On Equilibrium Number of Firms

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2006

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JEL: D4, L1

Introduction:

Of valued interest to economists is the extent of market entry evolving towards equilibrium co-existence of firms until all economic profits are dissipated. The number of firms, which can coexist together reaching saturated market equilibrium, is termed "the equilibrium number of firms". With an incumbent duopoly, competition in prices, products differentiated by quality, and sequential firm entry, an equilibrium number of firms is reached. Market saturation occurs when all quality locations are "fully covered". In addition, consumers obey a love for quality utility hence demand always seeks a higher quality product if offered at similar prices. Analysis of market concentration follows the sequential accommodation of market entry and is studied based on non-collusive industry wide profitability. The level of available production technology is implicit in maximum quality location possible.

Convergence and the Marginal Firm:

A converging solution is reached for the equilibrium number of firms such that "marginal firm" is defined as the N^{th} firm which earns non-zero economic profits. A given quality spectrum of L specifies the location of product offerings; such that the market will be fully covered when no demand is left uncovered by the "marginal firm". To obtain the "equilibrium number of firms", define $P_R \equiv \left(P_N \big/ P_{N-1} \right)$ as the *relative price ratio* between the two highest quality levels offered at market saturation, and from this, the "equilibrium number of firms" is solved by:

$$N^* = \left[2 + \frac{L}{1 + \frac{P_R}{3}} \right] \equiv \left(2 + \frac{3L}{3 + \left(P_N \big/ P_{N-1} \right)} \right).$$

Furthermore, an increase in production technology, implicit in maximum quality location, is accompanied by a *less-than-proportionate* increase in the equilibrium number of firms:

$$0 < \left[\frac{\partial N}{\partial L} \right] < 1 \text{ corresponding to } \left\{ 0 < \left(P_N \big/ P_{N-1} \right) < \infty \right\}.$$

Differentiation and "Technology-Neutral" Market Structures:

In the limit, several categories of behavior are studied. Changes in technology on equilibrium number of firms result in more competitive, less competitive, or “technology-neutral” market structures. These are established by:

(i) Advances in production technology, implicit in maximum quality location, result in more differentiation asymmetry between firms (heterogeneity) yielding a deeper (negative) impact of technology on industry concentration.

(ii) Less differentiation asymmetry (homogeneity) between firms leads to a “technology-neutral” market structure.

(iii) Under the assumption of no collusion and no exit strategy, a more competitive market structure is achieved when marginal impact of technology on long run concentration is accompanied by wider quality scales upon an evolving market structure.

Concluding Remark:

Equilibrium number of firms increase with technology, however, increases in technology are accompanied by a less than proportionate increase in equilibrium number of firms. Less competitive market behavior result from more intensity of differentiation between firms, with wider quality scales on the level of the market leading towards a more competitive market structure. Extensions to relax no exit strategies did not lead to a converging solution.

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Abstract

This article provides a simple account of the effect of quality competition on the extent of sequential entry accommodation for a differentiated oligopoly market characterized by locational differentiation. The model is solved with consumers seeking a "love for quality" surplus utility while firms maximize economic profits constrained by their chosen level of quality location as endogenized within a given spectrum of locational quality differentiation. Initially, a duopoly market is considered, followed by successive market entry until a differentiated oligopoly market is completely saturated, or "fully covered". Analysis of market concentration follows the sequential accommodation of market entry and is studied based on non-collusive industry-wide profitability using an augmented form of the Hirschman-Herfindahl concentration index. The level of available production technology is implicit in maximum quality location possible. In general, it is found that the degree of quality differentiation greatly affects the extent of market saturation and long run concentration. More differentiation asymmetry between firms deepens the marginal (negative) technological impact on long run concentration; with a less-than-proportionate change in the equilibrium number of firms. In the limit, several categories of behavior are studied, which imply a more competitive, less competitive, or "technology-neutral" market structure.

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I. INTRODUCTION

The objective of this article is to outline the impact of quality competition on market saturation and industry concentration for a differentiated oligopoly market structure, where products are differentiated by a non-linear form of locational quality differentiation. The analysis of market saturation with entry accommodation is followed by industry concentration as a measure of the competitive structure of markets in the long run. The oligopoly market is differentiated by quality choice.

Instead of assuming a pre-specified market structure as set and given to the model, and then study the choice of quality differentiation as an area of research within an arranged or prescribed market structure; the opposite direction of causality is taken towards such an analysis. That is, the *effect* of quality competition *on* entry accommodation and *on* the competitive structure of markets in the long run is the direction of research investigation, and not vice-versa. Current analysis, however, is limited to non-linear quality location under exact locational differentiation by quality choice; as a special form of quality competition for a differentiated oligopoly market structure with an incumbent duopoly assumed.

In general, several research assessments within this topic have been successful in the past. From the available volume of literature concerning this issue, there has been numerous articles studying how product differentiation affects the "equilibrium number of brands" within an oligopoly - or monopolistic competitive market structure. These include the earlier works of Chamberlin (1933), Galbraith (1957) and Dixit and Stiglitz (1977). As variants of these insights, Stoneman (1990, 1996), Budd, Harris and Vickers (1993), Vickers (1986), and Reinganum (1984, 1985) offer numerous insights into this interesting topic.

Kwon and Stoneman (1996), Stoneman (1990), and Ireland and Stoneman (1983) are a series of inter-related articles concentrating on the dynamic adoption of technology and its diffusion into industrial markets with adaptation costs. In Ireland and Stoneman (1983), the speed of technological diffusion is shown to depend on the existing number of brands in the market and the dynamic diffusion path is modeled as a function of adaptation costs. The diffusion of new process technology is further elaborated theoretically in Stoneman (1990) and in Kwon and Stoneman (1996) empirically for the UK engineering industry over the period 1983-96. The adoption follows an S-curve path where technology is adapted selectively at earlier stages (with adopted firms monopolizing some aspect of patent or R&D innovations) followed by faster adaptations as the cost of innovation declines and until complete saturation of the new process technology into the market. The question of how market behavior evolves when there is a sequence of opportunities to innovate in a process of "action-reaction" has been explored by Vickers (1986) and Budd, Harris and Vickers (1993). The dynamic state of market equilibrium tends to evolve in the direction where joint payoffs are greater based on the respective effort rates of the firms. Since joint payoffs are related to joint product-market profits less joint effort costs, their analysis suggests influence upon the pattern of efforts is a mutually self-reinforcing manner illuminated by asymptotic expansions. Such a result is found by numerical simulation.

The work of Reinganum (1984, 1985), later utilized in Eaton and Ware (1987) and Eaton and Schmitt (1994), generally considers a market in which one firm is the current incumbent, while the remaining firms are challengers with an imposed sequence of opportunistic innovations. The critical assumption is that success does not imply that the successful firm reaps monopoly profits forever, but only until the next, better innovation is developed. A fully optimizing behavioral model derives the equivalent of the Schumpeterian "process of creative destruction." That is, a firm enjoys temporary monopoly power but is soon overthrown by a more inventive challenger. The speed and nature of creative destruction is then analyzed and several conditions are attempted where such a scenario may not occur. The essential point here is that capitalistic markets are evolutionary by nature and that the fundamental impulse for innovation is essentially a decaying process with time (when markets are completely free from regulation).

