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6 March 2023

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**Abstract**

This paper studies credit using a search-theoretic model with anonymity in which traders cannot reveal their true identities to the public but can create transaction accounts as identities to borrow and store their trade histories. A transaction account that is used to borrow would be excluded from future transactions as a punishment when default occurs, but a defaulter can create a new account to trade again. We show that increasing-credit-limit schemes connected to account ages, as captured by accumulated repayment records, emerges endogenously to ensure debt repayment. We extend the model to consider a situation in which a trader may create multiple accounts to borrow and default intentionally. Requiring that proof of a deterrence activity is provided when an account is created can help deter multi-account fraud and enhance the lifetime value of traders.

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[§]We thank Steve Williamson, Cyril Monnet, Guillaume Rocheteau, Michael Choi, Isaac Swift, participants in the seminar at UC Irvine, the 2020 Summer Workshop on Money, Banking, Payments and Finance, Baptist-Monash-NTU joint workshop on the digital economy for helpful comments and suggestions.

# 1 Introduction

Conventional wisdom suggests that true identities are crucial for obtaining unsecured credit. In traditional banking systems, borrowers are required to disclose their true identities, and transaction records are attached to borrowers' identities. Recent developments in digital technologies such as blockchain and cryptocurrencies allow traders to engage in financial activities while keeping their true identities hidden. Traders can freely create anonymous digital accounts as identities for transactions, and the resulting transaction records are subsequently attached to these digital accounts.[1] Although such technology can promise greater privacy, how does privacy influence traders' capacity to engage in financial activities?

The central issue is a problem of limited commitment. In the literature on unsecured credit, in order to motivate repayment of a debt, a defaulter is punished by being permanently excluded from future credit transactions, and a credit limit is imposed to ensure that borrowers would rather meet their contractual obligations than default.[2] However, if traders use digital accounts as identities to incur debt, exclusion can be imposed on only these digital accounts. An immediate observation is that if borrowers can create anonymous digital accounts for free, a constant-credit-limit scheme cannot generate an equilibrium because a borrower will have the incentive to default and create a new account to be able to borrow again. Is there a credit-limit scheme or any other mechanisms that can be applied to motivate the repayment of debt given the issues caused by anonymity?

To shed light on this question, we construct a search-theoretic model to study how credit can be generated when true identities are absent. In the model, households cannot reveal their true identities, but they can freely and without cost create transaction accounts as identities to incur debt, and their transaction records are linked to these accounts rather than to their actual identities. We call this system an *anonymous credit* system. We first study a benchmark model in which a household and a merchant meet bilaterally in each period. A period is divided into two subperiods, day and night. The merchant can produce

[1] For example, in Bitcoin, an account is an identifier of alphanumeric addresses, and an account holder can apply a cryptographic algorithm to prove the ownership of the addresses by using digital signatures. See Berentsen and Schär (2018) and Sanches (2018) for a comprehensive introduction to Bitcoin and cryptocurrencies.

[2] See Kehoe and Levine (1993) and, more recently, Kocherlakota (1996) and Kehoe and Levine (2001).

for the household during the day, and the household can produce for the merchant during the night. A transaction can be facilitated by a debt contract that specifies the merchant's day production and the household's production during the consecutive night as repayment, and the contract is determined through a simple bargaining game in which the household proposes a take-it-or-leave-it offer to the merchant. New households join the economy in each period, and existing households randomly leave the economy at the end of each period. The transaction account that a defaulter used to borrow are excluded for future transactions permanently; however, given the anonymity of account holders, a defaulter can then create a new account and mimics a new entrant to borrow again immediately following default.

Although true identities are omitted in anonymous credit systems, the availability of transaction records allows traders to store repayment records on transaction accounts. The credit limit therefore can be subject to *the number of repayments* that the account has fulfilled, and we term this the *account age*. A key insight of our analysis is that increasing-credit-limit schemes regarding the account age are generated endogenously to guarantee repayment of the debt. To observe this, let us consider a household that enters the economy and does not hold an account with a repayment record. To motivate repayment of a debt, if a household repays, the account that the household used to trade must be promised a greater value at the next period as a reward so that maintaining the current account is more profitable than defaulting and creating a new account. To generate a greater value, the credit limit of an age-one account must be greater than that of a newly created account. Furthermore, to ensure that the borrower is most likely to repay the greater amount of debt, the value provided by an age-two account must be greater than that provided by an age-one account, a situation which implies that the credit limit of an age-two account must also be greater than that of an age-one account. This iterative relationship generates increasing credit limits over the account age.

In the anonymous credit system, there exists a continuum of credit equilibria driven purely by beliefs. An equilibrium that generates a greater lifetime value for households is associated with a scheme with greater credit limits. There is an upper bound of lifetime values that can be generated by anonymous credit systems, and the upper bound represents the efficiency of the system. We show that the upper bound of lifetime values generated by an anonymous credit system is strictly smaller than that generated by a true-identity credit system wherein defaulters are excluded permanently from the credit system. The reason is that the punishment of excluding transaction accounts is less severe than that of excluding borrowers' true identities.

In reality, a household may have opportunities to engage in multiple transactions and borrow from multiple merchants. Given that creating accounts is free, a household may have the incentive to create a large number of accounts and to incur a massive amount of debt even if their ability to repay is limited; we name this activity *multi-account fraud*. Does the possibility of engaging in multi-account fraud threaten the credit trades and impair the lifetime value a household can obtain in anonymous credit systems? And, if so, are there mechanisms to incentivize repayment of debt and improve lifetime value? To answer these questions, we extend the model by assuming that other than the regular meetings described in the benchmark model, a household can, by paying an entry cost, engage in meetings in which they can consume the merchant's product at the day but cannot repay at the night. Merchants cannot distinguish whether their counterparties can repay the debt or not, so households have opportunities to conduct multi-account fraud. We characterize the bargaining under the threat of multi-account fraud as a sequential game with incomplete information. In equilibrium, a merchant will not accept offers that make a household's gain from conducting fraud greater than its entry cost, and a household never finds it optimal to propose a contract that will not be accepted. This threat generates a *no-fraud constraint* on the terms of credit contract, a circumstance which places a limit on the volume of trades to ensure that households have no incentive to conduct fraud; the smaller the entry cost, the lower the trade volume.

Although a household can create a large number of accounts, its ability to work and make payments is limited, so a repayment record can be applied to reduce the number of accounts that a household can use to borrow and can thus mitigate the multi-account fraud problem. In our model, a household can create multiple new accounts at each period, but a household can produce for only one merchant and maintain only one accounts with a repayments record. Thus, the threat of multi-account fraud vanishes in meetings wherein the household is holding an account with a record but arises only if a household is using a new account to trade, and this result generates a punishment for defaulting because a defaulter must encounter the no-fraud constraint again. Consequently, an increasing-credit-limit scheme is not the only way to motivate the repayment of debt, but the credit limit can decrease or remain constant over the account age if the punishment is sufficiently severe, that is, if the trade volume is low for the newly created account because of the sufficiently small entry cost.

Online shopping and online payment services have significantly decreased the cost of engaging in multiple transactions, and our model implies that trade volume and the efficiency of anonymous credit systems will diminish as a consequence. We further explore whether there are mechanisms that can be applied to deter multi-account fraud and further improve trade volume and efficiency of anonymous credit systems. In reality, some activities are costly to conduct and are verifiable by others, and we name these activities *deterrence activities*. Associating these activities with the creation of accounts can further increase the cost of conducting fraud and improve the trade volume. An example of such deterrence activity is proof-of-work, which is a solution to a complex hash problem, but solving the problem requires substantial time and computing power. Proof-of-work has been applied to overcome spam attacks and distributed denial of service (DDoS) attacks and to ensure the safety of cryptocurrencies such as Bitcoin.[3] By requiring a deterrence activity when an account is created, households' cost of conducting multi-account fraud increases, and this requirement relaxes the no-fraud constraint and can increase trade volume. However, requiring a deterrence activity also generates a cost to households when they create accounts for regular transactions. Our model shows that when the entry cost is sufficiently small, the marginal benefit of imposing a deterrence activity in relaxing the no-fraud constraint can dominate the cost, and, consequently, a deterrence activity can be applied to improve the upper bound of households' lifetime values in this circumstance.

[3]In Bitcoin, a record maker (referred to as a miner) is required to provide a proof-of-work when she updates the public ledger, a strategy which aims to prevent miners from conducting double-spending fraud through manipulating the public ledger.

## 1.1 Related Literature

The model with repeated borrowing and lending we apply is mostly related to Gu et al. (2013). The main difference between our model and theirs is that in theirs, traders' true identities are known, but in ours they are unknown. The literature has compared money with credit by considering that credit works in environments with true identities and record-keeping technology, and money works in environments when true identities and record-keeping technology are both absent (For example, Andolfatto (2013); Sanchez and Willimson (2010); Gu et al. (2016)). We fill the gap by considering anonymous credit, in which true identities of borrowers are unknown, but the trade record can be preserved.

Our model is closely related to the literature of counterfeiting by Nosal and Wallace (2007), Li and Rocheteau (2011), and Li et al. (2012). In the counterfeiting literature, fraud is conducted through counterfeiting assets, but in our paper, fraud is conducted through creating multiple accounts to incur debt that the borrower cannot repay. Similar to these studies in which a counterfeiting cost generates the liquidity constraint of an asset, in our paper, the cost of engaging in multi-account fraud sets an upper bound of trade, and fraud does not occur in equilibrium. The use of deterrence activities to deter multi-account fraud in our paper is closely related to the message cost in Li and Wang (2022). In their model, transactions are completed through sending messages, and payers can send a double-spending message after conducting transactions in order to divert a payment for another use. Requiring message senders to pay a message cost whenever a transaction message is sent will increase the cost of conducting double-spending fraud and extend the upper bound of the trade volume.

Our paper is also related to the recent literature on privacy in central bank digital currency (CBDC). In Keister and Sanches (2023) and Williamson (2022), CBDC as a means of payment can preserve privacy just as physical currency does and can be used in transactions wherein privacy is needed. Keister and Sanches (2023) shows that CBDC can provide greater liquidity and improve welfare, and the policy also disintermediates banks and decreases investments. In Williamson (2022), the central bank can more efficiently use safe assets to provide liquidity than private banks do, and by issuing CBDC, the central bank can compete with private banks and improve welfare. Ahnert et al. (2022) study how banks use the information contained in deposit flows to extract rents from merchants and determine that CBDC generates a more efficient allocation than bank deposits by providing greater privacy to agents. While the literature focuses on CBDC as a payment system, our model focuses on credit and contributes to the literature by considering how digital accounts and transaction records can be applied to incur debt while maintaining the privacy of traders.

# 2 Benchmark Model

Time is discrete and continues forever, and each period contains two subperiods: day and night. There are two types of agents: households and merchants. In each period, a unit measure of households and a unit measure of merchants are in the economy, and there are a large number of households and merchants outside the economy. A household and a merchant in the economy meet bilaterally and randomly at the beginning of each period, and they separate at the end of the period. There are two perishable goods, the day good and the night good; goods are perishable across subperiods. Merchants produce the day good during the day and consume the night good during the night. Conversely, households consume the day good during the day and produce the night good during the night.

A household's instantaneous utility in a bilateral meeting is

$$\mathbb{E}\left[u(x) - z\right],$$

where $x$ is the household's consumption of the day good and $z$ is its production of the night good. We assume that $u(x)$ is twice continuously differentiable with $u(0) = 0$, $u'(x) > 0$, $\lim_{x\to 0} u'(x) = \infty$, $\lim_{x\to\infty} u'(x) = 0$, and $u''(x) < 0$. At the beginning of the night, a household receives a shock that determines whether it will leave the economy or not at the end of the night. With probability $\delta \in (0,1)$, a household will leave the economy at the end of the night, and measure $\delta$ of new households enter the economy at the beginning of the next day. A household discounts utility across periods by the rate $\beta \in (0,1)$.

A merchant's instantaneous expected utility in a bilateral meeting is

$$\mathbb{E}\left[-x+z\right],$$

where $x$ is the merchant's production of the day good and $z$ is her consumption of the night good. A merchant stays in the economy for only one period. A merchant in the economy leaves at the end of the night, and measure 1 of new merchants enters the economy at the beginning of the next day. Households and merchants that leave the economy never enter again.

Trade in a bilateral meeting is facilitated by a debt contract assigning the merchant's transfer of the day good, $x$, to the household, and the household's repayment in terms of the night good, $z$. The terms of a contract, $(x, z)$, are determined through a simple bargaining game in which the household proposes a take-it-or-leave-it offer and the merchant decides whether to accept or reject it. We assume that actions including the offers and the transfers of goods are fully observable by agents that are inside or outside the economy.

Households are anonymous in the sense that they cannot reveal their true identities to others, but they can create digital accounts as identities to trade and preserve transaction records. Although households' true identities are unknown, defaulters' digital accounts can be excluded as a punishment to motivate the payment of debt.[4] We confine our analysis to the mechanism wherein the repayment record of an account must be continuous, that is, the repayments of an account must be made in every period since the account has been created. Under this mechanism, a household can hold up to one account with a repayment record because it can produce for only one merchant. We denote by $s$ the number of repayments an account has made, and refer to $s$ as the *account age*. A meeting in which the household is holding an age $s$ account is called an "age $s$ meeting." The terms of trade in a meeting can be subject to the account age, and a household's expected value can also depend on the age of the account it is holding. Let $x_s$ and $z_s$ denote the day and night productions in an age $s$ meeting, and $V_s$ the expected value of a household that is holding an age $s$ account.

[4] A global punishment cannot generate an equilibrium with credit trades because households that will leave must decline to repay at night, so the only equilibrium under global punishments will be a no-trade equilibrium.

We name $V_s$ the *account value* of the age $s$ account.

## 2.1 Anonymous Credit Equilibria

We solve the bargaining game backward. Consider a household holding an age $s$ account. At the beginning of the night, the shock is realized and the household knows whether it will leave the economy or not, and it decides whether to repay the debt or default. With probability $1-\delta$, the household will not leave. In this case, if the household repays the debt, it incurs $z_s$ units of disutility generated by producing the night good, but it can proceed and become a holder of an account with age $s+1$, and this consequence generates the expected future value $V_{s+1}$ to the household. If a household does not repay its debt, the account that is used to raise the debt will be banned from future transactions; however, the household can create a new account with no repayment record to trade again, so its value will be $V_0$. We impose a tie-break rule such that a household repays the debt if it is indifferent to doing so or not. Consequently, a household repays the debt at night if and only if $-z_s + \beta V_{s+1} \geq \beta V_0$. Rearranging the inequality, we obtain the following limit for the night production:

$$z_s \leq \beta(V_{s+1} - V_0). \tag{1}$$

With probability $\delta$, a household will leave at the end of the period. In this case, the household will not repay the debt because its future value will be 0 regardless of its actions at night. Thus, given a contract satisfying (1), the household will repay the debt with probability $1-\delta$.

Now we consider the merchant's decision. If a household proposes an offer that satisfies (1), the merchant's expected payoff will be equal to $-x_s + (1-\delta)\, z_s$ if she accepts the offer, which comprises the disutility of producing the day good, $-x_s$, plus her expected gain from the household's debt repayment, $(1-\delta)z_s$. If the merchant rejects the offer, her value will be 0. We assume a tie-break rule that a merchant accepts an offer if her expected payoff is greater than or equal to zero; thus, a merchant accepts an offer $(x_s, z_s)$ if and only if (1) and the following rationality condition holds:

$$x_s \leq (1-\delta)z_s. \tag{2}$$

Given the merchant's response, if a household proposes a contract satisfying (1) and (2), its expected utility in the meeting will be equal to $u(x_s) - (1-\delta)z_s$, which comprises the utility gained from consuming the day production, $u(x_s)$, minus the expected disutility generated by repaying the debt, $-(1-\delta)z_s$. In an age $s$ meeting, a household makes an offer that maximizes its expected utility from contracts satisfying (1) and (2), so the equilibrium debt contract solves

$$\max_{x_s, z_s \in \mathbb{R}_+} \quad u(x_s) - (1-\delta)z_s \tag{P1}$$

$$\text{subject to} \quad \begin{cases} x_s \leq (1-\delta)z_s, \\ z_s \leq \beta(V_{s+1} - V_0). \end{cases}$$

Finally, given the terms of trade, $(x_s, z_s)$, a household's expected utility in an age $s$ meeting is equal to $u(x_s) - (1-\delta)z_s$, so the value of an age $s$ account can be characterized by

$$V_s = \sum_{j=0}^{\infty}(1-\delta)^j \beta^j \left[u(x_{s+j}) - (1-\delta)z_{s+j}\right] \text{ for all } s \geq 0, \tag{3}$$

where $(1-\delta)\beta$ is the effective discount factor.

