MPRA
Munich Personal RePEc Archive

# Health, basic research, human capital accumulation, and RD-based economic growth

Parui, Pintu

XIM University, Bhubaneswar

13 October 2023

Online at https://mpra.ub.uni-muenchen.de/118769/
MPRA Paper No. 118769, posted 14 Oct 2023 07:09 UTC

# Health, basic research, human capital accumulation, and R&D-based economic growth

**Pintu Parui**

*School of Economics, XIM University, Bhubaneswar*

**Abstract**

We construct a broad R&D-based endogenous growth model that incorporates the importance of children's health on human capital accumulation and publicly-funded basic research investments required to produce new goods. Although an increment in the number of healthcare professionals creates a shortage of workers for final goods production, the novelty of this paper is to demonstrate the significance of healthcare workers in enhancing the productivity of inputs of various sectors, along with its long-run consequences.

**Keywords: R&D-based growth, Basic science, Children's Health, Education, Fertility.**

**JEL Code: H41; J24; O31; O32; O41**

This draft: **October 13, 2023**

*Email id: pintuparui6@gmail.com*

***Acknowledgment**: I am grateful to Subrata Guha, Sugata Marjit, Gogol Mitra Thakur and Sandip Sarkar for insightful comments in an earlier draft. The usual disclaimer applies.*

# 1 Introduction

A large body of R&D-based endogenous growth literature focuses on the quality-quantity trade-off between fertility and human capital accumulation (cf. Prettner et al., 2013; Strulik et al., 2013; Prettner, 2014; Prettner and Werner, 2016). We observe that the issues related to children's health on human capital accumulation have been substantially undermined. Nevertheless, Baldanzi et al. (2021) is an exception, where the authors address this issue but ignore the importance of publicly funded basic research investments in the innovation of new goods.[1] On the other hand, Prettner and Werner (2016) address the importance of basic research,[2] but disregard the significance of children's health on human capital accumulation.

Constructing a general model, we extend the existing literature that captures ($i$) children's health on human capital accumulation and ($ii$) the significance of government-financed basic research investments in innovating new goods. Moreover, we assume that the government seeks to improve people's health by providing healthcare facilities to them. This, in turn, can enhance productivity in the related sectors by reducing production losses caused by sick employees. On the other hand, a rise in the number of healthcare workers means fewer workers are available for final goods production. The novelty of our paper lies in capturing this trade-off, along with the quality-quantity trade-offs in fertility and human capital accumulation. Besides, we contend that our model is more general than Prettner and Werner (2016) and Baldanzi et al. (2021).

# 2 The model

## 2.1 Consumption side

We consider an economy with three overlapping generations: children, adults, and retirees. Adults decide upon the consumption level $c_t$, savings for retirement $s_t$, the number of children $n_t$, education ($e_t$) and health ($m_t$) of each child. The time adults do not spend on raising, educating, and caring for their children's health is supplied to the labor

[1] For the importance of health investment in creating human capital and its long-run consequences, see Prettner et al. (2013) and Kuhn and Prettner (2016). For the empirical evidence, see Madsen (2016).

[2] Gersbach et al. (2013), Gersbach and Schneider (2015), Gersbach et al. (2018), and Gersbach et al. (2023) are the other contributors. For empirical evidence, see Czarnitzki and Thorwarth (2012), Toole (2012), Coad et al. (2021), and Mulligan et al. (2022).

market. While children don't participate in any economic decision, retirees consume their entire savings carried over from adulthood. Following Prettner and Werner (2016), we assume a single-parent household with the following utility function:

$$u_t = \ln c_t + \beta \ln [(R_{t+1} - 1)s_t] + \xi \ln n_t + \theta \ln e_t + \sigma \ln m_t \tag{1}$$

$\beta \in (0, 1)$ represents the inter-generational discount factor. $R_{t+1}$ represents the gross interest rate on assets between generation $t$ and $t+1$. $\xi \in (0, 1)$, $\theta \in (0, 1)$, and $\sigma \in (0, 1)$ are utility weights on the number of children, child's education and health respectively.[3] We assume that the next generation's human capital is a multiplicative function of education and health. Therefore a part of the utility function ($\xi \ln n_t + \theta \ln e_t + \sigma \ln m_t$) captures the trade-offs parents face in deciding the number of children and parental time expenditure on children's education and health. To simplify the model, we assume an exogenously given mortality of parents and to avoid nonsensical solutions we impose the restriction that $\xi > \theta + \sigma$.

Following Prettner and Werner (2016) we assume that the cost of raising children, educating them, and providing them the basic health facilities requires time costs of households.[4] Therefore, the budget constraint of the household reads:

$$(1 - \tau)(1 - \psi n_t - \eta e_t n_t - \chi m_t n_t) w_t h_t = c_t + s_t \tag{2}$$

where $\tau \in (0, 1)$ represents the income tax rate, $\psi > 0$, $\eta > 0$, and $\chi > 0$ denote opportunity cost (in terms of time) of child-rearing, per child education and health investment respectively, $w_t$ is the wage rate and $h_t$ represents the effective labor (i.e., the human capital). Optimal choices of consumption, savings, fertility, education, and health are (see Appendix A for the derivation)

[3]This type of utility function is often found in the literature (cf. Strulik et al., 2013; Prettner and Werner, 2016; Baldanzi et al., 2021) and is based on the "warm-glow motive of giving" (see Andreoni, 1989) and is a special case of utility formulation used in Galor and Weil (2000), and Galor (2005, 2011).

[4]For example, parental involvement in a child's physical development by assigning time for the child to participate in different sports and games, dance, and other physical activities will aid in developing the child's health. Different mental games will support a child's mental growth while also helping the child develop mental acuity, improving their ability to absorb their essential education. The time parents drive their children to get the necessary vaccines will also contribute to improving their health.

$$c_t = \frac{(1-\tau)w_t h_t}{1+\beta+\xi} \qquad s_t = \frac{\beta(1-\tau)w_t h_t}{1+\beta+\xi} \qquad n_t = \frac{(\xi-\theta-\sigma)}{\psi(1+\beta+\xi)}$$
$$e_t = \frac{\theta\psi}{\eta(\xi-\theta-\sigma)} \qquad m_t = \frac{\sigma\psi}{\chi(\xi-\theta-\sigma)} \tag{3}$$

Population size at time $t+1$ is

$$L_{t+1} = n_t L_t = \frac{(\xi-\theta-\sigma)}{\psi(1+\beta+\xi)} L_t \tag{4}$$

We assume that the individual human capital of the next generation depends positively on ($i$) educational effort by the parents, $e_t$ ($ii$) parents' productivity in education, $A_E$ ($iii$) healthcare effort by parents for their children, $m_t$[5] ($iv$) parents' productivity in healthcare for their children, $A_M$, and ($v$) the level of parents' individual human capital $h_t$ in the following way:

$$h_{t+1} = (A_E e_t h_t)^{\nu} (A_M m_t h_t)^{1-\nu} = \left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu} \frac{\psi}{(\xi-\theta-\sigma)} h_t \tag{5}$$

(5) also captures the trade-off between child quantity and quality that is summarised in the following proposition.