In our analysis of accommodating entry, we assume that the Bain-Sylos postulate holds good, following Bain (1951, 1972), Cowling and Waterson (1977) and Schmalensee (1987), where incumbent firms do not change capacity levels (production quantities) due to changes in market entry conditions. The basic assumptions of the Bain-Sylos hypothesis, as augmented in Cowling and Waterson (1977), Schmalensee (1987), and summarized by Tirole (1998) attest that the Herfindahl index yields a proportional (increasing) measure of industry-wide profitability if firm behavior is non-collusive such that optimal capacity levels (production quantities) are maintained. The Bain-Sylos hypothesis is also in line with our assumption of exact locational differentiation by quality choice, where consumers are differentiated by their marginal utility to consume ("love for quality" characteristics) rather than due to their disutility from purchasing a non-ideal brand caused by unit transportation costs. In essence, Bain's pioneering work distinguishes between three types of entry barriers: cost advantages of incumbent firms, production technology (e.g. economies of scale), and product differentiation. In our model, we are analyzing a specific case of product differentiation by quality choice, but with no allowance for asymmetric costs yet allow for changes in maximum quality scale (as a proxy for production technology). In our proposed model, we further add the critical assumption of no exit strategy for any incumbent firm in the market. This follows the spirit of Demsetz (1973) in his non-collusive argument for Bain's hypothesis about the positive correlation between market concentration and industry profitability. Also, Schmalensee (1987) re-explains Bain's hypothesis with Demsetz's conclusion by noting that differentiation and cost asymmetries between firms yield output asymmetries, increasing the concentration index, and at the same time, allow differentiated firms to enjoy a rent, thus increasing industry-wide profit.

Schmalensee (1987) and Geroski, Gilbert, and Jacquemin (1990) find the correlation between concentration and profitability most evident under Cournot competition and not so evident under Bertrand competition. In addition, the equilibrium number of firms is found unrelated to social welfare (as measured by consumer surplus plus industry profit) for Bertrand competition whereas it is positively related to social welfare for Cournot competition. In our model, although we do not provide social welfare calculations, we find that the equilibrium number of firms towards full market saturation is directly related to the level of production technology (i.e. the extent of maximum quality location) adopted in quality competition. Regarding this topic, Von-Weizsacher (1980) hypothesizes that substantial increases in production technologies may create an equilibrium number of firms beyond the number of firms required at the social optimum.

Our assessment here deals with a special case of quality competition with sequential entry where demand behaves according to a "love for quality" consumption behavior contingent on a particular non-linear form of surplus value functions. The adoption of technology is implicit in terms of quality location in the sense that a higher level of production technology implies a higher level of maximum quality location attainable in the market. An incumbent duopoly is assumed, and the analysis begins by exploring the impact of such form of quality differentiation upon the equilibrium number of firms until full market saturation followed by an assessment of long run concentration using the Hirschman-Herfindahl concentration index. In general, we find three categories of market behavior: (1) a *more competitive* (less concentrated) market structure is achieved as the marginal impact of technology on long run concentration is accompanied by wider quality scales (more differentiation asymmetry between firms) upon an evolving market structure; (2) a *less competitive* (more concentrated) market structure is achieved with less differentiation asymmetry between firms relative to their existing level of quality locations, and with a sequential entry bias towards higher quality locations; and finally, (3) a "*technology-neutral*" market structure could evolve, in the limit, as the marginal impact of technology on long run concentration decreases with quality differentiation.

These findings also contrast those in Geroski, Gilbert, and Jacquemin (1990) and Reinganum (1984, 1985), and have a different research direction than that of Kwon and Stoneman (1996) and Stoneman (1990). Although the degree of quality differentiation affecting long run concentration is essentially analyzed, the proposed model is a static assessment with sequential entry, in contrast to the dynamic nature of such models in the abstract literature.

II. THE MODEL

Consider an oligopoly market setting characterized by locational quality differentiation with an initial duopoly market structure. Both firms and consumers are differentiated by quality location. Quality competition is non-probabilistic with an endogenous choice of non-linear quality locations based on surplus value functions which exhibit the "love for quality" consumption behavior[1]. The oligopoly market is composed of a distinct number of firms (products), such that firms choose location and then compete in quantities. There is perfect information and a continuous credible threat of market entry until the oligopoly market becomes completely saturated, or "fully

[1] The "love for quality" consumption behavior is a characteristic of the assumed surplus value functions in (1) below. Surplus utility is non-linear in choice of quantities for a given level of endogenous quality location. In contrast, a "love for variety" consumption behavior would either include linearity in location or some kind of Lancasterian quality characteristics with hedonic prices. The latter form of consumption behavior is more typical of general equilibrium models.

covered"[2]. There are no exit strategies for any firm in the market once entry is established and there is no collusion between firms. Consumers simultaneously reside on a quality street of firm (product) locations, and each consumer buys one product which is of most similitude to his liking based on non-linear surplus value with respect to quality, whereas each firm produces a single product line[3] located on a previously unfilled market segment to maximize economic profits.

The decision to enter or not to enter the market is based on reduced-form profit levels and is contingent on the market not being completely saturated by the incumbent firms. The model is solved where consumers maximize surplus utility, while firms maximize economic profits constrained by their chosen level of quality locations. Combined industry profits determine the extent of market saturation for a given level of maximum quality location, the latter taken as an implicit measure of production technology. Analysis of market concentration follows the sequential accommodation of market entry and is studied based on industry-wide profitability based on non-collusive market segmentation using an augmented form of the Hirschman-Herfindahl concentration index[4].

Assuming separable discrete choice behavior characterized by locational quality differentiation, let consumers reside on an exact quality street[5] with *surplus value functions* as follows:

$$S(\theta_k) = \theta_k q_k^2 - P_k \tag{1}$$

where $\theta_k \in [0, L]$; $q_k \geq 0$; $P_k \geq 0$; $k = 1,2,3,\ldots,N$

[2] A "fully covered market" is synonymous with a "completely saturated market". A "fully covered" market necessarily implies that there are no profit incentives for any firm to enter the market (i.e. there is no more room for market entry); such that a prospective entrant will either earn zero or negative economic profits after entry, hence suggesting that any entry beyond market saturation follows an irrational strategy.

[3] i.e., there is no multi-branding in the market, and each firm produces a differentiated product located within the given spectrum of available quality locations. Each product location fills up a certain amount of market segment until the oligopoly market becomes completely saturated.

[4] The *Hirschman-Herfindahl concentration index* is used as a measure of market concentration, and is found proportional to industry-wide profitability if the Bain-Sylos hypothesis of non-collusion is assumed.

[5] Consumers are assumed to reside on a 'quality street' where products are differentiated by locational quality differentiation. The surplus value functions exhibit non-linearity in location and linearity in price. There is a "love-for-quality" surplus utility (i.e. the marginal utility of consumption is strictly positive and increases linearly with quality location). In (1), *N* is the endogenously determined "equilibrium number of firms", θ_k is quality location for firm (product) *k*, where $k = 1,2,3,\ldots,N$; and P_k and q_k are the price and quantity vectors for different levels of quality location $k \in [1, N]$, respectively.

N is the endogenously determined "equilibrium number of firms". The oligopoly market becomes completely saturated with no more room for entry when there is no profit incentive for an additional firm to enter the market. The endogenous choice of quality is embodied into the parameter θ_k such that $\theta_k \in [0, L]$; where L is the maximum possible quality attainable using current levels of production technology. That is, L is taken to be an implicit measure of production technology. The surplus value differential (extent of quality differentiation) between two locations k and $(k+1)$ is $S(\theta_{k+1}) - S(\theta_k) = (\theta_{k+1} q_{k+1}^2 - \theta_k q_k^2) - (P_{k+1} - P_k)$; where $0 \le (\theta_{k+1} - \theta_k) \le L$. In addition, consumer surplus is under separable discrete choice behavior with exact (locational) quality differentiation[6]. P_k and q_k are the price and quantity vectors for different levels of quality locations $k \in [1, N]$, respectively.

Firms choose location followed by quantities to maximize economic profits:

$$\pi_k(\theta_k | L) = P_k(q_k(\theta_k | L)) q_k(\theta_k | L) \tag{2}$$

where $k = 1,2,3,\ldots,N$; $\theta_k \in [0, L]$; $q_k \ge 0$; $P_k \ge 0$; $\{q(\theta_k *, L)\} \in \{Q*(\theta_i) | L |\}$; $i = 1,2,3,\ldots,k$, with $\{P(\theta_k *, L)\} \in \{P*(\theta_k) | (\overline{Q}*(\theta_k)), L\}$; and $L > 0$.

Any optimal allocation of locational quality choice assumes that consumers *always* maximize surplus utility; while firms *always* maximize economic profits.

