Token economics scoping review: Annotated bibliography

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DRAFT: Comments welcome.

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Abstract

This project highlights some tools and techniques relevant to the design of a token economy under a rational expectations hypothesis. Six articles are reviewed, these enumerate some conditions for equilibrium pricing of specific effervescent-tokens, dampened-tokens, breakable-tokens, and redeemable-tokens. A review of Polkadot parachain token designs indicates there are differences in the issues and risks addressed by token equilibrium pricing models and those addressed by the designs practiced in the Polkadot ecosystem.

Contents

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1 Background

As a scoping review, see Table 1, the premise of this work is the interested reader will refer to the published articles and cited references for additional details. The aim of this project is to point to ideas and references that initiate further investigation. A scoping review cannot be considered a source for specific detail on the relation between the economic concepts canvassed here. For the impatient, one can start with Darrell Duffie's text[1] along side Ljungqvist and Sargent's text.[2]

Table 1: Scoping Review: Extract from Table 1 of Grant, M.J. and Booth, A. (2009)

Description	Search	Appraisal	Synthesis	Analysis
Preliminary assessment of potential size and scope of available research literature. Aims to identify the nature and extent of research evidence (usually including ongoing research)	Completeness of searching determined by time/scope constraints. May include research in progress	No formal quality assessment	Typically tabular with some narrative commentary	Characterizes quantity and quality of literature, perhaps by study design and other key features. Attempts to specify a viable review

This project emerged from an effort to take a token-economy design and bootstrap some settings for a launch state. Our analytic framework is a competitive rational expectations equilibrium setting. The definition of "rational expectations" in economics is due to John Muth:[3]

> *In order to explain fairly simply how expectations are formed, we advance the hypothesis that they are essentially the same as the predictions of the relevant economic theory.*

Calculating an equilibrium in these settings is generally done via fixed-point theorems, such as Kakutani's Fixed Point Theorem,[4] or the Equivalent Martingale Methodology[5]. Both approaches are demonstrated in the articles annotated here. Since then, a multitude of introductory textbook treatments have emerged. Darrell Duffie's text[6] is considered a sound introduction, with an orientation exercise in Chapter 1 illustrating the use of Kakutani's Fixed Point Theorem to establish an equilibrium exists. Similarly, the text[7] by Ljungqvist and Sargent is an essential reference.

1.1 Terminology

The terms "token" and "coin" are not clearly defined or consistently used in the blockchain industry. Most obviously, we frequently observe this when the term 'tokenomics' refers to both tokens and coins. Here I

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[1] Darrell Duffie, "Dynamic Asset Pricing Theory: Third Edition," Princeton Series in Finance, 2001,

[2] Lars Ljungqvist and Thomas J. Sargent, *Recursive Macroeconomic Theory, Fourth Edition*, 4th ed., vol. 1, MIT Press Books 0262018748 (The MIT Press, December 2018).

[3] John F. Muth, "Rational Expectations and the Theory of Price Movements," *Econometrica* 29 (1961): 315, https://doi.org/10.2307/1909635.

[4] Shizuo Kakutani, "A generalization of Brouwer's fixed point theorem," *Duke Mathematical Journal* 8, no. 3 (1941): 457–459, https://doi.org/10.1215/S0012-7094-41-00838-4.

[5] Kenneth J Arrow and Gérard Debreu, "Existence of an equilibrium for a competitive economy," *Econometrica: Journal of the Econometric Society* 22, no. 3 (1954): 265–290, https://doi.org/10.2307/1907353; J. Michael Harrison and David M. Kreps, "Martingales and Arbitrage in Multiperiod Securities Markets," *Journal of Economic Theory* 20, no. 3 (1979): 381–408, https://doi.org/10.1016/0022-0531(79)90043-7; J. Michael Harrison and Stanley R. Pliska, "Martingales and Stochastic Integrals in the Theory of Continuous Trading," *Stochastic Processes and their Applications* 11, no. 3 (1981): 215–260, https://doi.org/10.1016/0304-4149(81)90026-0.

[6] Duffie, "Dynamic Asset Pricing Theory: Third Edition."

[7] Ljungqvist and Sargent, *Recursive Macroeconomic Theory, Fourth Edition*.

use the term tokens, apart from the annotations, where I try to adopt the author's usage.

The GDF Taxonomy for Cryptographic Assets[8] is a useful reference on token descriptions. However, their observation is:

> *Clearly defining what constitutes a token can be a surprisingly daunting task.*

Naturally, if the definition of what constitutes a token is fraught, the definition of a "Token Economy" should be doubly so. That is the case. While the literature surveyed here systematically analyses their valuation, the terms "token" and "coin" are, in some papers, used interchangeably. Yet, in others, they represent different concepts.

Consequently, you will find that each of the papers referenced here analyzes token economies that are subtly different. The detail of those subtle distinctions matters more than the nouns used.

2 Token-Economy

The Polkadot parachain tokens reviewed are listed in Table 4. None of the reviewed Polkadot parachain tokens were designed with an explicit equilibrium objective, nor using a rational expectations framework (with the possible exception of the Equilibrium parachain, see Table 2). No Polkadot parachain identifies an expression for even *stylized* token price dynamics. Here, and in the articles annotated, the term *stylized* is used in the ordinary sense: *represented in a way that simplifies details rather than trying to show naturalness or reality.*[9]

Hence, it was not possible to identify, across multiple Polkadot parachains, those elements of rational expectations modeling used in the design of Polkadot parachain tokens.

Table 2: Native Token-Economy Types: Polkadot Ecosystem

Chain	Token	Economy	Model	Equilibrium	Sector	Production	Monetary
Equilibrium	EQ	Open[a]	Structural	Partial[b]	52[c]	None	None

[a] This economy is Open in terms of borrowing and lending tokens.

[b] The state price process isn't explicitly specified (some references suggest a Black-Scholes-Merton type, others a jump-diffusion), hence it is not possible to identify a single supporting partial equilibrium with confidence. Nonetheless, I describe the effort as a partial equilibrium.

[c] Financial Services in the North American Industry Classification System.

Reviewing implemented token designs is, by definition, an exercise limited to a specific blockchain. However, an overview of the token design elements/issues a developer might consider is not so limited. In place of the Polkadot-centric "summary of implementations", a series of flow charts/decision trees is presented that may help any blockchain developer identify the scope of their token design. There is nothing in these choices that is limited to the Substrate or Polkadot ecosystems. Rather, the purpose of these illustrations is to point out: the ideas on most topics are developed to a point that there are choices available. Consequently, the figures don't contain information about the subject beyond providing terms the interested reader might search for in the course of further research into a topic.

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[8] GBBC Digital Finance LTD, "GBBC Digital Finance Code of Conduct Taxonomy for Cryptographic Assets: From the Perspective of General Global Regulatory Standards," October 2022, https://web.archive.org/web/20230728010242*/https://www.gdf.io/wp-content/uploads/2019/08/0010_GDF_Taxonomy-for-Cryptographic-Assets_Proof-V2-260719-1.pdf.

[9] *stylized, adj.*, in *Cambridge Academic Content Dictionary* (Cambridge University Press, July 28, 2023), eprint: https://dictionary.cambridge.org/dictionary/english/stylized.

