



Munich Personal RePEc Archive

Health, basic research, human capital accumulation, and RD-based economic growth

Parui, Pintu

XIM University, Bhubaneswar, India

13 October 2023

Online at <https://mpra.ub.uni-muenchen.de/118854/>
MPRA Paper No. 118854, posted 14 Oct 2023 07:23 UTC

Health, basic research, human capital accumulation, and R&D-based economic growth

Pintu Parui

School of Economics, XIM University, Bhubaneswar

Abstract

We construct a broad R&D-based endogenous growth model that incorporates the importance of children's health on human capital accumulation and publicly-funded basic research investments required to produce new goods. Although an increment in the number of healthcare professionals creates a shortage of workers for final goods production, the novelty of this paper is to demonstrate the significance of healthcare workers in enhancing the productivity of inputs of various sectors, along with its long-run consequences.

Keywords: R&D-based growth, Basic science, Children's Health, Education, Fertility.

JEL Code: H41; J24; O31; O32; O41

This draft: **October 13, 2023**

Email id: pintuparui6@gmail.com

Acknowledgment: *I am grateful to Subrata Guha, Sugata Marjit, Gogol Mitra Thakur and Sandip Sarkar for insightful comments in an earlier draft. The usual disclaimer applies.*

1 Introduction

A large body of R&D-based endogenous growth literature focuses on the quality-quantity trade-off between fertility and human capital accumulation (cf. [Prettner et al., 2013](#); [Strulik et al., 2013](#); [Prettner, 2014](#); [Prettner and Werner, 2016](#)). We observe that the issues related to children's health on human capital accumulation have been substantially undermined. Nevertheless, [Baldanzi et al. \(2021\)](#) is an exception, where the authors address this issue but ignore the importance of publicly funded basic research investments in the innovation of new goods.¹ On the other hand, [Prettner and Werner \(2016\)](#) address the importance of basic research,² but disregard the significance of children's health on human capital accumulation.

Constructing a general model, we extend the existing literature that captures (i) children's health on human capital accumulation and (ii) the significance of government-financed basic research investments in innovating new goods. Moreover, we assume that the government seeks to improve people's health by providing healthcare facilities to them. This, in turn, can enhance productivity in the related sectors by reducing production losses caused by sick employees. On the other hand, a rise in the number of healthcare workers means fewer workers are available for final goods production. The novelty of our paper lies in capturing this trade-off, along with the quality-quantity trade-offs in fertility and human capital accumulation. Besides, we contend that our model is more general than [Prettner and Werner \(2016\)](#) and [Baldanzi et al. \(2021\)](#).

2 The model

2.1 Consumption side

We consider an economy with three overlapping generations: children, adults, and retirees. Adults decide upon the consumption level c_t , savings for retirement s_t , the number of children n_t , education (e_t) and health (m_t) of each child. The time adults do not spend on raising, educating, and caring for their children's health is supplied to the labor

¹For the importance of health investment in creating human capital and its long-run consequences, see [Prettner et al. \(2013\)](#) and [Kuhn and Prettner \(2016\)](#). For the empirical evidence, see [Madsen \(2016\)](#).

²[Gersbach et al. \(2013\)](#), [Gersbach and Schneider \(2015\)](#), [Gersbach et al. \(2018\)](#), and [Gersbach et al. \(2023\)](#) are the other contributors. For empirical evidence, see [Czarnitzki and Thorwarth \(2012\)](#), [Toole \(2012\)](#), [Coad et al. \(2021\)](#), and [Mulligan et al. \(2022\)](#).

43 market. While children don't participate in any economic decision, retirees consume their
 44 entire savings carried over from adulthood. Following [Prettner and Werner \(2016\)](#), we
 45 assume a single-parent household with the following utility function:

$$u_t = \ln c_t + \beta \ln [(R_{t+1} - 1)s_t] + \xi \ln n_t + \theta \ln e_t + \sigma \ln m_t \quad (1)$$

46 $\beta \in (0, 1)$ represents the inter-generational discount factor. R_{t+1} represents the gross
 47 interest rate on assets between generation t and $t+1$. $\xi \in (0, 1)$, $\theta \in (0, 1)$, and $\sigma \in (0, 1)$
 48 are utility weights on the number of children, child's education and health respectively.³
 49 We assume that the next generation's human capital is a multiplicative function of ed-
 50 ucation and health. Therefore a part of the utility function ($\xi \ln n_t + \theta \ln e_t + \sigma \ln m_t$)
 51 captures the trade-offs parents face in deciding the number of children and parental time
 52 expenditure on children's education and health. To simplify the model, we assume an
 53 exogenously given mortality of parents and to avoid nonsensical solutions we impose the
 54 restriction that $\xi > \theta + \sigma$.

55 Following [Prettner and Werner \(2016\)](#) we assume that the cost of raising children,
 56 educating them, and providing them the basic health facilities requires time costs of
 57 households.⁴ Therefore, the budget constraint of the household reads:

$$(1 - \tau)(1 - \psi n_t - \eta e_t n_t - \chi m_t n_t) w_t h_t = c_t + s_t \quad (2)$$

58 where $\tau \in (0, 1)$ represents the income tax rate, $\psi > 0$, $\eta > 0$, and $\chi > 0$ denote
 59 opportunity cost (in terms of time) of child-rearing, per child education and health in-
 60 vestment respectively, w_t is the wage rate and h_t represents the effective labor (i.e., the
 61 human capital). Optimal choices of consumption, savings, fertility, education, and health
 62 are (see [Appendix A](#) for the derivation)

³This type of utility function is often found in the literature (cf. [Strulik et al., 2013](#); [Prettner and Werner, 2016](#); [Baldanzi et al., 2021](#)) and is based on the "warm-glow motive of giving" (see [Andreoni, 1989](#)) and is a special case of utility formulation used in [Galor and Weil \(2000\)](#), and [Galor \(2005, 2011\)](#).

⁴For example, parental involvement in a child's physical development by assigning time for the child to participate in different sports and games, dance, and other physical activities will aid in developing the child's health. Different mental games will support a child's mental growth while also helping the child develop mental acuity, improving their ability to absorb their essential education. The time parents drive their children to get the necessary vaccines will also contribute to improving their health.

$$\begin{aligned}
c_t &= \frac{(1-\tau)w_t h_t}{1+\beta+\xi} & s_t &= \frac{\beta(1-\tau)w_t h_t}{1+\beta+\xi} & n_t &= \frac{(\xi-\theta-\sigma)}{\psi(1+\beta+\xi)} \\
e_t &= \frac{\theta\psi}{\eta(\xi-\theta-\sigma)} & m_t &= \frac{\sigma\psi}{\chi(\xi-\theta-\sigma)}
\end{aligned} \tag{3}$$

63 Population size at time $t + 1$ is

$$L_{t+1} = n_t L_t = \frac{(\xi - \theta - \sigma)}{\psi(1 + \beta + \xi)} L_t \tag{4}$$

64 We assume that the individual human capital of the next generation depends positively
65 on (i) educational effort by the parents, e_t (ii) parents' productivity in education, A_E
66 (iii) healthcare effort by parents for their children, m_t ⁵ (iv) parents' productivity in
67 healthcare for their children, A_M , and (v) the level of parents' individual human capital
68 h_t in the following way:

$$h_{t+1} = (A_E e_t h_t)^\nu (A_M m_t h_t)^{1-\nu} = \left(A_E \frac{\theta}{\eta}\right)^\nu \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu} \frac{\psi}{(\xi - \theta - \sigma)} h_t \tag{5}$$

69 (5) also captures the trade-off between child quantity and quality that is summarised
70 in the following proposition.

