Mathematical Analysis of an Industry When Cost of Principal Raw Materials Increase: A Nonlinear Budget Constraint Attempt

Mohajan, Devajit

Department of Civil Engineering, Chittagong University of Engineering Technology, Chittagong, Bangladesh

12 June 2023

Mathematical Analysis of an Industry When Cost of Principal Raw Materials Increase: A Nonlinear Budget Constraint Attempt

Devajit Mohajan

Department of Civil Engineering, Chittagong University of Engineering & Technology, Chittagong, Bangladesh

Email: devajit1402@gmail.com

Mobile: +8801866207021

Haradhan Kumar Mohajan

Department of Mathematics, Premier University, Chittagong, Bangladesh

Email: haradhan1971@gmail.com

Mobile: +8801716397232

Abstract

During profit maximization procedure an industry faces various difficulties; and increase of cost of principal raw material is one of them that happen frequently. In this study an attempt is taken to discuss economic effects on various inputs when the cost of principal raw material is increased. The economic sensitivity analysis of various inputs on the basis of principal raw material is an essential step for an industry when the industry is on the profit maximization track. This article deals with four variable inputs, such as capital, labor, principal raw materials, and irregular inputs of an industry, where nonlinear budget constraint is considered. This study stresses on mathematical formalities to show the economic predictions scientifically.

Keywords: Profit maximization, nonlinear budget constraint, principal raw material

JEL Codes: C61, C67, D21, D24, H32, I31

1. Introduction

Once global economy was very poor and did not depend on mathematics. But the situation is changed from the 19th century and at present mathematical modeling becomes an essential tool in economics [Samuelson, 1947]. Mathematics not only covers economics, but also covers many fields of social sciences, such as psychology, sociology, political science, etc. [Carter, 2001]. We do not demand confidently that the global economy is sustainable always. But every industry can proceed very carefully for the economic development and sustainability [Ferdous & Mohajan, 2022]. The aim of every firm is to achieve maximum profit with minimum cost [Eaton & Lipsey, 1975]. Scientific economic predictions help the industry to develop economic structure for future production [Mohajan & Mohajan, 2022b].

Actually the method of Lagrange multiplier is a valuable and dominant technique in multivariable calculus [Baxley & Moorhouse, 1984; Islam et al., 2010]. We have applied it to obtain higher dimensional unconstrained problem from the lower dimensional constrained one. Cobb-Douglas production function is used here to operate the determinant of 6×6 bordered Hessian matrix, and 6×6 Jacobian properly. We have also applied Implicit Function Theorem of multivariable calculus [Cobb & Douglas, 1928].

2. Literature Review

Literature review is considered as an introductory section that shows the works of previous researchers [Polit & Hungler, 2013]. Two US professors Charles W. Cobb (1875-1949) and Paul H. Douglas (1892-1976) have introduced concept of production functions, which is known as Cobb-Douglas production function [Cobb & Douglas, 1928]. Another two US scholars John V. Baxley and John C. Moorhouse have developed the optimization problems in mathematical economics [Baxley & Moorhouse, 1984].

Professor Jamal Nazrul Islam (1939-2013) and his coworkers have studied profit maximization for social welfare [Islam et al., 2010, 2011]. Pahlaj Moolio and his coauthors have used the Cobb-Douglas production functions in the optimization problems [Moolio et al., 2009]. Lia Roy and her coworkers have discussed cost minimization policy for the sustainable development of an industry [Roy et al., 2021]. Jannatul Ferdous and Haradhan Kumar Mohajan have discussed on a profit maximization problem [Ferdous & Mohajan, 2022]. In a series of papers Devajit Mohajan and

Haradhan Kumar Mohajan have studied on optimization analysis [Mohajan & Mohajan, 2022a-f, 2023a-g].

3. Research Methodology of the Study

Research is a systematic inquiry where researchers collect data and information; later analyze and interpret them efficiently to give a rational and sensible conclusion [Groh, 2018]. It searches for truth and tries to advance the stock of human knowledge [Pandey & Pandey, 2015]. It is the careful consideration of a problem that uses scientific methods to explain, predict, and control the observed phenomenon [Babbie, 2017]. Methodology is a guideline for the accomplishment of a good research [Kothari, 2008]. It consents the reader to critically evaluate overall validity and reliability of a study [Howell, 2012]. Therefore, research methodology is the specific procedures that are used to identify, select, process, and analyze materials related to the research matters [Somekh & Lewin, 2005; Schwandt, 2014].

We have used Cobb-Douglas production function to discuss economic predictions. To obtain the results we have stressed on two matrix algebras: determinant of 5×5 bordered Hessian and Jacobian matrices [Mohajan, 2017b, 2018a]. To prepare this paper we have depended on the secondary data sources. We have unsparingly consulted valuable articles and books of famous authors. We have also collected some materials from the internet and websites [Islam et al., 2009a,b; Mohajan, 2017a, 2018b, 2020].

4. Objective of the Study

The main objective of this article is to discuss the economic effects of various inputs when the cost of principal raw material is increased. Other minor objectives of the study are as follows:

- to explain the mathematical terms more clearly and elaborately, and
- to show the optimum results smoothly.

5. Lagrangian Function

Let us consider that an industrial firm is willing to make a maximum profit from its products. Let the firm uses E_1 amount of capital, E_2 quantity of labor, E_3 quantity of principal raw materials, and E_4 quantity of irregular input for its annual production. Let us consider the Cobb-Douglas production function $f\left(E_1, E_2, E_3, E_4\right)$ as a profit function for our model [Cobb & Douglas, 1928; Islam et al., 2011; Mohajan & Mohajan, 2022a],

$$P(E_1, E_2, E_3, E_4) = f\left(E_1, E_2, E_3, E_4\right) = AE_1^{\alpha} E_2^{\beta} E_3^{\gamma} E_4^{\delta}, \qquad (1)$$

where A is the efficiency parameter that reflects the level of technology, i.e., technical process, economic system, etc., which represents total factor productivity. Moreover, A reflects the skill and efficient level of the workforce. Here α, β, γ, and δ are parameters; α indicates the output of elasticity of capital measures the percentage change in $P(E_1, E_2, E_3, E_4)$ for 1% change in E_1, while E_2, E_3, and E_4 are held constants. Similarly, β indicates the output of elasticity of labor, γ indicates the output of elasticity of principal raw materials, and δ indicates the output of elasticity of irregular input. Now these four parameters α, β, γ, and δ must satisfy the following four inequalities [Islam et al., 2010; Mohajan, 2022]:

$$0 < \alpha < 1,\ 0 < \beta < 1,\ 0 < \gamma < 1, \text{ and } 0 < \delta < 1. \qquad (2)$$

A strict Cobb-Douglas production function, in which $\forall = \alpha + \beta + \gamma + \delta < 1$ indicates decreasing returns to scale, $\forall = 1$ indicates constant returns to scale, and $\forall > 1$ indicates increasing returns to scale. Now we consider that the profit function is subject to a nonlinear budget constraint as [Mohajan & Mohajan, 2022a, 2023b],

$$B(E_1, E_2, E_3, E_4) = kE_1 + lE_2 + mE_3 + n(E_4)E_4, \qquad (3)$$

where k is rate of interest or services of capital per unit of capital E_1; l is the wage rate per unit of labor E_2; m is the cost per unit of principal raw material E_3; and n is the cost per unit of irregular input E_4. In nonlinear budget equation (3) we consider [Mohajan & Mohajan, 2023c],

$$n(E_4) = n_0 E_4 - n_0, \qquad (4)$$

where n_0 being the discounted price of the irregular input E_4. Therefore, the nonlinear budget constraint (3) takes the form [Mohajan, 2021a; Mohajan & Mohajan, 2023d];

$$B(E_1, E_2, E_3, E_4) = kE_1 + lE_2 + mE_3 + n_0 E_4^2 - n_0 E_4. \qquad (5)$$

We now formulate the maximization problem for the profit function (1) in terms of single Lagrange multiplier λ by defining the Lagrangian function $R\left(E_1, E_2, E_3, E_4, \lambda\right)$ as [Moolio et al., 2009; Mohajan & Mohajan, 2023e],

$$R\left(E_1, E_2, E_3, E_4, \lambda\right) = AE_1^{\alpha} E_2^{\beta} E_3^{\gamma} E_4^{\delta} + \lambda\left\{B(E_1, E_2, E_3, E_4) - kE_1 - lE_2 - mE_3 - n_0 E_4^2 + n_0 E_4\right\}. \qquad (6)$$

Relation (6) is a 5-dimensional unconstrained problem that is obtained from (1) and 4-dimensional constrained problem (3), where Lagrange multiplier λ, is considered as a device in our profit maximization model.

6. Analysis on Four Inputs

For maximization, first order differentiation equals to zero; then from (6) we can write [Mohajan, 2021b; Mohajan & Mohajan, 2022e],

$$R_\lambda = B - kE_1 - lE_2 - mE_3 - n_0 E_4^2 + n_0 E_4 = 0, \tag{7a}$$

$$R_1 = \alpha A E_1^{\alpha-1} E_2^\beta E_3^\gamma E_4^\delta - \lambda k = 0, \tag{7b}$$

$$R_2 = \beta A E_1^\alpha E_2^{\beta-1} E_3^\gamma E_4^\delta - \lambda l = 0, \tag{7c}$$

$$R_3 = \gamma A E_1^\alpha E_2^\beta E_3^{\gamma-1} E_4^\delta - \lambda m = 0, \tag{7d}$$

$$R_4 = \delta A E_1^\alpha E_2^\beta E_3^\gamma E_4^{\delta-1} - \lambda n_0 \left(2E_4 - 1\right) = 0, \tag{7e}$$

where, $\frac{\partial R}{\partial \lambda} = R_\lambda$, $\frac{\partial R}{\partial E_1} = R_1$, $\frac{\partial R}{\partial E_2} = R_2$, etc. indicate first-order partial differentiations of multivariate Lagrangian function.

Using equations (2) to (7) we can decide the values of E_1, E_2, E_3, and E_4 as follows [Ferdous & Mohajan 2022; Mohajan & Mohajan, 2022f]:

$$E_1 = \frac{\alpha B}{k \forall}, \tag{8a}$$

$$E_2 = \frac{\beta B}{l \forall}, \tag{8b}$$

$$E_3 = \frac{\gamma B}{m \forall}, \tag{8c}$$

$$E_4 = \frac{\delta B}{n \forall}. \tag{8d}$$

7. Bordered Hessian Matrix Analysis

Let us consider the determinant of the 5×5 bordered Hessian matrix as [Islam et al. 2011; Mohajan & Mohajan, 2022g],

$$
|H| = \begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 & -B_4 \\ -B_1 & R_{11} & R_{12} & R_{13} & R_{14} \\ -B_2 & R_{21} & R_{22} & R_{23} & R_{24} \\ -B_3 & R_{31} & R_{32} & R_{33} & R_{34} \\ -B_4 & R_{41} & R_{42} & R_{43} & R_{44} \end{vmatrix}. \tag{9}
$$

Taking first-order partial differentiations of (5) we get,

$$
B_1 = k,\ B_2 = l,\ B_3 = m, \text{ and } \ B_4 = 2n_0 E_4 - n_0. \tag{10}
$$

Taking second-order and cross-partial derivatives of (6) we get [Roy et al., 2021; Mohajan & Mohajan, 2023a],

$$
\begin{aligned}
&R_{11} = \alpha(\alpha-1)AE_1^{\alpha-2}E_2^{\beta}E_3^{\gamma}E_4^{\delta}, \\
&R_{22} = \beta(\beta-1)AE_1^{\alpha}E_2^{\beta-2}E_3^{\gamma}E_4^{\delta}, \\
&R_{33} = \gamma(\gamma-1)AE_1^{\alpha}E_2^{\beta}E_3^{\gamma-2}E_4^{\delta}, \\
&R_{44} = \delta(\delta-1)AE_1^{\alpha}E_2^{\beta}E_3^{\gamma}E_4^{\delta-2}, \\
&R_{12} = R_{21} = \alpha\beta AE_1^{\alpha-1}E_2^{\beta-1}E_3^{\gamma}E_4^{\delta}, \\
&R_{13} = R_{31} = \alpha\gamma AE_1^{\alpha-1}E_2^{\beta}E_3^{\gamma-1}E_4^{\delta}, \\
&R_{14} = R_{41} = \alpha\delta AE_1^{\alpha-1}E_2^{\beta}E_3^{\gamma}E_4^{\delta-1}, \\
&R_{23} = R_{32} = \beta\gamma AE_1^{\alpha}E_2^{\beta-1}E_3^{\gamma-1}E_4^{\delta}, \\
&R_{24} = R_{42} = \beta\delta AE_1^{\alpha}E_2^{\beta-1}E_3^{\gamma}E_4^{\delta-1}, \\
&R_{34} = R_{43} = \gamma\delta AE_1^{\alpha}E_2^{\beta}E_3^{\gamma-1}E_4^{\delta-1}.
\end{aligned} \tag{11}
$$

where $\frac{\partial^2 R}{\partial E_1 \partial E_2} = R_{12} = R_{21}$, $\frac{\partial^2 R}{\partial E_2^2} = R_{22}$, etc. indicate cross-partial, second order differentiations of multivariate Lagrangian function, respectively, etc.

