Show Me the Money. Why Neglecting Money in Monetary Theory and Policy is a Bad Idea

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22 October 2023

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Abstract

This paper discusses numerous and serious conceptual criticisms of arguments and theories that consider that inflation and the price level are exclusively a fiscal phenomenon in which money plays no distinctive role. The price level, substantial acceleration of the inflation rate or sustained inflation rates of two digits or more cannot be explained by expectations or changes in expectations alone as Sargent (1982), Woodford (2008) and the FTPL proponents claim. The empirical evidence obtained using cointegration and error correction models estimated using linear and non-linear techniques provides robust indication that money plays a crucial role in understanding the long-run evolution of the price level and the short-run dynamics of inflation.

JEL N° E31, E52.

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Introduction

This paper is a new attempt to defend the Quantity Theory and the role of money in the determination of the price level and the rate of inflation. In *The role of money in economies with monetary policy regimes that ignore monetary aggregates* (Olivo, 2012), I focused mainly on price level determinacy. From a theoretical point of view, the dynamic Aggregate-Demand / Aggregate-Supply (AD/AS) model that I used as a framework produces the typical results that control of a monetary aggregate generates price level determinacy under conditions that are not very restrictive, while under an interest rate peg the price level is indeterminate. An interest rate rule that reacts to expected inflation also leaves the price level indeterminate in this AD/AS framework. From an empirical point of view, I tried to assess the relative importance of money against interest rate in explaining the evolution of the price level in six countries: Australia, Canada, Chile, South Korea, New Zealand, and the United States. I first pooled quarterly data for these countries for different periods from the 1990s up to 2007, and then proceed to a country-by-country analysis. The selection of these countries was primarily motivated by the fact that their central banks did not consider monetary aggregates in their monetary policy strategies during the period under study. The paper relies on single equations models and simple VAR models. I summarize the results with single equation models that appear more robust. Both —with panel data and individual countries' series—monetary aggregates have, in most cases, a positive and statistically significant impact on the price level: Panel (M1), Australia (M2), Chile (M1), Korea (Reserve Money), New Zealand (Reserve Money), and the USA (Reserve Money). The short-run interest rate was not statistically significant or exhibited a positive and statistically significant influence on the price level consistent with the so call "price puzzle" (Chile, Korea, and New Zealand). Although the time span of the empirical models is not enough for a long-run analysis, they capture a glimpse of the operation of the Quantity Theory. The positive relationship between monetary aggregates and the price level is a result expected from the Quantity Theory, while the nominal short-run interest rate has no impact on the price level or a positive effect that has no theoretical support. Thus, my conclusion was that the Quantity Theory continued to be relevant and that monetary policy strategies should not ignore completely the behavior of monetary aggregates.

Ignoring monetary aggregates during the period of low inflation between 2000 and 2020 might be somewhat understandable. However, that the profession has continued to neglect money after the resurgence of inflation in 2021 is simply stunning. For example, Finance & Development, the publication of the International Monetary Fund (IMF) intended to reach a broader audience outside the economics profession titled its March 2023 issue *New Directions for Monetary Policy*. None of the main ten articles included in the issue give any major role to money as an explanation of the resurgence of inflation in 2021, or as a variable that should be considered in models for monetary policy. There are two articles in the issue that I find especially remarkable. In *How we missed the recent inflation surge* (Christoffer Koch and Diaa Noureldin) state that: "Despite our repeated revisions to the inflation forecasts between the first quarter of 2021 and the second quarter of 2022, misses have been sizable and persistent. These inflation surprises preceded the Russian invasion of Ukraine." The behavior of the money supply never crosses the mind of Koch and Noureldin as a possible cause for the failure of their model (or models) to predict the reemergence of inflation in 2021. The second article, *The Very Model of Modern Monetary Policy* (Greg Kaplan, Benjamin Moll, and Giovanni L. Violante) holds that the future of monetary policy modeling rests in the development of HANK models that combine heterogeneous agent models, which capture income and wealth distribution, with New Keynesian models, which are the basic framework for studying monetary policy and movements in aggregate demand. Thus, these authors argue that we should proceed to study the redistributive aspects of inflation with a model that cannot either explain or predict inflation. We can go on and on with examples of theories that ignore money, from the attempt to resuscitate the Fiscal Theory of the Price Level (FTPL) to attribute the return of inflation to an "unbacked fiscal shock" (whatever that means), or even pure and simple greed.

This document aligns with the minority camp represented by King (2022) and Borio et al (2023) that considers that money continues to be key to understand inflation and therefore, in the design and implementation of monetary policy. However, it is worth noticing that this crucial role of money in the determination of inflation derives from its key role in the determination of the price level. After critically examining several approaches that downplay the role of money, the paper analyzes and supports the role of monetary aggregates in the determination of the price level and inflation both in the long run and short run using annual data from 1960 to 2021-22 (1950-2019 for Venezuela).

The document is organized into four sections plus conclusions. In section 1, I examine Sargent (1982) paper on *The Ends of Four Big Inflations*. This is one of the pioneer articles in the wave of neglecting money in macroeconomic analysis. I present econometric results using the same data that Sargent discusses to show that his dismissal of money based on the observations toward the end of the inflationary episodes is misleading. In section 2, I discuss Woodford's (2008) position that both inflation and the price level (in that order) can be completely determined without any consideration of the money supply. I develop several theoretical and empirical arguments against Woodford's contentions. Section 3 presents the basic elements of the Fiscal Theory of the Price Level (FTPL) and a detailed discussion of its numerous theoretical and empirical limitations. Section 4 contains the presentation of the main results from the cointegration and Error Correction Models estimated for eight countries (Argentina, Brazil, Colombia, Mexico, Turkey, Sweden, United States, and Venezuela) using linear and non-linear techniques.

1. Sargent's The Ends of Four Big Inflations

Interestingly, the current view about the irrelevance of money in macroeconomics and monetary policy did not start from the Keynesian front. In *The Ends of Four Big Inflations*, Sargent (1982) argues that: *"people expect high rates of inflation in the future precisely because the government's current and prospective monetary and fiscal policies warrant those expectations. Further, the current rate of inflation and people's expectations about future rates of inflation may seem to respond slowly to isolated actions of restrictive monetary and fiscal policy that are viewed as temporary departures from what is perceived as a long-term government policy involving high average rates of government deficits and monetary expansion in the future*."

Sargent's (1982) paper contains abundant data distributed in many tables throughout the text, but there is no attempt to explore formally the interrelation among the variables described. In the case of fiscal variables such as revenues, expenditures and deficits, the data is very limited, and only includes semi-annual and annual observations. But the data on prices and monetary aggregates available monthly can be used to explore the inflationary events in more detail. Instead, to support his contention that what matters is the perception of agents regarding fiscal and monetary policy in the future, Sargent put special emphasis on the relation of inflation and money growth towards the end of the inflationary episodes:

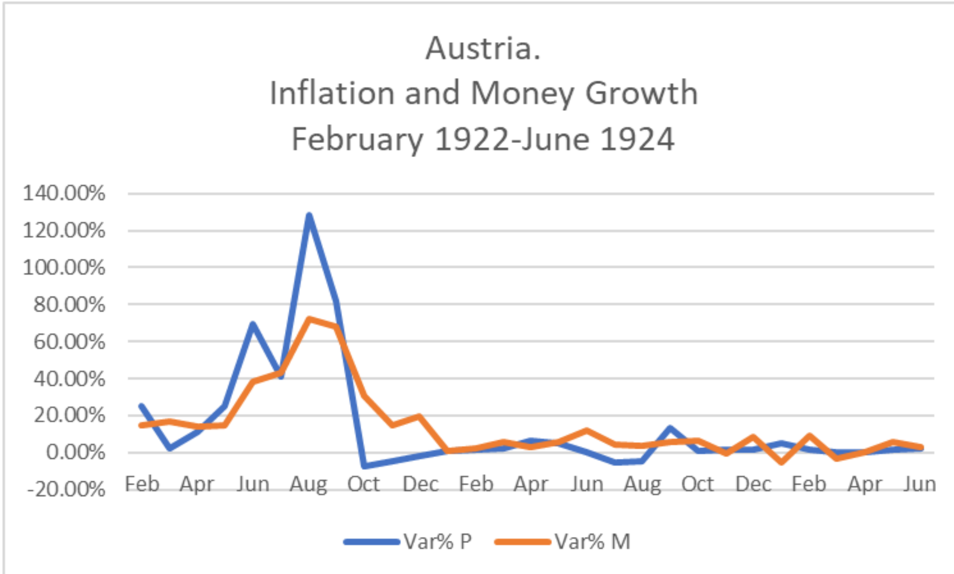
"Table A4 reveals that the Austrian crown abruptly stabilized in August 1922, while table A3 indicates that prices abruptly stabilized a month later. This occurred despite the fact that the central bank's note circulation continued to increase rapidly, as table Al indicates."

"Table H3 indicates that in March 1924, the rise in prices and the depreciation of the krone internationally both abruptly halted. The stabilization occurred in the face of continued expansion in the liabilities of the central bank, which increased by a factor of 3.15 between March 1924 and January 1925 (see table H2). This pattern parallels what occurred in Austria and has a similar explanation."

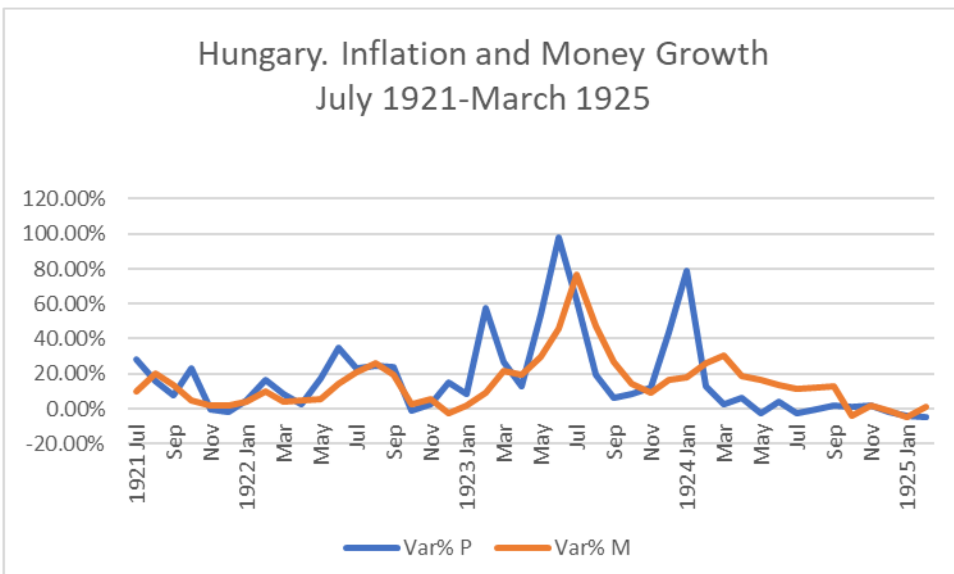
"Table P2 reveals that, from January 1924 to December 1924, the note circulation of the central bank increased by a factor of 3.2, in the face of relative stability of the price level and the exchange rate (see tables P3 and P4). This phenomenon matches what occurred in Austria and Hungary and has a similar explanation."

If one graphs the monthly data examined in Sargent (1982) for Austria, Hungary, Poland, and Germany, it can be easily seen that in the last stages of the inflationary episodes, money growth was also rapidly declining. But what is most notorious is Sargent's omission of the data before the international interventions and agreements that allowed these countries to stop the monetary financing of their fiscal deficits. Graphs 1 to 4 show clearly the close relationship between inflation and money growth during the entire hyperinflationary events.

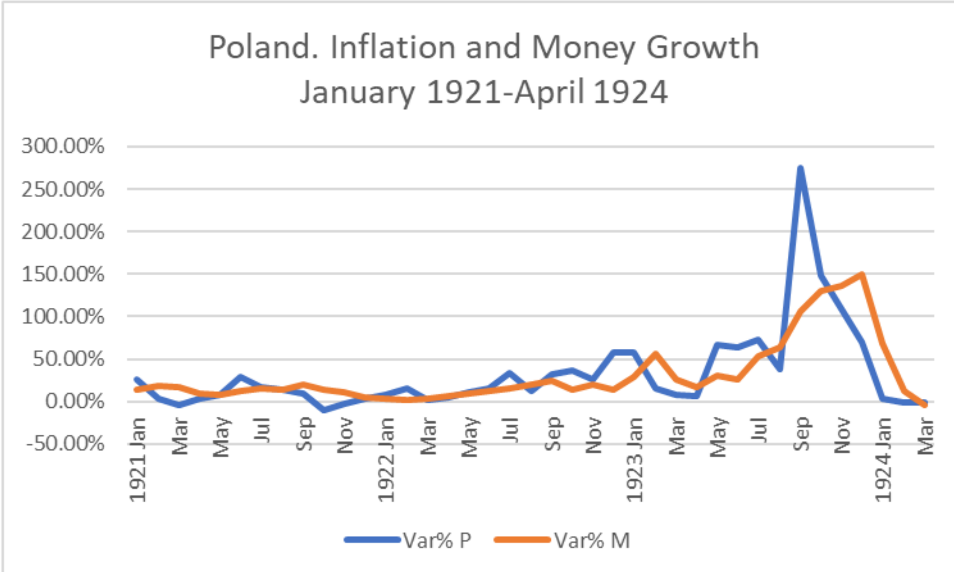
Graph 1



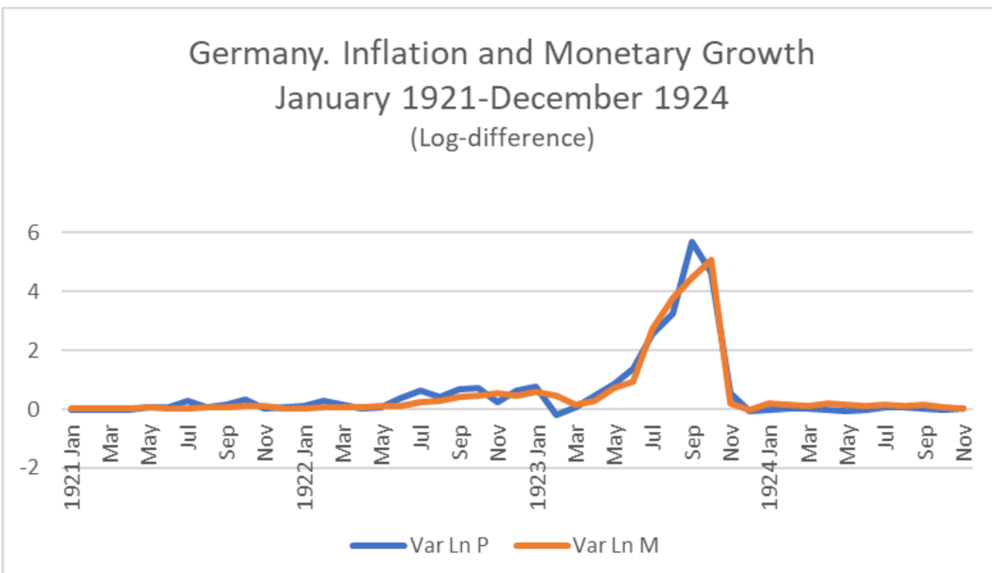
Graph 2



Graph 3



Graph 4



I also constructed a panel data set with the price level and money aggregates data contained in Sargent (1982). The result of estimating a fixed-effects regression (with a common intercept) between the monthly inflation rate (log-difference of the price level; ld_P) against the growth rate of money (ld_M) is shown in Table 1. The coefficient of ld_M is one and statistically different from zero (p-value<0.0001), and the coefficient of determination of the regression is 0.91.

Tabla 1

Model : Fixed-effects, using 159 observations
Included 4 cross-sectional units
Time-series length: minimum 29, maximum 47
Dependent variable: ld_P
Robust (HAC) standard errors

	Coefficient	*Std. Error*	*z*	*p-value*	
const	0.00569063	0.00208767	2.726	0.0064	***
ld_M	1.00724	0.00781505	128.9	<0.0001	***

Mean dependent var	0.274760	S.D. dependent var	0.684142
Sum squared resid	6.411714	S.E. of regression	0.204045
LSDV R-squared	0.913299	Within R-squared	0.907140
Log-likelihood	29.64559	Akaike criterion	−49.29117
Schwarz criterion	−33.94665	Hannan-Quinn	−43.05993
rho	−0.028485	Durbin-Watson	2.046419

Joint test on named regressors -
Test statistic: F(1, 3) = 16611.4
with p-value = P(F(1, 3) > 16611.4) = 1.02984e-06

Robust test for differing group intercepts -
Null hypothesis: The groups have a common intercept
Test statistic: Ilch F(3, 83.6) = 0.398343
with p-value = P(F(3, 83.6) > 0.398343) = 0.754534

My conclusion from this more formal examination of the data contained in Sargent (1982) is that although rational agents may consider the future evolution of fiscal and monetary policy in forming their inflation expectations, the contemporaneous evolution of the rate of growth of the money supply is the key variable that determines the behavior of inflation in episodes of very high inflation and hyperinflation.

1.1. Hyperinflation in Venezuela and fiscal adjustment

The close contemporaneous correlation between inflation and money growth can also be seen in a most recent hyperinflation episode observed in Venezuela during 1918-1920. This is illustrated in the following tables (2 to 5) extracted from Olivo (2021), that show frequency distributions, summary statistics and a linear regression using monthly data for the period

2017.12-2020.1 for the CPI inflation rate (vipc) and the rate of growth of the monetary base (vbm).

Table 2. Venezuela. Frequency distribution of the rate of inflation

Frequency distribution for vipc, obs 85-110
number of bins = 11, mean = 67.3603, sd = 42.4411

interval	midpt	frequency	rel.	cum.	
< 28.244	19.382	4	15.38%	15.38%	*****
28.244 - 45.969	37.107	7	26.92%	42.31%	*********
45.969 - 63.694	54.832	4	15.38%	57.69%	*****
63.694 - 81.419	72.557	3	11.54%	69.23%	****
81.419 - 99.144	90.282	3	11.54%	80.77%	****
99.144 - 116.87	108.01	2	7.69%	88.46%	**
116.87 - 134.59	125.73	2	7.69%	96.15%	**
134.59 - 152.32	143.46	0	0.00%	96.15%	
152.32 - 170.04	161.18	0	0.00%	96.15%	
170.04 - 187.77	178.91	0	0.00%	96.15%	
>= 187.77	196.63	1	3.85%	100.00%	*

Table 3. Venezuela. Frequency distribution of the rate of growth of the monetary base

Frequency distribution for vbm, obs 85-110
number of bins = 11, mean = 55.6876, sd = 32.5754

interval	midpt	frequency	rel.	cum.	
< 13.120	6.5599	1	3.85%	3.85%	*
13.120 - 26.240	19.680	3	11.54%	15.38%	****
26.240 - 39.360	32.800	4	15.38%	30.77%	*****
39.360 - 52.479	45.919	6	23.08%	53.85%	********
52.479 - 65.599	59.039	5	19.23%	73.08%	******
65.599 - 78.719	72.159	1	3.85%	76.92%	*
78.719 - 91.839	85.279	1	3.85%	80.77%	*
91.839 - 104.96	98.399	2	7.69%	88.46%	**
104.96 - 118.08	111.52	2	7.69%	96.15%	**
118.08 - 131.20	124.64	0	0.00%	96.15%	
>= 131.20	137.76	1	3.85%	100.00%	*

Table 4

Summary Statistics, using the observations 2017:12 - 2020:01

Variable	Mean	Median	S.D.	Min	Max
vbm	55.7	47.5	32.6	0.0802	131.
vipc	67.4	55.7	42.4	19.4	197.
vs2	66.2	48.6	69.5	-17.9	272.

Table 5

OLS, using observations 2017:12-2020:01 (T = 26)
Dependent variable: vipc
HAC standard errors, bandwidth 2 (Bartlett kernel)

	Coefficient	*Std. Error*	*z*	*p-value*	
const	11.7415	10.4508	1.124	0.2612	
vbm	0.998764	0.173747	5.748	<0.0001	***

Mean dependent var	67.36027	S.D. dependent var	42.44106
Sum squared resid	18567.78	S.E. of regression	27.81470
R-squared	0.587667	Adjusted R-squared	0.570487
F(1, 24)	33.04406	P-value(F)	6.35e-06
Log-likelihood	−122.3165	Akaike criterion	248.6331
Schwarz criterion	251.1492	Hannan-Quinn	249.3576
rho	−0.095535	Durbin-Watson	2.130419

But probably, the most interesting aspect of the Venezuelan hyperinflation experience for our present discussion is the rapid reduction in the inflation rate observed since 2019. As can be seen in table 6, annual inflation reached a peak of 130,060% in 2018 when the growth of M1 was 63,385%. Inflation fell rapidly to 9,586% in 2019 when M1 grew 4,951%, and in 2021 inflation and money growth were already below the values of 2017 (686.4% and 635.2%, respectively). This steep reduction in inflation occurred in the context of a very opaque fiscal adjustment forced by the rapid decline of seigniorage revenues, without any major institutional reform, no international financial support, and the continuing default on the US$ 160 billion of public sector foreign debt (more than 300% of the country estimated GDP in US dollars). Thus, a substantial reduction in money growth attained a strong decline in inflation without structural modifications in the fiscal and monetary institutions of the country.