**Definition 1** *An anonymous credit equilibrium is given by nonnegative sequences* $\{x_s, z_s, V_s\}_{s=0}^{\infty}$ *such that for* $s \geq 0$

*i. Given* $\{V_s\}_{s=0}^{\infty}$*, for* $s \geq 0$*, the debt contract* $(x_s, z_s)$ *solves (P1)*

*ii. Given* $\{x_s, z_s\}_{s=0}^{\infty}$*, for* $s \geq 0$*, the account value* $V_s$ *satisfies (3)*

Note that given $\{x_s, z_s\}_{s=0}^{\infty}$, if $\{V_s\}_{s=0}^{\infty}$ satisfies (3), the following recursive function of account values must also hold for all $s \geq 0$:

$$V_s = u(x_s) - (1-\delta)z_s + (1-\delta)\beta V_{s+1}. \tag{4}$$

Moreover, the recursive function (4) also implies (3) if the following boundedness condition holds:[5]

$$\lim_{S\to\infty}(1-\delta)^S\beta^S V_S = 0. \tag{5}$$

In the following analysis, we first solve for the paths that satisfy the optimization problem (P1) and the recursive form of account values (4), and we exclude paths that violate (5) to obtain the credit equilibria.

## 2.2 Dynamics of Anonymous Credit Equilibria

In this subsection, we solve for the dynamics of anonymous credit equilibria. We first solve the optimization problem (P1). Note that the rationality constraint, (2), must be binding. Combining the binding rationality constraint and the repayment constraint, (1), we obtain an upper bound on the day production, $(1-\delta)\beta(V_{s+1}-V_0)$, which we call the *credit limit* for an age $s$ account. Thus, the equilibrium debt contract, $(x_s, z_s)$, satisfies

$$\begin{aligned} x_s &= \min\{\tilde{x}, (1-\delta)\beta(V_{s+1}-V_0)\}, & (6)\\ (1-\delta)z_s &= x_s, & (7)\end{aligned}$$

where $\tilde{x}$ satisfies $u'(\tilde{x}) = 1$ and is the unconstrained optimal day production. By combining (4), (6), and (7), we define the forward-looking function of account values by

$$\begin{aligned} &f(V_{s+1}, V_0) \equiv u(x_s) - x_s + (1-\delta)\beta V_{s+1},\\ &\text{where } x_s = \min\{\tilde{x}, (1-\delta)\beta\,(V_{s+1}-V_0)\}. \end{aligned} \tag{8}$$

Then the relationship between $V_s$ and $V_{s+1}$ can be characterized by

$$V_s = f(V_{s+1}, V_0). \tag{9}$$

We observe from (9) that an increase in $V_{s+1}$ impacts $f(V_{s+1}, V_0)$ through two channels. First, a greater $V_{s+1}$ results in a strictly greater future value, $(1-\delta)\beta V_{s+1}$. Second, a greater

[5] See Appendix A.6 for the proof.

$V_{s+1}$ results in a greater credit limit, $(1-\delta)\beta(V_{s+1}-V_0)$, and this effect generates a greater volume of trade if the efficient quantity, $\tilde{x}$, is not achieved. Consequently, $f(V_{s+1}, V_0)$ is strictly increasing and concave in $V_{s+1}$.

Given $V_0$, let $g(V_s, V_0)$ denote the inverse function of $f(V_{s+1}, V_0)$, that is

$$g(V_s, V_0) \equiv \{V_{s+1} : f(V_{s+1}, V_0) = V_s\}. \tag{10}$$

The law of motion of account values is depicted by $V_{s+1} = g(V_s, V_0)$, and we illustrate the dynamic paths of account values using $g(V_s, V_0)$ in Figure 1. There exist multiple equilibrium paths $\{V_s\}_{s=0}^{\infty}$ that are driven purely by beliefs, and each equilibrium path is associated with a unique age 0 account value, $V_0$. We observe that a path that promises a greater value to an age 0 account (a greater $V_0$) is associated with a greater $V_s$ for all $s \geq 1$. Moreover, for $V_0 > 0$ (Figure 1 (II), (III), (IV)), the equilibrium account values are increasing over the account age, meaning that the credit limits are also increasing over the account age. For $V_0 = 0$ (Figure 1 (I)), the equilibrium degenerates, that is, $V_s = 0$ for all $s$, so the credit limits are equal to zero for all $s \geq 0$.

We formalize the above results regarding credit limits and analyze the underlying mechanisms. Given $V_0$, let $\tilde{v}_s(V_0)$ denote the value of an age $s$ account, then

$$\tilde{v}_s(V_0) = \begin{cases} V_0 & \text{for} \quad s = 0 \\ g(\tilde{v}_{s-1}(V_0), V_0) & \phantom{\text{for}} \quad s > 0 \end{cases}. \tag{11}$$

Let $\tilde{\phi}_s(V_0)$ denote the credit limits; then

$$\tilde{\phi}_s(V_0) = (1-\delta)\beta\left[\tilde{v}_{s+1}(V_0) - V_0\right].$$

**Proposition 1** *Account values and credit limits are increasing over the account age.*

*i. For $V_0 > 0$, we have $\tilde{v}_{s+1}(V_0) > \tilde{v}_s(V_0)$ and $\tilde{\phi}_{s+1}(V_0) > \tilde{\phi}_s(V_0)$ for all $s \geq 0$.*

*ii. For $V_0 = 0$, we have $\tilde{v}_s(V_0) = 0$ and $\tilde{\phi}_s(V_0) = 0$ for all $s \geq 0$.*

First, we prove case i of Proposition 1. We first show that if $V_0 > 0$, then the value of the age 1 account must be greater than that of the age 0 account; that is, $\tilde{v}_1(V_0) > V_0$. We prove this by contradiction. Suppose that $\tilde{v}_1(V_0) = V_0$ . By (9), $\tilde{v}_1(V_0) = V_0$ implies that the credit limit is 0 and therefore the trade surplus is 0. If $x_s = 0$, then $V_0 = (1-\delta)\beta V_1$, and because $V_0 > 0$, this result implies that $\tilde{v}_1(V_0) > V_0$, a contradiction. Moreover, by (9), if $V_1 > V_0$, either $u(x_1) - x_1 > u(x_0) - x_0$ or $(1-\delta)\beta V_2 > (1-\delta)\beta V_1$ must hold. In either case, we must have $V_2 > V_1$, and this iterative relationship generates the result that $\tilde{v}_{s+1}(V_0) > \tilde{v}_s(V_0)$ for all $s$. Thus, the credit limit must also satisfy $\tilde{\phi}_{s+1}(V_0) > \tilde{\phi}_s(V_0)$ for all $s \geq 0$. In case ii, if $V_0 = 0$, then $x_0 = 0$ and $V_1 = 0$; and $V_1 = 0$ also implies that $x_1 = 0$ and $V_2 = 0$. This iterative relationship generates the result that $V_s = 0$ and $\tilde{\phi}_s(V_0)$ for all $s \geq 0$.

From Figure 1 we also observe that a greater $V_0$ is associated with a greater difference between $\tilde{v}_s(V_0)$ and $V_0$ for $s \geq 1$ and, therefore, greater credit limits, $\tilde{\phi}_s(V_0)$. Moreover, when $V_0$ converges to zero, the credit limits converge to zero, and when $V_0$ goes to infinite, the credit limits also go to infinite. We formalize these properties in the following proposition.

**Proposition 2** *The credit limits, $\tilde{\phi}_s(V_0)$, are positively related to $V_0$*

i. The credit limits increase as $V_0$ increases. That is, $d\tilde{\phi}_s/dV_0(V_0) > 0$ for all $s \geq 0$.

ii. $\tilde{\phi}_s(V_0) \to 0$ as $V_0 \to 0$. Moreover, $\tilde{\phi}_s(V_0) \to \infty$ as $V_0 \to \infty$.

Having characterized all paths that solve (P1) and (4), we show that if $V_0$ is too large, the path will diverge to infinite and violates the boundedness condition (5). That is, there is an upper bound, $\bar{V}$, on the age 0 account values such that for $V_0 > \bar{V}$, credit equilibria do not exist. From Figure 1 (I) and (II) we see that when $V_0$ is small, there is at least one intersection between $g(V_s, V_0)$ and the 45-degree line. For a given $V_0$, let $\tilde{v}^h(V_0)$ and $\tilde{v}^l(V_0)$ denote the higher and lower intersections, respectively, between $g(V_s, V_0)$ and the 45-degree line, and thus we have $\tilde{v}^h(V_0) > \tilde{v}^l(V_0) \geq V_0$. As $V_0$ increases, the diagram of $g(V_s, V_0)$ shifts toward the upper-right along with the line $V_s = (1-\delta)\beta V_{s+1}$, and this shift results in a decrease in $\tilde{v}^h(V_0)$ and an increase in $\tilde{v}^l(V_0)$. We observe from Figure 1 (III) that the upper bound of the age 0 account values, $\bar{V}$, is equal to the value of $V_0$ that makes the function $g(V_s, V_0)$ tangent to the 45-degree line. For $V_0 \leq \bar{V}$, the law of motion function $g(V_s, V_0)$ and the 45-degree line intersect, as shown in Figure 1 (I), (II), (III), so the dynamic path of account values converges to $\tilde{v}^l(V_0)$ and generates a credit equilibrium. For $V_0 > \bar{V}$, there is no intersection, as shown in Figure 1 (IV), so the dynamic path of the account value, $\tilde{v}_s(V_0)$, goes to infinite as $s$ goes to infinite. The divergent paths of account values violate the boundedness condition (5) and, therefore, do not generate a credit equilibrium.[6]

## 2.3 Efficiency

In this section, we compare the upper bound of lifetime value generated by anonymous credit systems and that generated by true-identity credit systems. In anonymous credit equilibria, a household's expected lifetime value is equal to the value of an age 0 account, $V_0$, because a household must repay the debt until it leaves the economy. Recall that $\bar{V}$ is the upper bound of age 0 account values for a credit equilibrium to exist, so $\bar{V}$ is also the upper bound of households' expected lifetime value that can be generated by anonymous credit equilibria. We focus on households' expected lifetime values because merchants' expected lifetime values must be zero under the take-it-or-leave-it bargaining procedure.

The true-identity credit system considered here is one wherein households borrow with their true identities and exclusion of defaulters from the economy is feasible. Let $t$ denote the number of repayments that a household has made, and we name $t$ the age of the household. The expected value of the household and the terms of trade in a bilateral meeting can be subject to the age of the household, and we denote by $V_t$ the expected value of an age $t$ household and by $(x_t, z_t)$ the debt contract proposed by the household in bilateral meetings. The key difference between true-identity credit systems and anonymous credit systems is that in true-identity systems, if a household defaults, it will be permanently excluded from future transactions, so the expected utility of a defaulting household is 0. Thus, a household that will stay repays the debt if and only if $-z_t + \beta V_{t+1} \geq 0$, so the repayment constraint is $z_t \leq \beta V_{t+1}$. Together with the merchant's rationality constraint, $x_t \leq (1-\delta)z_t$, the

[6] See Appendix A.7 for the proof of excluding a divergent path from the credit equilibrium.

equilibrium debt contract, $(x_t, z_t)$, solves

$$\max_{x_t, z_t \in \mathbb{R}_+} \quad u(x_t) - (1-\delta) z_t \tag{12}$$
$$\text{subject to} \quad \begin{cases} x_t \leq (1-\delta) z_t \\ z_t \leq \beta V_{t+1} \end{cases}.$$

Households' values in the true-identity credit equilibrium follows

$$V_t = \sum_{j=0}^{\infty} (1-\delta)^j \beta^j \left[ u(x_{t+j}) - (1-\delta) z_{t+j} \right] \text{ for all } t \geq 0. \tag{13}$$

**Definition 2** *A true-identity credit equilibrium is given by nonnegative sequences* $\{x_t, z_t, V_t\}_{t=0}^{\infty}$ *such that*

*i. given* $\{V_t\}_{t=0}^{\infty}$*, the debt contract* $(x_t, z_t)$ *solves (12) for* $t \geq 0$;

*ii. given* $\{x_t, z_t\}_{s=0}^{\infty}$*, for* $t \geq 0$*, the account value* $V_t$ *satisfies (3).*

A detailed analysis of true-identity credit equilibria is in the Appendix. In a true-identity credit system, there are also multiple equilibrium paths generated by beliefs, and each equilibrium path is associated with a certain value of $V_0$. Note that $V_0$ represents a household's lifetime value in the true-identity credit equilibrium, and there is also an upper bound for households' lifetime values. The following proposition compares the upper bound of lifetime values that can be generated by anonymous credit systems and true-identity credit systems.

**Proposition 3** *The upper bound of households' lifetime values in anonymous credit equilibria is strictly smaller than that in true-identity credit equilibria.*

Proposition 3 is intuitive because in a true-identity credit equilibrium defaulters will be excluded from future transactions forever. Compared with anonymous credit equilibria, the more severe punishment generates a larger cost of default and generates higher credit limits, a situation which results in a greater lifetime value.

# 3 Extended Model of Multi-Account Fraud

To study multi-account fraud, we consider that a household comprises multiple agents, and agents replace households as participants in bilateral meetings. There are two types of agents, genuine and virtual, and a household comprises one genuine agent and $L \in \mathbb{N}$ virtual agents. The genuine agent is meant to represent a household's original identity that a household may want to maintain a good reputation and consider repaying the debt it incurs, while virtual agents represent the identities that a household uses to conduct multi-account fraud. The difference between a genuine and a virtual agent is three-fold. First, a genuine agent can produce at night, but a virtual agent cannot. Thus, only genuine agents can repay debt, and virtual agents cannot. Second, a genuine agent can propose an offer of the debt contract to the merchant matched to her for free, but a virtual agent must pay an entry cost, $c$, to meet the merchant and deliver the offer to her. This assumption is made to capture the observation that a household needs to pay costs or undertake efforts to obtain additional trading opportunities. Third, a virtual agent receives a discount on the utility compared with a genuine agent. A genuine agent receives $u(x)$ units of utility by consuming $x$ units of the day good, but a virtual agent receives $v(x) \equiv \alpha u(x)$ units of utility, where $\alpha \in (0, 1)$. This assumption captures the consideration that a household's priority is to purchase its favorite products, but it may attempt to purchase less-preferred products by conducting multi-account fraud. Agents need to create transaction accounts as identities to trade, but merchants in bilateral meetings cannot distinguish whether an account holder is a genuine or virtual agent.

Although a household can create a large number of accounts, its ability to produce and repay is limited. Consequently, the number of accounts with repayment records is limited in the economy, so repayment records can be applied to reduce the number of accounts that are used for trade and to mitigate the concern of multi-account fraud. In our model, because only genuine agents can make payments, a household can hold up to one account with a repayment record. We assume that accounts are not transferrable between agents; this assumption simplifies our analysis because a household does not need to consider the allocation of the account with a repayment record between genuine and virtual agents. Consequently, if an agent is holding an account with age $s \geq 1$, the merchant can infer that the agent must be genuine, meaning that the threat of fraud vanishes for meetings with age $s \geq 1$, but incomplete information about the type of agents only occurs in age 0 meetings.

As discussed in the introduction, associating the creation of an account with a deterrence activity may help deter multi-account fraud and improve the volume of trades. In our model, a deterrence activity is an activity that is costly to conduct and is observable by all households and merchants. We consider a one-time effort mechanism in which an agent is required to conduct $d$ units of deterrence activity when she is creating an account, and each unit of deterrence activity generates 1 unit of disutility to its provider.[7]

## 3.1 Bargaining with Incomplete Information

In this section we solve for the equilibrium of the bargaining game for age 0 meetings. Consider a meeting wherein the agent is not holding an account with a repayment record. Let $j$ denote the agent's type; if $j = g$, the agent is genuine, and if $j = v$, the agent is virtual. The timeline of the game is as follows. In the first stage, Nature randomly assigns an agent to a merchant. In the second stage, an agent makes decisions about whether to propose an offer to the merchant. The offer must be feasible in the sense that the genuine agent is willing to repay at night; thus, offers must be in the set $\Omega = \{(x, z) \in \mathbb{R}_+ \times \mathbb{R}_+ : z \leq \beta(V_1 - V_0)\}$, where $V_0$ and $V_1$ are taken as given in the bargaining game, and $V_1 - V_0 \geq 0$. An agent is idle if she does not propose an offer, and we denote this action by $I$. Let $a^j$ denote a type $j$ agent's action at this stage, then $a^j \in A \equiv \Omega \cup \{I\}$ for $j \in \{g, v\}$. In the third stage, if the merchant receives an offer, she decides to accept or reject it. Let $r : \Omega \to \{0, 1\}$ denote the merchant's response upon receiving an offer, where $r(x, z) = 1$ represents that the merchant accepts the offer, and $r(x, z) = 0$ represents that she rejects the offer. Finally, $\lambda : \Omega \to [0, 1]$ denotes the merchant's belief about the probability that the agent is genuine given the offer she received. The structure of the game is illustrated in Figure 2.