**Proposition 1.** *An increase in desire for a large family ($\xi$) increases fertility and reduces the next generation's human capital. An increase in the desire for having better educated ($\theta$) or healthy children ($\sigma$) increases the human capital of the next generation and reduces fertility.*

*Proof.* See Appendix B. □

$H_t$, the aggregate human capital stock of the economy is the product of individual human capital ($h_t$) and the total population size ($L_t$). Therefore, the human capital stock available for production, research, and healthcare facility ($\tilde{H}_t$) is given by the aggregate human capital stock adjusted for the time parents spend raising, educating, and caring for their children's health ($\psi n_t + \eta e_t n_t + \chi m_t n_t$) as

$$\tilde{H}_t = [1 - \psi n_t - \eta e_t n_t - \chi m_t n_t] H_t = \frac{1+\beta}{1+\beta+\xi} h_t L_t \tag{6}$$

[5]Note that along with the level of education, a better health condition is also an essential component in the individual human capital (cf. Rivera and Currais, 2004; Baldanzi et al., 2021).

## 2.2 Production side

The final goods sector, intermediate goods sector, applied research sector, basic research sector, and healthcare sector constitute the production side of the economy. The first three sectors are based on the standard Romer (1990) and Jones (1995) R&D-driven growth literature. We modify this structure to account for $(i)$ a tax-financed basic research sector that employs scientists to discover and explain the natural laws and phenomena required for applied research, $(ii)$ a tax-financed healthcare sector which enhances the productivity of human capital, and $(iii)$ the endogenous evolution of aggregate human capital in the production process.

The perfectly competitive final goods sector employs workers and machines to produce output $Y_t$ according to

$$Y_t = \left(H_{t,M}^{\varepsilon_0} H_{t,Y}\right)^{1-\alpha} \int_0^{A_t} x_{t,i}^{\alpha}\, di \tag{7}$$

where $H_{t,Y}$ and $H_{t,M}$ are the human capital (workers) employed in the final good and healthcare sectors respectively, $A_t$ is the technological frontier, $x_{t,i}$ is the amount of the blueprint-specific machine $i$ used in production, and $\alpha$ is the elasticity of output with respect to machines. $H_{t,M}^{\varepsilon_0}$ affects the productivity of workers, $H_{t,Y}$, while $\varepsilon_0$ measures the strength of the effect.[6] For a given total factor productivity (i.e., $H_{t,M}^{\varepsilon_0(1-\alpha)}$), (7) exhibits constant returns to scale in $H_{t,Y}$ and $x_{t,i}$. Perfect competition implies the wage rate $(w_{t,Y})$ and the machines' prices $(p_{t,i})$ are, respectively,

$$w_{t,Y} = (1-\alpha)\left(H_{t,M}^{\varepsilon_0} H_{t,Y}\right)^{-\alpha} H_{t,M}^{\varepsilon_0} \int_0^{A_t} x_{t,i}^{\alpha}\, di = (1-\alpha)\frac{Y_t}{H_{t,Y}} \tag{8}$$

[6] Let us take an example. An individual's human capital level at the time of entry into the labour force in period $t$ is $h_t$. This human capital depends on her parents' decision (in period $t-1$) to devote time to her education and health care when she was young. However, if the individual becomes ill, even though she continues to work, she may not be able to perform to her full potential. The healthcare facilities will assist her in regaining her capacity as soon as possible. As a result, she will be more productive than if she did not have access to this healthcare facility. In this context, it should be noted that healthcare facilities may have an impact on children's health. However, for the sake of simplicity, we are ignoring this possibility. One intriguing extension of the current model would be integrating this issue and investigating its long-run implications. One may also argue that the intensive form of human capital (i.e., $\frac{H_{t,M}}{H_t}$), rather than the amount of human capital employed in the healthcare sector $(H_{t,M})$, should play a role in determining the productivity of workers in the various sectors. Nonetheless, for comparable types of basic health concerns, people frequently take basic therapies on their own, without even consulting a healthcare expert, while observing the treatment of other sick people. In other words, healthcare practitioners not only directly address the health issues of those seeking treatment, but they also indirectly assist other sick people. As a result, we employ $H_{t,M}$ in our model instead of $\frac{H_{t,M}}{H_t}$ to reflect the spillover effect/positive externality provided by healthcare personnel.

$$p_{t,i} = \alpha x_{t,i}^{\alpha-1} \left(H_{t,M}^{\varepsilon_0} H_{t,Y}\right)^{1-\alpha} \tag{9}$$

Raw physical capital ($k_{t,i}$) serves as variable input and one machine-specific blueprint serves as fixed input in the production of the monopolistically competitive intermediate goods sector, which manufactures the machines for the final goods sector. We assume full depreciation of physical capital over the course of one generation. Therefore operating profits are $\pi_{t,i} = p_{t,i}k_{t,i} - R_t k_{t,i}$. Profit maximization then leads to the monopolistic pricing rule for each firm as

$$p_{t,i} = \frac{R_t}{\alpha} \tag{10}$$

Due to symmetry, each firm employs $k_t = \frac{K_t}{A_t}$ units of physical capital, where $K_t$ represents the aggregate physical capital stock. The aggregate production function can then be re-written as

$$Y_t = \left(A_t H_{t,M}^{\varepsilon_0} H_{t,Y}\right)^{1-\alpha} K_t^{\alpha} \tag{11}$$

The applied research sector employs scientists with human capital stock $H_{t,A}$ to create new blueprints that can be patented and sold to the intermediate goods sector. In the field of applied research, a firm's production function is defined as

$$A_{t+1} - A_t = \delta_1 H_{t,M}^{\varepsilon_1} A_t^{\phi_1} B_t^{\mu_1} H_{t,A} \tag{12}$$

where $\delta_1 H_{t,M}^{\varepsilon_1}$ is the productivity of inputs in the applied research sector, $B_t$ represents society's stock of basic knowledge discovered by basic researchers and forms the epistemic base for the stock of patented knowledge $A_t$. $\phi_1 \in [0,1]$ and $\mu_1 \in [0,1]$ measure the extent of intertemporal knowledge spillovers in the applied research sector and intersectoral knowledge spillovers from basic to applied research, respectively.[7] For a given stock of basic and applied knowledge, $\varepsilon_1$ assesses how strongly healthcare professionals enhance the productivity of applied research sector workers. Similar to Prettner and Werner (2016),

[7]While knowledge spillovers happen intertemporally in the applied research sector, intersectoral knowledge spillovers occur between basic and applied research. Like Prettner and Werner (2016), given that patents are partially excludable, whereas the laws of nature, once discovered, can be exploited by scientists freely, one can expect that the spillovers from basic research to applied research are greater than the opposite.

no technique can be developed without any propositional knowledge, $B_t$. Therefore, to begin with, we assume that $B_0 > 0$ and $A_0 > 0$. Moreover, we assume $H_{t,M} > 0$. Our framework nests both the endogenous and semi-endogenous growth models of Romer (1990) and Jones (1995) as special cases (see Remark 1).

