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Towards an Aggregate Social Welfare Function with Application to Developing Countries

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ABSTRACT

An aggregate social welfare function is theoretically proposed and empirically applied. The proposed social welfare function obeys monotonicity, symmetry, continuity, independence, common scale, and the Pigou-Dalton equity principle. It is built on a cardinal multi-dimensional framework. Critical socio-economic factors in the social welfare formulation involve Atkinson-adjusted per capita income, poverty eradication, social entitlements for clean water access and health care, education attainment, income inequality, gender equity, and life expectancy. Applying the social welfare formulation to 79 developing countries, data sensitivity charts reveal important welfare effects: (i) marginal utility of income is 0.532, (ii) an increasing welfare trigger effect is induced when income inequality (Gini index) falls below the threshold level of 0.24, (iii) social welfare becomes invariant to inequality when Gini surpasses the threshold level of 0.39, (iv) welfare variance rises linearly with clean water access, yet increases rapidly beyond 57% population access, (v) gender is highly elastic to welfare at an elasticity of 4.34, (vi) welfare is sensitive to education more than gender and health care, but less than water access and life expectancy, (vii) welfare is most sensitive to life expectancy when compared to any other factor, (viii) welfare falls when more expenditures are needed to eradicate poverty, and each \$1m increase in required expenditures to eradicate poverty leads to a welfare reduction of 0.8% (PPP adjusted), and finally, (ix) welfare variance decreases with more intensive poverty. Levels of country development are then clustered around three-level categories of social welfare.

Keywords – social welfare, utility, equity, empirical, socio-economic

JEL – I3, D6, C8, O2

Towards an Aggregate Social Welfare Function with Application to Developing Countries

1. Introduction

The pursuit of happiness and the good life has always been the target and interest of the human race, since the start of time. Plato's *Republic*, Aristotle's *Nicomachean Ethics*, and Smith's *Moral Sentiments* are examples of classical works in this domain. Consequently, one may ask what exactly is happiness and the good life? Defining the good life is sometimes seen as an incomprehensible task (Arrow, 1950). Yet, the socio-economic analysis of human welfare can be taken as an indicator of communal utility (Lange, 1942), perhaps itself an indicator of the good life. In essence, the concept of social welfare is defined as "the well-being of the society or community at large" (Pearce, D. 1992). Yet the *MIT Dictionary of Modern Economics* summarizes two sets of problems in dealing with social welfare:

".. in defining social welfare we face two sets of problems. The first problem concerns the 'social' aspect. In general, social welfare is seen as some aggregation of the welfare of individual members of a society - this raises the question of how the aggregation is to be achieved. The second problem relates to the concept of 'welfare'. I.M.D. Little has argued (see his *Critique of Welfare Economics*, 2nd ed., Oxford University Press, Oxford, 1957) that 'welfare' is an ethical concept since to define something as contributing to welfare is to make a *Value Judgment* about whether that thing is good or bad. Alternatively it has been argued that welfare should be equated with the satisfaction of individual preferences and regarded as a 'technical' term. On the whole, Little's argument is the more widely accepted".

In dealing with these problems in our research, the technical definition of social welfare is emphasized given value judgments in its mathematical formulation, taking the "both sides" approach attributed to Bergson (Samuelson 1977 and Pearson 1992). In our proposition, a socio-economic factors approach is undertaken for tackling critical dimensions of social welfare. Largely, a composite cardinal indicator of social welfare is the approach undertaken.

2. Social Welfare Functions: A Basic Review

The mathematical formulation of social welfare has taken many strides in the economic literature (see Bergson (1938), Arrow (1950), Little (1957), Samuelson (1977), Harsanyi (1987), and Moulin (2004)). From the literature, there are three approaches to social welfare formulation: (1) a constitutional mapping of preferences, (2) utilitarianism welfare,

and (3) cardinal welfare. The first approach has been pioneered by Kenneth Arrow leading to his famous *Impossibility Theorem* of mapping individual preferences to social preferences. The second approach has been pioneered by Bergson and Samuelson, based on Benthamite utilitarianism, and usually includes the maximization of self-utility with a *social possibilities constraint* similar to the budget constraint in Neoclassical literature. Consequences of Pareto improvement and Pareto social optimality (i.e. when applied to the *highest collective happiness*) are direct applications of the Bergson-Samuelson utilitarian welfare. The third approach is based on cardinal welfare which is grounded on a common scale of preferences which is multi-dimensional. It is seen as the most flexible approach. It has gained wide acclaim because of its flexibility, and has gathered pioneer contributions from Amartya Sen, John Rawls, Hugh Dalton, and others. Moulin (2004) identifies *six axioms for cardinal welfare functions* as follows:

- 1- Monotonicity: *if a person is better off while others are the same, welfare improves.*
- 2- Symmetry: *all people in the society are treated equal in valuing their welfare. So, if we re-order Person X after Person Y, compared to Person Y after Person X initially, and both persons' welfare did not change, then social welfare must not change.*
- 3- Continuity: *the feasible social welfare function is a closed set. This means that the spectrum of social welfare profiles have possible numerical values within a closed continuum ordering, which may or may not be linear.*
- 4- Independence: *this is also referred to as independence of irrelevant preferences, or independence of unconcerned agents. Basically, social welfare ordering should not change (i.e. welfare is "independent") from individuals whose utility did not change and who do not change others' utility. This relates to utility separability across individuals of the same society if and only if none of the other individuals are affected. Conversely, an increase or decrease of one person utility, if induced by the welfare of another, must change the social welfare outcome.*
- 5- Common Scale: *comparing two social welfare states must not change if both are multiplied by the same positive scalar number.*
- 6- Equity ("Pigou-Dalton Principle"): *a transfer from the rich to the poor is desirable, as long as it does not bring the rich to a poorer situation than the poor.*