Other assumptions related to entry and market concentration include the *Bain-Sylos hypothesis*[7] and the assumption of non-collusive behavior among firms. Specifically, it is assumed that an

[6] The quality street is 'exact' in the sense that there are no probability expectations in the assumed surplus value function; thereby implying that each consumer knows exactly his quality type and knows exactly his position in preferred quality locations relative to other consumers. Hence, we are assuming the simplest form of locational quality differentiation. For the purposes of our proposed model, we are only considering experience goods in consumption. However, search goods may also be assumed except for the added constraints that they always reveal their own true information and that there is no diminishing surplus value for repeat purchases. Also, "separable" means discrete choice behavior with no utility cross-effects.

[7] The basic assumptions of the Bain-Sylos hypothesis, as augmented in Cowling and Waterson (1977), Schmalensee (1987), and summarized by Tirole (1998) are as follows: If $C_i(q_i) = c_i q_i$ and $Q = k / p$ with constant price elasticity of consumer demand; then:

$$\Pi = \sum_{i=1}^{N} \Pi^i = \sum_{i=1}^{N} (p - c_i) q_i = \sum_{i=1}^{N} \frac{p \alpha_i q_i}{\varepsilon} = \frac{pQ}{\varepsilon} \left(\sum_{i=1}^{N} \alpha_i^2 \right) = kH$$; where k is a positive constant.

increase in maximum quality location suggests an incremental (proportionate) increase in production technology; while an increase in combined industry-wide profits signals an incentive for market entry. In addition, market saturation is assumed to occur only when there is no more residual demand left uncovered by the incumbent firms, such that any fixed quality spectrum becomes "fully covered". For the purpose of measuring market concentration after entry, the *Herfindahl concentration index* is used. The Herfindahl concentration index is a convex function of an unequal distribution of market shares and obeys the Lorenz axiomatic conditions[8] for an asymmetric market structure. A critical assumption in model analysis is that of the Bain-Sylos hypothesis, as augmented by the Cowling-Schmalensee postulate of non-collusion[9], such that incumbent firms do not necessarily change capacity levels (production quantities) in response to changes in market entry conditions; hence implying a continuous positive correlation between market concentration and industry-wide profitability. Finally, it will be assumed throughout that there are no exit strategies for any incumbent firm in the market.

Formally, general model assumptions are summarized by:

Thus, if consumers spend a fixed amount of income on their purchase of particular goods, then the Herfindahl index yields a proportional (increasing) measure of industry-wide profitability if firms are non-collusive and if optimal capacity levels (production quantities) are maintained.

[8] The Herfindahl index is a convex function of unequal market shares and obeys the axiomatic Lorenz conditions, namely in non-mathematical terms: (1) it is invariant to permutations of market shares between firms; (2) it has a mean-preserving spread (i.e. a further spread of the distribution of market shares towards its tails increases the index); and (3) the aggregate index decreases when the number of firms in the industry increase.

[9] In our analysis of accommodating entry, we assume that the Bain-Sylos postulate holds good, following Bain (1951, 1972), Cowling and Waterson (1977) and Schmalensee (1987), where incumbent firms do not change capacity levels (production quantities) due to changes in market entry conditions.

The Bain-Sylos hypothesis is in line with our assumption of exact locational differentiation by quality choice, where consumers are differentiated by their marginal utility to consume ("love for quality" characteristics) rather than due to their disutility from purchasing a non-ideal brand caused by unit transportation costs.

In essence, Bain's pioneering work distinguishes between three types of entry barriers: cost advantages of incumbent firms, production technology (e.g. economies of scale), and product differentiation. Demsetz (1973) offers a non-collusive argument for Bain's hypothesis about the positive correlation between market concentration and industry profitability. Schmalensee (1987) re-explains Bain's hypothesis with Demsetz's conclusion by noting that cost asymmetries between firms yield output asymmetries, increasing the concentration index, and at the same time, allow differentiated firms to enjoy a rent, thus increasing industry-wide profit. The spirit of the augmented Bain-Sylos hypothesis has also been analyzed by Geroski, Gilbert, and Jacquemin (1990). In addition to consumers being "hungry for additional quality characteristics", Geroski et.al. (1990) attest that consumers will also be loyal to brands already consumed. In addition to this advantage, incumbent firms have a goodwill advantage in comparison to new entrants, due to the intangible assets of the learning experience possessed by the existing firms.

Assumptions

A1. *Consumers are rational and reside on an exact quality street with perfect information; such that consumption behavior obeys* $S(\theta_j > \theta_i | P_j = P_i) > S(\theta_i)$ *;* $\forall j > i$ *;* $i, j \in k$ *.*

A2. *Products are differentiated by quality choice; such that each firm chooses optimal quality location according to* $\{\theta_1^*, \theta_2^*, \theta_3^*, \ldots\} \in \theta_k^* \in (0, L)$ *; with* $\theta_{k+1}^* \geq \theta_k^*$ *and* $\theta_N^* \leq L$ *.The surplus value differential (extent of quality differentiation) between two locations k and (k+1) is* $S(\theta_{k+1}) - S(\theta_k) = (\theta_{k+1} q_{k+1}^2 - \theta_k q_k^2) - (P_{k+1} - P_k)$ *; where* $0 \leq (\theta_{k+1} - \theta_k) \leq L$ *.*

A3. *Consumers maximize surplus value functions:* $S(\theta_k) = \theta_k q_k^2 - P_k$ *; while firms maximize economic profits:* $\pi_k(\theta_k | L) = P_k(q_k(\theta_k | L)) q_k(\theta_k | L)$ *.*

A4. *Firms compete in quality location (first stage) followed by competition in quantities (second stage). Competition (at the second stage) is described by optimal Cournot strategies* $\{q(\theta_k^*, L)\} \in \{Q^*(\theta_i) | L|\}$*;* $i = 1,2,3,\ldots,k$ *with* $\{P(\theta_k^*, L)\} \in \{P^*(\theta_k) | (\overline{Q}^*(\theta_k)), L\}$*;* *where* $k = 1,2,3,\ldots,N$ *;* $\theta_k \in [0, L]$*;* $q_k \geq 0$*;* $P_k \geq 0$ *; and* $L > 0$ *.*

A6. $L_2 > L_1$ *signifies* $\psi_2(L_2) > \psi_1(L_1)$ *; where* $\psi(\bullet)$ *is an implicit measure of production technology.*

A7. *Entry occurs if* $\sum_{k=1}^{N} \pi(k) > 0$ *until* $\pi(N^*) = 0$*. Residual demand is determined by uncovered quality locations:* $\left[\sum_{k=1}^{N} q_k(\theta_k) \right] / L$ *. Market saturation occurs when* $\sum_{k=1}^{N^*} q^*_{k(N)} \left(\theta^*_{k(N)} \right) \equiv L$ *.*

A9. *The Hirshman-Herfindahl index - defined by* $H(\alpha_1, \ldots, \alpha_N) \equiv \sum_{i=1}^{N} \alpha_i^2 (q_k(\theta_k))$ *- where* α_i *'s are relative market shares; is an increasing measure of market concentration.*

A10. *Utilizing the Bain-Sylos hypothesis, as augmented by the Cowling-Schmalensee postulate of non-collusion, the Herfindahl index is a proportionate measure of industry-wide profitability, such that* $\Pi = \sum_{i=1}^{N} \Pi^i = \sum_{i=1}^{N} \frac{p \alpha_i q_i}{\varepsilon} = \frac{pQ}{\varepsilon} \left(\sum_{i=1}^{N} \alpha_i^2 \right) = \lambda H(\bullet)$*;* $\lambda > 0$ *.*

A12. *For any time* $t > 0$ *; the number of firms existing in the market either increases or stays the same, but never decreases:* $N(t+1) > N(t)$ *;* $\forall N < N^*$*,* $t > 0$*; with* $[N(t+1) | N = N^*] \equiv N(t)$ *; hence imposing the constraint of no exit strategy for any incumbent firm in the market.*

These assumptions are imposed rather than derived, and therefore reflect on the strict limitations of the model proposed (see Figure 1).

