Endogenous privacy and heterogeneous price sensitivity

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Abstract

This study considers an incumbent firm and a newcomer competing in an old market and a new one in a modified Hotelling model. Consumers must pay a privacy cost to conceal their personal information. Otherwise, their personal information is left in the old market, and only the incumbent firm can utilize it. If consumers have homogeneous price sensitivity, both consumer and total surpluses are increasing with the privacy cost. However, if consumers' price sensitivity is sufficiently heterogeneous, both consumer and total surpluses are maximized when the privacy cost is zero. If heterogeneity is intermediate, consumer (total) surplus is maximized with zero (at a positive) privacy cost. Clearly, authorities should pay close attention to the heterogeneity in price sensitivity while deciding on privacy regulation.

JEL codes: D43, L10, L13.

Keywords: personalized pricing, privacy, personal information, heterogeneous consumers, Hotelling model.

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1 Introduction

In recent years, with the spread of the Internet, firms have gained ability to offer discriminatory prices based on consumers' personal information (Mattioli 2012).[1] In response, the European Union (EU) implemented the General Data Protection Regulation (GDPR) in 2018 to protect consumers' personal data. However, previous studies find counter-intuitive results that both consumer and total surpluses are highest when policymakers take little or no action on data privacy (Montes et al. 2019; Valletti and Wu 2020).[2]

The logic in prior research is as follows: As the privacy cost increases, consumers who remain in the anonymous market must pay higher privacy cost to keep their anonymity. In addition, more consumers are perfectly profiled by the firm, and consequently, forced to purchase the good at personalized prices. However, as the privacy cost increases, the reservation price (net of the privacy cost) of consumers who remain in the anonymous market decreases. This leads to fierce competition, and thus, an increase in consumer surplus in the anonymous market. The latter effect dominates the former if consumers' price sensitivity is sufficiently homogeneous so that an equilibrium price is high and hence a reduction in a reservation price results in a large price reduction. This logic explains why prior research finds the aforementioned result.

This study demonstrates that both consumer and total surpluses DECREASE with the privacy cost if consumers have sufficiently heterogeneous price sensitivity. Since consumers in developed countries have diverse needs (i.e., heterogeneous price sensitivity), the regulators in these countries should protect consumers' personal information and reduce the privacy cost as much as possible (like the EU).[3] Conversely, consumers in

[1]Mattioli (2012) reported that Orbitz Worldwide, a travel agency, offers higher hotel prices to Mac users than other PC users.

[2]Montes et al. (2019) show that consumer surplus in a duopolistic market is an increasing function of the privacy cost, which is the cost of concealing personal information. Valletti and Wu (2020) show that consumer surplus is a U-shaped function of the privacy cost and is maximized when the privacy cost is the highest.

[3]Choe et al. (2023) also justify the GDPR using a model different from ours.

developing countries are homogeneous because their incomes are low and they tend to focus on price.[4] Therefore, the best policy is to keep the privacy cost high. Indeed, the Chinese government has taken little action to reduce the privacy cost even though the Chinese e-commerce market is larger than that of the EU.[5] Our model succeeds in explaining these policy differences.

We explain the model in detail below. Following Montes et al. (2019), we consider an online competition between an incumbent firm and a newcomer in new and old markets.[6] The new market is the market for consumers who have not been active online in the past, while the old market is the market for consumers who have left information online in the past.[7] The consumers in the old market can conceal their personal information by paying the privacy cost. The firms offer uniform prices to consumers in the new market and consumers who conceal their personal information in the old market. If consumers do not conceal their personal information, only the incumbent firm can observe it and offers them personalized prices.[8] We assume that consumers in the new market are more price sensitive than consumers in the old market,[9] whereas Montes et al. (2019) suppose that their price sensitivity is the same.

Our study presents two findings. First, the optimal privacy regulation for consumers depends on the heterogeneity in their price sensitivity. If price sensitivity is sufficiently heterogeneous, that is, if the consumers in the new market are sufficiently price sensitive compared to those in the old market, the equilibrium price in the new market is low.

[4]IKEA, one of the largest multinational furniture retailers in the world, has recognized this market characteristic and adopted dramatically low prices in China compared to those in the EU (Chen 2021).

[5]China's e-commerce market is more than half the size of the global market (Sohaib et al. 2022).

[6]Unlike Montes et al. (2019), our model assumes that the newcomer also sells in the old market.

[7]We can also interpret the new (old) market to represent the younger (older) generation.

[8]Our study considers a context in which only one of two firms has access to consumers' personal information. For example, Microsoft has acquired LinkedIn and successfully gained access to its consumer data, while Salesforce, which could not acquire LinkedIn, is forced to sell its goods without knowledge of the characteristics of existing consumers (Montes et al. 2019). This denotes an asymmetry in data access between competing firms.

[9]Dedehayir et al. (2017) and Goldsmith and Newell (1997), extending Rogers' (1983) Diffusion of Innovations theory, empirically show that consumers who purchase goods later are more price sensitive than consumers who purchase them earlier.

Consequently, the positive effect of an increase in the privacy cost on the consumer surplus in the new market is limited. Therefore, consumer surplus is maximized when the government keeps the privacy cost as low as possible. Conversely, if price sensitivity is homogeneous enough, that is, if the consumers in the new market are as price insensitive as those in the old market, consumer surplus is maximized at the highest possible privacy cost.

Second, a privacy regulation maximizing consumer surplus may differ from a regulation maximizing total surplus. If the price sensitivity among consumers is sufficiently heterogeneous (homogeneous), both consumer and total surpluses are maximized at the lowest (highest) possible privacy cost. However, if the heterogeneity among consumers is intermediate, consumer (total) surplus is maximized at the lowest (highest) possible privacy cost. The intuition behind this is as follows: When the privacy cost is high, many consumers abandon concealing their personal information and purchase the good at personalized prices. This negatively affects consumer surplus, but total surplus is unaffected. Therefore, compared to total surplus, consumer surplus is maximized at lower privacy cost. Our second finding explains why the head of the EU's competition authority stated that "as data becomes increasingly important for competition, it may not be long before the Commission [the EU-level competition authority] has to deal with cases where granting access to data is the best way to restore competition," whereas Europe's data protection authority promotes the exact opposite regulation (i.e., the GDPR).[10]

The literature closely related to our research is that on consumers' endogenous privacy choices. Montes et al. (2019) and Valletti and Wu (2020) have considered a two-markets model similar to ours and assume homogeneous price sensitivity among consumers. Other studies have examined consumers' endogenous privacy choices in a single market (Casadesus-Masanell and Hervas-Drane 2015; Conitzer et al. 2012; Koh et

[10]Margrethe Vestager, Comm'r of Competition, Eur. Comm'n, Defending Competition in a Digitised World, Address at the European Consumer and Competition Day (Apr. 4, 2019), `https://wayback.archive-it.org/12090/20191129202059/https://ec.europa.eu/commission/commissioners/2014-2019/vestager/announcements/defending-competition-digitised-world_en`

al. 2017). Several studies assume consumers' exogenous privacy choices (Acquisti and Varian 2005; Esteves 2022; Shy and Stenbacka 2016; Taylor 2004; Taylor and Wagman 2014) but only consider two extreme cases: consumers cannot be anonymous, or they can conceal their personal information at no cost.

