theta &> 0 \\
E[\theta|x_1] - E[\theta|x_2] - \int_{\underline{\theta}}^{\bar{\theta}} \left\{((1-F(\theta|x_1)) - (1-F(\theta|x_2)))\, e^M(\theta|x_2)\right\} d\theta &> 0 \\
\int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta|x_1) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta|x_2) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} \left\{((1-F(\theta|x_1)) - (1-F(\theta|x_2)))\, e^M(\theta|x_2)\right\} d\theta &> 0
\end{aligned}$$

$$\begin{aligned}
\int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta|x_1) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} \left\{(1-F(\theta|x_1))\, e^M(\theta|x_2)\right\} d\theta &> \int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta|x_2) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} \left\{(1-F(\theta|x_2))\, e^M(\theta|x_2)\right\} d\theta \\
\int_{\underline{\theta}}^{\bar{\theta}} \left\{\theta - \frac{1}{H(\theta|x_1)} e^M(\theta|x_2)\right\} dF(\theta|x_1) &> \int_{\underline{\theta}}^{\bar{\theta}} \left\{\theta - \frac{1}{H(\theta|x_2)} e^M(\theta|x_2)\right\} dF(\theta|x_2)
\end{aligned}$$

Combining the two inequalities, we can say that

$$\begin{aligned}
&\int_{\underline{\theta}}^{\bar{\theta}} \left\{e^M(\theta|x_2) + \theta - \frac{e^M(\theta|x_2)^2}{2} - \frac{1}{H(\theta|x_1)} e^M(\theta|x_2)\right\} dF(\theta|x_1) \\
> &\int_{\underline{\theta}}^{\bar{\theta}} \left\{e^M(\theta|x_2) + \theta - \frac{e^M(\theta|x_2)^2}{2} - \frac{1}{H(\theta|x_2)} e^M(\theta|x_2)\right\} dF(\theta|x_2)
\end{aligned}$$

So even if after receiving signal x_1, the principal suboptimally chooses $e^M(\theta|x_2)$, her expected payoff is higher than $\pi(\theta|x_2)$. This means,

$$\pi(x_1) = \max_{e(\theta|x_1)} \int_{\underline{\theta}}^{\bar{\theta}} \left\{ e(\theta|x_1) + \theta - \frac{e(\theta|x_1)^2}{2} - \frac{1}{H(\theta|x_1)} e(\theta|x_1) \right\} dF(\theta|x_1)$$

$$\geq \int_{\underline{\theta}}^{\bar{\theta}} \left\{ e^M(\theta|x_2) + \theta - \frac{e^M(\theta|x_2)^2}{2} - \frac{1}{H(\theta|x_1)} e^M(\theta|x_2) \right\} dF(\theta|x_1)$$

$$> \int_{\underline{\theta}}^{\bar{\theta}} \left\{ e^M(\theta|x_2) + \theta - \frac{e^M(\theta|x_2)^2}{2} - \frac{1}{H(\theta|x_2)} e^M(\theta|x_2) \right\} dF(\theta|x_2) \quad = \quad \pi(x_2)$$

That is, $\pi(x_1) > \pi(x_2)$, which concludes the proof.

Proof of Proposition 3

Take an arbitrary $\theta > \underline{\theta}$. For the ease of exposition, we shall henceforth omit the argument θ from this proof and the proof of existence of equilibrium, and just write $\hat{R}(x, y)$ as the actual value for winning the agent of type θ given signals (x, y).

Suppose P_{-i} is playing the strategy $U^*_{-i}(\cdot)$ as described in proposition 3. P_i's expected payoff from bidding u after receiving signal x is therefore

$$\pi_i(x; u) = \int_{-\infty}^{\phi_{-i}(u)} \left\{ \hat{R}_i(x, y) - u \right\} dG_{-i}(y|x)$$

Differentiating with respect to u, we get

$$\frac{\partial \pi_i(x; u)}{\partial u} = \left[\left\{ \hat{R}(x, \phi_{-i}(u)) - u \right\} g_{-i}\left((\phi_{-i}(u)|x \right) \right] \phi^{'}_{-i}(u) - G_{-i}\left(\phi_{-i}(u)|x \right)$$