71 **Proposition 1.** *An increase in desire for a large family (ξ) increases fertility and reduces*
72 *the next generation's human capital. An increase in the desire for having better educated*
73 *(θ) or healthy children (σ) increases the human capital of the next generation and reduces*
74 *fertility.*

75 *Proof.* See Appendix B. □

76 H_t , the aggregate human capital stock of the economy is the product of individual
77 human capital (h_t) and the total population size (L_t). Therefore, the human capital stock
78 available for production, research, and healthcare facility (\tilde{H}_t) is given by the aggregate
79 human capital stock adjusted for the time parents spend raising, educating, and caring
80 for their children's health ($\psi n_t + \eta e_t n_t + \chi m_t n_t$) as

$$\tilde{H}_t = [1 - \psi n_t - \eta e_t n_t - \chi m_t n_t] H_t = \frac{1 + \beta}{1 + \beta + \xi} h_t L_t \tag{6}$$

⁵Note that along with the level of education, a better health condition is also an essential component in the individual human capital (cf. Rivera and Currais, 2004; Baldanzi et al., 2021).

2.2 Production side

The final goods sector, intermediate goods sector, applied research sector, basic research sector, and healthcare sector constitute the production side of the economy. The first three sectors are based on the standard Romer (1990) and Jones (1995) R&D-driven growth literature. We modify this structure to account for (i) a tax-financed basic research sector that employs scientists to discover and explain the natural laws and phenomena required for applied research, (ii) a tax-financed healthcare sector which enhances the productivity of human capital, and (iii) the endogenous evolution of aggregate human capital in the production process.

The perfectly competitive final goods sector employs workers and machines to produce output Y_t according to

$$Y_t = (H_{t,M}^{\varepsilon_0} H_{t,Y})^{1-\alpha} \int_0^{A_t} x_{t,i}^\alpha di \quad (7)$$

where $H_{t,Y}$ and $H_{t,M}$ are the human capital (workers) employed in the final good and healthcare sectors respectively, A_t is the technological frontier, $x_{t,i}$ is the amount of the blueprint-specific machine i used in production, and α is the elasticity of output with respect to machines. $H_{t,M}^{\varepsilon_0}$ affects the productivity of workers, $H_{t,Y}$, while ε_0 measures the strength of the effect.⁶ For a given total factor productivity (i.e., $H_{t,M}^{\varepsilon_0(1-\alpha)}$), (7) exhibits constant returns to scale in $H_{t,Y}$ and $x_{t,i}$. Perfect competition implies the wage rate ($w_{t,Y}$) and the machines' prices ($p_{t,i}$) are, respectively,

$$w_{t,Y} = (1 - \alpha) (H_{t,M}^{\varepsilon_0} H_{t,Y})^{-\alpha} H_{t,M}^{\varepsilon_0} \int_0^{A_t} x_{t,i}^\alpha di = (1 - \alpha) \frac{Y_t}{H_{t,Y}} \quad (8)$$

⁶Let us take an example. An individual's human capital level at the time of entry into the labour force in period t is h_t . This human capital depends on her parents' decision (in period $t - 1$) to devote time to her education and health care when she was young. However, if the individual becomes ill, even though she continues to work, she may not be able to perform to her full potential. The healthcare facilities will assist her in regaining her capacity as soon as possible. As a result, she will be more productive than if she did not have access to this healthcare facility. In this context, it should be noted that healthcare facilities may have an impact on children's health. However, for the sake of simplicity, we are ignoring this possibility. One intriguing extension of the current model would be integrating this issue and investigating its long-run implications. One may also argue that the intensive form of human capital (i.e., $\frac{H_{t,M}}{H_t}$), rather than the amount of human capital employed in the healthcare sector ($H_{t,M}$), should play a role in determining the productivity of workers in the various sectors. Nonetheless, for comparable types of basic health concerns, people frequently take basic therapies on their own, without even consulting a healthcare expert, while observing the treatment of other sick people. In other words, healthcare practitioners not only directly address the health issues of those seeking treatment, but they also indirectly assist other sick people. As a result, we employ $H_{t,M}$ in our model instead of $\frac{H_{t,M}}{H_t}$ to reflect the spillover effect/positive externality provided by healthcare personnel.

$$p_{t,i} = \alpha x_{t,i}^{\alpha-1} (H_{t,M}^{\varepsilon_0} H_{t,Y})^{1-\alpha} \quad (9)$$

99 Raw physical capital ($k_{t,i}$) serves as variable input and one machine-specific blueprint
 100 serves as fixed input in the production of the monopolistically competitive intermediate
 101 goods sector, which manufactures the machines for the final goods sector. We assume full
 102 depreciation of physical capital over the course of one generation. Therefore operating
 103 profits are $\pi_{t,i} = p_{t,i}k_{t,i} - R_t k_{t,i}$. Profit maximization then leads to the monopolistic
 104 pricing rule for each firm as

$$p_{t,i} = \frac{R_t}{\alpha} \quad (10)$$

105 Due to symmetry, each firm employs $k_t = \frac{K_t}{A_t}$ units of physical capital, where K_t
 106 represents the aggregate physical capital stock. The aggregate production function can
 107 then be re-written as

$$Y_t = (A_t H_{t,M}^{\varepsilon_0} H_{t,Y})^{1-\alpha} K_t^\alpha \quad (11)$$

108 The applied research sector employs scientists with human capital stock $H_{t,A}$ to create
 109 new blueprints that can be patented and sold to the intermediate goods sector. In the
 110 field of applied research, a firm's production function is defined as

$$A_{t+1} - A_t = \delta_1 H_{t,M}^{\varepsilon_1} A_t^{\phi_1} B_t^{\mu_1} H_{t,A} \quad (12)$$

111 where $\delta_1 H_{t,M}^{\varepsilon_1}$ is the productivity of inputs in the applied research sector, B_t represents
 112 society's stock of basic knowledge discovered by basic researchers and forms the epistemic
 113 base for the stock of patented knowledge A_t . $\phi_1 \in [0, 1]$ and $\mu_1 \in [0, 1]$ measure the ex-
 114 tent of intertemporal knowledge spillovers in the applied research sector and intersectoral
 115 knowledge spillovers from basic to applied research, respectively.⁷ For a given stock of ba-
 116 sic and applied knowledge, ε_1 assesses how strongly healthcare professionals enhance the
 117 productivity of applied research sector workers. Similar to [Prettner and Werner \(2016\)](#),

⁷While knowledge spillovers happen intertemporally in the applied research sector, intersectoral knowledge spillovers occur between basic and applied research. Like [Prettner and Werner \(2016\)](#), given that patents are partially excludable, whereas the laws of nature, once discovered, can be exploited by scientists freely, one can expect that the spillovers from basic research to applied research are greater than the opposite.

118 no technique can be developed without any propositional knowledge, B_t . Therefore, to
 119 begin with, we assume that $B_0 > 0$ and $A_0 > 0$. Moreover, we assume $H_{t,M} > 0$. Our
 120 framework nests both the endogenous and semi-endogenous growth models of [Romer](#)
 121 [\(1990\)](#) and [Jones \(1995\)](#) as special cases (see Remark 1).