This study is also related to the literature on behavior-based price discrimination (Choe et al. 2022; Esteves 2010; Fudenberg and Tirole 2000; Fudenberg and Villas-Boas 2012; Villas-Boas 1999, 2004).[11] In these studies, consumers do not make an explicit choice. Therefore, they cannot analyze the effect of the cost associated with this choice. For further literature on privacy, see Acquisti et al. (2016).

The remainder of this study is organized as follows. The next section describes the model. Section 3 provides the equilibrium calculations. Section 4 presents the comparative statics. Finally, Section 5 provides the conclusion.

2 Model

We consider a situation in which two firms, firm A and firm B, compete with each other. Following Montes et al. (2019), we assume that there are two markets, a new and an old market, and that consumers in both markets are uniformly distributed on $[0, 1]$.[12] Firm A is located at 0 and firm B is located at 1 in both the markets.

For the consumers in the new market, both firms observe only their distribution. These consumers represent new Internet users who have not previously left their personal information. Firms A and B offer the consumers uniform prices p_A and p_B, respectively, because neither firm can observe any personal information other than their distribution. Thus, the utilities of consumers at location $\theta \in [0, 1]$ purchasing from firms A, B are $u_A^N = v - p_A - \alpha t(\theta - 0)$ and $u_B^N = v - p_B - \alpha t(1 - \theta)$, respectively. Here, the superscript

[11]For more studies, comprehensive reviews can be found in Fudenberg and Villas-Boas (2006) and Esteves (2009).

[12]We can consider different market sizes. If we assume that the new market size is 1 and the old market size is λ, the main results of this study are robust in the range $0.5 \leq \lambda \leq 1.5$. Hence, for simplicity, we assume $\lambda = 1$.

"N" means "new market." Furthermore, v is the utility of consuming the goods, and t is the parameter of transportation cost; $\alpha \in (0, 1]$ is the parameter for the consumers in the new market, which gives less weightage to the transportation cost.[13] Therefore, α represents the price insensitivity of the consumers in the new market, and when α is small (large), the consumers are price sensitive (insensitive). Note that several studies have interpreted the coefficient parameter of transportation costs as price sensitivity (Coughlan and Soberman 2005, Ishibashi and Matsushima 2009, Mehra et al. 2020, Shaffer and Zettelmeyer 2004). This allows for an alternative interpretation of α as the brand orientation of the consumers in the new market.

For the consumers in the old market, firm B only knows their distribution and therefore it offers uniform price p_B. The utility of purchasing from firm B is $u_B^O = v - p_B - t(1-\theta)$. The superscript "$O$" means "old market." However, firm A may know both the distribution and personal information of the consumers, that is, their types. If the consumers do not protect their privacy, firm A observes their types and offers them personalized prices $p_A(\theta)$ accordingly. Thus, the utility of consumers who reveal personal information to firm A is $u_{RA}^O = v - p_A(\theta) - t(\theta - 0)$. Here, the subscript "$R$" means "revealing personal information." If the consumers pay the privacy cost c to conceal their types from firm A, firm A observes only their distribution and offers uniform price p_A. Thus, the utility of consumers concealing their types is $u_{CA}^O = v - p_A - t(\theta - 0) - c$. Here, the subscript "$C$" means "concealing personal information." We also assume $c < (3-\alpha)(2\alpha+1)t/(4\alpha+3) \overset{\text{def}}{=} c_H$ to guarantee that the number of consumers who conceal personal information is positive.

The behavioral categories of the consumers in both markets are as follows. Let θ_N be the type of consumer who is indifferent between purchasing from firms A and B in the new market. Then, the consumers with $\theta \in [0, \theta_N]$ purchase from firm A and those with $\theta \in (\theta_N, 1]$ purchase from firm B. Similarly, let θ_O be the type of consumer who

[13] For simplicity, we assume $0 < \alpha \leq 1$. Although the calculation becomes highly complex, we can show similar results in $1 < \alpha < 3$.

is indifferent between purchasing from firms A and B in the old market. Then, the consumers with $\theta \in [0, \theta_O]$ purchase from firm A and those with $\theta \in (\theta_O, 1]$ purchase from firm B. Among the consumers in the old market purchasing from firm A, let θ_{CR} be the type of consumer who is indifferent between concealing and revealing their personal information.[14] Then, consumers with $\theta \in [0, \theta_{CR}]$ conceal their information because firm A will charge higher discriminatory prices to them if it knows that they are located near it. Meanwhile, consumers with $\theta \in (\theta_{CR}, \theta_O]$ reveal their personal information.

Firms A and B produce their goods without any cost. Firm A offers uniform price p_A to the consumers in the new market and the consumers in the old market who conceal their information. It offers personalized prices $p_A(\theta)$ to the revealing consumers in the old market. Accordingly, the profit of firm A is as follows:

$$\pi_A = \int_0^{\theta_N} p_A d\theta + \int_0^{\theta_{CR}} p_A d\theta + \int_{\theta_{CR}}^{\theta_O} p_A(\theta) d\theta. \tag{1}$$

Since Firm B cannot observe the types of old market consumers, it offers the uniform price p_B across both the new and old markets. Thus, the profit of firm B is expressed as follows:

$$\pi_B = \int_{\theta_N}^{1} p_B d\theta + \int_{\theta_O}^{1} p_B d\theta. \tag{2}$$

Finally, we define the consumer, producer, and total surpluses. Consumer surplus is defined as follows.

$$CS = \int_0^{\theta_N} u_A^N d\theta + \int_{\theta_N}^{1} u_B^N d\theta + \int_0^{\theta_{CR}} u_{CA}^O d\theta + \int_{\theta_{CR}}^{\theta_O} u_{RA}^O d\theta + \int_{\theta_O}^{1} u_B^O d\theta.$$

The producer surplus is $PS = \pi_A + \pi_B$, and the total surplus is $TS = CS + PS$.

The stages of this game are as follows. In the first stage, the consumers in the old market decide whether to conceal or reveal their personal information to firm A. In the second stage, each firm determines a uniform price for the consumers who do not

[14] We assume that $\theta_{CR} < [(4\alpha + 3)\sqrt{1+\alpha} - 6\alpha^2 - 3\alpha + 3]/[4\alpha(1+\alpha)]$ as a condition for firm B to enter the old market. For some parameter ranges in our analysis, we have more than two equilibria. We focus on their interior solution. In Online Appendix, we provide the condition that the interior solution exists.

leave or reveal their personal information. In the third stage, firm A determines the personalized prices for the consumers who have revealed their personal information.[15] In the fourth stage, the consumers purchase and consume. We solve this game using backward induction.

3 Calculating Equilibrium

First, consider the fourth stage. The type θ_N is indifferent between purchasing from firms A and B in the new market. Considering that this consumer satisfies $u_A^N(\theta_N) = u_B^N(\theta_N)$, solving this equation yields θ_N as follows.

$$\theta_N = \frac{1}{2} + \frac{p_B - p_A}{2t\alpha}. \tag{3}$$

Therefore, in the new market, consumers at $\theta \leq \theta_N$ purchase from firm A, and consumers at $\theta > \theta_N$ purchase from firm B. The type θ_O is indifferent between purchasing from firms A and B in the old market. The consumers at $\theta \leq \theta_O$ purchase from firm A, and consumers at $\theta > \theta_O$ purchase from firm B.