The first order condition is derived by equating this derivative to 0. Therefore, in equilibrium we have

$$\frac{1}{\phi^{'}_{-i}(u)} = \left\{ \hat{R}_i(x, \phi_{-i}(u)) - u \right\} \frac{g_{-i}\left(\phi_{-i}(u)|x \right)}{G_{-i}\left(\phi_{-i}(u)|x \right)}$$

Because in equilibrium $x = \phi_i(u)$, and analogously for signal y of P_{-i}, $y = \phi_{-i}(u)$, we can write down the first order conditions for each principal as

$$\frac{1}{\phi'_{-i}(u)} = \left\{\hat{R}_i(\phi_i(u), \phi_{-i}(u)) - u\right\} \frac{g_{-i}\left(\phi_{-i}(u)|\phi_i(u)\right)}{G_{-i}\left(\phi_{-i}(u)|\phi_i(u)\right)} \textit{for i=1,2}$$

These, together with the common boundary conditions

$$\lim_{x \to -\infty} U_i^*(\theta|x) = \underline{\mathrm{U}}^*(\theta) = \lim_{y \to -\infty} U_{-i}^*(\theta|y)$$

characterizes the equilibrium bids. That the equilibrium bids characterized in the exposition of proposition 3 form a solution to this system is the same as in Milgrom & Weber (1982).

Using the aforementioned first order conditions, one can write

$$\begin{aligned}\frac{\phi'_{-i}(u)}{\phi'_i(u)} &= \frac{\left\{\hat{R}_{-i}(\phi_{-i}(u), \phi_i(u)) - u\right\} \frac{g_i(\phi_i(u)|\phi_{-i}(u))}{G_i(\phi_i(u)|\phi_{-i}(u))}}{\left\{\hat{R}_i(\phi_i(u), \phi_{-i}(u)) - u\right\} \frac{g_{-i}(\phi_{-i}(u)|\phi_i(u))}{G_{-i}(\phi_{-i}(u)|\phi_i(u))}} \\ &= \left\{\frac{\hat{R}_{-i}(\phi_{-i}(u), \phi_i(u)) - u}{\hat{R}_i(\phi_i(u), \phi_{-i}(u)) - u}\right\} \frac{s_i(\phi_i(u))}{s_{-i}(\phi_i(u))} \frac{G_{-i}\left(\phi_{-i}(u)|\phi_i(u)\right)}{G_i\left(\phi_i(u)|\phi_{-i}(u)\right)}\end{aligned}$$

Where the second line follows using Bayes' rule.

Now, by definition of the tying function,

$$Q_i(\phi_i(u)) = \phi_{-i}(u)$$

Taking derivative with respect to u and using the chain rule, we get

$$Q'_i(\phi_i(u)) = \frac{\phi'_{-i}(u)}{\phi'_i(u)}$$

Which then means

$$Q_i^{'}(x) = \left\{ \frac{\hat{R}_{-i}(\phi_{-i}(u), \phi_i(u)) - u}{\hat{R}_i(\phi_i(u), \phi_{-i}(u)) - u} \right\} \frac{s_i(\phi_i(u))}{s_{-i}(\phi_i(u))} \frac{G-_i\left(\phi_{-i}(u)|\phi_i(u)\right)}{G_i\left(\phi_i(u)|\phi_{-i}(u)\right)}$$

$$\text{or, } \frac{dQ_i(x)}{dx} = \left\{ \frac{\hat{R}_{-i}\left(\theta|Q_i(x), x\right) - U^*_{-i}(\theta|Q_i(x))}{\hat{R}_i\left(\theta|x, Q_i(x)\right) - U_i^*(\theta|x)} \right\} \frac{s_i(x)}{s_{-i}(Q(x))} \frac{G_{-i}(Q_i(x)|x)}{G_i(x|Q_i(x))}$$

Now going back to the first order conditions, and rewriting them in terms of bids instead of inverse bids, we get

$$U_i^{*'}(x) = \left[\hat{R}_i(x, Q_i(x)) - U_i^*(x)\right] \frac{g_{-i}(Q_i(x)|x)}{G_{-i}(Q_i(x)|x)}$$

