Opportunity Cost of Capital, Marginal Cost of Funds and Numeraires in Cost-Benefit Analysis

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Abstract: The question of choice of social discount rate, which is related to the choice of numeraire in CBA, has been unsettled for decades. The solution lies in using both the social time preference rate (STPR) and the social opportunity cost rate (SOCR) simultaneously but in different roles. There are two proposed methods of using the two rates, however, one of which places a great emphasis on the marginal cost of funds (MCF). This paper explores the interaction between these concepts using a numerical example to show how the alternative discounting methods compare and how one of them works even if the SOCR differs from the rate of fall of the value of the possible numeraires.

Keywords: Social discount rate; STP discounting; SOC discounting; Descriptive discounting; Prescriptive discounting; Two-rate discounting; Shadow Price of Capital; Marginal Cost of Funds.

JEL classification: D61; H43

1. Introduction

Drèze and Stern (1990) wrote that the social discount rate (SDR) is the rate at which the social value of the numeraire falls over time. "We cannot, therefore, answer the question, 'What should be the shadow discount rate?' without being told, or without our choosing, what the numeraire is to be. And the apparent difference between the shadow discount rates proposed in alternative methods of cost-benefit analysis should not mislead us into thinking that the differences are necessarily real – alternative methods may simply involve different units of account." They provided the example of foreign exchange in the hands of the government being a natural numeraire, in which case the discount rate is the interest rate in the world capital markets. Indeed, this was the numeraire used in CBA until about the mid to late 1970s, after which increasingly consumption became the numeraire of choice.

The dichotomy between Social Time Preference (STP) discounting versus Social Opportunity Cost (SOC) discounting approaches that had been unresolved for many decades reflects a difference in numeraires. The social Time Preference Rate (STPR) measures the rate of fall of the social value of consumption, whereas the Social Opportunity Cost Rate (SOCR) can be seen as reflecting the social cost of public borrowing.

Szekeres (2022, 2023) proposed a two-rate discounting method that reconciles the STP and SOC discounting approaches by using the SOCR to explicitly calculate the capital costs of projects and then discounting the after-capital-costs net flows by the STPR. In this way the requirements of both discounting approaches are simultaneously fulfilled.

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This was not the first proposal to simultaneously use both rates, however. Liu (2003) wrote: "We should use both market rates in a more direct way. The gross rate representing the opportunity cost of government revenues [assumed to equal the SOCR in this paper] should be used for the discounting of project costs while the net rate representing the valuation of consumers [assumed to equal the STPR in this paper] should be used for the discounting of consumer benefit."

The SOCR is usually calculated assuming that the public sector borrows to finance its investments. This is not the only source of funds for public projects, however, as tax revenues are also used. Raising taxes results in welfare losses that are captured by the concept of marginal cost of funds (MCF). Liu (2003) places great emphasis on using the MCF correction in a dynamic context, combining it with the simultaneous use of the two alternative discount rates. Harberger (2007) agreed with the joint use of the MCF and the SOCR by stating that the MCF correction "would not come *instead of* the conventional assumption of capital market sourcing, but rather as a necessary *supplement* to that assumption." Szekeres (2023) concurred, explaining that the welfare cost of raising taxes "is proportional to the investments made (the increments of the stock of capital), whereas the SOCR measures the cost of tying down the stock of capital for one year. […] The MCF and the SOCR have different dimensionalities (without and with reference to units of time, respectively) because they measure different things and are in no way interchangeable, nor does the use of one affect the use of the other."

Szekeres (2023) stated that the value of the SOCR might be different when taxing is the source of funds than when borrowing is, considering this to be a question in need of empirical estimation, while some proponents of the MCF correction assert that using MCF should replace shadow pricing capital, effectively stating that SOCR = STPR, given that they would discount with the latter. See Spackman (2020).

This paper will use four alternative discounting methods to compute the NPV of an illustrative long-lived project, in order to explore the assertion of Drèze and Stern that the choice of discount rate is intimately related to the choice of numeraire and that the differences in discounting methods may be more apparent than real. This paper will also show that the two-rate discounting method proposed by Szekeres (2022, 2023) decouples valuing capital cost from the choice of numeraire.