Cardinal social welfare has the advantage of multiple input possibilities leading to an output ordering. For example, the social welfare function can be based on per capita income (Y) and access to clean water (W) using Cobb-Douglas multiplicative formulation:

$$SWF = (Y)^a(W)^b, \text{ given } 0 < a, b < 1 \quad (1)$$

It can also take very simple formulations, such as the sum total of individual incomes:

$$SWF = \sum_{i=1}^n Y_i \quad (2)$$

Alternatively, a simple cardinal social welfare function can be formulated as the average life expectancy (LE) of individuals in society:

$$SWF = \frac{1}{n} \sum_{i=1}^n LE_i \quad (3)$$

Sen (1987) proposed an *inequality-adjusted* income measure of social welfare:

$$SWF = \bar{Y}(1 - G_Y) \quad (4)$$

where \bar{Y} is average income and G_Y is the Gini index of income inequality.

On the other hand, John Rawls (1971) proposed an even more distributive justice function whereby social welfare ordering is akin to the least-off segment of the population:

$$SWF = \min (Y_1, Y_2, Y_3, \dots, Y_n) \quad (5)$$

where n are the groups or segments of the population, and in the extreme distributive case, n is taken as the number of people in the society. The latter is implying that social welfare is a changing function of the welfare of the least-off individual in society.

In Tresch (2015), a comprehensive formulation of Atkinson's (1971) original social welfare function is beautifully laid out as follows:

“Atkinson sought a very simple specification of welfare—one that could easily be applied to the income data and yet would capture the full range of ethical judgments from the utilitarian to Rawlsian. He achieved this with three highly simplified and heroic assumptions: the social welfare function is utilitarian, everyone has identical tastes, and utility exhibits diminishing private marginal utility of income. Atkinson's assumptions (are) widely adopted in applied social welfare analysis.”

The Atkinson welfare function is based on the *diminishing utility of wealth* concept:

$$W(y) = y^* + 2y^{*\frac{1}{2}} + 3y^{*\frac{1}{3}} + \dots + (n-1)y^{*\frac{1}{n-1}} + n[y - (n-1)y^*]^{\frac{1}{n}} \quad (6)$$

where y^* is world average income and n is the number of countries with available data.

More modern formulation of cardinal welfare functions are extensions of the above formulations with an emphasis on a specific domain. For example, an extension of Sen's social welfare function with a focus on *inequality aversion* is given by Champernowne and Cowell (1998) as follows:

$$SWF = \frac{1}{n} \left[\frac{1}{1-e} \sum_{i=1}^n (Y_i)^{1-e} \right] \quad (7)$$

where e is a representation of *inequality aversion* (an “iso-elastic” measure), and $e \in (0, +\infty)$. Low values of e would imply a low aversion to inequality (such as utilitarian welfare of total income), and high values of e would imply a Rawlsian distributive welfare. The Food and Drug Administration (FAO) propose $e = 0.5$ as a median recommended measure of inequality aversion (Giovanni and Liberati (2006)).

On the other hand, a social welfare function with focus on *environmental resources* usually employ a dynamic welfare formulation (*see* Krabbe (1989), Frank and Babunovic (1984), Hartwick (1977), Kennedy (1964), Lewis et. al. (1979), and Hediger (2000)). This is due to factor-saving and factor-substitution elements across time, both leading to a dynamic change in the factor shares of income between resources and between peoples (or “generations”). The pressure of industrialization on the environment also carries a dynamic approach to welfare, since static corrections of social pollution losses are insufficient to achieve social optimality (Kolstad and Krautkraemer 1993). Marginal external cost analysis is therefore found insufficient to fully absorb resource degradation and inter-generational equity.

In an article by Wang and Lu (2019), a formulation of social welfare with environmentally friendly goods with externalities is proposed. An extended dynamic version is:

$$SWF = e^{-rt} \int_{t=0}^T \left[\sum_{i=1}^n U_{i,t} - \sum_{j=1}^m C_t^Y(Q^Y) - \sum_{k=1}^p C_t^N(Q^N) + \phi_t + \varphi_t \right] dt \quad (8)$$

whereby r is the social rate of time preference, T is the long run time horizon, n individuals compose the society with U as Hicksian monetary utility, m firms produce environmentally friendly goods Y at cost C^Y , and p firms produce non-environmentally friendly goods N at cost C^N , with $C^Y > C^N$ assumed due to the added cost of clean technology, and ϕ and φ are positive and negative externalities at time t respectively, such that $\phi_t > 0$ and $\varphi_t < 0$.

3. Proposed Formulation

We propose a multi-dimensional cardinal social welfare function. It is not a comprehensive set of all possible socio-economic dimensions, but rather intends to grasp the most critical socio-economic factors leading to social welfare from our perspective. We also try to include variables which have readily available data on a country scale. Furthermore, the proposed function is static and not dynamic. We also adhere to the strict definition of social welfare as the *total well-being* of the society or community at large, rather than the quality of life of an individual citizen living in that society. Thus, we take the approach of Pearce (1992), Moulin (2004), Krabbe (1989), and Tresch (2015), in contrast to the traditional utilitarian views of Harsanyi (1987), Samuelson (1983), and I.M.D. Little (1957).

Given the above, we see this research as an attempt or initial proposition towards social welfare which is not exhaustive nor intended to be so. Nonetheless, we attempt a cardinal measure of social welfare encompassing critical dimensions of collective human welfare, rather than individually separable utilities.