**Remarks 1.** *For $\tau = 0$, $\theta = 0$, $\sigma = 0$, $\xi > \psi(1 + \beta + \xi)$, $\mu_1 = 0$, $\varepsilon_0 = \varepsilon_1 = 0$, and $\phi_1 \in (0, 1)$, our model nests the Jones (1995) framework, while for $\tau = 0$, $\theta = 0$, $\sigma = 0$, $\xi > \psi(1 + \beta + \xi)$, $\mu_1 = 0$, $\varepsilon_0 = \varepsilon_1 = 0$, and $\phi_1 = 1$, our model nests the Romer (1990) framework.*

Firms in the applied research sector hire the human capital $H_{t,A}$ so as to maximize theirs profits

$$\pi_{t,A} = p_{t,A}\delta_1 H_{t,M}^{\varepsilon_1} A_t^{\phi_1} B_t^{\mu_1} H_{t,A} - w_{t,A} H_{t,A} \tag{13}$$

with $p_{t,A}$, the price of a blueprint and $w_{t,A}$, applied researchers' wage rate. This leads to the optimality condition

$$w_{t,A} = p_{t,A}\delta_1 H_{t,M}^{\varepsilon_1} A_t^{\phi_1} B_t^{\mu_1} \tag{14}$$

Following Strulik et al. (2013) and Prettner and Werner (2016), we assume that patent protection lasts for one generation. Once the patent expires, the right to sell the blueprint is handed over to the government, which can either consume or invest the associated proceeds unproductively. For a blueprint, applied research sector firms charge the entire operating profit of an intermediate goods producer, that is,

$$p_{t,A} = \pi_{t,i} = \alpha(1-\alpha)\frac{Y_t}{A_t} \tag{15}$$

A part ($\tau_0$) of the government's revenue is spent on employing scientists to discover the propositional knowledge in the basic research sector so that

$$\frac{\tau_0\tau(1+\beta)}{(1+\beta+\xi)} w_t h_t L_t = w_t h_t L_{t,B} \tag{16}$$

Therefore, the amount of human capital employed in the basic research sector is

$$H_{t,B} = L_{t,B} h_t = \frac{\tau_0\tau(1+\beta)}{(1+\beta+\xi)} H_t \quad \left(\equiv \tau_0\tau\tilde{H}_t\right) \tag{17}$$

Propositional knowledge evolves according to

$$B_{t+1} - B_t = \delta_2 H_{t,M}^{\varepsilon_2} A_t^{\phi_2} B_t^{\mu_2} H_{t,B} = \frac{\delta_2 \tau_0 \tau (1+\beta)}{(1+\beta+\xi)} H_{t,M}^{\varepsilon_2} A_t^{\phi_2} B_t^{\mu_2} H_t \tag{18}$$

where $\delta_2 H_{t,M}^{\varepsilon_2}$ is the productivity of inputs in the basic research sector, $\mu_2 \in [0,1]$ and $\phi_2 \in [0,1]$ measure the extent of intertemporal knowledge spillovers in the basic research sector and intersectoral knowledge spillovers from applied to basic research, and $H_{t,B}$ represents the amount of aggregate human capital that government employs by chossing the $\tau_0$ fraction of tax rate, $\tau$. For a given stock of applied and propositional knowledge, $\varepsilon_2$ indicates the strength of the effect of the healthcare professionals in enhancing the productivity of the basic research sector workers.

We assume that the government budget is balanced so that the rest $(1-\tau_0)$ of the government revenue is spent on employing the healthcare workers in the healthcare sector, i.e.,

$$\frac{(1-\tau_0)\tau(1+\beta)}{(1+\beta+\xi)} w_t h_t L_t = w_t h_t L_{t,M}$$

Therefore, the amount of human capital employed in the healthcare sector is

$$H_{t,M} = h_t L_{t,M} = \frac{(1-\tau_0)\tau(1+\beta)}{(1+\beta+\xi)} H_t \quad \left(\equiv (1-\tau_0)\tau \tilde{H}_t\right) \tag{19}$$

We assume that the government aims to improve people's health by providing healthcare facilities to them and, while doing so, affects the productivity of human capital.

## 2.3 Market clearing and balanced growth path (BGP)

Labor market clearing conditions are $\tilde{H}_t = h_t[L_{t,Y} + L_{t,A} + L_{t,B} + L_{t,M}] = H_{t,Y} + H_{t,A} + H_{t,B} + H_{t,M}$, and $w_{t,Y} = w_{t,A} = w_{t,B} = w_{t,M} = w_t$. (8), (14), (15), (17) and (19) yield the demand for human capital in the final goods and applied research sectors as, respectively,

$$H_{t,Y} = \frac{A_t^{1-\phi_1} B_t^{-\mu_1} H_{t,M}^{-\varepsilon_1}}{\alpha \delta_1} \tag{20}$$

$$H_{t,A} = \tilde{H}_t - H_{t,B} - H_{t,M} - H_{t,Y}$$

$$\Longrightarrow H_{t,A} = \frac{(1-\tau)(1+\beta)}{(1+\beta+\xi)} h_t L_t - \frac{A_t^{1-\phi_1} B_t^{-\mu_1}}{\alpha \delta_1} \left[ \frac{(1-\tau_0)\tau(1+\beta)}{(1+\beta+\xi)} h_t L_t \right]^{-\varepsilon_1} \tag{21}$$

The development of new blueprints is then given by

$$A_{t+1} = \left(\frac{1+\beta}{1+\beta+\xi}\right)^{1+\varepsilon_1} \left[(1-\tau_0)\tau\right]^{\varepsilon_1} (1-\tau)\delta_1 A_t^{\phi_1} B_t^{\mu_1} (h_t Lt)^{1+\varepsilon_1} - \left(\frac{1-\alpha}{\alpha}\right) A_t \tag{22}$$