Now we expand the Hessian (9) as $|H| > 0$ [Islam et al., 2010; Mohajan et al., 2013; Mohajan & Mohajan, 2022d],

$$
|H| = \frac{A^3\alpha\beta\gamma\delta E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}B^2}{E_1^2E_2^2E_3^2E_4^2\forall^2}(\alpha+\beta+\gamma+\delta)(\delta+3) > 0, \tag{12}
$$

where efficiency parameter, $A > 0$, and budget of the firm, $B > 0$; E_1, E_2, E_3, and E_4 are four different types of inputs; and consequently, $E_1, E_2, E_3, E_4 > 0$. Parameters, $\alpha, \beta, \gamma, \delta > 0$; also in the model either $0 < \forall = \alpha+\beta+\gamma+\delta < 1$, $\forall = 1$ or $\forall > 1$. Hence, equation (12) gives; $|H| > 0$ [Islam et al., 2011; Mohajan & Mohajan, 2022c].

8. Determination of Lagrange Multiplier λ

Now using the necessary values from (8) in (7a) we get [Moolio et al., 2009; Islam et al., 2011; Mohajan & Mohajan, 2022c],

$$B=\frac{\alpha AE_1^{\alpha}E_1^{\beta}E_1^{\gamma}E_1^{\delta}}{\lambda}+\frac{\beta AE_1^{\alpha}E_1^{\beta}E_1^{\gamma}E_1^{\delta}}{\lambda}+\frac{\gamma AE_1^{\alpha}E_1^{\beta}E_1^{\gamma}E_1^{\delta}}{\lambda}+\frac{\delta AE_1^{\alpha}E_1^{\beta}E_1^{\gamma}E_1^{\delta}}{\lambda}$$

$$\lambda=\frac{AE_1^{\alpha}E_1^{\beta}E_1^{\gamma}E_1^{\delta}\forall}{B}. \tag{13}$$

We have observed that the second-order condition is satisfied, so that the determinant of (5) survives at the optimum, i.e., $|J|=|H|$; and hence, we can apply the implicit function theorem. Now we compute twenty-five partial derivatives, such as $\frac{\partial\lambda}{\partial k}$, $\frac{\partial E_1}{\partial k}$, $\frac{\partial E_3}{\partial l}$, $\frac{\partial E_4}{\partial B}$, etc. that are referred to as the comparative statics of the model [Chiang, 1984; Mohajan & Mohajan, 2023f].

Let **G** be the vector-valued function of ten variables $\lambda^*, E_1^*, E_2^*, E_3^*, E_4^*, k, l, m, n$, and B, and we define the function **G** for the point $\left(\lambda^*, E_1^*, E_2^*, E_3^*, E_4^*, k, l, m, n, B\right)\in R^{10}$, and take the values in R^5. By the Implicit Function Theorem of multivariable calculus, the equation [Mohajan & Mohajan, 2022e, 2023a],

$$F\left(\lambda^*, E_1^*, E_2^*, E_3^*, E_4^*, k, l, m, n, B\right)=0, \tag{14}$$

may be solved in the form of

$$\begin{bmatrix}\lambda\\ E_1\\ E_2\\ E_3\\ E_4\end{bmatrix}=\mathbf{G}\left(k, l, m, n, B\right). \tag{15}$$

Now the 5×5 Jacobian matrix for $\mathbf{G}\left(k, l, m, n, B\right)$; regarded as $J_G=\frac{\partial\left(\lambda, E_1, E_2, E_3, E_4\right)}{\partial\left(k, l, m, n_0, B\right)}$, and is represented by;

$$J_G = \begin{bmatrix} \frac{\partial \lambda}{\partial k} & \frac{\partial \lambda}{\partial l} & \frac{\partial \lambda}{\partial m} & \frac{\partial \lambda}{\partial n_0} & \frac{\partial \lambda}{\partial B} \\ \frac{\partial E_1}{\partial k} & \frac{\partial E_1}{\partial l} & \frac{\partial E_1}{\partial m} & \frac{\partial E_1}{\partial n_0} & \frac{\partial E_1}{\partial B} \\ \frac{\partial E_2}{\partial k} & \frac{\partial E_2}{\partial l} & \frac{\partial E_2}{\partial m} & \frac{\partial E_2}{\partial n_0} & \frac{\partial E_2}{\partial B} \\ \frac{\partial E_3}{\partial k} & \frac{\partial E_3}{\partial l} & \frac{\partial E_3}{\partial m} & \frac{\partial E_3}{\partial n_0} & \frac{\partial E_3}{\partial B} \\ \frac{\partial E_4}{\partial k} & \frac{\partial E_4}{\partial l} & \frac{\partial E_4}{\partial m} & \frac{\partial E_4}{\partial n_0} & \frac{\partial E_4}{\partial B} \end{bmatrix}. \tag{16}$$

$$= -J^{-1} \begin{bmatrix} -E_1 & -E_2 & -E_3 & -E_4^2 + E_4 & 1 \\ -\lambda & 0 & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 & 0 \\ 0 & 0 & -\lambda & 0 & 0 \\ 0 & 0 & 0 & -2\lambda E_4 + \lambda & 0 \end{bmatrix}.$$

The inverse of Jacobian matrix is, $J^{-1} = \frac{1}{|J|} C^T$, where $C = (C_{ij})$, the matrix of cofactors of J, where T for transpose, then (16) becomes [Moolio et al., 2009; Mohajan, 2021b],

$$= -\frac{1}{|J|} \begin{bmatrix} C_{11} & C_{21} & C_{31} & C_{41} & C_{51} \\ C_{12} & C_{22} & C_{32} & C_{42} & C_{52} \\ C_{13} & C_{23} & C_{33} & C_{43} & C_{53} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{54} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} \end{bmatrix} \begin{bmatrix} -E_1 & -E_2 & -E_3 & -E_4^2 + E_4 & 1 \\ -\lambda & 0 & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 & 0 \\ 0 & 0 & -\lambda & 0 & 0 \\ 0 & 0 & 0 & -2\lambda E_4 + \lambda & 0 \end{bmatrix}$$

$$J_G = -\frac{1}{|J|} \begin{bmatrix} -E_1C_{11} - \lambda C_{21} & -E_2C_{11} - \lambda C_{31} & -E_3C_{11} - \lambda C_{41} & -E_4^2C_{11} + E_4C_{11} - 2\lambda E_4C_{51} + \lambda C_{51} & C_{11} \\ -E_1C_{12} - \lambda C_{22} & -E_2C_{12} - \lambda C_{32} & -E_3C_{12} - \lambda C_{42} & -E_4^2C_{12} + E_4C_{12} - 2\lambda E_4C_{52} + \lambda C_{52} & C_{12} \\ -E_1C_{13} - \lambda C_{23} & -E_2C_{13} - \lambda C_{33} & -E_3C_{13} - \lambda C_{43} & -E_4^2C_{13} + E_4C_{13} - 2\lambda E_4C_{53} + \lambda C_{53} & C_{13} \\ -E_1C_{14} - \lambda C_{24} & -E_2C_{14} - \lambda C_{34} & -E_3C_{14} - \lambda C_{44} & -E_4^2C_{14} + E_4C_{14} - 2\lambda E_4C_{54} + \lambda C_{54} & C_{14} \\ -E_1C_{15} - \lambda C_{25} & -E_2C_{15} - \lambda C_{35} & -E_3C_{15} - \lambda C_{45} & -E_4^2C_{15} + E_4C_{15} - 2\lambda E_4C_{55} + \lambda C_{55} & C_{15} \end{bmatrix}. \tag{17}$$

In (17) total 25 comparative statics are available, and in this study we deal only with four of them when wage rate is increased. The firm always attempts for the profit maximization production [Baxley & Moorhouse, 1984; Islam et al., 2010].

Now we analyze the effect on capital E_1 when per unit cost of principal raw material, *m* increases. Taking T_{23} (i.e., term of 2nd row and 3rd column) from both sides of (17) we get [Moolio et al., 2009; Islam et al., 2011; Roy et al., 2021],

$$\frac{\partial E_1}{\partial m} = \frac{E_3}{|J|}[C_{12}] + \frac{\lambda}{|J|}[C_{42}]$$

$$= \frac{E_3}{|J|}\text{Cofactor of } C_{12} + \frac{\lambda}{|J|}\text{Cofactor of } \mathrm{C}_{42}$$

$$= -\frac{E_3}{|J|}\begin{vmatrix} -B_1 & R_{12} & R_{13} & R_{14} \\ -B_2 & R_{22} & R_{23} & R_{24} \\ -B_3 & R_{32} & R_{33} & R_{34} \\ -B_4 & R_{42} & R_{43} & R_{44} \end{vmatrix} + \frac{\lambda}{|J|}\begin{vmatrix} 0 & -B_2 & -B_3 & -B_4 \\ -B_1 & R_{12} & R_{13} & R_{14} \\ -B_2 & R_{22} & R_{23} & R_{24} \\ -B_4 & R_{42} & R_{43} & R_{44} \end{vmatrix}$$

$$= -\frac{E_3}{|J|}\left\{ -B_1\begin{vmatrix} R_{22} & R_{23} & R_{24} \\ R_{32} & R_{33} & R_{34} \\ R_{42} & R_{43} & R_{44} \end{vmatrix} - R_{12}\begin{vmatrix} -B_2 & R_{23} & R_{24} \\ -B_3 & R_{33} & R_{34} \\ -B_4 & R_{43} & R_{44} \end{vmatrix} + R_{13}\begin{vmatrix} -B_2 & R_{22} & R_{24} \\ -B_3 & R_{32} & R_{34} \\ -B_4 & R_{42} & R_{44} \end{vmatrix} - R_{14}\begin{vmatrix} -B_2 & R_{22} & R_{23} \\ -B_3 & R_{32} & R_{33} \\ -B_4 & R_{42} & R_{43} \end{vmatrix} \right\}$$

$$+ \frac{\lambda}{|J|}\left\{ B_2\begin{vmatrix} -B_1 & R_{13} & R_{14} \\ -B_2 & R_{23} & R_{24} \\ -B_4 & R_{43} & R_{44} \end{vmatrix} - B_3\begin{vmatrix} -B_1 & R_{12} & R_{14} \\ -B_2 & R_{22} & R_{24} \\ -B_4 & R_{42} & R_{44} \end{vmatrix} + B_4\begin{vmatrix} -B_1 & R_{12} & R_{13} \\ -B_2 & R_{22} & R_{23} \\ -B_4 & R_{42} & R_{43} \end{vmatrix} \right\}$$