It is natural to wonder what is the scope of a token economy. I leave that to the developer (team) to define. For example, which blockchain is covered by the "payment of transaction fees"? Is it the main relay chain of said token economy, such as in the Substrate ecosystem? Or does the developer refer to a solo chain? Or to a parachain? a parathread? A close reading of the articles annotated here, and any that follow, should provide a developer with an opinion on which model bears the closest resemblance to what they have in mind. But only the developer (team) knows what they have in mind. Similarly, prudence, legislative and regulatory provisions mean that it is inappropriate to provide advice about the relevance of the studies to any specific application in Polkadot. Opining about the tangible implications for practitioners are also out of scope for the same reasons.

It is important to remember that what follows represents the author's subjective choices of elements or characteristics to model. The subsequent appearance of some of those same elements or characteristics in the articles selected for the annotated bibliography should not be interpreted as evidence that those elements or characteristics are indispensable or ideal. It is worth noting that the development of token designs is still in its early stages, and significant technical advancements often require substantial time.

Choices/tradeoffs that a token designer may wish to consider have been grouped under the following topics:

Model: structural vs reduced form

Economy: open vs ajar vs closed

Equilibrium: partial vs general

Sectors: public vs private, financial vs real and their interactions

Production: Cobb-Douglas vs constant elasticity of substitution

Monetary: Neutral vs Non-neutral Money, Quantity Theory of Money vs Fiscal Theory of the Price Level, etc.

Other attributes/specifications of a native token economy that are generally absent for Polkadot parachains are Agent Utility, Market Rates (Riskless Rates + Risk Premia), Rate Curves (by maturity), Borrowing/Lending, and Lender of Last Resort.

2.1 Economy Type

A token economy can be classified as open if it allows unrestricted staking and rewarding using the native token (NT) of other token economies, as well as accepting the native tokens of other token economies as a means of payment for transaction fees. On the other hand, if a token economy does not meet these criteria partially or fully, it can be categorized as either ajar or closed. In this context, staking refers to the actions taken to ensure the security of the blockchain.

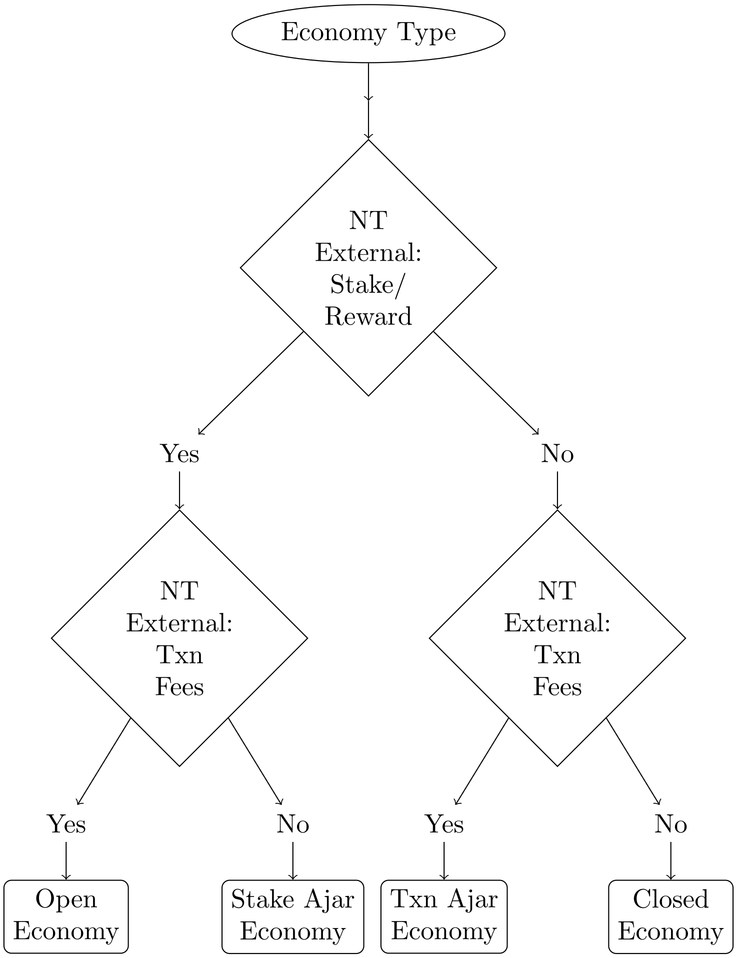

Figure 1: Token-Economy Economy Types

2.2 Model Type

Reduced-form models establish relationships between endogenous variables based on observable variables. For example, an endogenous variable might be user adoption, proxied by observed daily active accounts. Structural models, on the other hand, originate from theories and provide a deeper understanding of behavioral patterns, which may involve unobservable parameters. The selection of an approach is frequently influenced by data availability and considerations related to estimation and calibration. In the absence of data, simulations are often employed as an alternative method.

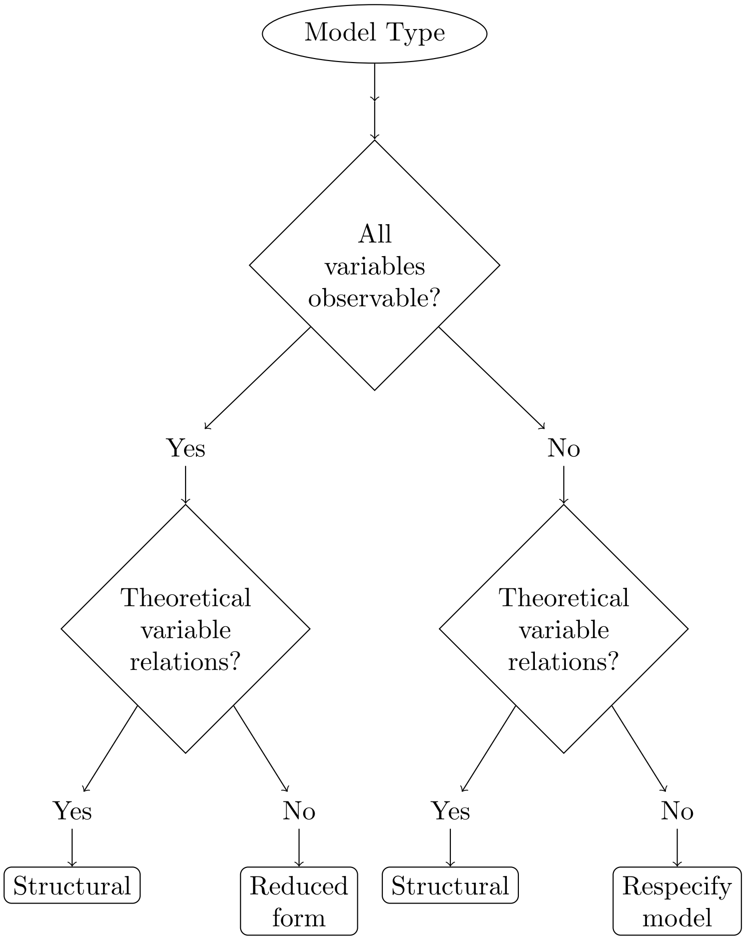

Figure 2: Token-Economy Model Types

2.3 Equilibrium Type

General equilibrium theory aims to provide an explanation of the behavior of supply, demand, and prices in an entire economy comprising multiple interacting markets. Its objective is to demonstrate that the interaction between demand and supply leads to an overall state of general equilibrium. This contrasts the theory of partial equilibrium, which focuses on analyzing a specific part of the economy while assuming other factors remain constant.