One can check that the equilibrium bidding strategy $U_i^*(x)$ in the exposition of proposition 3 satisfies this differential equation. We can also show that for the symmetric case, that is, when $S_i(\cdot|\theta) \equiv S_{-i}(\cdot|\theta)$, the equilibrium bidding strategy is unique up to the choice of the lowest contract. Suppose, for a contradiction, that $U_i(\cdot)$ and $\hat{U}_i(\cdot)$ are both solutions to this equation, and for some x, $U_i(x) < \hat{U}_i(x)$. Because in the symmetric case, $Q_i(x) = x$, this implies, from the differential equation, that $U_i^{'}(x) > \hat{U}_i^{'}(x)$, which means that they cannot both be solutions to the first order condition. Hence, the equilibrium bidding strategies in the exposition of proposition 3 form the unique regular equilibrium of the bidding game for the agent of type θ, taking $\hat{R}_i(\theta|x, y)$ as given.

Proof of Proposition 4

In this equilibrium both principals get a payoff of 0. Given P_{-i} is offering the first-best contract for all types and giving the agent all the surplus, any contract offered by P_i that is distorted downwards will be rejected by the agent, hence P_i cannot do better by offering a distorted contract. Offering a contract that distorts upwards from the first-best contract reduces total surplus, and incentive compatibility requires that the agent has to be offered more surplus than under P_{-i}'s contract. In this case the agent will accept P_i's contract but because total surplus decreases and the agent's rent increases, this means P_i is strictly worse off, and hence no deviation from offering the first-best contract is strictly beneficial for P_i. This proves the existence of the price war equilibrium.

Proof of Proposition 5

Let us first denote the bidding strategies in the symmetric equilibrium (where $\alpha_1 = \alpha_2 = \alpha_S$). Let's say for the particular θ, $U^S(\cdot)$ is the symmetric bidding function, which maps the principal's signal to the bid. We proceed by analyzing how the best reaction functions change from the symmetric setting as one principal (P_1) gets a signal with a higher accuracy level $\alpha_1 > \alpha_S$.

When P_2 has accuracy level α_S, and bidding with the symmetric equilibrium strategy $U^S(\cdot)$, P_1's maximization problem is the same when she has accuracy $\alpha_1 > \alpha_S$ as opposed to α_S as in both cases the payoff function to be maximized is $\pi_1(x, u)$. By lemma 3, we can say that against the symmetric bidding strategy by P_2, P_1's expected payoff will be higher under accuracy α_1 compared to α_S[10]. However, because we start with a common prior distribution of the agent's type, $F(\cdot)$ is the same no matter the accuracy levels of the signals. Hence, taking the expectation over all realizations of signals, the expected posterior for any α_i must equal the common prior for both principals. That is,

$$\int_{-\infty}^{\infty} f_i(\theta|x) dS_i^{\alpha_i}(x) = f(\theta)$$

Now, because the less informed principal's accuracy is the same as in the symmetric case, her signal distribution conditional on any type is also the same. That is, $s_2(y|\theta)$ is the same in both cases, for any y and θ.

Which means that for P_1, ex ante the distribution of her opponent's signal is the same under both cases. That is,

$$\int_{\underline{\theta}}^{\bar{\theta}} \left\{ \int_{-\infty}^{\infty} g_2(y|x) dS_1^{\alpha_1}(x|\theta) \right\} dF(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} s_2(y|\theta) dF(\theta) = s_2(y)$$

Now we analyze the bidding behavior of the less informed principal. Consider P_2's best response, after seeing signal y. Suppose that she is facing the same bids from her opponent as in the symmetric case, that is, $U_1(\theta|\cdot) = U^S(\theta|\cdot)$. In the asymmetric case, after received the realized signal y, the less informed principal's calculation of the interim expected value conditional on the opponent's signal being less than or equal to y, is

[10] This is presented as Fact1 in Persico (2000).

$$V_2^D(\theta|y) = \int_{-\infty}^{y} \hat{R}_2(\theta|y,x)dG_1^{\alpha_1}(x|y)$$

Because in the asymmetric case the opponent gets more accurate signals, and calculating $\hat{R}_2(\theta|y,x)$ involves putting probabilities over $[\theta, \bar{\theta}]$, and $V_2^D(\theta|y)$ is the expected value of $\hat{R}_2(\theta|y,x)$ taken only over signals of the opponent smaller than y, in calculating $V_2^D(\theta|y)$ the less informed principal puts higher probabilities on lower values of the agent's type compared to the symmetric case. Therefore, using the same calculation as in the proof of part 3 of proposition 1, we get

$$V_2^D(\theta|y) < V_2^S(\theta|y)$$

Where $V_2^S(\theta|y)$ is P_2's expected value under the symmetric information structure.

