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Abstract

This paper considers an international mixed duopoly model in which a state-owned public firm competes against a foreign labor-managed firm. The paper investigates endogenous roles of the firms by adopting the observable delay game and shows that the state-owned public firm should never play the role of Staclkelberg leader.

Keywords: Endogenous timing; Foreign labor-managed firm; International mixed duopoly; Stackelberg
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1. Introduction

This paper examines the endogenous order of moves in an international mixed duopoly by adopting the observable delay game by Hamilton and Slutsky (1990), where each firm first chooses the timing of choosing its output level. Pal (1998) considers the endogenous timing in a mixed oligopoly model in which a state-owned public firm competes with domestic private firms, and shows that the state-owned public firm should be the follower. Matsumura (2003) examines the endogenous order of moves in a mixed duopoly model where a state-owned public firm competes against a foreign private firm, and shows that, in contrast to Pal (1998), the state-owned public firm should become the leader. Lu (2007) examines the issue of endogenous order of moves in a mixed oligopoly consisting of a single state-owned public firm and foreign private competitors, and shows that there is no subgame perfect equilibrium outcome where the state-owned public firm produces simultaneously with all foreign private firms. Lu and Poddar (2009) investigate a game of endogenous timing with observable delay in the context of sequential capacity and quantity choice, and show that the state-owned public firm and the domestic private firm choose capacity and quantity sequentially in all possible equilibria. In addition, Bárcena-Ruiz (2007) considers a mixed duopoly in which a state-owned public firm and a domestic private firm choose whether to set prices simultaneously or sequentially, and shows that the firms choose prices simultaneously. He finds that the result obtained in the mixed duopoly under price competition differs from the one under quantity competition. However, these studies do not include labor-managed profit-per-worker maximizing firms.

Therefore, Ohnishi (2012) considers the endogenous timing in a mixed duopoly model where a state-owned public firm competes with a domestic labor-managed firm, and shows that the unique equilibrium coincides with the Stackelberg solution where the domestic labor-managed firm is the leader. As a result, it is found that the state-owned public firm cannot play the role of Stackelberg leader.

In the present paper, we investigate the issue of endogenous order of moves in an international mixed duopoly where a state-owned public firm competes against a foreign labor-managed firm. The timing of the game is as follows. Both firms first announce in which stage they will choose output. Next, if both firms decide to choose output in the same stage, a simultaneous move game occurs, whereas if both firms decide to choose output in different stages, a sequential move game arises. We present the subgame perfect equilibrium of the international mixed duopoly model.

The remainder of this paper proceeds as follows. In Section 2, we describe the model. Section 3 gives supplementary explanations of the model. Section 4 presents the equilibrium of the model. Finally, a conclusion is stated in Section 5.

2. Model

A mixed duopoly model is considered with one state-owned public firm (firm S) and one foreign labor-managed firm (firm FL), both producing perfectly substitutable commodities. Throughout this paper, subscripts S and FL represent firm S and firm FL, respectively. In addition, when i and j are used to represent firms in an expression, they should be understood to refer to S and FL with $i \neq j$. There is no possibility of entry or exit. The market price is determined by the inverse demand (price) function $P(X)$, where $X = x_{\mathrm{S}} + x_{\mathrm{FL}}$. We assume that $P'' \leq 0$.

Each firm's profit π_i is given by

$$\pi_i(x_{\mathrm{S}}, x_{\mathrm{FL}}) = P(X)x_i - c_i(x_i) - f_i$$

where $c_i(x_i)$ represents the production cost of firm i and f_i is the fixed cost of firm i. We assume that $c_i'' \geq 0$.

Domestic social surplus S is given by

$$S(x_{\mathrm{S}}, x_{\mathrm{FL}}) = \int_0^{x_{\mathrm{S}}+x_{\mathrm{FL}}} P(q)dq - \left[P(X)x_{\mathrm{FL}} + c_{\mathrm{S}}(x_{\mathrm{S}})\right]. \tag{1}$$

Firm S chooses x_{S} to maximize (1).