Our equilibrium concept is the Perfect Bayesian Equilibrium (PBE): actions are sequen-

[7] We can also allow the units of deterrence activity, $d$, to be chosen by agents, and this setting will also generate the result that deterrence activities can help deter multi-account fraud. We focus on the scenario in which $d$ is chosen by the mechanism instead of agents so that we can analyze how $d$ influences the upper bound of lifetime values.

tially rational, and beliefs accord with Bayes's rule. Let $w^g(a^g, r)$ and $w^v(a^v, r)$ denote the payoff functions of genuine agents and virtual agents, respectively. Then,

$$w^g(a^g, r) = \begin{cases} -d + r(x,z)\left[u(x) - (1-\delta)z + (1-\delta)\beta(V_1 - V_0)\right] & \text{if } a^g \in \Omega \\ 0 & \text{if } a^g = I, \end{cases} \tag{14}$$

$$w^v(a^v, r) = \begin{cases} -c - d + r(x,z)v(x) & \text{if } a^v \in \Omega \\ 0 & \text{if } a^v = I. \end{cases} \tag{15}$$

The genuine agent's payoff function, $w^g(a, r)$, says that if a genuine agent proposes an offer ($a \in \Omega$), she first incurs the disutility of conducting the deterrence activities, $d$; moreover, with probability $r(x, z)$, the merchant accepts the offer, and the agent's gain from the bilateral trade is $u(x) - (1-\delta)z$, and her expected payoff from accumulating the number of repayments is $(1-\delta)\beta(V_1 - V_0)$. If a genuine agent chooses to be idle ($a = I$), her payoff is 0. The virtual agent's payoff function $w^v(a^v, r)$ has a similar interpretation. The difference is that, first, a virtual agent needs to pay an entry cost, $c$, if she proposes an offer. Moreover, a virtual agent cannot repay at night so she cannot accumulate the number of repayments of her account. Thus, if the merchant accepts the offer, the virtual agent's trade surplus is simply equal to the utility from consuming the day good, $v(x)$. We impose a tie-break rule under which given a merchant's belief, $\lambda(x, z)$, the merchant accepts the offer if and only if her expected payoff is greater or equal to zero. That is

$$r(x,z) = \begin{cases} 1 & \text{if } -x + \lambda(x,z)(1-\delta)z \geq 0 \\ 0 & \text{if } -x + \lambda(x,z)(1-\delta)z < 0. \end{cases} \tag{16}$$

The merchant's belief, $\lambda(x, z)$, must be rational in the sense that it is derived from Bayes's rule.

Without further restrictions on out-of-equilibrium beliefs, there can be infinitely many equilibria. We apply forward induction to exclude equilibria caused by unreasonable beliefs, in the spirit of the Intuitive Criterion of Cho and Kreps (1987). That is, we adopt the consideration that a candidate outcome cannot form an equilibrium if a genuine agent may deviate to an out-of-equilibrium offer and expect to obtain a higher payoff than she receives in the given equilibrium, provided that the merchant applies forward induction whenever she observes an unexpected offer.

To formalize the refinement, we denote by $\tilde{w}^j(x,z)$ the payoff obtained by a genuine ($j=g$) or virtual ($j=v$) agent if she proposes $(x,z)$ and the offer is accepted by the merchant. That is,

$$\begin{aligned} \tilde{w}^g(x,z) &= -d+u(x)-(1-\delta)z+(1-\delta)\beta(V_1-V_0), \\ \tilde{w}^v(x,z) &= -c-d+v(x). \end{aligned}$$

An equilibrium outcome must not be disqualified by the following refinement:

**Refinement.** Given an outcome with a pair of genuine and virtual agents' strategies $(a^g,a^v)$ and the merchant's response $r$. The outcome is disqualified if there is an offer $(x',z')$ such that

$$\tilde{w}^v(x',z') < w^v(a^v,r), \tag{ICv}$$

$$\tilde{w}^g(x',z') > w^g(a^g,r), \tag{ICg}$$

$$-x'+(1-\delta)z' \geq 0, \tag{ICm}$$

The refinement says that an outcome is disqualified if there is an alternative offer $(x',z')$ that satisfies the following criterion. First, the virtual agent would not like to deviate to $(x',z')$ even if the merchant will accept the offer $(x',z')$ (condition (ICv)). Second, the genuine agent would like to deviate to $(x',z')$ if $(x',z')$ will be accepted by the merchant (condition (ICg)). Note that given the holding of (ICv) and (ICg), only a genuine agent has the incentive to propose $(x',z')$. Condition (ICm) says that under the belief that the offer is proposed by a genuine agent $\lambda(x',z')=1$, the merchant is willing to accept $(x',z')$. If the three criteria are satisfied, a genuine agent will be better off by deviating to $(x',z')$, so the proposed outcome should not be an equilibrium outcome.

We focus on the nondegenerate equilibrium in which a genuine agent makes an offer ($a^g=(x,z)\in\Omega$). We define a nondegenerate equilibrium of the bargaining game as follows:

**Definition 3** *A nondegenerate equilibrium of the bargaining game with incomplete informa-*

*tion is a tuple $(a^g, a^v, r, \lambda)$ where $a^g \in \Omega$, and the following conditions are satisfied*

i. Rationality of genuine agents: given $r$, genuine agents' action satisfies $a^g \in \arg\max_{a \in A} w^g(a^g, r)$.

ii. Rationality of virtual agents: given $r$, virtual agents' action satisfies $a^v \in \arg\max_{a \in A} w^v(a^v, r)$.

iii. Rationality of merchant: given $\lambda(x, z)$, the merchant's response $r(x, z)$ follows (16).

iv. Consistency of beliefs: given the actions $(a^g, a^v)$, the merchant's belief $\lambda(x, z)$ is derived from Bayes's rule.

v. Refinement: there does not exist an alternative offer $(x', z') \in \Omega$ that satisfies conditions (ICv) to (ICg).

We name a nondegenerate equilibrium an *equilibrium with fraud* if virtual agents and genuine agents propose the same offer ($a^v = a^g$); and we name a nondegenerate equilibrium an *equilibrium with no fraud* if a virtual agent chooses to propose an offer different from that proposed by a genuine agent ($a^v \neq a^g$) or to be idle ($a = I$).

**Lemma 1** *There is no equilibrium with fraud.*

Let $(x^*, z^*)$ form an equilibrium with fraud, and we show that the genuine agent can obtain a greater payoff by deviating to an out-of-equilibrium offer, say $(x', z')$. That is, $(x', z')$ satisfies conditions (ICv) to (ICm). First, we set $x'$ to be strictly smaller than $x^*$, then because virtual agents value only the day good, they will never deviate to $(x', z')$, so (ICv) must hold. Given $x' < x^*$, merchants should believe that the out-of-equilibrium offer is proposed by a genuine agent and be willing to accept the offer with a greater ratio of day-to-night productions than that of $(x^*, z^*)$. A genuine agent can thereby reduce her night production by sacrificing an arbitrarily small day production and thus can obtain a greater payoff from the deviation. Thus, there is a pair of $(x', z')$ that satisfies (ICg) and (ICm).

**Lemma 2** *In an equilibrium with no fraud, virtual agents choose to be idle ($a^v = I$).*

In an equilibrium with no fraud, if genuine and virtual agents propose different offers, merchants know whether the offer is proposed by a genuine or virtual agent. Because virtual agents will not repay at night, merchants will accept an offer only if the day production is zero. Because of the entry cost, the virtual agent will obtain a negative payoff from either proposing an offer with a zero day production or an offer that will be rejected. Thus, a virtual agent must choose to be idle in an equilibrium with no fraud. Note that the offer $(x, z)$ proposed by a genuine agent in an equilibrium with no fraud must satisfy the following *no-fraud constraint*

$$-d - c + v(x) \leq 0; \tag{17}$$

otherwise, a virtual agent will deviate to the offer proposed by the genuine agent. The no-fraud constraint, (17), places an upper bound on the day production, $x$. Let $(x^{\dagger}, z^{\dagger})$ denote the *optimal no-fraud offer,* the offer that generates the greatest payoff to the genuine agent among all offers that satisfy the no-fraud constraint and merchants' rationality constraint. Then $(x^{\dagger}, z^{\dagger})$ solves

$$\max_{x,z\in\mathbb{R}_+} \quad -d + u(x) - (1-\delta)z \tag{P2}$$

$$\text{subject to} \quad \begin{cases} -c - d + v(x) \leq 0 \\ x \leq (1-\delta)z \\ z \leq \beta(V_1 - V_0) \end{cases}$$

The first constraint is the no-fraud constraint; the second constraint is the merchant's rationality constraint that ensures that the merchant would like to accept the offer under the belief that her counterparty is genuine; and the third constraint is the feasibility constraint that guarantees the genuine agent would like to repay the debt. Because $u(x)$ is strictly concave and $\lim_{x\to\infty} u(x) = 0$, the solution of (P2) exists and is unique. The following lemma shows that the optimal no-fraud offer forms a nondegenerate equilibrium if it generates a positive payoff to the genuine agent.

**Lemma 3** *The optimal no-fraud offer* $(x^{\dagger}, z^{\dagger})$ *forms an equilibrium with no fraud if and only if*

$$\tilde{w}^g(x^{\dagger}, z^{\dagger}) \geq 0. \tag{18}$$

First, given that $(x^{\dagger}, z^{\dagger})$ satisfies the constraints in (P2), we can construct a merchant's belief under which the rationality conditions for genuine agents, virtual agents, and merchants hold and the consistency condition is also satisfied. The remaining is to show that the refinement is also satisfied. Suppose not, then there is an out-of-equilibrium offer that satisfies conditions (ICv) to (ICm). Let $(x', z')$ be the offer, then by (ICg), $(x', z')$ generates a greater payoff to the genuine agent than $(x^\dagger, z^\dagger)$ does; and by (ICm), $(x', z')$ satisfies the merchant's rationality constraint in (P2). Thus, given that $(x^\dagger, z^\dagger)$ solves (P2), $(x', z')$ must violate the no-fraud constraint, so (ICv) will not hold, producing a contradiction. Note that we need to check that the optimal no-fraud offer generates a nonnegative payoff to genuine agents, that is, (18) must hold; otherwise, a genuine agent would like to deviate to be idle and obtain 0 payoff.

**Lemma 4** *If $(x^*, z^*)$ forms an equilibrium with no fraud, then $(x^*, z^*)$ solves (P2) and satisfies (18).*

Suppose that $(x^*, z^*)$ does not solve (P2), we show that there is an alternative offer that satisfies conditions (ICv) to (ICm), so $(x^*, z^*)$ will violate the refinement. Consider an alternative offer, $(x', z')$, such that the day production, $x'$, is smaller than that of the optimal no-fraud offer, $x^\dagger$, and $z'$ satisfies $x' \leq (1-\delta)z'$. Because $(x^*, z^*)$ does not solve (P2), the offer must provide a payoff smaller than $\tilde{w}^g(x^\dagger, z^\dagger)$ to genuine agents. First, because $x'$ is smaller than $x^\dagger$ and $x^\dagger$ satisfies the no-fraud constraint, virtual agents will not deviate to $(x', z')$, and (ICv) holds. Moreover, because $x' \leq (1-\delta)z'$, a merchant will accept this alternative offer given the belief that the offer is proposed by a genuine agent, meaning that (ICm) holds. Finally, if $x'$ is sufficiently close to $x^\dagger$, a genuine agent will be better off by deviating to the alternative offer, $(x', z')$, given that the merchant will accept the offer, so (ICg) holds. Finally, $(x^*, z^*)$ must satisfy (18) because otherwise a genuine agent will choose to be idle.

Combining Lemma 1, 2, 3, and 4, we have the following proposition:

**Proposition 4** *An offer $(x, z)$ forms a nondegenerate equilibrium of the bargaining game with incomplete information if and only if $(x, z)$ solves (P2) and satisfies (18). Moreover, the nondegenerate equilibrium is an equilibrium with no fraud in which virtual agents do not enter the market and propose an offer.*

## 3.2 Anonymous credit equilibria under the threat of fraud

We now incorporate the bargaining game with incomplete information into the credit equilibrium under the threat of fraud. For age 0 meetings, by Proposition 4, the debt contract $(x_0, z_0)$ solves (P2) and satisfies (18). For bargaining in meetings with age $s \geq 1$, there is no asymmetric information problem, so the bargaining solution is the same as that in the benchmark model, and the equilibrium offer $(x_s, z_s)$ solves the optimization problem (P1).

To study the credit equilibrium under the threat of fraud, we first characterize the value of households. Note that a household's value comes from the genuine agent's payoff only because virtual agents choose to be idle and obtain zero payoffs. If a household does not have an account with a repayment record, its genuine agent must incur a cost of conducting the deterrence activity, $d$, to create a new account. Thus, given the terms of trade, $(x_s, z_s)$, the value of a household without an account with a repayment record is

$$V_0 = -d + \sum_{j=0}^{\infty}(1-\delta)^j \beta^j \left[u\left(x_j\right) - (1-\delta)z_j\right]. \tag{19}$$

Finally, because no deterrence activity is required in meetings with an account age greater than one, the account value $V_s$ satisfies (3) for $s \geq 1$.

**Definition 4** *Given the entry cost, $c$, and the deterrence activity, $d$, an anonymous credit equilibrium under the threat of multi-account fraud is given by nonnegative sequences $\{x_s, z_s, V_s\}_{s=0}^{\infty}$ such that*

*i. given $\{V_s\}_{s=0}^{\infty}$, for age 0 meetings, the term of trade, $(x_0, z_0)$, solves (P2) and satisfies (18);*

*ii. given $\{V_s\}_{s=0}^{\infty}$, for meetings with age $s \geq 1$, the term of trade, $(x_s, z_s)$, solves (P1);*

*iii. given $\{x_s, z_s\}_{s=0}^{\infty}$, the value $V_0$ satisfies (19);*

*iv. given $\{x_s, z_s\}_{s=0}^{\infty}$, for $s \geq 1$, the value $V_s$ satisfies (3).*

As in the benchmark model, we transform the system into the recursive form. Note that (3) and (19) imply

$$V_0 = -d + u\left(x_0\right) - (1-\delta)z_0 + (1-\delta)\beta V_1. \tag{20}$$

Thus, an equilibrium path of account values, $\{V_s\}_{s=0}^{\infty}$, satisfies (P1), (P2), (4), (20), and the boundedness condition (5) must also hold.

In age 0 meetings, the debt contract solves problem (P2), so the day and night good transfers satisfy

$$x_0 = \min\{\tilde{x}, (1-\delta)\beta (V_1 - V_0), u^{-1}(c/\alpha + d/\alpha)\}, \tag{21}$$

$$(1-\delta) z_0 = x_0. \tag{22}$$

The day production in age 0 meetings, $x_0$, is subject to the credit limit, $(1-\delta)\beta(V_1 - V_0)$, and the limit generated by the no-fraud constraint, $u^{-1}(c/\alpha + d/\alpha)$. This additional limit says that if the cost for virtual agents to create an account to make an offer, $c + d$, is smaller, or if virtual agents' gain from conducting fraud, which is captured by the discount on utility, $\alpha$, is greater, the equilibrium day transfer should be smaller to deter multi-account fraud. Let $C \equiv c/\alpha$ and $D \equiv d/\alpha$ denote the normalized entry cost and the normalized cost of deterrence activity, respectively, then the limit generated by the no-fraud constraint is determined by $C + D$. For conciseness, we call $C$ and $D$ the entry costs and the deterrence activity hereafter.