In a general equilibrium model, the overall equilibrium quantities and prices are determined endogenously for the entire economy. This involves considering initial endowments and modeling agent behavior, with the objective of describing changes in prices and quantities that result in a "Pareto Optimal" outcome. On the other hand, a partial equilibrium model focuses on analyzing a specific part of the economy while assuming other parts to be constant or even absent. In such cases, either the price or quantity process is typically specified, and the other variable is derived from it, which necessitates a reduced-form model. It is worth noting that a partial equilibrium approach can sometimes lead to outcomes where no trade occurs. When possible, it can be beneficial to calculate an equivalent structural equilibrium that supports a reduced-form partial equilibrium specification, and vice versa.

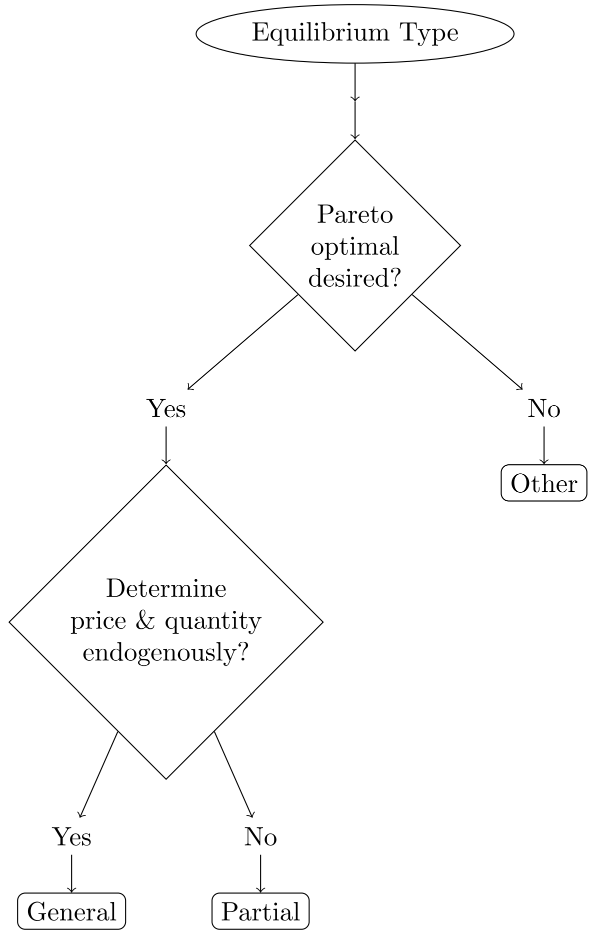

Figure 3: Token-Economy Equilibrium Types

2.4 Sector Type

Multisector models are used to explore the allocation of resources across different activities, e.g. validator staking vs non-validator bonding. Commonly studied are consumption vs investment, public vs private, and real vs financial sectors. It is becoming more common to model more than two sectors, the number and character of each sector are either self-evident or too unique to be summarized in a decision tree.

2.5 Production Type

A production function is a specification of how the quantity of output behaves as a function of the inputs used in production. In both general and partial equilibrium settings, it is used to connect different parts of an economy, for example, the Production-CAPM.[10] Only two production functions are considered here: The Cobb-Douglas (C-D) and the Constant Elasticity of Substitution[11] (CES). The elasticity of substitution (ES) is a measure of how easy it is to shift between factor inputs - the percentage change in the ratio of the two inputs relative to the percentage change in their prices.

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[10] John H. Cochrane, "Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations," *The Journal of Finance* 46, no. 1 (1991): 209–237, https://doi.org/10.1111/j.1540-6261.1991.tb03750.x.

[11] Leontief, linear and Cobb–Douglas functions are special cases of the CES production function.

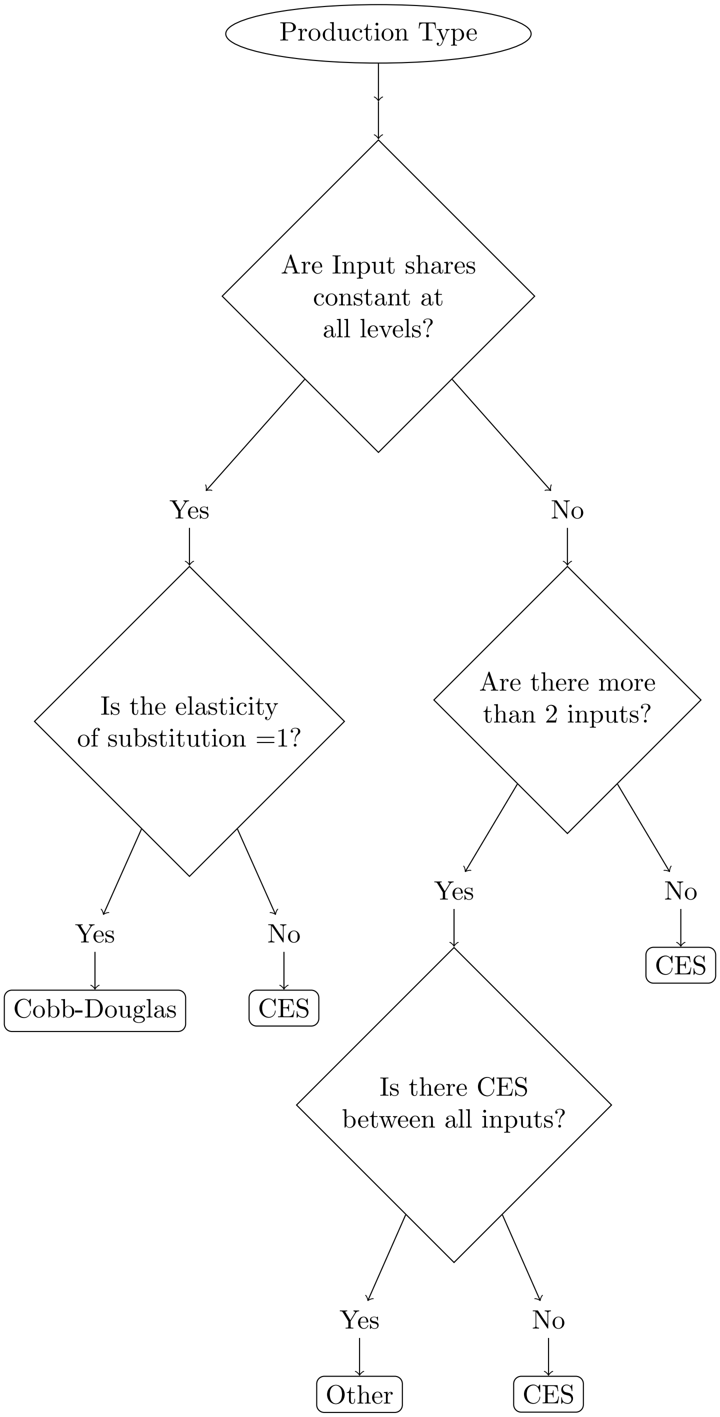

Figure 4: Token-Economy Production Types

2.6 Monetary Type

Monetary Policy is one of the most fraught topics in macroeconomics. There is no consensus on an unconditionally optimal policy. An important choice will be between adopting non-neutral or neutral monetary policies. While there is no consensus on the optimal choice, there is extensive literature on both. The monetary policy-related lectures for The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel,[12] provide a useful review of the background, progress, and controversies. Generally, the designer of a token economy will be left to choose from among the alternatives shown in Table 3.