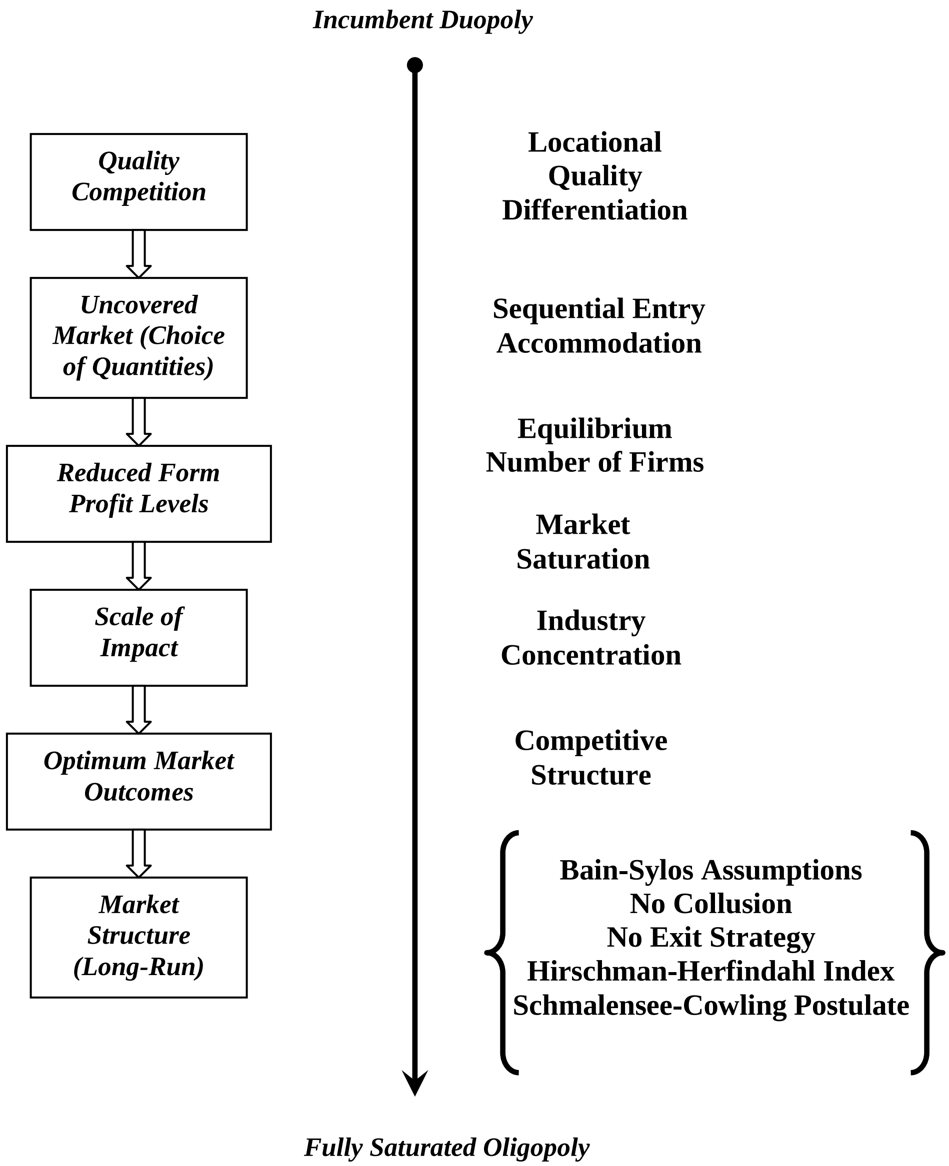

Figure 1: Methodology of Analysis on the Effects of Quality Competition on Market Saturation and Industry Concentration

III. ANALYSIS

A. Effect of Quality Competition on Sequential Entry Accommodation and on "The Equilibrium Number of Firms"

Assuming rational choice and utility-maximizing behavior for an equilibrium solution, the demand level of surplus value differentials for any two quality levels *k* and *(k+1)* imply:

$$(\theta_{k+1} q_k^2(\theta_k) - \theta_k q_k^2(\theta_k)) = (P_{k+1}(\theta_{k+1}) - P_k(\theta_k)) \tag{3}$$

such that indirect demand for any quality level θ_k is:

$$q_k(\theta_k) = \left(\frac{P_{k+1}(\theta_{k+1}) - P_k(\theta_k)}{\theta_{k+1} - \theta_k} \right)^{1/2} \tag{4}$$

where $0 \le (\theta_{k+1} - \theta_k) \le L$.

This leads to

$$P_{k+1}(\theta_{k+1}) = P_k(\theta_k) + (\theta_{k+1} - \theta_k) q_k^2(\theta_k) \tag{5}$$

Whence for quality locations $\theta_{k+j} \to \theta_N$ ($j = 2,3,\ldots,N-k$) we have:

$$\begin{aligned} P_{k+2}(\theta_{k+2}) &= P_{k+1}(\theta_{K+1}) + (\theta_{k+2} - \theta_{k+1}) q_{k+1}^2(\theta_{k+1}) \\ P_{k+3}(\theta_{k+3}) &= P_{k+2}(\theta_{k+2}) + (\theta_{k+3} - \theta_{k+2}) q_{k+2}^2(\theta_{k+2}) \\ P_N(\theta_N) &= P_{N-1}(\theta_{N-1}) + (\theta_N - \theta_{N-1}) q_{N-1}^2(\theta_{N-1}) \end{aligned} \tag{6}$$

Therefore,

$$P_k(\theta_k) = P_N(\theta_N) - (\theta_N - \theta_{N-1})q_{N-1}^2(\theta_{N-1}) - (\theta_{N-1} - \theta_{N-2})q_{N-2}^2(\theta_{N-2}) - \dots$$
$$\dots - (\theta_{k+3} - \theta_{k+2})q_{k+2}^2(\theta_{k+2}) - (\theta_{k+2} - \theta_{k+1})q_{k+1}^2(\theta_{k+1}) - (\theta_{k+1} - \theta_k)q_k^2(\theta_k) \quad (7)$$

Substituting (7) into the profit function in (2), and maximizing with respect to quantities at the second stage of competition, after simplification, leads to the following *implicit stability conditions*[10]:

$$\left(\frac{P_{k+1}(\theta_{k+1})}{P_k(\theta_k)}\right) \geq \left(\frac{3(\theta_{k+1} - \theta_k)}{\theta_k}\right) \quad (8)$$

$$\left(\frac{\theta_k}{3(\theta_{k+1} - \theta_k)}\right) \leq \left(\frac{P_k(q_k(\theta_k)) + \theta_{k+1}}{3(\theta_{k+1} - \theta_k)} - \frac{P_{k+1}(q_{k+1}(\theta_{k+1}))}{\theta_{k+1}}\right) \quad (9)$$

Using (7), (8), and (9), it can be shown that *reduced form* profits are of the form:

$$\frac{\pi_k(\theta_k)}{q_k(\theta_k)} = 3q_{N-1}^2(\theta_{N-1})[\theta_N - \theta_{N-1}] - (\theta_N - \theta_{N-1})q_{N-1}^2(\theta_{N-1}) - (\theta_{N-1} - \theta_{N-2})q_{N-2}^2(\theta_{N-2}) - \dots$$
$$\dots - (\theta_{k+3} - \theta_{k+2})q_{k+2}^2(\theta_{k+2}) - (\theta_{k+2} - \theta_{k+1})q_{k+1}^2(\theta_{k+1}) - (\theta_{k+1} - \theta_k)q_k^2(\theta_k) \quad (10)$$