The following factors are principally taken into consideration:

- (1). *Per capita income* (adjusted values based on purchasing power parity, and using Atkinson's diminishing social utility of wealth concept)
- (2). *Poverty* (poverty incidence, poverty depth, and expenditures needed to eradicate poverty)
- (3). *Income inequality* (Gini index)
- (4). *Clean water* (as a social entitlement)
- (5). *Access to health services* (as a social entitlement)
- (6). *Education attainment* (human development)
- (7). *Life expectancy* (livelihood aspect of human welfare)
- (8). *Gender equality* (social equity and cohesion).

Indirect factors which may cause, or be caused by social welfare, are not directly included in the social welfare formulation. However, causality of social welfare to those factors can be addressed as separate data sets. For example, the World Economic Forum's *Global Competitiveness Index* which basically measures a country's aggregate productivity, can be viewed as a consequence to social welfare, or be caused by it. Similarly speaking, other indicators related to climate change, innovation, openness index, and the World Bank's *Doing Business* index, all can be addressed using the same logic. Moreover, it must be mentioned that the approach taken in this research is a strict socio-economic formulation, rather than a political-cultural context of social welfare. Hence, political freedom, legal independence, and other related aspects are not directly incorporated into our formulation.

An aggregate social welfare function is proposed as follows:

$$\text{SWF} = [\text{W}^{\text{adj}}(\mathbf{Y}) - \text{P}^e(\mathbf{P}_0, \mathbf{P}_1, \mathbf{Z})] (1 - \text{G}_Y) (\text{G}_g) [\text{f}^{-1}(\text{S}_D)] \quad (9)$$

where

- a) $\text{W}^{\text{adj}}(\mathbf{Y})$ is the adjusted per-capita income using Atkinson's formula:

$$\begin{aligned} W(y) = & y^* + 2y^{*\frac{1}{2}} + 3y^{*\frac{1}{3}} + \dots \\ & + (n-1)y^{*\frac{1}{n-1}} + n \left[y - (n-1)y^* \right]^{\frac{1}{n}} \end{aligned}$$

where y^* is world average income and n is the number of countries.

- b) $P^e(P_0, P_1, Z)$ is total expenditures needed to eradicate poverty, which depends on the head-count ratio P_0 , the poverty gap P_1 , and the poverty line Z .
- c) G_Y is the Gini index for income inequality.
- d) G_g is a composite index to reflect aversion to gender inequality.
- e) $f(S_D)$ is a vector of social deprivation (an inverse of livelihood and social entitlements) which includes access to water, access to health, education attainment, and life expectancy.

The characteristics of this social welfare formulation are numerous. First, it is a direct extension of per-capita income formulation, by inclusion of *non-income* social entitlements pioneered by Amartya Sen as measured by social deprivation in livelihood, gender, and education. This is explicitly formulated as a vector of variables in the $f(S_D)$ sub-function above, which have four non-income components: (1) access to clean water, (2) access to health, (3) education attainment, and (4) life expectancy. Second, such a social welfare formulation includes Bernoulli's *diminishing social utility of wealth* by the income adjustment in Atkinson's formula, which is given by $W^{adj}(Y)$. This adjustment makes sure that the level of welfare development is not just a linear extension of income, but rather measures utility of wealth as a sub-dimension of social welfare. Third, $P^e(P_0, P_1, Z)$ is a measure of the *total expenditures needed to eradicate poverty* which is dependent on poverty count, poverty gap (depth of poverty), and is sensitive to a given cut-off poverty line. It has a negative connotation to social welfare since more resources needed to eradicate poverty implies a lesser level of welfare development historically attained until present time. Fourth, *inequality in human development* has two core dimensions: inequality of gender G_g and inequality of income G_Y , both of importance to human development, and hence social welfare. Basically, equal opportunity and equal income entitlements are seen as the benchmarks to which actual performance deviates. Fifth, *social deprivation* is reflected as an inverse measure of human development giving rise to $f^{-1}(S_D)$. Such a methodology measures social losses by lack of entitlements as the mirror image of social welfare gains to their access. Finally, the general social welfare formulation is given using a *Cobb-Douglas* style *multiplicative* formulation (in contrast to Samuelson's additive function) such that the relative sensitivity of each variable to social welfare can be feasibly evaluated using percentage changes or elasticity measures.

Table (1) summarizes the included dimensions in the social welfare function, along with descriptive assessment on the role of each factor to the collective welfare of a society.

Table (1): Factors Included in the Social Welfare Formulation

<i>Factor</i>	<i>Value/Adjustments</i>	<i>Measurement Role to Social Welfare</i>	<i>Mathematical Input</i>
Per Capita Income	Atkinson diminishing utility of wealth	Capabilities	$W(Y)$
Poverty	Expenditures needed to eradicate poverty	Eradication	$- P^e$
Income Inequality	Gini index derived from the Lorenz curve	Minimization	$(1-G_Y)$
Clean Water	Percentage of population with access to clean or improved water source	Social Entitlement (Universal)	Within $f^{-1}(S_D)$
Health Services	Percentage of population with access to health services	Social Entitlement (Universal)	Within $f^{-1}(S_D)$
Education	Schooling education attainment (enrollment relative to those eligible)	Human Development and Social Entitlement (Age Groups)	Within $f^{-1}(S_D)$
Life Expectancy	Mean life years of an individual person in society	Human Livelihood	Within $f^{-1}(S_D)$
Gender	Based on the Gender Development Index	Gender Equity	G_g

4. Applying the Six Axioms of Cardinal Welfare

Based on the proposed social welfare function in (9) above, the 6 axioms of cardinal welfare are analytically tested, as follows:

1- *Monotonicity*

The function has $dW/dY > 0$ (even though it is diminishing) from (6), and with $d(SWF)/d(W) > 0$ in (9), then by the simple chain rule: $d(SWF)/dY > 0$. Note here that the increase in welfare is not proportional to the increase in income due to Atkinson's adjustment. It is, nevertheless, monotonic. The rest of the parameters enforce this positive monotonicity.