122 **Remarks 1.** For $\tau = 0$, $\theta = 0$, $\sigma = 0$, $\xi > \psi(1 + \beta + \xi)$, $\mu_1 = 0$, $\varepsilon_0 = \varepsilon_1 = 0$, and
 123 $\phi_1 \in (0, 1)$, our model nests the [Jones \(1995\)](#) framework, while for $\tau = 0$, $\theta = 0$, $\sigma = 0$,
 124 $\xi > \psi(1 + \beta + \xi)$, $\mu_1 = 0$, $\varepsilon_0 = \varepsilon_1 = 0$, and $\phi_1 = 1$, our model nests the [Romer \(1990\)](#)
 125 framework.

126 Firms in the applied research sector hire the human capital $H_{t,A}$ so as to maximize
 127 their profits

$$\pi_{t,A} = p_{t,A} \delta_1 H_{t,M}^{\varepsilon_1} A_t^{\phi_1} B_t^{\mu_1} H_{t,A} - w_{t,A} H_{t,A} \quad (13)$$

128 with $p_{t,A}$, the price of a blueprint and $w_{t,A}$, applied researchers' wage rate. This leads
 129 to the optimality condition

$$w_{t,A} = p_{t,A} \delta_1 H_{t,M}^{\varepsilon_1} A_t^{\phi_1} B_t^{\mu_1} \quad (14)$$

130 Following [Strulik et al. \(2013\)](#) and [Prettner and Werner \(2016\)](#), we assume that
 131 patent protection lasts for one generation. Once the patent expires, the right to sell
 132 the blueprint is handed over to the government, which can either consume or invest the
 133 associated proceeds unproductively. For a blueprint, applied research sector firms charge
 134 the entire operating profit of an intermediate goods producer, that is,

$$p_{t,A} = \pi_{t,i} = \alpha(1 - \alpha) \frac{Y_t}{A_t} \quad (15)$$

135 A part (τ_0) of the government's revenue is spent on employing scientists to discover
 136 the propositional knowledge in the basic research sector so that

$$\frac{\tau_0 \tau (1 + \beta)}{(1 + \beta + \xi)} w_t h_t L_t = w_t h_t L_{t,B} \quad (16)$$

137 Therefore, the amount of human capital employed in the basic research sector is

$$H_{t,B} = L_{t,B} h_t = \frac{\tau_0 \tau (1 + \beta)}{(1 + \beta + \xi)} H_t \quad \left(\equiv \tau_0 \tau \tilde{H}_t \right) \quad (17)$$

138 Propositional knowledge evolves according to

$$B_{t+1} - B_t = \delta_2 H_{t,M}^{\varepsilon_2} A_t^{\phi_2} B_t^{\mu_2} H_{t,B} = \frac{\delta_2 \tau_0 \tau (1 + \beta)}{(1 + \beta + \xi)} H_{t,M}^{\varepsilon_2} A_t^{\phi_2} B_t^{\mu_2} H_t \quad (18)$$

139 where $\delta_2 H_{t,M}^{\varepsilon_2}$ is the productivity of inputs in the basic research sector, $\mu_2 \in [0, 1]$ and
 140 $\phi_2 \in [0, 1]$ measure the extent of intertemporal knowledge spillovers in the basic research
 141 sector and intersectoral knowledge spillovers from applied to basic research, and $H_{t,B}$
 142 represents the amount of aggregate human capital that government employs by choosing
 143 the τ_0 fraction of tax rate, τ . For a given stock of applied and propositional knowledge,
 144 ε_2 indicates the strength of the effect of the healthcare professionals in enhancing the
 145 productivity of the basic research sector workers.

146 We assume that the government budget is balanced so that the rest $(1 - \tau_0)$ of the
 147 government revenue is spent on employing the healthcare workers in the healthcare sector,
 148 i.e.,

$$\frac{(1 - \tau_0)\tau(1 + \beta)}{(1 + \beta + \xi)} w_t h_t L_t = w_t h_t L_{t,M}$$

149 Therefore, the amount of human capital employed in the healthcare sector is

$$H_{t,M} = h_t L_{t,M} = \frac{(1 - \tau_0)\tau(1 + \beta)}{(1 + \beta + \xi)} H_t \quad \left(\equiv (1 - \tau_0)\tau \tilde{H}_t \right) \quad (19)$$

150 We assume that the government aims to improve people's health by providing health-
 151 care facilities to them and, while doing so, affects the productivity of human capital.

152 2.3 Market clearing and balanced growth path (BGP)

153 Labor market clearing conditions are $\tilde{H}_t = h_t[L_{t,Y} + L_{t,A} + L_{t,B} + L_{t,M}] = H_{t,Y} + H_{t,A} +$
 154 $H_{t,B} + H_{t,M}$, and $w_{t,Y} = w_{t,A} = w_{t,B} = w_{t,M} = w_t$. (8), (14), (15), (17) and (19) yield the
 155 demand for human capital in the final goods and applied research sectors as, respectively,

$$H_{t,Y} = \frac{A_t^{1-\phi_1} B_t^{-\mu_1} H_{t,M}^{-\varepsilon_1}}{\alpha \delta_1} \quad (20)$$

$$H_{t,A} = \tilde{H}_t - H_{t,B} - H_{t,M} - H_{t,Y}$$

$$\implies H_{t,A} = \frac{(1 - \tau)(1 + \beta)}{(1 + \beta + \xi)} h_t L_t - \frac{A_t^{1-\phi_1} B_t^{-\mu_1}}{\alpha \delta_1} \left[\frac{(1 - \tau_0)\tau(1 + \beta)}{(1 + \beta + \xi)} h_t L_t \right]^{-\varepsilon_1} \quad (21)$$

156

The development of new blueprints is then given by

$$A_{t+1} = \left(\frac{1 + \beta}{1 + \beta + \xi} \right)^{1+\varepsilon_1} [(1 - \tau_0)\tau]^{\varepsilon_1} (1 - \tau)\delta_1 A_t^{\phi_1} B_t^{\mu_1} (h_t L_t)^{1+\varepsilon_1} - \left(\frac{1 - \alpha}{\alpha} \right) A_t \quad (22)$$

157

As there is full depreciation of physical capital, capital market clearing implies that aggregate savings are used for physical capital accumulation and purchasing new blueprints

158

for intermediate goods production, i.e., $K_{t+1} = s_t L_t - p_{t,A}(A_{t+1} - A_t) = \frac{\beta(1-\tau)}{1+\beta+\xi} w_t h_t L_t -$

159

160

$p_{t,A}(A_{t+1} - A_t)$. (8), (11), (15), (19), (20), and (22) yield the aggregate physical capital

161

stock of the next period as

$$K_{t+1} = \left[\frac{\beta(1-\tau)(1-\alpha) [(1-\tau_0)\tau(1+\beta)]^{\alpha\varepsilon_1+(1-\alpha)\varepsilon_0}}{(1+\beta+\xi)^{1+\alpha\varepsilon_1+(1-\alpha)\varepsilon_0}} K_t^\alpha \right] \times \left[\left(\frac{A_t^{2-\phi_1} B_t^{-\mu_1}}{\alpha\delta_1} \right)^{-\alpha} A_t (h_t L_t)^{1+\alpha\varepsilon_1+(1-\alpha)\varepsilon_0} \right] - \alpha(1-\alpha) \frac{Y_t}{A_t} \left[\left(\frac{1+\beta}{1+\beta+\xi} \right)^{1+\varepsilon_1} [(1-\tau_0)\tau]^{\varepsilon_1} (1-\tau)\delta_1 A_t^{\phi_1} B_t^{\mu_1} (h_t L_t)^{1+\varepsilon_1} - \frac{A_t}{\alpha} \right] \quad (23)$$