$$= -\frac{E_3}{|J|}\left[-B_1\{R_{22}(R_{33}R_{44} - R_{43}R_{34}) + R_{23}(R_{42}R_{34} - R_{32}R_{44}) + R_{24}(R_{32}R_{43} - R_{42}R_{33})\}\right.$$

$$- R_{12}\{-B_2(R_{33}R_{44} - R_{43}R_{34}) + R_{23}(-B_4R_{34} + B_3R_{44}) + R_{24}(-B_3R_{43} + B_4R_{33})\}$$

$$+ R_{13}\{-B_2(R_{32}R_{44} - R_{42}R_{34}) + R_{22}(-B_4R_{34} + B_3R_{44}) + R_{24}(-B_3R_{42} + B_4R_{32})\}$$

$$\left. - R_{14}\{-B_2(R_{32}R_{43} - R_{42}R_{33}) + R_{22}(-B_4R_{33} + B_3R_{43}) + R_{23}(-B_3R_{42} + B_4R_{32})\}\right]$$

$$+ \frac{\lambda}{|J|}\left[B_2\{-B_1(R_{23}R_{44} - R_{43}R_{24}) - R_{13}(-B_2R_{44} + B_4R_{24}) + R_{14}(-B_2R_{43} + B_4R_{23})\}\right.$$

$$- B_3\{-B_1(R_{22}R_{44} - R_{42}R_{24}) - R_{12}(-B_2R_{44} + B_4R_{24}) + R_{14}(-B_2R_{42} + B_4R_{22})\}$$

$$\left. + B_4\{-B_1(R_{22}R_{43} - R_{42}R_{23}) - R_{12}(-B_2R_{43} + B_4R_{23}) + R_{13}(-B_2R_{42} + B_4R_{22})\}\right]$$

$$= -\frac{E_3}{|J|}\{-B_1R_{22}R_{33}R_{44} + B_1R_{22}R_{43}R_{34} - B_1R_{23}R_{42}R_{34} + B_1R_{23}R_{32}R_{44} - B_1R_{24}R_{32}R_{43} + B_1R_{24}R_{42}R_{33}$$

$$+ B_2R_{12}R_{33}R_{44} - B_2R_{12}R_{43}R_{34} + B_4R_{12}R_{23}R_{34} - B_3R_{12}R_{23}R_{44} + B_3R_{12}R_{24}R_{43} - B_4R_{12}R_{24}R_{33}$$

$$- B_2R_{13}R_{32}R_{44} + B_2R_{13}R_{42}R_{34} - B_2R_{13}R_{22}R_{34} + B_3R_{13}R_{22}R_{44} - B_3R_{13}R_{24}R_{42} + B_4R_{13}R_{24}R_{32}$$

$$+ B_2R_{14}R_{32}R_{43} - B_2R_{14}R_{42}R_{33} + B_4R_{14}R_{22}R_{33} - B_3R_{14}R_{22}R_{43} + B_3R_{14}R_{23}R_{42} - B_4R_{14}R_{23}R_{32}\}$$

$$+ \frac{\lambda}{|J|}\{-B_1B_2R_{23}R_{44} + B_1B_2R_{24}R_{34} + B_2^2R_{13}R_{44} - B_2B_4R_{13}R_{24} - B_2^2R_{14}R_{34} + B_2B_4R_{14}R_{23} + B_1B_3R_{22}R_{44}$$

$$- B_1B_3R_{24}^2 - B_2B_3R_{12}R_{44} + B_3B_4R_{12}R_{24} + B_2B_3R_{14}R_{24} - B_3B_4R_{14}R_{22} - B_1B_4R_{22}R_{34} + B_1B_4R_{23}R_{24}$$

$$+ B_2B_4R_{12}R_{34} - B_4^2R_{12}R_{23} - B_2B_4R_{13}R_{24} + B_4^2R_{13}R_{22}\}$$

$$=-\frac{E_3}{|J|}\frac{A^3E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}}{E_1^2E_2^2E_3^2E_4^2}\{-kE_1^2\beta(\beta-1)\gamma(\gamma-1)\delta(\delta-1) +kE_1^2\beta(\beta-1)\gamma^2\delta^2 -kE_1^2\beta^2\gamma^2\delta^2$$
$$+kE_1^2\beta^2\gamma^2\delta(\delta-1) -kE_1^2\beta^2\gamma^2\delta^2 +kE_1^2\beta^2\gamma(\gamma-1)\delta^2 +lE_1E_2\alpha\beta\gamma(\gamma-1)\delta(\delta-1) -lE_1E_2\alpha\beta\gamma^2\delta^2$$
$$+nE_1E_4\alpha\beta^2\gamma^2\delta -mE_1E_3\alpha\beta^2\gamma\delta(\delta-1) -nE_1E_4\alpha\beta^2\gamma(\gamma-1)\delta -lE_1E_2\alpha\beta\gamma^2\delta(\delta-1) +lE_1E_2\alpha\beta\gamma^2\delta^2$$
$$-nE_1E_4\alpha\beta(\beta-1)\gamma^2\delta +mE_1E_3\alpha\beta(\beta-1)\gamma\delta(\delta-1) -mE_1E_3\alpha\beta^2\gamma\delta^2 +nE_1E_4\alpha\beta^2\gamma^2\delta +lE_1E_2\alpha\beta\gamma^2\delta^2$$
$$-lE_1E_2\alpha\beta\gamma(\gamma-1)\delta^2 +nE_1E_4\alpha\beta(\beta-1)\gamma(\gamma-1)\delta -mE_1E_3\alpha\beta(\beta-1)\gamma\delta^2 +mE_1E_3\alpha\beta^2\gamma\delta^2$$
$$-nE_1E_4\alpha\beta^2\gamma^2\delta\} +\frac{\lambda}{|J|}\frac{A^2E_1^{2\alpha}E_2^{2\beta}E_3^{2\gamma}E_4^{2\delta}}{E_1^2E_2^2E_3^2E_4^2}\{-klE_1^2E_2E_3\beta\gamma\delta(\delta-1) +klE_1^2E_2E_3\beta\gamma\delta^2$$
$$+l^2E_1E_2^2E_3\alpha\gamma\delta(\delta-1) -nlE_1E_2E_3E_4\alpha\beta\gamma\delta -l^2E_1E_2^2E_3\alpha\gamma\delta^2 +nlE_1E_2E_3E_4\alpha\beta\gamma\delta$$
$$+kmE_1^2E_3^2\beta(\beta-1)\delta(\delta-1) -kmE_1^2E_3^2\beta^2\delta^2 -lmE_1E_2E_3^2\alpha\beta\delta(\delta-1) +mnE_1E_3^2E_4\alpha\beta^2\delta$$
$$+lmE_1E_2E_3^2\alpha\beta\delta^2 -mnE_1E_3^2E_4\alpha\beta(\beta-1)\delta -knE_1^2E_3E_4\beta(\beta-1)\gamma\delta +knE_1^2E_3E_4\beta^2\gamma\delta$$
$$+nlE_1E_2E_3E_4\alpha\beta\gamma\delta -n^2E_1E_3E_4^2\alpha\beta^2\gamma -nlE_1E_2E_3E_4\alpha\beta\gamma\delta +n^2E_1E_3E_4^2\alpha\beta(\beta-1)\gamma\}$$

$$=-\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}B}{E_1E_2^2E_3E_4^2\forall}\{-(\beta-1)(\gamma-1)(\delta-1) +\beta(\gamma-1)(\delta-1) -\beta\gamma(\delta-1) +(\beta-1)\gamma(\delta-1)$$
$$-(2E_4-1)\beta(\gamma-1)\delta -(2E_4-1)(\beta-1)\gamma\delta +2(2E_4-1)\beta\gamma\delta +(2E_4-1)(\beta-1)(\gamma-1)\delta\}$$
$$+\frac{1}{|J|}\frac{A^2\alpha\beta\gamma\delta E_1^{2\alpha}E_2^{2\beta}E_3^{2\gamma}E_4^{2\delta}}{E_1E_2^2E_3E_4^2}\frac{AE_1^{2\alpha}E_2^{2\beta}E_3^{2\gamma}E_4^{2\delta}\forall}{B}\{(\beta-1)(\delta-1) -\beta(\delta-1) -2(2E_4-1)(\beta-1)\delta$$
$$+2(2E_4-1)\beta\delta -(2E_4-1)^2\beta\delta +(2E_4-1)^2(\beta-1)\delta\}$$

$$\frac{\partial E_1}{\partial m}=\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}B}{E_1E_2^2E_3E_4^2\forall}\{-4E_4^2\delta+E_4(10\delta-2\beta\gamma\delta)+\beta\gamma\delta-4\delta\}. \quad (18)$$

Now using $\alpha=\beta=\gamma=\delta=\frac{1}{4}$ then we get, $\forall=1$, i.e., for constant returns to scale, in (18) we get,

$$\frac{\partial E_1}{\partial m}\approx-\frac{1}{|J|}\frac{A^3E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}B}{2^{13}E_1E_2^2E_3E_4^2\forall}(4E_4-5)^2<0. \quad (19)$$

From the relation (19) we see that when per unit cost of principal raw material increases, the level of capital is decreased. It seems that principal raw material is essential for the firm and more capital is used for buying the principal raw material. Consequently, the firm may think to reduce the level of capital, which is reasonable. In this situation the firm may face difficulties to move profit maximization.

Now using $\alpha=\beta=\gamma=\delta=\frac{1}{2}$ then we get, $\forall=2$, i.e., for increasing returns to scale, in (18) we get,

$$\frac{\partial E_1}{\partial m}=\frac{1}{|J|}\frac{A^3E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}B}{2^7E_1E_2^2E_3E_4^2\forall}(8E_4-15)(1-2E_4). \quad (20)$$

From (19) we see that if $E_4 > \frac{1}{2}$ and $E_4 < \frac{15}{8}$ we get,

$$\frac{\partial E_1}{\partial m} > 0. \qquad (21)$$

From the inequality (21) we see that when per unit cost of principal raw material increases, the level of capital also increases. It seems that although cost of principal raw material increases, the demand of the products of the firm is also increased in the society, and the industry increases its capital structure. We believe that for increasing returns to scale the industry is in sustainable environment and also is in profit maximization.

From (19) we see that if $E_4 < \frac{1}{2}$ or $E_4 > \frac{15}{8}$ we get,

$$\frac{\partial E_1}{\partial m} < 0. \qquad (22)$$

From the inequality (22) we see that when per unit cost of principal raw material increases, the level of capital decreases, which is reasonable. In this situation the industry may not achieve profit maximization policy.