2- *Symmetry*

Re-ordering will not change the value of the function in (9), since $W(Y)$ is a collective aggregation of incomes, i.e. $W(Y_i+Y_j)=W(Y_j+Y_i)$, $i \neq j$. Also, the function is static and not dynamic, hence the problem of people migration away or into the country is invariant to the static form of the welfare function proposed. However, one should caution that a dynamic extension of the proposed function may not have the symmetry axiom in place due to migration and other factors affecting the number of individuals in society. Another dimension is that of poverty (P^e) and inequality (G_Y). Both have the symmetry axiom satisfied. This is because we are not including poverty count *per se* but rather the total expenditures needed to eradicate poverty at a certain point of time. Basically, the reordering of two poor people's monetary distance from the poverty line will not change the total poverty eradication expenditure included in the formula. The same logic applies to inequality. The other parameters also carry the same logic, albeit with the rare exception of gender change within the same period of time in calculating G_g .

3- *Continuity*

The social welfare function proposed is a closed set and cannot take infinite forms in positive or negative scales. See column (2) in Table (1) for an explanation regarding the continuum of each factor used in the proposed function.

4- *Independence*

The independence axiom is the most complicated to apply to our welfare proposition. Let us discuss its implications regarding one welfare domain after another.

a) Independence in income & income transfers

The independence axiom when applied to individual income levels can be ascertained by per-capita income, which is itself an average number by definition, and hence encompasses utility separability. Family dependency expenditures, such as *parent-to-child* income transfers, will entail positive cross-utility spillover effects which do not violate the independence axiom, so long as the child is a non-income bearer and/or the child is not included in an official employment roster. For more general income transfers, certain restrictions apply. These are further discussed within the Equity axiom of Pigou-Dalton Principle.

b) Independence in poverty alleviation and income inequality

The independence axiom when applied to poverty count may be violated only if the poverty line changes, since *non-poverty individuals* (before or after the

change in poverty line) will now no longer be independently separable. However, within a well-defined poverty line, the independence axiom holds. Further on this issue, independence as it relates to the *Gini index* (income inequality) will always hold true since the Lorenz curve is derived from cumulative incomes, which are utility separable.

c) Independence in negative externalities

Through the income factor within our welfare proposition, negative externalities may exist. Negative spill-over effects, such as pollution, will violate the independence axiom unless: (i) informal corruption leading to abatement is significantly reduced, and (ii) environmental corrective mechanisms are efficient, such that adequate penal enforcements fully correct the externality.

d) Independence for social entitlements in water & health

Water and health are part of social entitlements, expressed within aversion to social deprivation sub-function $f^{-1}(S_D)$. Since they are expressed in percentage terms in raw data, the independence axiom is not violated. However, this assumes that the number of people with access to clean water or health services are equally enforced to do so thru “equal opportunity” of access. This is a limitation on the independence axiom, and although the axiom itself is not violated, yet the equity dimension of social entitlements may not be fully enforced.

e) Independence in education attainment

Regarding education attainment, the independence axiom will be binding, even if there is child school leakage to informal labor markets. This is because the number of students enrolled relative to those eligible is the education factor chosen, and both numbers are individually separable. The education leakage of children working in informal markets due to poverty is a major development challenge that needs to be addressed. A deeper leakage will lead to lower social welfare in our proposed formulation.

f) Independence in gender and female-child positive externalities

There is positive spill-over effects in health and education between mother and child. This positive cross-utility behavior is re-enforced in the social entitlement metrics used in our formulation. Gender, religion, and race are invariant to the independence axiom, so long as such data are correctly reported. Similar to the symmetry axiom, the only exception is gender change within a period of reporting.

5- Common Scale

This axiom directly fits with our welfare formulation: $(\lambda W_1) > (\lambda W_2)$ is always true given an initial condition $W_1 > W_2$, for $\lambda > 0$. Since $d(SWF)/d(W) > 0$ then this also applies to two social welfare states. In retrospect, all the factors given in Table (1) have positive monotonicity to social welfare (see Equation (9)).

6- Equity (Pigou-Dalton Principle)

The Pigou-Dalton Principle states that a transfer from the rich to the poor is desirable, as long as it does not bring the rich to a poorer situation than the poor (see Pigou (1912), Dalton (1920) and Moulin (2004); also see Blum (2012) for empirical tests on the principle). Assuming rationality of self-interest, and cross-utility altruism does not lead to major self-utility losses, which is a fair and logical assumption, then this axiom holds true for our proposition. More formally, our welfare propositions in (6) and (9) will need the following two conditions for this particular axiom to be binding:

$$U_R(Y_R^0 - \tau) \geq U_P(Y_P^0 + \tau) \quad (10)$$

$$W(U_R) > W(U_P) \quad (11)$$

The above two equations basically state that an *income transfer* τ from a rich individual (R) to a poor individual (P), with original income levels Y_R^0 and Y_P^0 respectively, must have their utility *after* transfer as weakly dominated by the rich over the poor, whereas the corresponding welfare states must be strongly dominated. These two conditions make sure that (i) positive cross-utility effect of income transfer does not lead to drastic impact reduction on self-utility such that aggregate welfare is not reduced, and (ii) strong monotonicity holds true (axiom 1) for an income transfer transaction within society. Also it should be noted that the Atkinson diminishing marginal utility of welfare in (6) is also a critical condition since $W(Y^\alpha)$, $\alpha > 1$ immediately violates the Pigou-Dalton Principle. In our formulation, $\alpha < 1$.