As there is full depreciation of physical capital, capital market clearing implies that aggregate savings are used for physical capital accumulation and purchasing new blueprints for intermediate goods production, i.e., $K_{t+1} = s_t L_t - p_{t,A}(A_{t+1} - A_t) = \frac{\beta(1-\tau)}{1+\beta+\xi} w_t h_t L_t - p_{t,A}(A_{t+1} - A_t)$. (8), (11), (15), (19), (20), and (22) yield the aggregate physical capital stock of the next period as

$$\begin{aligned} K_{t+1} = & \left[\frac{\beta(1-\tau)(1-\alpha)\left[(1-\tau_0)\tau(1+\beta)\right]^{\alpha\varepsilon_1+(1-\alpha)\varepsilon_0}}{(1+\beta+\xi)^{1+\alpha\varepsilon_1+(1-\alpha)\varepsilon_0}} K_t^{\alpha}\right] \times \\ & \left[\left(\frac{A_t^{2-\phi_1} B_t^{-\mu_1}}{\alpha\delta_1}\right)^{-\alpha} A_t \left(h_t L_t\right)^{1+\alpha\varepsilon_1+(1-\alpha)\varepsilon_0}\right] \\ & -\alpha(1-\alpha)\frac{Y_t}{A_t}\left[\left(\frac{1+\beta}{1+\beta+\xi}\right)^{1+\varepsilon_1}\left[(1-\tau_0)\tau\right]^{\varepsilon_1}(1-\tau)\delta_1 A_t^{\phi_1} B_t^{\mu_1}(h_t Lt)^{1+\varepsilon_1} - \frac{A_t}{\alpha}\right] \end{aligned} \tag{23}$$

(18) and (19) yield

$$B_{t+1} = \frac{\delta_2 \tau_0 (1-\tau_0)^{\varepsilon_2}\left[\tau(1+\beta)\right]^{1+\varepsilon_2}}{(1+\beta+\xi)^{1+\varepsilon_2}} A_t^{\phi_2} B_t^{\mu_2} (h_t L_t)^{1+\varepsilon_2} + B_t \tag{24}$$

## 2.4 Analytical results for the long-run balanced growth path

We restrict ourselves to the following assumption to ensure the BGP and rule out the empirically improbable scenario of hyper-exponential growth.

**Assumption 1.** *The intertemporal and intersectoral knowledge spillovers are given by* $\phi_1 \in [0,1)$, $\phi_2 \in [0,1)$, $\mu_1 \in [0,1)$, *and* $\mu_2 \in [0,1)$. *Moreover, it holds that* $\phi_1 + \mu_1 < 1$ *and* $\phi_2 + \mu_2 < 1$.

The growth rates of blueprints, and the propositional knowledge are given by, respectively,

$$g_{t,A} \equiv \frac{A_{t+1} - A_t}{A_t} = \frac{(1-\tau)(1+\beta)^{1+\varepsilon_1}\left[(1-\tau_0)\tau\right]^{\varepsilon_1}}{(1+\beta+\xi)^{1+\varepsilon_1}} \delta_1 A_t^{\phi_1 - 1} B_t^{\mu_1} (h_t L t)^{1+\varepsilon_1} - \frac{1}{\alpha} \tag{25}$$

$$g_{t,B} \equiv \frac{B_{t+1} - B_t}{B_t} = \frac{\delta_2 \tau_0 (1-\tau_0)^{\varepsilon_2}\left[\tau(1+\beta)\right]^{1+\varepsilon_2}}{(1+\beta+\xi)^{1+\varepsilon_2}} B_t^{\mu_2 - 1} A_t^{\phi_2} (h_t L_t)^{1+\varepsilon_2} \tag{26}$$

The balanced growth factors (henceforth BGFs) of individual human capital, population size, and aggregate human capital are given by, respectively,[8]

$$\tilde{h} \equiv \frac{h_{t+1}}{h_t} = \left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu} \frac{\psi}{(\xi - \theta - \sigma)} \tag{27}$$

$$\tilde{L} \equiv \frac{L_{t+1}}{L_t} = n_t = \frac{(\xi - \theta - \sigma)}{\psi(1+\beta+\xi)} \tag{28}$$

$$\Omega \equiv \tilde{h}\tilde{L} = \frac{\left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu}}{(1+\beta+\xi)} \tag{29}$$

From now on, we assume that $\psi \in \left( \frac{(\xi-\theta-\sigma)}{1+\beta+\xi}, \frac{(\xi-\theta-\sigma)}{\left(A_E \frac{\theta}{\eta}\right)^{\nu}\left(A_M \frac{\sigma}{\chi}\right)^{1-\nu}} \right)$. This condition ensures that the individual human capital as well as population will both grow over time. As a result, $\Omega = \tilde{h}\tilde{L} > 1$ holds unambiguously. The following proposition introduces the main results of this paper.

**Proposition 2.** *(i) The BGFs of $A$, $B$, $K$, and $Y$ are given by*

$$\tilde{A} \equiv \frac{A_{t+1}}{A_t} = \Omega^{\frac{(1+\varepsilon_1)(1-\mu_2)+(1+\varepsilon_2)\mu_1}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}}; \qquad \tilde{B} \equiv \left(\frac{B_{t+1}}{B_t}\right) = \Omega^{\frac{(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}};$$

$$\tilde{K} \equiv \left(\frac{K_{t+1}}{K_t}\right) = \Omega^{\left[\frac{(1-\mu_2)(2-\phi_1+\varepsilon_1)+\mu_1(1+\varepsilon_2-\phi_2)}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}\right]+\varepsilon_0} = \tilde{Y} \equiv \left(\frac{Y_{t+1}}{Y_t}\right)$$

*(ii) These BGFs increase with aggregate human capital accumulation ($\Omega$), and with the knowledge spillovers $\mu_1$, $\mu_2$, $\phi_1$, $\phi_2$, and the strength of the effect of healthcare workers that enhances the productivity of workers employed in the applied research sector ($\varepsilon_1$), and the basic research sector ($\varepsilon_2$). The BGF of GDP also increases in the strength of the effect*

[8] Note that $R_t = \alpha p_t = \alpha^2 \frac{Y_t}{K_t}$. As in the BGP, $Y_t$ and $K_t$ are growing at the same rate, $R_t$ must be constant in the balanced growth path, i.e., $R_{t+1} = R_t = R$, $\forall t$.