We define the forward-looking function under the threat of fraud by:

$$\begin{aligned} &f^{\dagger}(V_{s+1}, V_0) \equiv -\alpha D + u(x_s^*) - x_s^* + (1-\delta)\beta V_{s+1}, \\ &\text{where } x_s^* = \min\{\tilde{x}, (1-\delta)\beta (V_{s+1} - V_0), u^{-1}(C+D)\}. \end{aligned} \tag{23}$$

The relationship between $V_0$ and $V_1$ can be characterized by taking $s = 0$ into the equation $V_s = f^{\dagger}(V_{s+1}, V_0)$.[8] For meetings with age $s \geq 1$, because there is no threat of fraud and a household holding an account with an age greater than or equal to one does not need to create a new account to trade, the relationship between $V_s$ and $V_{s+1}$, for $s \geq 1$, can be characterized by $V_s = f(V_{s+1}, V_0)$, as in (9). To summarize, the relationship between $V_s$ and

[8] The equation $V_s = f^{\dagger}(V_{s+1}, V_0)$ is obtained by combining (20), (21), and (22) and replacing $V_0$ and $V_1$ with $V_s$ and $V_{s+1}$, respectively. This substitution allows us to compare the forward-looking function with the threat of fraud, $f^{\dagger}(V_{s+1}, V_0)$, and without the threat of fraud, $f(V_{s+1}, V_0)$.

$V_{s+1}$ in the extended model under the threat of multi-account fraud satisfies:[9]

$$\begin{cases} V_s = f^{\dagger}(V_{s+1}, V_0) \text{ for } s = 0, \\ V_s = f(V_{s+1}, V_0) \text{ for } s \geq 1. \end{cases} \tag{24}$$

As in the benchmark model, there can be multiple equilibria driven by beliefs. Given $V_0$, let $g^{\dagger}(V_s, V_0)$ denote the inverse function of $f^{\dagger}(V_{s+1}, V_0)$; that is,

$$g^{\dagger}(V_s, V_0) \equiv \left\{V_{s+1} : f^{\dagger}(V_{s+1}, V_0) = V_s\right\}. \tag{25}$$

The law of motion of account values follows:

$$\begin{cases} V_{s+1} = g^{\dagger}(V_s, V_0) \text{ for } s = 0, \\ V_{s+1} = g(V_s, V_0) \text{ for } s \geq 1. \end{cases} \tag{26}$$

Thus, the changing of account values from $V_0$ to $V_1$ follows $g^{\dagger}(V_s, V_0)$, and the changing of account values from $V_s$ to $V_{s+1}$ follows $g(V_s, V_0)$ for $s \geq 1$.

## 3.3 Discussion

In the following section, we discuss how the entry costs and deterrence activity influence the dynamics of equilibrium paths and the upper bound of households' lifetime values. We take the entry cost as exogenously determined by the environment and let the size of the deterrence activity be chosen by the mechanism designer.

**Entry Costs** To focus on the role of the entry cost, we take as given that the size of deterrence activity is set at 0. Figure 3 demonstrates the law of motion functions, $g(V_s, V_0)$ and $g^{\dagger}(V_s, V_0)$, described in (26) and the equilibrium paths, $\{V_0, V_1, V_2, \dots\}$, under a given $V_0$

[9] The equilibrium condition (18) holds automatically given that all other conditions hold. To see this, letting $V_0$ and $V_1$ be nonnegative and satisfy (20), we have

$$\begin{aligned} & -d + u(x_0) - (1-\delta)z_0 + (1-\delta)\beta(V_1 - V_0) \\ = \; & V_0 - (1-\delta)\beta V_0 \geq 0. \end{aligned}$$

and various entry costs. We find that, first, different from the benchmark model, when the threat of fraud is present, equilibrium credit limits can not only increase, but also decrease or remain constant over the account age. This occurs because the threat of multi-account fraud generates an additional constraint—the no-fraud constraint—on the trade volume for age 0 meetings, and an agent encounters the constraint on the trade volume once she defaults and create a new account for future transactions. This effect serves as a punishment and generates an additional cost for default, so increasing-credit-limit schemes are not the only mechanism that can be applied to motivate the repayment of debt. Given $V_0$, if the entry cost is smaller, the trade volume for age 0 meetings will be smaller, meaning that the punishment for default will be more severe. If the punishment is severe enough, using an increasing-debt-limit scheme as a reward is not needed to motivate the repayment of the debt, and the credit limit can decrease or remain constant over the account age.

To formalize these results, we first observe from (23) that if $C$ is larger, the limit of trade imposed by the no-fraud constraint will be greater, so the required $V_1$ to generate a given $V_0$ will be smaller. Figure 3 (V) demonstrates a case wherein $C$ is sufficiently large such that $V_1$ is smaller than than $V^l$, the lower intersection between the law of motion function, $g(V_s, V_0)$, and the 45-degree line. In this case, the equilibrium account values follow $V_1 < V_2 < V_3 < \dots$, and the credit limits increase over the account age. The intuition is that if $C$ is sufficiently large, the punishment for default is not severe enough, so an increase-credit-limit scheme is required to generate the given $V_0$. If $C$ is smaller such that $V_1$ is between the lower and higher intersections, $V_1 \in [V^l, V^h]$, the punishment for default is sufficiently severe, so the credit limit can be constant or decreasing over the account age. If $V_1 = V^l$ (Figure 3 (IV)) or $V_1 = V^h$ (Figure 3 (II)), the equilibrium account values follow $V_1 = V_2 = V_3 = \dots$ , and credit limits are constant over the account age in these cases. If $V_1 \in (V^l, V^h)$ (Figure 3 (III)), the equilibrium account values follow $V_1 > V_2 > V_3 > \dots$, and credit limits are decreasing over the account age. When $C$ is sufficiently small such that $V_1$ is greater than the higher intersection, $V_1 > V^h$, the dynamic path will explode and violate the boundedness condition, so $V_0$ cannot be generated by a credit equilibrium, as shown in Figure 3 (I). For $V_0 > \bar{V}$, where $\bar{V}$ is the upper bound of lifetime values in the economy without the threat of fraud, there is no intersection between $g(V_s, V_0)$ and the 45-degree line, so the dynamic path must explode, and $V_0$ cannot be generated by a credit equilibrium. In summary, a credit equilibrium exists if and only if, first, there is an intersection between $g(V_s, V_0)$ and the 45-degree line and this occurs when $V_0 \leq \bar{V}$; and, second, the value of an age 1 account is smaller than the higher intersection between $g(V_s, V_0)$ and the 45-degree line, that is, $V_1 \leq V^h$.

To explore how entry costs influence the upper bound of lifetime value, we denote by $\tilde{v}_s^M(V_0)$ the value of a household holding an age $s$ account given the lifetime values, $V_0$; then $\tilde{v}_s^M(V_0)$ can be derived iteratively by:

$$\tilde{v}_s^M(V_0) = \begin{cases} V_0 & \\ g^\dagger(\tilde{v}_{s-1}^M(V_0), V_0) & \text{for} \\ g(\tilde{v}_{s-1}^M(V_0), V_0) & \end{cases} \begin{array}{l} s = 0 \\ s = 1 \\ s > 1. \end{array} \tag{27}$$

Let $\bar{V}^M$ denote the upper bound of lifetime values that can be generated by a credit equilibrium under the threat of multi-account fraud, then $\bar{V}^M$ is the greatest $V_0$ that satisfies $V_0 \leq \bar{V}$ and $\tilde{v}_1^M(V_0) \leq \tilde{v}^h(V_0)$.

We focus on the parameter set under which $\tilde{v}_1^M(\bar{V}) > \tilde{v}^h(\bar{V})$ when $C = 0$ and $D = 0$, because in this case, $\bar{V}$ cannot be achieved when the entry cost and the deterrence activity are absent, so this situation provides a space for the entry cost and the deterrence activity to improve the upper bound of lifetime values of households.[10] We illustrate the relationship between $C$ and $\bar{V}^M$ in Figure 4. We observe that there is a threshold value of entry costs $\bar{C}$ such that for $C < \bar{C}$, a greater $C$ implies a greater upper bound of lifetime values, $\bar{V}^M$; for $C \geq \bar{C}$, the upper bound of lifetime values reaches its greatest possible values, $\bar{V}$, and a further increase in $C$ does not increase $\bar{V}^M$.

To understand the result shown in Figure 4, we demonstrate the functions $\tilde{v}_1^M(V_0)$, $\tilde{v}^h(V_0)$, and $\tilde{v}^l(V_0)$ under various entry costs in Figure 5. Because the no-fraud constraint only impacts the terms of trade for age 0 meetings, the entry cost has no impact on $\tilde{v}^h(V_0)$

[10]Under the parameter set wherein $\tilde{v}_1^M(\bar{V}) \leq \tilde{v}^h(\bar{V})$ when $C = 0$ and $D = 0$, $\bar{V}$ can be generated by a credit equilibrium even if $C = 0$ and $D = 0$, so a greater entry cost cannot further improve the upper bound of lifetime values. Note that whether $\tilde{v}_1^M(\bar{V}) > \tilde{v}^h(\bar{V})$ or $\tilde{v}_1^M(\bar{V}) \leq \tilde{v}^h(\bar{V})$ when $C = 0$ and $D = 0$ depends on the time discount factor, $\beta$, the leaving rate, $\delta$, and the functional form of the utility function. See Appendix A.13 for a detailed discussion.

and $\tilde{v}^l(V_0)$, but it influences $\bar{V}^M$ through changing $\tilde{v}_1^M(V_0)$. We observe from Figure 5 that whether the no-fraud constraint is binding implies two branches of $\tilde{v}_1^M(V_0)$, and there is a threshold of account values, say $V^c$, such that for $V_0 < V^c$, the no-fraud constraint is nonbinding, and we name this branch of $\tilde{v}_1^M(V_0)$ the *nonbinding branch*; for $V_0 > V^c$, the no-fraud constraint is binding, and we name this branch of $\tilde{v}_1^M(V_0)$ the *binding branch.*[11]

When the entry cost, $C$, is sufficiently small under which $\tilde{v}_1^M(\bar{V}) > \tilde{v}^h(\bar{V})$ (Figure 5 (I) and (II)), the upper bound of lifetime values, $\bar{V}^M$, satisfies $\tilde{v}_1^M(\bar{V}^M) = \tilde{v}^h(\bar{V}^M)$ and is smaller than $\bar{V}$ because $\tilde{v}_1^M(V_0)$ is increasing and $\tilde{v}^h(V_0)$ is decreasing in $V_0$. In this case, $\bar{V}^M$ must be located at the binding branch of $\tilde{v}_1^M(V_0)$ because otherwise we will have $\tilde{v}_1^M(\bar{V}^M) = \tilde{v}_1(\bar{V}^M) < \tilde{v}^h(\bar{V}^M)$, and the fact that $\tilde{v}_1(\bar{V}^M) = \tilde{v}^h(\bar{V}^M)$ will be contradicted. As $C$ increases, the binding branch of $\tilde{v}_1^M(V_0)$ shifts downward because an increase in $C$ makes the no-fraud constraint less stringent and allows greater gains from trade, so the required $V_1$ to generate a given $V_0$ is smaller. Thus, a greater $C$ implies a greater $\bar{V}^M$ in this circumstance, and this result can be seen by comparing Figure 5 (I) and (II). The threshold value of entry costs, $\bar{C}$, is equal to the entry cost under which $\tilde{v}_1(\bar{V}) = \tilde{v}^h(\bar{V})$. If the entry cost is greater than the threshold value ($C \geq \bar{C}$), we will have $\tilde{v}_1^M(\bar{V}) \leq \tilde{v}^h(\bar{V})$ (Figure 5 (III) and (IV)); thus, $\bar{V}$ can be generated by a credit equilibrium, and $\bar{V}^M = \bar{V}$.

**Deterrence Activity** In this subsection, we take as given the entry cost and analyze the roles of the deterrence activity. Figure 6 demonstrates the law of motion functions $g(V_s, V_0)$ and $g^{\dagger}(V_s, V_0)$, and shows the impacts of the deterrence activity on the functions and equilibrium paths. We observe that the credit limits can increase, decreasing, or remain constant over the account age. The reason is that when a deterrence activity is imposed, a defaulter encounters not only the restriction on the trade volume for age 0 meeting but also the disutility of conducting the deterrence activity again. These two effects both generate costs to default, so increasing-credit-limit schemes are not the only mechanism that can be applied to motivate repayment.

[11] To see this result, suppose that the no-fraud constraint is nonbinding; in that case, then according to Proposition 2, the credit limit for age 0 meetings will approach 0 when $V_0$ approaches 0, and the day production will be determined by the credit limit and equal to $\tilde{\phi}_0(V_0)$ when $V_0$ is sufficiently small, but the no-fraud constraint will be nonbinding. If $V_0$ is sufficiently large, the credit limit will be greater than the limit generated by the no-fraud constraint, meaning that the no-fraud constraint will be binding.

We analyze how the size of deterrence activities influences the upper bound of lifetime values. As in the analysis of the entry cost, we also focus on the parameter set in which $\tilde{v}_1^M(\bar{V}) > \tilde{v}^h(\bar{V})$ when $C = 0$ and $D = 0$. Moreover, we focus on the entry costs that are smaller than $\bar{C}$, under which $\bar{V}^M < \bar{V}$ when $D = 0$, so there is a space for the deterrence activity to improve $\bar{V}^M$.

Figure 7 illustrates the impact of the deterrence activity on $\bar{V}^M$ under various values of utility discounts for virtual agents, $\alpha$, and entry costs, $C$, that are the key parameters that determine the effectiveness of the deterrence activity. The figure shows that different from the entry cost, a greater deterrence activity may result in a smaller upper bound of lifetime values. For instance, in Figure 7 (I) wherein $\alpha$ is taken as given, an increase in $D$ from zero decreases $\bar{V}^M$ when the entry cost $C$ is large, while it increases $\bar{V}^M$ when $C$ is small. Moreover, if $D$ is sufficiently large, an increase in $D$ decreases $\bar{V}^M$ regardless of the size of $C$.

To understand this result, we illustrate in Figure 8 functions $\tilde{v}_1(V_0)$, $\tilde{v}^h(V_0)$, and $\tilde{v}^l(V_0)$ under various sizes of deterrence activities. We observe that given $V_0$, an increase in $D$ may increase or decrease $\tilde{v}_1(V_0)$ because a greater deterrence activity not only provides a greater limit of trade for age 0 meetings but also generates a greater disutility when a household is creating an account.

We start from the case wherein no deterrence activity is imposed ($D = 0$) and analyze under what circumstances increasing the size of the deterrence activity from 0 can improve $\bar{V}^M$. As in the discussion of the entry cost, for $C < \bar{C}$ and $D = 0$, the upper bound of lifetime value, $\bar{V}^M$, satisfies $\tilde{v}_1^M(\bar{V}^M) = \tilde{v}^h(\bar{V}^M)$, and $\bar{V}^M$ is at the binding branch of $\tilde{v}_1^M(V_0)$. Thus, studying how $D$ influences $\bar{V}^M$ is equivalent to studying how $D$ influences the binding branch of $\tilde{v}_1^M(V_0)$.

The binding branch of $\tilde{v}_1^M(V_0)$ is equal to the value of $V_1$ that solves

$$V_0 = -\alpha D + (C + D) - u^{-1}(C + D) + (1 - \delta)\beta V_1, \tag{28}$$

and (28) is obtained by setting the age 0 day production to the limit generated by the no-fraud constraint $x_0^* = u^{-1}(C+D)$. In (28), the first term, $-\alpha D$, is the disutility of conducting the deterrence activity, and the second term, $(C+D)-u^{-1}(C+D)$, is the household's gain from trade, a gain which is equal to the utility generated by consuming the day goods, $(C+D)$, minus the disutility generated by producing the night goods, $u^{-1}(C+D)$.

By (28), the marginal benefit of increasing the size of the deterrence activity is equal to the marginal gain from trade, $1-\left(u^{-1}\right)'(C+D)$, and the marginal cost is equal to the marginal disutility, $\alpha$. An increase in $D$ shifts downward the binding branch of $\tilde{v}_1^M(V_0)$ and increases $\bar{V}^M$ if and only if the marginal benefit is greater than the marginal cost. We obtain the following results.

1) The role of the entry cost, $C$ (Figure 7 (I)): When the entry cost approaches 0 and when $D=0$, the marginal benefit of increasing the size of the deterrence activity is equal to $\lim_{C\to 0}\left[1-\left(u^{-1}\right)'(C)\right]=1$ and is greater than the marginal cost, $\alpha$. Thus, imposing a deterrence activity will increase $\bar{V}^M$ as long as $C$ is sufficiently small. As $C$ increases, the marginal benefit decreases because the marginal utility of consumption decreases; thus, imposing a deterrence activity is less effective on increasing $\bar{V}^M$.