Now we analyze the effect on worker E_2 when per unit cost of principal raw material, m increases. Taking T_{33} (i.e., term of 3rd row and 3rd column) from both sides of (17) we get [Mohajan, 2021c; Wiese, 2021],

$$\frac{\partial E_2}{\partial m} = \frac{E_3}{|J|}[C_{13}] + \frac{\lambda}{|J|}[C_{43}]$$

$$= \frac{E_3}{|J|}\text{Cofactor of } C_{13} + \frac{\lambda}{|J|}\text{Cofactor of C}_{43}$$

$$= \frac{E_3}{|J|}\begin{vmatrix} -B_1 & R_{11} & R_{13} & R_{14} \\ -B_2 & R_{21} & R_{23} & R_{24} \\ -B_3 & R_{31} & R_{33} & R_{34} \\ -B_4 & R_{41} & R_{43} & R_{44} \end{vmatrix} - \frac{\lambda}{|J|}\begin{vmatrix} 0 & -B_1 & -B_3 & -B_4 \\ -B_1 & R_{11} & R_{13} & R_{14} \\ -B_2 & R_{21} & R_{23} & R_{24} \\ -B_4 & R_{41} & R_{43} & R_{44} \end{vmatrix}$$

$$= \frac{E_3}{|J|}\left\{ -B_1\begin{vmatrix} R_{21} & R_{23} & R_{24} \\ R_{31} & R_{33} & R_{34} \\ R_{41} & R_{43} & R_{44} \end{vmatrix} - R_{11}\begin{vmatrix} -B_2 & R_{23} & R_{24} \\ -B_3 & R_{33} & R_{34} \\ -B_4 & R_{43} & R_{44} \end{vmatrix} + R_{13}\begin{vmatrix} -B_2 & R_{21} & R_{24} \\ -B_3 & R_{31} & R_{34} \\ -B_4 & R_{41} & R_{44} \end{vmatrix} - R_{14}\begin{vmatrix} -B_2 & R_{21} & R_{23} \\ -B_3 & R_{31} & R_{33} \\ -B_4 & R_{41} & R_{43} \end{vmatrix} \right\}$$

$$-\frac{\lambda}{|J|}\left\{B_1\begin{vmatrix}-B_1 & R_{13} & R_{14}\\ -B_2 & R_{23} & R_{24}\\ -B_4 & R_{43} & R_{44}\end{vmatrix}-B_3\begin{vmatrix}-B_1 & R_{11} & R_{14}\\ -B_2 & R_{21} & R_{24}\\ -B_4 & R_{41} & R_{44}\end{vmatrix}+B_4\begin{vmatrix}-B_1 & R_{11} & R_{13}\\ -B_2 & R_{21} & R_{23}\\ -B_4 & R_{41} & R_{43}\end{vmatrix}\right\}$$

$$=\frac{E_3}{|J|}\left[-B_1\{R_{21}(R_{33}R_{44}-R_{43}R_{34})+R_{23}(R_{41}R_{34}-R_{31}R_{44})+R_{24}(R_{31}R_{43}-R_{41}R_{33})\}\right.$$
$$-R_{11}\{-B_2(R_{33}R_{44}-R_{43}R_{34})+R_{23}(-B_4R_{34}+B_3R_{44})+R_{24}(-B_3R_{43}+B_4R_{33})\}$$
$$+R_{13}\{-B_2(R_{31}R_{44}-R_{41}R_{34})+R_{21}(-B_4R_{34}+B_3R_{44})+R_{24}(-B_3R_{41}+B_4R_{31})\}$$
$$\left.-R_{14}\{-B_2(R_{31}R_{43}-R_{41}R_{33})+R_{21}(-B_4R_{33}+B_3R_{43})+R_{23}(-B_3R_{41}+B_4R_{31})\}\right]$$

$$-\frac{\lambda}{|J|}\left[B_1\{-B_1(R_{23}R_{44}-R_{43}R_{24})+R_{13}(-B_4R_{24}+B_2R_{44})+R_{14}(-B_2R_{43}+B_4R_{23})\}\right.$$
$$-B_3\{-B_1(R_{21}R_{44}-R_{41}R_{24})+R_{11}(-B_4R_{24}+B_2R_{44})+R_{14}(-B_2R_{41}+B_4R_{21})\}$$
$$\left.+B_4\{-B_1(R_{21}R_{43}-R_{41}R_{23})+R_{11}(-B_4R_{23}+B_2R_{43})+R_{13}(-B_2R_{41}+B_4R_{21})\}\right]$$

$$=-\frac{E_3}{|J|}\{B_1R_{21}R_{33}R_{44}\quad -B_1R_{21}R_{43}R_{34}\quad +B_1R_{23}R_{41}R_{24}\quad -B_1R_{23}R_{31}R_{44}\quad +B_1R_{24}R_{31}R_{43}\quad -B_1R_{24}R_{41}R_{33}$$
$$-B_2R_{11}R_{33}R_{44}\ +B_2R_{11}R_{43}R_{34}\ -B_4R_{11}R_{23}R_{34}\ +B_3R_{11}R_{23}R_{44}\ -B_3R_{11}R_{24}R_{43}\ +B_4R_{11}R_{24}R_{33}\ +B_2R_{13}R_{31}R_{44}$$
$$-B_2R_{13}R_{41}R_{34}\ +B_4R_{13}R_{21}R_{34}\ -B_3R_{13}R_{21}R_{44}\ +B_3R_{13}R_{24}R_{41}\ -B_4R_{13}R_{24}R_{31}\ -B_2R_{14}R_{31}R_{43}\ +B_2R_{14}R_{41}R_{33}$$
$$-B_4R_{14}R_{21}R_{33}+B_3R_{14}R_{21}R_{43}-B_3R_{14}R_{23}R_{41}+B_4R_{14}R_{23}R_{31}\}$$

$$-\frac{\lambda}{|J|}\{-B_1^2R_{23}R_{44}\quad +B_1^2R_{24}R_{43}\quad -B_1B_4R_{13}R_{24}\quad +B_1B_2R_{13}R_{44}\quad -B_1B_2R_{14}R_{43}\quad +B_1B_4R_{14}R_{23}\quad +B_1B_3R_{21}R_{44}$$
$$-B_1B_3R_{41}R_{24}\quad +B_3B_4R_{11}R_{24}\quad -B_2B_3R_{11}R_{44}\quad +B_2B_3R_{14}R_{41}\quad -B_3B_4R_{14}R_{21}\quad -B_1B_4R_{21}R_{43}\quad +B_1B_4R_{41}R_{23}$$
$$-B_4^2R_{11}R_{23}+B_2B_4R_{11}R_{43}-B_2B_4R_{13}R_{41}+B_4^2R_{13}R_{21}\}$$

$$=-\frac{E_3}{|J|}\frac{A^3E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}}{E_1^2E_2^2E_3^2E_4^2}\qquad\{kE_1E_2\alpha\beta\gamma(\gamma-1)\delta(\delta-1)\qquad -kE_1E_2\alpha\beta\gamma^2\delta^2\qquad +kE_1E_2\alpha\beta\gamma^2\delta^2$$
$$-kE_1E_2\alpha\beta\gamma^2\delta(\delta-1)\qquad +kE_1E_2\alpha\beta\gamma^2\delta^2\qquad -kE_1E_2\alpha\beta\gamma(\gamma-1)\delta^2\qquad -lE_2^2\alpha(\alpha-1)\gamma(\gamma-1)\delta(\delta-1)$$
$$+lE_2^2\alpha(\alpha-1)\gamma^2\delta^2\qquad -nE_2E_4\alpha(\alpha-1)\beta\gamma^2\delta\qquad +mE_2E_3\alpha(\alpha-1)\beta\gamma\delta(\delta-1)\qquad -mE_2E_3\alpha(\alpha-1)\beta\gamma\delta^2$$
$$+nE_2E_4\alpha(\alpha-1)\beta\gamma(\gamma-1)\delta\ +lE_2^2\alpha^2\gamma^2\delta(\delta-1)\ -lE_2^2\alpha^2\gamma^2\delta^2\ +nE_2E_4\alpha^2\beta\gamma^2\delta\ -mE_2E_3\alpha^2\beta\gamma\delta(\delta-1)$$
$$+mE_2E_3\alpha^2\beta\gamma\delta^2\qquad -nE_2E_4\alpha^2\beta\gamma^2\delta\qquad -lE_2^2\alpha^2\gamma^2\delta^2\qquad +lE_2^2\alpha^2\gamma(\gamma-1)\delta^2\qquad -nE_2E_4\alpha^2\beta\gamma(\gamma-1)\delta$$
$$+mE_2E_3\alpha^2\beta\gamma\delta^2\qquad -mE_2E_3\alpha^2\beta\gamma\delta^2\qquad +nE_2E_4\alpha^2\beta\gamma^2\delta\}\qquad -\frac{1}{|J|}\frac{A^2E_1^{2\alpha}E_2^{2\beta}E_3^{2\gamma}E_4^{2\delta}}{E_1^2E_2E_3E_4^2}\frac{A\beta\gamma E_1^{\alpha}E_2^{\beta}E_3^{\gamma}E_4^{\delta}\forall}{B}$$
$$\{-k^2E_1^2E_2E_3\beta\gamma\delta(\delta-1)\ +k^2E_1^2E_2E_3\beta\gamma\delta^2\ -knE_1E_2E_3E_4\alpha\beta\gamma\delta\ +klE_1E_2^2E_3\alpha\gamma\delta(\delta-1)\ +klE_1E_2^2E_3\alpha\gamma\delta^2$$
$$+knE_1E_2E_3E_4\alpha\beta\gamma\delta\qquad +kmE_1E_2E_3^2\alpha\beta\delta(\delta-1)\qquad -kmE_1E_2E_3^2\alpha\beta\delta^2\qquad +mnE_2E_3^2E_4\alpha(\alpha-1)\beta\delta$$
$$-lmE_2^2E_3^2\alpha(\alpha-1)\delta(\delta-1)\ +lmE_2^2E_3^2\alpha^2\delta^2\ -mnE_2E_3^2E_4\alpha^2\beta\delta\ -knE_1E_2E_3E_4\alpha\beta\gamma\delta\ +knE_1E_2E_3E_4\alpha\beta\gamma\delta$$
$$-n^2E_2E_3E_4^2\alpha(\alpha-1)\beta\gamma\ +nlE_2^2E_3E_4\alpha(\alpha-1)\gamma\delta\ -nlE_2^2E_3E_4\alpha^2\gamma\delta\ +n^2E_2E_3E_4^2\alpha^2\beta\gamma\}$$

$$= -\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}}{E_1^2E_2E_3E_4^2}\{kE_1(\gamma-1)(\delta-1) - kE_1\gamma(\delta-1) + kE_1\gamma\delta - kE_1(\gamma-1)\delta + lE_2\alpha\beta^{-1}\gamma(\delta-1)$$
$$- lE_2\alpha\beta^{-1}\gamma\delta + lE_2\alpha\beta^{-1}(\gamma-1)\delta - lE_2\alpha\beta^{-1}\gamma\delta - lE_2(\alpha-1)\beta^{-1}(\gamma-1)(\delta-1) + lE_2\alpha\beta^{-1}\gamma\delta - mE_3(\alpha-1)\delta$$
$$+ mE_3(\alpha-1)(\delta-1) - mE_3\alpha(\delta-1) + mE_3\alpha\delta + nE_4(\alpha-1)(\gamma-1) - nE_4(\alpha-1)\gamma + nE_4\alpha\gamma - nE_4\alpha(\gamma-1)\}$$
$$-\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}\forall}{E_1^2E_2E_3E_4^2B}\left\{-k^2E_1^2\beta\gamma\delta(\delta-1) + k^2E_1^2\beta\gamma\delta^2 + klE_1E_2\alpha\gamma\delta(\delta-1) + klE_1E_2\alpha\gamma\delta^2\right.$$
$$+ kmE_1E_3\alpha\beta\delta(\delta-1) - kmE_1E_3\alpha\beta\delta^2 + mnE_3E_4\alpha(\alpha-1)\beta\delta - mnE_3E_4\alpha^2\beta\delta - lmE_2E_3\alpha(\alpha-1)\delta(\delta-1)$$
$$\left. + lmE_2E_3\alpha^2\delta^2 + nlE_2E_4\alpha(\alpha-1)\gamma\delta - nlE_2E_4\alpha^2\gamma\delta - n^2E_4^2\alpha(\alpha-1)\beta\gamma + n^2E_4^2\alpha^2\beta\gamma\right\}$$

$$= -\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}B}{E_1^2E_2E_3E_4^2\forall}\{\alpha(\gamma-1)(\delta-1) - (\alpha-1)(\gamma-1)(\delta-1) + (\alpha-1)\gamma(\delta-1) - \alpha\gamma(\delta-1)$$
$$-(\alpha-1)\gamma\delta + \alpha\gamma\delta + (2E_4-1)(\alpha-1)(\gamma-1)\delta - (2E_4-1)(\alpha-1)\gamma\delta + (2E_4-1)\alpha\gamma\delta - (2E_4-1)\alpha(\gamma-1)\delta\}$$
$$-\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}B}{E_1^2E_2E_3E_4^2\forall}\{\alpha(\delta-1) - (\alpha-1)(\delta-1)$$
$$-2(2E_4-1)\alpha\delta + 2(2E_4-1)(\alpha-1)\delta - (2E_4-1)^2(\alpha-1)\delta + (2E_4-1)^2\alpha\delta\}.$$
$$\frac{\partial E_2}{\partial m} = -\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}B}{E_1^2E_2E_3E_4^2\forall}\left(4E_4^2 - 6E_4\delta + 2\delta + \gamma\delta\right). \quad (23)$$

Now using $\alpha = \beta = \gamma = \delta = \frac{1}{4}$ then we get, $\forall = 1$, i.e., for constant returns to scale, in (23) we get,

$$\frac{\partial E_2}{\partial m} = -\frac{1}{|J|}\frac{A^3E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}B}{2^{12}E_1E_2^2E_3E_4^2\forall}(8E_4-3)^2 < 0. \quad (24)$$

From the equation (24) we have realized that when per unit cost of principal raw material increases, wage of laborers decreases. It seems that less principal raw material is purchased after increase price of it and more labors become unemployed, and they agree to work with lower wage.