Table 3: Monetary Policy Types

Monetary Target	Target Variable	Objective
Inflation	Interest rate	A given rate of change in an index
Price Level	Interest rate	A specific index number
Monetary Aggregate	Growth in money supply	A given rate of change in an index
Exchange Rate	The spot price of the currency	The spot price of the currency
Collateral peg	Collateral spot price	Low inflation as measured by the collateral price

2.7 Native Token Functions

This analysis is limited to publicly available network whitepapers or token-economy/tokenomics documentation. None of the reviewed Polkadot parachain tokens were designed using a rational expectations framework (with the possible exception of the Equilibrium parachain), and none derive an expression for their token price dynamics. Consequently, the range of functionality for which tokens can be used has been summarized using the classification scheme by Burnie, Burnie, and Henderson(2018).[13] In their scheme, differentiating tokens based on functional attributes, tokens are categorized into crypto-transaction tokens (cash substitutes); crypto-fuel tokens (generic blockchain applications); and crypto-voucher tokens (exchangeable for consumption goods). This classification helps identify issues to consider when participating in a cryptocurrency system. For crypto-transaction and crypto-fuel tokens is the token a better form of money? For crypto-fuel tokens, the application user base and the utility of the token system are more important. For crypto-voucher tokens, the value of the numeraire good and the token's exchangeability are important considerations. The results are summarized in Table 4.

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[12] Nobel Prize Committee, *The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel*, technical report (Nobel Prize Committee).

[13] Andrew Burnie, James Burnie, and Andrew Henderson, "Developing a Cryptocurrency Assessment Framework: Function over Form," *Ledger* 3 (July 2018), https://doi.org/10.5195/ledger.2018.151.

Table 4: Native Token Functions: Polkadot Ecosystem

Chain	Token	Fuel	Transaction	Voucher
Polkadot	DOT Y	Y	N	N
Acala	ACA	Y	N	N
Astar	ASTR	Y	N	N
Bifrost	BFC	Y	Y	N
Centrifuge	CFG	Y	N	N
Clover	CLV	Y	Y	N
Composable	LAYR	Y	N	N
Crust	CRU	Y	Y	Y
Darwinia	RING	Y	N	N
Efinity	EFI	Y	N	N
Equilibrium	EQ	Y	Y	N
HydraDX	HDX	Y	N	N
Interlay	INTR	Y	N	N
KILT	KILT	Y	N	N
Moonbeam	GLMR	Y	N	N
Nodle	NODL	Y	N	N
OriginTrail	TRAC/OTP	Y	Y	N
Parallel	PARA	Y	N	N
Phala	PHA	Y	N	Y
Statemint	DOT	Y	N	N
Unique	UNQ	Y	N	N

3 Token Price Models

While the articles included here may be the first generation of Token pricing models, they nontheless encompass a reasonably broad range of Token characteristics and price dynamics. These are categorized as:

1. **Effervescent-tokens:** Lin William Cong, Ye Li, and Neng Wang, "Tokenomics: Dynamic Adoption and Valuation," *The Review of Financial Studies* 34, no. 3 (March 2021): 1105–1155, https://doi.org/10.1093/rfs/hhaa089
2. **Dampened-tokens:** Lin William Cong, Ye Li, and Neng Wang, "Token-based platform finance," *Journal of Financial Economics* 144, no. 3 (2022): 972–991, https://doi.org/10.1016/j.jfineco.2021.10
3. **Breakable-tokens:** Michael Sockin and Wei Xiong, "Decentralization through Tokenization," *The Journal of Finance* 78, no. 1 (2023): 247–299, https://doi.org/10.1111/jofi.13192 and Michael Sockin and Wei Xiong, "A Model of Cryptocurrencies," *Management Science*, Forthcoming, https://doi.org/10.1287/mnsc.2023.4756
4. **Redeemable-tokens:** Kenneth Rogoff and Yang You, "Redeemable Platform Currencies," *The Review of Economic Studies* 90, no. 2 (May 2022): 975–1008, https://doi.org/10.1093/restud/rdac028

4 Summary

The annotated articles can be viewed in two ways. The first is a description of what platforms do. The second is a description of how platforms should be designed. Taking the first view, the results can help set expectations about platform functionality, strengths, and weaknesses. Taking the second view, the results can suggest how to configure a blockchain, point to possible users, and the functionality they may desire. As well as set expectations about the platform. Both views require mapping real-world properties and behaviors to abstract descriptions. And this requires an abundance of pragmatism and the willingness to live with approximations.

It is true that neither Polkadot nor the parachains currently on Polkadot have been designed nor conceived, using a rational expectations hypothesis. That does not mean it is not possible to do so. However, it is important to bear in mind that the models surveyed here are first-generation models. Consequently, it is very likely a blockchain that strives to implement one of the models described here will require substantial revisions as well as fine-tuning.

5 Annotated Bibliography

Chen, Long, Lin William Cong, and Yizhou Xiao. “A Brief Introduction to Blockchain Economics.” Chap. 1 in *Information for Efficient Decision Making*, 1–40. 2020. https://doi.org/10.1142/9789811220470_0001.

The authors provide a succinct introduction to blockchain economics. They distinguish a blockchain from crypto-tokens and currencies. Introduce the key concepts and features of blockchains. Highlight its decentralized nature, cryptographic security, and immutability. The authors also explore the possible impact of a blockchain on different sectors. They focus on technical improvements and applications. And adopt an economic perspective to identify useful and desirable features of blockchains. Section 2 defines the general concept of a blockchain. The authors consider decentralized consensus to be its main advantage over traditional systems. They explore the skeptical view that a blockchain is only a database upgrade. In their view, blockchains provide a trust system using the following building blocks. 1) Prevent a single point of failure. 2) Reduce market power and enable stakeholding. And 3) enable value exchange, asset traceability, and information interaction. Further, zero-knowledge-proofs on top of blockchains enable multi-party computation applications. Section 3 considers games under consensus protocols. Decentralization, scalability, and consensus are identified as desirable features of a blockchain. Also identified, is a tension between them - the blockchain impossibility triangle. Protocols canvased are Proof-of-work, Byzantine fault tolerance, Proof-of-stake, and Proof-of-burn. In each use case, the authors suggest finding the need for decentralization, scalability, and consensus. Then design the protocols and business models as required. Section 4 considers network security, excess concentration, energy use, adoption, and smart contracts. There is an emphasis on information sharing and gathering in decentralized systems. As well as how permissioned blockchains enable better multi-party computation and information exchange. Section 5 summarizes promising future directions for industry development.

Cong, Lin William, Ye Li, and Neng Wang. “Tokenomics: Dynamic Adoption and Valuation.” *The Review of Financial Studies* 34, no. 3 (March 2021): 1105–1155. https://doi.org/10.1093/rfs/hhaa089.