*of healthcare workers that enhances the productivity of workers employed in the final goods sector* ($\varepsilon_0$)*.*

*(iii) The BGFs are independent of the tax rates,* $\tau\tau_0$*, and* $\tau(1-\tau_0)$*.*

*(iv) The BGF of individual human capital* ($\tilde{h}$) *increases with the utility weight of children's education* ($\theta$)*, and health* ($\sigma$)*, and decreases with the utility weight of the number of children* ($\xi$)*.*

*(v) The BGF of the population* ($\tilde{L}$) *decreases with with* $\theta$ *and* $\sigma$*, and rises with* $\xi$*.*

*(vi)The BGF of aggregate human capital* ($\Omega$) *increases with* $\theta$ *and* $\sigma$*, and decreases with* $\xi$*.*

*(vii) The BGF of per capita GDP is given by*

$$\tilde{y} = \frac{\tilde{Y}}{\tilde{L}} = \frac{\left(\frac{\left(A_E \frac{\theta}{\eta}\right)^{\nu}\left(A_M \frac{\sigma}{\chi}\right)^{1-\nu}}{(1+\beta+\xi)}\right)^{\left(\frac{(1-\mu_2)(2-\phi_1+\varepsilon_1)+\mu_1(1+\varepsilon_2-\phi_2)}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}\right)+\varepsilon_0}}{\left(\frac{\xi-\theta-\sigma}{\psi(1+\beta+\xi)}\right)}$$

*The per capita GDP growth factor increases with the utility weight of children's education* ($\theta$) *and health* ($\sigma$)*, and decreases with the utility weight of the number of children* ($\xi$)*. It also increases with the knowledge spillovers* $\mu_1$*,* $\mu_2$*,* $\phi_1$*,* $\phi_2$*, and the strength of the effect of healthcare workers that enhances the productivity of workers employed in the final goods sector* ($\varepsilon_0$)*, applied research sector* ($\varepsilon_1$)*, and the basic research sector* ($\varepsilon_2$)*.*

*Proof.* See Appendix C. □

Furthermore, Prettner and Werner (2016) and Baldanzi et al. (2021) growth models are nested as special cases within our very general model (see Remark 2).

**Remarks 2.** *For* $\varepsilon_0 = \varepsilon_1 = \varepsilon_2 = 0$*, and* $\nu = 1$ *our model nests the Prettner and Werner (2016) framework, while for* $\varepsilon_0 = \varepsilon_1 = \varepsilon_2 = \mu_1 = \mu_2 = \phi_2 = 0$ *our model nests the Baldanzi et al. (2021) framework.*

One implication of Proposition 2 is that human capital accumulation is a primary factor for long-run economic growth. A second line of implication of this proposition is although aggregate human capital accumulation is increasing with the desire for educated and healthy children, it is decreasing in population growth. Furthermore, higher intertemporal and intersectoral knowledge spillovers and the strength of the effect of healthcare employees in enhancing the productivity of workers employed in the applied research, basic research and final goods sectors lead to a rise in balanced growth rates.

The effect of $\theta$ and $\sigma$ on per capita GDP growth that emerges from our model is significantly higher than Baldanzi et al. (2021). In Baldanzi et al.'s model the per capita GDP growth factor is influenced only due to $\phi_1$. Contrarily, along with $\phi_1$, in our model per capita GDP growth factor is influenced by intertemporal and intersectoral knowledge spillovers like $\phi_2$, $\mu_1$ and $\mu_2$. Another reason for the difference between Baldanzi et al.'s findings and ours is because unlike them we incorporate the impact of healthcare employees in enhancing the productivity of workers in various sectors (i.e., $\varepsilon_0$, $\varepsilon_1$, $\varepsilon_2$). The impact of $\theta$ on per capita GDP growth in our model is significantly larger than that of Prettner and Werner (2016), particularly when $(i)$ $\nu = 1$ and $(ii)$ $A_E\theta = A_M\sigma$. Inclusion of $\varepsilon_0$, $\varepsilon_1$, and $\varepsilon_2$ plays a crucial role for this difference. We would also like to highlight that unlike Prettner and Werner (2016), a rise in $\sigma$ increases the per capita GDP growth factor in our model.

# 3 Conclusion

We present an R&D-based endogenous growth model emphasizing the role of patentable applied research, publicly-funded basic research, and publicly-funded healthcare sectors. One may also perceive this contribution as a step towards the reconciliation between two recent contributions by Prettner and Werner (2016) and Baldanzi et al. (2021). Our second contribution is to illustrate healthcare workers' role and long-run consequences in enhancing productivity in various sectors. A future research problem might be investigating the varying impacts of healthcare workers and basic/applied research on the medium-run growth during the transition. Furthermore, interested researchers may also consider extending the proposed model in the context of a developing country.

# References

Andreoni, J. (1989). Giving with impure altruism: Applications to charity and ricardian equivalence. *Journal of political Economy*, 97(6):1447–1458.

Baldanzi, A., Bucci, A., and Prettner, K. (2021). Children's health, human capital accumulation, and r&d-based economic growth. *Macroeconomic Dynamics*, 25(3):651–668.

Coad, A., Segarra-Blasco, A., and Teruel, M. (2021). A bit of basic, a bit of applied? r&d strategies and firm performance. *The Journal of Technology Transfer*, 46(6):1758–1783.

Czarnitzki, D. and Thorwarth, S. (2012). Productivity effects of basic research in low-tech and high-tech industries. *Research policy*, 41(9):1555–1564.

Galor, O. (2005). From stagnation to growth: unified growth theory. *Handbook of economic growth*, 1:171–293.

Galor, O. (2011). *Unified growth theory*. Princeton University Press.

Galor, O. and Weil, D. N. (2000). Population, technology, and growth: From malthusian stagnation to the demographic transition and beyond. *American economic review*, 90(4):806–828.

Gersbach, H., Schetter, U., and Schmassmann, S. (2023). From local to global: A theory of public basic research in a globalized world. *European Economic Review*, page 104530.

Gersbach, H. and Schneider, M. T. (2015). On the global supply of basic research. *Journal of Monetary Economics*, 75:123–137.

Gersbach, H., Schneider, M. T., and Schneller, O. (2013). Basic research, openness, and convergence. *Journal of Economic Growth*, 18:33–68.

Gersbach, H., Sorger, G., and Amon, C. (2018). Hierarchical growth: Basic and applied research. *Journal of Economic Dynamics and Control*, 90:434–459.

Jones, C. I. (1995). R&d-based models of economic growth. *Journal of political Economy*, 103(4):759–784.

Kuhn, M. and Prettner, K. (2016). Growth and welfare effects of health care in knowledge-based economies. *Journal of Health Economics*, 46:100–119.