Next, in the third stage, we derive the personalized prices $p_A(\theta)$ that firm A offers to consumers who have revealed their personal information. Solving $u_{RA}^O = u_B^O$, we obtain the personalized prices $p_A(\theta)$ as follows.

$$p_A(\theta) = p_B + (1 - 2\theta)t. \tag{4}$$

Firm A offers personalized prices $p_A(\theta)$ to consumers at $\theta > \theta_{CR}$; however, it refrains from selling to consumers with $p_A(\theta) < 0$ due to the zero marginal cost of the good. Thus, the consumer furthest from firm A to whom this firm sells the good at a personalized price corresponds to the consumer at θ_O who satisfies $p_A(\theta_O) = 0$. Solving $p_A(\theta_O) = 0$, we obtain θ_O as follows:

$$\theta_O = \frac{p_B + t}{2t}. \tag{5}$$

[15] Since a firm can adjust personalized prices more flexibly than the list price, we assume that firm A chooses the personalized prices after the uniform price. This pricing structure is standard in the literature on personalized pricing (Choe et al. 2018; Shaffer and Zhang 2002; Thisse and Vives 1988).

Thirdly, we consider the second stage. Substituting (3) and (5) into (1) and (2) for each firm's profit, we obtain the following maximization problems.

$$\max_{p_A} \int_0^{\frac{1}{2}+\frac{p_B-p_A}{2t\alpha}} p_A d\theta + \int_0^{\theta_{CR}} p_A d\theta + \int_{\theta_{CR}}^{\frac{p_B+t}{2t}} p_A(\theta) d\theta,$$

$$\max_{p_B} \int_{\frac{1}{2}+\frac{p_B-p_A}{2t\alpha}}^{1} p_B d\theta + \int_{\frac{p_B+t}{2t}}^{1} p_B d\theta.$$

Calculating the first-order condition for each firm, we obtain the uniform prices as follows.

$$p_A = \frac{2\alpha t[(2\alpha+2)\theta_{CR}+\alpha+2]}{4\alpha+3}, \quad p_B = \frac{\alpha t(\theta_{CR}+5)}{4\alpha+3}. \tag{6}$$

Finally, we consider the first stage. By substituting (6) into u_{CA}^O and u_{RA}^O, we obtain (7) and (8).

$$u_{CA}^O = v - \frac{2\alpha t[(2\alpha+2)\theta_{CR}+\alpha+2]}{4\alpha+3} - t\theta - c, \tag{7}$$

$$u_{RA}^O = v - \frac{\alpha t(\theta_{CR}+5)}{4\alpha+3} - t(1-\theta). \tag{8}$$

From (7) and (8), we obtain the type θ_{CR}^* of the consumer who is indifferent between concealing and revealing personal information:

$$\theta_{CR}^* = \frac{(-2\alpha^2+5\alpha+3)\,t-(4\alpha+3)c}{2(\alpha+1)(2\alpha+3)t}. \tag{9}$$

Substituting (9) into (6), we obtain the equilibrium uniform prices as follows.

$$p_A^* = \frac{2\alpha(3t-c)}{2\alpha+3}, \quad p_B^* = \frac{\alpha[2(\alpha+3)t-c]}{(\alpha+1)(2\alpha+3)}. \tag{10}$$

Furthermore, by substituting (10) into (4), we obtain the equilibrium personalized prices as follows.

$$p_A^*(\theta) = \frac{\alpha[2(\alpha+3)t-c]}{(\alpha+1)(2\alpha+3)} + (1-2\theta)t.$$

From the aforementioned results, the equilibrium profit for each firm is as follows.

$$\pi_A^* = \frac{(8\alpha+9)c^2-12\alpha tc+36\alpha(\alpha+2)t^2}{4(2\alpha+3)^2 t}, \quad \pi_B^* = \frac{\alpha[2(\alpha+3)t-c]^2}{2(\alpha+1)(2\alpha+3)^2 t}.$$

Similarly, we find the consumer and total surpluses in equilibrium as follows.

$$CS^* = 2v + \frac{\left[\begin{array}{l} (4\alpha^2+16\alpha+9)\,c^2 + 2\,(4\alpha^3+16\alpha^2-3\alpha-9)\,tc \\ -\,(4\alpha^4+68\alpha^3+229\alpha^2+174\alpha+9)\,t^2 \end{array}\right]}{4(\alpha+1)(2\alpha+3)^2 t},$$

$$TS^* = 2v + \frac{\left[\begin{array}{l} (12\alpha^2+35\alpha+18)\,c^2 + 2\,(4\alpha^3+6\alpha^2-21\alpha-9)\,tc \\ -\,(4\alpha^4+24\alpha^3+73\alpha^2+30\alpha+9)\,t^2 \end{array}\right]}{4(\alpha+1)(2\alpha+3)^2 t}.$$

4 Comparative Statics

This section investigates the optimal privacy regulation by changing c (privacy cost) and α (price insensitivity). Differentiating the uniform prices p_A^* and p_B^*, and personalized price $p_A^*(\theta)$ with respect to c, we obtain the following.

Lemma 1 *As the privacy cost increases, firm A reduces both its uniform and personalized prices, and firm B reduces its uniform price.*

Proof.

$$\frac{\partial p_A^*}{\partial c} = -\frac{2\alpha}{2\alpha+3} < 0, \;\; \frac{\partial p_B^*}{\partial c} = -\frac{\alpha}{(\alpha+1)(2\alpha+3)} < 0, \;\text{ and }\; \frac{\partial p_A^*(\theta)}{\partial c} = -\frac{\alpha}{(\alpha+1)(2\alpha+3)} < 0$$

because $0 < \alpha < 1$. □

The intuition behind Lemma 1 is as follows. As the privacy cost increases, the consumers remaining in the anonymous market must pay the higher privacy cost to remain anonymous. This leads to the lower reservation price (net of the privacy cost), and thus, firm A lowers the uniform price. In response, firm B lowers its uniform price. Then, the personalized prices of firm A decrease because they are an increasing function of the uniform price of firm B.

Next, by differentiating the uniform prices p_A^* and p_B^*, and personalized price $p_A^*(\theta)$ with respect to α (price insensitivity of the consumers in the new market), we obtain the following Lemma 2.

Lemma 2 *As the consumers in the new market become more price sensitive, firm A reduces its uniform and personalized prices, and firm B reduces its uniform price.*

Proof.

$$\frac{\partial p_A^*}{\partial \alpha} = \frac{6(3t-c)}{(2\alpha+3)^2}, \quad \frac{\partial p_B^*}{\partial \alpha} = \frac{-\left(3-2\alpha^2\right)c - 2\left(\alpha^2-6\alpha-9\right)t}{(\alpha+1)^2(2\alpha+3)^2},$$
$$\text{and} \quad \frac{\partial p_A^*(\theta)}{\partial \alpha} = \frac{-\left(3-2\alpha^2\right)c - 2\left(\alpha^2-6\alpha-9\right)t}{(\alpha+1)^2(2\alpha+3)^2}.$$

$\partial p_A^*/\partial\alpha > 0$ if and only if $c < 3t$. Since we assume $c < (3-\alpha)(2\alpha+1)t/(4\alpha+3) = c_H$, $c < 3t$ is satisfied for any $\alpha \in (0,1]$. Therefore, we obtain $\partial p_A^*/\partial\alpha > 0$. Since $\partial p_B^*/\partial\alpha = \partial p_A^*(\theta)/\partial\alpha$, solving $\partial p_B^*/\partial\alpha > 0$ or $\partial p_A^*(\theta)/\partial\alpha > 0$ yields $c < 2\left(-\alpha^2+6\alpha+9\right)t/(3-2\alpha^2)$. Since $2\left(-\alpha^2+6\alpha+9\right)t/(3-2\alpha^2) - c_H = (\alpha+1)(2\alpha+3)\left(-2\alpha^2+6\alpha+15\right)t/[(4\alpha+3)\left(3-2\alpha^2\right)]$, we obtain $\partial p_B^*/\partial\alpha > 0$ and $\partial p_A^*(\theta)/\partial\alpha > 0$ for any $\alpha \in (0,1]$. □

The intuition behind Lemma 2 is as follows. When α is small (i.e., consumers in the new market are more price sensitive), the uniform prices of firms A and B are low because of fierce price competition in the new market. Therefore, the personalized prices of firm A are correspondingly low. The opposite is true when α is large (i.e., when the consumers are price insensitive).