5. Data Charts and Country Welfare

Using a data sample of 79 developing countries, we apply the aggregate welfare function and provide data sensitivity graphs. This is important for two reasons: (i) a robust check on the logical inference of the proposed aggregate welfare function, and (ii) welfare sensitivity measures (such as *welfare elasticity indicators*) can be inferred relative to critical socio-economic factors of human well-being. In this sample, we use the World Bank (2020) definition of “low income, lower middle income, and upper middle income [industrialized]

economies”. Developing countries are highly sensitive to social welfare, and global implications can be inferred in terms of sensitivity to human well-being when *decomposed* into various factors. These factors include life expectancy, education, water access, poverty, income per capita, inequality, health and gender equity.

Table (2): Welfare Sensitivity to Socio-Economic Factors

<i>Socio-Economic Factor</i>	<i>Welfare Sensitivity</i>	<i>Value</i>	<i>Figure</i>	<i>Comments (All \$ values are PPP adjusted)</i>
Per Capita Income	Marginal Utility of Income	0.532	Figure 1	a) \$1 gain in income generates 0.532 increase in aggregate welfare, or “social utility”. b) Positively satisfied societies (positive welfare) require a minimum per capita income threshold of \$1,109/person. c) There is a mild increase of variance in this sensitivity measure for high levels of income (i.e. higher than \$4,000/person). d) zero income (psychologically attributed to zero income unemployment) generates negative welfare.
Gini Index (Income Inequality)	Welfare “Trigger Effect”	See Comments	Figure 2	a) Welfare is continuously reduced with higher inequality, up to Gini of 0.39 b) Welfare is insensitive to inequality for $Gini > 0.39$ c) A welfare trigger effect is asymptotic as $Gini \rightarrow (0.24)^-$
Gender Development Index	Gender Elasticity	4.34	Figure 3	Welfare is highly sensitive (i.e. highly elastic) to gender development.
Improved Water Source	Water Sensitivity	5.55	Figure 4	a) Welfare is very sensitive to clean water access. b) Water sensitivity has wide dispersion beyond 57% population access to clean water.
Health Care	Health Sensitivity	3.7	Figure 5	Although significant, health sensitivity is inferior to water and gender. Moreover, there is high variance across the entire welfare spectrum.
Poverty Eradication	$\frac{d(\ln SWF)}{dP^e}$	- 0.8%	Figure 6	a) Welfare (<i>SWF</i>) falls when more expenditures are needed to eradicate poverty (P^e). Specifically, a \$1m increase in required expenditures to eradicate poverty leads to a welfare reduction of 0.8%. b) Unlike per capita income, welfare does not fall below zero: if $P^e \rightarrow +\infty$, then $SWF \rightarrow 0$. c) Welfare variance decreases with more P^e .
Education Enrollment	Education Sensitivity	5.4	Figure 7	Welfare is sensitive to education, more than gender and health care, but less than water access and life expectancy.
Life Expectancy	Sensitivity (Highest)	5.77	Figure 8	Welfare is most sensitive to life expectancy as compared to any other factor.

Table (2) summarizes the data charts and welfare sensitivity to the different socio-economic factors. It should be noted that elasticity in Table (2) is defined as $\frac{d(\ln(SWF))}{d(\ln x)}$ where x is the chosen factor, whereas “sensitivity” is defined as $\frac{d(\ln(SWF))}{dx}$, i.e. percentage change in social welfare for a one unit change in the factor.

The *marginal utility of income* is $0.532 < 1$. This reinforces Atkinson’s formulation of diminishing marginal utility of wealth. The highest sensitivity in social entitlements is due to *life expectancy*, closely followed by *clean water access*. There are three *threshold levels* we arrived at, given our aggregate welfare formulation:

- (1) An increasing *welfare trigger effect* is induced when income inequality (Gini index) falls below the threshold level of 0.24
- (2) Social welfare becomes *invariant* to inequality when Gini surpasses the threshold level of 0.39
- (3) Welfare is highly sensitive to clean water access, yet welfare variance increases rapidly beyond the threshold level of 57% population access.

Additional information and important details are provided in Table (2) along with sensitivity comparisons.

Three categories (high, average, and low) regarding *country welfare* are conceived from the data charts and aggregate social welfare formulation. Table (3) below summarizes these three categories using relative cut-off welfare scores and their corresponding socio-economic factors. The results in Table (3) are meant to be a cross-comparison between welfare states. This can be used in country welfare positioning, such as target versus initial welfare states for a given country, or welfare comparisons between different countries.

Table (3): Country Welfare Characteristics: Three Categories

	<i>High Country Welfare</i>	<i>Average Country Welfare</i>	<i>Low Country Welfare</i>
	Socio-economic factors are usually <i>superior</i> to:	Socio-economic factors are usually <i>parallel</i> to:	Socio-economic factors are usually <i>inferior</i> to:
Per capita income (Atkinson adjusted) in PPP \$	\$14,267/capita	\$10,507/capita	\$2,989/capita
Gini Index	0.25	0.3	0.4
Gender Development	0.85	0.8	0.6
Clean Water	0.97	0.85	0.75
Health Care	1.0	0.85	0.65
Education	0.9	0.8	0.6
Life Expectancy	85 years	76 years	62 years
Cut-off Welfare Score	SWF=7000 Max. SWF = 10,000 SWF% = 0.7	SWF=5000 Max. SWF = 10,000 SWF% = 0.5	SWF=1000 Max. SWF=10,000 SWF% = 0.1