Madsen, J. B. (2016). Health, human capital formation and knowledge production: Two centuries of international evidence. *Macroeconomic Dynamics*, 20(4):909–953.

Mulligan, K., Lenihan, H., Doran, J., and Roper, S. (2022). Harnessing the science base: Results from a national programme using publicly-funded research centres to reshape firms' r&d. *Research Policy*, 51(4):104468.

Prettner, K. (2014). The non-monotonous impact of population growth on economic prosperity. *Economics Letters*, 124(1):93–95.

Prettner, K., Bloom, D. E., and Strulik, H. (2013). Declining fertility and economic well-being: do education and health ride to the rescue? *Labour economics*, 22:70–79.

Prettner, K. and Werner, K. (2016). Why it pays off to pay us well: The impact of basic research on economic growth and welfare. *Research Policy*, 45(5):1075–1090.

Rivera, B. and Currais, L. (2004). Public health capital and productivity in the spanish regions: A dynamic panel data model. *World Development*, 32(5):871–885.

Romer, P. M. (1990). Endogenous technological change. *Journal of political Economy*, 98(5, Part 2):S71–S102.

Strulik, H., Prettner, K., and Prskawetz, A. (2013). The past and future of knowledge-based growth. *Journal of Economic Growth*, 18:411–437.

Toole, A. A. (2012). The impact of public basic research on industrial innovation: Evidence from the pharmaceutical industry. *Research Policy*, 41(1):1–12.

# Appendix

## A Derivation of the optimal values of $c_t$, $s_t$, $n_t$, $e_t$ and $m_t$

Using (1) and (2) we set the Lagrangian as

$$\begin{aligned}\mathcal{L} = &\ \ln c_t + \beta \ln\left[(R_{t+1} - 1)s_t\right] + \xi \ln n_t + \theta \ln e_t + \sigma \ln m_t \\ &+ \lambda[(1-\tau)(1 - \psi n_t - \eta e_t n_t - \chi m_t n_t)w_t h_t - c_t - s_t]\end{aligned}$$

The first-order conditions are given by

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \implies \frac{1}{c_t} = \lambda \tag{A.1}$$

$$\frac{\partial \mathcal{L}}{\partial s_t} = 0 \implies \frac{\beta}{s_t} = \lambda \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = 0 \implies \frac{\xi}{n_t} = \lambda(1-\tau)(\psi + \eta e_t + \chi m_t)w_t h_t \tag{A.3}$$

$$\frac{\partial \mathcal{L}}{\partial e_t} = 0 \implies \frac{\theta}{e_t} = \lambda(1-\tau)\eta n_t w_t h_t \tag{A.4}$$

$$\frac{\partial \mathcal{L}}{\partial m_t} = 0 \implies \frac{\sigma}{m_t} = \lambda(1-\tau)\chi n_t w_t h_t \tag{A.5}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \implies (1-\tau)(1 - \psi n_t - \eta e_t n_t - \chi m_t n_t)w_t h_t - c_t - s_t = 0 \tag{A.6}$$

Dividing (A.4) by (A.5) we obtain

$$\chi m_t = \frac{\eta\sigma}{\theta} e_t \tag{A.7}$$

Dividing (A.1) by (A.2) we obtain

$$s_t = \beta c_t \tag{A.8}$$

Dividing (A.3) by (A.4) and using (A.7) we obtain

$$e_t = \frac{\theta\psi}{\eta(\xi - \theta - \sigma)} \tag{A.9}$$

Inserting the value of $e_t$ into (A.7) we get

$$m_t = \frac{\sigma\psi}{\chi(\xi - \theta - \sigma)} \tag{A.10}$$

Dividing (A.3) by (A.1) and rearranging it we obtain

$$(1-\tau)(\psi+\eta e_t+\chi m_t)n_t w_t h_t=\xi c_t \tag{A.11}$$

Inserting (A.11) into (A.6) we get

$$(1-\tau)w_t h_t-(1-\tau)(\psi+\eta e_t+\chi m_t)n_t w_t h_t-c_t-s_t=0$$

$$\implies c_t=\frac{(1-\tau)w_t h_t}{1+\beta+\xi} \tag{A.12}$$

Therefore,

$$s_t=\frac{\beta(1-\tau)w_t h_t}{1+\beta+\xi} \tag{A.13}$$

Inserting the values of $c_t$, $s_t$, $e_t$ and $m_t$ into (A.6) and rearranging it we obtain

$$n_t=\frac{\xi-\theta-\sigma}{\psi(1+\beta+\xi)} \tag{A.14}$$

# B Proof of Proposition 1

The partial derivatives of fertility $n_t$ with respect to $\xi$, $\theta$, and $\sigma$ are

$$\begin{aligned}
&\frac{\partial n_t}{\partial \xi}=\frac{1+\beta+\theta+\sigma}{\psi\left(1+\beta+\xi\right)^2}>0 \qquad \frac{\partial n_t}{\partial \theta}=-\frac{1}{\psi\left(1+\beta+\xi\right)}<0\\
&\frac{\partial n_t}{\partial \sigma}=-\frac{1}{\psi\left(1+\beta+\xi\right)}<0
\end{aligned} \tag{B.1}$$

The partial derivatives of the individual human capital $h_{t+1}$ with respect to $\xi$, $\theta$, and $\sigma$ are

$$\begin{aligned}
\frac{\partial h_{t+1}}{\partial \xi}&=-\left(A_E\frac{\theta}{\eta}\right)^{\nu}\left(A_M\frac{\sigma}{\chi}\right)^{1-\nu}\frac{\psi}{(\xi-\theta-\sigma)^2}h_t<0\\
\frac{\partial h_{t+1}}{\partial \theta}&=\frac{\left(A_E\frac{\theta}{\eta}\right)^{\nu}\left(A_M\frac{\sigma}{\chi}\right)^{1-\nu}\psi h_t}{(\xi-\theta-\sigma)}\left[\frac{\nu A_E}{\eta}\left(A_E\frac{\theta}{\eta}\right)^{-1}+\frac{1}{(\xi-\theta-\sigma)}\right]>0\\
\frac{\partial h_{t+1}}{\partial \sigma}&=\frac{\left(A_E\frac{\theta}{\eta}\right)^{\nu}\left(A_M\frac{\sigma}{\chi}\right)^{1-\nu}\psi h_t}{(\xi-\theta-\sigma)}\left[\frac{(1-\nu)A_M}{\chi}\left(A_M\frac{\sigma}{\chi}\right)^{-1}+\frac{1}{(\xi-\theta-\sigma)}\right]>0
\end{aligned} \tag{B.2}$$