On the other hand, firm FL chooses x_{FL} to maximize its profit per worker:

$$\omega_{\mathrm{FL}}(x_{\mathrm{S}}, x_{\mathrm{FL}}) = \frac{P(X)x_{\mathrm{FL}} - c(x_{\mathrm{FL}}) - f_{\mathrm{FL}}}{l(x_{\mathrm{FL}})}, \tag{2}$$

where l denotes the quantity of labor utilized. We assume that $l'' > 0$. This assumption meanes that the marginal quantity of labor utilized is increasing.

The timing of the game is as follows. In the first stage, each firm i simultaneously and independently chooses $e_i \in (2,3)$, where e_i indicates when to produce x_i. $e_i = 2$ impllies that firm i produces in the second stage, and $e_i = 3$ implies that it produces in the third stage. At the end of the first stage, each firm observes e_{S} and e_{FL}. In the second stage, firm i choosing $e_i = 2$ selects its output x_i in this stage. At the end of the second stage, each firm observes the output of the rival if the rival chooses to produces in the second stage. In the third stage, firm i choosing $e_i = 3$ selects its output x_i. At the end of the game, the market opens and each firm i sells its output x_i. Throughout this paper, we use subgame perfection as our equilibrium concept.

3. Supplementary explanations

First, we consider the reaction function of firm S. Firm S aims to maximize domestic social surplus with respect to x_{S}, given x_{FL}. The reaction function of firm S is derived from the following first-order condition:

$$P - c_{\mathrm{S}}' - P' x_{\mathrm{FL}} = 0. \tag{3}$$

Furthermore, the following second-order condition is satisfied:

$$P' - c_{\mathrm{S}}'' - P'' x_{\mathrm{FL}} < 0. \tag{4}$$

Therefore, we obtain the reaction function of firm S:

$$R_{\mathrm{S}}'(x_{\mathrm{FL}}) = \frac{P'' x_{\mathrm{FL}}}{P' - c_{\mathrm{S}}'' - P'' x_{\mathrm{FL}}}. \tag{5}$$

We can now state the following lemma.

Lemma 1: If $P'' < 0$*, the slope of* $R_{\mathrm{S}}(x_{\mathrm{FL}})$ *is positive, while if* $P'' = 0$*, the slope of* $R_{\mathrm{S}}(x_{\mathrm{FL}})$ *is zero.*

Second, we consider the reaction function of firm FL. Firm FL aims to maximize its profit per worker with respect to x_{FL}, given x_{S}. The reaction function of firm FL is derived from the following first-order condition:

$$\left(P + P' x_{\mathrm{FL}} - c_{\mathrm{FL}}'\right) l_{\mathrm{FL}} - \left(P x_{\mathrm{FL}} - c_{\mathrm{FL}} - f_{\mathrm{FL}}\right) l_{\mathrm{FL}}' = 0. \tag{6}$$

Furthermore, the second-order condition is satisfied:

$$\left(2P' + P'' x_{\mathrm{FL}} - c_{\mathrm{FL}}''\right) l_{\mathrm{FL}} - \left(P x_{\mathrm{FL}} - c_{\mathrm{FL}} - f_{\mathrm{FL}}\right) l_{\mathrm{FL}}'' < 0. \tag{7}$$

Therefore, we have the reaction function of firm FL:

$$R_{\mathrm{FL}}'(x_{\mathrm{S}}) = -\frac{P'' x_{\mathrm{FL}} l_{\mathrm{FL}} + \left(l_{\mathrm{FL}} - x_{\mathrm{FL}} l_{\mathrm{FL}}'\right) P'}{\left(2P' + P'' x_{\mathrm{FL}} - c_{\mathrm{FL}}''\right) l_{\mathrm{FL}} - \left(P x_{\mathrm{FL}} - c_{\mathrm{FL}} - f_{\mathrm{FL}}\right) l_{\mathrm{FL}}''}. \tag{8}$$

Since $l_{\mathrm{FL}}'' > 0$, $l_{\mathrm{FL}} - x_{\mathrm{FL}} l_{\mathrm{FL}}'$ is negative. Hence, we can present the following lemma.