Table 6

Venezuela Inflation and Money Growth		
	Var% CPI	Var% M1
2017	862.6	1,129.6
2018	130,060.2	63,384.9
2019	9,585.5	4,951.4
2020	2,959.8	1,287.1
2021	686.4	635.2
2022	234.1	353.8

Source: Central Bank of Venezuela and author's own calculations

Of course, a sustainable reduction of inflation toward levels consistent with price stability is not possible without a strong macroeconomic program that includes reforms that promote fiscal and monetary discipline in the present and the future. But such a program must produce a rapid reduction in the rate of growth of the money supply even in a context where the demand for money starts to recover. Similar to the experiences described by Sargent in the European post World War I hyperinflations, it is possible that the Venezuelan economy will need a transition period until foreign resources are available to finance the fiscal deficits that will be inevitable for some time until a macroeconomic program starts to repair the systematic destruction that the economy has endured for many years. During this transition period, the monetary financing of the fiscal deficit may be necessary and a high rate of growth of the money supply may persist, but that period must be very short. This monetary financing should be given under very precise and transparent conditions and repaid to the central bank once the government has received external financing. Any attempt to prolong this transition period could substantially diminish the credibility of a program and its effectiveness.

2. The absence of money in the New Keynesian models

This section is mainly based on the article by Michael Woodford (Woodford, 2008) *How Important is Money in the Conduct of Monetary Policy?* which is probably the most elaborate presentation in defense of the structure of the New Keynesian model that completely ignores money.[1]

2.1. The historical significance of monetarism

Woodford's (2008) article begins by acknowledging monetarism at least two important lessons regarding the conduct of monetary policy that remain current:

-Monetarism established that monetary policy can do something about inflation, and that the central bank can be reasonably responsible for controlling this variable.

-Monetarism emphasized a verifiable commitment by the central bank to an anti-inflationary policy. The monetarists were the first to stress the importance of containing inflationary expectations, and to stress the role that a commitment to a policy rule could play in creating the kind of expectations necessary for macroeconomic stability. Research from the past few decades has only added further support to these claims.

But Woodford affirms emphatically that none of these Monetarist recommendations depends on the thesis about the importance of monetary aggregates in the conduct of monetary policy. Therefore, Woodford considers that the "two pillars" strategy followed by the European Central Bank (ECB), in which monetary aggregates continue to play a relevant role, is not justified from the perspective of the lessons derived from monetarism.[2]

2.2. Can inflation be understood without money?

Woodford (2008) addresses whether it is possible to understand inflation without money. To develop this theme, Woodford starts from a standard forward-looking New Keynesian model:

$$y_{t+1} = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n) \text{ (1)}$$

$$\pi_t - \bar{\pi}_t = k y_t + \beta E_t(\pi_{t+1} - \bar{\pi}_{t+1}) + u_t \text{ (2)}$$

[1] McCallum (2001) also extensively discusses this topic.

[2] The first pillar is what the ECB calls "economic analysis", which evaluates the short- and medium-term determinants of price developments. According to the ECB, this analysis takes into account the fact that the evolution of prices in this horizon is significantly influenced by the interaction between demand and supply in the markets for goods and services. The second pillar is called "monetary analysis", and it assesses the medium- and long-term outlook for inflation, exploiting the long-term link between money and prices.

Where:

$y =$ output gap; the logarithmic difference between observed output and the trend or natural output.

$\pi =$ inflation rate.

$\bar{\pi} =$ perceived rate of trend inflation.

$i =$ short-term nominal interest rate (the risk-free rate generated by a money market instrument that is maintained between the periods t and $t+1$).

r^n= "Wicksellian" natural real interest rate (a function of exogenous real factors, similar to natural output).

The additional equation required to close the system specifies a Taylor-type monetary policy rule, in terms of the nominal interest rate:

$$i_t = r_t^* + \bar{\pi}_t + \phi_\pi(\pi_t - \bar{\pi}_t) + \phi_y y_t \quad (3)$$

where r^*represents the central bank's perception of the natural real interest rate, and $\bar{\pi}$ is the central bank's target inflation rate.

Note that Woodford (2008) assumes that the central bank's target inflation rate coincides with the trend inflation rate ($\bar{\pi}$), to which suppliers that do not re-optimize index their prices. A possible interpretation of this assumption proposed by Smets and Wouters (Woodford, 2008) is that the private sector observes the central bank's inflation target and indexes prices to it.

A fundamental assumption in this approach by Woodford is that $\bar{\pi}_t$ and r_t^* are exogenous processes, whose evolution represents changes in the attitude of the central bank that are taken as independent of what is happening with the evolution of inflation and real activity. Woodford following Smets and Wouters (2003) assumes that the inflation target follows a random walk:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \nu_t^\pi \quad (4)$$

Where ν_t^π is a shock *i.i.d* (independently and identically distributed), with zero mean. For its part, r_t^* is a stationary variable.

Using the policy rule (3) to substitute the nominal interest rate i_t in equation (1), equations (1) and (2) can be written in the following form:

$z_t = AE_t z_{t+1} + a(r_t^n - r_t^*)$ (5)

Where:

$$z_t \equiv \begin{bmatrix} \pi_t - \bar{\pi}_t \\ y_t \end{bmatrix}$$

A is a 2X2 matrix of coefficients and a is a vector of (2X1) coefficients.

A solution for this system can be found by applying the forward iteration method to equation (5). This would result in the following expression:

$z_t = \sum_{j=0}^{\infty} A^j a E_t\left(r_{t+j}^n - r_{t+j}^*\right) + \lim_{j\to\infty} A^j E_t z_{t+j+1}$ (6)

The solution of this system will be non-explosive (a solution in which both elements of z_t are stationary processes, under the assumption that the exogenous process $r_t^n - r_t^*$is stationary), if both eigenvalues of A are inside the unit circle. If this condition is satisfied (as expected in the empirical Taylor rules in which the Taylor principle is satisfied), the unique non-explosive solution is given by:

$z_t = \sum_{j=0}^{\infty} A^j a E_t\left(r_{t+j}^n - r_{t+j}^*\right)$; $\lim_{j\to\infty} A^j E_t z_{t+j+1} \to 0$ (7)

This implies a solution for the equilibrium inflation rate of the following form:

$\pi_t = \bar{\pi}_t + \psi_j E_t\left(r_{t+j}^n - r_{t+j}^*\right)$ (8)

Where:

$$\psi_j \equiv [1 \quad 0] A^j a$$

For each j.

According to Woodford (2008), this shows that inflation is determined by the inflation target of the central bank, and by current and future discrepancies between the natural real interest rate and the equilibrium real interest rate perceived by the monetary authority. If the intercept in the Taylor rule r_t^* fits perfectly to r_t^n, the central bank must exactly achieve its inflation target.

But not only does the model determine the inflation rate, but it also implies a certain trajectory for the price level, given an initial price level that is a historical datum at the time the policy represented by the Taylor rule begins to be implemented. Woodford 's (2008) reasoning to support that this model determines the price level is the following: if it t_o is the first period in which the policy based on the Taylor rule begins to be implemented, a higher price level P_{t0} will correspond to a higher inflation rate π_{t0}, and will trigger a higher target interest rate from the central bank. Given the value of P_{t0-1}, which is at t_o a historically given datum for the central bank, there is a unique equilibrium value determined for P_{t0}, and similarly for P_t for any period $t \geq t_0$. Thus, Woodford (2008) concludes that equation (3), illustrates how a monetary policy strategy by the central bank that does not involve a target for the quantity of money, and that can be implemented without even measuring any monetary aggregate, can determine the general price level.

Woodford (2008) also discusses whether the omission of money from the model may distort the basic relationships relevant to an analysis of the effects of alternative monetary policy decisions. As formulated, the model is consistent with a world in which there is no special role for money in facilitating transactions, and thus there is no reason why money should not be perfectly substitutable for any other similar nominal asset without risk. According to Woodford, the derivation of the model in this case without frictions is a way of clarifying that the basic relationships in the model do not have an intrinsic connection with the evolution of the money supply. However, Woodford argues that the model does not require assuming that open market operations are irrelevant, or that there is no single defined path for the money supply associated with the policy rule. This is because the model is consistent with the existence of a well-defined money demand function that gives rise to an equilibrium relationship of the form:

$$\log\left({}^{M_t}/_{P_t}\right) = \eta_y logY_t - \eta_i i_t + \epsilon_t^m \quad (9)$$

In which M_t is the nominal money supply, η_y is the income elasticity of money demand, $\eta_i$is the semi-elasticity of money demand with respect to the interest rate, Y_t is real income, and $\epsilon_t^m$is an exogenous demand shock of money. This additional equation, however, is not needed for the model to determine the evolution of inflation, prices, output, and the interest rate under a given interest rate rule.

2.3. Implications of the long-run relationship between money and prices

The last point that Woodford (2008) addresses with respect to the role of money in the New Keynesian model, refers to the implications of the abundant empirical evidence available about the existence of a long-term relationship between monetary growth and inflation. Several analysts argue that this evidence is robust and sufficient to justify controlling the growth rate of money, given the reasonable concern of a central bank with the evolution of the inflation trend in the long term. Woodford briefly reviews different types of empirical studies of the long-run or low-frequency relationship between money and prices, and finally focuses on the evidence from an application of cointegration analysis to data from the Euro area. Woodford (2008) builds on the evidence provided by Assenmacher-Ische and Gerlach (Gerlach and Svensson, 2003) which indicates that the growth rate of the broad money concept and the inflation rate are both non-stationary series, but that these series cointegrate. Taking this evidence, Woodford (2008) assumes that there is a reliable structural equation of the form for the Euro zone:

$$logM_t - logP_t = f(X_t) \quad (10)$$

This equation represents the demand for money, and it holds regardless of the monetary policy followed by the central bank. $f(X_t)$ is a general function of real and nominal variables, with the property that $f(X_t)$ will be a first difference stationary process (integrated of order 1, *I(1)*) in the case of any monetary policy that makes the inflation rate a stationary process in first difference. In this case, inflation is stationary in first difference (integrated of order 1, *I(1)*), the growth rate of the money stock would also have to be stationary in first difference, and the growth rate of money and inflation would have to cointegrate with a cointegration vector [1 -1]:

$$\mu_t - \pi_t = \Delta f(X_t);\ \Delta f(X_t) \sim I(0) \quad (11)$$

Woodford shows that the New Keynesian model (equations (1)-(2)) with the Taylor rule (3), extended to include the money demand equation (equation (9)) is consistent with the cointegration relation (11). By differentiating equation (9):

$\mu_t - \pi_t = \eta_y \gamma_t - \eta_i \Delta i_t + \Delta\epsilon_t^m$ (12)

where $\gamma_t = \Delta log Y_t$.

Assuming that r_t^n and r_t^* are stationary processes or that the difference $r_t^n - r_t^*$ is stationary, and that if $\bar{\pi}_t$ is a random walk (equation (4)), the inflation rate π_t is a variable *I(1)*, it is reasonable to assume that all terms on the right hand side of (12), γ_t, Δi_t, $\Delta\epsilon_t^m$are stationary variables (*I(0)*). From all of the above, it follows that μ_t must be a variable *I(1)*, as π_t, and that these variables are cointegrated with a cointegration vector [1 -1].

The conclusion that Woodford draws from all this analysis is that the New Keynesian model is consistent with long-term or low-frequency evidence, and that therefore these facts, no matter how well established, do not provide evidence against the validity of non-monetary models. Additionally, if a structural relationship such as (10) exists, then it follows that any policy that is successful in achieving an inflation rate equal to some target value $\bar{\pi}_t$ on average in the long run would also generate a rate of monetary growth equal to $\bar{\pi}_t + \Delta f(X_t)$ on average in the long run. But this, according to Woodford, does not imply that a successful policy must involve a goal of monetary growth, indeed, it does not even require a measurement of the money supply.

2.4. Answers to Woodford's (2008) position

Although the New Keynesian model has attained a status of dominance in academia and central banks, some economists have tried to call attention to its multiple inconsistencies. Thus, it is important to briefly present some of the arguments that have been developed to answer Woodford's position on the irrelevance of money both from a theoretical and empirical perspective.

2.4.1. Can inflation be understood without money?

Nelson (2003) points out that the New Keynesian model by taking the trend or steady state inflation rate ($\bar{\pi}$) as an exogenous variable, can only explain the deviations of observed inflation from the trend. Nelson (2003) argues that the steady state inflation rate ($\bar{\pi}$) is not an exogenous variable, but rather is determined by the economy's steady state rate of monetary growth. Therefore, Friedman's claim that inflation is always and everywhere a monetary phenomenon remains valid in the New Keynesian model. It is in fact a steady state property of the model. This statement by Nelson is supported by the fact that in the money in utility function model, from which the *IS* equation of the New Keynesian model is derived, the steady state inflation rate is equal to the steady state money growth rate (π^{ss}= θ^{ss}). Thus, as Nelson (2003) points out the monetary growth rate / inflation link does not have a counterpart in the equations that describe the dynamics of inflation in the New Keynesian model. This long run relationship is "buried" in the constant terms of the structural relationships that underlie the New Keynesian model equations and has therefore been completely omitted from the dynamic equations that are expressed in terms of deviations from the stationary state. Consequently, the steady state link between monetary growth and inflation has a special status that deserves separate consideration from other long run relationships. Nelson (2003) comments that monetarists recognize that the policy-relevant rate of money growth may change over time, but the recognition that the steady-state relationship between the rate of money growth and the rate of inflation may be subject to changes, must be distinguished from the view that the long-term relationship does not deserve attention in the formulation of monetary policy. It follows from this discussion that McCallum's (2004) and Woodford (2003, 2008) position that in the New Keynesian model the long-term average inflation rate is entirely determined by the target value set by the central bank ($\bar{\pi} = \pi^*$) should be taken with skepticism.

A corollary of the previous discussion is that since the New Keynesian model only determines the deviations of the inflation rate with respect to its steady state value, then it cannot determine the trend or steady state price level either. From this follows that monetary rules designed to keep the inflation rate close to an objective value, do not determinate the general level of prices in the economy (Olivo, 2011).

2.4.2. Implications of the empirical long run relationship between money and prices

Olivo (2011) rejects Woodford 's (2008) position that the New Keynesian model is consistent with the empirical evidence supporting the existence of a long run relationship between money and prices and considers that the results that Assenmacher-Ische and Gerlach (Gerlach and Svensson, 2003) report for the Euro zone are not robust. As described previously, the approach of Woodford (2008) based on the empirical analysis of Assenmacher-Ische and Gerlach, implies that

$logM_t - logP_t = f(X_t)$ is a stationary process in first difference, that is $f(X_t) \sim I(1)$. This in turn implies that the inflation rate and the growth rate of the nominal money supply are $I(1)$ variables, and that $\mu_t - \pi_t = \Delta f(X_t);\ \Delta f(X_t) \sim I(0)$.

In the context of the Monetarist analysis, a more plausible hypothesis is that $logM_t$ and $logP_t$ must be $I(1)$ variables, and if there is a cointegration relationship between them, $f(X_t) \sim I(0)$. It follows then that, $\mu_t$and π_t must be $I(0)$ variables. The Monetarist approach is the most plausible from a theoretical point of view because the inflation rate and the growth rate of the quantity of money may exhibit some persistence, but not contain a unit-root. A series that contains a unit-root presents a variance that increases with time, so that when $t \rightarrow \infty$, its variance also tends to infinity. This is inconsistent with the existence of a steady state equilibrium. The inflation rate can behave like a random walk during the initial phase of a period of high or very high inflation, or during an episode of hyperinflation, but in a sustained process of high - very high inflation, and even more so in contexts of moderate-low inflation, the inflation rate should behave as a stationary variable.

From an empirical point of view, the examination of quarterly data for the period 1990-2005 for six countries (Australia, Canada, Chile, Korea, New Zealand, and the United States) indicates that both the inflation rate and the growth rates of M1 and M2 are stationary variables. Additionally, annual data for Germany for the period 1961-1999 suggest that the inflation rate is a stationary variable (at a significance level of 10% using the adjusted Dickey-Fuller test), while the growth rate of M3 is stationary (at a significance level of 5% using the adjusted Dickey-Fuller test) for the period 1970-1999. To cite another source, Aksoy and Piskorski (2006) find, using US quarterly data for the period 1965:1 – 1998:2, that the rate of inflation and the rate of growth of various definitions of money are stationary.

In general, it is very important to keep in mind the suggestion of Granger (1997), that if the analysis of a series in levels indicates that it is $I(2)$ (its growth rate is $I(1)$), it is a good idea to plot it against time and to conduct tests of unit roots that take into account possible structural changes. This advice is more relevant as the period of analysis becomes longer, as this increases the likelihood of structural changes.

3. The Fiscal Theory of the Price Level

Some influential academics, including Leeper (1991), Sims (1994), Woodford (1995), Cochrane (2007), stand out as the original promoters of what has been called the "Price Level Theory of Fiscal" (TFNP). In contrast to previous literature (for example, Sargent and Wallace, 1981; Aiyagari and Gertler, 1985), a "non-Ricardian" regime implies that the government's intertemporal budget constraint does not always hold. FTPL proponents do not accept the fundamental proposition that the government's intertemporal budget constraint imposes limits on government instruments that must be satisfied for all admissible values of the endogenous variables of the economy. In contrast, this theory holds that the government's intertemporal budget constraint must be satisfied only in equilibrium.

Woodford (1995) defines a "Ricardian" fiscal policy regime as one in which the inter-temporal budget constraint is always fulfilled, regardless of the path followed by the price level. Woodford (1995) argues, however, that there is no institution that would impose such a budget constraint on the government in an economy that is expected to continue indefinitely. Therefore, the definition of a "non-Ricardian" fiscal policy regime in the FTPL is based on the idea that the government's inter-temporal budget constraint only holds in equilibrium. Using this definition of a "non-Ricardian" regime, Woodford (1995) argues that a change in the current or future government deficit will affect the equilibrium price level, while a change in the current or future value of the money supply will not influence the equilibrium price level in the absence of a change in fiscal variables. Woodford (1995) argues that his theory embodies the spirit of Sargent and Wallace's unpleasant monetarist arithmetic (1981), but he quickly acknowledges that these theories are not the same. In the FTPL, a permanent reduction in the money supply, without a change in the expected trajectory of the fiscal variables, implies a permanently higher trajectory for the price level. This is because the increase in the face value of government liabilities is inflationary even if monetization never occurs. The connection between a higher value of government liabilities and a higher price level is direct and does not depend on an eventual increase in the money supply.

Buiter (2004) argues that the FTPL is very different from Sargent and Wallace's fiscal theory of inflation. Buiter characterizes unpleasant monetarist arithmetic as a conventional theory of the price level, in the sense that the intertemporal budget constraint always holds, and the quantity theory determines the price level. In contrast, the FTPL breaks the direct connection between the money supply and the price level.

The FTPL approach can be explained starting from the government's inter-temporal budget constraint:

$$B_t / P_t = E_t\left(\sum_{i=0}^{\infty}\frac{sp_{t+i}}{(1+\rho)^i}\right)+E_t\left(\sum_{i=0}^{\infty}\frac{s_{t+i}}{(1+\rho)^i}\right)$$

In the case of the FTPL, if the government sets the present value of the primary surplus $\left(\sum_{i=0}^{\infty}\frac{sp_{t+i}}{(1+\rho)^i}\right)$, and the central bank the present value of seigniorage $\left(\sum_{i=0}^{\infty}\frac{s_t}{(1+\rho)^i}\right)$ at levels that do not satisfy the budget constraint, the price level (P) is adjusted so that the constraint is met. Hence the direct connection between a higher balance of government liabilities and a higher level of prices, which does not depend on an eventual increase in the money supply.