This paper is organized as follows: Section 2 shows how NPVs are computed using the STP and SOC approaches as well as by the Liu and Szekeres two-rate methods; Section 3 compares the four methods and shows how they relate to one another; Section 4 draws conclusions.

2. Four discounting methods

For the illustrative analyses that follow we assume that STPR = 3%, SOCR = 7% and MCF =1.5. We follow current CBA practice, which is often called efficiency analysis, in which benefits are defined by willingness to pay, costs by willingness to supply and no income distribution effects or other preferences of social planners are given weight. Any such additional considerations, including indirect secondary effects, are assumed to have been included in the specification of project flows or in the values given to the STPR, the SOCR or the MCF.

The following illustrative project will be analyzed:

Table 1
Sample project benefits and costs

Year	**0**	**100**
Benefits		20 000
Operating costs		8 000

Investment	1	
Net flow	**-1**	**12 000**

Because we would like to use the MCF correction in the illustrative project, we make the further assumption that the source of funds for the investment cost is tax revenues and that the operating costs of the project will also be funded by tax revenues, as it would be the case, for example, in an environment improving project that generates no cash revenues. Because the MCF measures a welfare cost that is contemporary with the raising of tax revenues, the project's net flow is modified as shown in Table 2. Both the investment and the operating cost items are multiplied by the MCF factor, which increases the initial investment cost from 1 to 1.5 in Year 0 and the reduces the net flow in year 100 from 12,000 to 8,000 because the operating costs of 8,000 increase to 12,000 by virtue of their multiplication by 1.5.

Table 2
Net Flow after MCF correction

Year	**0**	**100**
Net flow	-1.5	8 000

2.1 STP discounting

The NPV of the project using the STPR as a discount rate can be computed as follows. NPV = –1.5 + 8,000 / $(1.03)^{100}$ = 414.763.

2.2 SOC discounting

The NPV of the project using the SOCR as a discount rate can be computed as follows. NPV = –1.5 + 8,000 / $(1.07)^{100}$ = 7.720.

2.3 Liu two-rate discounting

Liu proposed discounting as follows; "the MCF approach to multi-period project evaluation consists of the following components. (i) A project should be represented as a stream of direct investments [interpreted by Liu (2003) to include all government outlays, that is investments and operating costs], a stream of direct benefits measured as contemporaneous willingness to pay and a stream of indirect revenue benefits [we assume that there are none in the illustrative project]; (ii) future project direct benefits should be discounted at the net rate of return while future project costs, including indirect revenue benefits as negative costs [we assume no indirect revenues], should be discounted at the gross rate of return; (iii) the present value of net costs should be multiplied by the marginal cost of funds before being compared to the present value of the direct benefits."

The above quote describes the following project feasibility criterion proposed by Liu (2003, expression 15):

$$\sum_{t=0}^{\infty} \frac{B_t}{(1+r_n)^t} - \mathrm{MCF} \cdot \sum_{t=0}^{\infty} \frac{\Delta I_t - \Delta R_t}{(1+r_g)^t} > 0$$

where B stands for benefits, I for both investments and operating costs and R for revenues.

We assume that Liu's net rate of return r_n is the STPR and that the gross rate of return r_g is the SOCR. The latter assumption is justified by the fact that the SOCR equals the actual financial cost of the public sector when it borrows. It is the sum of interest payments and indirect tax effects. This was observed by Burgess and Zerbe (2013): "the [SOCR] criterion measures the impact of the project on the government's budget when the private sector is kept at pre-project utility," was corroborated experimentally by Szekeres (2022) and was explained by Harberger (2007).

Following the Liu calculation recipe, the present value of the benefits is 20,000 / $(1.03)^{100}$ = 1,040.656, the present value of the operating costs is 8,000 / $(1.07)^{100}$ = 9.220 and NPV = 1,040.656 – 1.5 (1 + 9.220) = 1,025.327. These values are summarized in the following table. Notice that the operating cost was taken from Table 1, as the multiplication by the MCF factor is done in the last step of the Liu calculation. Despite this, the effects of the MCF corrections are shown separately in Table 3 for later use.

Table 3
Liu two-rate discounting

Year	**Present Values**
Benefits	1 040.656
Operating costs	9.220
MCF surcharge on operating costs	4.610
Investment	1.000
MCF surcharge on investment	0.500
Net Present Value after MCF correction	1 025.327

2.4 Szekeres two-rate discounting

Szekeres two-rate discounting consists of replacing the money costs of the initial investments by their opportunity costs (measured by the SOCR) and discounting the resulting project flow by the STPR.