Consider a specific blockchain, with a wide range of users, where each user has diverse needs for value transfers and smart contracts. In that setting, the authors calculate a formula for the effervescent-token price,

$$P\left(A_t\right) = \frac{N\left(A_t\right) S\left(A_t\right) A_t}{M} \left(\frac{1-\alpha}{r-\mu_t^P}\right)^{\frac{1}{\alpha}}, \tag{1}$$

$$N_t = 1 - G_t\left(\underline{u}_t\right), \tag{2}$$

where M is the token supply, $S\left(A_t\right)$ measures the aggregate transaction needs, and could be approximated by daily transaction volume, μ_t^P is the endogenously determined instantaneous drift of the price diffusion, r is the required rate of return on user token holdings, and α is a constant parameter related to the flow of utility (or convenience yield) generated by user token holdings (over dt). $N\left(A_t\right)$ is the platform user base and could be approximated by daily active addresses. Decreasing marginal utility is captured by a constant $\alpha \in (0,1)$. Finally, A_t evolves according to a geometric Brownian motion stochastic differential equation with constant instantaneous drift, μ^A, and instantaneous diffusion, σ^A, $G_t(\dots)$ is a cumulative probability function, and $\underline{u}_t$ is the marginal user platform adoption threshold,

$$\underline{u}_t = -\ln(N_t) + \ln\left(\frac{\phi}{A_t\alpha}\right) - \left(\frac{1-\alpha}{\alpha}\right)\ln\left(\frac{1-\alpha}{r-\mu_t^P}\right), \tag{3}$$

where ϕ is the cost of joining the platform. When platform productivity, A_t, is sufficiently high, all agents participate with probability one at all times, $s \geq t$, such that

$$\overline{P}\left(A_t\right) = \frac{\overline{S}A_t}{M}\left(\frac{1-\alpha}{r-\mu^A}\right)^{\frac{1}{\alpha}} \tag{4}$$

where $\overline{S}$ is the aggregate transaction needs of all agents. This model shows that token prices can elevate, crash then stabilize. The authors describe this as "bubbly". But, the term "bubble", in financial economics, has a meaning different from that here. Hence, I use the term "effervescent". This forecloses the conclusion that such prices reflect a deviation from the fundamentals. The authors show that when endogenous adoption drives the correlation between the stochastic discount factor and token return, then, these "effervescent" price dynamics in a rational expectations model are a reflection of the fundamentals. And not a deviation. The authors build a dynamic feedback loop between user adoption and token prices. Here token prices reflect expectations about the future growth of platform use. They use a framework that highlights user heterogeneity. Then study the simultaneous calculation of user adoption and token valuation. In this setting, tokens are valuable because they allow users to make transactions on the digital platform. This makes the token a hybrid of money and investable assets. Users make a two-step decision. First, decide whether to become a platform user by paying a participation cost. And, if so, decide the real token balance. The token market clears by equating user demand and fixed supply. Owning tokens incurs a carrying cost, the return lost by not investing in other financial assets. This cost is partly offset by the expected token price appreciation. Some assumed and derived properties of the platform are:

1. Fixed token supply.
2. Monetary neutrality ensures that the token market is always stable.
3. There are network effects for the platform from user adoption.
4. Users are heterogeneous.
5. User utility from platform activities makes up only a small part of their total utility.
6. A token payment is for specific economic transactions (i.e. the token is not a medium for generic payments).

The model contains financial and real sectors. The financial sector operates through the endogenous determination of token prices. The real sector manifests itself in user adoption. A tokenless model is analogous to the standard money models of holdings. The model is solved via the change-of-measure technique (see Darrell Duffie, *Dynamic Asset Pricing Theory: Third Edition*, Princeton Series in Finance (Princeton University Press, 2001)). The user network effect and intertemporal feedback results apply to platforms owned by trusted third parties, permissioned blockchains, and permissionless blockchains. Key platform properties that flow from the assumptions and analysis are as follows. Token value depends on platform productivity. User adoption follows a log-Normal S-curve. And platform adoption dynamics influence token asset pricing dynamics. Given a tokenless platform with the same productivity as a tokenized platform, but uses a numeraire good for payments. Then the user base of the tokenized platform is larger, and more stable, than the tokenless platform. Various figures illustrate the model dynamics and parameters. Eight (8) Propositions are proved in the course of developing the models.

Cong, Lin William, Ye Li, and Neng Wang. "Token-based platform finance." *Journal of Financial Economics* 144, no. 3 (2022): 972–991. https://doi.org/10.1016/j.jfineco.2021.10.

Consider three types of users in a dynamic continuous-time economy. 1) Platform owners. 2) Contributors (miners/validators, distributed-application developers, etc.). And 3) Platform users. With a generic consumption good as the numeraire, and an optimal token supply, the equilibrium price for a dampened token is, in general,

$$P_t = \frac{N_t^{\gamma} U_t A_t}{M_t} \left(\frac{1-\alpha}{r-\mu_t^P} \right)^{\frac{1}{\alpha}}, \tag{5}$$

$$N_t = 1 - G_t\left(\underline{u}_t\right), \tag{6}$$

where A_t is platform productivity (synonymous with quality), U_t is the transaction need aggregated over participating users, N_t is the endogenously determined platform user base, M_t is the total amount of circulating tokens, μ_t^P is the endogenously determined instantaneous drift of the price diffusion, r is the required rate of return on user token holdings, and α is a constant parameter related to the flow of utility (or convenience yield) generated by user token holdings (over dt), $G_t(\dots)$ is a cumulative probability function, and $\underline{\mathrm{u}}_t$ is the marginal user platform adoption threshold,

$$\underline{\mathrm{u}}_t = \frac{\phi}{N_t^{\gamma} A_t \alpha} \left(\frac{r-\mu_t^p}{1-\alpha} \right)^{\frac{1+\alpha}{\alpha}}, \tag{7}$$

where ϕ is the cost of joining the platform, $\gamma \in (0,1)$ relates to the network effect of user adoption (when there is no network effect $\gamma = 0$). When parameterized explicit expressions can be obtained for N_t, U_t, and P_t, regardless of the consensus protocol and level of decentralization. Compared to the authors' other work, annotated herein, in this paper, both A_t and the token supply M_t are endogenous. In fact, stabilized token prices result from the dynamic, optimal payout and buyback decisions of the platform owners. To illustrate the size of this effect, a 200% per annum volatility of productivity yields token price volatility below 0.15% per annum. A platform is a currency area (economy) where a unique set of economic activities take place. Its tokens derive value by facilitating associated transactions. Stylized features of the economy are:

1. Tokens are monetary assets.
2. Current platform productivity and normalized token supply are the key state variables.
3. Platform transactions are in native tokens.
4. The marginal value of extra platform productivity is positive.
5. The marginal value of the extra token supply is negative.

The authors analyze the optimal token supply. Explore the dynamics of platform investment and financing. And the conflict of interest between the entrepreneur and users. They show a blockchain can allow a commitment not to expropriate value through excessive seignorage. The authors offer a corporate finance perspective of protocol design. It connects work on platform economics to work on the role of financial slack and issuance costs in setting capital structure. Platform token supply management involves investment, payout, and buyback decisions. And they analyze such platform token supply management, instead of cash management. When investment induces user network effects, the token price varies as users respond to supply variations. This perspective applies to both traditional and blockchain-based platforms. Token issuances or buy-backs finance the platform owner's investment in productivity. In return for tokens, contributors commit effort and resources to improve platform productivity. Users desire tokens to pay on the platform (convenience yield) and they buy tokens from contributors. User holdings expose them to the fluctuation of token price. This leads to an intertemporal complementarity that amplifies the effects of platform productivity changes on user adoption. The amount of resources the platform can raise by issuing tokens depends on the token price. User token demand and platform owner token supply set the token price. Furthermore, the optimal token supply strategy stabilizes the token price. Some insights that result from the model are

1. Tokens are akin to durable goods but defy Coase's conjecture (see Ronald H Coase, "Durability and Monopoly," *Journal of Law and Economics* 15, no. 1 (April 1972): 143–149, https://doi.org/10.1086/466731)
2. Underinvestment arises from the conflict of interest between the entrepreneur and platform users. That is, an entrepreneurial time inconsistency problem arises. This is when the optimal ex-ante level of investment is suboptimal ex-post.
3. Blockchain enables commitment to predetermined rules of investment. This can add value by addressing the entrepreneurial time inconsistency problem.
4. Platforms with endogenous productivity growth have stable tokens.