Just as Leeper, Sims, Woodford, Cochrane and other renowned scholars have written extensively in favor of FTPL, another influential group (McCallum and Nelson, 2006; Buiter 1998, 1999, 2004, 2017) has come forward with serious criticisms of this theory.

Buiter (1999) argues that there are two ways to refute the fiscal theory of the price level. The first is based on a priori economic arguments. Buiter considers it axiomatic that only those models of a market economy that rule out the possibility of default by all agents, including the government, are correctly posed. The budget constraints of households, firms, and the government must be satisfied for all admissible values of the endogenous variables of the economy. It does not matter if the government or private agents are small (price takers) or large (monopolies or monopsonies). Nor does it matter if the government optimizes or what it optimizes, or if it acts according to ad-hoc rules. According to this "Ricardian" postulate about the correct specification of budget constraints, a "non-Ricardian" fiscal rule that does not rule out the possibility of default is erroneously stated.

The second way to refute the FTPL according to Buiter applies even if the a priori postulate that budget constraints must always be satisfied and not only in equilibrium is not accepted. In this case, a “non-Ricardian” fiscal rule only makes sense if an endogenously determined *default discount* factor is explicitly introduced. Buiter (1999) shows that it is not true that the general price level can replicate the role of the discount factor for public debt default. When the discounted value of the primary surpluses plus seigniorage differs from the *default-free notional value* of the public debt, it is not possible to guarantee that the debt will be serviced as specified in the contracts. Buiter introduces the default discount factor on the notional value of the current debt (D_t). This factor determines the fraction of the contractual payments for the period t that are effectively cancelled.[3]

A “Ricardian” fiscal rule is defined by the requirement that $D_t \equiv 1$. With a "Ricardian" rule there can be no discount or premium for default. In this case, taxes, government spending, or seigniorage must be residually adjusted to satisfy the budget constraint at the notional price of debt free of the possibility of default.

With a “non-Ricardian” fiscal rule, the government is allowed to over-determine its fiscal-monetary program. The default discount factor D_t is now determined endogenously. In general, the expected present value of future primary surpluses plus seigniorage will not equal the value of outstanding debt valued at the notional default-free price. If the government follows a "non-Ricardian" rule, the government's intertemporal budget constraint must be specified as follows:

$$D_t({}^{B_t}\!/_{P_t}) = E_t\left(\sum_{i=0}^{\infty}\frac{sp_{t+i}}{(1+\rho)^i}\right) + E_t\left(\sum_{i=0}^{\infty}\frac{s_{t+i}}{(1+\rho)^i}\right)$$

In principle D_t and P_t are interchangeable to satisfy the government's budget constraint, but they are not interchangeable when considering the rest of the equilibrium relations of the economy. In this case, only the default discount factor D_t can balance the government's intertemporal budget constraint in a well-conceived general equilibrium model with an overdetermined fiscal-monetary program.

[3]The value of D should generally lie between 0 and 1, but Buiter (1999) does not rule out the possibility of $D < 0$ or $D > 1$.

Under a “non-Ricardian” fiscal rule and a monetary policy that specifies a path for the money supply, P_t is determined by equilibrium conditions in the money market, and the budget constraint (20) determines the discount factor on public debt D_t:

$$D_t = \left[E_t \left(\sum_{i=0}^{\infty} \frac{sp_{t+i}}{(1+\rho)^i} \right) + E_t \left(\sum_{i=0}^{\infty} \frac{s_{t+i}}{(1+\rho)^i} \right) \right] / \left({B_t}/{P_t} \right)$$

With a “non-Ricardian” fiscal rule and a monetary policy that specifies a nominal interest rate rule, the price level remains undetermined. If $B_t \neq 0$ the indeterminacy of the price level also implies that the default discount factor remains undetermined. However, the intertemporal budget constraint always determines the real effective value of the public debt $D_t({B_t}/{P_t})$, although it does not specify the discount factor and the price level separately.

Buiter 's (1999) main conclusion is that the introduction of the discount factor for government debt invalidates the fiscal theory of the price level.

Buiter (2004) presents additional arguments against the FTPL. Of the criticisms elaborated in detail by Buiter (2004), one of the most important refers to the fact that the FTPL transforms the inter-temporal budget constraint of the consolidated public sector into a behavioral equation, which adjusts the price level towards an equilibrium that equals demand with supply. "Economists think of equilibrium prices as a mechanism that equalizes demand and supply, not budget constraints." (Buiter 2004). In this sense, Buiter asks what feasible story can an economist imagine if the general level of prices in period 1 is below the value necessary to equalize both sides of the inter-temporal budget constraint? Why should there be some upward pressure on the general price level in period 1, given that the observed real value of the debt in period 1 exceeds the present value of the primary surplus plus seigniorage?

Another point refers to the impossibility of deriving a theory of inflation from the FTPL. Thus, the FTPL is a theory of price level determination but not a theory of inflation. This is a theoretical inconsistency that evidently does not happen in the case of the Quantity Theory.

An additional major problem of the FTPL is that it is unlikely that it can determine the price level in a country where government financing depends significantly on foreign debt, because in this case the stock of nominal debt to GDP ratio cannot be stabilized through adjustments in the price level. Foreign debt as indexed public debt, as Buiter (1998) notes, also invalidates the FTPL.

Hence, there are serious logical inconsistencies in the FTPL that undermine its potential validity. Additionally, from an empirical point of view, the FTPL is practically impossible to test. Until now it has not been possible to introduce in an empirical model the relevant restrictions that allow us to clearly differentiate the FTPL from the conventional theory of the price level based on the Quantity Theory. Canzoneri, Cumby and Diba (1998) impose certain restrictions on a Vector Autoregressive (VAR) model to try to test the validity of the FTPL for the United States. Assuming that the restrictions imposed by the authors are valid, in the sense of capturing the fundamental aspects of the FTPL, the work does not find evidence in favor of this theory using data after the Second World War. The authors find that a positive innovation in the fiscal surplus reduces liabilities for several periods and increases future surpluses. A "Ricardian" regime offers a very straightforward explanation for these results: surpluses pay debt in this regime. In contrast, the correlation between the current surplus and future surpluses is difficult to explain in a "non-Ricardian" regime, in which surpluses are governed by exogenous political processes.

Mendoza and Ostry (2008) performed an empirical analysis of fiscal solvency based on conditions consistent with a dynamic stochastic general equilibrium model. The results obtained by these authors show evidence of fiscal solvency, in the form of a robust conditional response of the primary fiscal balance to changes in public debt. This result is obtained using panel data for emerging economies (34 countries for the period 1990-2005) and industrialized economies (22 countries for the period 1970-2005) separately, and in a combined panel. As Canzonery, Cumby, and Diba (1998) point out, these types of results are easy to explain in the context of a "Ricardian" fiscal regime.

Additionally, Canzonery, Cumby and Diba (1998) point out that it would probably not be reasonable to hold central banks responsible for the objective of price stability under a "non-Ricardian" regime. Therefore, if, as its proponents argue, "non-Ricardian" regimes are frequently observed, the widespread practice of assigning responsibility for achieving and maintaining price stability to central banks would be incorrect.

In principle, there would not be reasons to be very concerned with the FTPL. In contrast to the New Keynesian approach, the FTPL has not enjoyed widespread popularity in academia and central banks. But as Buiter (2017) points out, the FTPL is making an unexpected come back. Buiter (2017) reported that in 2016 many of the originators of the FTPL participated in a conference whose theme was “Next Steps for the Fiscal Theory of the Price Level” held at the Becker Friedman Institute for Research on Economics at the University of Chicago. John Cochrane has been a very active promoter of the theory through his blog “The Grumpy Economist” and his recent book The Fiscal Theory of the Price Level (Princeton University Press, 2023).

However, I stick to Buiter string of works that maintain that the FTPL is a theory plagued by inconsistencies that make it untenable as theory of the price level. As Buiter (2017) points out, the error at the roots of the FTPL is the confusion of the intertemporal budget constraint of the State with a behavioral equation: a misspecified government bond pricing equilibrium condition. From the previous discussion, I consider that it is still safe to affirm that, from the point of view of determining the price level, the quantity of money continues to play a fundamental role in a monetary economy.

4.1. The Fiscal Theory of Monetary Policy

Cochrane (2023) develops what he calls the “fiscal theory of monetary policy”. He characterizes this theory as “models that incorporate fiscal theory, yet in their other ingredients incorporate standard DSGE (dynamic stochastic general equilibrium) models, including price stickiness or other non-neutralities of new-Keynesian models that are most commonly used to analyze monetary policy.” In the simplest example of the fiscal theory of monetary policy the interest rate target (or the interest rate rule) sets expected inflation ($i_t = E_t \pi_{t+1}$), and fiscal news sets unexpected inflation:

$$\Delta E_{t+1}\pi_{t+1} = -\Delta E_{t+1}\pi_{t+1} \sum_{j=0}^{\infty} \beta^j \, \tilde{s}_{t+1+j}$$

Where:

$\Delta E_{t+1} = E_{t+1} - E_t$

$\tilde{s}_t = s_t / V$. The surplus scaled by steady-state debt.

The general observation against this type of model is that it combines two models with serious theoretical weaknesses for explaining the price level and inflation, and zero empirical support. Evidence obtained from calibration exercises is not a substitute for real empirical tests.

4. Recent Evidence on the Long run Relationship between Money and Prices

In this section, I present more recent international evidence on the important role of money in the determination of the price level and inflation. I analyzed the relationship between money and prices and inflation and money growth in eight countries (Argentina, Brazil, Colombia, Mexico, Sweden, Turkey, United States, and Venezuela), using annual data for periods over 50 years. The relationship between money and prices is analyzed using the Engle and Granger (EG) cointegration framework in its standard form and adding endogenous threshold effects defined by inflation when relevant. To evaluate the presence of cointegration between money and prices, I follow McCallum (2010) who argues that a regression between two *I(1)* variables will very likely not be spurious if its residuals are not autocorrelated [4]. The relationship between money growth and inflation is examined through the estimation of error correction models (ECM) with thresholds effects defined by inflation when relevant.

The choice of countries under study is not formally random, but I have tried to include countries that have experienced diverse inflationary processes. Excepting the cases of Sweden and the United States, all other countries have exhibited extended periods of inflation with rates of two digits or more. All countries, except Argentina, Turkey, and Venezuela, have been able to attain inflation rates below 10% during the current century until the onset of the Covid-19 pandemic. In the cases of Argentina, Brazil and Venezuela, there have been episodes of sustained very high inflation (with three-digit rates) and hyperinflation events as defined in Cagan (1956). The countries that have achieved one-digit inflation rates during this century implement monetary policy strategies that follow the New Keynesian approach where monetary aggregates are completely ignored.

In what follows, I will present a brief review of the econometric results obtained for the selected countries. Detailed results are presented in the appendix. This appendix includes unit-root tests

[4] In addition, the econometric appendix shows the results of applying de Augmented Dicke-Fuller (ADF) test to the residuals of the cointegrating vectors.

run before the estimation of the ECMs. For all countries, the inflation rate and the growth rate of Broad Money can be considered as stationary processes, in contrast to the their characterization in New Keynesian models as *I(1)* processes.

Argentina

For Argentina, I used annual data of Broad Money and the GDP Deflator obtained from the World Bank database for the period 1960-2018.

I found a cointegration relationship between the logarithm of the GDP deflator (LGDPDEFARG) and the logarithm of Broad Money (LBMARG) for Argentina for the period 1960-2022. The cointegrating vector was estimated using Maximum Likelihood as it contains ARMA terms to correct autocorrelation. I could not find a cointegration relation between LGDPDEFARG and LBMARG estimating the cointegrating vector with thresholds defined by the inflation rate. The coefficient of LBMARG is close to one and statistically significant.

For the Error Correction Model (ECM), the threshold regression indicates two thresholds defined by the inflation rate (LDGDPDEFARG) or three regimes. In the ECM when the inflation rate is below 16.9% both the coefficient of the rate of growth of Broad Money (LDBMARG) and the coefficient of the cointegration residuals lagged one period – COINTRES(-1) – are not statistically significant. When inflation is equal to or greater than 16.9% but less than 96%, the coefficient of LDBMARG is 0.67 and statistically significant, and the coefficient of COINTRES(-1) is -0.67 and statistically significant. In the third regime when inflation is equal to or larger than 96%, the coefficient of LDBMARG is 0.75 and statistically significant, and the coefficient of COINTRES(-1) is -1 and statistically significant. Thus, for Argentina, Broad Money growth Granger cause inflation when inflation is equal to or greater than 16.9%.

Brazil

In the case of Brazil, I used annual data (1960-2022) of Broad Money and the GDP deflator obtained from the World Bank database.

I detected cointegration between the logarithm of Broad Money (LBMBRA) and the logarithm of the GDP deflator (LGDPDEFBRA) using OLS and including additional terms to correct autocorrelation. The coefficient of LBM is statistically significant with a value of 0.59.

The ECM was estimated using a threshold regression with the inflation rate (LDGDPDEFBRA) as the threshold variable. When inflation is below 125%, the coefficient of the rate of growth of Broad Money (LDBMBRA) is 0.78 and statistically significant. When inflation is equal to or greater than 125%, the coefficient of LDBMBRA decreases to 0.2 but is still statistically significant. The coefficient of the residuals from the cointegrating vector lagged one period – COINTRES(-1) – is not statistically significant in the first regime when inflation is below 125%, but in the second regime when the inflation rate is equal to or greater than 125%, this coefficient has the expected negative sign and is statistically significant. This last result indicates that when inflation is equal to or greater than 125%, Broad Money growth Granger causes inflation.

Colombia

For Colombia, I used annual data from 1960 to 2022 of Broad Money and the Consumer Price Index. The source for both series is the World Bank.

The cointegrating vector was estimated with a threshold regression where the thresholds were defined by the inflation rate measured as the log difference of the CPI (LDCPICOL) and including additional terms to correct autocorrelation. In the cointegrating vector, the coefficient of the logarithm of Broad Money (LBMCOL) is not statistically significant when the inflation rate (LDCPICOL) is below 7.2%, but it is statistically significant for the other three regimes identified when inflation is equal to or greater than 7.2%.

The ECM for Colombia was also estimated with thresholds defined by inflation. This ECM shows that the coefficient of the rate of growth of Broad Money lagged one period – LDBMCOL(-1) – is only statistically significant when the inflation rate is between 11.3% and 20.3%. When inflation is above 20.3%, only the coefficient of the cointegration residuals lagged one period – COINTRES(-1) – is statistically significant and has the expected negative sign. This indicates that when inflation is above 20% Broad Money growth Granger causes inflation.

Mexico

The econometric estimations for Mexico are based on annual data from 1960 to 2022 of Broad Money and the Consumer Price Index. The source for both series is the World Bank database.

Estimating the cointegrating vector using OLS and adding terms to correct autocorrelation, I found that the logarithm of Broad Money (LBMMEX) and the logarithm of the Consumer Price

Index (LCPIMEX) cointegrate. The coefficient of LBMMEX has a value of 0.27 and is statistically significant.

The ECM was estimated using a threshold regression with thresholds defined by the inflation rate. I found that the growth rate of Broad Money (LDBMMEX) is statistically significant when the inflation rate is less than 18.2% (coefficient=0.16), and when the inflation rate is equal to or above 29.5% (coefficient=0.6). The coefficient of LDBMMEX is not statistically significant when inflation is in the [18.2%-29.5%) range. In contrast, the coefficient of the cointegration residuals lagged one period – COINTRES(-1) – is negative as expected and statistically significant in all the regimes determined by the inflation thresholds. Thus, the growth rate of Broad Money Granger causes inflation at all levels.

Sweden

In the case of Sweden, I used annual data of Broad Money and the Consumer Price Index extracted from the World Bank database for the period 1960-2021.

With a threshold regression with thresholds defined by inflation and additional terms to correct autocorrelation, I obtained a cointegration relation between the logarithm of Broad Money (LBMSWE) and the logarithm of the CPI (LCPISWE) when the inflation rate is equal to or greater than 9%. When inflation is above this threshold the coefficient of LBMSWE is 0.41 and statistically significant. For the regimes identified by the thresholds below 9%, the coefficient of LBM is not statistically different from zero, except for the case when inflation is below 1,8%. In the latter case, the coefficient of LBMSWE is relatively small (0.04) but statistically significant (p-value=0.0957).

The ECM model was also estimated using a threshold regression with thresholds defined by the inflation rate. For all the regimes identified by the thresholds, the coefficient of the rate of growth of Broad Money (LDBMSWE) is not statistically significant. The coefficient of the residuals from the cointegrating vector lagged one period – COINTRES(-1) – is clearly statistically significant with a value of -3.31 when inflation is equal to or greater than 8.5%. The coefficient of COINTRES(-1) is also statistically significant (p-value=0.1) with a value of -0.66, when the inflation rate is below 1.8% (p-value=0.1). The results indicate that Broad Money growth Granger-cause inflation clearly when the latter is equal to or above 8.5%.

Turkey

For Turkey, I used annual data from the World Bank for the period 1960-2022 for the variables Broad Money and the Consumer Price Index.

A cointegration relation was found between the logarithm of Broad Money (LBMTUR) and the logarithm of the Consumer Price Index (LCPITUR) for the whole sample period, using an ARMA Maximum Likelihood method as some ARMA terms were included to correct autocorrelation. The coefficient of the logarithm of Broad money (LBMTUR) is 0.39 and statistically significant.

The ECM model was estimated using a threshold regression with the inflation rate used to define the thresholds. When LDCPITUR is below 37.1%, the coefficient of the rate of growth of Broad Money (LDBMTUR) is not statistically significant, but the coefficient of the lagged values of the cointegration residuals –COINTRES1(-1) – has the expected negative sign and is statistically significant. When the inflation rate is above 37.1%, LDBMTUR has a coefficient of 0.42 and is statistically significant. Additionally, the coefficient of COINTRES1(-1) has the expected negative sign and is also statistically significant. These results indicate that the rate of growth of Broad Money Granger causes inflation at any level.

United States

Econometric estimations for the U.S. are based on annual data for the period 1960-2021 of the variables Broad Money and the Consumer Price Index. The source of both series is the World Bank database.

The cointegrating vector was estimated using a threshold regression with the inflation rate defining the thresholds and adding additional terms to correct autocorrelation. I found a cointegration relationship between de logarithm of Broad Money (LBMUSA) and the logarithm of the Consumer Price Index (LCPIUSA) considering two inflation thresholds: 2.6% and 5.3%. When inflation is under 2.6%, the coefficient of LBMUSA is not statistically significant. For the other two regimes the coefficient of LBMUSA is statistically significant. In the U.S. case, the coefficients of LBMUSA in the cointegrating vector are relatively small compared to those reported previously for countries that have experienced higher inflation rates. However, the coefficient of LBMUSA increases substantially when inflation trespasses the 5.3% threshold.

The ECM was estimated using a threshold regression with thresholds defined by inflation. In the regimes identified when the inflation rate is below 5.7%, the coefficient of the growth rate of Broad Money (LDBMUSA) is not statistically significant. But when inflation is equal to or greater than 5.7%, the coefficient of LDBMUSA is statistically significant with a value of 0.47. Similarly, the coefficient of the cointegration residuals lagged one period – COINTRES(-1) – is not statistically significant when the inflation rate is below 5.7%, but when the inflation rate is higher than 5.7% the coefficient of COINTRES(-1) has the expected negative sign and is statistically significant. Thus, the growth rate of Broad Money Granger causes inflation in the regime where the inflation rate is equal to or above 5.7%.

Venezuela

The relationship between money and prices, and money growth and inflation in Venezuela is examined for the period 1950-2019. Money is measured as M1 and the price level is represented by the Consumer Price Index (CPI). The source of both annual series is the Central Bank of Venezuela.

The estimation of the cointegrating vector using a threshold regression with inflation (LD_CPI) as the threshold variable, indicates cointegration between the logarithm of the Consumer Price Index (L_CPI) and the logarithm of M1 (L_M1) when inflation is equal to or greater than 24.7%. The coefficient of L_M1 is 0.56 (p-value=0) when inflation is in the [24.75-42.3%) range and increases to 1.1 (p-value=0) when inflation is equal to or greater than 42.3%.

The ECM was estimated using a threshold regression with the inflation rate (LD_CPI) defining the thresholds. The coefficient of the growth rate of M1 (LD_M1) is statistically significant in both regimes: when inflation is lower than 47.5%, and when inflation is equal to or greater than 47.5%. However, when inflation is above 47.5% the coefficient of LD_M1 is close to 1, versus a value of 0.15 when inflation is under 47.5%. When inflation is below 47.5%, the coefficient of the cointegration residuals lagged one period – COINTTR(-1) – has a value of -0.25 and is statistically significant, if we slightly relax the statistical criterion to reject the null hypothesis (p-value=0.18). When inflation is equal to or greater than 47.5%, the coefficient of the cointegration residuals lagged one period has a value of -1.28 and is statistically significant. Thus, M1 growth Granger-cause inflation clearly when inflation is above 47.5%.