Table 4
Szekeres two-rate discounting (STPR, SOCR)

Year	**0**	**100**
Net flow	-1.5	8 000.000
Funding flow	-1	867.716
Net flow after capital costs	-0.5	7 132.283

An easy way to implement this method is to define a funding flow that consists of the provision of the required investment amount and of its repayment, interest included, whenever feasible. Capital cost recovery in Year 100 includes the principal amount financed and accumulated compound interest costs, which can be calculated by multiplying the amount financed by (1 + SOCR) raised to the power of the number of years, which in this case is 1.07^{100} = 867.715. Subtracting the funding flow (line 2 in Table 4) from the project's net flow (line 1) will yield the net flow after capital costs (line 3). In this case this net flow does not have a zero value in Year 0 because capital is only needed to cover the investment cost but not the extra welfare cost that raising the funds through taxation entailed. That extra cost is a dead-weight loss that does not increase the required capital stock and therefore does not need financing. From the net flow after capital costs we can compute NPV = – 0.5 + 7,132.283 / $(1.03)^{100}$ = 370.613.

3. Comparing the results

The four alternative NPVs calculated are the following:

Table 5
Comparing the results

Discounting method	Present Values
STP	414.763
SOC	7.720
Liu two-rate	1 025.327
Szekeres two-rate	370.613

We now proceed to compare the cases.

3.1 STP discounting

It is not evident to all that discounting by the STPR implies a capital cost equal to that rate. This can be shown by using Szekeres two-rate discounting with both rates equal to the STPR. We can see that the capital cost in the Year 100 is $(1.03)^{100} = 19.219$.

Table 6
Szekeres two-rate discounting (STPR, STPR)

Year	0	100
Net flow	-1.5	8 000.000
Funding flow	-1	19.219
Net flow after capital costs	-0.5	7 980.781

The NPV of the net flow after capital costs is NPV = $-0.5 + 7{,}980.781 / (1.03)^{100} = 414.763$, the same as when computed directly, which shows that in fact discounting the original project net flow by the STPR implies an opportunity cost of capital given by the discount rate used.

Those who believe that in the case of tax funded projects SOCR = 0 must perform the following analysis:

Table 7
Szekeres two-rate discounting (STPR, 0)

Year	0	100
Net flow	-1.5	8 000.000
Funding flow	-1	1
Net flow after capital costs	-0.5	7 999.000

Notice that the funding flow contains no interest as SOCR = 0. In this case NPV = $-0.5 + 7{,}999.000 / (1.03)^{100} = 415.711$. Admittedly this is not all that different, but still, it is not conceptually the same.

The problem with STP discounting is that it implicitly undervalues the opportunity cost of capital. If we know that the opportunity cost of capital is given by the SOCR, then the way to correct STP discounting is to explicitly include the right capital costs. One way of doing it is by using Szekeres two-rate discounting. This was done in Table 4, which gave the welfare correct NPV = 370.613.

Another way to correct STP discounting is to use the Shadow Price of Capital correction (SPC). This can be done by computing the present value, discounted at the STPR, of the project's opportunity costs of capital. Taking this value from Table 4 we get SPC = $867.716 / (1.03)^{100} = 45.150$. As the STP approach

NPV already computed had an initial investment of 1, the correction needed is 45.150 – 1 = 44.150. The corrected NPV is therefore NPV = 414.763 – 44.150 = 370.613, the same value that was obtained by Szekeres two-rate discounting.

3.2 SOC discounting

The problem with SOC discounting is that it undervalues future benefits. If we take the net flow of the project in year 100 and discount it at the STPR, we obtain a present value of 8,000 / $(1.03)^{100}$ = 416.263, whereas if we discount it at the SOCR the present value becomes 8,000 / $(1.07)^{100}$ = 9.220.

If we believe that the larger value is the correct one from a welfare point of view, we must adjust the SOC calculation method to reflect this. But we can only adjust the net benefits above capital costs. As a first step we perform the Szekeres two-rate analysis to determine the value of net benefits left over in Year 100 after capital costs have been covered.