162

(18) and (19) yield

$$B_{t+1} = \frac{\delta_2 \tau_0 (1 - \tau_0)^{\varepsilon_2} [\tau(1 + \beta)]^{1+\varepsilon_2}}{(1 + \beta + \xi)^{1+\varepsilon_2}} A_t^{\phi_2} B_t^{\mu_2} (h_t L_t)^{1+\varepsilon_2} + B_t \quad (24)$$

163

2.4 Analytical results for the long-run balanced growth path

164

We restrict ourselves to the following assumption to ensure the BGP and rule out the

165

empirically improbable scenario of hyper-exponential growth.

166

Assumption 1. *The intertemporal and intersectoral knowledge spillovers are given by*

167

$\phi_1 \in [0, 1)$, $\phi_2 \in [0, 1)$, $\mu_1 \in [0, 1)$, and $\mu_2 \in [0, 1)$. Moreover, it holds that $\phi_1 + \mu_1 < 1$

168

and $\phi_2 + \mu_2 < 1$.

169

The growth rates of blueprints, and the propositional knowledge are given by, respectively,

170

tively,

$$g_{t,A} \equiv \frac{A_{t+1} - A_t}{A_t} = \frac{(1 - \tau)(1 + \beta)^{1+\varepsilon_1} [(1 - \tau_0)\tau]^{\varepsilon_1}}{(1 + \beta + \xi)^{1+\varepsilon_1}} \delta_1 A_t^{\phi_1 - 1} B_t^{\mu_1} (h_t L_t)^{1+\varepsilon_1} - \frac{1}{\alpha} \quad (25)$$

$$g_{t,B} \equiv \frac{B_{t+1} - B_t}{B_t} = \frac{\delta_2 \tau_0 (1 - \tau_0)^{\varepsilon_2} [\tau(1 + \beta)]^{1+\varepsilon_2}}{(1 + \beta + \xi)^{1+\varepsilon_2}} B_t^{\mu_2 - 1} A_t^{\phi_2} (h_t L_t)^{1+\varepsilon_2} \quad (26)$$

171 The balanced growth factors (henceforth BGFs) of individual human capital, popu-
172 lation size, and aggregate human capital are given by, respectively,⁸

$$\tilde{h} \equiv \frac{h_{t+1}}{h_t} = \left(A_E \frac{\theta}{\eta} \right)^\nu \left(A_M \frac{\sigma}{\chi} \right)^{1-\nu} \frac{\psi}{(\xi - \theta - \sigma)} \quad (27)$$

$$\tilde{L} \equiv \frac{L_{t+1}}{L_t} = n_t = \frac{(\xi - \theta - \sigma)}{\psi(1 + \beta + \xi)} \quad (28)$$

$$\Omega \equiv \tilde{h}\tilde{L} = \frac{\left(A_E \frac{\theta}{\eta} \right)^\nu \left(A_M \frac{\sigma}{\chi} \right)^{1-\nu}}{(1 + \beta + \xi)} \quad (29)$$

173 From now on, we assume that $\psi \in \left(\frac{(\xi - \theta - \sigma)}{1 + \beta + \xi}, \frac{(\xi - \theta - \sigma)}{\left(A_E \frac{\theta}{\eta} \right)^\nu \left(A_M \frac{\sigma}{\chi} \right)^{1-\nu}} \right)$. This condition ensures
174 that the individual human capital as well as population will both grow over time. As a
175 result, $\Omega = \tilde{h}\tilde{L} > 1$ holds unambiguously. The following proposition introduces the main
176 results of this paper.

177 **Proposition 2.** (i) The BGFs of A , B , K , and Y are given by

$$\begin{aligned} \tilde{A} \equiv \frac{A_{t+1}}{A_t} &= \Omega^{\frac{(1+\varepsilon_1)(1-\mu_2)+(1+\varepsilon_2)\mu_1}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}}; & \tilde{B} \equiv \left(\frac{B_{t+1}}{B_t} \right) &= \Omega^{\frac{(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}}; \\ \tilde{K} \equiv \left(\frac{K_{t+1}}{K_t} \right) &= \Omega^{\left[\frac{(1-\mu_2)(2-\phi_1+\varepsilon_1)+\mu_1(1+\varepsilon_2-\phi_2)}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1} \right] + \varepsilon_0} = \tilde{Y} \equiv \left(\frac{Y_{t+1}}{Y_t} \right) \end{aligned}$$

178 (ii) These BGFs increase with aggregate human capital accumulation (Ω), and with
179 the knowledge spillovers μ_1 , μ_2 , ϕ_1 , ϕ_2 , and the strength of the effect of healthcare workers
180 that enhances the productivity of workers employed in the applied research sector (ε_1), and
181 the basic research sector (ε_2). The BGF of GDP also increases in the strength of the effect

⁸Note that $R_t = \alpha p_t = \alpha^2 \frac{Y_t}{K_t}$. As in the BGP, Y_t and K_t are growing at the same rate, R_t must be constant in the balanced growth path, i.e., $R_{t+1} = R_t = R, \forall t$.

182 of healthcare workers that enhances the productivity of workers employed in the final goods
183 sector (ε_0).

184 (iii) The BGFs are independent of the tax rates, $\tau\tau_0$, and $\tau(1 - \tau_0)$.

185 (iv) The BGF of individual human capital (\tilde{h}) increases with the utility weight of
186 children's education (θ), and health (σ), and decreases with the utility weight of the number
187 of children (ξ).

188 (v) The BGF of the population (\tilde{L}) decreases with θ and σ , and rises with ξ .

189 (vi) The BGF of aggregate human capital (Ω) increases with θ and σ , and decreases
190 with ξ .

191 (vii) The BGF of per capita GDP is given by

$$\tilde{y} = \frac{\tilde{Y}}{\tilde{L}} = \frac{\left(\frac{(A_E \frac{\theta}{\eta})^\nu (A_M \frac{\sigma}{\chi})^{1-\nu}}{(1+\beta+\xi)} \right)^{\left(\frac{(1-\mu_2)(2-\phi_1+\varepsilon_1)+\mu_1(1+\varepsilon_2-\phi_2)}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1} \right) + \varepsilon_0}}{\left(\frac{\xi - \theta - \sigma}{\psi(1+\beta+\xi)} \right)}$$

192 The per capita GDP growth factor increases with the utility weight of children's edu-
193 cation (θ) and health (σ), and decreases with the utility weight of the number of children
194 (ξ). It also increases with the knowledge spillovers μ_1 , μ_2 , ϕ_1 , ϕ_2 , and the strength of
195 the effect of healthcare workers that enhances the productivity of workers employed in the
196 final goods sector (ε_0), applied research sector (ε_1), and the basic research sector (ε_2).