2) The role of the utility discount for virtual agent, $\alpha$ (Figure 7 (II)): For $C<\bar{C}$ and $D=0$, because the no-fraud constraint is binding, the marginal benefit of increasing the size of the deterrence activity, $1-\left(u^{-1}\right)'(C)$, must be positive. Thus, if $\alpha$ is sufficiently small, the marginal cost will be smaller than the marginal benefit, so an increase in the size of the deterrence activity from 0 increases the upper bound of lifetime value. As $\alpha$ increases, the marginal cost of increasing $D$ increases, so an increase in $D$ is less effective on increasing $\bar{V}^M$.

Finally, given $C$ and $\alpha$, if the size of the deterrence activity is larger, the marginal benefit of increasing $D$ will be smaller. If $D$ is sufficiently large, the marginal benefit will be smaller than the marginal cost, $\alpha$. In this circumstance, an increase in $D$ will decrease the upper bound of the lifetime value, $\bar{V}^M$. Note that there will be no credit equilibrium if the deterrence activity is too large such that $\tilde{v}_1^M(0)>\tilde{v}^h(0)$. In this circumstance, the cost of engaging in the credit trade will be too large, so no equilibrium path can generate a nonnegative lifetime value to households.

# 4 Conclusion

We study credit in an environment wherein traders' true identities are hidden, but they can freely create transaction accounts as identities to engage in transactions and incur debt, and repayment records can be preserved and attached to transaction accounts. The main issue raised by this greater privacy is two-fold. First, a defaulter can create a new account to borrow again right after default and cannot be excluded from the economy indefinitely no matter how many times the borrow defaults. Second, a borrower can create multiple accounts to borrow and default intentionally. We propose multiple mechanisms to solve these issues generated by anonymity. We show that increasing credit limits over the account age can motivate borrowers to repay the debt rather than defaulting and starting over again. If the threat of multi-account fraud is present, requiring a deterrence activity can help deter households from conducting fraud and permit a greater trade volume. Finally, repayment records can also be applied to diminish the number of accounts that can be maintained by households and mitigate the multi-account fraud problem.

Note that although actions are fully observable in our model, in many transactions using digital accounts, such as transactions on blockchains, whether the transactions occurred or not is not observable by the public. Under this situation, if participants are all anonymous, a trader may have opportunities to fake transaction records and manipulate the reputation of accounts, for instance, by acting as both borrower and lender simultaneously. A potential solution to this problem is to introduce into the credit system financial intermediaries with publicly known identities. If a trader must borrow from a publicly known commercial bank or the central bank, it is not possible for the trader to forge transaction records. While the incentive problem of the intermediary can be a further issue, we leave analysis of these issues for future research.

In summary, the raising of anonymous credit greatly relies on the credibility of record-keeping technology and the methods available to help identify a trader's true identity. The results of this paper provide usable insight into the design of future blockchain protocol and CBDC.

# References


Ahnert, T., Hoffmann, P., and Monet, C. (2022). The digital economy, privacy, and CBDC.

Andolfatto, D. (2013). Incentives and the limits to deflationary policy. *Journal of Monetary Economics*, 60(4):383–390.

Berentsen, A. and Schär, F. (2018). A short introduction to the world of cryptocurrencies. *Federal Reserve Bank of St. Louis Review*, 100(1):1–16.

Cho, I.-K. and Kreps, D. M. (1987). Signaling games and stable equilibria. *The Quarterly Journal of Economics*, 102(2):179–221.

Gu, C., Mattesini, F., Monnet, C., and Wright, R. (2013). Endogenous credit cycles. *Journal of Political Economy*, 121(5):940–965.

Gu, C., Mattesini, F., and Wright, R. (2016). Money and credit redux. *Econometrica*, 84(1):1–32.

Kehoe, T. and Levine, D. (1993). Debt-constrained asset markets. *Review of Economic Studies*, 60(4):865–888.

Kehoe, T. and Levine, D. (2001). Liquidity constrained markets versus debt constrained markets. *Econometrica*, 69(3):575–598.

Keister, T. and Sanches, D. (2023). Should central banks issue digital currency. *The Review of Economic Studies*, 90(1):404–431.

Kocherlakota, N. R. (1996). Implications of efficient risk sharing without commitment. *Review of Economic Studies*, 63:595–609.

Li, Y. and Rocheteau, G. (2011). On the threat of counterfeiting. *Macroeconomic Dynamics*, 15(s1):10–41.

Li, Y., Rocheteau, G., and Weill, P.-O. (2012). Liquidity and the threat of fraudulent assets. *Journal of Political Economy*, 120(5):815–846.

Li, Y. and Wang, C.-C. (2022). A search-theoretic model of double-spending fraud. *Journal of Economic Dynamics and Control*, 142:104157.

Nosal, E. and Wallace, N. (2007). A model of (the threat of) counterfeiting. *Journal of Monetary Economics*, 54:229–246.

Sanches, D. (2018). Bitcoin vs. the buck: Is currency competition a good thing? *Federal Reserve Bank of Philadelphia Economic Insights*, pages 9–14.

Sanchez, D. and Willimson, S. D. (2010). Money and credit with limited commitment and theft. *Journal of Economic Theory*, 415(5).

Williamson, S. (2022). Central bank digital currency: Welfare and policy implications. *Journal of Political Economy*, 130(11):2829–2861.

# A Proofs of Propositions and Lemmas

## A.1 Proof of Lemma 1

We prove the lemma by induction.

i. For $V_0 > 0$. We first show that $\tilde{v}_1(V_0) > \tilde{v}_0(V_0) = V_0$. Note that we must have $\tilde{v}_1(V_0) \geq V_0$ in a credit equilibrium because the credit limit $(1-\delta)\beta(\tilde{v}_1(V_0) - V_0)$ must be nonnegative. By (9), if $\tilde{v}_1(V_0) = V_0$, we will have $f(\tilde{v}_1(V_0), V_0) = (1-\delta)\beta V_0 < V_0$. Moreover, $f(V_1, V_0)$ is increasing in $V_1$ and goes to infinity as $V_1$ goes to infinity, so there is a unique $V_1 > 0$ that solves $f(V_1, V_0) = V_0$. Thus, $\tilde{v}_1(V_0) > V_0$. Second, given $s > 0$, we assume that $\tilde{v}_s(V_0) > \tilde{v}_{s-1}(V_0)$ holds. Because $f(V_{s+1}, V_0)$ is strictly increasing in $V_{s+1}$, the presumption $\tilde{v}_s(V_0) > \tilde{v}_{s-1}(V_0)$ implies $f(\tilde{v}_s(V_0), V_0) > f(\tilde{v}_{s-1}(V_0), V_0)$. This result implies $\tilde{v}_{s+1}(V_0) > \tilde{v}_s(V_0)$. Thus, $\tilde{v}_s(V_0^*)$ is strictly increasing in $s$.

ii. For $V_0 = 0$. By (9), given that $V_0 = 0$, we must have $\tilde{v}_1(V_0) = 0$. Moreover, by (9), for $s > 0$, if $\tilde{v}_s(V_0) = V_0 = 0$, we must also have $\tilde{v}_{s+1}(V_0) = 0$. Thus, $\tilde{v}_s(V_0) = 0$ for all $s \geq 0$.

## A.2 Proof of Lemma 2

i. We prove by induction.

For $s = 0$: Recall that $\tilde{v}_1(V_0)$ is equal to the $V_1$ that solves

$$f(V_1, V_0) - V_0 = 0. \tag{29}$$

By combining (8) and (29) and applying implicit differentiation, we obtain

$$\begin{gathered}\frac{d\tilde{v}_1(V_0)}{dV_0} = \frac{(1-\delta)\beta[u'(\tilde{x}_0(V_0))-1]+1}{(1-\delta)\beta[u'(\tilde{x}_0(V_0))-1]+(1-\delta)\beta} \\ \text{where } \tilde{x}_0(V_0) = \min\{(1-\delta)\beta\left(\tilde{v}_1(V_0) - V_0\right), \tilde{x}\}\end{gathered} \tag{30}$$

Because $(1-\delta)\beta < 1$, by (30), we have $d\tilde{v}_1(V_0)/dV_0 > 1$. Thus, $d\tilde{\phi}_0/dV_0(V_0) = (1-\delta)\beta(d\tilde{v}_1(V_0)/dV_0 - 1) > 0$.

For $s \geq 1$: we assume that $d\tilde{v}_s(V_0)/dV_0 > 1$ holds. Given $V_s \equiv \tilde{v}_s(V_0)$, $\tilde{v}_{s+1}(V_0)$ is equal to the $V_{s+1}$ that solves

$$f(V_{s+1}, V_0) - V_s = 0. \tag{31}$$

By combining (8) and (31) and applying implicit differentiation, we obtain

$$\begin{gathered} \frac{d\tilde{v}_{s+1}(V_0)}{dV_0} = \frac{(1-\delta)\beta[u'(\tilde{x}_s(V_0))-1] + \frac{d\tilde{v}_s(V_0)}{dV_0}}{(1-\delta)\beta[u'(\tilde{x}_s(V_0))-1]+(1-\delta)\beta} \\ \text{where } \tilde{x}_s(V_0) = \min\{(1-\delta)\beta\left(\tilde{v}_{s+1}(V_0) - V_0\right), \tilde{x}\} > 0 \end{gathered} \tag{32}$$

Because $d\tilde{v}_s(V_0)/dV_0 > 1 > (1-\delta)\beta$, by (32), we have $d\tilde{v}_{s+1}(V_0)/dV_0 > 1$. Consequently, $d\tilde{\phi}_s/dV_0(V_0) = (1-\delta)\beta(d\tilde{v}_{1+1}(V_0)/dV_0 - 1) > 0$.

ii. First, we prove that $\lim_{V_0 \to 0} \tilde{v}_s(V_0) = 0$ and $\lim_{V_0 \to 0} \tilde{\phi}_s(V_0) = 0$. Given $V_0$, let $V_s = \tilde{v}_s(V_0)$, because $u(x_s) - x_s \geq 0$ for all $s \geq 0$, we have

$$\begin{aligned} V_0 &= \sum_{\tau=0}^{s-1}(1-\delta)^\tau \beta^\tau \left[u(x_\tau) - x_\tau\right] + (1-\delta)^s \beta^s V_s \\ &\geq (1-\delta)^s \beta^s V_s. \end{aligned}$$

Thus, $V_s \leq (1-\delta)^{-s}\beta^{-s}V_0$. Given $\epsilon > 0$, we have $V_s < \epsilon$ if $V_0 < (1-\delta)^s\beta^s\epsilon$. Moreover, $\phi_{s-1} = (1-\delta)\beta\left(V_s - V_0\right) \leq (1-\delta)\beta V_s \leq (1-\delta)^{-(s-1)}\beta^{-(s-1)}V_0$, and thus, $\phi_{s-1} < \epsilon$ if $V_0 < (1-\delta)^{s-1}\beta^{s-1}\epsilon$.

Second, we prove that $\lim_{V_0 \to \infty} \tilde{v}_s(V_0) = \infty$ and $\lim_{V_0 \to \infty} \tilde{\phi}_s(V_0) = \infty$. Because $V_s \geq V_0$, we have $V_s \to \infty$ as $V_0 \to \infty$. Moreover, given $s \geq 1$, because $u(x_s) - x_s$ is bounded above by $\tilde{f} \equiv u(\tilde{x}) - \tilde{x}$, we have $(1-\delta)^\tau\beta^\tau V_s + M_s \geq V_0$, where $M_s \equiv \sum_{\tau=0}^{s-1}(1-\delta)^\tau\beta^\tau\tilde{f}$. Thus,

$$\begin{aligned} \phi_{s-1} &\equiv (1-\delta)\beta\left(V_s - V_0\right) \\ &\geq (1-\delta)\beta\left\{\left[(1-\delta)^{-\tau}\beta^{-\tau} - 1\right]V_0 - (1-\delta)^{-\tau}\beta^{-\tau}M_s\right\} \end{aligned}$$

Because $(1-\delta)^{-\tau}\beta^{-\tau} > 1$, $\tilde{\phi}_{s-1}(V_0) \to \infty$ as $V_0 \to \infty$.

## A.3 The Upper Bound of Lifetime Values in Anonymous Credit Equilibrium

**Lemma 5** *In an anonymous credit equilibrium, the upper bound of $V_0$ that can be generated by a credit equilibrium is*

$$\begin{aligned}\bar{V} =\ & \frac{1}{1-(1-\delta)\beta}u(\hat{x}) - \left\{\frac{1}{(1-\delta)\beta\left[1-(1-\delta)\beta\right]}\right\}\hat{x},\\ \textit{where } u'(\hat{x}) =\ & \frac{1}{(1-\delta)\beta}.\end{aligned}$$

***Proof.*** *First, we observe that given $V_0$, the function $f(V_{s+1}, V_0)$ is an increasing, concave, continuous differentiable function with $\lim_{V_{s+1}\to V_0} f(V_{s+1}, V_0) = \infty$ and*

$$\frac{\partial}{\partial V_{s+1}} f(V_{s+1}, V_0) = \begin{cases} u'\left[(1-\delta)\beta\left(V_{s+1}-V_0\right)\right](1-\delta)\beta & \textit{for } (1-\delta)\beta(V_{s+1}-V_0) \leq \tilde{x} \\ (1-\delta)\beta & \textit{for } (1-\delta)\beta(V_{s+1}-V_0) \geq \tilde{x} \end{cases}. \tag{33}$$

*Second, as $V_0$ increases, the function moves up and to the right along $V_{s+1} = V_s/\left[(1-\delta)\beta\right]$. The existence of credit equilibrium requires that there is an intersection between the graph of $V_s = f(V_{s+1}, V_0)$ and the 45 degree line. Thus, $\bar{V}$ is equal to the value of $V_0$ under which the graph of $V_s = f(V_{s+1}, V_0)$ and the 45-degree line is tangent. Let $\hat{V}$ denote the value of $V_{s+1}$ such that $f(V_{s+1}, \bar{V})$ tangent the 45-degree line, and thus, $\partial f(\hat{V}, \bar{V})/\partial V_{s+1} = 1$ and $f(\hat{V}, \bar{V}) = \hat{V}$. We denote by $\hat{x}$ the day production when $V_{s+1} = \hat{V}$. By (33), because $\partial f(\hat{V}, \bar{V})/\partial V_{s+1} = 1$, we must have $(1-\delta)\beta(\hat{V} - V_0) \leq \tilde{x}$; thus,*

$$\hat{x} = (1-\delta)\beta\left(\hat{V} - \bar{V}\right). \tag{34}$$

*By (33), $\partial f(\hat{V}, \bar{V})/\partial V_{s+1} = 1$ if and only if $u'\left[(1-\delta)\beta\left(V_{s+1}-V_0\right)\right](1-\delta)\beta = 1$; thus, $\hat{x}$ also satisfies*

$$u'(\hat{x}) = \frac{1}{(1-\delta)\beta}. \tag{35}$$

*Moreover, $f(\hat{V}, \bar{V}) = \hat{V}$ if and only if*

$$\hat{V} = u(\hat{x}) - \hat{x} + (1-\delta)\beta\hat{V}. \tag{36}$$

*Combining (34) and (36), we obtain*

$$\bar{V} = \frac{1}{1-(1-\delta)\beta} u(\hat{x}) - \left[\frac{1}{(1-\delta)\beta\,[1-(1-\delta)\beta]}\right] \hat{x}.$$

■

## A.4 The Upper Bound of Lifetime Values in True-Identity Credit Equilibria

Let $f^R(V_{t+1})$ denote the forward-looking function in the true-identity credit system. The equilibrium value of households satisfies

$$\begin{aligned} &V_t = f^R(V_{t+1}) \equiv u(x_t) - x_t + (1-\delta)\beta V_{t+1}, \\ &\text{where } x_t = \min\{\tilde{x}, (1-\delta)\beta V_{t+1}\}. \end{aligned} \tag{37}$$

Let $g^R(V_t)$ denote the $V_{t+1}$ that solves $f^R(V_{t+1}) = V_t$. The dynamics of households' value can be characterized by $V_{t+1} = g^R(V_t)$. We illustrate the dynamic paths under the rue-identity credit equilibrium in Figure 9. We observe that there are two intersections between $g^R(V_t)$ and the 45-degree line. Let $V^h$ denote the upper intersection. Then for $V_0 > V^h$, households' values will go to infinite as time goes to infinite and will violate the boundedness condition. Thus, let $\bar{V}^R$ denote the upper bound of households' lifetime values, and then $\bar{V}^R = V^h$. We solve $\bar{V}^R$ in the following lemma:

**Lemma 6** *In a true-identity credit equilibrium, if* $(1-\delta)\beta u(\tilde{x}) \geq \tilde{x}$, *then* $\bar{V}^R = [u(\tilde{x}) - \tilde{x}] / [1-(1-\delta)\beta]$, *where* $\tilde{x}$ *solves* $u'(\tilde{x}) = 1$. *Otherwise,* $\bar{V}^R = \left[u(x^R) - x^R\right] / [1-(1-\delta)\beta]$, *where* $x^R = (1-\delta)\beta u(x^R)$.