6. Limitations

The vital topic of social welfare has an analytical universal appeal and has the potential to include endless variables in its composition. This research covers both theoretical and empirical domains of social welfare. There are numerous limitations in our research which can be taken as grounds for further research. In terms of theoretical formulation, the aggregate welfare function proposed abstains from the political-legal dimension. Therefore, it is not an institutional welfare proposition. Human rights and corruption are not directly expressed. Whether cultural norms are cause or effect to social welfare is an interesting dimension to be researched as well. The political, institutional, cultural, and historical domains of social welfare can be addressed as an extension or reformulation of our proposition. From the data and empirical side, we have been constrained by the static welfare formulation, and hence time series data has not been applied. Time series analysis will be very useful when a dynamic welfare function is used. Extensions to socio-economic factors such as environmental GDP, information literacy, digital infrastructure, and rule of law, are some ideas that come to mind which can sharpen the research findings. Another dimension is a discussion of causality between social welfare and quality of life. In our research, we have assumed that quality of life is a consequence to social welfare. This assumption can actually

be reversed. Concerning the country selections, we have only applied our formulation to developing countries, and hence a natural extension would be an application to all countries and to a global level across time as well. Finally, in the analysis of cut-off welfare scores, we assumed three distinct welfare states, whereas a simulation of continuous welfare states is also possible.

7. Conclusion

An aggregate social welfare function has been feasibly proposed. Critical socio-economic factors in the welfare formulation involve an Atkinson-adjusted per capita income, poverty eradication, social entitlements for clean water access and health care, education attainment, income inequality, gender equity, and life expectancy. The six axioms of cardinal welfare have been analytically tested and generally proven valid for the proposed function. The social welfare function obeys monotonicity, symmetry, continuity, independence, common scale, and the Pigou-Dalton equity principle. Applying the social welfare formulation to 79 developing countries, data sensitivity charts and factor decomposition reveal many welfare implications for various socio-economic elements. Three “welfare state” scenarios are conceived from the data charts: high country welfare, average country welfare, and low country welfare. Overall, this research contains several limitations which can be taken as grounds for further research, such as extending welfare from static to dynamic formulation, and the inclusion of institutional and political-cultural factors in its application.