# C Proof of Proposition 2

***(i)*** Growth rates of the endogenous variables have to be constant along the balanced growth path. therefore,

$$\frac{g_{t+1,A}-g_{t,A}}{g_{t,A}}=0 \implies g_{t+1,A}=g_{t,A}$$

$$\Longrightarrow \frac{(1-\tau)(1+\beta)^{1+\varepsilon_1}\left[(1-\tau_0)\tau\right]^{\varepsilon_1}}{(1+\beta+\xi)^{1+\varepsilon_1}}\delta_1 A_{t+1}^{\phi_1-1}B_{t+1}^{\mu_1}(h_{t+1}L_{t+1})^{1+\varepsilon_1}-\frac{1}{\alpha}$$
$$=\frac{(1-\tau)(1+\beta)^{1+\varepsilon_1}\left[(1-\tau_0)\tau\right]^{\varepsilon_1}}{(1+\beta+\xi)^{1+\varepsilon_1}}\delta_1 A_{t}^{\phi_1-1}B_{t}^{\mu_1}(h_{t}L_{t})^{1+\varepsilon_1}-\frac{1}{\alpha}$$
$$\Longrightarrow \left(\frac{h_{t+1}}{h_t}\frac{L_{t+1}}{L_t}\right)^{1+\varepsilon_1}\left(\frac{A_{t+1}}{A_t}\right)^{\phi_1-1}\left(\frac{B_{t+1}}{B_t}\right)^{\mu_1}=1$$
$$\Omega^{1+\varepsilon_1}\left(\frac{A_{t+1}}{A_t}\right)^{\phi_1-1}\left(\frac{B_{t+1}}{B_t}\right)^{\mu_1}=1 \tag{C.1}$$
$$\frac{g_{t+1,B}-g_{t,B}}{g_{t,B}}=0 \Longrightarrow g_{t,B}=g_{t-1,B}$$
$$\Longrightarrow \frac{\delta_2\tau_0(1-\tau_0)^{\varepsilon_2}\left[\tau(1+\beta)\right]^{1+\varepsilon_2}}{(1+\beta+\xi)^{1+\varepsilon_2}}B_{t+1}^{\mu_2-1}A_{t+1}^{\phi_2}(h_{t+1}L_{t+1})^{1+\varepsilon_2}$$
$$=\frac{\delta_2\tau_0(1-\tau_0)^{\varepsilon_2}\left[\tau(1+\beta)\right]^{1+\varepsilon_2}}{(1+\beta+\xi)^{1+\varepsilon_2}}B_{t}^{\mu_2-1}A_{t}^{\phi_2}(h_{t}L_{t})^{1+\varepsilon_2}$$
$$\Longrightarrow \left(\frac{h_{t+1}}{h_t}\frac{L_{t+1}}{L_t}\right)^{1+\varepsilon_2}\left(\frac{A_{t+1}}{A_t}\right)^{\phi_2}\left(\frac{B_{t+1}}{B_t}\right)^{\mu_2-1}=1$$
$$\left(\frac{B_{t+1}}{B_t}\right)=\Omega^{\frac{1+\varepsilon_2}{1-\mu_2}}\left(\frac{A_{t+1}}{A_t}\right)^{\frac{\phi_2}{1-\mu_2}} \tag{C.2}$$

Inserting (C.2) into (C.1) we obtain

$$\tilde{A}\equiv\left(\frac{A_{t+1}}{A_t}\right)=\Omega^{\frac{(1+\varepsilon_1)(1-\mu_2)+(1+\varepsilon_2)\mu_1}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}} \tag{C.3}$$

Therefore,

$$\tilde{B}\equiv\left(\frac{B_{t+1}}{B_t}\right)=\Omega^{\frac{(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}} \tag{C.4}$$

In the BGP, $Y_t$ and $K_t$ must grow at the same rate. Therefore, equation (11) suggests

$$\tilde{Y}\equiv\left(\frac{Y_{t+1}}{Y_t}\right)=\left(\frac{A_{t+1}}{A_t}\right)^{2-\phi_1}\left(\frac{B_{t+1}}{B_t}\right)^{-\mu_1}\left(\frac{H_{t+1,M}}{H_{t,M}}\right)^{\varepsilon_0-\varepsilon_1}=\left(\frac{K_{t+1}}{K_t}\right)\equiv\tilde{K} \tag{C.5}$$
$$\Longrightarrow \left(\frac{Y_{t+1}}{Y_t}\right)\equiv\tilde{Y}=\tilde{K}\equiv\left(\frac{K_{t+1}}{K_t}\right)=\Omega^{\left[\frac{(1-\mu_2)(2-\phi_1+\varepsilon_1)+\mu_1(1+\varepsilon_2-\phi_2)}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}\right]+\varepsilon_0} \tag{C.6}$$

***(ii)***

$$\frac{\partial\tilde{A}}{\partial\mu_1}=\tilde{A}\ln(\Omega)\frac{(1-\mu_2)\left[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2}>0$$
$$\frac{\partial\tilde{B}}{\partial\mu_1}=\tilde{B}\ln(\Omega)\frac{\phi_2\left[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2}\geq 0$$
$$\frac{\partial\tilde{K}}{\partial\mu_1}=\tilde{K}\ln(\Omega)\frac{(1-\mu_2)\left[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2}>0$$

$$\frac{\partial \tilde{A}}{\partial \mu_2} = \tilde{A}\ln(\Omega)\frac{\mu_1\left[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} \geq 0$$

$$\frac{\partial \tilde{B}}{\partial \mu_2} = \tilde{B}\ln(\Omega)\frac{(1-\phi_1)\left[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} > 0$$

$$\frac{\partial \tilde{K}}{\partial \mu_2} = \tilde{K}\ln(\Omega)\frac{\mu_1\left[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} \geq 0$$

$$\frac{\partial \tilde{A}}{\partial \phi_1} = \tilde{A}\ln(\Omega)\frac{(1-\mu_2)\left[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} > 0$$

$$\frac{\partial \tilde{B}}{\partial \phi_1} = \tilde{B}\ln(\Omega)\frac{\phi_2\left[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} \geq 0$$

$$\frac{\partial \tilde{K}}{\partial \phi_1} = \tilde{K}\ln(\Omega)\frac{(1-\mu_2)\left[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} > 0$$

$$\frac{\partial \tilde{A}}{\partial \phi_2} = \tilde{A}\ln(\Omega)\frac{\mu_1\left[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} \geq 0$$

$$\frac{\partial \tilde{B}}{\partial \phi_2} = \tilde{B}\ln(\Omega)\frac{(1-\phi_1)\left[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} > 0$$