Lemma 3 investigates the effect of the price insensitivity α on the price reduction caused by the increase in the privacy cost.

Lemma 3 *If the consumers in the new market are price sensitive, as the privacy cost increases, firm A slightly reduces both its uniform and personalized prices, and firm B slightly reduces its uniform price.*

Proof.

$$\frac{\partial^2 p_A^*}{\partial\alpha\partial c} = -\frac{6}{(2\alpha+3)^2}, \quad \frac{\partial^2 p_B^*}{\partial\alpha\partial c} = -\frac{3-2\alpha^2}{[(\alpha+1)(2\alpha+3)]^2}, \text{ and } \frac{\partial^2 p_A^*(\theta)}{\partial\alpha\partial c} = -\frac{3-2\alpha^2}{[(\alpha+1)(2\alpha+3)]^2}.$$

Since $0 < \alpha < 1$, we obtain $\partial^2 p_A^*/(\partial\alpha\partial c) < 0$, $\partial^2 p_B^*/(\partial\alpha\partial c) < 0$ and $\partial^2 p_A^*(\theta)/(\partial\alpha\partial c) < 0$. □

The intuition for Lemma 3 is as follows. As shown in Lemma 2, when the consumers in the new market are sensitive to prices, the uniform and personalized prices of firm A and uniform price of firm B are low. Thus, the prices slightly decline as the privacy cost rises.

Based on the aforementioned implications, Proposition 1 provides the results of comparative statics on the consumer surplus with respect to privacy cost c.

Proposition 1 *(i) If the consumers in the new market are price sensitive, that is, if* $0 < \alpha < 0.7699$*, the consumer surplus is a U-shaped function of the privacy cost. Otherwise, that is, if* $0.7699 \leq \alpha \leq 1$*, the consumer surplus is an increasing function of the privacy cost. (ii) If the consumers are sufficiently price sensitive, that is, if* $0 < \alpha \leq 0.3604$*, the consumer surplus is maximized at* $c = 0$*. Otherwise, that is, if* $0.3604 < \alpha \leq 1$*, the consumer surplus is maximized at* $c = c_H$*.*

Proof. See the Appendix. □

We consider the intuition behind Proposition 1. First, we discuss Proposition 1 (i). An increase in c has two effects on the consumer surplus. The first effect is that it reduces the utility of consumers who conceal personal information. In the following section, we refer to this as the "direct effect." Note that the direct effect is small for large c because few consumers conceal their personal information. The second effect is that the increase in c reduces the equilibrium prices (as shown in Lemma 1). We refer to this effect as the "price reduction effect." Crucially, this effect increases the consumer surplus. With small c, the direct effect dominates the price reduction effect. Meanwhile, at sufficiently high c, the price reduction effect dominates the direct effect due to the latter's diminutive effect caused by fewer consumers concealing their personal information. Furthermore, Lemma 3 shows that the price reduction effect becomes larger if the consumers in the new market are sufficiently price insensitive. Thus, the consumer surplus is an increasing function of c if the consumers in the new market are sufficiently price insensitive.

Proposition 1 (i) shows that the consumer surplus is a U-shaped or increasing function of c. Therefore, consumer surplus is maximized at either $c = 0$ or $c = c_H$. From Lemma 3, when α is large, the price reduction effect is large. Accordingly, the region in which the price reduction effect dominates the direct effect is large. Therefore, the area of increasing consumer surplus, characterized by a U-shaped function or an increasing function, expands. In this case, consumer surplus tends to be maximized at $c = c_H$. Conversely, when α is small, the direct effect dominates. Consumer surplus tends to be maximized at $c = 0$ because the area of increasing consumer surplus shrinks.

In Montes et al. (2019), who consider a duopolistic market, along with Valletti and Wu (2020), the price reduction effect always dominates. Thus, these studies justify not protecting consumer privacy. Meanwhile, our study shows that the direct effect can dominate considering heterogeneous price sensitivity among consumers. If consumers in the new market are sufficiently price sensitive, privacy protection should be enforced (such as the EU's GDPR). Therefore, regulators should consider the heterogeneity in price sensitivity while discussing the optimal privacy regulation for consumers.

Second, the comparative statics on the firm's profits with respect to c are summarized in Proposition 2.

Proposition 2 *(i) The profit of firm A is a U-shaped function of the privacy cost. The profit is maximized at $c = c_H$. (ii) The profit of firm B is a decreasing function of the privacy cost. The profit is maximized at $c = 0$.*

Proof. See the Appendix. □

We discuss the intuition of Proposition 2. First, we consider Proposition 2 (ii). The price reduction effect reduces the profit of firm B. Therefore, the profit of firm B is a decreasing function of the cost of privacy and is maximized at $c = 0$. Next, we consider Proposition 2 (i). An increase in c has two effects on the profit of firm A: price reduction and "personalized price" effects. The personalized price effect is that more consumers purchase at personalized prices. The price reduction (personalized price) effect reduces (increases) the profit of firm A. For higher c, consumers who newly reveal their personal information have higher a willingness-to-pay. Thus, for high (low) c, the personalized price (price reduction) effect dominates. Based on the above, we argue that the profit of firm A is a U-shaped function of the privacy cost. Finally, regardless of the price insensitivity of the consumers in the new market, the personalized price effect is larger than the price reduction effect. Therefore, the profit of firm A is maximized with $c = c_H$.

Third, Proposition 3 provides the comparative statics on the total surplus with respect to c.

Proposition 3 *The total surplus is a U-shaped function of the privacy cost. If the consumers in the new market are sufficiently price sensitive, that is, if* $0 < \alpha \leq 0.0638$*, the total surplus is maximized at* $c = 0$*. Otherwise, that is, if* $0.0638 < \alpha \leq 1$*, the total surplus is maximized at* $c = c_H$*.*

Proof. See the Appendix. □

The intuition behind Proposition 3 is as follows. We argue that the total surplus is a U-shaped function of c. An increase in c has two effects on the total surplus. The first is the direct effect, which reduces the total surplus. If c is high, the direct effect is small because fewer consumers conceal their personal information. The second effect is that the price reduction effect improves the asymmetry of market share between firms. We refer to this effect as the "asymmetry improvement effect." This effect increases the total surplus because it reduces consumers' transportation costs. Considering the given information, for small c, the direct effect dominates, while for large c, the asymmetry improvement effect dominates.