197 *Proof.* See Appendix C. □

198 Furthermore, [Prettner and Werner \(2016\)](#) and [Baldanzi et al. \(2021\)](#) growth models
199 are nested as special cases within our very general model (see Remark 2).

200 **Remarks 2.** For $\varepsilon_0 = \varepsilon_1 = \varepsilon_2 = 0$, and $\nu = 1$ our model nests the [Prettner and Werner](#)
201 [\(2016\)](#) framework, while for $\varepsilon_0 = \varepsilon_1 = \varepsilon_2 = \mu_1 = \mu_2 = \phi_2 = 0$ our model nests the
202 [Baldanzi et al. \(2021\)](#) framework.

203 One implication of Proposition 2 is that human capital accumulation is a primary
204 factor for long-run economic growth. A second line of implication of this proposition
205 is although aggregate human capital accumulation is increasing with the desire for edu-
206 cated and healthy children, it is decreasing in population growth. Furthermore, higher
207 intertemporal and intersectoral knowledge spillovers and the strength of the effect of

208 healthcare employees in enhancing the productivity of workers employed in the applied
209 research, basic research and final goods sectors lead to a rise in balanced growth rates.

210 The effect of θ and σ on per capita GDP growth that emerges from our model is
211 significantly higher than Baldanzi et al. (2021). In Baldanzi et al.’s model the per capita
212 GDP growth factor is influenced only due to ϕ_1 . Contrarily, along with ϕ_1 , in our model
213 per capita GDP growth factor is influenced by intertemporal and intersectoral knowledge
214 spillovers like ϕ_2 , μ_1 and μ_2 . Another reason for the difference between Baldanzi et al.’s
215 findings and ours is because unlike them we incorporate the impact of healthcare em-
216 ployees in enhancing the productivity of workers in various sectors (i.e., ε_0 , ε_1 , ε_2). The
217 impact of θ on per capita GDP growth in our model is significantly larger than that of
218 Prettnner and Werner (2016), particularly when (i) $\nu = 1$ and (ii) $A_E\theta = A_M\sigma$. Inclusion
219 of ε_0 , ε_1 , and ε_2 plays a crucial role for this difference. We would also like to highlight
220 that unlike Prettnner and Werner (2016), a rise in σ increases the per capita GDP growth
221 factor in our model.

222 3 Conclusion

223 We present an R&D-based endogenous growth model emphasizing the role of patentable
224 applied research, publicly-funded basic research, and publicly-funded healthcare sectors.
225 One may also perceive this contribution as a step towards the reconciliation between two
226 recent contributions by Prettnner and Werner (2016) and Baldanzi et al. (2021). Our
227 second contribution is to illustrate healthcare workers’ role and long-run consequences
228 in enhancing productivity in various sectors. A future research problem might be in-
229 vestigating the varying impacts of healthcare workers and basic/applied research on the
230 medium-run growth during the transition. Furthermore, interested researchers may also
231 consider extending the proposed model in the context of a developing country.

References

- Andreoni, J. (1989). Giving with impure altruism: Applications to charity and ricardian equivalence. *Journal of political Economy*, 97(6):1447–1458.
- Baldanzi, A., Bucci, A., and Prettnner, K. (2021). Children’s health, human capital accumulation, and r&d-based economic growth. *Macroeconomic Dynamics*, 25(3):651–668.

- Coad, A., Segarra-Blasco, A., and Teruel, M. (2021). A bit of basic, a bit of applied? r&d strategies and firm performance. *The Journal of Technology Transfer*, 46(6):1758–1783.
- Czarnitzki, D. and Thorwarth, S. (2012). Productivity effects of basic research in low-tech and high-tech industries. *Research policy*, 41(9):1555–1564.
- Galor, O. (2005). From stagnation to growth: unified growth theory. *Handbook of economic growth*, 1:171–293.
- Galor, O. (2011). *Unified growth theory*. Princeton University Press.
- Galor, O. and Weil, D. N. (2000). Population, technology, and growth: From malthusian stagnation to the demographic transition and beyond. *American economic review*, 90(4):806–828.
- Gersbach, H., Schetter, U., and Schmassmann, S. (2023). From local to global: A theory of public basic research in a globalized world. *European Economic Review*, page 104530.
- Gersbach, H. and Schneider, M. T. (2015). On the global supply of basic research. *Journal of Monetary Economics*, 75:123–137.
- Gersbach, H., Schneider, M. T., and Schneller, O. (2013). Basic research, openness, and convergence. *Journal of Economic Growth*, 18:33–68.
- Gersbach, H., Sorger, G., and Amon, C. (2018). Hierarchical growth: Basic and applied research. *Journal of Economic Dynamics and Control*, 90:434–459.
- Jones, C. I. (1995). R&d-based models of economic growth. *Journal of political Economy*, 103(4):759–784.
- Kuhn, M. and Prettnner, K. (2016). Growth and welfare effects of health care in knowledge-based economies. *Journal of Health Economics*, 46:100–119.
- Madsen, J. B. (2016). Health, human capital formation and knowledge production: Two centuries of international evidence. *Macroeconomic Dynamics*, 20(4):909–953.
- Mulligan, K., Lenihan, H., Doran, J., and Roper, S. (2022). Harnessing the science base: Results from a national programme using publicly-funded research centres to reshape firms’ r&d. *Research Policy*, 51(4):104468.
- Prettnner, K. (2014). The non-monotonous impact of population growth on economic prosperity. *Economics Letters*, 124(1):93–95.
- Prettnner, K., Bloom, D. E., and Strulik, H. (2013). Declining fertility and economic well-being: do education and health ride to the rescue? *Labour economics*, 22:70–79.
- Prettnner, K. and Werner, K. (2016). Why it pays off to pay us well: The impact of basic research on economic growth and welfare. *Research Policy*, 45(5):1075–1090.
- Rivera, B. and Currais, L. (2004). Public health capital and productivity in the spanish regions: A dynamic panel data model. *World Development*, 32(5):871–885.
- Romer, P. M. (1990). Endogenous technological change. *Journal of political Economy*, 98(5, Part 2):S71–S102.

Strulik, H., Prettnner, K., and Prskawetz, A. (2013). The past and future of knowledge-based growth. *Journal of Economic Growth*, 18:411–437.

Toole, A. A. (2012). The impact of public basic research on industrial innovation: Evidence from the pharmaceutical industry. *Research Policy*, 41(1):1–12.