Now we test the effect on principal raw material E_3 when per unit cost of it increases. Taking T_{43} (i.e., term of 4th row and 3rd column) from both sides of (17) we get [Roy et al., 2021; Mohajan & Mohajan, 2023f],

$$\frac{\partial E_3}{\partial m} = \frac{E_3}{|J|}[C_{14}] + \frac{\lambda}{|J|}[C_{44}]$$

$$= \frac{E_3}{|J|}\text{Cofactor of } C_{14} + \frac{\lambda}{|J|}\text{Cofactor of } C_{44}$$

$$
= -\frac{E_3}{|J|}\begin{vmatrix} -B_1 & R_{11} & R_{12} & R_{14} \\ -B_2 & R_{21} & R_{22} & R_{24} \\ -B_3 & R_{31} & R_{32} & R_{34} \\ -B_4 & R_{41} & R_{42} & R_{44} \end{vmatrix} + \frac{\lambda}{|J|}\begin{vmatrix} 0 & -B_1 & -B_2 & -B_4 \\ -B_1 & R_{11} & R_{12} & R_{14} \\ -B_2 & R_{21} & R_{22} & R_{24} \\ -B_4 & R_{41} & R_{42} & R_{44} \end{vmatrix}
$$

$$
= -\frac{E_3}{|J|}\left\{ -B_1\begin{vmatrix} R_{21} & R_{22} & R_{24} \\ R_{31} & R_{32} & R_{34} \\ R_{41} & R_{42} & R_{44} \end{vmatrix} - R_{11}\begin{vmatrix} -B_2 & R_{22} & R_{24} \\ -B_3 & R_{32} & R_{34} \\ -B_4 & R_{42} & R_{44} \end{vmatrix} + R_{12}\begin{vmatrix} -B_2 & R_{21} & R_{24} \\ -B_3 & R_{31} & R_{34} \\ -B_4 & R_{41} & R_{44} \end{vmatrix} - R_{14}\begin{vmatrix} -B_2 & R_{21} & R_{22} \\ -B_3 & R_{31} & R_{32} \\ -B_4 & R_{41} & R_{42} \end{vmatrix} \right\}
$$

$$
+ \frac{\lambda}{|J|}\left\{ B_1\begin{vmatrix} -B_1 & R_{12} & R_{14} \\ -B_2 & R_{22} & R_{24} \\ -B_4 & R_{42} & R_{44} \end{vmatrix} - B_2\begin{vmatrix} -B_1 & R_{11} & R_{14} \\ -B_2 & R_{21} & R_{24} \\ -B_4 & R_{41} & R_{44} \end{vmatrix} + B_4\begin{vmatrix} -B_1 & R_{11} & R_{12} \\ -B_2 & R_{21} & R_{22} \\ -B_4 & R_{41} & R_{42} \end{vmatrix} \right\}
$$

$$
= -\frac{E_3}{|J|}\left[-B_1\left\{ R_{21}\left(R_{32}R_{44} - R_{42}R_{34}\right) + R_{22}\left(R_{41}R_{34} - R_{31}R_{44}\right) + R_{24}\left(R_{31}R_{42} - R_{41}R_{32}\right) \right\} \right.
$$

$$
- R_{11}\left\{ -B_2\left(R_{32}R_{44} - R_{42}R_{34}\right) + R_{22}\left(-B_4R_{34} + B_3R_{44}\right) + R_{24}\left(-B_3R_{42} + B_4R_{32}\right) \right\}
$$

$$
+ R_{12}\left\{ -B_2\left(R_{31}R_{44} - R_{41}R_{34}\right) + R_{21}\left(-B_4R_{34} + B_3R_{44}\right) + R_{24}\left(-B_3R_{41} + B_4R_{31}\right) \right\}
$$

$$
\left. - R_{14}\left\{ -B_2\left(R_{31}R_{42} - R_{41}R_{32}\right) + R_{21}\left(-B_4R_{32} + B_3R_{42}\right) + R_{22}\left(-B_3R_{41} + B_4R_{31}\right) \right\} \right]
$$

$$
+ \frac{\lambda}{|J|}\left[B_1\left\{ -B_1\left(R_{22}R_{44} - R_{42}R_{24}\right) + R_{12}\left(-B_4R_{24} + B_2R_{44}\right) + R_{14}\left(-B_2R_{42} + B_4R_{22}\right) \right\} \right.
$$

$$
- B_2\left\{ -B_1\left(R_{21}R_{44} - R_{41}R_{24}\right) + R_{11}\left(-B_4R_{24} + B_2R_{44}\right) + R_{14}\left(-B_2R_{41} + B_4R_{21}\right) \right\}
$$

$$
\left. + B_4\left\{ -B_1\left(R_{21}R_{42} - R_{41}R_{22}\right) + R_{11}\left(-B_4R_{22} + B_2R_{42}\right) + R_{12}\left(-B_2R_{41} + B_4R_{21}\right) \right\} \right]
$$

$$
= -\frac{E_3}{|J|}\left\{ -B_1R_{21}R_{32}R_{44} \quad + B_1R_{21}R_{42}R_{34} \quad - B_1R_{22}R_{41}R_{34} \quad + B_1R_{22}R_{31}R_{44} \quad - B_1R_{24}R_{31}R_{42} \quad + B_1R_{24}R_{41}R_{32} \right.
$$

$$
+ B_2R_{11}R_{32}R_{44} \quad - B_2R_{11}R_{42}R_{34} \quad + B_4R_{11}R_{22}R_{34} \quad - B_3R_{11}R_{22}R_{44} \quad + B_3R_{11}R_{24}R_{42} \quad - B_4R_{11}R_{24}R_{32}
$$

$$
- B_2R_{12}R_{31}R_{44} \quad + B_2R_{12}R_{41}R_{34} \quad - B_4R_{12}R_{21}R_{34} \quad + B_3R_{12}R_{21}R_{44} \quad - B_3R_{12}R_{24}R_{41} \quad + B_4R_{12}R_{24}R_{31}
$$

$$
\left. + B_2R_{14}R_{31}R_{42} \quad - B_2R_{14}R_{41}R_{32} \quad + B_4R_{14}R_{21}R_{32} \quad - B_3R_{14}R_{21}R_{42} \quad + B_3R_{14}R_{22}R_{41} - B_4R_{14}R_{22}R_{31} \right\}
$$

$$
+ \frac{\lambda}{|J|}\left\{ -B_1^2R_{22}R_{44} \quad + B_1^2R_{42}R_{24} \quad - B_1B_4R_{12}R_{24} \quad + B_1B_2R_{12}R_{44} \quad - B_1B_2R_{14}R_{42} \quad + B_1B_4R_{14}R_{22} \quad + B_1B_2R_{21}R_{44} \right.
$$

$$
- B_1B_2R_{41}R_{24} + B_2B_4R_{11}R_{24} - B_2^2R_{11}R_{44} + B_2^2R_{14}R_{41} - B_2B_4R_{14}R_{21} - B_1B_4R_{21}R_{42} + B_1B_4R_{41}R_{22} - B_4^2R_{11}R_{22}
$$

$$
\left. + B_2B_4R_{11}R_{42} - B_2B_4R_{12}R_{41} + B_4^2R_{12}R_{21} \right\}
$$

$$
= -\frac{E_3}{|J|}\frac{A^3E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}}{E_1^2E_2^2E_3^2E_4^2} \qquad \left\{ -kE_1E_3\alpha\beta^2\gamma\delta(\delta-1) \qquad + kE_1E_3\alpha\beta^2\gamma\delta^2 \qquad - kE_1E_3\alpha\beta(\beta-1)\gamma\delta^2 \right.
$$

$$
+ kE_1E_3\alpha\beta(\beta-1)\gamma\delta(\delta-1) \qquad - kE_1E_3\alpha\beta^2\gamma\delta^2 \qquad + kE_1E_3\alpha\beta^2\gamma\delta^2 \qquad + lE_2E_3\alpha(\alpha-1)\beta\gamma\delta(\delta-1)
$$

$$
- lE_2E_3\alpha(\alpha-1)\beta\gamma\delta^2 \quad + nE_3E_4\alpha(\alpha-1)\beta(\delta-1)\gamma\delta \quad - mE_3^2\alpha(\alpha-1)\beta(\beta-1)\delta(\delta-1) \quad + mE_3^2\alpha(\alpha-1)\beta^2\delta^2
$$

$$
+ nE_3E_4\alpha(\alpha-1)\beta(\beta-1)\gamma\delta \quad - nE_3E_4\alpha(\alpha-1)\beta^2\gamma\delta \quad - lE_2E_3\alpha\beta\gamma\delta(\delta-1) + lE_2E_3\alpha\beta\gamma\delta^2 \quad - nE_3E_4\alpha^2\beta^2\gamma\delta
$$

$$+mE_3^2\alpha^2\beta^2\delta(\delta-1) \; -mE_3^2\alpha^2\beta^2\delta^2 \; +nE_3E_4\alpha^2\beta^2\gamma\delta \; +lE_2E_3\alpha\beta\gamma\delta^2 \; -lE_2E_3\alpha\beta\gamma\delta^2 \; +nE_3E_4\alpha^2\beta^2\gamma\delta$$
$$-mE_3^2\alpha^2\beta^2\delta^2 \; +mE_3^2\alpha^2\beta(\beta-1)\delta^2 \; -nE_3E_4\alpha^2\beta(\beta-1)\gamma\delta\} \; +\frac{1}{|J|}\frac{A^2E_1^{2\alpha}E_2^{2\beta}E_3^{2\gamma}E_4^{2\delta}}{E_1^2E_2^2E_3^2E_4^2}\frac{AE_1^{\alpha}E_2^{\beta}E_3^{\gamma}E_4^{\delta}\forall}{B}$$
$$\{-k^2E_1^2E_3^2\beta(\beta-1)\delta(\delta-1) \; +k^2E_1^2E_3^2\beta^2\delta^2 \; -knE_1E_3^2E_4\alpha\beta^2\delta \; +klE_1E_2E_3^2\alpha\beta\delta(\delta-1) \; -klE_1E_2E_3^2\alpha\beta\delta^2$$
$$+knE_1E_3^2E_4\alpha\beta(\beta-1)\delta \; +klE_1E_2E_3^2\alpha\beta\delta(\delta-1) \; -klE_1E_2E_3^2\alpha\beta\delta^2 \; +nlE_2E_3^2E_4\alpha(\alpha-1)\beta\delta$$
$$-l^2E_2^2E_3^2\alpha(\alpha-1)\delta(\delta-1) \; +l^2E_2^2E_3^2\alpha^2\delta^2 \; -nlE_2E_3^2E_4\alpha^2\beta\delta \; -knE_1E_3^2E_4\alpha\beta^2\delta \; +knE_1E_3^2E_4\alpha\beta(\beta-1)\delta$$
$$-n^2E_3^2E_4^2\alpha(\alpha-1)\beta(\beta-1)+nlE_2E_3^2E_4\alpha(\alpha-1)\beta\delta \; -nlE_2E_3^2E_4\alpha^2\beta\delta \; +n^2E_3^2E_4^2\alpha^2\beta^2\}$$