Since the total surplus is a U-shaped function of c, the total surplus is maximized at either $c = 0$ or $c = c_H$. Lemma 3 shows that if α is higher, the price reduction effect is larger. Therefore, with a higher α, the asymmetry improvement effect is also larger and the area of increasing total surplus expands. In this case, the total surplus tends to be maximized at $c = c_H$. If α is small, the smallest privacy cost, $c = 0$, has the maximum total surplus because the direct effect dominates the asymmetry improvement effect.

Comparing Proposition 1 and Proposition 3, we obtain Corollary 1.

Corollary 1 *(i) If the price sensitivity of the consumers in the new market is intermediate, that is, if* $0.0638 < \alpha \leq 0.3604$*, the consumer surplus is maximized at* $c = 0$ *and total surplus is maximized at* $c = c_H$*. (ii) Otherwise, the arguments maximizing the consumer and total surpluses are the same.*

We consider the intuition behind Corollary 1. From Propositions 1 and 3, at $c = 0$, the consumer and total surpluses are maximized when α is sufficiently small. For sufficiently large α, the largest c yields the maximum consumer and total surpluses. Hence, we obtain Corollary 1 (ii). Next, we consider Corollary 1 (i). As noted in Proposition 3, an increase in c has two effects on the total surplus: the direct and asymmetry improvement effects. When considering the consumer surplus, we consider an additional effect. Specifically, an increase in c decreases the consumer surplus because more consumers purchase at personalized prices. Therefore, the area in which consumer surplus is maximized at $c = 0$ is wider than that in which total surplus is maximized at $c = 0$, leading to Corollary 1 (i).

Finally, we analyze the effect of α (price insensitivity) on the consumer surplus, each firm's profit, and the total surplus. Differentiating each equilibrium value with respect to α, we obtain Proposition 4.

Proposition 4 *(i) The consumer surplus monotonically decreases as the consumers in the new market become price insensitive. (ii) As they become price insensitive, the profits of both firms monotonically increase. (iii) As they become price insensitive, the total surplus monotonically decreases.*

Proof. See the Appendix. □

The intuition behind Proposition 4 is as follows. First, we consider Propositions 4 (i) and (ii). From Lemma 2, if the consumers in the new market are price insensitive, the firms set higher prices. Thus, if the consumers are price insensitive, the profits increase. Conversely, this leads to a decrease in consumer surplus.

Next, we discuss Proposition 4 (iii). In terms of the total surplus, a rise or fall in prices is simply an income transfer between the consumers and firms. Therefore, the consumer's transportation costs determine the effect on the total surplus. A larger α leads to an increase in transportation costs for all consumers in the new market. Therefore, if the consumers become price insensitive, the total surplus decreases.

5 Conclusion

This study considers a model where an incumbent firm and a newcomer compete for two markets: a new market and an old one. In the new market, neither firm has access to consumers' personal information. In the old market, the incumbent (newcomer) firm can (cannot) observe consumers' personal information. We assume that consumers in the old market can conceal their personal information by paying the privacy cost. We also allow for heterogeneity in price sensitivity among consumers in these markets.

We find: First, the optimal privacy regulation for consumers depends on the heterogeneity in their price sensitivities. If the price sensitivity is sufficiently heterogeneous, the consumer surplus is maximized at the lowest possible privacy cost. This finding is contrary to the results of previous studies. Second, the privacy regulation maximizing consumer surplus may differ from that maximizing total surplus. With intermediate heterogeneity in consumers' price sensitivity, the consumer (total) surplus is maximized at the lowest (highest) possible privacy cost. These findings indicate that national authorities should consider heterogeneity in price sensitivity among consumers when deciding on privacy regulation.

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Appendix

Proof of Proposition 1.

We prove Proposition 1 (i). Differentiating the consumer surplus CS^* with respect to c yields the following equation.

$$\frac{\partial CS^*}{\partial c} = \frac{2\left(4\alpha^2+16\alpha+9\right)c+2\left(4\alpha^3+16\alpha^2-3\alpha-9\right)t}{4(\alpha+1)(2\alpha+3)^2 t}. \tag{A1}$$

Next, we consider the sign of (A1). Solving $\partial CS^*/\partial c < 0$ yields $c < (-4\alpha^3-16\alpha^2+3\alpha+9)t/(4\alpha^2+16\alpha+9) \stackrel{\text{def}}{=} c_{CS}$. Thus, if we ignore the range of c, CS^* is a U-shaped function with a minimum at $c = c_{CS}$.

Here, we consider the range of c. First, we check the sign of c_{CS}. The sign of c_{CS} corresponds to the sign of $-4\alpha^3-16\alpha^2+3\alpha+9$. Let us define α_1 as the solution to $-4\alpha^3-16\alpha^2+3\alpha+9=0$ which is between 0 and 1. Then, we can show that:

$$c_{CS} > 0 \quad \text{if} \quad 0 < \alpha < \alpha_1. \tag{A2}$$

Since $\alpha_1 \approx 0.7699$, the consumer surplus is an increasing function of c if $0.7699 \leq \alpha \leq 1$.

Second, we compare c_H and c_{CS}. From $c_H - c_{CS}$, we obtain the following equation.

$$c_H - c_{CS} = \frac{2\alpha\left(4\alpha^3+32\alpha^2+55\alpha+24\right)t}{\left(4\alpha+3\right)\left(4\alpha^2+16\alpha+9\right)}. \tag{A3}$$

From (A3), we obtain $c_H > c_{CS}$ because $\alpha \in (0,1]$. Therefore, if $0 < \alpha < 0.7699$, the consumer surplus is a U-shaped function of c.

Finally, we prove Proposition 1 (ii). We examine whether $c = 0$ or $c = c_H$ maximizes CS^*. If $0.7699 \leq \alpha \leq 1$, the consumer surplus is an increasing function of c; therefore, the consumer surplus is maximized at $c = c_H$. Next, we consider when $0 < \alpha < 0.7699$. Let us denote the consumer surplus when $c = 0$ as CS_0^* and the consumer surplus when $c = c_H$ as CS_H^*. Calculating $CS_H^* - CS_0^*$ yields the following equation.

$$CS_H^* - CS_0^* = -\frac{\left(48\alpha^5+112\alpha^4-592\alpha^3-552\alpha^2+45\alpha+81\right)t}{4(2\alpha+3)^2(4\alpha+3)^2}. \tag{A4}$$

Examine the sign of (A4). The sign of (A4) is the same as the sign of $-48\alpha^5 - 112\alpha^4 + 592\alpha^3 + 552\alpha^2 - 45\alpha - 81$. Solving $-48\alpha^5 - 112\alpha^4 + 592\alpha^3 + 552\alpha^2 - 45\alpha - 81 > 0$ in the range $\alpha \in (0, 1]$ yields the following result.

$$CS_H^* > CS_0^* \quad \text{if} \quad \frac{1}{6}\left(\sqrt{10} - 1\right) < \alpha \leq 1. \tag{A5}$$

From (A2) and (A5), if the consumers in the new market are sufficiently price sensitive, that is, if $0 < \alpha \leq \left(\sqrt{10} - 1\right)/6 \approx 0.3604$, the consumer surplus is maximized at $c = 0$. Additionally, if the consumers are as price insensitive as the consumers in the old market, that is, if $\left(\sqrt{10} - 1\right)/6 < \alpha \leq 1$, the consumer surplus is maximized at $c = c_H$. $\square$

Proof of Proposition 2.