The results obtained for all countries are summarized in Tables 7.1 and 7.2.

Table 7.1. Summary of Econometric Results

	Cointegration/LBM Coefficient>0	**ECM/LDBM Coefficient>0**	**ECM/ COINTRES(-1) Coefficient<0**
Argentina	$\pi \geq 0$	$\pi \geq 16.9\%$ Thresholds 16.9%, 96%	$\pi \geq 16.9\%$
Brazil	$\pi \geq 0$	$\pi \geq 0$ Thresholds 124.9%	$\pi \geq 124.9\%$
Colombia	$\pi \geq 7.2\%$ Thresholds 7.2%, 15.5%, 21.6%	$11.3\% \leq \pi < 20.3\%$ Thresholds 6.2%, 11.3%, 20.3%	$\pi \geq 20.3\%$
Mexico	$\pi \geq 0$	$0 \leq \pi < 18.3\%$/ $\pi \geq 29.5\%$ Thresholds 18.3%, 29.5%	$\pi \geq 0$
Sweden	$0 \leq \pi < 1.8\%$ $\pi \geq 9\%$ Thresholds 1.8%, 4.6%, 9%	No significant Thresholds 1.8%, 4.6%, 8.5%	$\pi \geq 8.5\%$
Turkey	$\pi \geq 0$	$\pi \geq 37.1\%$ Thresholds 37.1%	$\pi \geq 0$
United States	$\pi \geq 2.6\%$ Thresholds 2.6%, 5.3%	$\pi \geq 5.7\%$ Thresholds 1.9%, 3.3%, 5.7%	$\pi \geq 5.7\%$

Table 7.2. Summary of Econometric Results (Venezuela)

	Cointegration/L_M1 Cofficient>0	**ECM/LD_M1 Coefficient>0**	**ECM/ COINTTR(-1) Coefficient<0**
Venezuela	$\pi \geq 24.7$ Thresholds 13.4, 24.7, 42.3	$\pi \geq 0$ Thresholds 47.5	$\pi \geq 0$

Conclusions

From Sargent's contention that the end of four hyperinflations in Europe during the 1920's was due exclusively to fiscal adjustment, to the canonical New Keynesian model developed by Woodford (2008), and finally, to the Fiscal Theory of the Price Level, the common thread in this literature is the idea that money can be completely neglected in the analysis of the price level and inflation, and therefore, in monetary policy. This paper discusses numerous and serious conceptual criticisms of arguments and theories that consider that the price level and inflation are exclusively a fiscal phenomenon in which money plays no distinctive role. The determination of the price level cannot be explained by expectations, and substantial accelerations of the inflation rate or sustained inflation rates of two digits or more cannot be explained by changes in expectations, as Sargent (1982), Woodford (2008) and the FTPL proponents claim. As Klein and Shambaugh (2010) emphasize regarding the ability of PEGs scheme to control inflation, the credibility effect of PEGs is not enough. It is essential to provide discipline in the form of prudent and stable growth of the money supply to deliver low inflation in the long run. I believe that a similar argument applies to the capacity of promises of fiscal-monetary discipline to control inflation.

It is also important to reject views such as Leeper's (2023) interpretation of the monetary nature of inflation as meaning that inflation can in principle always be controlled by monetary policy. I think that Milton Friedman was quite aware that the line separating fiscal and monetary policy is tenuous as can be easily seen in this paragraph from A Program for Monetary Stability (1960):

The attention devoted to the "independence" of the Federal Reserve System tends to obscure the essential fact that open market operations and debt management are different names for the same monetary tool, wielded in one case by the Federal Reserve System, in the other, by the Treasury. The fiction that the Federal Reserve System is only quasi-governmental and its separation from the departmental organization of the federal administration no doubt alter the impact of political influences and lead to different actions than would be taken if the Reserve System were administratively consolidated with the Treasury. As an economic matter, however, the accounts of the Federal Reserve and the Treasury must be consolidated to determine what monetary action government is taking or to judge what the effects of such actions are likely to be.

Thus, I think it is not very controversial to conjecture that Friedman's statement of the monetary nature of inflation is not related to which governmental agency manages monetary policy or if monetary policy can be completely separated from fiscal policy, but to the necessary presence of money and the attention to its behavior to understand the dynamics of the price level and inflation.

However, beyond the theoretical arguments, this document places a strong emphasis in providing empirical evidence to support the Quantity Theory and Friedman's contention that money is always and everywhere a monetary phenomenon. The monetary nature of inflation, as understood by Friedman, appears clearly in the data of eight countries with very different economic characteristics and inflationary experiences. The empirical evidence obtained using cointegration and error correction models estimated using linear and non-linear techniques (Threshold Regression) provides robust indication that money plays a crucial role in understanding the long-run evolution of the price level and the short-run dynamics of inflation. The definition of money used in the models is Broad Money (M1 in the case of Venezuela), no bank reserves, or the Monetary Base. I follow Brunner and Meltzer (1997) general conception of money: *To protect against uncertainty, to reduce costs of acquiring information and to shift the costs of bearing uncertainty, society develops institutions, including money, price setting and wage settings arrangements*. Money is a very special kind of asset. To argue that money can be perfectly substituted by other assets or that its demand originates only from legal constraints, ignores the costs that economic agents face in acquiring information in a context of uncertainty, even if, in general, their behavior is rational.

From the theoretical arguments against models that dismiss money and specially from the empirical evidence obtained, my conclusion is that neglecting money in macroeconomic models and in the design and implementation of monetary policy deprives policy makers of valuable information that is vital to attain and preserve price level and macroeconomic stability.

Appendix. Econometric Results

Argentina

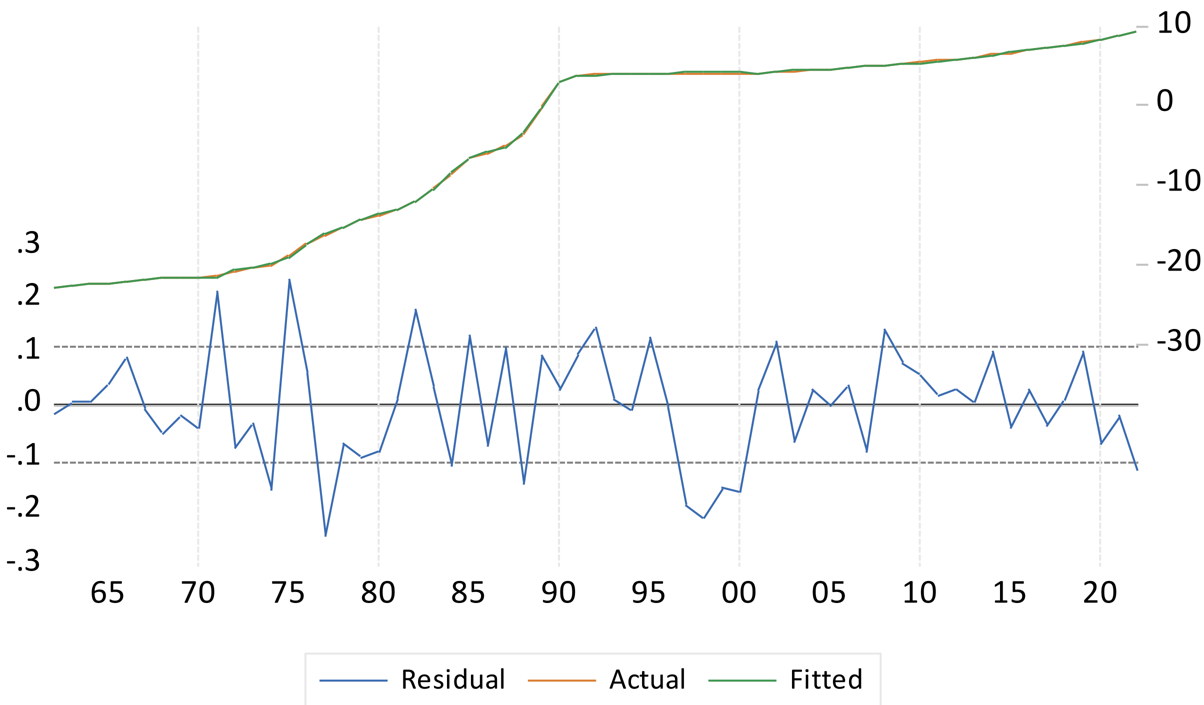
1. Cointegration

Dependent Variable: LGDPDEFARG
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 08/25/23 Time: 19:48
Sample: 1962 2022
Included observations: 61
Convergence achieved after 19 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-18.13273	0.554953	-32.67436	0.0000
LBMARG	1.013767	0.033308	30.43640	0.0000
@TREND	-0.112208	0.013269	-8.456113	0.0000
@TREND^2	0.000715	0.000126	5.696655	0.0000
LGDPDEFARG(-1)	0.258457	0.050142	5.154462	0.0000
LGDPDEFARG(-2)	-0.213218	0.028523	-7.475349	0.0000
MA(1)	0.238028	0.160383	1.484123	0.1438
MA(4)	-0.347216	0.134220	-2.586917	0.0125
SIGMASQ	0.010412	0.002249	4.630365	0.0000

R-squared	0.999922	Mean dependent var	-4.295588
Adjusted R-squared	0.999910	S.D. dependent var	11.67049
S.E. of regression	0.110516	Akaike info criterion	-1.421634
Sum squared resid	0.635119	Schwarz criterion	-1.110194
Log likelihood	52.35985	Hannan-Quinn criter.	-1.299578
F-statistic	83628.40	Durbin-Watson stat	1.894750
Prob(F-statistic)	0.000000		

Inverted MA Roots	.71	-.06+.76i	-.06-.76i	-.83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Date: 09/23/23 Time: 17:49
Sample (adjusted): 1962 2022
Q-statistic probabilities adjusted for 2 ARMA terms and 2 dynamic regressors

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.040	0.040	0.1033	
		2	0.018	0.016	0.1233	
		3	-0.108	-0.110	0.8987	0.343
		4	0.039	0.048	1.0028	0.606
		5	-0.069	-0.070	1.3324	0.721
		6	-0.093	-0.102	1.9306	0.749
		7	0.030	0.052	1.9945	0.850
		8	-0.243	-0.270	6.2784	0.393
		9	-0.037	-0.030	6.3799	0.496
		10	0.106	0.143	7.2330	0.512
		11	-0.076	-0.198	7.6719	0.568
		12	-0.119	-0.101	8.7759	0.553
		13	-0.191	-0.190	11.686	0.388
		14	-0.025	-0.149	11.738	0.467
		15	-0.166	-0.174	14.041	0.371
		16	0.010	-0.135	14.049	0.446
		17	0.026	-0.108	14.107	0.517
		18	-0.079	-0.202	14.669	0.549
		19	0.169	0.030	17.282	0.435
		20	0.172	0.008	20.069	0.329
		21	0.127	-0.098	21.619	0.304
		22	-0.045	-0.086	21.821	0.350
		23	0.086	-0.037	22.560	0.368
		24	0.072	-0.050	23.103	0.396
		25	-0.015	-0.048	23.129	0.453
		26	0.083	-0.042	23.894	0.468
		27	0.057	0.032	24.258	0.505
		28	-0.093	-0.137	25.260	0.504

*Probabilities may not be valid for this equation specification.

Null Hypothesis: COINTRES has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.281166	0.0000
Test critical values:	1% level	-2.604073	
	5% level	-1.946348	
	10% level	-1.613293	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(COINTRES)
Method: Least Squares
Date: 09/23/23 Time: 17:50
Sample (adjusted): 1963 2022
Included observations: 60 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
COINTRES(-1)	-0.958753	0.131676	-7.281166	0.0000

R-squared	0.473199	Mean dependent var	-0.001827
Adjusted R-squared	0.473199	S.D. dependent var	0.142804
S.E. of regression	0.103649	Akaike info criterion	-1.679089
Sum squared resid	0.633843	Schwarz criterion	-1.644183
Log likelihood	51.37266	Hannan-Quinn criter.	-1.665435
Durbin-Watson stat	1.976710		

2. Error Correction Model (ECM)

Null Hypothesis: LDGDPDEFARG has a unit root
Trend Specification: Intercept only
Break Specification: Intercept only
Break Type: Innovational outlier

Break Date: 1989
Break Selection: Minimize Dickey-Fuller t-statistic
Lag Length: 2 (Automatic - based on Schwarz information criterion, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.928754	0.0111
Test critical values:	1% level	-4.949133	
	5% level	-4.443649	
	10% level	-4.193627	

*Vogelsang (1993) asymptotic one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: LDGDPDEFARG
Method: Least Squares
Date: 10/27/23 Time: 12:43
Sample (adjusted): 1964 2022
Included observations: 59 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LDGDPDEFARG(-1)	0.658949	0.069196	9.522920	0.0000
D(LDGDPDEFARG(-1))	0.336965	0.086384	3.900782	0.0003
D(LDGDPDEFARG(-2))	-0.262391	0.093350	-2.810835	0.0069
C	0.294038	0.077255	3.806079	0.0004
INCPTBREAK	-0.247845	0.082772	-2.994304	0.0042
BREAKDUM	2.180274	0.317266	6.872078	0.0000

R-squared	0.838091	Mean dependent var	0.542888
Adjusted R-squared	0.822817	S.D. dependent var	0.715495
S.E. of regression	0.301174	Akaike info criterion	0.533889
Sum squared resid	4.807416	Schwarz criterion	0.745164
Log likelihood	-9.749731	Hannan-Quinn criter.	0.616362
F-statistic	54.86888	Durbin-Watson stat	2.208704
Prob(F-statistic)	0.000000		

Null Hypothesis: LDBMARG has a unit root
Trend Specification: Intercept only
Break Specification: Intercept only
Break Type: Innovational outlier

Break Date: 1989
Break Selection: Minimize Dickey-Fuller t-statistic
Lag Length: 2 (Automatic - based on Schwarz information criterion, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.771041	0.0193
Test critical values:	1% level	-4.949133	
	5% level	-4.443649	
	10% level	-4.193627	

*Vogelsang (1993) asymptotic one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: LDBMARG
Method: Least Squares
Date: 10/27/23 Time: 12:42
Sample (adjusted): 1964 2022
Included observations: 59 after adjustments

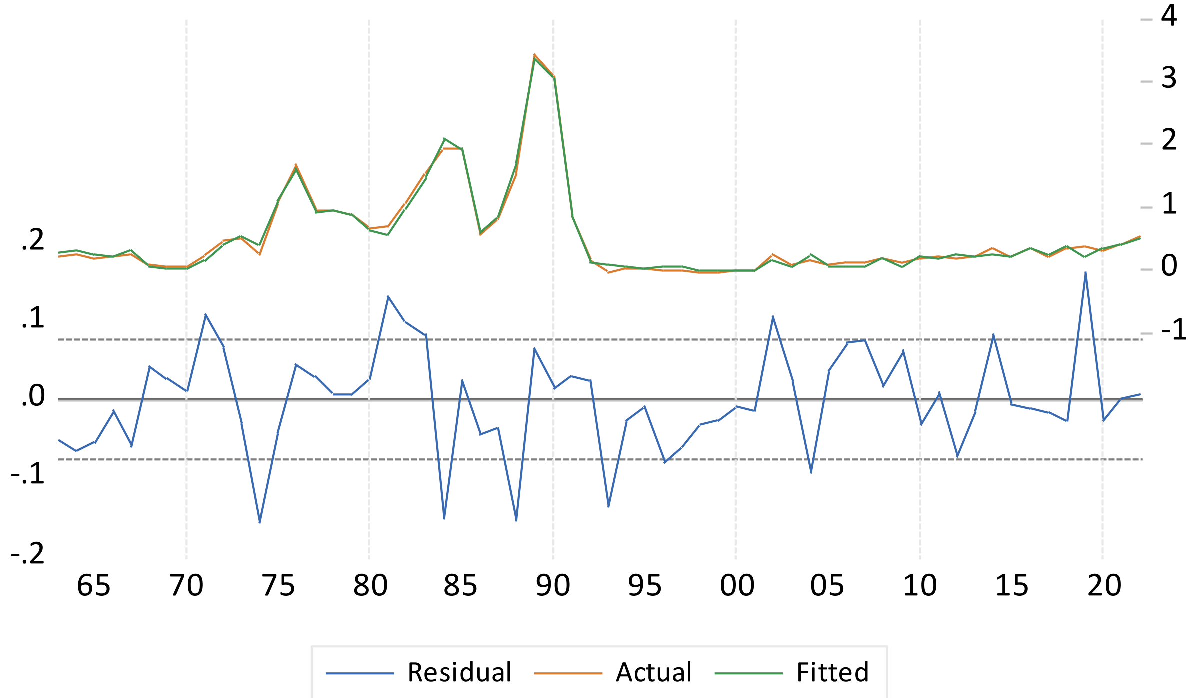
Variable	Coefficient	Std. Error	t-Statistic	Prob.
LDBMARG(-1)	0.692156	0.064523	10.72721	0.0000
D(LDBMARG(-1))	0.297586	0.088607	3.358484	0.0015
D(LDBMARG(-2))	-0.298905	0.093631	-3.192361	0.0024
C	0.288203	0.069569	4.142716	0.0001
INCPTBREAK	-0.235112	0.070975	-3.312596	0.0017
BREAKDUM	1.774445	0.274317	6.468590	0.0000

R-squared	0.853295	Mean dependent var	0.578002
Adjusted R-squared	0.839455	S.D. dependent var	0.637213
S.E. of regression	0.255319	Akaike info criterion	0.203536
Sum squared resid	3.454946	Schwarz criterion	0.414811
Log likelihood	-0.004317	Hannan-Quinn criter.	0.286009
F-statistic	61.65398	Durbin-Watson stat	1.920507
Prob(F-statistic)	0.000000		

Dependent Variable: LDGDPDEFARG
Method: Discrete Threshold Regression
Date: 08/25/23 Time: 19:55
Sample (adjusted): 1963 2022
Included observations: 60 after adjustments
Selection: Trimming 0.15, Max. thresholds 5, Sig. level 0.05
Threshold variable: LDGDPDEFARG

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LDGDPDEFARG < 0.1685889 -- 18 obs				
C	0.044574	0.038045	1.171612	0.2475
LDBMARG	0.057083	0.248882	0.229357	0.8196
COINTRES(-1)	0.178358	0.242652	0.735035	0.4661
LDBMARG(-1)	-0.014403	0.213247	-0.067542	0.9464
LDGDPDEFARG(-2)	0.015338	0.052067	0.294576	0.7697
0.1685889 <= LDGDPDEFARG < 0.9607735 -- 32 obs				
C	0.067762	0.026661	2.541608	0.0145
LDBMARG	0.674461	0.071777	9.396597	0.0000
COINTRES(-1)	-0.666643	0.188811	-3.530749	0.0010
LDBMARG(-1)	0.042585	0.066277	0.642536	0.5238
LDGDPDEFARG(-2)	0.028572	0.046933	0.608789	0.5457
0.9607735 <= LDGDPDEFARG -- 10 obs				
C	0.337426	0.116537	2.895438	0.0058
LDBMARG	0.750139	0.072354	10.36758	0.0000
COINTRES(-1)	-1.043414	0.255150	-4.089411	0.0002
LDBMARG(-1)	0.645198	0.104242	6.189399	0.0000
LDGDPDEFARG(-2)	-0.686482	0.141922	-4.837043	0.0000

R-squared	0.991129	Mean dependent var	0.537637
Adjusted R-squared	0.988369	S.D. dependent var	0.710570
S.E. of regression	0.076632	Akaike info criterion	-2.087274
Sum squared resid	0.264264	Schwarz criterion	-1.563688
Log likelihood	77.61823	Hannan-Quinn criter.	-1.882471
F-statistic	359.1224	Durbin-Watson stat	1.723602
Prob(F-statistic)	0.000000		



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Date: 09/23/23 Time: 17:53
Sample (adjusted): 1963 2022
Q-statistic probabilities adjusted for 3 dynamic regressors

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.133	0.133	1.1149	0.291
		2	-0.033	-0.052	1.1862	0.553
		3	-0.119	-0.110	2.1168	0.549
		4	0.012	0.043	2.1265	0.713
		5	-0.018	-0.034	2.1481	0.828
		6	-0.132	-0.142	3.3485	0.764
		7	-0.178	-0.144	5.5795	0.590
		8	-0.027	-0.000	5.6302	0.689
		9	0.057	0.020	5.8690	0.753
		10	0.143	0.105	7.3879	0.688
		11	0.092	0.071	8.0326	0.710
		12	0.021	-0.002	8.0656	0.780
		13	0.016	0.005	8.0852	0.838
		14	-0.030	-0.051	8.1604	0.881
		15	-0.251	-0.259	13.352	0.575
		16	-0.168	-0.092	15.734	0.472
		17	-0.061	-0.005	16.054	0.520
		18	-0.186	-0.250	19.119	0.385
		19	0.050	0.088	19.342	0.435
		20	0.103	0.081	20.330	0.437
		21	-0.036	-0.231	20.451	0.493
		22	-0.050	-0.127	20.691	0.540
		23	-0.029	-0.064	20.775	0.595
		24	0.133	0.033	22.590	0.544
		25	0.026	-0.004	22.664	0.597
		26	-0.039	0.075	22.827	0.643
		27	-0.012	0.061	22.842	0.693
		28	-0.022	-0.049	22.898	0.738

*Probabilities may not be valid for this equation specification.