Lemma 2: The slope of $R_{\mathrm{FL}}(x_{\mathrm{S}})$ *is positive.*

Third, we consider Stackelberg games. If firm i is the Stackelberg leader, then firm i selects x_i, and firm j selects x_j after observing x_i. Firm i maximizes $(x_i, R_j(x_i))$ with respect to x_i. We present the following lemma, where the superscripts L, F and C denote the Stackelberg equilibrium outcome where firm S is the leader, the Stackelberg equilibrium outcome where firm S is the follower, and the Cournot equilibrium outcome, respectively.

Lemma 3: (i) $x_{\mathrm{S}}^L > x_{\mathrm{S}}^C$*, (ii)* $x_{\mathrm{FL}}^C \ge x_{\mathrm{FL}}^F$*, (iii)* $x_{\mathrm{S}}^C \ge x_{\mathrm{S}}^F$*, and (iv)* $x_{\mathrm{FL}}^L > x_{\mathrm{FL}}^C$.

Proof: (i) If firm S is the leader, then it chooses x_{S} so as to maximize $S(x_{\mathrm{S}}, R_{\mathrm{FL}}(x_{\mathrm{S}}))$, which satisfies the first-order condition:

$$P - c_{\mathrm{S}}' - P'x_{\mathrm{FL}} - P'x_{\mathrm{FL}}R_{\mathrm{FL}}' = 0 . \tag{9}$$

Here, $P' < 0$, and $R_{\mathrm{FL}}' > 0$ (Lemma 2). To satisfy (9), $P - c_{\mathrm{S}}' - P'x_{\mathrm{FL}}$ must be negative.

(ii) If firm FL is the leader, then it chooses x_{FL} so as to maximize $\omega_{\mathrm{FL}}(R_{\mathrm{S}}(x_{\mathrm{FL}}), x_{\mathrm{FL}})$, which satisfies the first-order condition:

$$\left(P + P'x_{\mathrm{FL}} - c_{\mathrm{FL}}'\right)l_{\mathrm{FL}} - \left(Px_{\mathrm{FL}} - c_{\mathrm{FL}} - f_{\mathrm{FL}}\right)l_{\mathrm{FL}}' + P'x_{\mathrm{FL}}R_{\mathrm{S}}' = 0 . \tag{10}$$

Here, $P' < 0$, and $R_{\mathrm{S}}' \ge 0$ (Lemma 1), and therefore, $\left(P + P'x_{\mathrm{FL}} - c_{\mathrm{FL}}'\right)l_{\mathrm{FL}} - \left(Px_{\mathrm{FL}} - c_{\mathrm{FL}} - f_{\mathrm{FL}}\right)l_{\mathrm{FL}}' \ge 0$.

(iii) This follows from Lemma 1 and Lemma 3 (ii).

(iv) This follows from Lemma 2 and Lemma 3 (i). Q.E.D.

4. Equilibrium

Before discussing the equilibrium in the model presented in Section 2, we prove the following two propositions.

Proposition 1: $S(x_{\mathrm{S}}^L, x_{\mathrm{FL}}^L) > S(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C) \ge S(x_{\mathrm{S}}^F, x_{\mathrm{FL}}^F)$.

Proof: First, we prove that $S(x_{\mathrm{S}}^L, x_{\mathrm{FL}}^L) > S(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C)$. If firm S is the leader, then it maximizes domestic social surplus with respect to x_{S}. Since firm S can choose $x_{\mathrm{S}} = x_{\mathrm{S}}^C$, we obtain $S(x_{\mathrm{S}}^L, x_{\mathrm{FL}}^L) \ge S(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C)$. We now show that $S(x_{\mathrm{S}}^L, x_{\mathrm{FL}}^L) \ne S(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C)$ by showing that $x_{\mathrm{S}}^L \ne x_{\mathrm{S}}^C$. From Lemma 3 (i), we see that $S(x_{\mathrm{S}}^L, x_{\mathrm{FL}}^L) > S(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C)$.