***Proof.*** *In a true-identity credit equilibrium, the greatest lifetime value is generated by the stationary equilibrium, in which the trade volume* $x^*$ *and the account value* $V^{R*}$ *satisfy*

$$\bar{V}^R = \frac{1}{1-(1-\delta)\beta}\left[u(x^R) - x^R\right] \tag{38}$$

$$x^R = \min\{\tilde{x}, (1-\delta)\beta\bar{V}^R\}. \tag{39}$$

■

We first consider the case that $(1-\delta)\beta\bar{V}^R \geq \tilde{x}$. In this case, $x^R = \tilde{x}$, and $\bar{V}^R = [u(\tilde{x}) - \tilde{x}]/[1-(1-\delta)\beta]$. We need to check that $(1-\delta)\beta\bar{V}^R = (1-\delta)\beta[u(\tilde{x}) - \tilde{x}]/[1-(1-\delta)\beta] \geq \tilde{x}$ and the inequality holds if and only if $(1-\delta)\beta u(\tilde{x}) \geq \tilde{x}$.

Second, we consider the case that $(1-\delta)\beta\bar{V}^R < \tilde{x}$. In this case, $x^R = (1-\delta)\beta\bar{V}^R$. By (38), we have

$$\begin{aligned} & \frac{1}{(1-\delta)\beta}x^R = \frac{1}{1-(1-\delta)\beta}\left[u(x^R) - x^R\right] \\ \Rightarrow \quad & x^R = (1-\delta)\beta u(x^R). \end{aligned}$$

## A.5 Proof of Proposition

3

We apply Lemma 5 and Lemma 6 to prove the proposition. First, if $(1-\delta)\beta u(\tilde{x}) \geq \tilde{x}$, we have $x^R = \tilde{x}$, and $\bar{V}^R = [u(\tilde{x}) - \tilde{x}]/[1-(1-\delta)\beta]$. By (35) and (36), $\hat{V} = [u(\hat{x}) - \hat{x}]/[1-(1-\delta)\beta]$ and $\hat{x} < \tilde{x}$, so $\bar{V}^R > \hat{V}$. Second, if $(1-\delta)\beta u(\tilde{x}) < \tilde{x}$, then $x^R$ satisfies $x^R = (1-\delta)\beta u(x^R)$, and by the concavity of $u(x)$, we have $u'(x^R) < 1/[(1-\delta)\beta]$; thus, $x^R > \hat{x}$, and this also implies that $\bar{V}^R > \hat{V}$. Finally, by (34), $\hat{V} > \bar{V}$ holds. Thus, we must have $\bar{V}^R > \bar{V}$.

## A.6 Equilibrium Condition

First, we show that (3) implies (4). Let $\{V_s\}_{s=0}^{\infty}$ be a sequence that satisfies (3) for all $s \geq 0$, then

$$\begin{aligned} V_s &= u(x_s) - (1-\delta)z_s + (1-\delta)\beta\sum_{j=1}^{\infty}(1-\delta)^j\beta^j\left[u(x_{s+j}) - (1-\delta)z_{s+j}\right] && (40) \\ &= u(x_s) - (1-\delta)z_s + (1-\delta)\beta V_{s+1}, && (41) \end{aligned}$$

so (4) holds.

Second, we show that if (5) holds, then (4) implies (3). Let $\{V_s\}_{s=0}^{\infty}$ satisfy (4) for all $s$, then

$$\begin{aligned} V_s &= u(x_s) - (1-\delta)z_s + (1-\delta)\beta V_{s+1} \\ &= \sum_{j=0}^{S}(1-\delta)^j\beta^j\left[u(x_{s+j}) - (1-\delta)z_{s+j}\right] + (1-\delta)^{S+1}\beta^{S+1}V_S. \end{aligned}$$

Taking $S$ to infinity, we obtain

$$V_s = \sum_{j=0}^{\infty}(1-\delta)^j\beta^j\left[u(x_{s+j}) - (1-\delta)z_{s+j}\right] + \lim_{S\to\infty}(1-\delta)^{S+1}\beta^{S+1}V_S. \tag{42}$$

Given that (5) holds, (42) implies (3).

## A.7 Divergent Path

We show that for a path $\{V_s\}_{s=0}^{\infty}$ that satisfies (8) for all $s \geq 0$ and $V_s \to \infty$ as $s \to \infty$, then $\{V_s\}_{s=0}^{\infty}$ violates the boundedness condition (5). Because $V_s$ diverges to infinity, there is a $\tilde{s} > 0$ such that $(1-\delta)\beta(V_s - V_0) \geq \tilde{x}$ for all $s \geq \tilde{s}$. Let $\tilde{f} = u(\tilde{x}) - \tilde{x}$, then for $s \geq \tilde{s}$, we have

$$V_s = \tilde{f} + (1-\delta)\beta V_{s+1}. \tag{43}$$

By (43), for $T > s > \tilde{s}$, we have

$$\begin{aligned} V_s &= \sum_{t=0}^{T-s}(1-\delta)^t\beta^t\tilde{f} + (1-\delta)^{T-s}\beta^{T-s}V_T, \\ V_T &= \sum_{t=0}^{T-s}\frac{-\tilde{f}}{(1-\delta)^t\beta^t} + \frac{V_s}{(1-\delta)^{T-s}\beta^{T-s}},. \\ (1-\delta)^T\beta^T V_T &= \left[-\sum_{t=s}^{T}(1-\delta)^t\beta^t\tilde{f}\right] + (1-\delta)^s\beta^s V_s. \end{aligned}$$

Taking $T$ to infinity, we obtain

$$\lim_{T\to\infty}(1-\delta)^T\beta^T V_T = (1-\delta)^s\beta^s\left[\frac{-\tilde{f}}{1-(1-\delta)\beta}+V_s\right]. \tag{44}$$

Because $V_{s+1} > V_s$, (43) implies

$$V_s > \frac{\tilde{f}}{[1-(1-\delta)\beta]}. \tag{45}$$

Thus, (44) and (45) imply

$$\lim_{T\to\infty}(1-\delta)^T\beta^T V_T > 0,$$

so the path $\{V_s\}_{s=0}^{\infty}$ violates the boundedness condition and does not generate a credit equilibrium.

## A.8 Proof of Lemma 1

We prove by contradiction. Suppose that there is an equilibrium with fraud, and let $(x^*, z^*)$ denote the offer proposed by genuine and virtual agents in the equilibrium, and let $W^{g*}$ and $W^{v*}$ denote the genuine and virtual agents' payoffs in the equilibrium, respectively. We construct an offer $(x', z')$ that satisfies (ICg) to (ICm). The virtual agent's payoff in the equilibrium with fraud must be nonnegative, and this implies that $-d - c + v(x^*) \geq 0$; otherwise the virtual agent would deviate to being idle. Thus, because $c > 0$, we must have $x^* > 0$. Let $x' = x^* - \epsilon > 0$, where $\epsilon > 0$, and let $z'$ satisfy $-x' + (1-\delta)z' = 0$. First, we show that $(x', z')$ satisfies (ICg) if $\epsilon$ is sufficiently small. Because both genuine and virtual agents propose $(x^*, z^*)$ in the equilibrium with fraud, we have $\lambda(x^*, z^*) < 1$; and by (16), we have $-x^* + (1-\delta)\lambda(x^*, z^*)z^* \geq 0$. Moreover, because $x^* > 0$, we have $-d + u(x^*) - x^* > -d + u(x^*) - x^*/\lambda(x^*, z^*)$. If $\epsilon$ is set sufficiently small, we will have

$-d + u(x') - x' > -d + u(x^*) - x^*/\lambda(x^*, z^*)$. Therefore,

$$\begin{aligned} & -d + u(x') - (1-\delta)z' + (1-\delta)\beta(V_1 - V_0) \\ = \quad & -d + u(x') - x' + (1-\delta)\beta(V_1 - V_0) \\ > \quad & -d + u(x^*) - x^*/\lambda(x^*, z^*) + (1-\delta)\beta(V_1 - V_0) \\ \geq \quad & -d + u(x^*) - (1-\delta)z + (1-\delta)\beta(V_1 - V_0) \\ = \quad & W^{g*}. \end{aligned}$$

Moreover, because $x' < x$, we have $-d - c + v(x') < -d - c + v(x) = W^{v*}$, so (ICv) holds. Third, (ICm) holds because by construction, $-x' + (1-\delta)z' = 0$. Finally, because $x' < x$, we have $z' < z \leq \beta(V_1 - V_0)$, so $(x', z') \in \Omega$. Thus, $(x', z')$ satisfies (ICv) to (ICm), and this result excludes the existence of an equilibrium with fraud.

## A.9 Proof of Lemma 2

Suppose that a genuine agent proposes an offer $(x^*, z^*)$ in an equilibrium with no fraud. If a virtual agent proposes an offer $(\tilde{x}, \tilde{z})$ in the equilibrium, the offer must be different from $(x^*, z^*)$; thus, the merchant must believe that an agent that proposes $(\tilde{x}, \tilde{z})$ is a virtual agent, so $\lambda(\tilde{x}, \tilde{z}) = 0$. If $\tilde{x} > 0$, by (16), the offer will be rejected by the merchant, and the virtual agent will obtain a value equal to $-d - c < 0$. Thus, a virtual agent does not propose an offer with $\tilde{x} > 0$. If $\tilde{x} = 0$, by (16), the offer will be accepted by the merchant, but this implies that the virtual agent obtains a value equal to $-d - c + v(\tilde{x}) = -d - c < 0$. Thus, a virtual agent does not propose $\tilde{x} = 0$. Consequently, in an equilibrium with no fraud, a virtual agent must choose to be idle.

## A.10 Proof of Lemma 3

We first prove that the optimal no-fraud offer $(x^\dagger, z^\dagger)$ forms an equilibrium with no fraud if $\tilde{w}^g(x^\dagger, z^\dagger) \geq 0$. Suppose that $(x^\dagger, z^\dagger)$ satisfies (18). We construct the merchant's beliefs as follows. Let the merchant believes that the agent is genuine if the agent propose $(x, z)$ such that $-d - c + v(x) \leq 0$ and $-x + (1-\delta)z \geq 0$; otherwise, the merchant believes that the agent is virtual. If the offer satisfies $-d - c + v(x) \leq 0$ and $-x + (1 - \delta)z \geq 0$, the merchant will accept the offer. If $-d - c + v(x) > 0$, we must have $x > 0$, and because the merchant believes that the agent is virtual in this case, the merchant must reject the offer. If $-x + (1 - \delta)z < 0$, we must also have $x > 0$, and because the merchant believes that the agent is virtual, the merchant must also reject the offer. Thus, given that $-d - c + v(x^\dagger) \leq 0$ and $-x^\dagger + (1 - \delta)z^\dagger \geq 0$, the merchant will accept the offer $(x^\dagger, z^\dagger)$. The genuine agent's payoff from proposing $(x^\dagger, z^\dagger)$ will be equal to $\tilde{w}^g(x^\dagger, z^\dagger)$, and her payoff from proposing any other offer will be 0. Thus, it is the genuine agent's best response to propose $(x^\dagger, z^\dagger)$ if $\tilde{w}^g(x^\dagger, z^\dagger) \geq 0$. Moreover, a virtual agent obtains at most zero by proposing any offer, so it is the virtual agent's best response to be idle.

We prove that the refinement is also satisfied. First, suppose that $V_1 - V_0 > 0$ and that there is an alternative offer $(x', z') \in \Omega$ satisfying (ICv) to (ICm). Let $W^{g\dagger}$ denote the genuine agent's payoff in the original equilibrium. By (ICg), the alternative offer $(x', z')$ generates a payoff greater than $W^{g\dagger}$. By (ICg), $(x', z')$ satisfies the constraint $-x' + (1 - \delta)z' \geq 0$. Moreover, because $(x', z') \in \Omega$, $(x', z')$ also satisfies $z' \leq \beta(V_1 - V_0)$. Because $W^{g\dagger}$ is the solution of problem (P2), given that $(x', z')$ generates a payoff greater than $W^{g\dagger}$ and satisfies constraints $-x' + (1-\delta)z' \geq 0$ and $z' \leq \beta(V_1 - V_0)$, the offer $(x', z')$ must violate the no-fraud constraint, $-d - c + v(x') \leq 0$, and this result contradicts (ICv), so the refinement is satisfied. Second, suppose that $V_1 - V_0 = 0$, then there is only one offer in the feasible set, and this must be the equilibrium offer, and an alternative offer that satisfies (ICv) to (ICm) does not exist, so the refinement is satisfied.

We now show that the optimal no-fraud offer $(x^\dagger, z^\dagger)$ forms an equilibrium with no fraud only if $\tilde{w}^g(x^\dagger, z^\dagger) \geq 0$. We prove by contradiction. Suppose that $(x^\dagger, z^\dagger)$ does not satisfy (18), a genuine agent must deviate to be idle ($a^g = I$) and obtain zero payoff, so $(x^\dagger, z^\dagger)$ cannot form an equilibrium with no fraud.

## A.11 Proof of Lemma 4

We first consider the case under which $V_1 - V_0 > 0$. We prove by contradiction. If $(x^*, z^*)$ forms an equilibrium with no fraud, we must have $-d - c + v(x^*) \leq 0$, $x^* \leq (1 - \delta)z^*$, and $z^* \leq \beta(V_1 - V_0)$. We assume that $(x^*, z^*)$ does not solve (P2), and we show that

$(x^*, z^*)$ will violate the refinement. Because $(x^*, z^*)$ does not solve (P2), we have $W^{g*} < W^{g\dagger}$. Moreover, we have $W^{v*} = 0$ because the value of virtual agents in an equilibrium with no fraud must be zero. Because $\lim_{x\to 0} u'(x) = \infty$ and $V_1 - V_0 > 0$, the optimal no-fraud offer $(x^\dagger, z^\dagger)$ satisfies $x^\dagger, z^\dagger > 0$. Let $x' = x^\dagger - \epsilon > 0$ for some $\epsilon > 0$ and let $z' = x'/(1-\delta)$, then $(x', z')$ satisfies (ICm). Moreover, we take $\epsilon$ to be sufficiently small such that $-d + u(x') - (1-\delta)z' + (1-\delta)\beta(V_1 - V_0) > W^{v*}$, so (ICg) is satisfied. Moreover, because $(x^\dagger, z^\dagger)$ solves (P2), we have $-d - c + v(x^\dagger) \leq 0$; thus, because $x' < x^\dagger$, we have $-d - c + v(x') < 0 = W^{v*}$, so (ICv) holds. Consequently, because $(x', z')$ satisfies (ICv) to (ICm), $(x^*, z^*)$ does not form an equilibrium with no fraud.

We then consider the case that $V_1 - V_0 = 0$. In this case, the only feasible offer is $x = z = 0$, so the equilibrium offer $(x^*, z^*)$ must satisfy $x^* = z^* = 0$, and $(x^*, z^*)$ must solve (P2).

Finally, $(x^*, z^*)$ must satisfy (18) in either case; otherwise the genuine agent would rather choose to be idle.

## A.12 Proof of Proposition 4

We first prove the "if" part of the Proposition. Suppose that $(x, z)$ forms a nondegenerate equilibrium. By Lemma 1, the equilibrium must be no-fraud. Thus, by Lemma 4, the offer must solve (P2) and satisfy (18).

We now prove the "only if" part of the Proposition. Suppose that $(x, z)$ solves (P2) and satisfies (18). By Lemma 3, $(x, z)$ must form a nondegenerate equilibrium.

Finally, by Lemma 2, virtual agents are idle in the nondegenerate equilibrium.