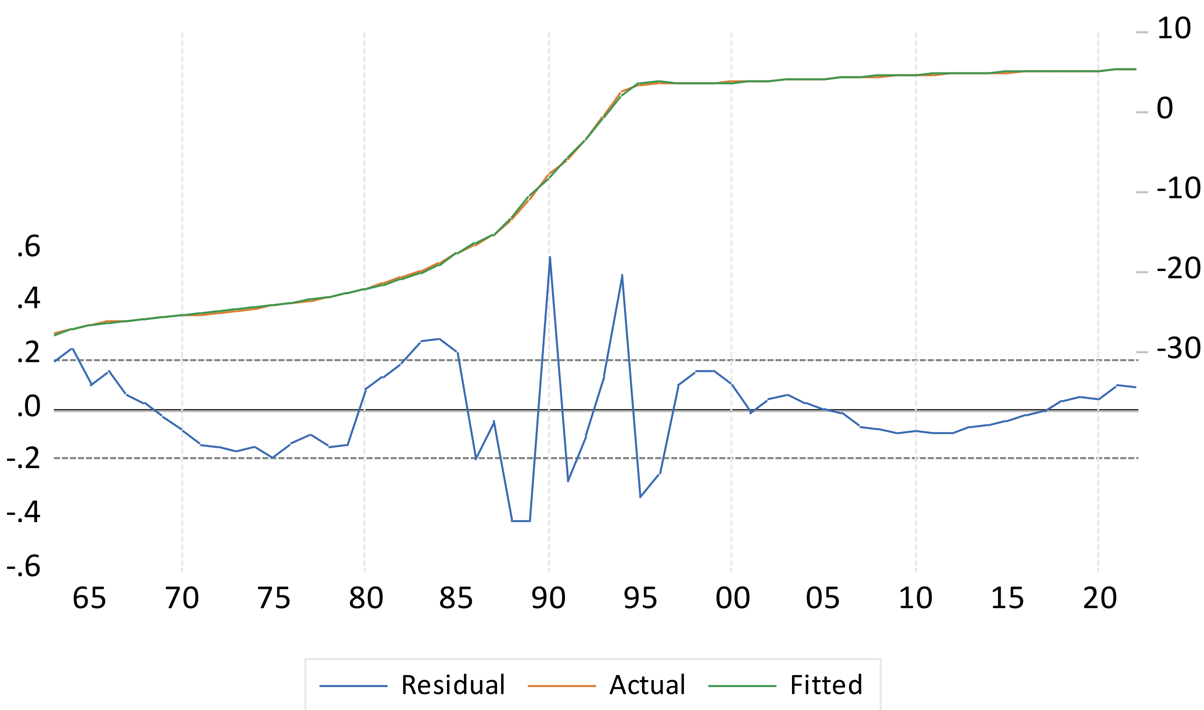
Brazil

1. Cointegration

Dependent Variable: LGDPDEFBRA
Method: Least Squares
Date: 08/26/23 Time: 19:22
Sample (adjusted): 1963 2022
Included observations: 60 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-13.15465	0.694025	-18.95414	0.0000
LBMBRA	0.588643	0.031205	18.86352	0.0000
LGDPDEFBRA(-1)	0.509689	0.050596	10.07372	0.0000
LGDPDEFBRA(-3)	-0.130607	0.023586	-5.537381	0.0000
@TREND^2	-0.000314	5.59E-05	-5.609735	0.0000

R-squared	0.999823	Mean dependent var	-8.229506
Adjusted R-squared	0.999810	S.D. dependent var	13.42484
S.E. of regression	0.185207	Akaike info criterion	-0.455025
Sum squared resid	1.886599	Schwarz criterion	-0.280496
Log likelihood	18.65075	Hannan-Quinn criter.	-0.386757
F-statistic	77484.80	Durbin-Watson stat	1.671682
Prob(F-statistic)	0.000000		



Date: 09/23/23 Time: 17:58
Sample (adjusted): 1963 2022
Q-statistic probabilities adjusted for 2 dynamic regressors

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<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob*</th></tr>
<tr><td>. |*. |</td><td>. |*. |</td><td>1</td><td>0.153</td><td>0.153</td><td>1.4835</td><td>0.223</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>2</td><td>-0.025</td><td>-0.050</td><td>1.5248</td><td>0.467</td></tr>
<tr><td>. |*. |</td><td>. |*. |</td><td>3</td><td>0.113</td><td>0.129</td><td>2.3645</td><td>0.500</td></tr>
<tr><td>. |*. |</td><td>. | . |</td><td>4</td><td>0.087</td><td>0.048</td><td>2.8635</td><td>0.581</td></tr>
<tr><td>**| . |</td><td>**| . |</td><td>5</td><td>-0.230</td><td>-0.252</td><td>6.4341</td><td>0.266</td></tr>
<tr><td>**| . |</td><td>.*| . |</td><td>6</td><td>-0.225</td><td>-0.171</td><td>9.9303</td><td>0.128</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>7</td><td>-0.054</td><td>-0.033</td><td>10.132</td><td>0.181</td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>8</td><td>-0.148</td><td>-0.116</td><td>11.703</td><td>0.165</td></tr>
<tr><td>. | . |</td><td>. |*. |</td><td>9</td><td>-0.040</td><td>0.087</td><td>11.820</td><td>0.224</td></tr>
<tr><td>. | . |</td><td>.*| . |</td><td>10</td><td>-0.055</td><td>-0.087</td><td>12.044</td><td>0.282</td></tr>
<tr><td>**| . |</td><td>**| . |</td><td>11</td><td>-0.210</td><td>-0.289</td><td>15.389</td><td>0.165</td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>12</td><td>-0.136</td><td>-0.151</td><td>16.824</td><td>0.156</td></tr>
<tr><td>. | . |</td><td>.*| . |</td><td>13</td><td>-0.031</td><td>-0.115</td><td>16.902</td><td>0.204</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>14</td><td>-0.006</td><td>-0.001</td><td>16.905</td><td>0.261</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>15</td><td>-0.013</td><td>0.034</td><td>16.920</td><td>0.324</td></tr>
<tr><td>. |*. |</td><td>. | . |</td><td>16</td><td>0.099</td><td>-0.032</td><td>17.749</td><td>0.339</td></tr>
<tr><td>. |*. |</td><td>. | . |</td><td>17</td><td>0.136</td><td>-0.054</td><td>19.352</td><td>0.309</td></tr>
<tr><td>. |*. |</td><td>. | . |</td><td>18</td><td>0.121</td><td>-0.023</td><td>20.658</td><td>0.297</td></tr>
<tr><td>. |*. |</td><td>. | . |</td><td>19</td><td>0.088</td><td>-0.042</td><td>21.357</td><td>0.317</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>20</td><td>0.073</td><td>0.037</td><td>21.850</td><td>0.349</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>21</td><td>0.037</td><td>0.012</td><td>21.981</td><td>0.401</td></tr>
<tr><td>. | . |</td><td>.*| . |</td><td>22</td><td>-0.030</td><td>-0.091</td><td>22.070</td><td>0.456</td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>23</td><td>-0.067</td><td>-0.119</td><td>22.519</td><td>0.489</td></tr>
<tr><td>. | . |</td><td>.*| . |</td><td>24</td><td>-0.058</td><td>-0.088</td><td>22.861</td><td>0.528</td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>25</td><td>-0.115</td><td>-0.125</td><td>24.257</td><td>0.505</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>26</td><td>-0.043</td><td>0.030</td><td>24.462</td><td>0.550</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>27</td><td>-0.021</td><td>0.002</td><td>24.510</td><td>0.602</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>28</td><td>-0.040</td><td>-0.061</td><td>24.700</td><td>0.644</td></tr>
</table>

*Probabilities may not be valid for this equation specification.

Null Hypothesis: COINTRES has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.565781	0.0000
Test critical values:	1% level	-2.604746	
	5% level	-1.946447	
	10% level	-1.613238	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(COINTRES)
Method: Least Squares
Date: 09/23/23 Time: 18:00
Sample (adjusted): 1964 2022
Included observations: 59 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
COINTRES(-1)	-0.845978	0.128847	-6.565781	0.0000

R-squared	0.426337	Mean dependent var	-0.001584
Adjusted R-squared	0.426337	S.D. dependent var	0.233181
S.E. of regression	0.176612	Akaike info criterion	-0.612917
Sum squared resid	1.809128	Schwarz criterion	-0.577704
Log likelihood	19.08105	Hannan-Quinn criter.	-0.599171
Durbin-Watson stat	1.999514		

2. Error Correction Model (ECM)

Null Hypothesis: LDGDPDEFBRA has a unit root
Trend Specification: Trend and intercept
Break Specification: Intercept only
Break Type: Innovational outlier

Break Date: 1994
Break Selection: Minimize Dickey-Fuller t-statistic
Lag Length: 0 (Automatic - based on Schwarz information criterion, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.556718	< 0.01
Test critical values:	1% level	-5.347598	
	5% level	-4.859812	
	10% level	-4.607324	

*Vogelsang (1993) asymptotic one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: LDGDPDEFBRA
Method: Least Squares
Date: 10/27/23 Time: 12:52
Sample (adjusted): 1962 2022
Included observations: 61 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LDGDPDEFBRA(-1)	0.611163	0.069976	8.733885	0.0000
C	-0.020212	0.109745	-0.184172	0.8545
TREND	0.025169	0.005568	4.520649	0.0000
INCPTBREAK	-1.204527	0.211253	-5.701811	0.0000
BREAKDUM	1.687393	0.431884	3.907049	0.0003

R-squared	0.826179	Mean dependent var	0.558656
Adjusted R-squared	0.813763	S.D. dependent var	0.820770
S.E. of regression	0.354205	Akaike info criterion	0.840530
Sum squared resid	7.025822	Schwarz criterion	1.013552
Log likelihood	-20.63616	Hannan-Quinn criter.	0.908339
F-statistic	66.54247	Durbin-Watson stat	1.622769
Prob(F-statistic)	0.000000		

Null Hypothesis: LDBMBRA has a unit root
Trend Specification: Intercept only
Break Specification: Intercept only
Break Type: Additive outlier

Break Date: 1997
Break Selection: Minimize Dickey-Fuller t-statistic
Lag Length: 6 (Automatic - based on Schwarz information criterion, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.573760	0.0355
Test critical values:	1% level	-4.949133	
	5% level	-4.443649	
	10% level	-4.193627	

*Vogelsang (1993) asymptotic one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: RESID
Method: Least Squares
Date: 10/27/23 Time: 13:00
Sample (adjusted): 1968 2022
Included observations: 55 after adjustments

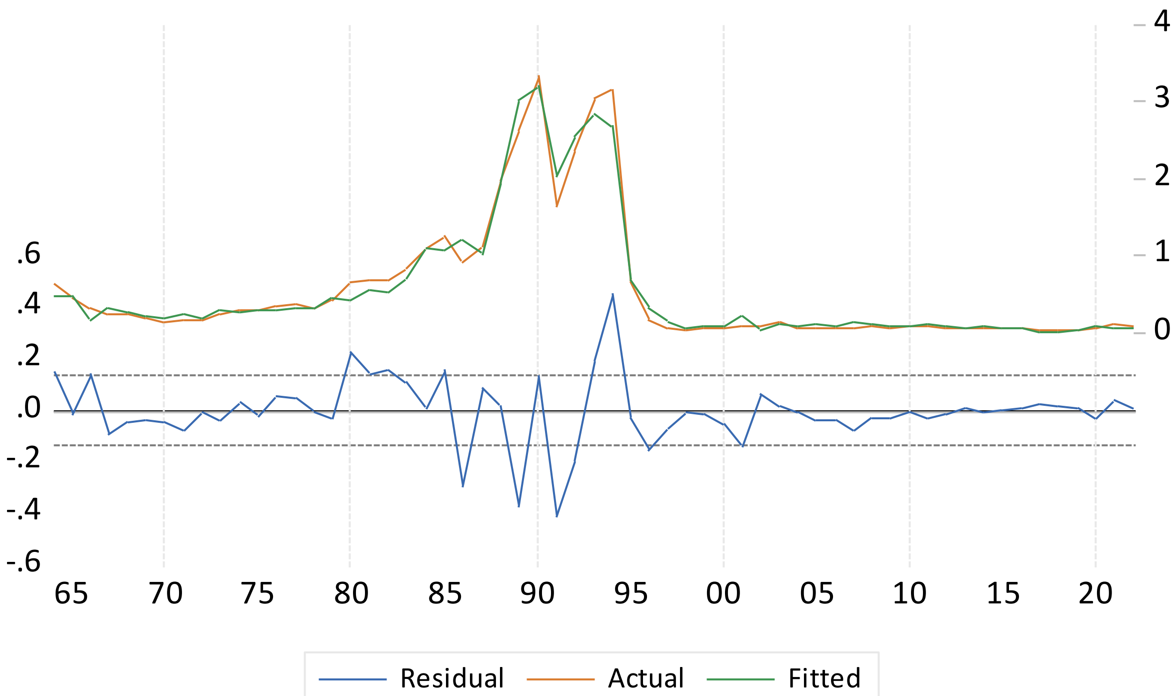
Variable	Coefficient	Std. Error	t-Statistic	Prob.
RESID(-1)	0.582098	0.091370	6.370803	0.0000
D(RESID(-1))	0.065571	0.140572	0.466457	0.6434
D(RESID(-2))	-0.110691	0.168455	-0.657096	0.5148
D(RESID(-3))	0.204702	0.177960	1.150268	0.2567
D(RESID(-4))	0.853997	0.177165	4.820360	0.0000
D(RESID(-5))	1.009016	0.191834	5.259838	0.0000
D(RESID(-6))	0.699829	0.197266	3.547632	0.0010
BREAKDUM	-1.093319	0.486525	-2.247199	0.0301
BREAKDUM1	-0.171674	0.523087	-0.328194	0.7444
BREAKDUM2	2.576706	0.511751	5.035075	0.0000
BREAKDUM3	2.815734	0.600607	4.688147	0.0000
BREAKDUM4	1.147101	0.523142	2.192715	0.0341
BREAKDUM5	-0.821641	0.349202	-2.352912	0.0235
BREAKDUM6	-0.391022	0.361753	-1.080910	0.2861

R-squared	0.857131	Mean dependent var	0.066295
Adjusted R-squared	0.811831	S.D. dependent var	0.753811
S.E. of regression	0.326991	Akaike info criterion	0.817563
Sum squared resid	4.383856	Schwarz criterion	1.328521
Log likelihood	-8.482995	Hannan-Quinn criter.	1.015155
Durbin-Watson stat	2.026636		

Dependent Variable: LDGDPDEFBRA
Method: Discrete Threshold Regression
Date: 08/26/23 Time: 19:26
Sample (adjusted): 1964 2022
Included observations: 59 after adjustments
Selection: Trimming 0.15, Max. thresholds 5, Sig. level 0.05
Threshold variable: LDGDPDEFBRA

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LDGDPDEFBRA < 1.248593 -- 51 obs				
C	-0.033838	0.029734	-1.138013	0.2604
LDBMBRA	0.783034	0.077554	10.09665	0.0000
COINTRES(-1)	-0.115791	0.159716	-0.724983	0.4718
LDGDPDEFBRA(-1)	0.157178	0.053212	2.953792	0.0047
1.248593 <= LDGDPDEFBRA -- 8 obs				
C	0.546455	0.261549	2.089307	0.0417
LDBMBRA	0.201082	0.080074	2.511200	0.0152
COINTRES(-1)	-1.570409	0.178273	-8.809021	0.0000
LDGDPDEFBRA(-1)	0.609833	0.064733	9.420738	0.0000

R-squared	0.975864	Mean dependent var	0.560880
Adjusted R-squared	0.972551	S.D. dependent var	0.834557
S.E. of regression	0.138268	Akaike info criterion	-0.993777
Sum squared resid	0.975014	Schwarz criterion	-0.712077
Log likelihood	37.31643	Hannan-Quinn criter.	-0.883813
F-statistic	294.5711	Durbin-Watson stat	1.936218
Prob(F-statistic)	0.000000		



Date: 09/23/23 Time: 18:04
Sample (adjusted): 1964 2022
Q-statistic probabilities adjusted for 2 dynamic regressors

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
. | . |	. | . |	1	0.020	0.020	0.0258	0.872
. | . |	. | . |	2	-0.045	-0.046	0.1545	0.926
. | . |	. | . |	3	-0.008	-0.007	0.1591	0.984
.*| . |	.*| . |	4	-0.113	-0.115	0.9955	0.910
. | . |	. | . |	5	0.044	0.049	1.1261	0.952
.*| . |	.*| . |	6	-0.073	-0.088	1.4862	0.960
.*| . |	.*| . |	7	-0.109	-0.104	2.3119	0.941
.*| . |	.*| . |	8	-0.124	-0.145	3.3952	0.907
. | . |	. | . |	9	-0.045	-0.046	3.5392	0.939
. | . |	. | . |	10	0.041	0.002	3.6633	0.961
.*| . |	.*| . |	11	-0.084	-0.119	4.1943	0.964
. | . |	. | . |	12	0.035	0.006	4.2881	0.978
. | . |	. | . |	13	0.009	-0.025	4.2944	0.988
. | . |	. | . |	14	0.039	0.015	4.4172	0.992
. | . |	.*| . |	15	-0.047	-0.122	4.5971	0.995
. | . |	. | . |	16	0.011	0.003	4.6070	0.997
. | . |	. | . |	17	0.034	-0.005	4.7078	0.998
. |*. |	. | . |	18	0.077	0.072	5.2215	0.998
. | . |	. | . |	19	0.072	0.030	5.6879	0.999
. | . |	. | . |	20	-0.022	-0.005	5.7328	0.999
. | . |	. |*. |	21	0.053	0.076	5.9962	0.999
. | . |	. | . |	22	0.006	0.005	5.9997	1.000
. | . |	. | . |	23	-0.058	-0.048	6.3336	1.000
. | . |	. | . |	24	0.019	0.025	6.3717	1.000

*Probabilities may not be valid for this equation specification.