Appendix

A Derivation of the optimal values of c_t , s_t , n_t , e_t and m_t

Using (1) and (2) we set the Lagrangian as

$$\begin{aligned} \mathcal{L} = & \ln c_t + \beta \ln [(R_{t+1} - 1)s_t] + \xi \ln n_t + \theta \ln e_t + \sigma \ln m_t \\ & + \lambda [(1 - \tau)(1 - \psi n_t - \eta e_t n_t - \chi m_t n_t)w_t h_t - c_t - s_t] \end{aligned}$$

The first-order conditions are given by

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \implies \frac{1}{c_t} = \lambda \quad (\text{A.1})$$

$$\frac{\partial \mathcal{L}}{\partial s_t} = 0 \implies \frac{\beta}{s_t} = \lambda \quad (\text{A.2})$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = 0 \implies \frac{\xi}{n_t} = \lambda(1 - \tau)(\psi + \eta e_t + \chi m_t)w_t h_t \quad (\text{A.3})$$

$$\frac{\partial \mathcal{L}}{\partial e_t} = 0 \implies \frac{\theta}{e_t} = \lambda(1 - \tau)\eta n_t w_t h_t \quad (\text{A.4})$$

$$\frac{\partial \mathcal{L}}{\partial m_t} = 0 \implies \frac{\sigma}{m_t} = \lambda(1 - \tau)\chi n_t w_t h_t \quad (\text{A.5})$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \implies (1 - \tau)(1 - \psi n_t - \eta e_t n_t - \chi m_t n_t)w_t h_t - c_t - s_t = 0 \quad (\text{A.6})$$

Dividing (A.4) by (A.5) we obtain

$$\chi m_t = \frac{\eta \sigma}{\theta} e_t \quad (\text{A.7})$$

Dividing (A.1) by (A.2) we obtain

$$s_t = \beta c_t \quad (\text{A.8})$$

Dividing (A.3) by (A.4) and using (A.7) we obtain

$$e_t = \frac{\theta \psi}{\eta(\xi - \theta - \sigma)} \quad (\text{A.9})$$

Inserting the value of e_t into (A.7) we get

$$m_t = \frac{\sigma \psi}{\chi(\xi - \theta - \sigma)} \quad (\text{A.10})$$

Dividing (A.3) by (A.1) and rearranging it we obtain

$$(1 - \tau)(\psi + \eta e_t + \chi m_t)n_t w_t h_t = \xi c_t \quad (\text{A.11})$$

Inserting (A.11) into (A.6) we get

$$\begin{aligned} (1 - \tau)w_t h_t - (1 - \tau)(\psi + \eta e_t + \chi m_t)n_t w_t h_t - c_t - s_t &= 0 \\ \implies c_t &= \frac{(1 - \tau)w_t h_t}{1 + \beta + \xi} \end{aligned} \quad (\text{A.12})$$

Therefore,

$$s_t = \frac{\beta(1 - \tau)w_t h_t}{1 + \beta + \xi} \quad (\text{A.13})$$

Inserting the values of c_t , s_t , e_t and m_t into (A.6) and rearranging it we obtain

$$n_t = \frac{\xi - \theta - \sigma}{\psi(1 + \beta + \xi)} \quad (\text{A.14})$$

B Proof of Proposition 1

The partial derivatives of fertility n_t with respect to ξ , θ , and σ are

$$\begin{aligned} \frac{\partial n_t}{\partial \xi} &= \frac{1 + \beta + \theta + \sigma}{\psi(1 + \beta + \xi)^2} > 0 & \frac{\partial n_t}{\partial \theta} &= -\frac{1}{\psi(1 + \beta + \xi)} < 0 \\ \frac{\partial n_t}{\partial \sigma} &= -\frac{1}{\psi(1 + \beta + \xi)} < 0 \end{aligned} \quad (\text{B.1})$$

The partial derivatives of the individual human capital h_{t+1} with respect to ξ , θ , and σ are

$$\begin{aligned} \frac{\partial h_{t+1}}{\partial \xi} &= -\left(A_E \frac{\theta}{\eta}\right)^\nu \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu} \frac{\psi}{(\xi - \theta - \sigma)^2} h_t < 0 \\ \frac{\partial h_{t+1}}{\partial \theta} &= \frac{\left(A_E \frac{\theta}{\eta}\right)^\nu \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu} \psi h_t}{(\xi - \theta - \sigma)} \left[\frac{\nu A_E}{\eta} \left(A_E \frac{\theta}{\eta}\right)^{-1} + \frac{1}{(\xi - \theta - \sigma)} \right] > 0 \\ \frac{\partial h_{t+1}}{\partial \sigma} &= \frac{\left(A_E \frac{\theta}{\eta}\right)^\nu \left(A_M \frac{\sigma}{\chi}\right)^{1-\nu} \psi h_t}{(\xi - \theta - \sigma)} \left[\frac{(1 - \nu) A_M}{\chi} \left(A_M \frac{\sigma}{\chi}\right)^{-1} + \frac{1}{(\xi - \theta - \sigma)} \right] > 0 \end{aligned} \quad (\text{B.2})$$

C Proof of Proposition 2

(i) Growth rates of the endogenous variables have to be constant along the balanced growth path. therefore,

$$\frac{g_{t+1,A} - g_{t,A}}{g_{t,A}} = 0 \implies g_{t+1,A} = g_{t,A}$$

$$\begin{aligned}
&\implies \frac{(1-\tau)(1+\beta)^{1+\varepsilon_1} [(1-\tau_0)\tau]^{\varepsilon_1}}{(1+\beta+\xi)^{1+\varepsilon_1}} \delta_1 A_{t+1}^{\phi_1-1} B_{t+1}^{\mu_1} (h_{t+1} L_{t+1})^{1+\varepsilon_1} - \frac{1}{\alpha} \\
&= \frac{(1-\tau)(1+\beta)^{1+\varepsilon_1} [(1-\tau_0)\tau]^{\varepsilon_1}}{(1+\beta+\xi)^{1+\varepsilon_1}} \delta_1 A_t^{\phi_1-1} B_t^{\mu_1} (h_t L_t)^{1+\varepsilon_1} - \frac{1}{\alpha} \\
&\implies \left(\frac{h_{t+1}}{h_t} \frac{L_{t+1}}{L_t} \right)^{1+\varepsilon_1} \left(\frac{A_{t+1}}{A_t} \right)^{\phi_1-1} \left(\frac{B_{t+1}}{B_t} \right)^{\mu_1} = 1 \\
&\Omega^{1+\varepsilon_1} \left(\frac{A_{t+1}}{A_t} \right)^{\phi_1-1} \left(\frac{B_{t+1}}{B_t} \right)^{\mu_1} = 1 \tag{C.1}
\end{aligned}$$

$$\frac{g_{t+1,B} - g_{t,B}}{g_{t,B}} = 0 \implies g_{t,B} = g_{t-1,B}$$

$$\begin{aligned}
&\implies \frac{\delta_2 \tau_0 (1-\tau_0)^{\varepsilon_2} [\tau(1+\beta)]^{1+\varepsilon_2}}{(1+\beta+\xi)^{1+\varepsilon_2}} B_{t+1}^{\mu_2-1} A_{t+1}^{\phi_2} (h_{t+1} L_{t+1})^{1+\varepsilon_2} \\
&= \frac{\delta_2 \tau_0 (1-\tau_0)^{\varepsilon_2} [\tau(1+\beta)]^{1+\varepsilon_2}}{(1+\beta+\xi)^{1+\varepsilon_2}} B_t^{\mu_2-1} A_t^{\phi_2} (h_t L_t)^{1+\varepsilon_2} \\
&\implies \left(\frac{h_{t+1}}{h_t} \frac{L_{t+1}}{L_t} \right)^{1+\varepsilon_2} \left(\frac{A_{t+1}}{A_t} \right)^{\phi_2} \left(\frac{B_{t+1}}{B_t} \right)^{\mu_2-1} = 1 \\
&\left(\frac{B_{t+1}}{B_t} \right) = \Omega^{\frac{1+\varepsilon_2}{1-\mu_2}} \left(\frac{A_{t+1}}{A_t} \right)^{\frac{\phi_2}{1-\mu_2}} \tag{C.2}
\end{aligned}$$