$$=-\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}}{E_1^2E_2^2E_4^2} \quad \{-kE_1\beta(\delta-1) \; -kE_1(\beta-1)\delta \; +kE_1(\beta-1)(\delta-1) \; +kE_1\beta\delta$$
$$+lE_2(\alpha-1)(\delta-1) \; -lE_2\alpha(\delta-1) \; +lE_2\alpha\delta \; -lE_2(\alpha-1)\delta \; -2mE_3\alpha\beta\gamma^{-1}\delta \; +mE_3\alpha(\beta-1)\gamma^{-1}\delta$$
$$-mE_3(\alpha-1)(\beta-1)\gamma^{-1}(\delta-1)+mE_3\alpha\beta\gamma^{-1}(\delta-1)+mE_3(\alpha-1)\beta\gamma^{-1}\delta \; +nE_4(\alpha-1)(\beta-1) \; -nE_4(\alpha-1)\beta$$
$$+nE_4\alpha\beta \; -nE_4\alpha(\beta-1)\} \quad +\frac{1}{|J|}\frac{A^3E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}B}{E_1^2E_2^2E_4^2\forall} \quad \{-k^2E_1^2\beta(\beta-1)\delta(\delta-1) \; +k^2E_1^2\beta^2\delta^2$$
$$-2knE_1E_4\alpha\beta^2\delta \; +2knE_1E_4\alpha\beta(\beta-1)\delta \; +2klE_1E_2\alpha\beta\delta(\delta-1) \; -2klE_1E_2\alpha\beta\delta^2 \; +nlE_2E_4\alpha(\alpha-1)\beta\delta$$
$$-l^2E_2^2\alpha(\alpha-1)\delta(\delta-1) \; +l^2E_2^2\alpha^2\delta^2 \; -nlE_2E_4\alpha^2\beta\delta \; +nlE_2E_4\alpha(\alpha-1)\beta\delta \; -nlE_2E_3^2E_4\alpha^2\beta\delta$$
$$-n^2E_4^2\alpha(\alpha-1)\beta(\beta-1)+n^2E_4^2\alpha^2\beta^2\}$$

$$=-\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}}{E_1^2E_2^2E_4^2} \; \{-\alpha\beta(\delta-1) \; +\alpha(\beta-1)(\delta-1) \; +(\alpha-1)\beta(\delta-1) \; -(\alpha-1)(\beta-1)(\delta-1)$$
$$+(2E_4-1)(\alpha-1)(\beta-1)\delta \; -(2E_4-1)(\alpha-1)\beta\delta \; +(2E_4-1)\alpha\beta\delta \; -(2E_4-1)\alpha(\beta-1)\delta\}$$
$$+\frac{1}{|J|}\frac{A^3\alpha\beta\delta E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}B}{E_1^2E_2^2E_4^2\forall} \; \{-\alpha(\beta-1)(\delta-1) \; +2\alpha\beta(\delta-1) \; -(\alpha-1)\beta(\delta-1) \; -4(2E_4-1)\alpha\beta\delta$$
$$+2(2E_4-1)\alpha(\beta-1)\delta+2(2E_4-1)(\alpha-1)\beta\delta-(2E_4-1)^2(\alpha-1)(\beta-1)\delta+(2E_4-1)^2\alpha\beta\delta\}$$

$$\frac{\partial E_3}{\partial m}=\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}}{E_1^2E_2^2E_4^2}\{E_4^2(4\alpha\delta+4\beta\delta-4\delta)-E_4(8\alpha\delta+8\beta\delta+2\gamma\delta-4\delta)$$
$$+4\alpha\delta+4\beta\delta+2\gamma\delta-\alpha-\beta-\gamma-\delta\}. \tag{25}$$

Now using $\alpha=\beta=\gamma=\delta=\frac{1}{4}$ then we get, $\forall=1$, i.e., for constant returns to scale, in (25) we get,

$$\frac{\partial E_3}{\partial m}=-\frac{1}{|J|}\frac{A^3E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}}{2^9E_1^2E_2^2E_4^2}\left\{\left(2E_4-\frac{3}{4}\right)^2+\frac{39}{16}\right\}. \tag{26}$$

In (26) if $\left(2E_4-\frac{3}{4}\right)^2>\frac{39}{16}$, i.e., $E_4>\left(3\pm\sqrt{39}\right)/8$, then we get,

$$\frac{\partial E_3}{\partial m}<0. \tag{27}$$

The relation (27) indicates that per unit cost of principal raw material, m increases; the tendency of purchasing principal raw material E_3 decreases, which is reasonable. In this situation, the industry may face difficulties in its production for profit maximization [Islam et al., 2011; Mohajan; 2021b, Mohajan & Mohajan, 2022c].

In (26) if $\left(2E_4 - \frac{3}{4}\right)^2 < \frac{39}{16}$, i.e., $E_4 < \left(3+\sqrt{39}\right)/8$, then we get,

$$\frac{\partial E_3}{\partial m} > 0. \tag{28}$$

The inequality (28) indicates that if per unit cost of principal raw material increases, the tendency of purchasing principal raw material E_3 also increases, which is not reasonable. From this study we observe that the revenue of the industry have increased and tends to profit maximization [Mohajan; 2022, Mohajan & Mohajan, 2022b].

Now we study the effects on irregular inputs E_4 when per unit cost of principal raw material, m increases. Taking T_{53} (i.e., term of 5th row and 3rd column) from both sides of (12) we get [Islam et al., 2010; Roy et al., 2021],

$$\frac{\partial E_4}{\partial m} = \frac{E_3}{|J|}[C_{15}] + \frac{\lambda}{|J|}[C_{45}]$$

$$= \frac{E_3}{|J|}\text{Cofactor of } C_{15} + \frac{\lambda}{|J|}\text{Cofactor of } \mathrm{C}_{45}$$

$$= \frac{E_3}{|J|}\begin{vmatrix} -B_1 & R_{11} & R_{12} & R_{13} \\ -B_2 & R_{21} & R_{22} & R_{23} \\ -B_3 & R_{31} & R_{32} & R_{33} \\ -B_4 & R_{41} & R_{42} & R_{43} \end{vmatrix} + \frac{\lambda}{|J|}\begin{vmatrix} 0 & -B_1 & -B_2 & -B_3 \\ -B_1 & R_{11} & R_{12} & R_{13} \\ -B_2 & R_{21} & R_{22} & R_{23} \\ -B_4 & R_{41} & R_{42} & R_{43} \end{vmatrix}$$

$$= \frac{E_3}{|J|}\left\{-B_1\begin{vmatrix} R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \\ R_{41} & R_{42} & R_{43} \end{vmatrix} - R_{11}\begin{vmatrix} -B_2 & R_{22} & R_{23} \\ -B_3 & R_{32} & R_{33} \\ -B_4 & R_{42} & R_{43} \end{vmatrix} + R_{12}\begin{vmatrix} -B_2 & R_{21} & R_{23} \\ -B_3 & R_{31} & R_{33} \\ -B_4 & R_{41} & R_{43} \end{vmatrix} - R_{13}\begin{vmatrix} -B_2 & R_{21} & R_{22} \\ -B_3 & R_{31} & R_{32} \\ -B_4 & R_{41} & R_{42} \end{vmatrix}\right\}$$

$$+ \frac{\lambda}{|J|}\left\{B_1\begin{vmatrix} -B_1 & R_{12} & R_{13} \\ -B_2 & R_{22} & R_{23} \\ -B_4 & R_{42} & R_{43} \end{vmatrix} - B_2\begin{vmatrix} -B_1 & R_{11} & R_{13} \\ -B_2 & R_{21} & R_{23} \\ -B_4 & R_{41} & R_{43} \end{vmatrix} + B_3\begin{vmatrix} -B_1 & R_{11} & R_{12} \\ -B_2 & R_{21} & R_{22} \\ -B_4 & R_{41} & R_{42} \end{vmatrix}\right\}$$

$$= \frac{E_3}{|J|}\left[-B_1\{R_{21}(R_{32}R_{43} - R_{42}R_{33}) + R_{22}(R_{41}R_{33} - R_{31}R_{43}) + R_{23}(R_{31}R_{42} - R_{41}R_{32})\}\right.$$

$$-R_{11}\{-B_2(R_{32}R_{43}-R_{42}R_{33})+R_{22}(-B_4R_{33}+B_3R_{43})+R_{23}(-B_3R_{42}+B_4R_{32})\}$$

$$+R_{12}\{-B_2(R_{31}R_{43}-R_{41}R_{33})+R_{21}(-B_4R_{33}+B_3R_{43})+R_{23}(-B_3R_{41}+B_4R_{31})\}$$

$$-R_{13}\{-B_2(R_{31}R_{42}-R_{41}R_{32})+R_{21}(-B_4R_{32}+B_3R_{42})+R_{22}(-B_3R_{41}+B_4R_{31})\}]$$

$$+\frac{\lambda}{|J|}[B_1\{-B_1(R_{22}R_{43}-R_{42}R_{23})+R_{12}(-B_4R_{23}+B_2R_{43})+R_{13}(-B_2R_{42}+B_4R_{22})\}$$

$$-B_2\{-B_1(R_{21}R_{43}-R_{41}R_{23})+R_{11}(-B_4R_{23}+B_2R_{43})+R_{13}(-B_2R_{41}+B_4R_{21})\}$$

$$+B_3\{-B_1(R_{21}R_{42}-R_{41}R_{22})+R_{11}(-B_4R_{22}+B_2R_{42})+R_{12}(-B_2R_{41}+B_4R_{21})\}]$$

$$-R_{13}\{-B_2(R_{31}R_{42}-R_{41}R_{32})+R_{21}(-B_4R_{32}+B_3R_{42})+R_{22}(-B_3R_{41}+B_4R_{31})\}]$$

$$=\frac{E_3}{|J|}\{-B_1R_{21}R_{32}R_{43}\quad +B_1R_{21}R_{42}R_{33}\quad -B_1R_{22}R_{41}R_{33}\quad +B_1R_{22}R_{31}R_{43}\quad -B_1R_{23}R_{31}R_{42}\quad +B_1R_{23}R_{41}R_{32}$$
$$+B_2R_{11}R_{32}R_{43}\ -B_2R_{11}R_{42}R_{33}\ +B_4R_{11}R_{22}R_{33}\ -B_3R_{11}R_{22}R_{43}\ +B_3R_{11}R_{23}R_{42}\ -B_4R_{11}R_{23}R_{32}\ -B_2R_{12}R_{31}R_{43}$$
$$+B_2R_{12}R_{41}R_{33}\ -B_4R_{12}R_{21}R_{33}\ +B_3R_{12}R_{21}R_{43}\ -B_3R_{12}R_{23}R_{41}\ +B_4R_{12}R_{23}R_{31}\ +B_2R_{13}R_{31}R_{42}\ -B_2R_{13}R_{41}R_{32}$$
$$+B_4R_{13}R_{21}R_{32}\ -B_3R_{13}R_{21}R_{42}\ +B_3R_{13}R_{22}R_{41}\ -B_4R_{13}R_{22}R_{31}\}\ +\frac{\lambda}{|J|}\{-B_1^2R_{22}R_{43}\ +B_1^2R_{42}R_{23}\ -B_1B_4R_{12}R_{23}$$
$$+B_1B_2R_{12}R_{43}\quad -B_1B_2R_{13}R_{42}\quad +B_1B_4R_{13}R_{22}\quad +B_1B_2R_{21}R_{43}\quad -B_1B_2R_{41}R_{23}\quad +B_2B_4R_{11}R_{23}\quad -B_2^2R_{11}R_{43}$$
$$+B_2^2R_{13}R_{41}\quad -B_2B_4R_{13}R_{21}\quad -B_1B_3R_{21}R_{42}\quad +B_1B_3R_{41}R_{22}\quad -B_3B_4R_{11}R_{22}\quad +B_2B_3R_{11}R_{42}\quad -B_2B_3R_{12}R_{41}$$
$$+B_3B_4R_{12}R_{21}\}$$