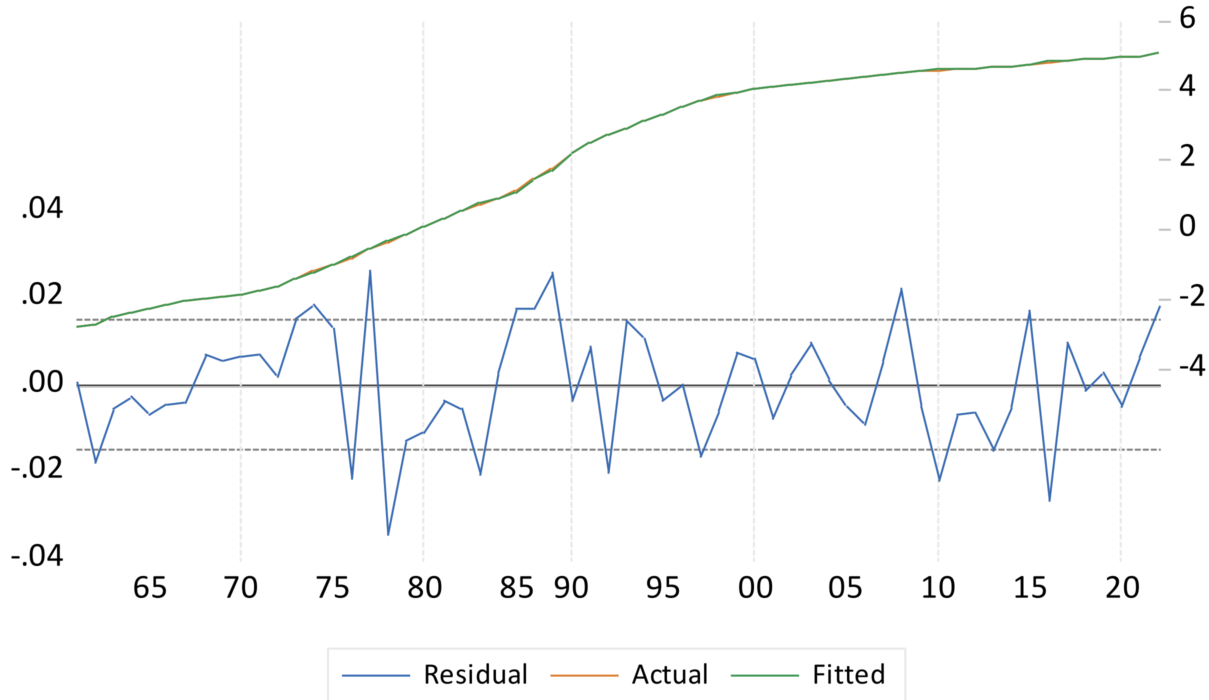
Colombia

1. Cointegration

Dependent Variable: LCPICOL
Method: Discrete Threshold Regression
Date: 08/26/23 Time: 19:46
Sample (adjusted): 1961 2022
Included observations: 60 after adjustments
Selection: Trimming 0.15, Max. thresholds 5, Sig. level 0.05
Threshold variable: LDCPICOL

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LDCPICOL < 0.07244586 -- 23 obs				
C	0.824587	0.859996	0.958827	0.3429
LBMCOL	-0.028897	0.033009	-0.875436	0.3861
LCPICOL(-1)	1.036617	0.037732	27.47346	0.0000
@TREND^2	1.80E-06	2.32E-05	0.077581	0.9385
0.07244586 <= LDCPICOL < 0.1546776 -- 11 obs				
C	-2.591981	0.647246	-4.004630	0.0002
LBMCOL	0.102920	0.024706	4.165778	0.0001
LCPICOL(-1)	0.880393	0.029646	29.69732	0.0000
@TREND^2	-6.54E-05	1.56E-05	-4.201618	0.0001
0.1546776 <= LDCPICOL < 0.216192 -- 16 obs				
C	-1.657767	0.885882	-1.871319	0.0680
LBMCOL	0.069365	0.031952	2.170941	0.0354
LCPICOL(-1)	0.914062	0.058220	15.70012	0.0000
@TREND^2	-2.07E-05	8.90E-05	-0.233140	0.8167
0.216192 <= LDCPICOL -- 10 obs				
C	-4.928517	1.817859	-2.711166	0.0095
LBMCOL	0.182085	0.064452	2.825131	0.0071
LCPICOL(-1)	0.640305	0.123914	5.167349	0.0000
@TREND^2	0.000612	0.000212	2.883647	0.0061

R-squared	0.999978	Mean dependent var	1.903832
Adjusted R-squared	0.999970	S.D. dependent var	2.764281
S.E. of regression	0.015158	Akaike info criterion	-5.317373
Sum squared resid	0.010110	Schwarz criterion	-4.758881
Log likelihood	175.5212	Hannan-Quinn criter.	-5.098916
F-statistic	130803.9	Durbin-Watson stat	1.960620
Prob(F-statistic)	0.000000		



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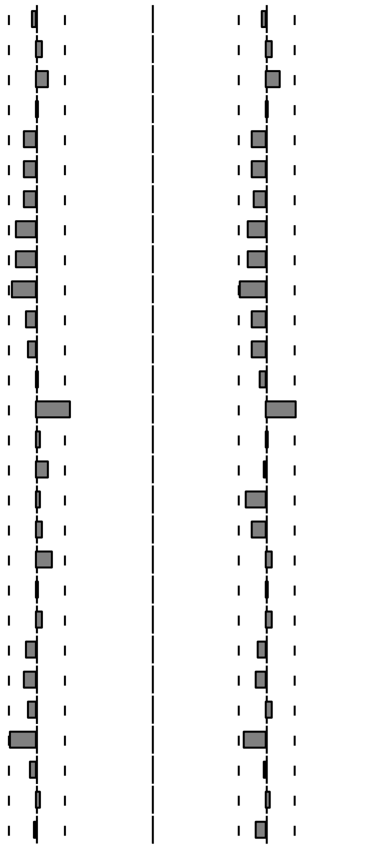
Date: 09/23/23 Time: 18:13
Sample (adjusted): 1961 2022
Q-statistic probabilities adjusted for 4 dynamic regressors

Autocorrelation / Partial Correlation



	AC	PAC	Q-Stat	Prob*
1	-0.029	-0.029	0.0540	0.816
2	0.050	0.049	0.2118	0.900
3	0.114	0.117	1.0536	0.788
4	-0.003	0.001	1.0544	0.901
5	-0.104	-0.118	1.7838	0.878
6	-0.101	-0.125	2.4804	0.871
7	-0.104	-0.105	3.2442	0.862
8	-0.178	-0.157	5.5150	0.701
9	-0.173	-0.168	7.7008	0.565
10	-0.213	-0.236	11.072	0.352
11	-0.089	-0.134	11.675	0.389
12	-0.076	-0.122	12.118	0.436
13	0.011	-0.054	12.128	0.517
14	0.311	0.271	19.935	0.132
15	0.025	0.006	19.988	0.172
16	0.103	-0.012	20.889	0.183
17	0.031	-0.177	20.974	0.227
18	0.061	-0.132	21.304	0.264
19	0.138	0.065	23.020	0.236
20	0.021	0.010	23.060	0.286
21	0.044	0.062	23.245	0.331
22	-0.090	-0.079	24.041	0.345
23	-0.102	-0.087	25.088	0.346
24	-0.074	0.046	25.648	0.371
25	-0.231	-0.189	31.322	0.179
26	-0.052	-0.024	31.614	0.206
27	0.035	0.028	31.750	0.241
28	-0.020	-0.084	31.796	0.283

*Probabilities may not be valid for this equation specification.

Null Hypothesis: COINTRES has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.330078	0.0000
Test critical values:	1% level	-2.606163	
	5% level	-1.946654	
	10% level	-1.613122	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(COINTRES)
Method: Least Squares
Date: 09/23/23 Time: 18:14
Sample (adjusted): 1962 2022
Included observations: 57 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
COINTRES(-1)	-1.033683	0.141019	-7.330078	0.0000

R-squared	0.488605	Mean dependent var	0.000831
Adjusted R-squared	0.488605	S.D. dependent var	0.018479
S.E. of regression	0.013214	Akaike info criterion	-5.797638
Sum squared resid	0.009779	Schwarz criterion	-5.761795
Log likelihood	166.2327	Hannan-Quinn criter.	-5.783708
Durbin-Watson stat	1.899183		

2. Error Correction Model (ECM)

Null Hypothesis: LDCPICOL has a unit root
Trend Specification: Intercept only
Break Specification: Intercept only
Break Type: Innovational outlier

Break Date: 1998
Break Selection: Minimize Dickey-Fuller t-statistic
Lag Length: 0 (Automatic - based on Schwarz information criterion, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.402464	0.0562
Test critical values:	1% level	-4.949133	
	5% level	-4.443649	
	10% level	-4.193627	

*Vogelsang (1993) asymptotic one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: LDCPICOL
Method: Least Squares
Date: 10/27/23 Time: 13:06
Sample (adjusted): 1962 2022
Included observations: 61 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LDCPICOL(-1)	0.564194	0.098991	5.699421	0.0000
C	0.079869	0.018898	4.226438	0.0001
INCPTBREAK	-0.059247	0.016406	-3.611367	0.0006
BREAKDUM	0.054987	0.044035	1.248729	0.2169

R-squared	0.749831	Mean dependent var	0.129252
Adjusted R-squared	0.736664	S.D. dependent var	0.081215
S.E. of regression	0.041677	Akaike info criterion	-3.454424
Sum squared resid	0.099006	Schwarz criterion	-3.316006
Log likelihood	109.3599	Hannan-Quinn criter.	-3.400177
F-statistic	56.94856	Durbin-Watson stat	2.267487
Prob(F-statistic)	0.000000		

Null Hypothesis: LDBMCOL has a unit root
Trend Specification: Intercept only
Break Specification: Intercept only
Break Type: Innovational outlier

Break Date: 1994
Break Selection: Minimize Dickey-Fuller t-statistic
Lag Length: 0 (Automatic - based on Schwarz information criterion, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.418261	0.0538
Test critical values:	1% level	-4.949133	
	5% level	-4.443649	
	10% level	-4.193627	

*Vogelsang (1993) asymptotic one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: LDBMCOL
Method: Least Squares
Date: 10/27/23 Time: 13:10
Sample (adjusted): 1962 2022
Included observations: 55 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LDBMCOL(-1)	0.559458	0.099709	5.610895	0.0000
C	0.104356	0.025534	4.086979	0.0002
INCPTBREAK	-0.054806	0.017706	-3.095273	0.0032
BREAKDUM	0.142797	0.058527	2.439836	0.0182

R-squared	0.661318	Mean dependent var	0.181412
Adjusted R-squared	0.641395	S.D. dependent var	0.090615
S.E. of regression	0.054264	Akaike info criterion	-2.919975
Sum squared resid	0.150172	Schwarz criterion	-2.773987
Log likelihood	84.29930	Hannan-Quinn criter.	-2.863520
F-statistic	33.19453	Durbin-Watson stat	2.174520
Prob(F-statistic)	0.000000		

Dependent Variable: LDCPICOL
Method: Discrete Threshold Regression
Date: 08/26/23 Time: 19:49
Sample (adjusted): 1962 2022
Included observations: 57 after adjustments
Selection: Trimming 0.15, Max. thresholds 5, Sig. level 0.05
Threshold variable: LDCPICOL

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LDCPICOL < 0.06157952 -- 17 obs				
C	0.031116	0.011532	2.698225	0.0098
LDBMCOL(-1)	0.057504	0.085931	0.669194	0.5068
COINTRES(-1)	0.122242	0.351392	0.347881	0.7296
0.06157952 <= LDCPICOL < 0.1127951 -- 13 obs				
C	0.090501	0.014170	6.386717	0.0000
LDBMCOL(-1)	-0.100958	0.098226	-1.027818	0.3095
COINTRES(-1)	-0.318304	0.689643	-0.461548	0.6466
0.1127951 <= LDCPICOL < 0.2031828 -- 14 obs				
C	0.109182	0.013379	8.160398	0.0000
LDBMCOL(-1)	0.246258	0.055794	4.413673	0.0001
COINTRES(-1)	-0.398602	0.318119	-1.252998	0.2167
0.2031828 <= LDCPICOL -- 13 obs				
C	0.214120	0.017001	12.59432	0.0000
LDBMCOL(-1)	0.047721	0.060242	0.792151	0.4324
COINTRES(-1)	-0.545128	0.261949	-2.081047	0.0432

R-squared	0.965288	Mean dependent var	0.121159
Adjusted R-squared	0.956803	S.D. dependent var	0.077577
S.E. of regression	0.016123	Akaike info criterion	-5.232415
Sum squared resid	0.011699	Schwarz criterion	-4.802299
Log likelihood	161.1238	Hannan-Quinn criter.	-5.065257
F-statistic	113.7618	Durbin-Watson stat	1.683096
Prob(F-statistic)	0.000000		

Date: 09/23/23 Time: 18:32
Sample (adjusted): 1962 2022
Included observations: 57 after adjustments

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.097	0.097	0.5623	0.453
		2	-0.171	-0.182	2.3425	0.310
		3	0.093	0.135	2.8760	0.411
		4	-0.107	-0.177	3.5977	0.463
		5	-0.175	-0.101	5.5771	0.350
		6	-0.004	-0.036	5.5784	0.472
		7	-0.044	-0.071	5.7089	0.574
		8	0.030	0.060	5.7703	0.673
		9	0.042	-0.032	5.8920	0.751
		10	0.062	0.076	6.1683	0.801
		11	0.115	0.081	7.1383	0.788
		12	0.029	0.021	7.2000	0.844
		13	-0.032	0.008	7.2766	0.887
		14	-0.180	-0.200	9.8018	0.777
		15	0.028	0.136	9.8635	0.828
		16	-0.020	-0.104	9.8973	0.872
		17	-0.146	-0.058	11.694	0.818
		18	-0.020	-0.070	11.729	0.861
		19	0.106	0.039	12.727	0.852
		20	-0.129	-0.153	14.246	0.818
		21	-0.143	-0.179	16.157	0.761
		22	0.068	0.026	16.596	0.785
		23	0.082	0.032	17.261	0.796
		24	-0.061	-0.044	17.641	0.820

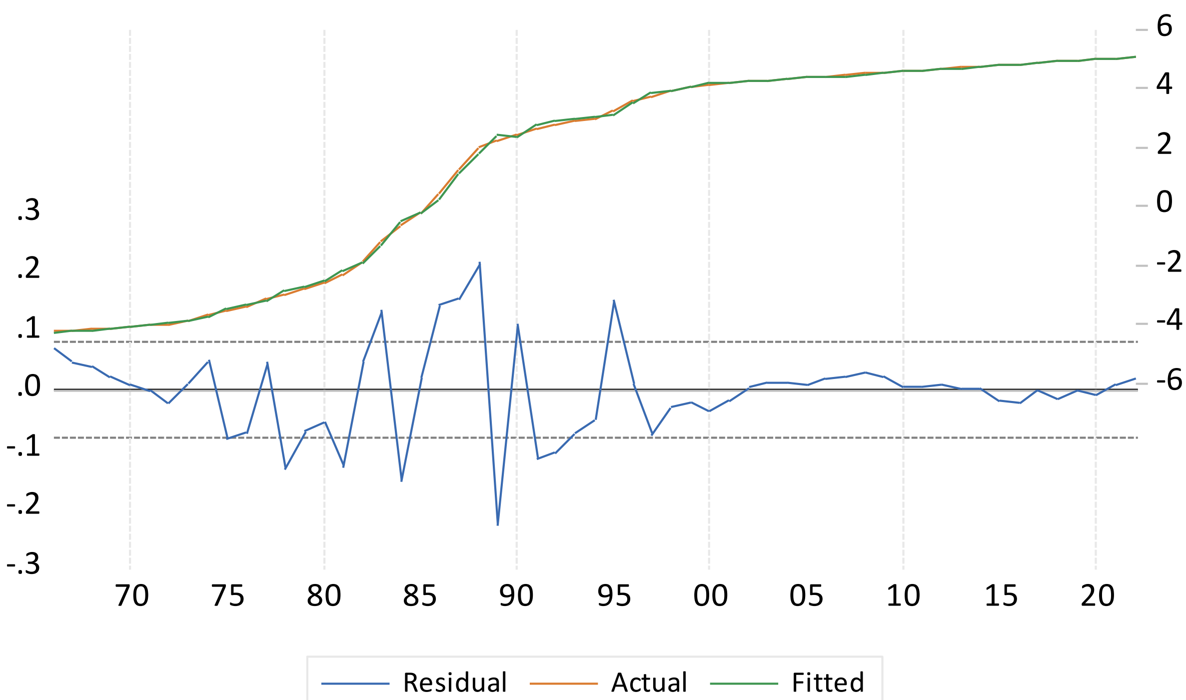
Mexico

1. Cointegration

Dependent Variable: LCPIMEX
Method: Least Squares
Date: 08/26/23 Time: 20:00
Sample (adjusted): 1966 2022
Included observations: 57 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-6.041277	1.094005	-5.522165	0.0000
LBMMEX	0.272329	0.050374	5.406177	0.0000
LCPIMEX(-1)	1.473026	0.128030	11.50533	0.0000
LCPIMEX(-2)	-0.916215	0.209225	-4.379085	0.0001
LCPIMEX(-3)	0.242587	0.139553	1.738314	0.0883
LCPIMEX(-6)	-0.062820	0.036418	-1.724956	0.0907
@TREND	-0.013369	0.004359	-3.066694	0.0035

R-squared	0.999519	Mean dependent var	1.477068
Adjusted R-squared	0.999461	S.D. dependent var	3.542111
S.E. of regression	0.082243	Akaike info criterion	-2.043680
Sum squared resid	0.338199	Schwarz criterion	-1.792779
Log likelihood	65.24487	Hannan-Quinn criter.	-1.946171
F-statistic	17304.10	Durbin-Watson stat	2.042269
Prob(F-statistic)	0.000000		



Date: 09/23/23 Time: 18:48
Sample (adjusted): 1966 2022
Q-statistic probabilities adjusted for 4 dynamic regressors

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.029	-0.029	0.0505	0.822
		2	0.011	0.011	0.0585	0.971
		3	0.037	0.038	0.1451	0.986
		4	-0.141	-0.139	1.4101	0.842
		5	0.117	0.111	2.2928	0.807
		6	-0.294	-0.298	8.0019	0.238
		7	-0.139	-0.145	9.3102	0.231
		8	0.088	0.065	9.8453	0.276
		9	-0.050	-0.001	10.021	0.349
		10	-0.147	-0.268	11.569	0.315
		11	-0.099	-0.098	12.281	0.343
		12	-0.109	-0.180	13.165	0.357
		13	0.128	0.012	14.408	0.346
		14	-0.023	-0.074	14.449	0.417
		15	-0.095	-0.115	15.168	0.439
		16	0.068	-0.150	15.551	0.485
		17	0.009	-0.098	15.558	0.555
		18	0.054	-0.116	15.813	0.606
		19	0.085	0.056	16.451	0.627
		20	0.054	0.018	16.719	0.671
		21	0.123	-0.029	18.125	0.641
		22	0.086	-0.021	18.833	0.656
		23	-0.055	-0.046	19.128	0.694
		24	0.000	-0.041	19.128	0.745

*Probabilities may not be valid for this equation specification.

Null Hypothesis: COINTRES has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.684547	0.0000
Test critical values:	1% level	-2.606911	
	5% level	-1.946764	
	10% level	-1.613062	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(COINTRES)
Method: Least Squares
Date: 09/23/23 Time: 18:21
Sample (adjusted): 1967 2022
Included observations: 56 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
COINTRES(-1)	-1.029042	0.133911	-7.684547	0.0000

R-squared	0.517737	Mean dependent var	-0.000850
Adjusted R-squared	0.517737	S.D. dependent var	0.112060
S.E. of regression	0.077820	Akaike info criterion	-2.251142
Sum squared resid	0.333077	Schwarz criterion	-2.214975
Log likelihood	64.03198	Hannan-Quinn criter.	-2.237120
Durbin-Watson stat	2.011340		

2. Error Correction Model (ECM)

Null Hypothesis: LDCPIMEX has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic
Elliott-Rothenberg-Stock DF-GLS test statistic		-2.109411
Test critical values:	1% level	-2.603423
	5% level	-1.946253
	10% level	-1.613346

*MacKinnon (1996)

DF-GLS Test Equation on GLS Detrended Residuals
Dependent Variable: D(GLSRESID)
Method: Least Squares
Date: 10/27/23 Time: 13:20
Sample (adjusted): 1962 2022
Included observations: 61 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
GLSRESID(-1)	-0.137271	0.065075	-2.109411	0.0391

R-squared	0.068961	Mean dependent var	0.000984
Adjusted R-squared	0.068961	S.D. dependent var	0.107654
S.E. of regression	0.103876	Akaike info criterion	-1.674982
Sum squared resid	0.647412	Schwarz criterion	-1.640377
Log likelihood	52.08694	Hannan-Quinn criter.	-1.661420
Durbin-Watson stat	1.776173		

Null Hypothesis: LDBMMEX has a unit root
Exogenous: Constant
Bandwidth: 5 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-5.776565	0.0000
Test critical values:	1% level	-3.542097	
	5% level	-2.910019	
	10% level	-2.592645	

*MacKinnon (1996) one-sided p-values.