Five (5) Propositions are proved in the course of developing the models.

Sockin, Michael, and Wei Xiong. "Decentralization through Tokenization." *The Journal of Finance* 78, no. 1 (2023): 247–299. https://doi.org/10.1111/jofi.13192.

Consider an online platform that facilitates bilateral transactions among a pool of users. A utility token entitles holders to services but not the cash flows of the platform. To take part in the platform requires only one token. Hence, because they only enable transactions, there is no incentive to hoard tokens. From this, the authors show, the introduction of non-users produces conflicts that are difficult to resolve. Introduce one of two non-user groups: equity holders or miners/validators. Now, with miners/validators or equity holders, the platform can break down. This means there are no users participating in the platform. The authors explore the conditions under which such breakable tokens deteriorate. The model is a rational expectations cutoff equilibrium in which the utility token price is

$$P = \exp\left((1-\eta_c)\tau_\varepsilon^{-1/2} z^T + A + \frac{1}{2}\eta_c^2\tau_\varepsilon^{-1}\right)\Phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^T\right) - \kappa, \tag{8}$$

and

$$z^T = \sqrt{\tau_\varepsilon}(\hat{A}^T - A) \tag{9}$$

where τ_ε is the dispersion term in a random walk model of user endowments, $\eta_c \in (0,1)$ represents the weight in a Cobb-Douglas utility function on his consumption of his trading partner's good C_j, and $1-\eta_c$ is the weight on the consumption of his own good C_i. A higher η_c means a stronger complementarity between the consumption of the two goods. A is the aggregate goods endowment across all users and could be approximated by total transaction fees. And $\hat{A}^T$ is the threshold value in the user's cutoff strategy for utility token purchases. When the platform has validators or miners present, strategic attacks have a known probability of success. The authors provide an expression for this probability. The authors also consider a conventional equity-based funding scheme. In this scheme, equity conveys both control and cash flow rights. They compare this equity scheme to several utility token schemes. Without investors, there are users only, and the hybrid equity-utility token price P is

$$P = \exp\left(A + \frac{1}{2}\left((1-\eta_c)^2 + \eta_c^2\right)\tau_\varepsilon^{-1}\right) - \kappa, \tag{10}$$

where κ is a user partition cost. With investors and users the hybrid equity-utility token price P is

$$P = \frac{\frac{1}{2}\delta_T U + (1 - s_I)\frac{1}{2}\delta_T U + s_I \gamma\, \Phi\big(-z_I^{ET}\big)}{n + N + \Phi\big(-z_I^{ET}\big)} - s_I \gamma + p_I^{ET}, \tag{11}$$

and

$$z_I^{ET} = \sqrt{\tau_\varepsilon}(\hat{A}_I^{ET} - A) \tag{12}$$

where $\hat{A}_I^{ET}$ is the cutoff endowment of the marginal investor, p_I^{ET} is a price discount/premium for the marginal user, δ_T is the developer profit maximizing transaction fee, U is a Cobb-Douglas utility function where users are risk-neutral with respect to the uncertainty of their wealth, N is number of tokens in the developer's retention policy, γ is harm to users from the owners subverting actions, and $s_I \in 0, 1$ is the subverting action at $t = 2$. For example, investors modify the platform and sell user data to third parties, $s_I = 1$. The investor's decision to subvert the platform is expressed via an inequality relation, which the authors derive. The aggregate endowment A is a key characteristic of the platform. A well-designed platform attracts users with strong needs to transact with each other. That is, users need to trade goods with each other, which can occur only on the platform. Together, these produce a network effect. In this telling, there are three dates. At time 0, the platform developer funds the platform. Based on a prior belief about the platform's fundamentals, they issue traditional equity or tokens. The choice of funding scheme also determines the control and ownership of the platform in the later periods. At a time of $t = 1$, potential users choose whether to join the platform, subject to a cost. Each Platform user can match with another user to make beneficial transactions at $t = 1$ and $t = 2$. These times are the short run, $t = 1$, and the long run, $t = 2$. Those who do not join at time 0 cannot take part in either round of trading. Issuing equity leads to an owner who can profit by charging transaction fees. The owner can commit to not exploiting users at time 0, and they choose to provide a subsidy at time 1 to attract the marginal user. Control of the platform allows the owner to exploit users at time 2 after the platform collects extensive data about them at time 1. Under the equity scheme, the owner can always choose, at times 2, to reverse any prior commitment. This owner-user tension is like owner-manager agency conflicts in corporate finance. Naturally, users want to find a way to bind the owner to their prior commitments. This demand for commitment motivates tokenization. With a hybrid equity-utility token or miners/validators, their cash flow rights can result in them taking control of the platform. This introduces the commitment problem, as described above. Without commitment devices, concern about the exploitation of users grows. User participation, owner profit, and social surplus are all lower, and the breakdown of the platform is more likely. Here a token is an asset that conveys a right to the services of the platform, participation in its governance, but not cash flow rights. Hence, this includes the "payment" and "consumer" ("utility") tokens in the taxonomy of Global Digital Finance (GDF). Finally, the authors show that weak platforms are more likely to adopt the utility token-based scheme. Ten (10) Propositions are proved in the course of developing the models.

Sockin, Michael, and Wei Xiong. "A Model of Cryptocurrencies." *Management Science*, Forthcoming. https://doi.org/10.1287/mnsc.2023.4756.

The authors consider an online platform made up of users, speculators, and validators. The interactions of these three groups lead to rich price dynamics. In a discrete-time setting, with infinitely many periods, overlapping generations, and fixed token supply, the rational expectations cut-off equilibrium token price is

$$P_t = \frac{1}{R} \exp \left(\frac{\sqrt{\tau_\varepsilon}}{\lambda} \left(A_t - A_t^* \right) - \frac{1}{\lambda} y_t + \frac{1}{\lambda} \zeta_t \right), \tag{13}$$

$$R = \frac{(1-\beta) U_t^*}{P_t} + \frac{E \left[P_{t+1} \mid \mathcal{I}_t \right]}{P_t} - \frac{\kappa}{P_t}, \tag{14}$$

$$U_t^* = \exp \left(A_t + \frac{1}{2}((1-\eta_c)^2 + \eta_c^2)\tau_\epsilon^{-1} \right) \Phi \left((1-\eta_c) \tau_\varepsilon^{-1/2} + \frac{A_t - A_t^*}{\tau_\varepsilon^{-1/2}} \right) \Phi \left(\eta_c \tau_\varepsilon^{-1/2} + \frac{A_t - A_t^*}{\tau_\varepsilon^{-1/2}} \right), \tag{15}$$