Because this is true for all θ, it must be that when the opponent plays the symmetric strategy $U^S(\theta|\cdot)$, in any regular equilibrium, for any type $\theta > \underline{\theta}$, based on the bidding strategies formulated in proposition 3, P_2's best response is to bid lower in the asymmetric case. That is, for any y, $U_2^D(\theta|y) < U^S(\theta|y)$.

Now from the better informed principal's perspective, the ex ante distribution of her opponent's signal, $s_2(\cdot)$, is the same as in the symmetric case. By lemma 2, she must have an ex ante higher payoff in the asymmetric case, which can only happen if either her ex ante expected value conditional on winning is higher, that is, $\int_{\underline{\theta}}^{\bar{\theta}} V_1^D(\theta)dF(\theta) > \int_{\underline{\theta}}^{\bar{\theta}} V_1^S(\theta)dF(\theta)$, or her ex ante expected bid is lower, that is, $\int_{\underline{\theta}}^{\bar{\theta}} U_1^D(\theta)dF(\theta) < \int_{\underline{\theta}}^{\bar{\theta}} U_1^S(\theta)dF(\theta)$; or both. Suppose towards a contradiction that her ex ante expected bid is higher in the asymmetric case, so $\int_{\underline{\theta}}^{\bar{\theta}} U_1^D(\theta)dF(\theta) > \int_{\underline{\theta}}^{\bar{\theta}} U_1^S(\theta)dF(\theta)$. Then her ex ante expected value must be higher, so at least for some signal realizations her interim value must be higher. Because under the more accurate signal structure, higher signal realizations are more correlated with higher types of the agent, this means that these signal realizations are the highest possible realizations. So we can find some x such that $\int_{x}^{\infty} \left\{ \int_{\underline{\theta}}^{\bar{\theta}} V_1^D(\theta|z)dF_1^{\alpha_1}(\theta|z) \right\} dS_1^{\alpha_1}(z) > \int_{x}^{\infty} \left\{ \int_{\underline{\theta}}^{\bar{\theta}} V_1^S(\theta|z)dF_1^{\alpha_S}(\theta|z) \right\} dS_1^{\alpha_S}(z)$. In order for her ex ante expected value to be higher, these signal realizations in $[x, \infty)$ must have a sufficiently higher probability under X^{α_1} compared to X^{α_S}. However, because X^{α_1} is more accurate, it is more correlated with θ, and a higher probability of realizations $[x, \infty)$ implies ex ante some subset of of highest types $[\theta, \bar{\theta}]$ has greater probability under X^{α_1} compared to X^{α_S}. This violates the fact that the type distribution $F(\cdot)$ has a common prior distribution, as we must have $\int\limits_{-\infty}^{\infty} f_i(\theta|x)dS_i^{\alpha_i}(x) = f(\theta)$, for all θ, as shown before. Hence the better informed principal's ex ante expected bid cannot be higher under the more accurate signal X^{α_1}.

For any type of the agent $\theta > \underline{\theta}$, because the less informed principal's reaction function moves towards lower bids when $\alpha_1 > \alpha_S$, and the more informed principal's average bid does not increase, both principals' expected bid in the asymmetric equilibrium must be lower than in the symmetric equilibrium, therefore the agent's ex ante expected payoff in the asymmetric equilibrium must be lower.

Proof of Proposition 6

Consider two first-period types of the agent $\theta_1^{'} > \theta_1$. Suppose, towards a contradiction, that a fully separating equilibrium exists. Denote by $U_i^{TP}(\theta_1|\theta_1^{'})$ as the sum of expected two-period payoff type $\theta_1^{'}$ could get by taking the allocated effort for type θ_1 at period 1, under the separating contract. As described in part 3, incentive compatibility requires that

$$U_i^{TP}(\theta_1^{'}) = U_i^{TP}(\theta_1) + \int\limits_{\theta_1}^{\theta_1^{'}} \left\{ e_{i1}(q) + \int\limits_{\underline{\theta}}^{\bar{\theta}} U_2(\theta_2) dF(\theta_2|\theta_1^{'}) \right\} dq$$