Residual variance (no correction)	0.028429
HAC corrected variance (Bartlett kernel)	0.043693

Phillips-Perron Test Equation
Dependent Variable: D(LDBMMEX)
Method: Least Squares
Date: 10/27/23 Time: 13:28
Sample (adjusted): 1962 2022
Included observations: 61 after adjustments

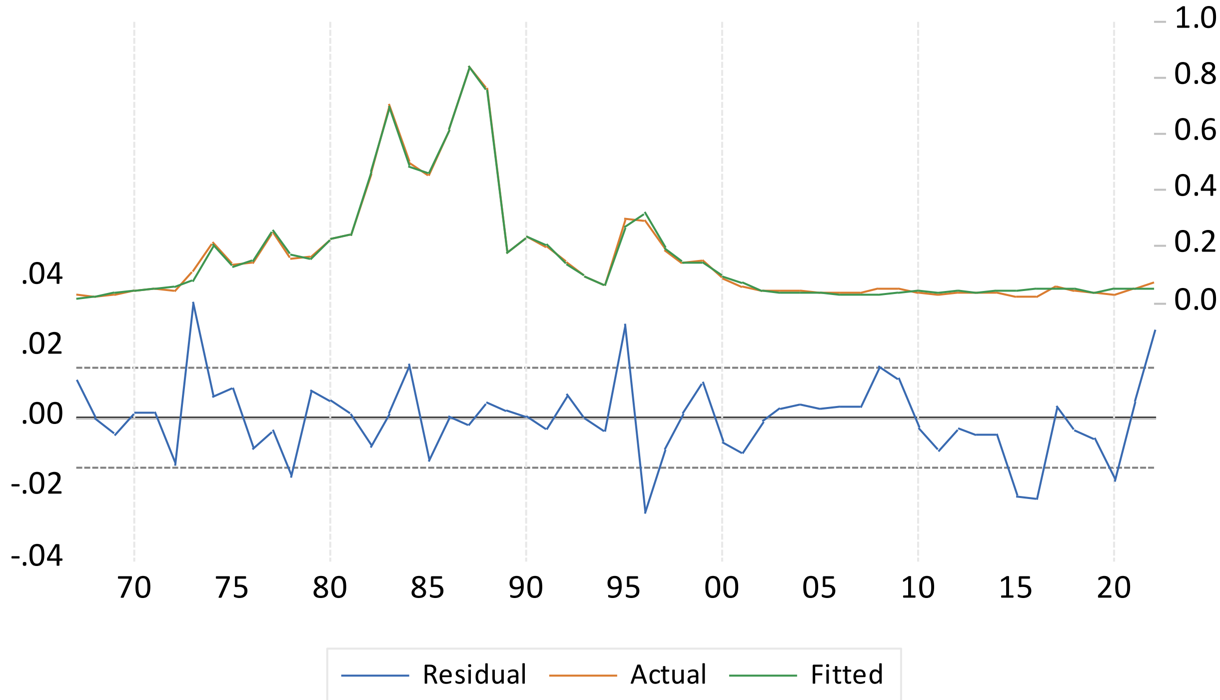
Variable	Coefficient	Std. Error	t-Statistic	Prob.
LDBMMEX(-1)	-0.632266	0.121474	-5.204968	0.0000
C	0.130086	0.033389	3.896046	0.0003

R-squared	0.314684	Mean dependent var	-0.000868
Adjusted R-squared	0.303069	S.D. dependent var	0.205364
S.E. of regression	0.171443	Akaike info criterion	-0.656897
Sum squared resid	1.734165	Schwarz criterion	-0.587688
Log likelihood	22.03536	Hannan-Quinn criter.	-0.629774
F-statistic	27.09169	Durbin-Watson stat	2.278523
Prob(F-statistic)	0.000003		

Dependent Variable: LDCPIMEX
Method: Discrete Threshold Regression
Date: 08/26/23 Time: 20:04
Sample (adjusted): 1967 2022
Included observations: 56 after adjustments
Selection: Trimming 0.15, Max. thresholds 5, Sig. level 0.05
Threshold variable: LDCPIMEX

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LDCPIMEX < 0.1823871 -- 39 obs				
C	0.014855	0.005561	2.671084	0.0114
LDBMMEX	0.163686	0.039100	4.186321	0.0002
COINTRES(-1)	-0.452274	0.117479	-3.849813	0.0005
LDCPIMEX(-1)	0.617723	0.090157	6.851642	0.0000
LDCPIMEX(-2)	-0.357208	0.149401	-2.390939	0.0223
LDCPIMEX(-3)	0.239447	0.081638	2.933020	0.0059
LDCPIMEX(-6)	-0.080244	0.017074	-4.699705	0.0000
0.1823871 <= LDCPIMEX < 0.2954893 -- 8 obs				
C	0.180748	0.028709	6.295893	0.0000
LDBMMEX	-0.020127	0.082787	-0.243119	0.8093
COINTRES(-1)	-0.373142	0.138517	-2.693834	0.0108
LDCPIMEX(-1)	0.337113	0.137280	2.455654	0.0192
LDCPIMEX(-2)	-0.134922	0.076753	-1.757880	0.0875
LDCPIMEX(-3)	0.149580	0.096593	1.548557	0.1305
LDCPIMEX(-6)	-0.202645	0.222314	-0.911526	0.3683
0.2954893 <= LDCPIMEX -- 9 obs				
C	-0.361811	0.032145	-11.25569	0.0000
LDBMMEX	0.599635	0.042784	14.01549	0.0000
COINTRES(-1)	-1.298067	0.125073	-10.37844	0.0000
LDCPIMEX(-1)	0.902456	0.125898	7.168142	0.0000
LDCPIMEX(-2)	-0.654871	0.147701	-4.433762	0.0001
LDCPIMEX(-3)	0.165579	0.087055	1.902017	0.0654
LDCPIMEX(-6)	2.108496	0.151249	13.94058	0.0000

R-squared	0.996764	Mean dependent var	0.166683
Adjusted R-squared	0.994915	S.D. dependent var	0.195750
S.E. of regression	0.013959	Akaike info criterion	-5.425376
Sum squared resid	0.006820	Schwarz criterion	-4.665869
Log likelihood	172.9105	Hannan-Quinn criter.	-5.130917
F-statistic	539.0318	Durbin-Watson stat	1.909789
Prob(F-statistic)	0.000000		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Date: 09/23/23 Time: 18:23
Sample (adjusted): 1967 2022
Q-statistic probabilities adjusted for 12 dynamic regressors

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
. | . |	. | . |	1	-0.010	-0.010	0.0064	0.936
.*| . |	.*| . |	2	-0.092	-0.092	0.5102	0.775
. | . |	.*| . |	3	-0.065	-0.067	0.7684	0.857
. |*. |	. |*. |	4	0.151	0.142	2.1885	0.701
. | . |	. | . |	5	0.022	0.014	2.2189	0.818
.*| . |	. | . |	6	-0.081	-0.062	2.6493	0.851
.*| . |	.*| . |	7	-0.170	-0.155	4.5666	0.713
.*| . |	.*| . |	8	-0.135	-0.180	5.7959	0.670
. | . |	. | . |	9	0.011	-0.040	5.8045	0.759
.*| . |	.*| . |	10	-0.074	-0.106	6.1895	0.799
. | . |	. | . |	11	0.025	0.051	6.2339	0.857
**| . |	**| . |	12	-0.257	-0.253	11.102	0.520
. | . |	. | . |	13	0.049	0.009	11.287	0.587
. |*. |	. |*. |	14	0.139	0.079	12.789	0.543
. |*. |	. | . |	15	0.137	0.070	14.276	0.505
. | . |	. |*. |	16	0.009	0.080	14.282	0.578
.*| . |	.*| . |	17	-0.114	-0.147	15.374	0.569
. |*. |	. | . |	18	0.078	0.011	15.897	0.600
. |*. |	. |*. |	19	0.185	0.093	18.913	0.462
. | . |	. | . |	20	0.072	0.029	19.376	0.498
**| . |	.*| . |	21	-0.210	-0.107	23.459	0.320
. |*. |	. |*. |	22	0.118	0.154	24.780	0.308
.*| . |	.*| . |	23	-0.151	-0.187	27.013	0.255
. |*. |	. | . |	24	0.092	0.056	27.874	0.265

*Probabilities may not be valid for this equation specification.

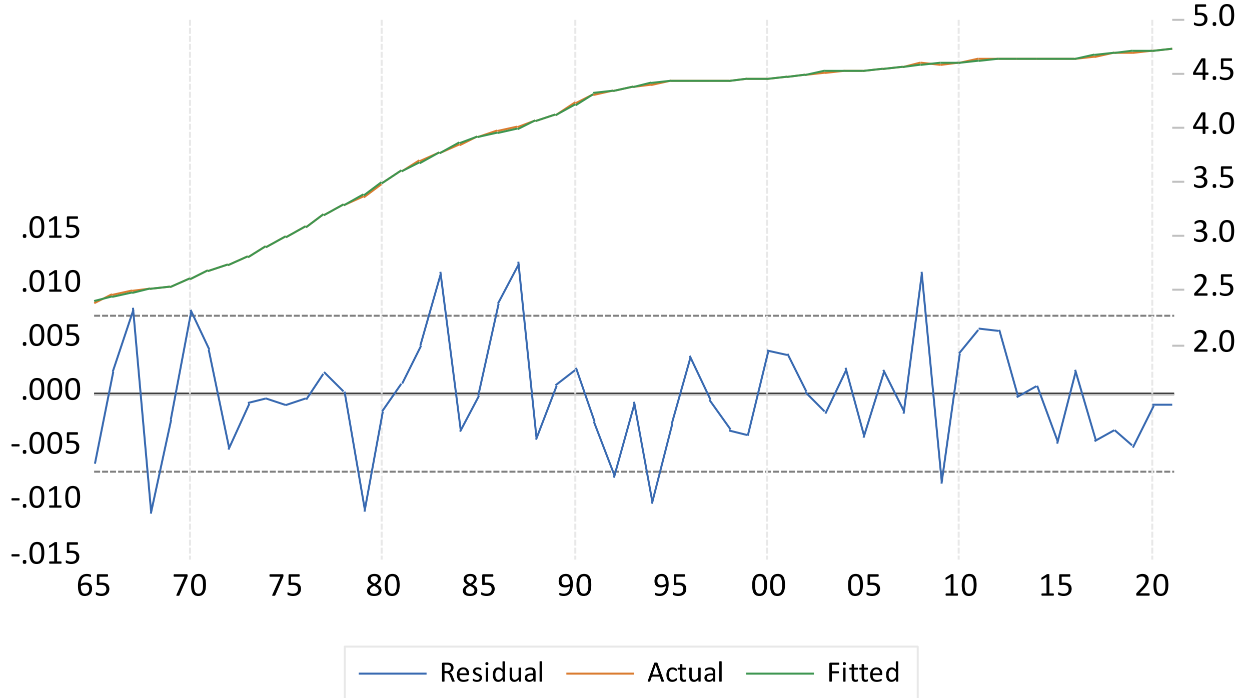
Sweden

1. Cointegration

Dependent Variable: LCPISWE
Method: Discrete Threshold Regression
Date: 08/26/23 Time: 20:20
Sample (adjusted): 1965 2021
Included observations: 57 after adjustments
Selection: Trimming 0.15, Max. thresholds 5, Sig. level 0.05
Threshold variable: LDCPISWE

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LDCPISWE < 0.01768421 -- 16 obs				
C	0.004640	0.111683	0.041543	0.9671
LBMSWE	0.043158	0.025063	1.721936	0.0957
LCPISWE(-1)	0.711935	0.193471	3.679811	0.0009
LCPISWE(-2)	0.152824	0.282029	0.541872	0.5920
LCPISWE(-3)	-0.286689	0.324156	-0.884416	0.3837
LCPISWE(-4)	0.224748	0.231051	0.972720	0.3387
LCPISWE(-5)	-0.069668	0.155998	-0.446594	0.6585
0.01768421 <= LDCPISWE < 0.04619797 -- 18 obs				
C	-0.042613	0.137057	-0.310913	0.7581
LBMSWE	0.003433	0.006275	0.547017	0.5886
LCPISWE(-1)	1.156934	0.164239	7.044220	0.0000
LCPISWE(-2)	-0.220364	0.282949	-0.778813	0.4424
LCPISWE(-3)	0.026998	0.175926	0.153465	0.8791
LCPISWE(-4)	0.132877	0.260380	0.510317	0.6137
LCPISWE(-5)	-0.104174	0.132390	-0.786874	0.4377
0.04619797 <= LDCPISWE < 0.09024854 -- 14 obs				
C	-0.400269	0.583512	-0.685965	0.4982
LBMSWE	0.020458	0.026538	0.770899	0.4470
LCPISWE(-1)	1.278912	0.214558	5.960682	0.0000
LCPISWE(-2)	-0.415541	0.333809	-1.244847	0.2232
LCPISWE(-3)	0.236019	0.262366	0.899581	0.3758
LCPISWE(-4)	-0.037861	0.215638	-0.175575	0.8618
LCPISWE(-5)	-0.090840	0.120517	-0.753747	0.4571
0.09024854 <= LDCPISWE -- 9 obs				
C	-9.326223	3.849569	-2.422667	0.0219
LBMSWE	0.414691	0.170258	2.435660	0.0212
LCPISWE(-1)	0.842294	0.136029	6.192005	0.0000
LCPISWE(-2)	-0.061164	0.312413	-0.195779	0.8461
LCPISWE(-3)	0.531770	0.462959	1.148633	0.2601
LCPISWE(-4)	-1.840381	0.833011	-2.209312	0.0352
LCPISWE(-5)	1.073994	0.566572	1.895600	0.0680

R-squared	0.999955	Mean dependent var	3.963100
Adjusted R-squared	0.999914	S.D. dependent var	0.765780
S.E. of regression	0.007117	Akaike info criterion	-6.745958
Sum squared resid	0.001469	Schwarz criterion	-5.742354
Log likelihood	220.2598	Hannan-Quinn criter.	-6.355924
F-statistic	24011.45	Durbin-Watson stat	1.992025
Prob(F-statistic)	0.000000		



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Date: 09/23/23 Time: 18:37
Sample (adjusted): 1965 2021
Q-statistic probabilities adjusted for 20 dynamic regressors

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
. | . |	. | . |	1	-0.010	-0.010	0.0055	0.941
.*| . |	.*| . |	2	-0.154	-0.154	1.4502	0.484
. | . |	. | . |	3	0.052	0.050	1.6205	0.655
. |** |	. |** |	4	0.248	0.231	5.5116	0.239
.*| . |	.*| . |	5	-0.141	-0.131	6.7898	0.237
. | . |	. | . |	6	-0.023	0.043	6.8243	0.337
.*| . |	**| . |	7	-0.147	-0.225	8.2731	0.309
.*| . |	**| . |	8	-0.183	-0.252	10.569	0.227
**| . |	**| . |	9	-0.216	-0.248	13.829	0.129
. | . |	. | . |	10	0.057	-0.036	14.061	0.170
. | . |	. | . |	11	-0.040	0.011	14.180	0.223
.*| . |	. | . |	12	-0.067	0.038	14.515	0.269
. | . |	. |*. |	13	0.020	0.106	14.547	0.337
. | . |	.*| . |	14	-0.043	-0.179	14.692	0.400
. | . |	. | . |	15	0.053	-0.029	14.914	0.458
. | . |	.*| . |	16	0.068	-0.146	15.288	0.504
. | . |	.*| . |	17	-0.039	-0.202	15.413	0.566
.*| . |	**| . |	18	-0.177	-0.274	18.120	0.448
. | . |	.*| . |	19	0.001	-0.194	18.120	0.514
. |*. |	. | . |	20	0.102	0.018	19.065	0.518
. | . |	. |*. |	21	0.042	0.098	19.228	0.571
.*| . |	. | . |	22	-0.144	-0.012	21.212	0.508
. |*. |	. | . |	23	0.077	0.022	21.795	0.533
. | . |	.*| . |	24	0.041	-0.150	21.967	0.581

*Probabilities may not be valid for this equation specification.

Null Hypothesis: COINTRES has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.586369	0.0000
Test critical values:	1% level	-2.606911	
	5% level	-1.946764	
	10% level	-1.613062	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(COINTRES)
Method: Least Squares
Date: 09/23/23 Time: 18:38
Sample (adjusted): 1966 2021
Included observations: 56 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
COINTRES(-1)	-1.009582	0.133078	-7.586369	0.0000

R-squared	0.511258	Mean dependent var	9.41E-05
Adjusted R-squared	0.511258	S.D. dependent var	0.007293
S.E. of regression	0.005099	Akaike info criterion	-7.701940
Sum squared resid	0.001430	Schwarz criterion	-7.665773
Log likelihood	216.6543	Hannan-Quinn criter.	-7.687918
Durbin-Watson stat	1.979343		

2. Error Correction Model (ECM)

Null Hypothesis: LDCPISWE has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic
Elliott-Rothenberg-Stock DF-GLS test statistic		-2.158776
Test critical values:	1% level	-2.604073
	5% level	-1.946348
	10% level	-1.613293

*MacKinnon (1996)

DF-GLS Test Equation on GLS Detrended Residuals
Dependent Variable: D(GLSRESID)
Method: Least Squares
Date: 10/27/23 Time: 13:38
Sample (adjusted): 1962 2021
Included observations: 60 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
GLSRESID(-1)	-0.146406	0.067819	-2.158776	0.0349

R-squared	0.073206	Mean dependent var	8.52E-07
Adjusted R-squared	0.073206	S.D. dependent var	0.019974
S.E. of regression	0.019229	Akaike info criterion	-5.048280
Sum squared resid	0.021815	Schwarz criterion	-5.013374
Log likelihood	152.4484	Hannan-Quinn criter.	-5.034627
Durbin-Watson stat	2.126003		

Null Hypothesis: LDBMSWE has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.740664	0.0000
Test critical values:	1% level	-3.544063	
	5% level	-2.910860	
	10% level	-2.593090	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LDBMSWE)
Method: Least Squares
Date: 10/27/23 Time: 13:41
Sample (adjusted): 1962 2021
Included observations: 60 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LDBMSWE(-1)	-0.724252	0.126162	-5.740664	0.0000
C	0.056005	0.011288	4.961569	0.0000

R-squared	0.362324	Mean dependent var	0.000515
Adjusted R-squared	0.351329	S.D. dependent var	0.056063
S.E. of regression	0.045153	Akaike info criterion	-3.324743
Sum squared resid	0.118251	Schwarz criterion	-3.254932
Log likelihood	101.7423	Hannan-Quinn criter.	-3.297436
F-statistic	32.95522	Durbin-Watson stat	1.997695
Prob(F-statistic)	0.000000		

Dependent Variable: LDCPISWE
Method: Discrete Threshold Regression
Date: 08/26/23 Time: 20:24
Sample (adjusted): 1968 2021
Included observations: 54 after adjustments
Selection: Trimming 0.15, Max. thresholds 5, Sig. level 0.05
Threshold variable: LDCPISWE

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LDCPISWE < 0.01768421 -- 16 obs				
C	0.004121	0.003904	1.055477	0.2986
LDBMSWE	0.007484	0.036479	0.205152	0.8387
COINTRES(-1)	-0.661254	0.391773	-1.687852	0.1006
LDCPISWE(-1)	0.049853	0.151565	0.328920	0.7442
LDCPISWE(-7)	-0.024342	0.056633	-0.429815	0.6700
0.01768421 <= LDCPISWE < 0.04619797 -- 17 obs				
C	0.018633	0.003810	4.890932	0.0000
LDBMSWE	0.017362	0.032128	0.540393	0.5924
COINTRES(-1)	0.122726	0.310602	0.395122	0.6952
LDCPISWE(-1)	-0.027304	0.081051	-0.336869	0.7383
LDCPISWE(-7)	0.169950	0.056465	3.009863	0.0049
0.04619797 <= LDCPISWE < 0.08501207 -- 11 obs				
C	0.054975	0.007203	7.632810	0.0000
LDBMSWE	-0.074259	0.056332	-1.318235	0.1962
COINTRES(-1)	0.236626	0.307137	0.770425	0.4464
LDCPISWE(-1)	0.327836	0.081146	4.040083	0.0003
LDCPISWE(-7)	-0.055742	0.075872	-0.734681	0.4676
0.08501207 <= LDCPISWE -- 10 obs				
C	0.076599	0.009709	7.889623	0.0000
LDBMSWE	-0.015921	0.054485	-0.292206	0.7719
COINTRES(-1)	-3.312400	0.552164	-5.998938	0.0000
LDCPISWE(-1)	0.164056	0.118405	1.385551	0.1749
LDCPISWE(-7)	0.140259	0.073800	1.900529	0.0659

R-squared	0.984282	Mean dependent var	0.041269
Adjusted R-squared	0.975498	S.D. dependent var	0.036870
S.E. of regression	0.005771	Akaike info criterion	-7.193751
Sum squared resid	0.001132	Schwarz criterion	-6.457091
Log likelihood	214.2313	Hannan-Quinn criter.	-6.909650
F-statistic	112.0587	Durbin-Watson stat	1.873397
Prob(F-statistic)	0.000000		

Date: 09/23/23 Time: 18:43
Sample (adjusted): 1968 2021
Q-statistic probabilities adjusted for 8 dynamic regressors

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.052	0.052	0.1542	0.695
		2	-0.078	-0.081	0.5081	0.776
		3	0.097	0.107	1.0703	0.784
		4	0.087	0.070	1.5309	0.821
		5	-0.039	-0.032	1.6238	0.898
		6	-0.120	-0.116	2.5308	0.865
		7	-0.161	-0.176	4.2000	0.756
		8	-0.031	-0.035	4.2640	0.833
		9	-0.140	-0.142	5.5776	0.781
		10	-0.094	-0.041	6.1911	0.799
		11	0.182	0.208	8.5168	0.666
		12	-0.153	-0.188	10.212	0.597
		13	0.018	0.076	10.235	0.675
		14	-0.128	-0.276	11.468	0.649
		15	-0.075	-0.107	11.905	0.686
		16	0.095	0.057	12.628	0.700
		17	-0.151	-0.214	14.497	0.632
		18	-0.239	-0.156	19.314	0.373
		19	0.019	-0.098	19.346	0.435
		20	0.029	-0.023	19.419	0.495
		21	0.031	-0.004	19.507	0.553
		22	0.161	0.105	21.948	0.463
		23	-0.082	-0.148	22.596	0.485
		24	0.117	-0.084	23.967	0.463

*Probabilities may not be valid for this equation specification.