By means of utilizing $P_k(\theta_k) = 3q_{k-1}^2(\theta_{k-1})[\theta_k - \theta_{k-1}]$ and $q_k(\theta_k) = \left(\frac{P_{k+1}(\theta_{k+1})}{3(\theta_{k+1} - \theta_k)}\right)^{1/2}$; where $k \in [1, N]$ and $0 \leq (\theta_{k+1} - \theta_k) \leq L$; $\theta_k \in [0, L]$; $q_k \geq 0$; $P_k \geq 0$; the optimum solution at the *first* stage of competition from (7) and (10) imply that for any new entrants, beyond an incumbent duopoly market where maximum quality location is set at $L > 0$; locational quality differentiation yield[11] $L > \theta_3^* > \theta_2^* > \theta_1^* \geq 0$ with $L > \theta_3^* > \left(\theta_1^* + \frac{P_2^*(\theta_2^*)\ \theta_1^*}{3P_1^*(\theta_1 *)}\right)$ for the third firm (first entrant);

[10] The implicit stability conditions are found by utilizing first-order and second-order conditions for maximization of profits in (2) with respect to quantities at the second stage of competition, and after substitution of (7) and (5) into the surplus value functions given in (1).
[11] This is found by some tedious calculations involving maximization of the reduced form profit levels in (10) with respect to quality location for each θ_k; $k = 1,2,3,\dots,N$.

given $\theta_1^* \geq \left(\theta_2^* - \frac{P_1^*(\theta_1^*)\ \theta_2^*}{3P_2^*(\theta_2^*) - \theta_2^*}\right)$ and $\theta_2^* - \theta_1^* \leq \left(\frac{P_2^*(\theta_2^*)\ \theta_1^*}{3P_1^*(\theta_1^*)}\right)$. Similarly, a fourth firm (second entrant) could locate at $L > \theta_4^* > \theta_3^* > \theta_2^* > 0$ with $L > \theta_4^* > \left(\theta_2^* + \frac{P_3^*(\theta_3^*)(\theta_2^*)}{3P_2^*(\theta_2^*)}\right)$, given $\theta_2^* \geq \left(\theta_3^* - \frac{P_2^*(\theta_2^*)\ \theta_3^*}{3P_3^*(\theta_3^*) - \theta_3^*}\right)$ and $\theta_3^* - \theta_2^* \leq \left(\frac{P_3^*(\theta_3^*)\ \theta_2^*}{3P_2^*(\theta_2^*)}\right)$;and a fifth firm (third entrant) would then locate at $L > \theta_5^* > \left(\theta_3^* + \frac{P_4^*(\theta_4^*)(\theta_3^*)}{3P_3^*(\theta_3^*)}\right)$, ... *etc.* By analogy and some minor modifications, the $(k-2)^{th}$ entrant (or k^{th} firm) locates at $L > \theta_k^* > \left(\theta_{k-2}^* + \frac{P_{k-1}^*(\theta_{k-1}^*)(\theta_{k-2}^*)}{3P_{k-2}^*(\theta_{k-2}^*)}\right)$; given $\theta_{k-2}^* \geq \left(\theta_{k-1}^* - \frac{P_{k-2}^*(\theta_{k-2}^*)\theta_{k-1}^*}{3P_{k-1}^*(\theta_{k-1}^*) - \theta_{k-1}^*}\right)$ and $\theta_{k-1}^* - \theta_{k-2}^* \leq \left(\frac{P_{k-1}^*(\theta_{k-1}^*)\theta_{k-2}^*}{3P_{k-2}^*(\theta_{k-2}^*)}\right)$.

In the limit[12], from (10) and utilizing the above argument, for $k = 1,2,3,\ldots,N$; it can be shown that $\pi_{N*}(\theta_{N*}) \to 0$ as[13]:

$$\left\{L - \left[(N^*-2) + \left(\frac{N^*-2}{3}\right)\left(P_N^*(\theta_N^*) \Big/ P_{N-1}^*(\theta_{N-1}^*)\right)\right]\right\} \to 0 \tag{11}$$

The "marginal firm" is the N^{th} firm (or equivalently the $(N-2)^{th}$ firm to enter the incumbent duopoly market) which would earn non-zero economic profits, given that optimum prices obey (7)

12 In the analysis of accommodating entry, we assume that the Bain-Sylos postulate holds good, following Bain (1951, 1972), Cowling and Waterson (1977) and Schmalensee (1987), and contrary to Spence (1977), where it will be assumed that incumbent firms do not change capacity levels (production quantities) due to changes in market entry conditions. This is also in line with the assumption of exact locational differentiation by quality choice, where consumers are differentiated by their marginal utility to consume ("love for quality" characteristics) rather than due to their disutility from purchasing a non-ideal brand caused by unit transportation costs. In essence, Bain's pioneering work distinguishes between three types of entry barriers: cost advantages of incumbent firms, production technology (e.g. economies of scale), and product differentiation. In our model, we are analyzing a specific case of quality differentiation models with an implicit allowance for changes in maximum quality scale (as a proxy for production technology) in costless production.

13 The concept of the "equilibrium number of firms" at market saturation is utilized here, whence it is assumed that there is no room for market entry if any new entrant receives below-zero economic profits for a previously uncovered quality location by the existing firms.

and given the implicit stability conditions in (8) and (9) leading towards the reduced form profit functions in (10).

Hence, for any quality spectrum $L > 0$; the market will be fully covered when no differentiated demand is left uncovered by the above "marginal firm" condition. Consequently, the cycle of entry could be repeated for a discrete number of entrants into the differentiated market, with the constraint that the market is not yet saturated by the maximum possible quality location desired by consumers.

Further, to obtain the "equilibrium number of firms" for a given level of maximum quality location (production technology), we can define[14] $P_R \equiv \left(P_N \big/ P_{N-1} \right)$ as the *relative price ratio* between the two highest quality levels offered at market saturation, and it follows that $L - \left[N^* - 2 + \frac{N^* - 2}{3} P_R \right] = 0$; hence $(N^* - 2) = \left[\frac{L}{1 + \frac{P_R}{3}} \right]$ which directly proves:

$$N^* = \left[2 + \frac{L}{1 + \frac{P_R}{3}} \right] \equiv \left(2 + \frac{3L}{3 + \left(P_N \big/ P_{N-1} \right)} \right). \tag{12}$$

The established "equilibrium number of firms", N^*, as given in (12), indicate the *total* number of firms a market could accommodate with sequential entry until full market saturation (i.e. until there is no profit incentive for any additional firm to enter the market, or equivalently based on our strict model limitations, until all available quality locations are fully covered for a given level of production technology implicit in L).

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[14] Strictly speaking, the definition of relative price ratio is $P_R \equiv \left(\frac{P_N^*(\theta_N^* | L)}{P_{N-1}^*(\theta_{N-1}^* | L)} \right)$; but for the sake of simplicity $P_R \equiv \left(P_N \big/ P_{N-1} \right)$ will be used for the rest of this article.

It can therefore be established:

PROPOSITION 1. *The "equilibrium number of firms" for a differentiated oligopoly market characterized by locational quality differentiation is given by:*

$$N^* = \left[2 + \frac{L}{1 + \frac{P_R}{3}} \right] \equiv 2 + \frac{3L}{3 + \left(P_N \big/ P_{N-1} \right)} \; ; \; with \; \left[\frac{\partial N}{\partial L} \right] > 0 \; and \; \left[\frac{\partial N}{\partial P_R} \right] = \frac{-3L}{\left\{ 3 + \left(P_N \big/ P_{N-1} \right) \right\}^2} < 0 \, .$$

Thus, sequential entry upon an incumbent duopoly accommodates at most N^* firms for a fully saturated market when products are differentiated by locational quality differentiation. The extent of market entry given in Proposition 1 is contingent upon a given level of maximum quality location, $L > 0$, used as an implicit (proxy) measure of production technology.