## A.13 Implementability of $\bar{V}$ in the Extensive Model

We analyze under what circumstance the greatest possible lifetime value $\bar{V}$ can be generated in a credit equilibrium with multi-account fraud when $C \to 0$ and $D = 0$. The steady state account value $\hat{V}$ satisfies

$$\hat{V} = u(\hat{x}) - \hat{x} + (1-\delta)\beta\hat{V}, \tag{46}$$

where $\hat{x}$ is the steady state trade volume, and $u'(\hat{x}) = 1/[(1-\delta)\beta]$. Moreover $\bar{V}$ satisfies

$$\hat{x} = (1-\delta)\beta\left(\hat{V} - \bar{V}\right), \tag{47}$$

and because $C \to 0$, $V_1 = \tilde{v}_1^M(\bar{V})$ satisfies

$$\bar{V} = (1-\delta)\beta V_1. \tag{48}$$

By (46), we obtain

$$\hat{V} = \frac{1}{1-(1-\delta)\beta}\left[u(\hat{x}) - \hat{x}\right]. \tag{49}$$

By (47), we obtain

$$\bar{V} = \hat{V} - \frac{\hat{x}}{(1-\delta)\beta}. \tag{50}$$

By (49) and (50),

$$V_1 = \frac{1}{(1-\delta)\beta}\left[\hat{V} - \frac{\hat{x}}{(1-\delta)\beta}\right]. \tag{51}$$

Thus,

$$\begin{aligned} \frac{\bar{V}}{\hat{V}} &= 1 - \frac{1}{(1-\delta)\beta}\frac{\hat{x}}{\hat{V}}, \\ \frac{V_1}{\hat{V}} &= \frac{1}{(1-\delta)\beta}\left[1 - \frac{1}{(1-\delta)\beta}\frac{\hat{x}}{\hat{V}}\right]. \end{aligned}$$

By (49),

$$\frac{V_1}{\hat{V}} \geq 1 \iff \frac{u(\hat{x}) - \hat{x}}{\hat{x}} \geq \frac{1}{(1-\delta)\beta}.$$

We observe that whether $\bar{V}$ can be generated depends on the relative size of the steady state trade surplus, $u(\hat{x}) - \hat{x}$, and the steady state trade volume, $\hat{x}$. Note that their relative size can be determined by the curvature of the utility function. If the curvature is greater, the trade surplus relative to the trade volume will be greater, and this will result in a smaller $\bar{V}$ to $\hat{V}$ ratio. Because $V_1$ is a fixed proportion of $\bar{V}$, a greater curvature also results in a smaller ratio of $V_1$ to $\hat{V}$. Thus, if the curvature of the utility function is sufficiently large, $V_1$ will be greater than $\hat{V}$, meaning that $\bar{V}$ will not be generated by a credit equilibrium. We take a CRRA utility function for example. Let

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma}, \gamma \in (0,1).$$

Then

$$\hat{x} = [(1-\delta)\beta]^{\frac{1}{\gamma}},$$

and

$$u(\hat{x}) - \hat{x} = [(1-\delta)\beta]^{\frac{1}{\gamma}} \left[ \frac{[(1-\delta)\beta]^{-1}}{1-\gamma} - 1 \right].$$

We observe that

$$\frac{V_1}{\hat{V}} \gtreqless 1 \iff \gamma \gtreqless 1 - \frac{1}{1+[(1-\delta)\beta]}.$$

# B Figures

In all figures, the utility function is $u(x) = x^{1-\gamma}/(1-\gamma)$, and the parameters are: $\beta = 0.9$, $\delta = 0.5$, $\gamma = 0.5$.

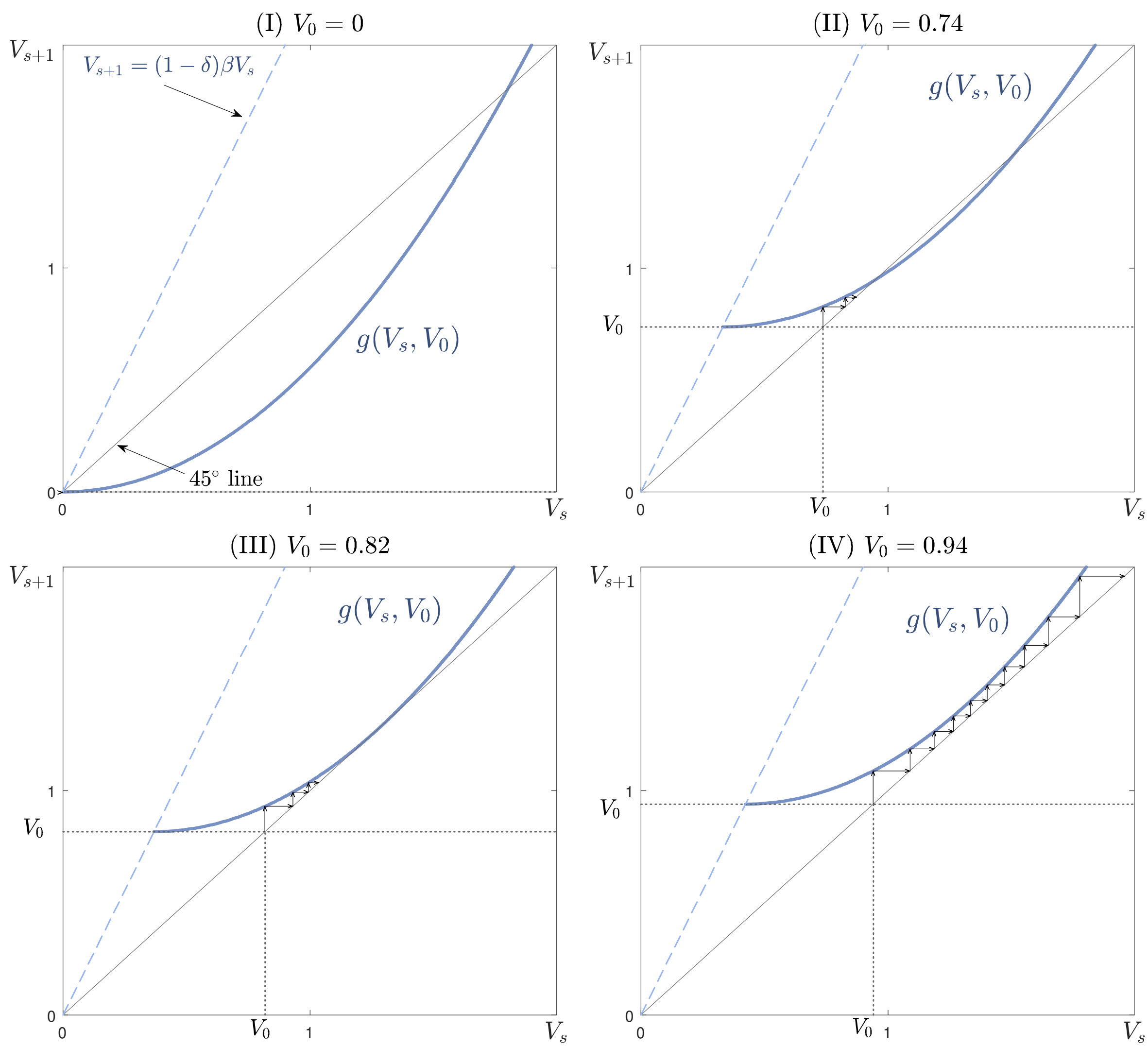


Figure 1: Dynamics of account values: benchmark model.

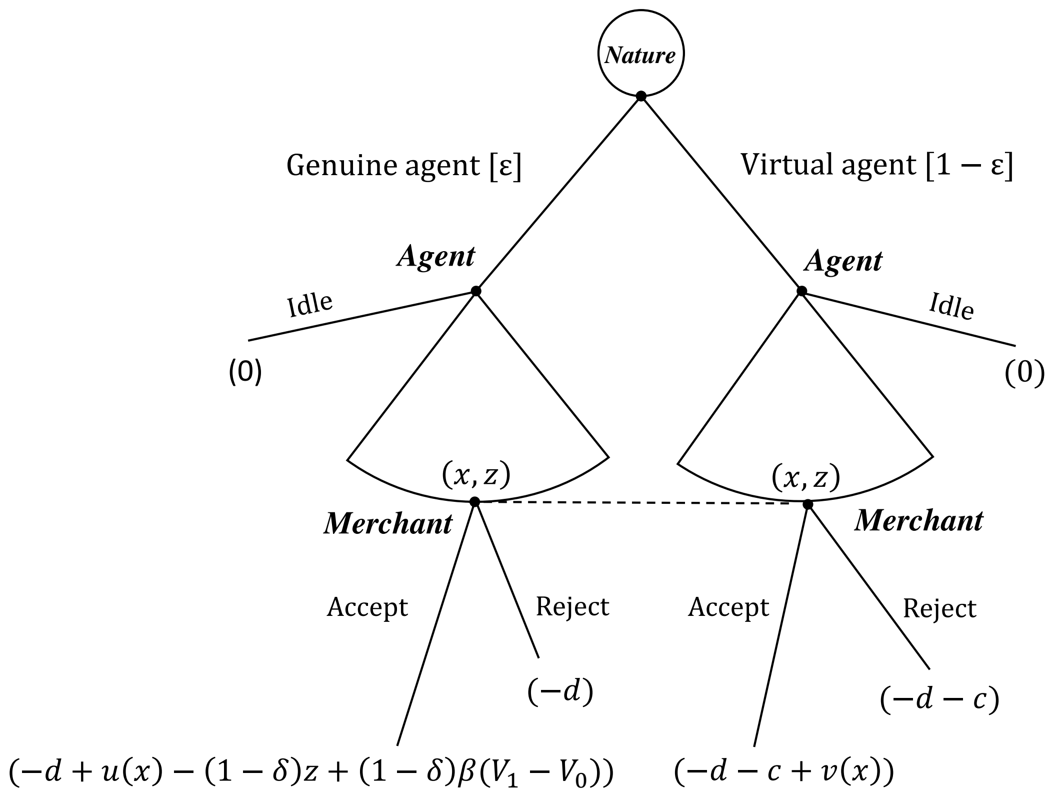


Figure 2: Bargaining game with incomplete information

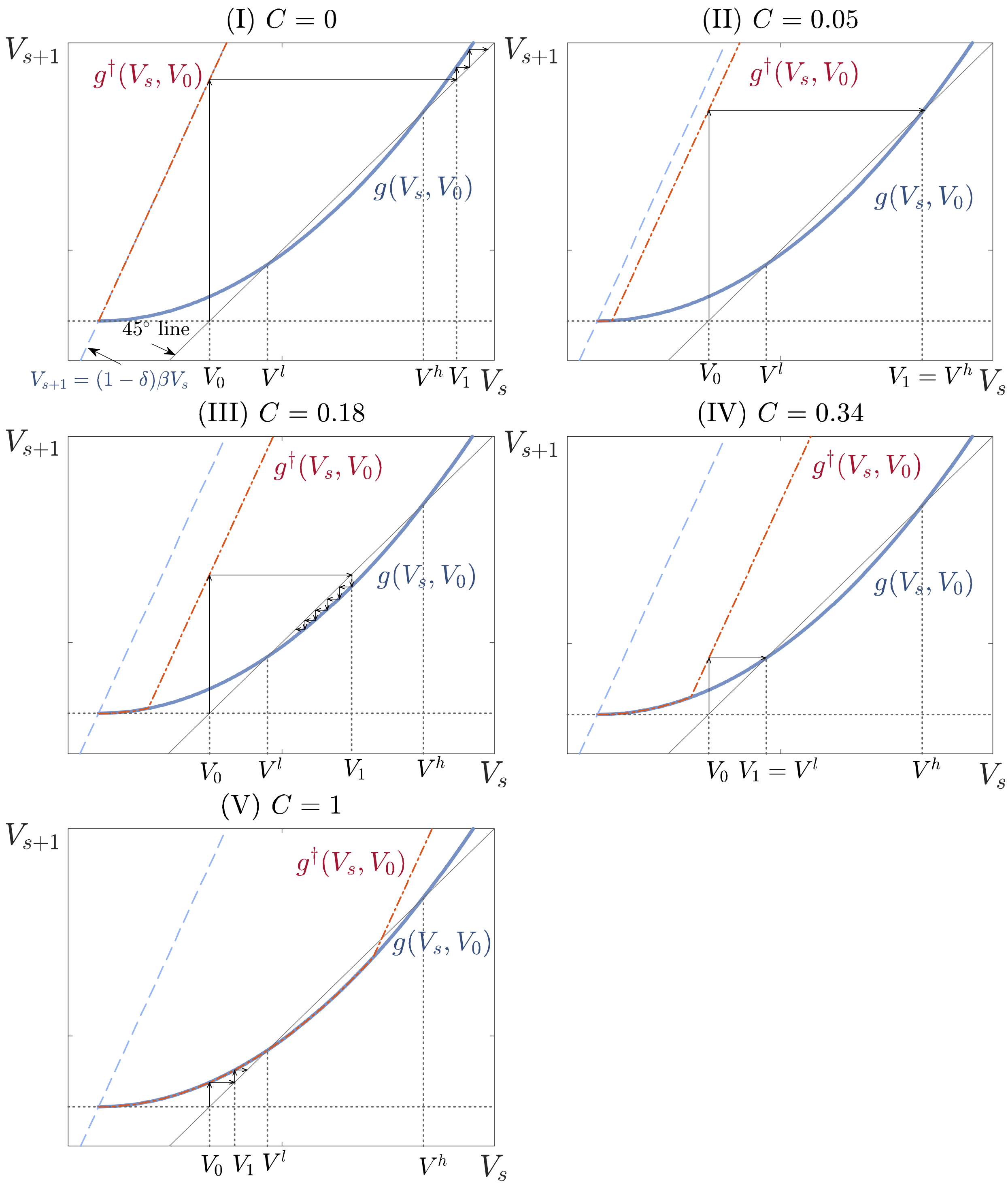


Figure 3: Dynamics of account values under various entry costs (virtual agents' utility discount: $\alpha = 0.4$).

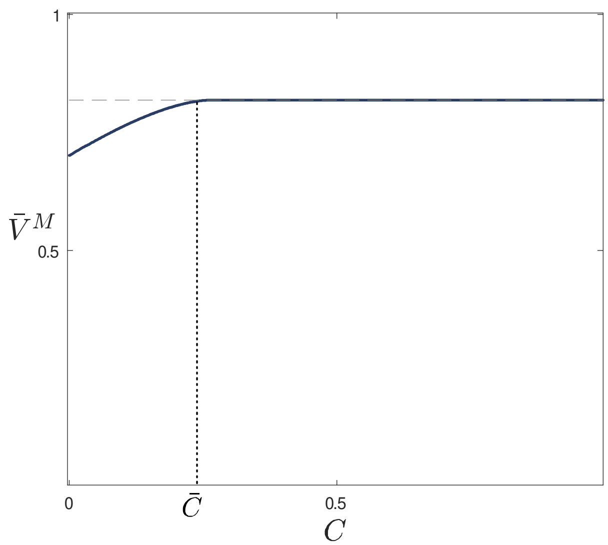


Figure 4: Effects of deterrence activity on the upper bound of lifetime values (virtual agents' utility discount: $\alpha = 0.4$).