Next, we prove that $S(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C) \ge S(x_{\mathrm{S}}^F, x_{\mathrm{FL}}^F)$. We consider the game where firm S is the follower and firm FL is the leader. Lemma 3 (ii) shows that $x_{\mathrm{FL}}^C \ge x_{\mathrm{FL}}^F$. If $x_{\mathrm{FL}}^C = x_{\mathrm{FL}}^F$, then $S(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C) = S(x_{\mathrm{S}}^F, x_{\mathrm{FL}}^F)$ from $S(R_{\mathrm{S}}(x_{\mathrm{FL}}), x_{\mathrm{FL}})$. If $x_{\mathrm{FL}}^F < x_{\mathrm{FL}}^C$, since $\partial S/\partial x_{\mathrm{FL}} = -P'x_{\mathrm{FL}} > 0$, decreasing x_{FL} decreases domestic social surplus. Q.E.D.

The intuition behind Proposion 1 is as follows. Since firm S (the leader) can choose $x_{\mathrm{S}} = x_{\mathrm{S}}^C$, we see that $S(x_{\mathrm{S}}^L, x_{\mathrm{FL}}^L) \geq S(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C)$. Furthermore, since $x_{\mathrm{S}}^L > x_{\mathrm{S}}^C$ (Lemma 3 (i)), firm S (the leader) increases its output, and hence domestic social surplus increases. Proposition 1 (i) means that firm S has the leader's advantage. On the other hand, since $x_{\mathrm{FL}}^C \geq x_{\mathrm{FL}}^F$ (Lemma 3 (ii)), we can easily guess that $S(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C) \geq S(x_{\mathrm{S}}^F, x_{\mathrm{FL}}^F)$.

Proposition 2: $\omega_{\mathrm{FL}}(x_{\mathrm{S}}^F, x_{\mathrm{FL}}^F) \geq \omega_{\mathrm{FL}}(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C) > \omega_{\mathrm{FL}}(x_{\mathrm{S}}^L, x_{\mathrm{FL}}^L)$.

Proof: First, we prove that $\omega_{\mathrm{FL}}(x_{\mathrm{S}}^F, x_{\mathrm{FL}}^F) \geq \omega_{\mathrm{FL}}(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C)$. By definition, Stackelberg leader's payoff is never smaller than the payoff at Cournot equilibrium.

Next, we prove that $\omega_{\mathrm{FL}}(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C) > \omega_{\mathrm{FL}}(x_{\mathrm{S}}^L, x_{\mathrm{FL}}^L)$. We now consider the game where firm S is the leader and firm FL is the follower. Lemma 3 (i) shows that $x_{\mathrm{S}}^L > x_{\mathrm{S}}^C$. Since $\partial\omega_{\mathrm{FL}} / \partial x_{\mathrm{S}} = P' x_{\mathrm{FL}} / l < 0$, increasing x_{S} decreases ω_{FL}. Q.E.D.

Proposition 2 indicates that firm FL should not be the follower. The intuition behind Proposition 2 is straightforward. If firm FL is the leader, since it can choose $x_{\mathrm{FL}} = x_{\mathrm{FL}}^C$, we have $\omega_{\mathrm{FL}}(x_{\mathrm{S}}^F, x_{\mathrm{FL}}^F) \geq \omega_{\mathrm{FL}}(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C)$. On the other hand, if firm S is the leader, then it increases x_{S} (Lemma 3 (i)). Increasing x_{S} decreases x_{FL} because of substitute goods, and moreover decreasing x_{FL} decreases ω_{FL}.

We now present the equilibrium of the international mixed duopoly model with observable delay.