First, consider Proposition 2 (i). Differentiating the profit of firm A, π_A^*, with respect to c, we obtain the following equation.

$$\frac{\partial \pi_A^*}{\partial c} = \frac{(8\alpha + 9)c - 6\alpha t}{2(2\alpha + 3)^2 t}. \tag{A6}$$

Solving $\partial \pi_A^* / \partial c < 0$, we obtain $c < 6\alpha t/(8\alpha + 9) \stackrel{\text{def}}{=} c_A$ from (A6). Thus, if we ignore the range of c, we find that π_A^* is a U-shaped function with a minimum at $c = c_A$.

$$c_H - c_A = \frac{(2\alpha + 3)\left(-8\alpha^2 + 11\alpha + 9\right)t}{(4\alpha + 3)(8\alpha + 9)}. \tag{A7}$$

The sign of (A7) corresponds to the sign of $-8\alpha^2 + 11\alpha + 9$. Solving $-8\alpha^2 + 11\alpha + 9 > 0$ yields $\left(11 - \sqrt{409}\right)/16 < \alpha < \left(11 + \sqrt{409}\right)/16$. Therefore, $c_H > c_A$ holds in the range $\alpha \in (0, 1]$. Accordingly, for any $\alpha \in (0, 1]$, π_A^* is a U-shaped function of c.

Second, we examine whether $c = 0$ or $c = c_H$ maximizes π_A^*. Let us denote the profit of firm A at $c = 0$ as π_{A0}^* and profit of firm A at $c = c_H$ as π_{AH}^*. Calculating $\pi_{AH}^* - \pi_{A0}^*$, we obtain the following equation.

$$\pi_{AH}^* - \pi_{A0}^* = \frac{(3 - \alpha)(2\alpha + 1)\left(-16\alpha^3 - 26\alpha^2 + 33\alpha + 27\right)t}{4(2\alpha + 3)^2(4\alpha + 3)^2}. \tag{A8}$$

Examine the sign of (A8). This sign corresponds to the sign of $-16\alpha^3-26\alpha^2+33\alpha+27$. In the range $\alpha \in (0,1]$, $-16\alpha^3-26\alpha^2+33\alpha+27>0$. Thus, for any $\alpha \in (0,1]$, we obtain $\pi^*_{AH} > \pi^*_{A0}$. Therefore, the profit of firm A, π^*_A, is always maximized at $c = c_H$.

Finally, we prove Proposition 2 (ii). By differentiating the equilibrium profit of firm B, π^*_B, with respect to c, we obtain the following equation.

$$\frac{\partial \pi^*_B}{\partial c} = -\frac{\alpha[2(\alpha+3)t-c]}{(\alpha+1)(2\alpha+3)^2 t}. \tag{A9}$$

From (A9), if $c < 2(\alpha+3)t \overset{\text{def}}{=} c_B$, we obtain $\partial\pi^*_B/\partial c < 0$. Thus, if we ignore the range of c, we obtain that π^*_B is a U-shaped function with a minimum at $c = c_B$.

Here, we consider the range of c. Calculating $c_B - c_H$ yields the following equation.

$$c_B - c_H = \frac{5(\alpha+1)(2\alpha+3)t}{4\alpha+3}. \tag{A10}$$

From (A10), we find that $c_B > c_H$. Therefore, π^*_B is a decreasing function of c. Accordingly, we find that π^*_B is maximized at $c = 0$. □

Proof of Proposition 3.

Differentiating the equilibrium total surplus TS^* with respect to c yields the following equation.

$$\frac{\partial TS^*}{\partial c} = \frac{(12\alpha^2+35\alpha+18)\,c + (4\alpha^3+6\alpha^2-21\alpha-9)\,t}{2(\alpha+1)(2\alpha+3)^2 t}. \tag{A11}$$

Solving $\partial TS^*/\partial c < 0$, we obtain $c < (-4\alpha^3-6\alpha^2+21\alpha+9)\,t/(12\alpha^2+35\alpha+18) \overset{\text{def}}{=} c_{TS}$ from (A11). Therefore, if we ignore the range of c, we find that TS^* is a U-shaped function with a minimum at $c = c_{TS}$.

Next, we consider the range of c. We identify the sign of c_{TS}. This sign corresponds to the sign of $-4\alpha^3-6\alpha^2+21\alpha+9$. For $\alpha \in (0,1]$, we find that $-4\alpha^3-6\alpha^2+21\alpha+9>0$, which is why we obtain $c_{TS} > 0$.

Next, we compare c_H and c_{TS}. Calculating $c_H - c_{TS}$ yields the following equation.

$$c_H - c_{TS} = \frac{(2\alpha+3)\,(-4\alpha^3+19\alpha^2+26\alpha+9)\,t}{(3\alpha+2)(4\alpha+3)(4\alpha+9)}. \tag{A12}$$

The sign of (A12) corresponds to the sign of $-4\alpha^3+19\alpha^2+26\alpha+9$. For $\alpha \in (0,1]$, we find $-4\alpha^3+19\alpha^2+26\alpha+9>0$, so we obtain $c_H > c_{TS}$. Therefore, we are certain that TS^* is a U-shaped function of c.

Finally, we examine whether TS^* is maximized at $c=0$ or $c=c_H$. Let us denote the total surplus at $c=0$ as TS_0^* and the total surplus at $c=c_H$ as TS_H^*. Calculating $TS_H^*-TS_0^*$, we obtain the following equation.

$$TS_H^*-TS_0^* = \frac{(3-\alpha)\alpha(2\alpha+1)\left(8\alpha^3+62\alpha^2+43\alpha-3\right)t}{4(\alpha+1)(2\alpha+3)^2(4\alpha+3)^2}. \tag{A13}$$

The sign of (A13) corresponds to the sign of $8\alpha^3+62\alpha^2+43\alpha-3$. Let us define α_2 as the solution to $8\alpha^3+62\alpha^2+43\alpha-3=0$ which is between 0 and 1. Then, we can show that

$$TS_H^* > TS_0^* \quad \text{if} \quad \alpha_2 < \alpha \leq 1.$$

Since $\alpha_2 \approx 0.0638$, the total is maximized at $c=0$ if $0<\alpha\leq 0.0638$. Conversely, if they are as price insensitive as the consumers in the old market, that is, if $0.0638 < \alpha \leq 1$, the total surplus is maximized at $c=c_H$. □

Proof of Proposition 4.

First, we prove Proposition 4 (i). By differentiating the consumer surplus CS^* with respect to α, we obtain the following equation.

$$\frac{\partial CS^*}{\partial \alpha} = \frac{\begin{bmatrix} -c^2(8\alpha^3+52\alpha^2+62\alpha+15)+12c(16\alpha^2+26\alpha+9)t \\ -(\alpha+1)^2(8\alpha^3+36\alpha^2+54\alpha+459)t^2 \end{bmatrix}}{4(\alpha+1)^2(2\alpha+3)^3 t}. \tag{A14}$$

The sign of (A14) corresponds to the sign of the numerator. The numerator is a convex upward quadratic function of c. Now, considering the discriminant D_{CS} of the numerator, we obtain the following result.

$$D_{CS} = -4(2\alpha+3)^2\left(16\alpha^6+160\alpha^5+552\alpha^4+1768\alpha^3+3205\alpha^2+2322\alpha+441\right)t^2 < 0. \tag{A15}$$

From (A15), we obtain $\partial CS^*/\partial\alpha < 0$ for any $\alpha \in (0, 1]$.