$$=\frac{E_3}{|J|}\frac{A^3E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}}{E_1^2E_2^2E_3^2E_4^2}\quad\{-kE_1E_4\alpha\beta^2\gamma^2\delta\quad +kE_1E_4\alpha\beta^2\gamma(\gamma-1)\delta\quad -kE_1E_4\alpha\beta(\beta-1)\gamma(\gamma-1)\delta$$
$$+kE_1E_4\alpha\beta(\beta-1)\gamma^2\delta\quad -kE_1E_4\alpha\beta^2\gamma^2\delta\quad +kE_1E_4\alpha\beta^2\gamma^2\delta\quad +lE_2E_4\alpha(\alpha-1)\beta\gamma^2\delta$$
$$-lE_2E_4\alpha(\alpha-1)\beta\gamma(\gamma-1)\delta\quad +nE_4^2\alpha(\alpha-1)\beta(\beta-1)\gamma(\gamma-1)\quad -mE_3E_4\alpha(\alpha-1)\beta(\beta-1)\gamma\delta$$
$$+mE_3E_4\alpha(\alpha-1)\beta^2\gamma\delta\ -nE_4^2\alpha(\alpha-1)\beta^2\gamma^2\ -lE_2E_4\alpha^2\beta\gamma^2\delta\ +lE_2E_4\alpha^2\beta\gamma(\gamma-1)\delta l\ -nE_4^2\alpha^2\beta^2\gamma(\gamma-1)$$
$$+mE_3E_4\alpha^2\beta^2\gamma\delta\ -mE_3E_4\alpha^2\beta^2\gamma\delta\ +nE_4^2\alpha^2\beta^2\gamma^2\ +lE_2E_4\alpha^2\beta\gamma^2\delta\ -lE_2E_4\alpha^2\beta\gamma^2\delta\ +nE_4^2\alpha^2\beta^2\gamma^2$$
$$-mE_3E_4\alpha^2\beta^2\gamma\delta\ +mE_3E_4\alpha^2\beta(\beta-1)\gamma\delta\ -nE_4^2\alpha^2\beta(\beta-1)\gamma^2\}\ +\frac{1}{|J|}\frac{A^2E_1^{2\alpha}E_2^{2\beta}E_3^{2\gamma}E_4^{2\delta}}{E_1^2E_2^2E_3^2E_4^2}\frac{AE_1^{\alpha}E_2^{\beta}E_3^{\gamma}E_4^{\delta}\forall}{B}$$
$$\{-k^2E_1^2E_3E_4\beta(\beta-1)\gamma\delta\ +k^2E_1^2E_3E_4\beta^2\gamma\delta\ -knE_1E_3E_4^2\alpha\beta^2\gamma\ +klE_1E_2E_3E_4\alpha\beta\gamma\delta\ -klE_1E_2E_3E_4\alpha\beta\gamma\delta$$
$$+knE_1E_3E_4^2\alpha\beta(\beta-1)\gamma\quad +klE_1E_2E_3E_4\alpha\beta\gamma\delta\quad -klE_1E_2E_3E_4\alpha\beta\gamma\delta\quad +nlE_2E_3E_4^2\alpha(\alpha-1)\beta\gamma$$
$$-l^2E_2^2E_3E_4\alpha(\alpha-1)\gamma\delta\ +l^2E_2^2E_3E_4\alpha^2\gamma\delta\ -nlE_2E_3E_4^2\alpha^2\beta\gamma\ -mE_1E_3^2E_4\alpha\beta^2\delta\ +kmE_1E_3^2E_4\alpha\beta(\beta-1)\delta$$
$$-mnE_3^2E_4^2\alpha(\alpha-1)\beta(\beta-1)+lmE_2E_3^2E_4\alpha(\alpha-1)\beta\delta\ -lmE_2E_3^2E_4\alpha^2\beta\delta\ +mnE_3^2E_4^2\alpha^2\beta^2\}$$

$$=\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}}{E_1^2E_2^2E_3E_4}\quad\{-2\alpha\beta\gamma\quad +2(\alpha-1)\beta\gamma\quad +2\alpha(\beta-1)\gamma\quad -(\alpha-1)(\beta-1)\gamma\quad +2\alpha\beta(\gamma-1)$$
$$-\alpha(\beta-1)(\gamma-1)\ -(\alpha-1)\beta(\gamma-1)\ +(2E_4-1)(\alpha-1)(\beta-1)(\gamma-1)\ -(2E_4-1)\alpha\beta(\gamma-1)\ +(2E_4-1)\alpha\beta\gamma$$
$$-(2E_4-1)\alpha(\beta-1)\gamma\}\qquad +\frac{1}{|J|}\frac{A^3\alpha\beta\gamma\delta E_1^{3\alpha}E_2^{3\beta}E_3^{3\gamma}E_4^{3\delta}}{E_1^2E_2^2E_3E_4}\qquad\{-(2E_4-1)\alpha\beta\qquad +(2E_4-1)\alpha(\beta-1)$$
$$+(2E_4-1)(\alpha-1)\beta\ -(2E_4-1)(\alpha-1)(\beta-1)\}$$

$$= \frac{1}{|J|} \frac{A^3 \alpha\beta\gamma\delta E_1^{3\alpha} E_2^{3\beta} E_3^{3\gamma} E_4^{3\delta}}{E_1^2 E_2^2 E_3 E_4} \{E_4(-2\beta\gamma + 2\alpha + 2\beta + 2\gamma - 4) + \alpha\beta\gamma - \beta\gamma - 2\alpha - 2\beta - 2\gamma + 2\}$$

$$\frac{\partial E_4}{\partial m} = -\frac{1}{|J|} \frac{A^3 \alpha\beta\gamma\delta E_1^{3\alpha} E_2^{3\beta} E_3^{3\gamma} E_4^{3\delta}}{E_1^2 E_2^2 E_3 E_4} (168 E_4 - 29). \quad (29)$$

If $E_4 > 29/168$ in (29) we get,

$$\frac{\partial E_4}{\partial m} < 0. \quad (30)$$

From the equation (30) we see that if per unit cost of principal raw material increases, the tendency of purchasing irregular inputs E_4 decreases. It seems that irregular input is complementary to principal raw material. Therefore, the price of E_3 goes up, citizens of the country buy less of it, and consequently level of consumption of E_4 also decreases [Islam et al., 2010; Mohajan, 2021a; Ferdous & Mohajan, 2022].

8. Conclusions

In this study we have discussed the economic effects of various inputs of an industry if the cost of principal raw material is increased. In this paper we have considered nonlinear budget constraint to provide economic predictions though the profit maximization. In the study we have included Cobb-Douglas productions function as our profit function. We have used 5×5 bordered Hessian matrix and 5×5 Jacobian to operate the mathematical formulations.

References

Babbie, E. R. (2017). *The Basics of Social Research*. Cengage Learning, Boston, MA, USA.

Baxley, J. V., & Moorhouse, J. C. (1984). Lagrange Multiplier Problems in Economics. *The American Mathematical Monthly*, 91(7), 404-412.

Bhattacharjee, N. R., Das, S., & Mohajan, H. K. (2014). Andrews-Garvan-Liang's sptcrank for Marked Overpartitions. *International Journal of Applied Science*, 1(2), 71–81.

Carter, M. (2001). *Foundations of Mathematical Economics*. MIT Press, Cambridge, London.

Chiang, A. C. (1984). *Fundamental Methods of Mathematical Economics* (3rd Ed.). Singapore: McGraw-Hill.

Cobb, C. W., & Douglass, P. H. (1928). A Theory of Production. *American Economics Review*, 18(1), 139-165.

Eaton, B., & Lipsey, R. (1975). The Principle of Minimum Differentiation Reconsidered: Some New Developments in the Theory of Spatial Competition. *Review of Economic Studies*, 42(1), 27-49.

Ferdous, J., & Mohajan, H. K. (2022). Maximum Profit Ensured for Industry Sustainability. *Annals of Spiru Haret University. Economic Series*, 22(3), 317-337.

Groh, A. (2018). Research Methods in Indigenous Contexts. New York: Springer.

Hossain, F., Das, S., & Mohajan, H. K. (2014). Bingmann-Lovejoy-Osburn's Generating Function in the Overpartitions. *Open Science Journal of Mathematics and Application*, 2(4), 37–43.

Hossain, F., Das, S., & Mohajan, H. K. (2015). The Rogers-Ramanujan Identities. *Turkish Journal of Analysis and Number Theory*, 3(2), 37–42.

Howell, K. E. (2012). An Introduction to the Philosophy of Methodology. Sage Publications Ltd.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2009a). Preference of Social Choice in Mathematical Economics. *Indus Journal of Management & Social Sciences*, 3(1), 17-38.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2009b). Political Economy and Social Welfare with Voting Procedure. *KASBIT Business Journal*, 2(1), 42-66.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2010). Utility Maximization Subject to Multiple Constraints. *Indus Journal of Management & Social Sciences*, 4(1), 15-29.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2011a). Median Voter Model cannot Solve all the Problems of Voting System. *International Journal of Economics and Research*, 2(6), 111–125.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2011b). Output Maximization Subject to a Nonlinear Constraint. *KASBIT Business Journal*, 4(1), 116-128.

Islam, J. N., Mohajan, H. K., & Moolio, P. (2011c). Methods of Voting System and Manipulation of Voting. *International Journal of Management and Transformation*, 5(1), 10–34.

Kothari, C. R. (2008). *Research Methodology: Methods and Techniques* (2nd Ed.). New Delhi: New Age International (P) Ltd.

Mohajan, D., & Mohajan, H. K. (2022a). Profit Maximization Strategy in an Industry: A Sustainable Procedure. *Law and Economy*, 1(3), 17-43. https://doi:10.56397/LE.2022.10.02

Mohajan, D. & Mohajan, H. K. (2022b). Sensitivity Analysis among Commodities and Coupons during Utility Maximization. Frontiers in Management Science, 1(3), 13-28.

Mohajan, D., & Mohajan, H. K. (2022c). Importance of Total Coupon in Utility Maximization: A Sensitivity Analysis. *Law and Economy*, 1(5), 65-67.

Mohajan, D., & Mohajan, H. K. (2022d). Utility Maximization Analysis of an Emerging Firm: A Bordered Hessian Approach. *Annals of Spiru Haret University. Economic Series*, 22(4), 292-308.

Mohajan, D., & Mohajan, H. K. (2022e). Sensitivity Analysis among Commodities and Coupons during Utility Maximization. *Frontiers in Management Science*, 1(3), 13-28.

Mohajan, D., & Mohajan, H. K. (2022f). Importance of Total Coupon in Utility Maximization: A Sensitivity Analysis. *Law and Economy*, 1(5), 65–67.

Mohajan, D., & Mohajan, H. K. (2023a). Utility Maximization Analysis of an Organization: A Mathematical Economic Procedure. *Law and Economy*, 2(1), 1-15.

Mohajan, D., & Mohajan, H. K. (2023b). Utility Maximization Investigation: A Bordered Hessian Method. *Annals of Spiru Haret University. Economic Series*, Manuscript Submitted.

Mohajan, D. & Mohajan, H. K. (2023c). Sensitivity Analysis among Commodities and Prices: Utility Maximization Perspectives. Unpublished Manuscript.

Mohajan, D., & Mohajan, H. K. (2023d). Effect of Various Inputs When Budget of an Organization Increases: A Profit Maximization Study. *Noble International Journal of Economics and Financial Research*, 7(4),

Mohajan, D., & Mohajan, H. K. (2023e). A Study on Nonlinear Budget Constraint of a Local Industrial Firm of Bangladesh: A Profit Maximization Investigation. *Law and Economy*, 2(5), 27-33.