Inserting (C.2) into (C.1) we obtain

$$\tilde{A} \equiv \left(\frac{A_{t+1}}{A_t} \right) = \Omega^{\frac{(1+\varepsilon_1)(1-\mu_2)+(1+\varepsilon_2)\mu_1}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}} \tag{C.3}$$

Therefore,

$$\tilde{B} \equiv \left(\frac{B_{t+1}}{B_t} \right) = \Omega^{\frac{(1+\varepsilon_2)(1-\phi_1)+(1+\varepsilon_1)\phi_2}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1}} \tag{C.4}$$

In the BGP, Y_t and K_t must grow at the same rate. Therefore, equation (11) suggests

$$\tilde{Y} \equiv \left(\frac{Y_{t+1}}{Y_t} \right) = \left(\frac{A_{t+1}}{A_t} \right)^{2-\phi_1} \left(\frac{B_{t+1}}{B_t} \right)^{-\mu_1} \left(\frac{H_{t+1,M}}{H_{t,M}} \right)^{\varepsilon_0-\varepsilon_1} = \left(\frac{K_{t+1}}{K_t} \right) \equiv \tilde{K} \tag{C.5}$$

$$\implies \left(\frac{Y_{t+1}}{Y_t} \right) \equiv \tilde{Y} = \tilde{K} \equiv \left(\frac{K_{t+1}}{K_t} \right) = \Omega^{\left[\frac{(1-\mu_2)(2-\phi_1+\varepsilon_1)+\mu_1(1+\varepsilon_2-\phi_2)}{(1-\phi_1)(1-\mu_2)-\phi_2\mu_1} \right] + \varepsilon_0} \tag{C.6}$$

(ii)

$$\frac{\partial \tilde{A}}{\partial \mu_1} = \tilde{A} \ln(\Omega) \frac{(1-\mu_2) [(1+\varepsilon_2)(1-\phi_1) + (1+\varepsilon_1)\phi_2]}{[(1-\phi_1)(1-\mu_2) - \phi_2\mu_1]^2} > 0$$

$$\frac{\partial \tilde{B}}{\partial \mu_1} = \tilde{B} \ln(\Omega) \frac{\phi_2 [(1+\varepsilon_2)(1-\phi_1) + (1+\varepsilon_1)\phi_2]}{[(1-\phi_1)(1-\mu_2) - \phi_2\mu_1]^2} \geq 0$$

$$\frac{\partial \tilde{K}}{\partial \mu_1} = \tilde{K} \ln(\Omega) \frac{(1-\mu_2) [(1+\varepsilon_2)(1-\phi_1) + (1+\varepsilon_1)\phi_2]}{[(1-\phi_1)(1-\mu_2) - \phi_2\mu_1]^2} > 0$$

$$\begin{aligned}
\frac{\partial \tilde{A}}{\partial \mu_2} &= \tilde{A} \ln(\Omega) \frac{\mu_1 [(1 + \varepsilon_2)(1 - \phi_1) + (1 + \varepsilon_1)\phi_2]}{[(1 - \phi_1)(1 - \mu_2) - \phi_2\mu_1]^2} \geq 0 \\
\frac{\partial \tilde{B}}{\partial \mu_2} &= \tilde{B} \ln(\Omega) \frac{(1 - \phi_1) [(1 + \varepsilon_2)(1 - \phi_1) + (1 + \varepsilon_1)\phi_2]}{[(1 - \phi_1)(1 - \mu_2) - \phi_2\mu_1]^2} > 0 \\
\frac{\partial \tilde{K}}{\partial \mu_2} &= \tilde{K} \ln(\Omega) \frac{\mu_1 [(1 + \varepsilon_2)(1 - \phi_1) + (1 + \varepsilon_1)\phi_2]}{[(1 - \phi_1)(1 - \mu_2) - \phi_2\mu_1]^2} \geq 0 \\
\frac{\partial \tilde{A}}{\partial \phi_1} &= \tilde{A} \ln(\Omega) \frac{(1 - \mu_2) [(1 + \varepsilon_2)\mu_1 + (1 + \varepsilon_1)(1 - \mu_2)]}{[(1 - \phi_1)(1 - \mu_2) - \phi_2\mu_1]^2} > 0 \\
\frac{\partial \tilde{B}}{\partial \phi_1} &= \tilde{B} \ln(\Omega) \frac{\phi_2 [(1 + \varepsilon_2)\mu_1 + (1 + \varepsilon_1)(1 - \mu_2)]}{[(1 - \phi_1)(1 - \mu_2) - \phi_2\mu_1]^2} \geq 0 \\
\frac{\partial \tilde{K}}{\partial \phi_1} &= \tilde{K} \ln(\Omega) \frac{(1 - \mu_2) [(1 + \varepsilon_2)\mu_1 + (1 + \varepsilon_1)(1 - \mu_2)]}{[(1 - \phi_1)(1 - \mu_2) - \phi_2\mu_1]^2} > 0 \\
\frac{\partial \tilde{A}}{\partial \phi_2} &= \tilde{A} \ln(\Omega) \frac{\mu_1 [(1 + \varepsilon_2)\mu_1 + (1 + \varepsilon_1)(1 - \mu_2)]}{[(1 - \phi_1)(1 - \mu_2) - \phi_2\mu_1]^2} \geq 0 \\
\frac{\partial \tilde{B}}{\partial \phi_2} &= \tilde{B} \ln(\Omega) \frac{(1 - \phi_1) [(1 + \varepsilon_2)\mu_1 + (1 + \varepsilon_1)(1 - \mu_2)]}{[(1 - \phi_1)(1 - \mu_2) - \phi_2\mu_1]^2} > 0 \\
\frac{\partial \tilde{K}}{\partial \phi_2} &= \tilde{K} \ln(\Omega) \frac{\mu_1 [(1 + \varepsilon_2)\mu_1 + (1 + \varepsilon_1)(1 - \mu_2)]}{[(1 - \phi_1)(1 - \mu_2) - \phi_2\mu_1]^2} \geq 0 \\
\frac{\partial \tilde{A}}{\partial \varepsilon_0} &= 0 \quad \frac{\partial \tilde{B}}{\partial \varepsilon_0} = 0 \quad \frac{\partial \tilde{K}}{\partial \varepsilon_0} = \tilde{K} \ln(\Omega) > 0 \\
\frac{\partial \tilde{A}}{\partial \varepsilon_1} &= \tilde{A} \ln(\Omega) \frac{(1 - \mu_2)}{[(1 - \phi_1)(1 - \mu_2) - \phi_2\mu_1]} > 0 \\
\frac{\partial \tilde{B}}{\partial \varepsilon_1} &= \tilde{B} \ln(\Omega) \frac{\phi_2}{[(1 - \phi_1)(1 - \mu_2) - \phi_2\mu_1]} \geq 0 \\
\frac{\partial \tilde{K}}{\partial \varepsilon_1} &= \tilde{K} \ln(\Omega) \frac{(1 - \mu_2)}{[(1 - \phi_1)(1 - \mu_2) - \phi_2\mu_1]} > 0 \\
\frac{\partial \tilde{A}}{\partial \varepsilon_2} &= \tilde{A} \ln(\Omega) \frac{\mu_1}{[(1 - \phi_1)(1 - \mu_2) - \phi_2\mu_1]} \geq 0 \\
\frac{\partial \tilde{B}}{\partial \varepsilon_2} &= \tilde{B} \ln(\Omega) \frac{(1 - \phi_1)}{[(1 - \phi_1)(1 - \mu_2) - \phi_2\mu_1]} > 0 \\
\frac{\partial \tilde{K}}{\partial \varepsilon_2} &= \tilde{K} \ln(\Omega) \frac{\mu_1}{[(1 - \phi_1)(1 - \mu_2) - \phi_2\mu_1]} \geq 0
\end{aligned}$$