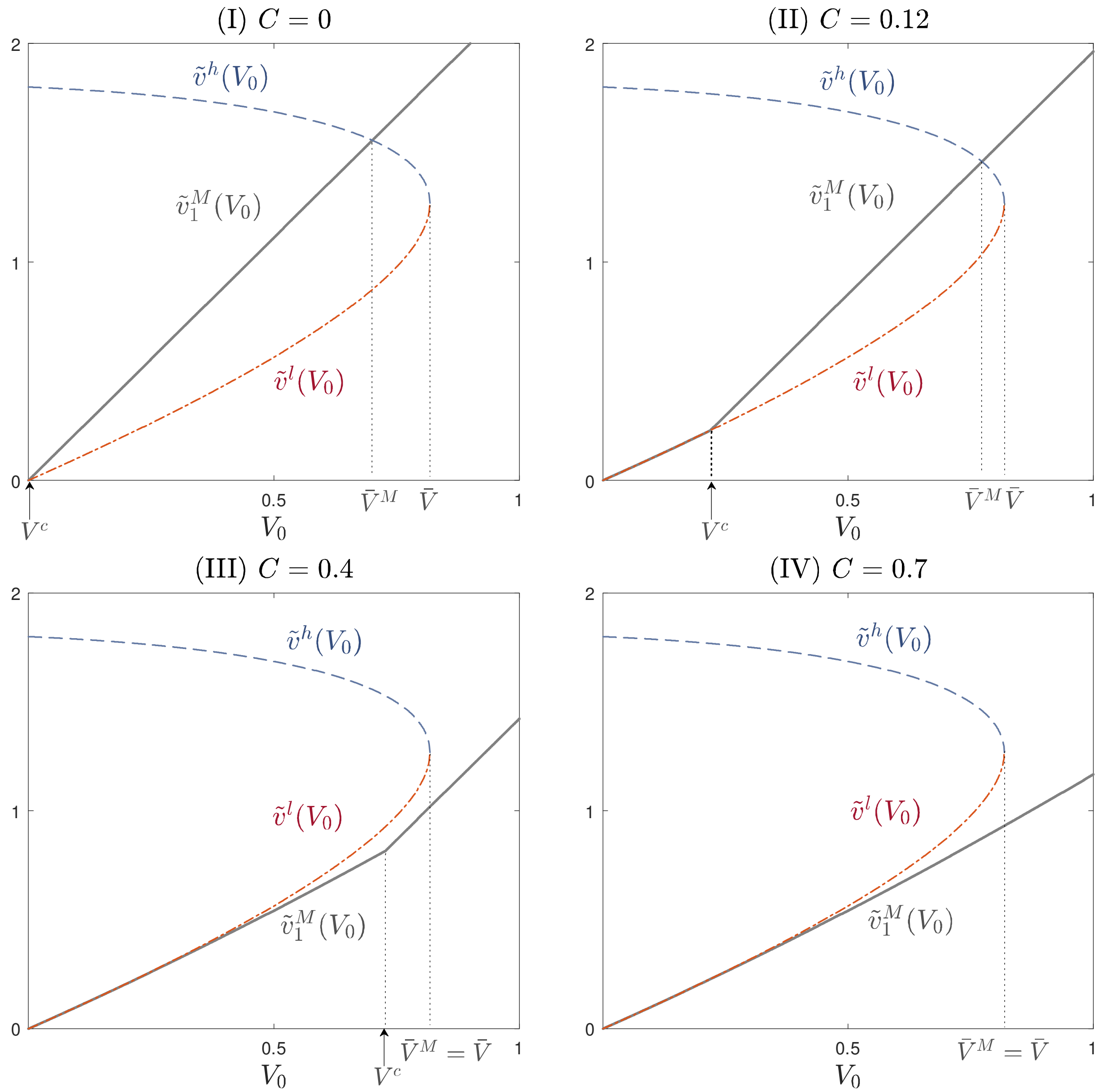


Figure 5: The functions $\tilde{v}_1^M(V_0)$, $\tilde{v}^h(V_0)$, and $\tilde{v}^l(V_0)$ under various entry costs (virtual agents' utility discount: $\alpha = 0.4$).

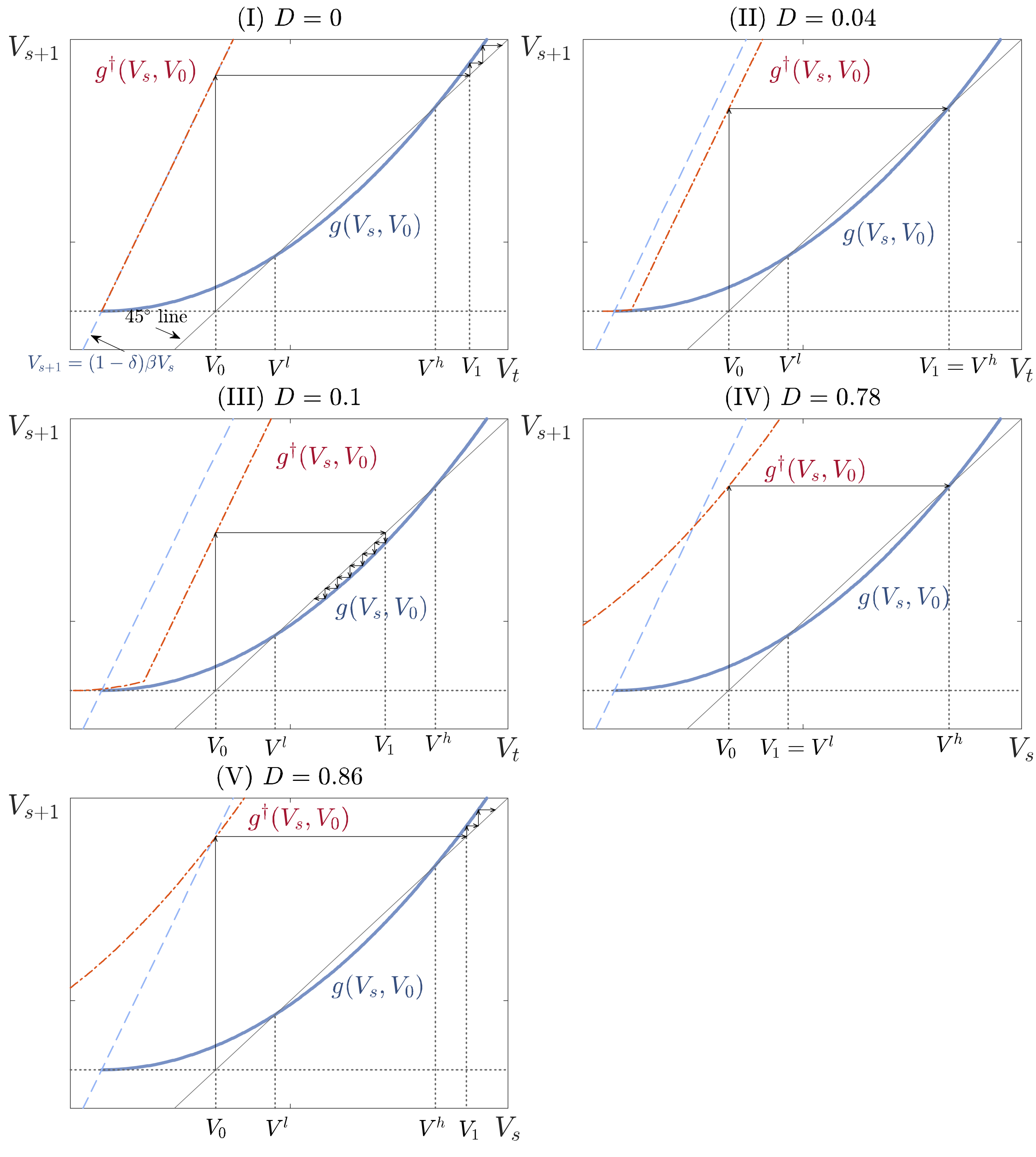


Figure 6: Dynamics under various deterrence activities (virtual agents' utility discount: $\alpha = 0.4$; the entry cost: $C = 0$).

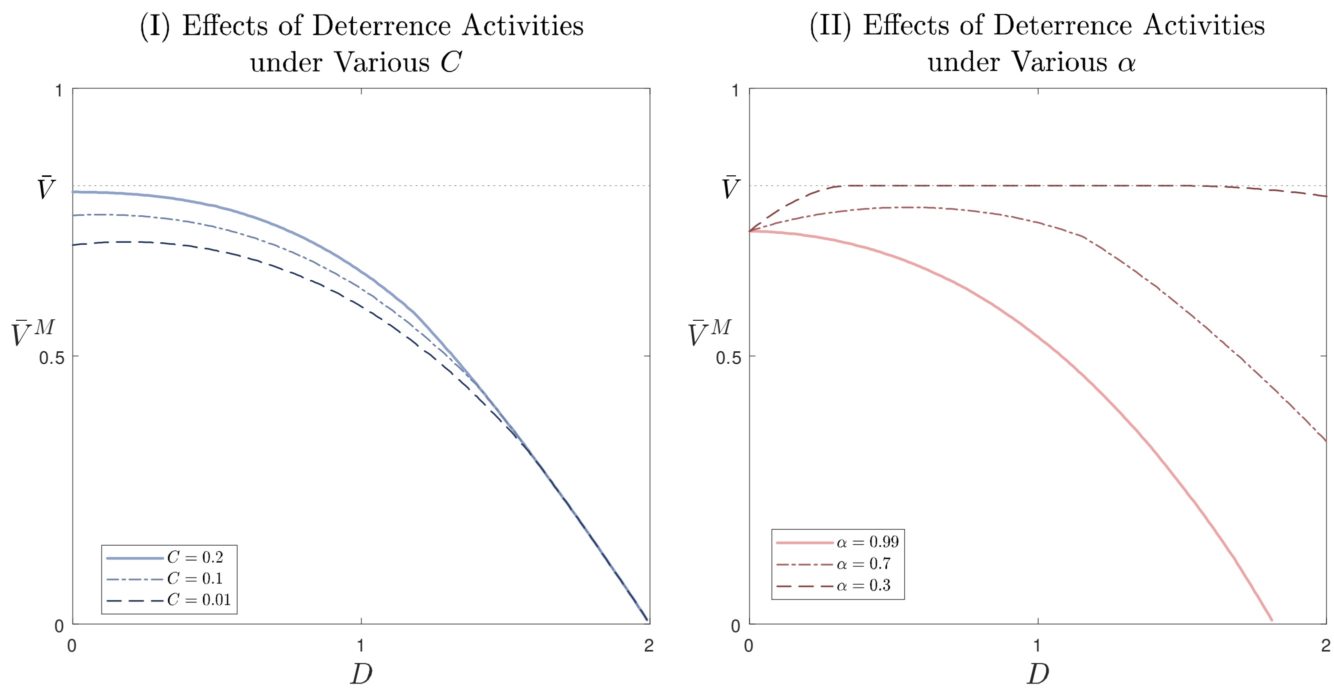


Figure 7: Effects of deterrence activity on the upper bound of lifetime values. (Left panel: $\alpha = 0.9$. Right panel: $C = 0.05$)

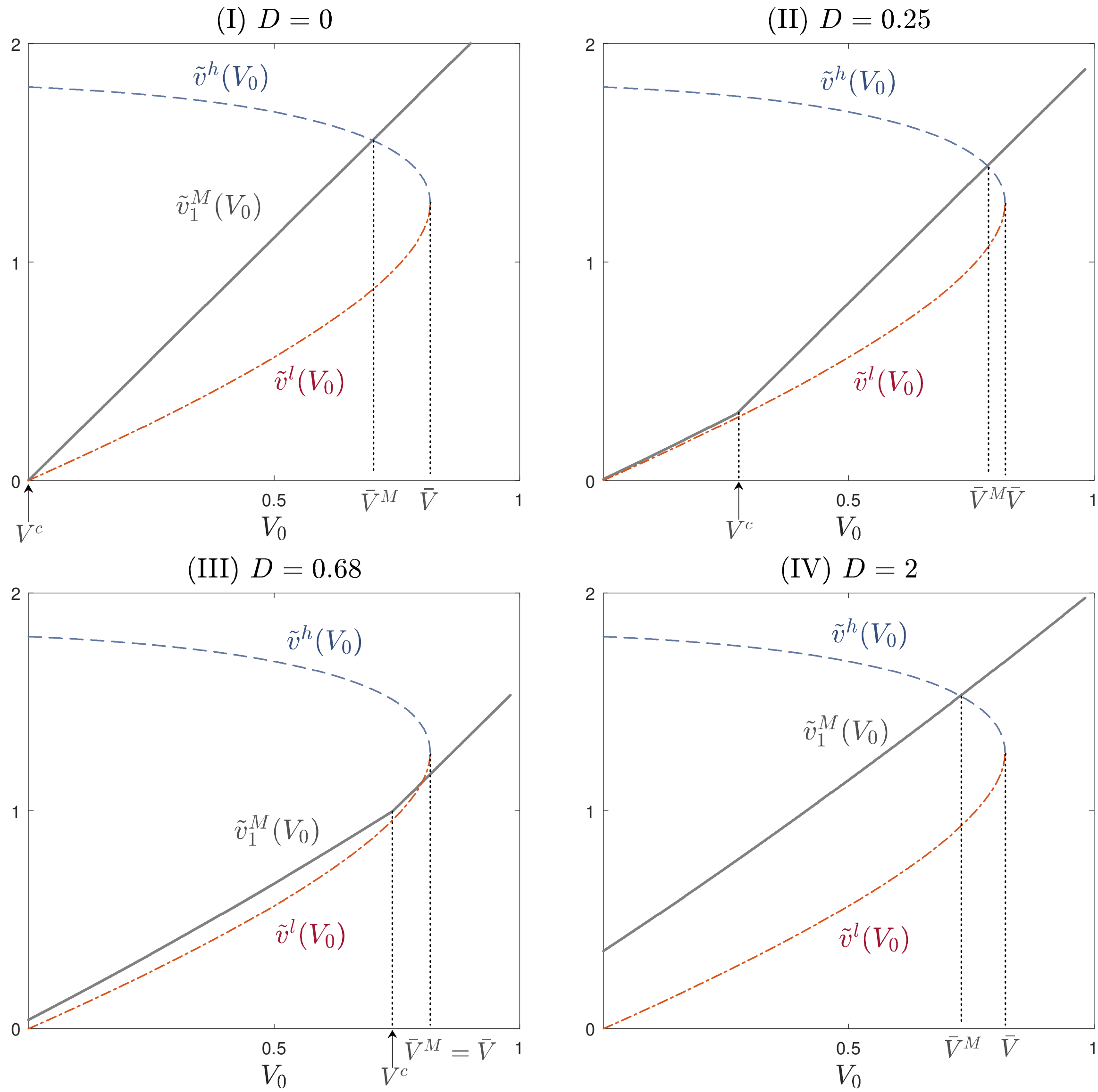


Figure 8: The functions $\tilde{v}_1^M(V_0)$, $\tilde{v}^h(V_0)$, and $\tilde{v}^l(V_0)$ under various size of deterrence activity (virtual agents' utility discount: $\alpha = 0.4$; the entry cost: $C = 0$).

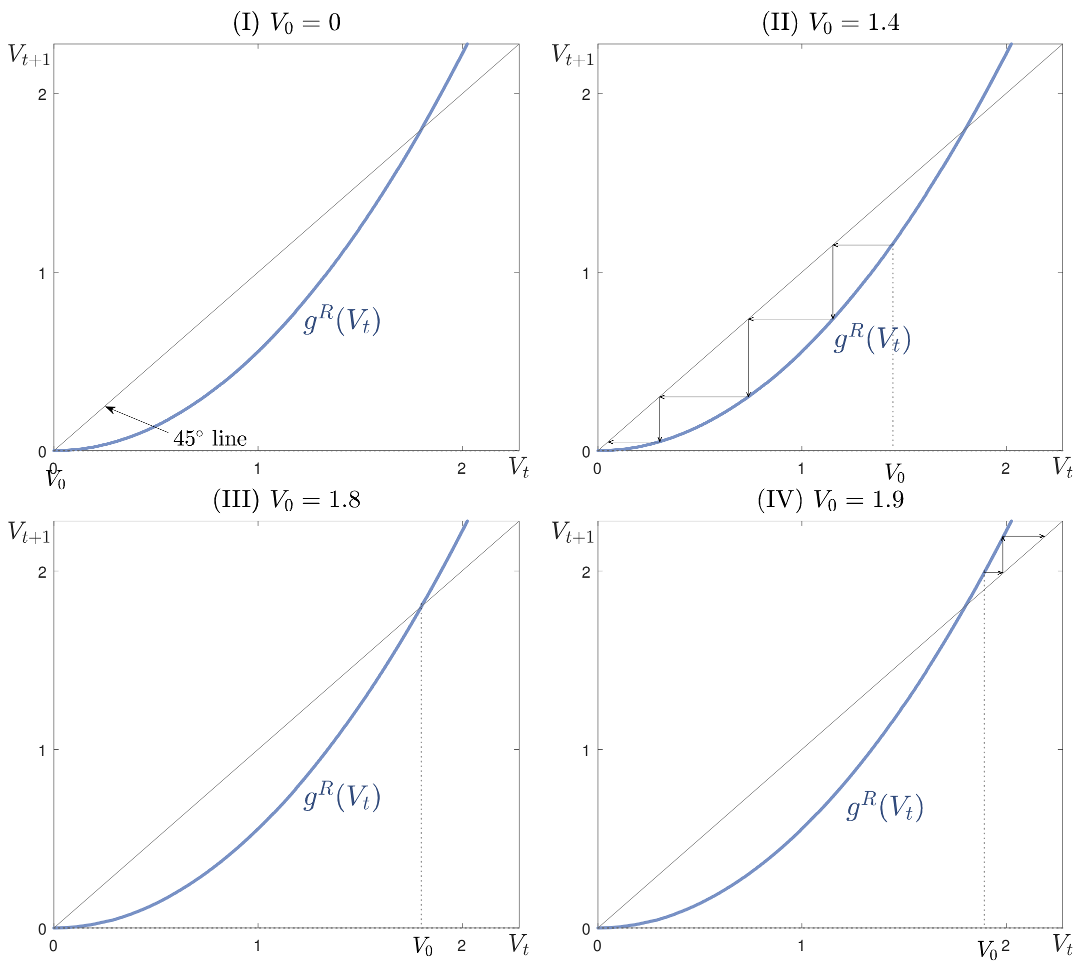


Figure 9: Dynamics of account values: true identity credit.