where $R \geq 1$ is the interest rate for the holding period, and U_t^* is the total transaction surplus on the platform. A_t is the aggregate endowment, and is a key characteristic of the platform, A_t^* is the marginal users' participation threshold that solves a fixed-point condition, τ_ε is a measure of endowment dispersion among users. $\lambda > 0$ parameterizes speculator short-selling cut-off policies, ζ_t is speculator sentiment, and y_t is token supply. The structure of public information for all users is $\mathcal{I}_t = \{A_t, y_t, Q_t, \zeta_t\}$; the demand fundamental, time-varying token supply, user optimism, and speculator sentiment. Cryptocurrency returns, R, have three components: 1) a convenience yield of the marginal user (dividend), 2) capital gain, and 3) user participation cost compensation. Where $\beta > 0$ is the fraction of the utility surplus paid as a platform service fee, $E \left[P_{t+1} \mid \mathcal{I}_t \right]$ is the token price expected in the next period, and κ is a user partition cost. For the aggregate transaction surplus, $\eta_c \in (0, 1)$ represents the weight in the Cobb-Douglas utility function on a user's consumption of her trading partner's good, and $1 - \eta_c$ is the weight on the consumption of her own good. A higher η_c indicates a stronger complementarity between the consumption of the goods obtained from trading. Here tokens ease transactions between platform users of certain goods or services. That is, these tokens are not a general means of exchange. Also, the authors focus on utility token price dynamics and platform stability. Nonetheless, their results also apply to coins and altcoins. In particular, the role of coins and altcoins in 1) Funding digital platforms. 2) Serving as investment assets for speculators. And 3) decentralized consensus protocols. This model shows a platform is fragile whenever there is a fixed token supply, a user network effect, and a token price that is not neutral. Retradability, elastic issuance, and user optimism all mitigate this instability. Reductions in the token's expected retrade value exacerbated the instability. Speculators crowding out users reduces the token retrade value. Users' expecting strategic miner attacks also reduce the token retrade value. Hence, while miner attacks do not lead the platform to fail, expected losses from miner attacks can increase platform fragility. This is especially true when the mining cost is high. In this model, blockchain speculation differs from other assets, such as stocks and commodities. In stocks and commodities, speculation can increase price volatility. But for blockchains, speculation can lead to price and demand collapsing to zero. The authors explore design choices that may make a platform more robust to breaking down. Token retradability is a powerful dual-edged tool for enhancing or harming platform performance. Token retradability enhances performance when it capitalizes on user optimism. Yet, this effect of user optimism declines as the platform matures. Given otherwise identical platforms. This suggests a large market value platform might be more fragile than a small market value platform. And thus, larger platforms may have more pronounced price volatility than smaller, identical, platforms. Token retradability harms performance when it incentivizes speculator enthusiasm. Speculation acts as a tax on user participation and exacerbates the platform's instability. Having outsiders with a conflict of interest with users increases cryptocurrency platform instability. By introducing mining and strategic attacks, the miners' common mining efficiency ξ_t is an extra state variable. Here, efficiency is the inverse of the miner's cost of mining. Strategic attacks occur when either A_t or ξ_t falls below a critical boundary. This boundary reveals it may be possible for both a no-attack equilibrium and an attack equilibrium to be self-fulfilling. Furthermore, expecting strategic attacks leads to an adverse feedback loop. This feedback loop is novel to decentralized cryptocurrency platforms. These insights are valid for other types of attacks. For example, a selfish mining attack. They also apply to other consensus protocols, such as proof of stake, if the interests of validators conflict with those of users. Several empirical implications for cryptocurrency return patterns flow from this model, both time-series patterns (e.g., momentum, reversal, life-cycle effects, relation to investor attention, chance of strategic attacks) and cross-sectional patterns (e.g., size effect). Five (5) propositions are proved in the course of the model development.

Rogoff, Kenneth, and Yang You. "Redeemable Platform Currencies." *The Review of Economic Studies* 90, no. 2 (May 2022): 975–1008. https://doi.org/10.1093/restud/rdac028.

> *. . . our read of the centuries-old history of money is that the government may initially allow or even foster private innovation in transaction technology, but eventually the government regulates and appropriates.*

The authors consider a stylized partial equilibrium model of using redeemable tokens in place of using fiat currency. The purpose is to explore the design, features, sales, and pricing strategies for issuing these redeemable tokens. The issue strategies are an 'initial coin offering' (ICO), a 'seasoned coin offerings' (SCO), or a combination of an ICO and SCO. They start with a simple price strategy, selling all non-tradable and tradable tokens for the same price. Then they move to more sophisticated pricing strategies. Here platforms use a price menu ("buy more and save more") to sell tokens. The most general result is a separating equilibrium for the price menu with diverse consumers. Using a price menu, the platform could exploit all the potential gains from inter-temporal trade, but only if the token is non-tradable. For tradable tokens, a price menu adds nothing to the platform's options. Non-tradable ICO optimal issuance: Assume the platform sets the issue quantity M, then the issue price is

$$P_{I,N} = \left[\frac{\beta p}{1-\beta(1-p)}\right]^{M} \tag{16}$$

where β is the users' time discount factor, and p is the probability the consumer demands one unit of the platform commodity. Let $R_{I,N}$ be the total revenue from a non-tradable ICO,

$$R_{I,N} = \underbrace{MP_{I,N}}_{Token\ Issuance} + \underbrace{\left[\frac{\beta^{*} p}{1-\beta^{*}(1-p)}\right]^{M} \frac{\beta^{*} p}{1-\beta^{*}}}_{Fiat\ Money}. \tag{17}$$

where β^{*} is the platform time discount factor, and $\beta^{*} < \beta$ reflects the assumption the platform has access to outside investment opportunities which are better than those the user has access to. The authors provide the two necessary and sufficient conditions to find the unique revenue-maximizing issuance quantity, M. Additional models considered are: 1) Tradable ICO, 2) Non-tradable ICO with price discrimination 3) Non-tradable ICO+SCO, 4) Tradable ICO+SCO, 5) Non-tradable ICO without price discrimination, 6) Tradable ICO with or without price discrimination, 6) Tradable/Non-tradable ICO+SCO without price discrimination. Finally, a platform chooses the pooling equilibrium if $\beta^{*} < \beta = 1$. Otherwise, the platform chooses a separating equilibrium if the platform can gain sufficiently large profit from high-frequency consumers. The optimal issuance policy (M_L, M_H, P_L, P_H) can be solved as a revenue maximization problem with incentive constraints and participation constraints.

The optimal token prices for frequent, P_H, and infrequent, P_L, consumers are

$$P_H = M_H^{-1}\left(M_L P_L + \sum_{i=M_L+1}^{M_H}\left(\frac{\beta p_H}{1-\beta(1-p_H)}\right)^i\right), \tag{18}$$

$$P_L = M_L^{-1}\left(\sum_{i=1}^{M_L}\left(\frac{\beta p_L}{1-\beta(1-p_L)}\right)^i\right). \tag{19}$$

Appendix 2.4 details how to find the unique revenue-maximizing issuance quantities, M_L and M_H. A central result is that platforms generally earn higher revenues via non-tradable tokens. That is, platforms should only create a new generic currency when off-platform use provides large, unique, benefits. The gains from trade appear when the platform has a higher rate of return on its outside investments than small retail consumers. Consumers' share of the gains from trade tends to be higher with traded tokens. Tradable tokens limit the ability of a platform to offer price-quantity tradeoffs or memory features. In general, SCOs are possible, and this leads to a time-consistency problem for issuers and users. In principle, there are three issuance strategies:

1. **A no information policy:** All consumers receive the same price in every SCO regardless of their purchasing and spending history.
2. **A history-dependent policy:** The platform charges an SCO price that is a function of the consumer's entire history with the platform.
3. **A Markov policy:** Issuance depends only on the consumer's current account information (holdings of tokens)

Policies (2) and (3) are 'un-money like' but reflect the richer possibilities that digital currencies offer. But, the "no information" policy is most likely to be regulatory-compliant and least likely to raise privacy concerns. Potential future sales affect the price of an initial ICO. Knowing how helps show how a lack of credibility might affect initial issuance and price. If one uses SCOs to maintain a constant supply of tokens, then the most coins a consumer will hold is one. This result is the same whether tokens are tradable or not. And in fact, the tradable and non-tradable cases become equal. The main results are about tradability versus non-tradability. And how demand for token holdings is sensitive to future issuance policy. These results appear to generalize to heterogeneous agents. Different regulatory treatment is appropriate when new assets offer functionality not currently available. Motivated by that observation, the paper presents a history of four (4) generations of redeemable platform assets.