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Annex: Data Charts

Figure 1: SWF Vs GDP per capita

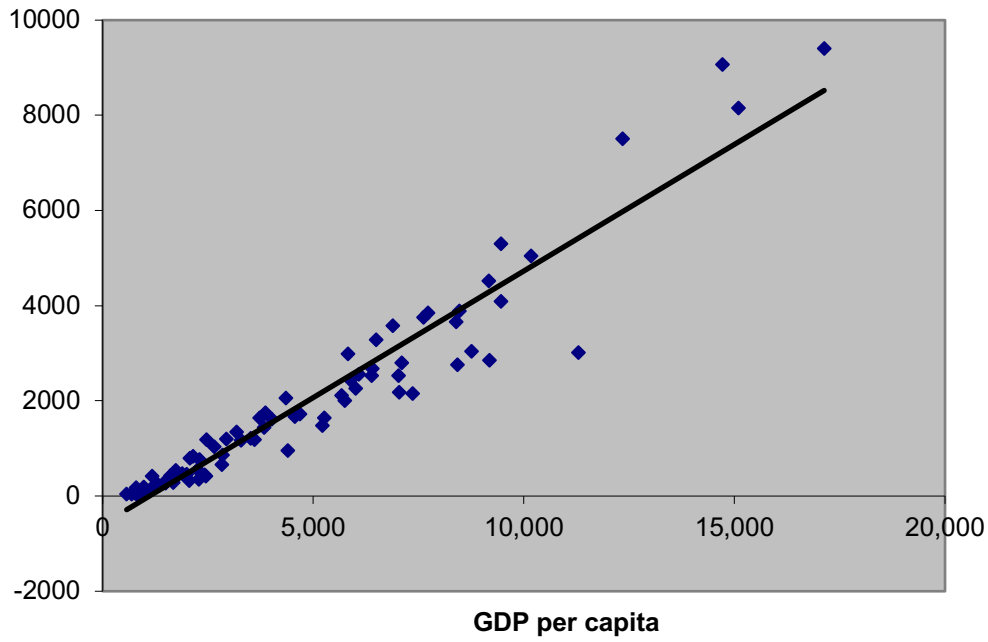


Figure 2: SWF Vs Gini Index

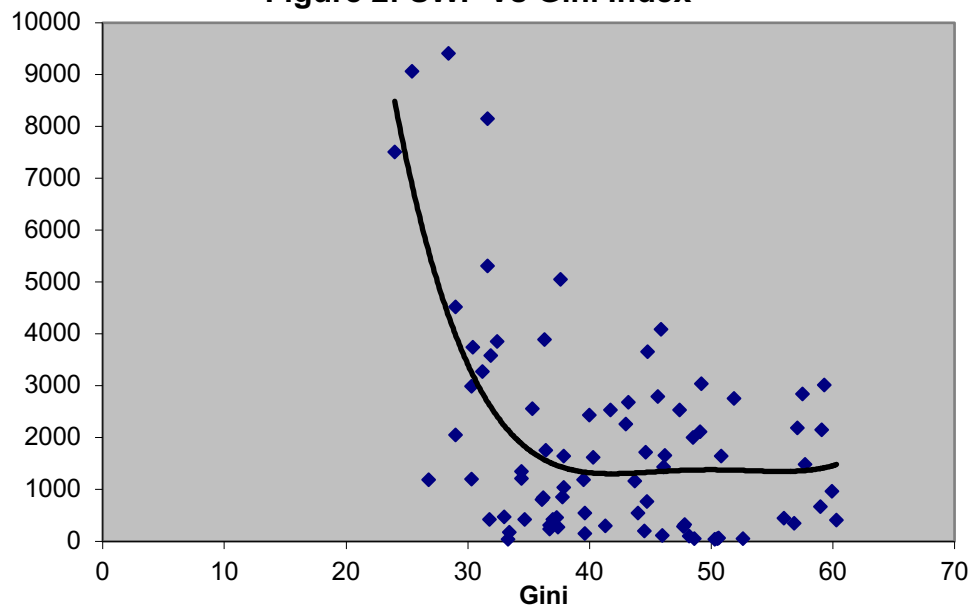


Figure 3: SWF Vs Gender Development Index

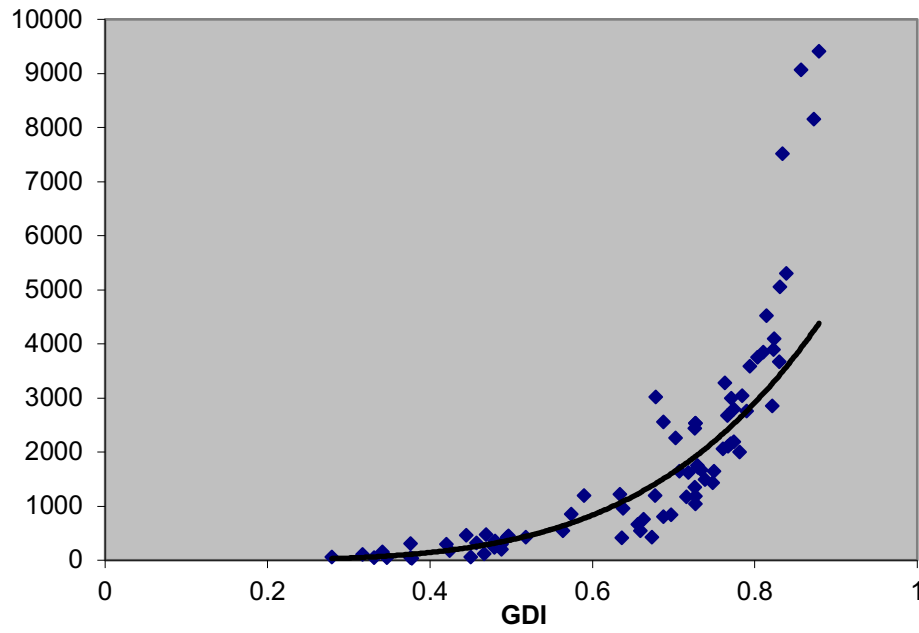


Figure 4: SWF Vs Access to Improved Water Source