$$\frac{\partial \tilde{K}}{\partial \phi_2} = \tilde{K}\ln(\Omega)\frac{\mu_1\left[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} \geq 0$$

$$\frac{\partial \tilde{A}}{\partial \varepsilon_0} = 0 \qquad \frac{\partial \tilde{B}}{\partial \varepsilon_0} = 0 \qquad \frac{\partial \tilde{K}}{\partial \varepsilon_0} = \tilde{K}\ln(\Omega) > 0$$

$$\frac{\partial \tilde{A}}{\partial \varepsilon_1} = \tilde{A}\ln(\Omega)\frac{(1-\mu_2)}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]} > 0$$

$$\frac{\partial \tilde{B}}{\partial \varepsilon_1} = \tilde{B}\ln(\Omega)\frac{\phi_2}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]} \geq 0$$

$$\frac{\partial \tilde{K}}{\partial \varepsilon_1} = \tilde{K}\ln(\Omega)\frac{(1-\mu_2)}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]} > 0$$

$$\frac{\partial \tilde{A}}{\partial \varepsilon_2} = \tilde{A}\ln(\Omega)\frac{\mu_1}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]} \geq 0$$

$$\frac{\partial \tilde{B}}{\partial \varepsilon_2} = \tilde{B}\ln(\Omega)\frac{(1-\phi_1)}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]} > 0$$

$$\frac{\partial \tilde{K}}{\partial \varepsilon_2} = \tilde{K}\ln(\Omega)\frac{\mu_1}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]} \geq 0$$

***(iii)***

$$\frac{\partial \tilde{A}}{\partial \tau\tau_0} = \frac{\partial \tilde{B}}{\partial \tau\tau_0} = \frac{\partial \tilde{K}}{\partial \tau\tau_0} = 0$$

$$\frac{\partial \tilde{A}}{\partial \tau(1-\tau_0)} = \frac{\partial \tilde{B}}{\partial \tau(1-\tau_0)} = \frac{\partial \tilde{K}}{\partial \tau(1-\tau_0)} = 0$$

***(iv)***

$$\frac{\partial \tilde{h}}{\partial \xi} = -\left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu} \frac{\psi}{(\xi-\theta-\sigma)^2} < 0$$

$$\frac{\partial \tilde{h}}{\partial \theta} = \frac{\left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu} \psi}{(\xi-\theta-\sigma)} \left[\frac{\nu A_E}{\eta}\left(A_E \frac{\theta}{\eta}\right)^{-1} + \frac{1}{(\xi-\theta-\sigma)}\right] > 0$$

$$\frac{\partial \tilde{h}}{\partial \sigma} = \frac{\left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu} \psi}{(\xi-\theta-\sigma)} \left[\frac{(1-\nu)A_M}{\chi}\left(A_M \frac{\sigma}{\chi}\right)^{-1} + \frac{1}{(\xi-\theta-\sigma)}\right] > 0$$

***(v)***

$$\frac{\partial \tilde{L}}{\partial \xi} = \frac{1+\beta+\theta+\sigma}{\psi(1+\beta+\xi)^2} > 0; \quad \frac{\partial \tilde{L}}{\partial \theta} = \frac{-1}{\psi(1+\beta+\xi)} < 0; \quad \frac{\partial \tilde{L}}{\partial \sigma} = \frac{-1}{\psi(1+\beta+\xi)} < 0$$

***(vi)***

$$\frac{\partial \Omega}{\partial \xi} = -\frac{\left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu}}{(1+\beta+\xi)^2} < 0$$

$$\frac{\partial \Omega}{\partial \theta} = \frac{\frac{\nu A_E}{\eta}\left(A_E \frac{\theta}{\eta}\right)^{\nu-1} \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu}}{(1+\beta+\xi)} > 0$$

$$\frac{\partial \Omega}{\partial \sigma} = \frac{\frac{(1-\nu)A_M}{\chi}\left(A_E \frac{\theta}{\eta}\right)^{\nu} \left(A_M \frac{\sigma}{\chi}\right)^{-\nu}}{(1+\beta+\xi)} > 0$$

***(vii)***

$$\frac{\partial \tilde{y}}{\partial \xi} = \tilde{y}\left[\frac{-1}{(1+\beta+\xi)}\right]\left[\frac{(1-\mu_2)(2-\phi_1+\varepsilon_1)+\mu_1(1+\varepsilon_2-\phi_2)}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1} + \varepsilon_0 + \frac{1+\beta+\theta+\sigma}{\xi-\theta-\sigma}\right] < 0$$

$$\frac{\partial \tilde{y}}{\partial \theta} = \tilde{y}\left[\left(\frac{(1-\mu_2)(2-\phi_1+\varepsilon_1)+\mu_1(1+\varepsilon_2-\phi_2)}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1} + \varepsilon_0\right)\left(\frac{\nu}{\theta}\right) + \left(\frac{1}{\xi-\theta-\sigma}\right)\right] > 0$$

$$\frac{\partial \tilde{y}}{\partial \sigma} = \tilde{y}\left[\left(\frac{(1-\mu_2)(2-\phi_1+\varepsilon_1)+\mu_1(1+\varepsilon_2-\phi_2)}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1} + \varepsilon_0\right)\left(\frac{1-\nu}{\sigma}\right) + \left(\frac{1}{\xi-\theta-\sigma}\right)\right] > 0$$

$$\frac{\partial \tilde{y}}{\partial \mu_1} = \tilde{y}\ln(\Omega)\frac{(1-\mu_2)\left[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} > 0$$

$$\frac{\partial \tilde{y}}{\partial \mu_2} = \tilde{y}\ln(\Omega)\frac{\mu_1\left[(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} \geq 0$$

$$\frac{\partial \tilde{y}}{\partial \phi_1} = \tilde{y}\ln(\Omega)\frac{(1-\mu_2)\left[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} > 0$$

$$\frac{\partial \tilde{y}}{\partial \phi_2} = \tilde{y}\ln(\Omega)\frac{\mu_1\left[(1+\varepsilon_2)\mu_1+(1+\varepsilon_1)(1-\mu_2)\right]}{\left[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1\right]^2} \geq 0$$

$$\frac{\partial \tilde{y}}{\partial \varepsilon_0} = \tilde{y}\ln(\Omega) > 0$$

$$\frac{\partial \tilde{y}}{\partial \varepsilon_1} = \tilde{y}\ln(\Omega)\frac{(1-\mu_2)}{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]} > 0$$

$$\frac{\partial \tilde{y}}{\partial \varepsilon_2} = \tilde{y}\ln(\Omega)\frac{\mu_1}{[(1-\phi_1)(1-\mu_2)-\phi_2\mu_1]} \geq 0$$