(iii)

$$\begin{aligned}
\frac{\partial \tilde{A}}{\partial \tau \tau_0} &= \frac{\partial \tilde{B}}{\partial \tau \tau_0} = \frac{\partial \tilde{K}}{\partial \tau \tau_0} = 0 \\
\frac{\partial \tilde{A}}{\partial \tau (1 - \tau_0)} &= \frac{\partial \tilde{B}}{\partial \tau (1 - \tau_0)} = \frac{\partial \tilde{K}}{\partial \tau (1 - \tau_0)} = 0
\end{aligned}$$

(iv)

$$\begin{aligned}\frac{\partial \tilde{h}}{\partial \xi} &= - \left(A_E \frac{\theta}{\eta} \right)^\nu \left(A_M \frac{\sigma}{\chi} \right)^{1-\nu} \frac{\psi}{(\xi - \theta - \sigma)^2} < 0 \\ \frac{\partial \tilde{h}}{\partial \theta} &= \frac{\left(A_E \frac{\theta}{\eta} \right)^\nu \left(A_M \frac{\sigma}{\chi} \right)^{1-\nu} \psi}{(\xi - \theta - \sigma)} \left[\frac{\nu A_E}{\eta} \left(A_E \frac{\theta}{\eta} \right)^{-1} + \frac{1}{(\xi - \theta - \sigma)} \right] > 0 \\ \frac{\partial \tilde{h}}{\partial \sigma} &= \frac{\left(A_E \frac{\theta}{\eta} \right)^\nu \left(A_M \frac{\sigma}{\chi} \right)^{1-\nu} \psi}{(\xi - \theta - \sigma)} \left[\frac{(1-\nu) A_M}{\chi} \left(A_M \frac{\sigma}{\chi} \right)^{-1} + \frac{1}{(\xi - \theta - \sigma)} \right] > 0\end{aligned}$$

(v)

$$\frac{\partial \tilde{L}}{\partial \xi} = \frac{1 + \beta + \theta + \sigma}{\psi(1 + \beta + \xi)^2} > 0; \quad \frac{\partial \tilde{L}}{\partial \theta} = \frac{-1}{\psi(1 + \beta + \xi)} < 0; \quad \frac{\partial \tilde{L}}{\partial \sigma} = \frac{-1}{\psi(1 + \beta + \xi)} < 0$$

(vi)

$$\begin{aligned}\frac{\partial \Omega}{\partial \xi} &= - \frac{\left(A_E \frac{\theta}{\eta} \right)^\nu \left(A_M \frac{\sigma}{\chi} \right)^{1-\nu}}{(1 + \beta + \xi)^2} < 0 \\ \frac{\partial \Omega}{\partial \theta} &= \frac{\frac{\nu A_E}{\eta} \left(A_E \frac{\theta}{\eta} \right)^{\nu-1} \left(A_M \frac{\sigma}{\chi} \right)^{1-\nu}}{(1 + \beta + \xi)} > 0 \\ \frac{\partial \Omega}{\partial \sigma} &= \frac{\frac{(1-\nu) A_M}{\chi} \left(A_E \frac{\theta}{\eta} \right)^\nu \left(A_M \frac{\sigma}{\chi} \right)^{-\nu}}{(1 + \beta + \xi)} > 0\end{aligned}$$

(vii)

$$\begin{aligned}\frac{\partial \tilde{y}}{\partial \xi} &= \tilde{y} \left[\frac{-1}{(1 + \beta + \xi)} \right] \left[\frac{(1 - \mu_2)(2 - \phi_1 + \varepsilon_1) + \mu_1(1 + \varepsilon_2 - \phi_2)}{(1 - \phi_1)(1 - \mu_2) - \phi_2 \mu_1} + \varepsilon_0 + \frac{1 + \beta + \theta + \sigma}{\xi - \theta - \sigma} \right] < 0 \\ \frac{\partial \tilde{y}}{\partial \theta} &= \tilde{y} \left[\left(\frac{(1 - \mu_2)(2 - \phi_1 + \varepsilon_1) + \mu_1(1 + \varepsilon_2 - \phi_2)}{(1 - \phi_1)(1 - \mu_2) - \phi_2 \mu_1} + \varepsilon_0 \right) \left(\frac{\nu}{\theta} \right) + \left(\frac{1}{\xi - \theta - \sigma} \right) \right] > 0 \\ \frac{\partial \tilde{y}}{\partial \sigma} &= \tilde{y} \left[\left(\frac{(1 - \mu_2)(2 - \phi_1 + \varepsilon_1) + \mu_1(1 + \varepsilon_2 - \phi_2)}{(1 - \phi_1)(1 - \mu_2) - \phi_2 \mu_1} + \varepsilon_0 \right) \left(\frac{1 - \nu}{\sigma} \right) + \left(\frac{1}{\xi - \theta - \sigma} \right) \right] > 0 \\ \frac{\partial \tilde{y}}{\partial \mu_1} &= \tilde{y} \ln(\Omega) \frac{(1 - \mu_2) [(1 + \varepsilon_2)(1 - \phi_1) + (1 + \varepsilon_1)\phi_2]}{[(1 - \phi_1)(1 - \mu_2) - \phi_2 \mu_1]^2} > 0 \\ \frac{\partial \tilde{y}}{\partial \mu_2} &= \tilde{y} \ln(\Omega) \frac{\mu_1 [(1 + \varepsilon_2)(1 - \phi_1) + (1 + \varepsilon_1)\phi_2]}{[(1 - \phi_1)(1 - \mu_2) - \phi_2 \mu_1]^2} \geq 0 \\ \frac{\partial \tilde{y}}{\partial \phi_1} &= \tilde{y} \ln(\Omega) \frac{(1 - \mu_2) [(1 + \varepsilon_2)\mu_1 + (1 + \varepsilon_1)(1 - \mu_2)]}{[(1 - \phi_1)(1 - \mu_2) - \phi_2 \mu_1]^2} > 0 \\ \frac{\partial \tilde{y}}{\partial \phi_2} &= \tilde{y} \ln(\Omega) \frac{\mu_1 [(1 + \varepsilon_2)\mu_1 + (1 + \varepsilon_1)(1 - \mu_2)]}{[(1 - \phi_1)(1 - \mu_2) - \phi_2 \mu_1]^2} \geq 0 \\ \frac{\partial \tilde{y}}{\partial \varepsilon_0} &= \tilde{y} \ln(\Omega) > 0\end{aligned}$$

$$\frac{\partial \tilde{y}}{\partial \varepsilon_1} = \tilde{y} \ln(\Omega) \frac{(1 - \mu_2)}{[(1 - \phi_1)(1 - \mu_2) - \phi_2 \mu_1]} > 0$$
$$\frac{\partial \tilde{y}}{\partial \varepsilon_2} = \tilde{y} \ln(\Omega) \frac{\mu_1}{[(1 - \phi_1)(1 - \mu_2) - \phi_2 \mu_1]} \geq 0$$