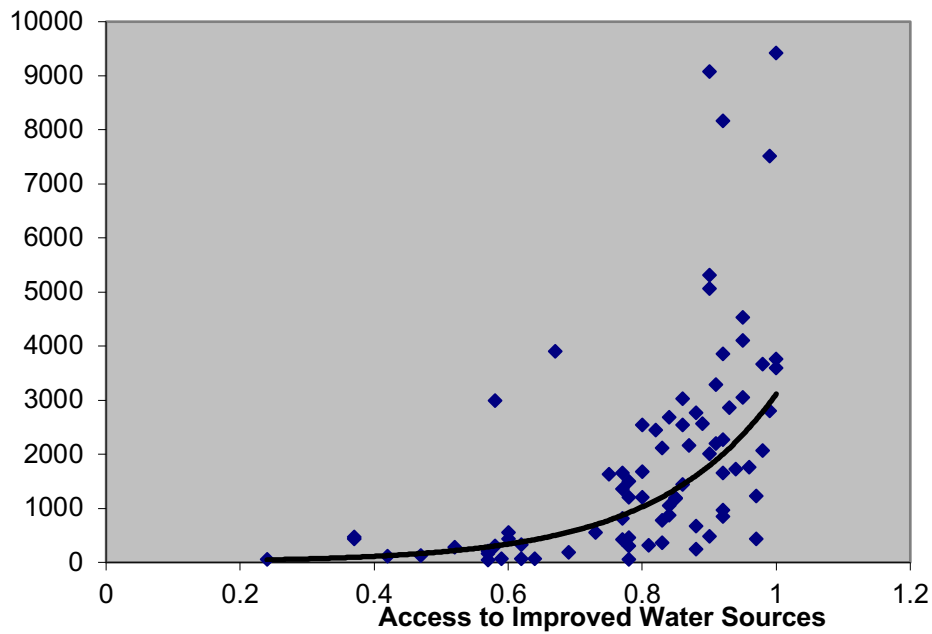


Figure 5: SWF Vs Healthcare

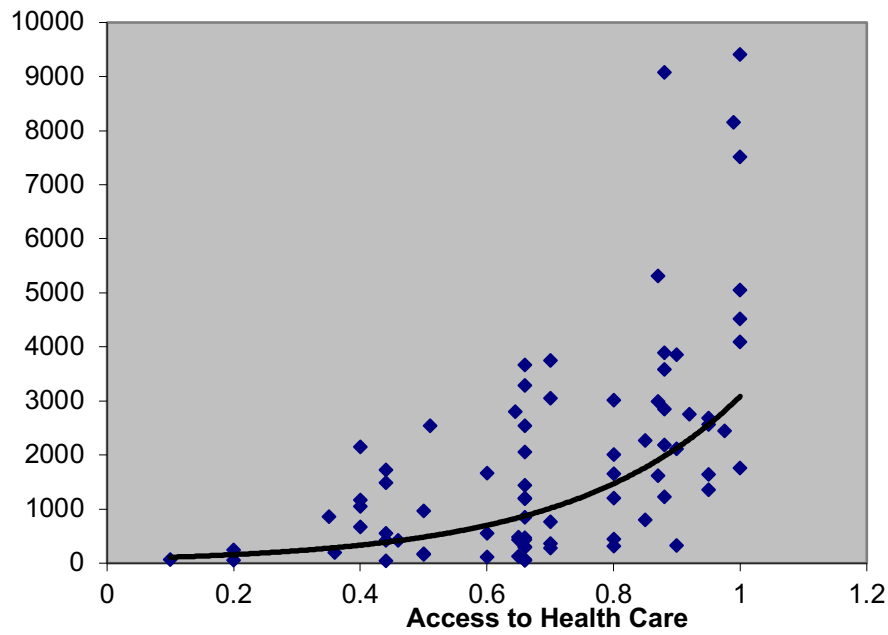


Figure 6: SWF Vs Poverty Eradication

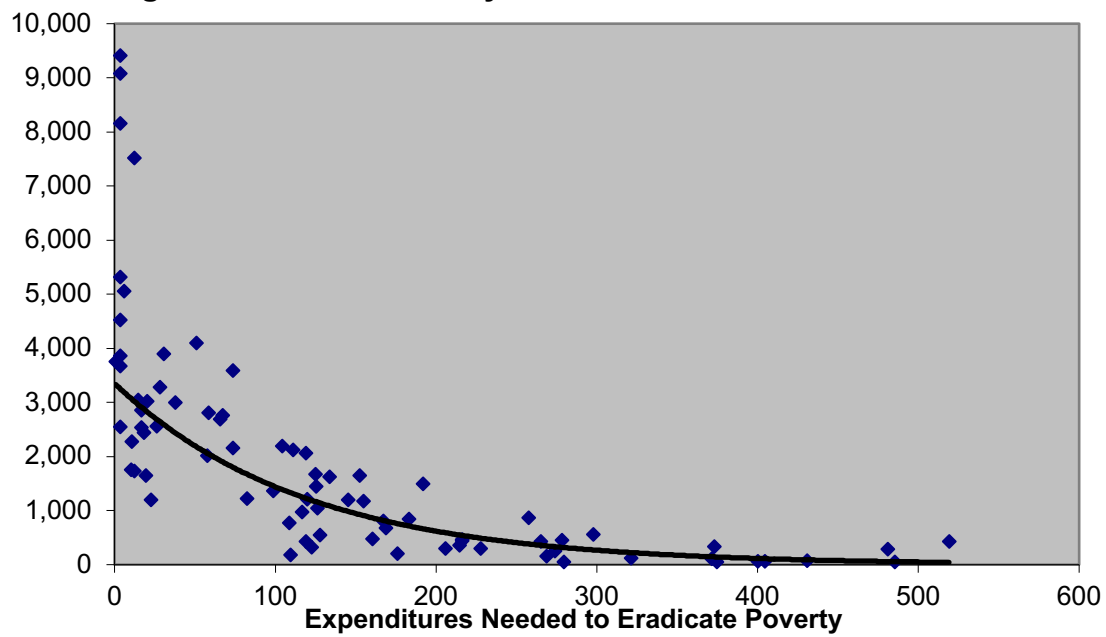


Figure 7: SWF Vs Education Enrolment

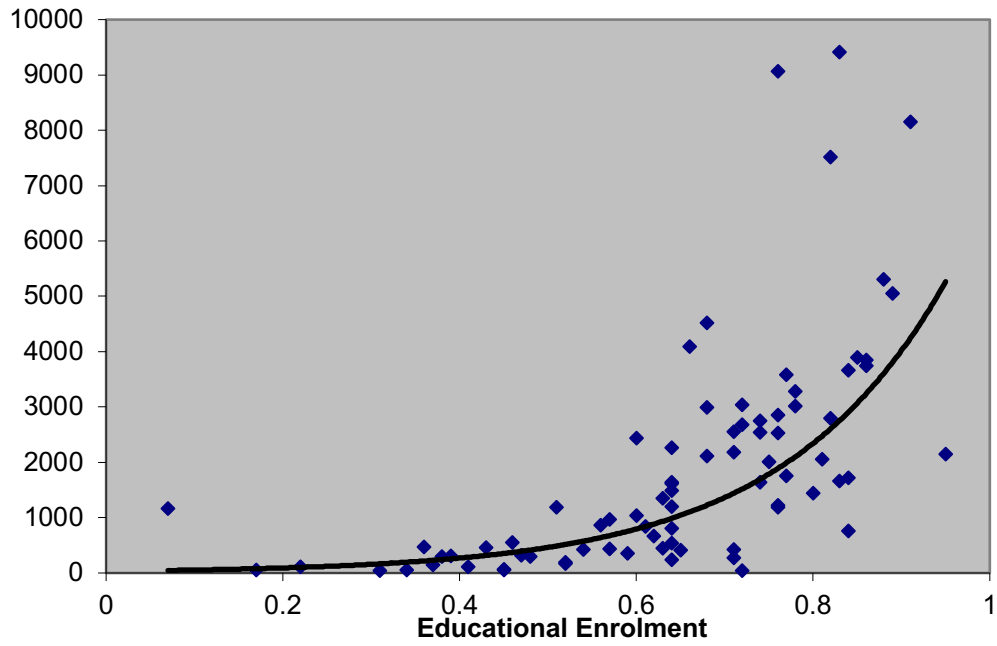


Figure 8: SWF Vs Life Expectancy