The amount left over, taken from Table 4, is 7,132.284. As a second step, we define a conversion factor that will take this value from a consumption numeraire to the public funds numeraire. This conversion factor is $((1 + \text{SOCR}) / (1 + \text{STPR}))^t$, with t being the number of years, which in this case is $(1.07/1.03)^{100}$ = 45.150. The effect of this factor is that it will give the desired present value when discounted at the SOCR. In Table 8 the conversion factor multiplies the net benefits after capital costs (7,132.284 × 45.150 = 32,2020.786), resulting in the corrected net flow shown in Table 8.

Table 8
SOC discounting with benefit correction

Year	0	100
Net flow after capital costs	-0.5	7 132.284
Corrected net flow to discount	-0.5	32 2020.786

The NPV of the corrected flow, discounted at the SOCR, which is the rate of fall of the public funds numeraire, can be calculated as follows: NPV = –0.5 + 32,2020.786 / $(1.07)^{100}$ = 370.613, which is the same result that was obtained in Section 2.4 with Szekeres two-rate discounting.

3.3 Liu two-rate discounting

This method of discounting is fatally flawed and cannot be corrected in its defined form. The basic reason is that it uses two numeraires simultaneously, which leads to consequences that Liu (2003) surely did not intend. What Liu two-rate discounting does in effect is to capitalize operating costs (and any future investments) using the SOCR and then uses the STPR to discount the result. On the one hand the method overstates the project's net operating benefits by understating operating costs relative to benefits, while on the other hand it implicitly quantifies the project's costs of capital with the STPR, contrary to its stated intention of letting the SOCR define the opportunity cost of public funds. More specifically:

- Discounting operating costs at a rate that differs from that used for benefits is problematic conceptually, because operating costs are a measure of foregone consumption, and therefore should be valued on the same basis as consumption. The fact that in this case the operating costs are public expenses and therefore should be corrected by the MCF factor does not require them to be discounted at a different rate. By discounting costs at a higher rate than benefits are discounted at, the Liu two-rate method overestimates net operating benefits, which is one of the reasons why it yields the highest computed NPV of all the reviewed discounting methods (See Table 5).

- Liu (2003) stated that "future project costs [which includes investments], including indirect revenue benefits as negative costs, should be discounted at the gross rate of return." This reveals the intention of using public funds as the numeraire of the analysis, with the consequent requirement of using the SOCR as the discount rate. However, discounting benefits at the STPR effectively results in discounting the initial investment of Year 0 at the STPR, not the SOCR. Recall the calculation formula from Section 2.3: NPV = 1,040.656 – 1.5 (1 + 9.220). The PV of the benefits, 1,040.656 = 20,000 / $(1.03)^{100}$, is obtained by discounting benefits at the STPR. Setting operating costs to zero for the moment would result in NPV = 20,000 / $(1.03)^{100}$ – 1.5, where the second term is the investment of 1 multiplied by the MCF. As far as the investment in Year 0 is concerned, the discount rate used is the STPR. This is the second reason why this method yields the highest computed NPV.

The discrepancy between Liu's (2003) apparent intentions and effectively obtained results will be demonstrated in the following analysis. Table 9 shows the net benefits of the project as would be presumably accepted by Liu, because he proposes using the SOCR to discount investments. In line with this, the opportunity cost of capital in Year 100 must be 1 × 1.07^{100} = 867.72.

Table 9
Net benefits in Year 100

Year	Values
Benefits	20 000.000
Operating costs	8 000.000
MCF surcharge on operating costs	4 000.000
Opportunity cost of capital	867.720
Net benefits	7 132.280

Discounting the above values at the STPR yields the following present values. To be able to compute the NPV, the MCF surcharge on the initial investment has been added.

Table 10
Comparison of present values

Year	Present Values of items in Table 9	Liu present values from Table 3	Percent under-valuation
Benefits	1 040.650	1 040.656	
Operating costs	416.260	9.220	97.76%
MCF surcharge on operating costs	208.130	4.610	97.76%
Opportunity cost of capital	45.150	1.000	97.76%
MCF surcharge on investments	0.500	0.500	
NPV	**370.613**	**1 025.327**	

The NPV in the first column corresponds to the Szekeres two-rate result, while that in the second column corresponds to the Liu two-rate result (values from Table 3). The Liu present value of the opportunity cost of capital is 1/45.150th of the value that it would have had if it had really been computed using the SOCR as desired, instead of using the STPR, as effectively computed. The percentage of underestimation corresponds to the following: 1 – (1.03/1.07)^100 = 0.977851. This is the same as 1 minus the numeraire conversion factor that converts values from the public funds numeraire to the consumption numeraire (1 – 1/45.12 = 0.977851).