where the agent's second-period expected contract is $U_2(\cdot)$. By proposition 6, for any second period realization of type $\theta_2 > \underline{\theta}$, $U_2^D(\theta_2) < U_2^S(\theta_2)$, so in order to compensate the agent for the loss of payoff in the second period, the first period contract must give the higher type agent

$$\begin{aligned}
U_1(\theta_1^{'}) &= U_1(\theta_1) + \int\limits_{\theta_1}^{\theta_1^{'}} \left\{ e_{i1}(q) + \mathbb{E}_{(\theta_2|\theta_1^{'})}\left[U_2^S(\theta_2) - U_2^D(\theta_2)\right] \right\} dq \\
&= U_1(\theta_1) + \int\limits_{\theta_1}^{\theta_1^{'}} \left\{ e_{i1}(q) + \mathbb{E}_{(\theta_2|\theta_1^{'})}\left[U_2^S(\theta_2) - U_2^D(\theta_2)\right] \right\} dq \\
\therefore U_1(\theta_1^{'}) - U_1(\theta_1) &= \int\limits_{\theta_1}^{\theta_1^{'}} e_{i1}(q) dq + \mathbb{E}_{(\theta_2|\theta_1^{'})}\left[U_2^S(\theta_2) - U_2^D(\theta_2)\right]
\end{aligned}$$

Proof. Here, the term $\mathbb{E}_{(\theta_2|\theta_1^{'})}\left[U_2^S(\theta_2) - U_2^D(\theta_2)\right]$ refers to the expected value of $\left[U_2^S(\theta_2) - U_2^D(\theta_2)\right]$, where the expectation is taken over all possible second period realizations of type θ_2, given the agent's first period type is θ_1.

Because $U_2^S(\theta_2) - U_2^D(\theta_2) > 0$ for any θ_2, we can say $\mathbb{E}_{(\theta_2|\theta_1^{'})}\left[U_2^S(\theta_2) - U_2^D(\theta_2)\right] > 0$. By picking a small enough $\epsilon > 0$, we can find type $\hat{\theta}_1 = \theta_1^{'} - \epsilon$ such that

$$
\begin{aligned}
C\left(e_{i1}(\theta_1^{'}|\hat{\theta}_1)\right) - C\left(e_{i1}(\theta_1^{'})\right) &= \int_{\hat{\theta}_1}^{\theta_1^{'}} \left\{C^{'}\left(e_{i1}(q)\right)\left(1 + e_{i1}^{'}(q)\right)\right\} dq \\
&= \int_{\hat{\theta}_1}^{\theta_1^{'}} \left\{e_{i1}(q) + e_{i1}(q)e_{i1}^{'}(q)\right\} dq \\
&= \int_{\hat{\theta}_1}^{\theta_1^{'}} e_{i1}(q)dq + \int_{\hat{\theta}_1}^{\theta_1^{'}} \left\{e_{i1}(q)e_{i1}^{'}(q)\right\} dq \\
&< \int_{\hat{\theta}_1}^{\theta_1^{'}} e_{i1}(q)dq + \mathbb{E}_{(\theta_2|\hat{\theta}_1)}\left[U_2^S(\theta_2) - U_2^D(\theta_2)\right] \\
&= \int_{\hat{\theta}_1}^{\theta_1^{'}} \left\{e_{i1}(q) + \mathbb{E}_{(\theta_2|\hat{\theta}_1)}\left[U_2^S(\theta_2) - U_2^D(\theta_2)\right]\right\} dq
\end{aligned}
$$

where the inequality follows from the fact that $\mathbb{E}_{(\theta_2|\hat{\theta}_1)}\left[U_2^S(\theta_2) - U_2^D(\theta_2)\right] > 0$, and the integral $\int_{\hat{\theta}_1}^{\theta_1^{'}} \left\{e_{i1}(q)e_{i1}^{'}(q)\right\} dq$ can be made small enough by picking a small enough ϵ. This means, we can find a type below $\theta_1^{'}$ who will strictly benefit by mimicking type $\theta_1^{'}$ in the first period and not taking the specified contract from P_i in the second period. This violates upward incentive compatibility for any subset of fully separating types in the first period, which means no fully separating contract can exist in the first period. □