In essence, "the equilibrium number of firms" entering a differentiated oligopoly market where products are differentiated by locational quality differentiation is *proportional* to the extent of quality location possible (i.e. to the level of production technology achievable)[15], whereas it is *inversely proportional* to the price ratio between the two highest quality levels attained in the market. Formally, $\left[\frac{\partial N}{\partial L} \right] > 0$ while $\left[\frac{\partial N}{\partial P_R} \right] = \frac{-3L}{\left\{ 3 + \left(P_N \big/ P_{N-1} \right) \right\}^2} < 0$. Thus, whenever there is a possibility for a larger quality spectrum to be available for consumers, there is always an economic incentive for additional market entry and, consequently, the market becomes relatively *more* saturated. On the other hand, whenever the *relative price ratio* between the two products of highest quality levels existing in the market widens (i.e. their price ratio increases), then the market is likely to be more easily saturated with a *fewer* number of entrants, hence suggesting that the market tends to become relatively *less* saturated.

[15] Here, we make no distinction between the maximum quality location possible, and the highest production technology achievable. Hence, quality location becomes a proxy for production technology along a locational quality differentiation 'street'. This is in general agreement with our basic model setup conditions. Refer to model assumptions.

In any case, it seems that the number of firms required to saturate a differentiated market depends on two main factors: (i) maximum quality location; and (ii) relative prices, with a positive dependence on the former and a negative dependence on the latter[16].

B. Market Saturation

Consequently, when firms engage in flexible capacity competition in a differentiated Cournot market where products are differentiated by quality choice, the market accommodates a discrete number of entrants, such that the *total* number of firms required to saturate the market is proportional to the maximum quality location possible (L) and inversely proportional to the relative price ratio between the two highest quality levels offered in the market (P_R). Such an assessment may imply that prices for low-quality location are less critical in shaping the market structure as compared to prices corresponding to the two highest quality products. However, it should be noted that all relative prices are linked by the implicit stability conditions required to attain a stable equilibrium solution. In addition, $\left(\frac{\partial N / \partial L}{\partial P_R}\right) = 0$ whereas $\left(\frac{\partial N / \partial P_R}{\partial L}\right) < 0$. This may have an additional implication for market dynamics such that the additional technology associated with a higher quality choice generates a negative effect on entry-price tradeoffs, such that a higher choice of quality levels, if induced by higher technologies in production, can deepen the relative price effect on market entry thus ultimately creating a fully saturated market more quickly, but on the other hand, relative prices have no effect on the relative degree of market saturation when a higher level of quality choice (induced by a higher level of technology) is initially selected.

In general, *accommodating entry* suggests that a higher quality scale, such as that induced by a higher level of production technology, creates additional economic incentives for market entry towards full market saturation. A differentiated oligopoly market characterized by locational quality differentiation accommodates more firms as the quality scale in the market rises, i.e. $\left[\frac{\partial N}{\partial L}\right] > 0$. On the other hand, the behavioral structure of such an accommodation differs in great

[16] Therefore, the equilibrium number of firms required to saturate the market is found to increase with maximum quality location and decrease with relative prices. By relative prices, we imply the relative price ratio between the two highest quality levels attained in the market, as given in the definition of P_R in (12).

degree as the relative price ratio changes. The maximum limit on N^* is achieved when $\left(P_N / P_{N-1}\right) \to 0$; whereas the minimum limit on N^* is achieved when $\left(P_N / P_{N-1}\right) \to \infty$.

Hence, in the limit, $0 < \left[\frac{\partial N}{\partial L}\right] < 1$ corresponds to $\left\{0 < \left(P_N / P_{N-1}\right) < \infty\right\}$ since:

$$\lim_{\left(\frac{P_N}{P_{N-1}}\right) \to 0} \left[2 + \frac{3L}{3 + \left(P_N / P_{N-1}\right)}\right] = 2 + L \tag{13}$$

hence implying

$$\lim_{\left(\frac{P_N}{P_{N-1}}\right) \to 0} \left[\frac{\partial N}{\partial L}\right] = 1; \tag{14}$$

and

$$\lim_{\left(\frac{P_N}{P_{N-1}}\right) \to \infty} \left[2 + \frac{3L}{3 + \left(P_N / P_{N-1}\right)}\right] = 2 \tag{15}$$

hence implying

$$\lim_{\left(\frac{P_N}{P_{N-1}}\right) \to \infty} \left[\frac{\partial N}{\partial L}\right] = 0. \tag{16}$$

Moreover, although $\left[\frac{\partial N}{\partial L}\right] > 0$ *is* valid for any choice of quality locations, its *scale of impact* on market saturation is found more powerful with a decline in relative prices[17]. This suggests that

[17] Regarding this topic, Von-Weizsacher (1980) hypothesizes that substantial increases in production technologies may create an equilibrium number of firms beyond the number of firms required at the social

even though a higher quality scale as implied by increases in production technology would induce more market entry, the scale of impact on market saturation is more evident for a decline in relative prices between the two highest quality levels attained in the saturated market. Hence, prices *do* have an important effect on market saturation in the sense that lower relative prices can accommodate relatively *more* entrants as compared to higher relative prices, given the same incremental increase in quality location (i.e. a marginal increase in production technology).

The analysis in (13)-(16), leading to $0 < \left[\dfrac{\partial N}{\partial L}\right] < 1$, suggest that increases in maximum quality location do not drastically accommodate an exponential number of market entrants. A higher technological base in production would allow more market entrants on a linear scale at most, even though consumer surplus is non-linear in assumed demand for quality locations. Accordingly, a given increase in maximum quality location is accompanied by a less-than-proportionate increase in the equilibrium number of firms.

This leads to:

PROPOSITION 2. *An increase in production technology (maximum quality location) is accompanied by a less-than-proportionate increase in the equilibrium number of firms:*

$$0 < \left[\frac{\partial N}{\partial L}\right] < 1 \textit{ corresponds to } \left\{0 < \left(P_N \big/ P_{N-1} \right) < \infty \right\} \textit{ with } \left(\frac{\partial N \big/ \partial L}{\partial P_R} \right) = 0 \textit{ and } \left(\frac{\partial N \big/ \partial P_R}{\partial L} \right) < 0 .$$

This finding is central to quality competition with locational differentiation. Yet, it is strictly confined to an oligopoly market characterized by quality differentiation of the form proposed in this model (i.e. where consumers obey specific surplus value functions characterized by "love for quality" in demand, and firms compete in flexible quantities, and market behavior is characterized by non-collusion, non-combative entry, costless production, etc.). A more critical analysis is

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optimum. If we take Von-Weizsacher's hypothesis as given, and further our assumption of the Bain-Sylos postulate, then markets competing under locational quality differentiation may be seen as able to accommodate more firms than required at the social optimum. This, however, does not imply that an increase in relative prices yields a better social outcome. For additional inquiries regarding the Bain-Sylos postulate, refer to Bain (1972), Demsetz (1973), and Schmalensee (1987).

necessary for the causes towards such a conclusion[18], but it is also critical to note that although the model assumes non-linearity in demand for quality location, the market outcome exhibits a *less-than-proportionate* increase in the equilibrium number of firms for an accompanying increase in production technology (given a proportionate change in maximum quality location). Excessive advances in production technologies, therefore, may inhibit market saturation for a proportionate increase in maximum quality location.

C. Industry Concentration

Beyond market entry, it is also interesting to analyze *market concentration* for different levels of maximum quality location, and to particularly compare an aggregate concentration index, after all entry conditions have been accounted for; in order to study the effect of quality competition and production technology on industry profitability and, hopefully, on the competitive structure of markets in the long run. There are a host of concentration indices available in the literature[19], but the most common (and most useful for our current analysis) is the *Hirschman-Herfindahl index* which summarizes the aggregate distribution of the squares of respective market shares among all firms in a particular oligopoly market, in a simple increasing measure of industry concentration.

By utilizing the Bain-Sylos assumptions of non-collusion and non-combative sequential entry, as initially presented in Bain (1951) and later augmented analytically by Cowling (1977) and Schmalensee (1987); the *Hirschman-Herfindahl concentration index* is taken to be an increasing measure of industry concentration for a differentiated oligopoly market[20].