Proposition 3: In the equilibrium,. (i) $e_{\mathrm{S}} = e_{\mathrm{FL}} = 2$*; (ii)* $e_{\mathrm{S}} = 3$ *and* $e_{\mathrm{FL}} = 2$.

Proof: At the first stage, each firm i simultaneously and independently chooses $e_i \in (2,3)$. At the second stage, firm i choosing $e_i = 2$ selects x_i in this stage. At the third stage, if firm i chooses $e_i = 3$, it selects x_i. At the end of the game, each firm i sells x_i. Our equilibrium concept is subgame perfection, and all information in the model is common knowledge. Hence, we can consider the following payoff matrix:

		Firm FL Stage 2	Firm FL Stage 3
Firm S	Stage 2	$S(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C), \omega_{\mathrm{FL}}(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C)$	$S(x_{\mathrm{S}}^L, x_{\mathrm{FL}}^L), \omega_{\mathrm{FL}}(x_{\mathrm{S}}^L, x_{\mathrm{FL}}^L)$
	Stage 3	$S(x_{\mathrm{S}}^F, x_{\mathrm{FL}}^F), \omega_{\mathrm{FL}}(x_{\mathrm{S}}^F, x_{\mathrm{FL}}^F)$	$S(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C), \omega_{\mathrm{FL}}(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C)$

From Propositions 1 and 2, we see that $S(x_{\mathrm{S}}^L, x_{\mathrm{FL}}^L) > S(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C) \geq S(x_{\mathrm{S}}^F, x_{\mathrm{FL}}^F)$ and $\omega_{\mathrm{FL}}(x_{\mathrm{S}}^F, x_{\mathrm{FL}}^F) \geq \omega_{\mathrm{FL}}(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C) > \omega_{\mathrm{FL}}(x_{\mathrm{S}}^L, x_{\mathrm{FL}}^L)$. If $P'' < 0$, then $S(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C) > S(x_{\mathrm{S}}^F, x_{\mathrm{FL}}^F)$ (Lemma 1, Lemma 3 (iii) and Proposition 1) and $\omega_{\mathrm{FL}}(x_{\mathrm{S}}^F, x_{\mathrm{FL}}^F) > \omega_{\mathrm{FL}}(x_{\mathrm{S}}^C, x_{\mathrm{FL}}^C)$ (Lemma 1, Lemma 3 (ii) and Proposition 2), so that there is an equilibrium where $e_{\mathrm{S}} = e_{\mathrm{FL}} = 2$. On the other hand, if $P'' = 0$, then $S(x_{\mathrm{S}}^{C}, x_{\mathrm{FL}}^{C}) = S(x_{\mathrm{S}}^{F}, x_{\mathrm{FL}}^{F})$ (Lemma 3 (iii) and Proposition 1) and $\omega_{\mathrm{FL}}(x_{\mathrm{S}}^{F}, x_{\mathrm{FL}}^{F}) = \omega_{\mathrm{FL}}(x_{\mathrm{S}}^{C}, x_{\mathrm{FL}}^{C})$ (Lemma 3 (ii) and Proposition 2), so that there are two equilibrium solutions: (i) $e_{\mathrm{S}} = e_{\mathrm{FL}} = 2$; (ii) $e_{\mathrm{S}} = 3$ and $e_{\mathrm{FL}} = 2$. Thus, this proposition is proved. Q.E.D.

Proposition 3 indicates that in the equilibrium outcomes firm S never plays the role of Stackelberg leader. Matsumura (2003) examines the endogenous timing in a mixed duopoly model where a state-owned public firm competes against a foreign profit-maximizing private firm, and shows that in the unique equilibrium the state-owned public firm plays the role of Stackelberg leader. Therefore, we find that our result makes a sharp contrast with that of Matsumura (2003).

5. Conclusion

In this paper, we have considered an international mixed duopoly model in which a state-owned public firm competes against a foreign labor-managed firm. We have examined endogenous roles of the firms by adopting the observable delay game and have shown that in the state-owned public firm should never play the role of Stackelberg leader.

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