Mohajan, D., & Mohajan, H. K. (2023f). Sensitivity Analysis for Profit Maximization with Respect to Per Unit Cost of other Raw Materials. Unpublished Manuscript.

Mohajan, D., & Mohajan, H. K. (2023g). Sensitivity Analysis of Inputs of an Organization: A Profit Maximization Exploration. Manuscript Submitted.

Mohajan, D., & Mohajan, H. K. (2023h). Bulimia Nervosa: A Psychiatric Problem of Disorder. *Innovation in Science and Technology*, 2(3), 26-32.

Mohajan, D., & Mohajan, H. K. (2023i). Abdominal Elephantiasis: An Obstructive Disease Due to Extreme Obesity. *Journal of Innovations in Medical Research*, 2(7), 13-15.

Mohajan, D., & Mohajan, H. K. (2023j). Panniculus Morbidus: A New Global Health Crisis Due to Extreme Obesity. Unpublished Manuscript.

Mohajan, D., & Mohajan, H. K. (2023k). Long-Term Regular Exercise Increases $\dot{V}O_2$max for Cardiorespiratory Fitness. *Innovation in Science and Technology*, 2(2), 38-43.

Mohajan, D., & Mohajan, H. K. (2023l). Broca Index: A Simple Tool to Measure Ideal Body Weight. *Innovation in Science and Technology*, 2(2), 21-24.

Mohajan, D., & Mohajan, H. K. (2023m). Obesity and Its Related Diseases: A New Escalating Alarming in Global Health. *Journal of Innovations in Medical Research*, 2(3), 12-23.

Mohajan, D., & Mohajan, H. K. (2023n). Body Mass Index (BMI) is a Popular Anthropometric Tool to Measure Obesity among Adults. *Journal of Innovations in Medical Research*, 2(4), 25-33.

Mohajan, D., & Mohajan, H. K. (2023o). A Study on Body Fat Percentage for Physical Fitness and Prevention of Obesity: A Two Compartment Model. *Journal of Innovations in Medical Research*, 2(4), 1-10.

Mohajan, D., & Mohajan, H. K. (2023p). Ponderal Index: An Important Anthropometric Indicator for Physical Growth. *Journal of Innovations in Medical Research*, 2(6), 15-19.

Mohajan, D., & Mohajan, H. K. (2023q). Anorexia Nervosa: A Dreadful Psychosocial Health Complication. Unpublished Manuscript.

Mohajan, D., & Mohajan, H. K. (2023r). Discovery of Insulin is a Great Achievement for the Diabetes Patients. *Studies in Social Science & Humanities*, 2(8), 8-16.

Mohajan, D., & Mohajan, H. K. (2023s). Basic Concepts of Diabetics Mellitus for the Welfare of General Patients. *Studies in Social Science & Humanities*, 2(6), 23-31.

Mohajan, D., & Mohajan, H. K. (2023t). Sodium-Glucose Cotransporter-2 (SGLT-2) Inhibitors: Antidiabetics Medications for Treating Diabetes. Unpublished Manuscript.

Mohajan, D., & Mohajan, H. K. (2023u). Mathematical Model for Nonlinear Budget Constraint: Economic Activities on Increased Budget. *Studies in Social Science & Humanities*, 2(5), 20-40.

Mohajan, D., & Mohajan, H. K. (2023v). Sulfonylureas: A Widely Used Oral Anti-Hyperglycaemic Medication for Type 2 Diabetes Management. Unpublished Manuscript.

Mohajan, D., & Mohajan, H. K. (2023w). Oral Hypoglycaemic Agents: Non-Insulin Medications for Type 2 Diabetes Patients. Unpublished Manuscript.

Mohajan, D., & Mohajan, H. K. (2023x). Effects of Metformin among Type 2 Diabetes Pregnant Women: A Preliminary Study. Unpublished Manuscript.

Mohajan, D., & Mohajan, H. K. (2023y). Metformin: An Oral Anti-hyperglycaemic Agent for the Treatment of Type 2 diabetes. Unpublished Manuscript.

Mohajan, D., & Mohajan, H. K. (2023z). Management of Type-I Diabetes: A Right Procedure to Normal Life Expectancy. Unpublished Manuscript.

Mohajan, D., & Mohajan, H. K. (2023A). Prevention and Management Strategies of Pre-diabetes. Peroxisome Proliferator-Activated Receptor γ (PPAR γ). Unpublished Manuscript.

Mohajan, D., & Mohajan, H. K. (2023B). Diabetic Ketoacidosis (DKA): A Severe Diabetes Mellitus Disorder. Unpublished Manuscript.

Mohajan, D., & Mohajan, H. K. (2023C). Hypoglycaemia among Diabetes Patients: A Preventive Approach. *Journal of Innovations in Medical Research*, 2(9), 29-35.

Mohajan, D., & Mohajan, H. K. (2023D). Hyperglycaemia among Diabetes Patients: A Preventive Approach. Unpublished Manuscript.

Mohajan, D., & Mohajan, H. K. (2023E). Dipeptidyl Peptidase-4 (DPP-4) Inhibitors: New Oral Medications for the Treatment of Type 2 Diabetes. Unpublished Manuscript.

Mohajan, D., & Mohajan, H. K. (2023F). Historical View of Diabetics Mellitus: From Ancient Egyptian Polyuria to Discovery of Insulin. *Studies in Social Science & Humanities*, 2(7), 26-34.

Mohajan, D., & Mohajan, H. K. (2023G). An Economical Study When Cost of Irregular Raw Materials of an Industry Increases for Nonlinear Budget Constraint. *Law and Economy*, 2(7), 24-43.

Mohajan, D., & Mohajan, H. K. (2023H). Effects of Various Inputs for Increased Interest Rate of Capital: A Nonlinear Budget Constraint Consideration. *Frontiers in Management Science*, 2(4), 15-33.

Mohajan, D., & Mohajan, H. K. (2023I). Economic Investigation of Lagrange Multiplier if Cost of Inputs and Budget Size of a Firm Increase: A Profit Maximization Endeavor. *Annals of Spiru Haret University*. *Economic Series*, 23(2), 340-364.

Mohajan, D., & Mohajan, H. K. (2023J). Bronze Diabetes: A Common Genetic Disorder Due to Systemic Iron Overload. *Journal of Innovations in Medical Research*, 2(8), 1-7.

Mohajan, D., & Mohajan, H. K. (2023K). The Responses of an Organization for the Increase in Wage Rates: Profit Maximization Cases. *Law and Economy*, 2(8), 14-29.

Mohajan, D., & Mohajan, H. K. (2023L). Various Problems Arise in Industrial Economics If Wage Rate Increases: A Study for Nonlinear Budget Constraint. *Law and Economy*, 2(6), 1-19.

Mohajan, D., & Mohajan, H. K. (2023M). Economic Aspects of Profit Maximization if Cost of Principal Raw Material Increases. *Frontiers in Management Science*, 2(3), 28-42.

Mohajan, H. K. (2012a). The Lease Financing in Bangladesh: A Satisfied Progress in Business and Industrialization. *International Journal of Finance and Policy Analysis*, 4(1), 9-24.

Mohajan, H. K. (2012b). Aspects of Green Marketing: A Prospect for Bangladesh. *International Journal of Economics and Research*, 3(3), 1-11.

Mohajan, H. K. (2013a). Friedmann, Robertson-Walker (FRW) Models in Cosmology. *Journal of Environmental Treatment Techniques*, 1(3), 158-164.

Mohajan, H. K. (2013b). Economic Development of Bangladesh. *Journal of Business Management and Administration*, 1(4), 41-48.

Mohajan, H. K. (2013c). Ethiopia: A Socio-economic Study. *Journal of Business Management and Administration*, 1(5), 59-74.

Mohajan, H. K. (2014). Chinese Sulphur Dioxide Emissions and Local Environment Pollution. *International Journal of Scientific Research in Knowledge*, 2(6), 265-276.

Mohajan, H. K. (2015a). Sustainable Development Policy of Global Economy. *American Journal of Environmental Protection*, 3(1), 12-29.

Mohajan, H. K. (2015b). Present and Future of Nestlé Bangladesh Limited. *American Journal of Food and Nutrition*, 3(2), 34-43.

Mohajan, H. K. (2016a). An Analysis of Knowledge Management for the Development of Global Health. *American Journal of Social Sciences*, 4(4), 38-57.

Mohajan, H. K. (2016b). A Comprehensive Analysis of Knowledge Management Cycles. *Journal of Environmental Treatment Techniques*, 4(4), 121-129.

Mohajan, H. K. (2017a). Optimization Models in Mathematical Economics. *Journal of Scientific Achievements*, 2(5), 30-42.

Mohajan, H. K. (2017b). Two Criteria for Good Measurements in Research: Validity and Reliability. *Annals of Spiru Haret University Economic Series*, 17(3), 58-82.

Mohajan, H. K. (2018a). Qualitative Research Methodology in Social Sciences and Related Subjects. *Journal of Economic Development, Environment and People*, 7(1), 23-48.

Mohajan, H. K. (2018b). *Aspects of Mathematical Economics, Social Choice and Game Theory*. PhD Dissertation, Jamal Nazrul Islam Research Centre for Mathematical and Physical Sciences (JNIRCMPS), University of Chittagong, Chittagong, Bangladesh.

Mohajan, H. K. (2020). Quantitative Research: A Successful Investigation in Natural and Social Sciences. *Journal of Economic Development, Environment and People*, 9(4), 52-79.

Mohajan, H. K. (2021a). Estimation of Cost Minimization of Garments Sector by Cobb-Douglass Production Function: Bangladesh Perspective. *Annals of Spiru Haret University Economic Series*, 21(2), 267-299. doi: https://doi.org/10.26458/21212

Mohajan, H. K. (2021b). Product Maximization Techniques of a Factory of Bangladesh: A Sustainable Procedure. *American Journal of Economics, Finance and Management*, 5(2), 23-44.

Mohajan, H. K. (2021c). Utility Maximization of Bangladeshi Consumers within Their Budget: A Mathematical Procedure. *Journal of Economic Development, Environment and People*, 10(3), 60-85.

Mohajan, H. K. (2022a). Cost Minimization Analysis of a Running Firm with Economic Policy. *Annals of Spiru Haret University. Economic Series*, 22(3), 317-337.

Mohajan, H. K. (2022b). An Overview on the Feminism and Its Categories. *Research and Advances in Education*, 1(3), 11-26.

Mohajan, H. K., Islam, J. N., & Moolio, P. (2013). *Optimization and Social Welfare in Economics*. Lambert Academic Publishing, Germany.

Moolio, P., Islam, J. N., & Mohajan, H. K. (2009). Output Maximization of an Agency. *Indus Journal of Management and Social Sciences*, 3(1), 39-51.

Pandey, P., & Pandey, M. M. (2015). *Research Methodology: Tools and Techniques*. Bridge Center, Romania, European Union.

Polit, D. F., & Hungler, B. P. (2013). *Essentials of Nursing Research: Methods, Appraisal, and Utilization* (8th Ed.). Philadelphia: Wolters Kluwer/Lippincott Williams and Wilkins.

Rahman, M. M., & Mohajan, H. K. (2019). Rohingya-The Stateless Community Becoming the Lost Generation. *Journal of Economic Development, Environment and People,* 8(2), 24–36.

Roy, L., Molla, R., & Mohajan, H. K. (2021). Cost Minimization is Essential for the Sustainability of an Industry: A Mathematical Economic Model Approach. *Annals of Spiru Haret University Economic Series*, 21(1), 37-69.

Samuelson, P. A. (1947). *Foundations of Economic Analysis*. Harvard University Press, Cambridge, MA.

Schwandt, T. A. (2014). *Methodology*. Sage Publications Ltd.

Somekh, B., & Lewin, C. (2005). *Research Methods in the Social Sciences*. Sage Publications Ltd.

Wiese, H. (2021). Cost Minimization and Profit Maximization. In Advanced Microeconomics. Springer Gabler, Wiesbaden. https://doi.org/10.1007/978-3-658-34959-2_9