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[18] In order to study this effect more closely, a dynamic form of the model has to be evaluated in order to obtain the behavioral aspects of market dynamics for changes in production technology (maximum quality location). More precisely, a dynamic price path has to be obtained with sequential entry where the level and utilization of available production technology is continuously changing, consequently leading towards a dynamic path of endogenous quality locations. This is outside the scope of the current analysis but is highly recommended for future research. Regarding dynamic analysis of quality competition, see Kwon and Stoneman (1996), Eaton and Schmitt (1994), Budd, Harris and Vickers (1993), Slade (1991), Stoneman (1990), Vickers (1986), Reinganum (1985), Nakao (1982), and with descriptive insights made earlier by Galbraith (1957).

[19] Besides the Hirschman-Herfindahl concentration index, two other famous concentration indices are the m-firm concentration index and the entropy concentration index, defined as:

$R(\alpha_1,\ldots,\alpha_M \,|\, M < N^*) \equiv \sum_{i=1}^{M} \alpha_i$ and $E(\alpha_1,\ldots,\alpha_N \,|\, N = N^*) \equiv \sum_{i=1}^{N} \alpha_i \ln \alpha_i$, respectively.

[20] The Herfindahl index is a convex function of unequal market shares and obeys the axiomatic Lorenz conditions, namely in non-mathematical terms: (1) it is invariant to permutations of market shares between firms; (2) it has a mean-preserving spread (i.e. a further spread of the distribution of market shares towards its

The Hirschman-Herfindahl concentration index is a convex function of an unequal distribution of market shares and obeys the Lorenz axiomatic conditions (see Assumptions).

Following Hirschman (1945) and Cowling and Waterson (1977), the Hirschman-Herfindahl concentration index yields a proportional measure of industry profitability when market behavior is characterized by non-collusion and product asymmetry (quality differentiation). This also follows Demsetz (1973) in his assertion that *intrinsic asymmetries* among firms are likely to produce less product variety and more industry profitability, even under the Bain-Sylos postulate of non-combative entry[21]. Within our particular model, intrinsic asymmetries between firms are mainly explained by different levels of quality choice (i.e. by the relative choices of locational quality differentiation)[22].

With $\alpha_i \equiv \frac{100 q_i}{L}$ denoting firm *i*'s relative market share per unit of technology [where i=1,2,3,…N^* and $\frac{\sum_{i=1}^{N^*} \alpha_i}{100} \equiv 1$], the *Hirschman-Herfindahl concentration index* is defined as:

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tails increases the index); and (3) the aggregate index decreases when the number of firms in the industry increase. This follows the assumptions of the Bain-Sylos hypothesis, as augmented in Cowling and Waterson (1977), Schmalensee (1987), and summarized by Tirole (1998): If $C_i(q_i) = c_i q_i$ and $Q = k / p$ with constant price elasticity of consumer demand; then:

$\Pi = \sum_{i=1}^{N} \Pi^i = \sum_{i=1}^{N} (p - c_i) q_i = \sum_{i=1}^{N} \frac{p \alpha_i q_i}{\varepsilon} = \frac{pQ}{\varepsilon} \left(\sum_{i=1}^{N} \alpha_i^2 \right) = kH$; where k is a positive constant.

Thus, if consumers spend a fixed amount of income on their purchase of particular goods, then the Herfindahl index yields a proportional (increasing) measure of industry-wide profitability.

See model assumptions for more details regarding the structure and utilization of the Hirschman-Herfindahl concentration index. Also see Hirschman (1945).

[21] Demsetz (1973) offers a non-collusive argument for Bain's hypothesis about the positive correlation between market concentration and industry profitability. Schmalensee (1987) tries to re-explain Bain's hypothesis with Demsetz's conclusion by noting that cost asymmetries between firms yield output asymmetries, increasing the concentration index, and at the same time, they allow differentiated firms to enjoy a rent, thus increasing industry-wide profit. However, such a correlation is most evident under Cournot competition and not so evident under Bertrand competition. In addition, the equilibrium number of firms is found unrelated to social welfare (as measured by consumer surplus plus industry profit) for Bertrand competition whereas it is positively related to social welfare for Cournot competition.

[22] This reduces the analysis of industry concentration towards a singular dimension of asymmetry since production is assumed costless (i.e. there are no cost asymmetries between firms).

$$H(\alpha_1,\ldots,\alpha_N \big| N = N^*) \equiv \left\{ \sum_{i=1}^{N^*} \alpha_i^2 (q_i^* (\theta_i^*)) \middle| \left\{ L - \left[N^* - 2 + \frac{N^* - 2}{3} P_R \right] = 0 \right\} \right\}, \tag{17}$$

and after some tedious calculations, it can be verified that for an N-market structure, having established the equilibrium number of firms in (12), such that $N = N^*$ for a given level of maximum quality location, the Hirschman-Herfindahl concentration index obeys:

$$H(\alpha_1,\ldots,\alpha_N \big| N = N^*) \equiv \sum_{i=1}^{N^*} \alpha_i^2 = \left(\frac{100}{L}\right)^2 \sum_{i=1}^{N^*} \left\{ q_1^{*2}(\theta_1^*), q_2^{*2}(\theta_2^*), q_3^{*2}(\theta_3^*), \ldots , q_{N-1}^{*\,2}(\theta_{N-1}^*), q_N^{*\,2}(\theta_N^*) \right\}$$

$$= \left(\frac{100}{L}\right)^2 \sum_{i=1}^{N^*} \left(\frac{P_{i+1}^*(\theta_{i+1}^* \big| L)}{3(\theta_{i+1}^* - \theta_i^*)\big| L} \right) = \left\{ \left(\frac{100}{L}\right)^2 \sum_{i=1}^{N^*} \left(\frac{P_{i+1}^*(\theta_{i+1}^* \big| L)}{3(\theta_{i+1}^* - \theta_i^*)\big| L} \right) \middle| N^* \equiv \left(2 + \frac{3L}{3 + \left(P_N^*(\theta_N^*) \big/ P_{N-1}^*(\theta_{N-1}^*) \right)} \right) \right\} \tag{18}$$

which ultimately leads to:

$$H^S(\alpha_1, \alpha_2, \ldots, \alpha_{N-1}, \alpha_N) \leq \left[\left(\frac{100\sqrt{N^*(L) - 2}}{L} \right) \left(\frac{3 + P_N^* \big/ P_{N-1}^*}{3} \right) \right]^2 \tag{19}$$

for $N > 2$ (oligopoly market).

It can also be verified that:

$$\left[\frac{\partial H^S}{\partial L} \right] \leq 0 \tag{20}$$

Hence, markets competing under locational quality differentiation have an upper-bound on their long-run concentration. The upper-bound is either reduced, or stays the same, as the level of available production technology improves.

But what are the conditions for such an outcome?

From the previous analysis, there are two categories of behavior:

(i) $\left[\frac{\partial H^S}{\partial L}\right] = 0$; or (ii) $\left[\frac{\partial H^S}{\partial L}\right] < 0$.

As $(\theta_{k+1} - \theta_k) \to 0$ with $k \to (N^* - 1)$; we have $(\theta_N^* \to \theta_{N-1}^*)$ and it follows that

$$\left[\left\{\lim_{(\theta_N^* \to \theta_{N-1}^*)}\left[\frac{\partial H^S(\bullet)}{\partial L}\right] = 0 \middle| \lim_{\left(\frac{P_N^*}{P_{N-1}^*}\right) \to 0}\left[\frac{\partial N}{\partial L}\right] = 1\right\} \middle| \left\{L - \left[(N^* - 2) + \left(\frac{N^* - 2}{3}\right)\left(P_N^*(\theta_N^*) \big/ P_{N-1}^*(\theta_{N-1}^*)\right)\right]\right\} \to 0\right]$$

hence suggesting:

$$\lim_{(\theta_N^* \to \theta_{N-1}^*)}\left[\frac{\partial H^S(\bullet)}{\partial L}\right] = 0 \quad \text{for } k = N^*. \tag{21}$$

On the other hand, as $(\theta_{k+1} - \theta_k)\uparrow$; with $k \to N^* - 1$; we have $(\theta_N^* - \theta_{N-1}^*)\uparrow$; and it follows that, in the limit:

$$\lim_{(\theta_N^* - \theta_{N-1}^*) \to L}\left[\frac{\partial H^S(\bullet)}{\partial L}\right] = \left(\frac{-2(N^* - 2)}{L^3}\right) Z_H < 0 \tag{22}$$

where $Z_H > 0$ is a positive constant.