Second, consider Proposition 4 (ii). Differentiating the profit of firm A, π_A^*, with respect to α, we obtain the following equation.

$$\frac{\partial \pi_A^*}{\partial \alpha} = \frac{(3t - c)[(4\alpha + 3)c + 6(\alpha + 3)t]}{(2\alpha + 3)^3 t}. \tag{A16}$$

(A16) is positive if $c < 3t$. Given that $c_H < 3t$, we obtain $\partial\pi_A^*/\partial\alpha > 0$. Next, differentiating the profit of firm B, π_B^*, with respect to α, we obtain the following equation.

$$\frac{\partial \pi_B^*}{\partial \alpha} = \frac{[2(\alpha + 3)t - c]\left[\left(4\alpha^2 + 2\alpha - 3\right)c + 2\left(-4\alpha^2 + 3\alpha + 9\right)t\right]}{2(\alpha + 1)^2(2\alpha + 3)^3 t}. \tag{A17}$$

For (A17), the denominator is always positive. Additionally, because $c_H < 2(\alpha + 3)t$, $2(\alpha+3)t-c > 0$. Therefore, the sign of (A17) corresponds to the sign of $[(4\alpha^2 + 2\alpha - 3)\,c + 2\,(-4\alpha^2 + 3\alpha + 9)\,t]$. It is clear that the second term in $[(4\alpha^2 + 2\alpha - 3)\,c + 2\,(-4\alpha^2 + 3\alpha + 9)\,t]$ is always positive. Next, if the first term of $[(4\alpha^2 + 2\alpha - 3)\,c + 2\,(-4\alpha^2 + 3\alpha + 9)\,t]$ is greater than or equal to 0, then $[(4\alpha^2 + 2\alpha - 3)\,c + 2\,(-4\alpha^2 + 3\alpha + 9)\,t] > 0$, we obtain $\partial\pi_B^*/\partial\alpha > 0$. Solving $4\alpha^2 + 2\alpha - 3 \geq 0$ for α yields $(\sqrt{13} - 1)/4 \leq \alpha \leq 1$. Therefore, if $(\sqrt{13} - 1)/4 \leq \alpha \leq 1$, then $\partial\pi_B^*/\partial\alpha > 0$. Next, consider when $0 < \alpha < (\sqrt{13} - 1)/4$. Solving $[(4\alpha^2 + 2\alpha - 3)\,c + 2\,(-4\alpha^2 + 3\alpha + 9)\,t] > 0$, we obtain $c < 2\,(4\alpha^2 - 3\alpha - 9)\,t/(4\alpha^2 + 2\alpha - 3) \overset{\text{def}}{=} c_b$. Next, we compare c_H and c_b. Calculating $c_b - c_H$, we obtain the following equation.

$$c_b - c_H = \frac{\left(8\alpha^4 + 16\alpha^3 - 28\alpha^2 - 81\alpha - 45\right)t}{(4\alpha + 3)\left(4\alpha^2 + 2\alpha - 3\right)}. \tag{A18}$$

In (A18), the denominator is negative because $4\alpha^2 + 2\alpha - 3 < 0$. Therefore, the sign of (A18) is the same as the sign of $-8\alpha^4 - 16\alpha^3 + 28\alpha^2 + 81\alpha + 45$. Since $-8\alpha^4 - 16\alpha^3 + 28\alpha^2 + 81\alpha + 45 > 0$ is always positive in $\alpha \in (0, 1]$, therefore $c_b > c_H$. Therefore, even for $4\alpha^2 + 2\alpha - 3 < 0$, $[(4\alpha^2 + 2\alpha - 3)\,c + 2\,(-4\alpha^2 + 3\alpha + 9)\,t] > 0$. Therefore, for any $\alpha \in (0, 1]$, we obtain $\partial\pi_B^*/\partial\alpha > 0$.

Finally, we prove Proposition 4 (iii). Differentiating the total surplus TS^* with respect to α, we obtain the following equation.

$$\frac{\partial TS^*}{\partial \alpha} = \frac{\left[\begin{array}{l} -c^2(24\alpha^3+104\alpha^2+106\alpha+21)+4c(10\alpha^2+69\alpha+66)\alpha t \\ -(8\alpha^5+52\alpha^4+94\alpha^3+315\alpha^2+324\alpha+27)\,t^2 \end{array}\right]}{4(\alpha+1)^2(2\alpha+3)^3 t}. \tag{A19}$$

The sign of (A19) corresponds to the sign of the numerator. Here, we find that the numerator is a convex upward quadratic function of c. Calculating the discriminant D_{TS} of the numerator, we obtain the following equation.

$$D_{TS} = -4(2\alpha+3)^2\left(48\alpha^6+376\alpha^5+792\alpha^4+1152\alpha^3+1579\alpha^2+990\alpha+63\right)t^2 < 0. \tag{A20}$$

Therefore, from (A20), we obtain $\partial TS^*/\partial\alpha < 0$. $\square$

Online Appendix (not for publication): condition for firm B to enter the old market.

In this part, we show the condition where firm B enters the old market.

Remark 1 *Firm B enters the old market if* $\theta_{CR} < [(4\alpha+3)\sqrt{1+\alpha} - 6\alpha^2 - 3\alpha + 3]/[4\alpha(1+\alpha)]$.

Proof. In the case that firm B enters the old market, we define the profit of each firm in Section 2. Thus, we obtain the following candidate best response functions of the firms in the second stage.

$$p_A = \frac{1}{2}[p_B + \alpha t(2\theta_{CR} + 1)] = BR_A, \tag{B1}$$

$$p_B = \frac{p_A + 2\alpha t}{2(\alpha+1)} = BR_B. \tag{B2}$$

Next, we consider the case in which firm B does not enter the old market. In this case, the profit of each firm is expressed as follows.

$$\pi'_A = \int_0^{\theta_N} p_A d\theta + \int_0^{\theta_{CR}} p_A d\theta + \int_{\theta_{CR}}^1 p_A(\theta) d\theta, \tag{B3}$$

$$\pi'_B = \int_{\theta_N}^1 p_B d\theta. \tag{B4}$$

From (B3) and (B4), we obtain the following candidate best response functions of the firms when firm B does not enter the old market.

$$p_A = \frac{1}{2}[p_B + \alpha t(2\theta_{CR} + 1)] = BR_A \tag{B5}$$

$$p_B = \frac{1}{2}(p_A + \alpha t) = BR'_B \tag{B6}$$

From (B1) and (B5), the candidate best response functions of firm A are identical, regardless of whether firm B enters the old market or not. Therefore, whether or not firm B enters the old market in equilibrium depends on the shape of firm B's profit.

Here, the share of firm B in the old market is positive when $\theta_O = (p_B + t)/(2t) < 1$, indicating that it enters the old market when $p_B < t$. Therefore, the shape of the profit of firm B can be the case (a)∼(d) in Figure 1 below.