We can ascertain that Liu two-rate discounting is internally inconsistent by computing the future value of the second column of Table 10 at the STPR in Year 100 and by comparing it to the assumptions about net benefits in Year 100 found in Table 9.

Table 11
Net benefits in Year 100

Year	Future values of Liu present values	Originally assumed values
Benefits	20 000.000	20 000.000
Operating costs	177.196	8 000.000
MCF surcharge on operating costs	88.598	4 000.000
Opportunity cost of capital	19.219	867.720
Net benefits	19 714.988	7 132.280

The percentages of underestimation that could be computed from these numbers would be the same here as in Table 10. A `number worth noting is the opportunity cost of capital, which has the same value as in Table 6, which is another way of confirming that, contrary to its assertion, the Liu method computes the opportunity cost of capital at the STPR rather than at the SOCR.

We can remove the source of operational benefits overestimation by the multiplication of the present value of the operational cost by the numeraire conversion factor we have already used, after which we are left with the same NPV as was displayed by STP discounting in Section 2.1.

Table 12
Liu two-rate discounting results
converted to the consumption numeraire

Year	Original present values	Corrected present values
Benefits	1 040.656	1 040.656
Operating costs	9.219	416.263
Net Present Value after MCF correction	**1 025.327**	**414.763**

Concerning the unintended discount rate problem, we should note that to rely on discounting to implicitly calculate the opportunity cost of capital we must discount the entire net flow at the same rate. Only then will the stated discount rate be effective. We can reproduce that through an alternative correction, by converting all benefits to the public funds numeraire, which means dividing their present value by the conversion factor of 45.150. Then we have the following:

Table 13
Liu two-rate discounting results
converted to the public-funds numeraire

Year	Original present values	Corrected present values
Benefits	1 040.656	23.049
Operating costs	9.219	9.219

Net Present Value after MCF correction	**1 025.327**	**7.720**

The division of the present value of benefits that were calculated using the STPR by the correction factor gives us the result that we would have had if we had calculated the present value using the SOCR. The NPV of this adjusted flow is the same as that of SOC discounting, see Section 2.2.

It was probably Liu's (2003) intention to remedy the undervaluation of benefits inherent in SOC discounting by specifying that benefits be discounted at the STPR, but in doing so he unwittingly ended up effectively discounting investments and operating costs at the STPR because unless sufficient benefits offset the return requirements of the desired discount rate, it will not have been effectively used. In other words, the discount rate applied to benefits is what determines the cost of capital implicitly computed through discounting. Only the benefits in excess of the return requirements of the discount rate used can be valued alternatively, if necessary.

4. Conclusions

Drèze and Stern (1990) were right in stating that it is possible to select arbitrarily the numeraire of the analysis, but the conversion from one to the other via conversion factors cannot be performed mechanically, because discounting imputes the capital costs implicit in the rate of fall of the numeraire, and these differ. This means that only with the public funds numeraire will the opportunity cost of capital be accurately measured, and only with the consumption numeraire will intertemporal weights correspond to social preferences. However, we have seen that if we establish the value of something in the numeraire in which its value is best established (the value of future consumption using the consumption numeraire and the cost of capital using the public funds numeraire) judicious analysis will lead to analogous results independently of the numeraire chosen. It is interesting to note that the SPC factor (45.150, see Section 3.1) is the same as the change of numeraire correction factor (45.150, see Section 3.2) because ultimately, they are computed using the same formula.

We have also seen that in all cases Szekeres two-rate discounting easily led to the correct result. By making the cost of capital explicit, this method allows treating capital as just any other input of projects, to be valued by its shadow price, the SOCR. After having accounted for capital costs explicitly, discounting no longer computes capital costs, it just defines intertemporal weighting. Consequently, the Szekeres two-rate discounting will work even if the opportunity cost of capital happened to be different from the rate of fall of any of the plausible numeraires that might be used.

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