1. Trading stamps (1930s-1980s).
2. Customer loyalty programs (1980s-current).
3. Platform cash/stored-value cards.
4. Crypto-coins/tokens.

The model used here most resembles platform cash/stored-value cards such as Amazon, Uber, and Alibaba. Nonetheless, some insights are relevant to the newer 4^{th} generation of redeemable assets. Consumer motivation arises, in general, from several sources. For example, tokens that offer a discount, pay interest or provide a money-like convenience. A critical issue is how a consumer values a credit that pays for her M^{th} unit of platform good. The consumer value depends on the exact timing of the consumer's needs for the platform good. Platform motivation arises, in general, from several sources. For example, tokens that introduce low-interest consumers, reduce transaction costs and strengthen consumer loyalty. For a given quantity, non-tradable tokens often sell at higher prices and yield higher profits to the platform. One possible reason is reminiscent of Coase's conjecture (see Ronald H Coase, "Durability and Monopoly," *Journal of Law and Economics* 15, no. 1 (April 1972): 143–149, https://doi.org/10.1086/466731). The reason is that tradability forces the platform to compete with future resale market values. These future values limit the power to charge a high price upfront. Several topics discussed in more detail are consumer surplus, embedded memory, interest payments, runs, proportional costs, convenience yields, and platform commitments. History-dependent issuance and issuance based on current holdings is reminiscent of Narayana R. Kocherlakota, "Money Is Memory," *Journal of Economic Theory* 81, no. 2 (1998): 232–251, https://doi.org/https://doi.org/10.1006/jeth.1997.2357. Nine (9) Propositions are proved in the course of developing the models.

A Methodology

The published research annotated is six articles selected from the commercial research databases available from the State Library of New South Wales, Australia. The workflow and article counts are shown in Figure 5. This selection was arrived at in the following stages:

1. Preparation: by operationalizing the inquiry *"Refereed articles on block-chain token-economics using rational expectations equilibrium (a.k.a. no-arbitrage) arguments/analysis, ranked by journal impact factors"*;
2. Retrieval: eliminating duplicates;
3. Screening: removing false positives; ranking by journal impact factor;
4. Selection: selecting the top-6; and
5. Write-up: reviewing remaining article abstracts and substituting where judged appropriate.

The annotated bibliography component of this project is closest to a "Scoping Review",[14] and Table 1.

Each section of the report/working paper was developed using some subset of the following iterative process:[15]

1. Review reporting guidelines, best practice handbooks, and training modules [preparation stage]
2. Formulate question and decide on review type [preparation stage]
3. Search for previously published literature [preparation stage]
4. Develop and test search strategies [preparation stage]
5. Review search strategies [preparation stage]
6. Execute search [retrieval stage]
7. De-duplicate data/information [retrieval stage]
8. Screen title and abstracts [screening stage]
9. Retrieve full-text articles [retrieval stage]

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[14] Maria J. Grant and Andrew Booth, "A typology of reviews: an analysis of 14 review types and associated methodologies," *Health Information & Libraries Journal* 26, no. 2 (2009): 91–108, https://doi.org/https://doi.org/10.1111/j.1471-1842.2009.00848.x, eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1471-1842.2009.00848.x.

[15] Guy Tsafnat et al., "Systematic review automation technologies," *Systematic Reviews* 3, no. 1 (July 2014): 74, https://doi.org/10.1186/2046-4053-3-74.

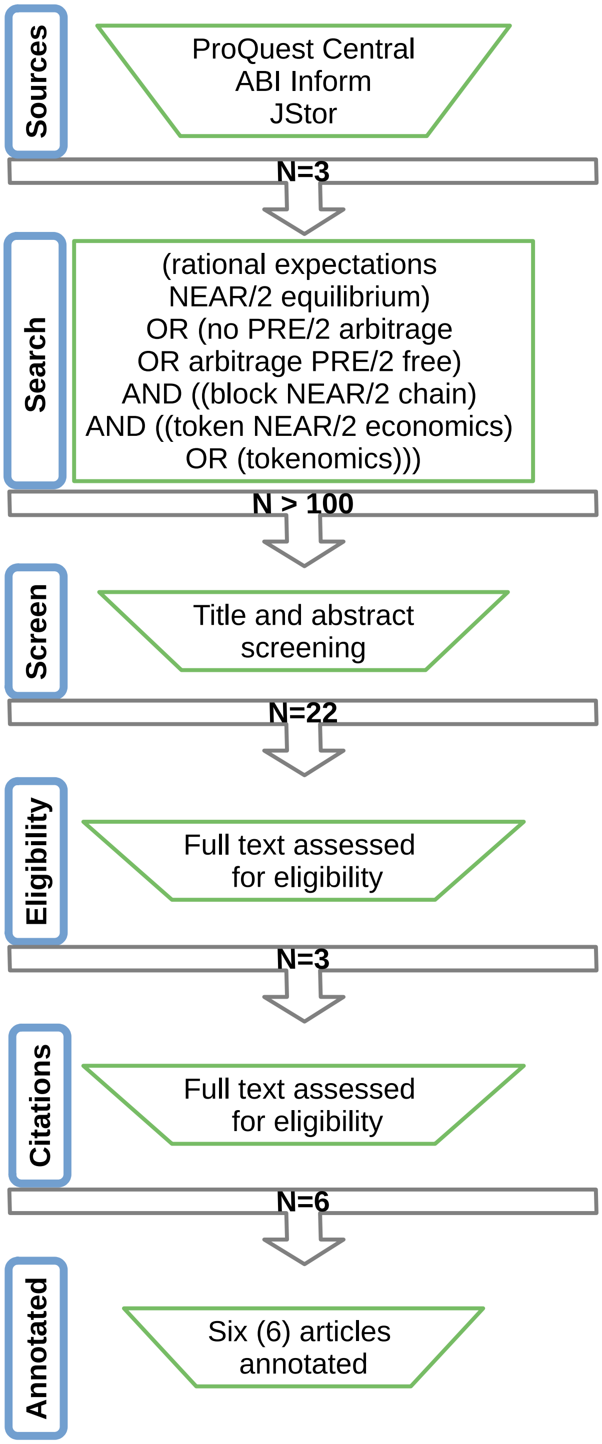

Figure 5: Flowchart of the selection process.

10. Screen articles in full-text [screening stage]

11. Search for grey literature (preprints, working papers) [retrieval stage]

12. Quality assessment and data/information extraction [synthesis stage]
13. Citation chasing [retrieval stage]
14. Update database searches [retrieval stage]
15. Synthesize data/information [synthesis stage]
16. Manuscript development [write-up stage]