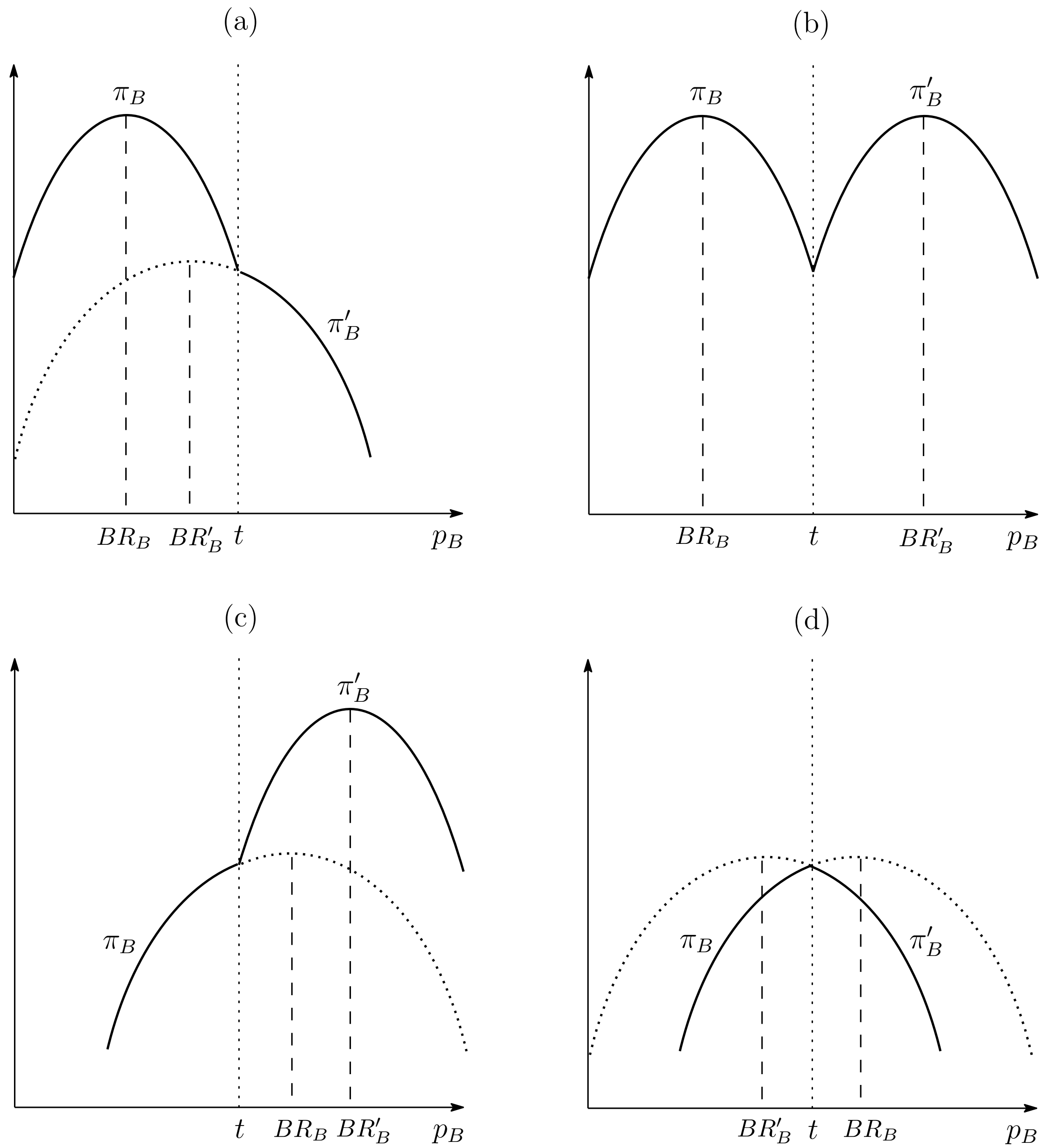

Figure 1. The profit of firm B

Find the condition for each case. First, let us consider Figure 1 (a). Case (a) is established when the following (B7) is satisfied.

$$BR_B < t \text{ and } BR'_B \leq t. \tag{B7}$$

Solving (B7), we obtain $p_A \leq t(2-\alpha)$. Therefore, if $p_A \leq t(2-\alpha)$, case (a) is established. In this case, the profit of firm B is maximized with $p_B = BR_B$.

Second, we consider Figure 1 (b). Case (b) holds if the following (B8) is satisfied.

$$BR_B < t \text{ and } t < BR'_B. \tag{B8}$$

Solving (B8), we obtain $t(2-\alpha) < p_A < 2t$. Thus, if $t(2-\alpha) < p_A < 2t$, case (b) is established. Next, we examine whether the profit of firm B is maximized when $p_B = BR_B$ or $p_B = BR'_B$. Substituting (B2) and (B6) into the profit of firm B, π_B, π'_B, respectively, we obtain the following two equations.

$$\pi_B|_{p_B=BR_B} = \frac{(p_A + 2\alpha t)^2}{8\alpha(\alpha+1)t}, \tag{B9}$$

$$\pi'_B|_{p_B=BR'_B} = \frac{(p_A + \alpha t)^2}{8\alpha t}. \tag{B10}$$

Solving (B9) > (B10), we obtain $p_A < \left(1 - \alpha + \sqrt{\alpha+1}\right) t$. Thus, in case (b), if $p_A < \left(1 - \alpha + \sqrt{\alpha+1}\right) t$, the profit of firm B is maximized with $p_B = BR_B$.

Third, consider Figure 1 (c). Case (c) is established when the following (B11) is satisfied.

$$t \leq BR_B \text{ and } t < BR'_B. \tag{B11}$$

Solving (B11), we obtain $2t \leq p_A$. Therefore, if $2t \leq p_A$, case (c) is established, and the profit of firm B is maximized given $p_B = BR'_B$.

Finally, consider Figure 1 (d). Case (d) holds if the following (B12) is satisfied.

$$t \leq BR_B \text{ and } BR'_B \leq t. \tag{B12}$$

Here, we can immediately see that (B12) is not satisfied.

Accordingly, if $p_A < \left(1-\alpha+\sqrt{\alpha+1}\right)t \overset{\text{def}}{=} \bar{p}_A$, the profit of firm B is maximized with $p_B = BR_B$. Thus, the best response of firm B is $p_B = BR_B$ if $p_A < \bar{p}_A$.

Next, we find the intersection of the best response $p_A = BR_A$ for firm A and the best response $p_B = BR_B$ for firm B under $p_A < \bar{p}_A$. Solving this for p_A and p_B, we obtain the intersection as follows.

$$\tilde{p}_A = \frac{2\alpha t[(2\alpha+2)\theta_{CR}+\alpha+2]}{4\alpha+3}, \quad \tilde{p}_B = \frac{\alpha t(2\theta_{CR}+5)}{4\alpha+3}.$$

Finding the condition that this intersection satisfies $p_A < \bar{p_A}$ and $p_B < t$, we obtain the following condition.

$$\theta_{CR} < \frac{(4\alpha+3)\sqrt{\alpha+1}-6\alpha^2-3\alpha+3}{4\alpha(\alpha+1)}.$$

Hence, firm B enters the old market if $\theta_{CR} < [(4\alpha+3)\sqrt{1+\alpha}-6\alpha^2-3\alpha+3]/[4\alpha(1+\alpha)]$. $\square$