It is also important to note that $\frac{\partial\left[\frac{\partial H^S(\bullet)}{\partial L}\right]}{\partial(\theta_N-\theta_{N-1})}<0$ and $\lim_{(\theta_N-\theta_{N-1})\to L}\left\{\frac{\partial\left[\frac{\partial H^S(\bullet)}{\partial L}\right]}{\partial(\theta_N-\theta_{N-1})}\right\}=-\infty$.

Hence, in the limit, less asymmetry in quality differentiation $[(\theta_{k+1}-\theta_k)\to 0]$ yields $\left[\frac{\partial H^S(\bullet)}{\partial L}\right]=0$; whereas more asymmetry in quality differentiation $[(\theta_{k+1}-\theta_k)\uparrow]$ generates $\left[\frac{\partial H^S(\bullet)}{\partial L}\right]<0$ with $\lim_{(\theta_N-\theta_{N-1})\to L}\left\{\frac{\partial\left[\frac{\partial H^S(\bullet)}{\partial L}\right]}{\partial(\theta_N-\theta_{N-1})}\right\}=-\infty$.

Consequently, the effect of production technology (maximum quality location) on industry concentration follows $\left[\frac{\partial H^S}{\partial L}\right]\le 0$, such that :

(i) $\left[\frac{\partial H^S(\bullet)}{\partial L}\right]=0$ is attained as $(\theta_{k+1}-\theta_k)\to 0$; and

(ii) $\left[\frac{\partial H^S(\bullet)}{\partial L}\right]<0$ is attained as $(\theta_{k+1}-\theta_k)\uparrow$ generating $\lim_{(\theta_N-\theta_{N-1})\to L}\left\{\frac{\partial\left[\frac{\partial H^S(\bullet)}{\partial L}\right]}{\partial(\theta_N-\theta_{N-1})}\right\}=-\infty$.

This leads to:

PROPOSITION 3. *Advances in production technology (maximum quality location) under locational quality differentiation imply that more differentiation asymmetry between firms yield a deeper (negative) impact of technology on concentration whereas less differentiation asymmetry between firms, in the limit, may lead towards a "technology-neutral" market structure:* $\left[\frac{\partial H^S(\bullet)}{\partial L}\right]=0$ *as* $(\theta_{k+1}-\theta_k)\to 0$*;*

$$\left[\frac{\partial H^{S}(\bullet)}{\partial L}\right]<0 \quad as \ (\theta_{k+1}-\theta_{k})\uparrow; \ and \lim_{(\theta_N-\theta_{N-1})\to L}\left\{\frac{\partial\left[\frac{\partial H^{S}(\bullet)}{\partial L}\right]}{\partial(\theta_N-\theta_{N-1})}\right\}=-\infty \ ;$$

given:

$$H^{S}(\alpha_1,\alpha_2,\ldots,\alpha_{N-1},\alpha_N)\leq\left[\left(\frac{100\sqrt{N^{*}(L)-2}}{L}\right)\left(\frac{3+P_N^{*}/P_{N-1}^{*}}{3}\right)\right]^{2} \quad for \ N>2\,.$$

As a result, advances in production technology tend to decrease aggregate market concentration for a given change in quality location as differentiation asymmetries between firms intensify.

An *increase* in relative quality locations creating *more* asymmetry between firms generates a *negative technological impact* on industry concentration, whereas a *decrease* in relative quality locations creating *less* asymmetry between firms, in the limit, leads toward a “technology-neutral” market structure.

If we take concentration as an *inverse* measure of the competitive structure of markets in the long run (i.e. highly concentrated markets are considered less competitive, whereas low concentrated markets are considered highly competitive); then Proposition 3 implies that advances in production technology affect industry concentration and the competitive structure of markets in the long run as follows:

(i). more differentiation asymmetry between firms lead to a deeper impact of technology on concentration and achieve a more competitive market structure;

(ii). less differentiation asymmetry between firms, in the limit, leads toward a “technology-neutral” market structure;

(iii). locational quality differentiation suggest intrinsic asymmetries between firms yield a more liberal technological impact on an evolving market structure.

Also, the *marginal* impact of technology on long run concentration *decreases* with quality differentiation[23]. This can be seen directly from $\frac{\partial\left[\frac{\partial H^S(\bullet)}{\partial L}\right]}{\partial(\theta_N - \theta_{N-1})} < 0$; with its limit as mentioned in Proposition 3[24].

Therefore, if market competition is characterized by the assumed model of locational quality differentiation, a *higher* level of production technology may ultimately *decrease* long run concentration and achieve a *more competitive* market structure if consequential technological improvements are met by wider quality scales through differentiation asymmetries between firms. If such a condition ceases to exist, then advances in production technology would yield a neutral – rather than positive – impact on the competitive structure of markets in the long run.

Thus, the established findings in this article seem to point to the notion that the nature of quality competition through locational quality differentiation not only affects the equilibrium number of firms towards market saturation, but also has a large impact on industry concentration and on the competitive structure of markets in the long run.

[23] Here, the marginal impact is taken as a function of the highest two quality locations attained in the market; $(\theta_N - \theta_{N-1})$, but this can also be proven valid for the general case of $(\theta_{k+1} - \theta_k)$ as long as $2 < k \leq N-2$.

[24] Advances in production technology tend to decrease aggregate market concentration for a given change in quality location as differentiation asymmetries between firms intensify. As a result, we may deduce that *higher* production technologies (proxied by *L*), under the conditions stated in Proposition 3, may *reduce* industry concentration and allow for a more competitive market structure in the long run, with the presumption that if the Bain-Sylos postulate remains valid for all market entries until market saturation, then the reduction in concentration would most probably yield a definite reduction in *combined* industry-wide profits, thus deterring entry beyond saturation (as technology improves). Therefore, even though advances in production technology promote competitive behavior in the long run, unwarranted advances in production technology may actually deter entry beyond saturation if technology improves continuously. Such an assessment needs an optimal control model of dynamic technological progress and is outside the scope of the current analysis. In any case, the suggested argument points to the established finding that markets cannot be too concentrated as available technology improves over time.

IV. CONCLUSION

The nature of quality competition affects the equilibrium number of firms, the extent of market saturation, and level of industry concentration in differentiated markets. Current analysis assumes a non-linear form of locational quality differentiation based on an incumbent duopoly market with sequential entry until full market saturation towards an oligopoly market structure; such that consumers obey a "love for quality" consumption behavior based on surplus value functions; and firms compete in location followed by quantities. There is no collusion among firms and level of technology is implicit in maximum possible quality location.

Of the effects of quality competition on the competitive structure of markets in the long run, as confined by the strict limitations of imposed model assumptions briefly described above, it has been established that:

(i) A higher level of production technology (implicit in maximum quality location) is met by a less-than-proportionate increase in "the equilibrium number of firms" in order to reach full market saturation.

(ii) Advances in production technology (maximum quality location) imply more differentiation asymmetry between firms yield a deeper (negative) impact of technology on concentration whereas less differentiation asymmetry between firms, in the limit, may lead towards a “technology-neutral” market structure.

(iii) Under the Bain-Sylos assumptions of no collusion and no exit strategy, a more competitive market structure is ultimately achieved as the marginal impact of technology on long run concentration is accompanied by wider quality scales upon an evolving market structure.

In essence, advances in production technology tend to decrease aggregate market concentration for a given change in quality location as differentiation asymmetries between firms intensify. Such a conclusion could be extended towards a dynamic market setting with technology diffusion, and with different assumptions about the nature of consumer surplus values (incorporation of demand effects upon supply factors). An empirical example of the proposed model would also prove the above conclusions more solid.

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