Turkey

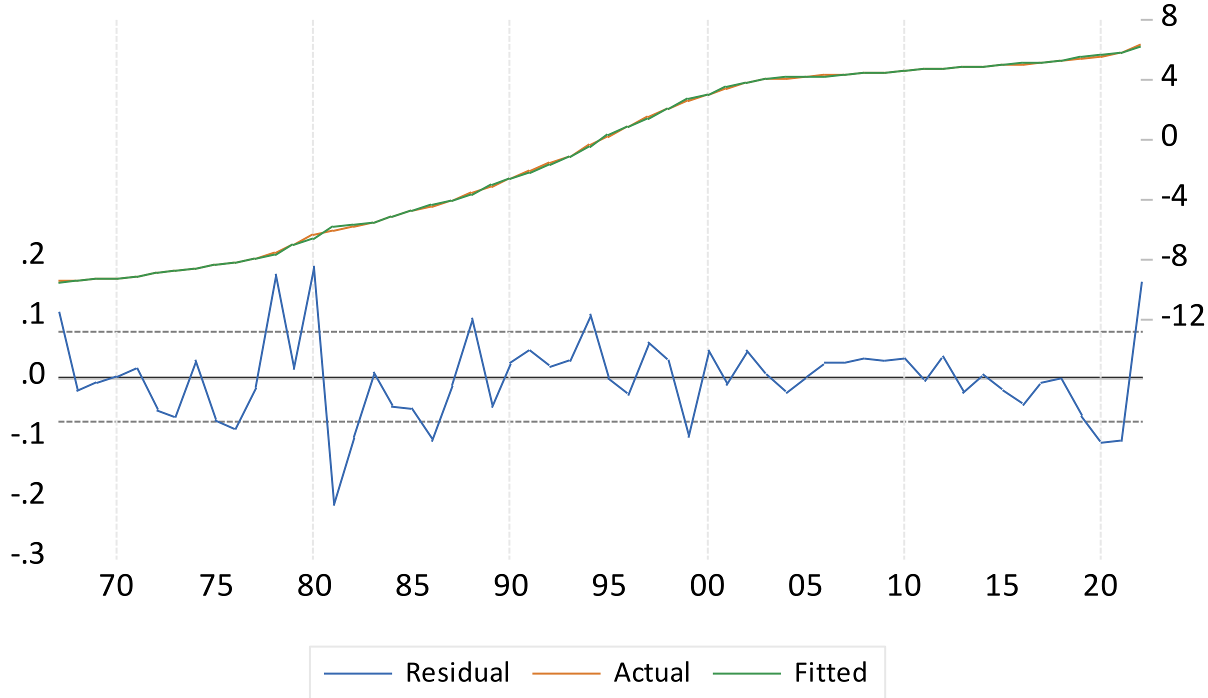
1. Cointegration

Dependent Variable: LCPITUR
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 08/26/23 Time: 22:06
Sample: 1967 2022
Included observations: 56
Convergence achieved after 30 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-8.609480	1.812920	-4.748957	0.0000
LBMTUR	0.394375	0.081630	4.831258	0.0000
LCPITUR(-1)	0.665282	0.105277	6.319362	0.0000
LCPITUR(-7)	-0.140030	0.021728	-6.444624	0.0000
MA(1)	0.700035	0.165157	4.238607	0.0001
MA(2)	0.415193	0.216206	1.920357	0.0606
SIGMASQ	0.004895	0.001085	4.511076	0.0000

R-squared	0.999840	Mean dependent var	-0.958611
Adjusted R-squared	0.999820	S.D. dependent var	5.580931
S.E. of regression	0.074797	Akaike info criterion	-2.219849
Sum squared resid	0.274133	Schwarz criterion	-1.966680
Log likelihood	69.15576	Hannan-Quinn criter.	-2.121696
F-statistic	51025.86	Durbin-Watson stat	1.930253
Prob(F-statistic)	0.000000		

Inverted MA Roots	-.35+.54i	-.35-.54i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Date: 09/23/23 Time: 18:52
Sample (adjusted): 1967 2022
Q-statistic probabilities adjusted for 2 ARMA terms and 2 dynamic regressors

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.035	-0.035	0.0728	
		2	-0.053	-0.055	0.2439	
		3	-0.063	-0.067	0.4861	0.486
		4	0.004	-0.004	0.4871	0.784
		5	0.004	-0.004	0.4879	0.922
		6	-0.080	-0.085	0.9045	0.924
		7	-0.162	-0.172	2.6513	0.754
		8	-0.042	-0.071	2.7699	0.837
		9	-0.032	-0.073	2.8411	0.899
		10	0.080	0.044	3.2896	0.915
		11	-0.031	-0.044	3.3592	0.948
		12	-0.020	-0.037	3.3887	0.971
		13	0.063	0.033	3.6912	0.978
		14	0.082	0.044	4.2055	0.979
		15	-0.017	-0.035	4.2279	0.989
		16	-0.022	-0.022	4.2679	0.994
		17	0.044	0.061	4.4308	0.996
		18	-0.014	-0.023	4.4477	0.998
		19	-0.155	-0.159	6.5489	0.989
		20	0.077	0.086	7.0830	0.989
		21	-0.110	-0.107	8.2115	0.984
		22	0.005	-0.020	8.2135	0.990
		23	-0.062	-0.080	8.5885	0.992
		24	0.018	-0.003	8.6220	0.995

*Probabilities may not be valid for this equation specification.

Null Hypothesis: COINTRES1 has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.414706	0.0000
Test critical values:	1% level	-2.607686	
	5% level	-1.946878	
	10% level	-1.612999	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(COINTRES1)
Method: Least Squares
Date: 09/23/23 Time: 18:53
Sample (adjusted): 1968 2022
Included observations: 55 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
COINTRES1(-1)	-1.037057	0.139865	-7.414706	0.0000

R-squared	0.504438	Mean dependent var	0.000970
Adjusted R-squared	0.504438	S.D. dependent var	0.098985
S.E. of regression	0.069682	Akaike info criterion	-2.471745
Sum squared resid	0.262199	Schwarz criterion	-2.435248
Log likelihood	68.97299	Hannan-Quinn criter.	-2.457631
Durbin-Watson stat	1.892941		

2. Error Correction Model (ECM)

Zivot-Andrews Unit Root Test
Date: 10/27/23 Time: 13:10
Sample: 1960 2022
Included observations: 63
Null Hypothesis: LDCPITUR has a unit root with a structural break in the intercept
Chosen lag length: 0 (maximum lags: 4)
Chosen break point: 2002

	t-Statistic	Prob. *
Zivot-Andrews test statistic	-4.050941	0.001925
1% critical value:	-5.34	
5% critical value:	-4.93	
10% critical value:	-4.58	

* Probability values are calculated from a standard t-distribution and do not take into account the breakpoint selection process

Zivot-Andrews Unit Root Test
Date: 10/27/23 Time: 13:10
Sample: 1960 2022
Included observations: 63
Null Hypothesis: LDBMTUR has a unit root with a structural break in the intercept
Chosen lag length: 1 (maximum lags: 4)
Chosen break point: 2002

	t-Statistic	Prob. *
Zivot-Andrews test statistic	-5.153618	5.37E-05
1% critical value:	-5.34	
5% critical value:	-4.93	
10% critical value:	-4.58	

* Probability values are calculated from a standard t-distribution and do not take into account the breakpoint selection process

Null Hypothesis: LDCPITUR is stationary
Exogenous: Constant
Bandwidth: 6 (Newey-West automatic) using Bartlett kernel

		LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic		0.192912
Asymptotic critical values*:	1% level	0.739000
	5% level	0.463000
	10% level	0.347000

*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

Residual variance (no correction)	0.041549
HAC corrected variance (Bartlett kernel)	0.220474

KPSS Test Equation
Dependent Variable: LDCPITUR
Method: Least Squares
Date: 10/27/23 Time: 14:26
Sample (adjusted): 1961 2022
Included observations: 62 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.260056	0.026098	9.964450	0.0000

R-squared	0.000000	Mean dependent var	0.260056
Adjusted R-squared	0.000000	S.D. dependent var	0.205499
S.E. of regression	0.205499	Akaike info criterion	-0.310754
Sum squared resid	2.576020	Schwarz criterion	-0.276445
Log likelihood	10.63336	Hannan-Quinn criter.	-0.297283
Durbin-Watson stat	0.225773		

Null Hypothesis: LDBMTUR is stationary
Exogenous: Constant
Bandwidth: 6 (Newey-West automatic) using Bartlett kernel

		LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic		0.189484
Asymptotic critical values*:	1% level	0.739000
	5% level	0.463000
	10% level	0.347000

*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

Residual variance (no correction)	0.040591
HAC corrected variance (Bartlett kernel)	0.206341

KPSS Test Equation
Dependent Variable: LDBMTUR
Method: Least Squares
Date: 10/27/23 Time: 14:28
Sample (adjusted): 1961 2022
Included observations: 62 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.331285	0.025796	12.84263	0.0000

R-squared	0.000000	Mean dependent var	0.331285
Adjusted R-squared	0.000000	S.D. dependent var	0.203116
S.E. of regression	0.203116	Akaike info criterion	-0.334086
Sum squared resid	2.516612	Schwarz criterion	-0.299777
Log likelihood	11.35666	Hannan-Quinn criter.	-0.320615
Durbin-Watson stat	0.375090		

Dependent Variable: LDCPITUR
Method: Discrete Threshold Regression
Date: 08/26/23 Time: 22:17
Sample (adjusted): 1968 2022
Included observations: 55 after adjustments
Selection: Trimming 0.15, Max. thresholds 5, Sig. level 0.05
Threshold variable: LDCPITUR

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LDCPITUR < 0.371316 -- 35 obs				
C	0.034753	0.015233	2.281357	0.0278
LDBMTUR	0.078238	0.091158	0.858267	0.3957
COINTRES1(-1)	-0.795242	0.136049	-5.845259	0.0000
LDCPITUR(-1)	0.881353	0.140683	6.264837	0.0000
LDCPITUR(-2)	-0.469152	0.122733	-3.822533	0.0004
LDCPITUR(-3)	0.093689	0.085389	1.097205	0.2790
LDCPITUR(-5)	0.017055	0.048926	0.348587	0.7292
0.371316 <= LDCPITUR -- 20 obs				
C	0.259755	0.045014	5.770583	0.0000
LDBMTUR	0.419532	0.068964	6.083338	0.0000
COINTRES1(-1)	-0.645656	0.238843	-2.703265	0.0099
LDCPITUR(-1)	0.531733	0.186587	2.849784	0.0068
LDCPITUR(-2)	-0.207468	0.175691	-1.180866	0.2445
LDCPITUR(-3)	0.024869	0.104734	0.237448	0.8135
LDCPITUR(-5)	-0.300048	0.083979	-3.572897	0.0009

R-squared	0.976676	Mean dependent var	0.285902
Adjusted R-squared	0.969280	S.D. dependent var	0.203728
S.E. of regression	0.035708	Akaike info criterion	-3.611582
Sum squared resid	0.052276	Schwarz criterion	-3.100624
Log likelihood	113.3185	Hannan-Quinn criter.	-3.413990
F-statistic	132.0632	Durbin-Watson stat	1.943865
Prob(F-statistic)	0.000000		

Date: 09/23/23 Time: 18:55
Sample (adjusted): 1968 2022
Q-statistic probabilities adjusted for 8 dynamic regressors

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.020	0.020	0.0238	0.877
		2	-0.175	-0.176	1.8386	0.399
		3	-0.041	-0.035	1.9417	0.585
		4	-0.047	-0.079	2.0796	0.721
		5	-0.031	-0.044	2.1391	0.830
		6	0.119	0.100	3.0448	0.803
		7	0.006	-0.017	3.0469	0.881
		8	0.045	0.083	3.1828	0.922
		9	0.081	0.087	3.6352	0.934
		10	0.049	0.085	3.8031	0.956
		11	-0.173	-0.139	5.9263	0.878
		12	-0.114	-0.095	6.8785	0.866
		13	-0.045	-0.091	7.0307	0.901
		14	0.080	0.029	7.5156	0.913
		15	0.096	0.040	8.2394	0.914
		16	0.120	0.113	9.4001	0.896
		17	-0.046	0.003	9.5715	0.921
		18	-0.076	-0.019	10.063	0.930
		19	-0.117	-0.099	11.257	0.915
		20	0.100	0.121	12.149	0.911
		21	-0.095	-0.128	12.975	0.909
		22	-0.139	-0.168	14.807	0.870
		23	0.154	0.092	17.143	0.802
		24	0.100	0.007	18.154	0.795

*Probabilities may not be valid for this equation specification.

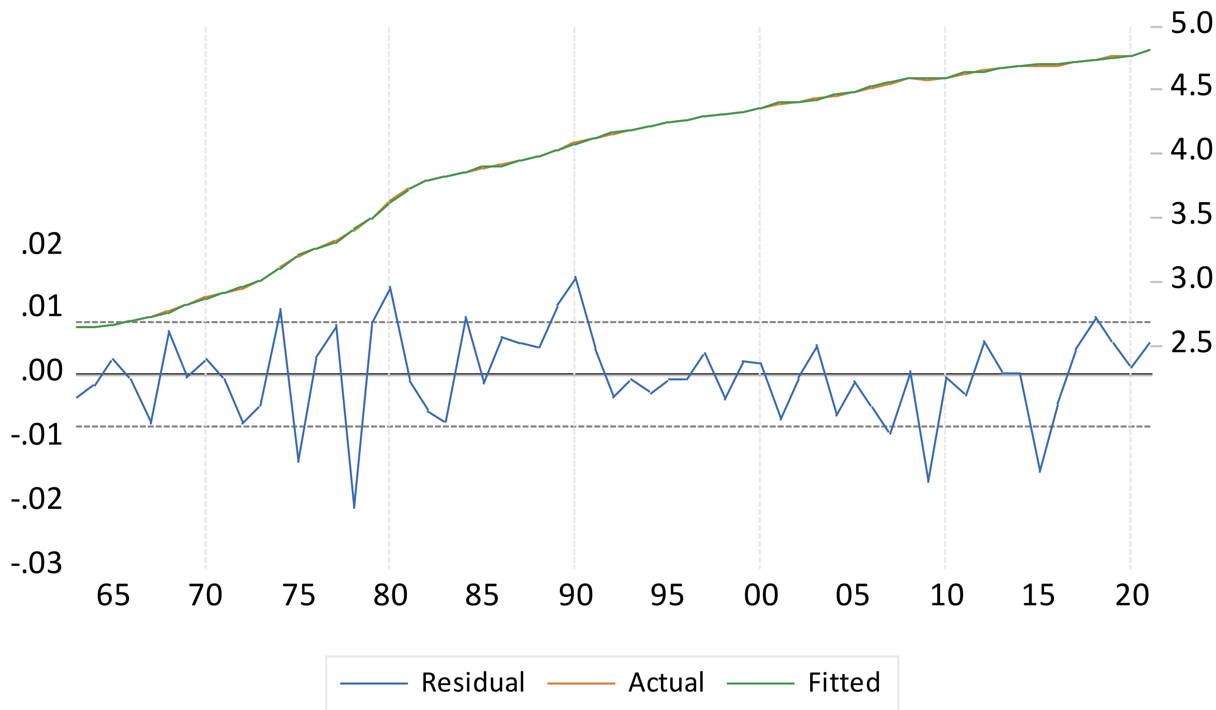
United States

1. Cointegration

Dependent Variable: LCPIUSA
Method: Discrete Threshold Regression
Date: 08/27/23 Time: 19:00
Sample (adjusted): 1963 2021
Included observations: 59 after adjustments
Selection: Trimming 0.15, Max. thresholds 5, Sig. level 0.05
Threshold variable: LDCPIUSA

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LDCPIUSA < 0.02574027 -- 20 obs				
C	0.407865	0.319892	1.275008	0.2090
LBMUSA	-0.017725	0.014400	-1.230848	0.2249
LCPIUSA(-1)	0.934179	0.189659	4.925572	0.0000
LCPIUSA(-2)	-0.019804	0.317090	-0.062457	0.9505
LCPIUSA(-3)	0.117778	0.220402	0.534375	0.5958
0.02574027 <= LDCPIUSA < 0.05318417 -- 27 obs				
C	-0.378848	0.206110	-1.838083	0.0728
LBMUSA	0.017801	0.008971	1.984228	0.0535
LCPIUSA(-1)	1.229154	0.204884	5.999272	0.0000
LCPIUSA(-2)	-0.258977	0.308378	-0.839802	0.4056
LCPIUSA(-3)	0.001697	0.124625	0.013618	0.9892
0.05318417 <= LDCPIUSA -- 12 obs				
C	-2.217579	0.863780	-2.567295	0.0137
LBMUSA	0.089437	0.037413	2.390514	0.0212
LCPIUSA(-1)	1.581852	0.125937	12.56064	0.0000
LCPIUSA(-2)	-1.542230	0.205811	-7.493425	0.0000
LCPIUSA(-3)	0.902234	0.146604	6.154233	0.0000

R-squared	0.999898	Mean dependent var	3.945747
Adjusted R-squared	0.999866	S.D. dependent var	0.698310
S.E. of regression	0.008079	Akaike info criterion	-6.583924
Sum squared resid	0.002872	Schwarz criterion	-6.055737
Log likelihood	209.2258	Hannan-Quinn criter.	-6.377741
F-statistic	30946.85	Durbin-Watson stat	2.019452
Prob(F-statistic)	0.000000		



Date: 09/23/23 Time: 19:00
Sample (adjusted): 1963 2021
Q-statistic probabilities adjusted for 9 dynamic regressors

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.016	-0.016	0.0168	0.897
		2	-0.061	-0.061	0.2510	0.882
		3	0.159	0.157	1.8728	0.599
		4	-0.071	-0.072	2.1988	0.699
		5	0.033	0.053	2.2695	0.811
		6	0.140	0.109	3.5945	0.731
		7	-0.041	-0.015	3.7108	0.812
		8	-0.029	-0.032	3.7686	0.877
		9	0.033	-0.002	3.8452	0.921
		10	0.047	0.070	4.0077	0.947
		11	0.021	0.018	4.0419	0.969
		12	-0.174	-0.199	6.3703	0.896
		13	-0.051	-0.059	6.5773	0.923
		14	-0.063	-0.081	6.8911	0.939
		15	-0.046	-0.007	7.0631	0.956
		16	-0.009	-0.048	7.0695	0.972
		17	-0.144	-0.135	8.8537	0.945
		18	-0.133	-0.096	10.411	0.918
		19	-0.079	-0.099	10.969	0.925
		20	-0.078	-0.070	11.525	0.931
		21	-0.057	-0.065	11.829	0.944
		22	-0.033	-0.015	11.933	0.959
		23	0.049	0.115	12.168	0.968
		24	-0.055	-0.055	12.479	0.974

*Probabilities may not be valid for this equation specification.

Null Hypothesis: COINTRES has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.653487	0.0000
Test critical values:	1% level	-2.605442	
	5% level	-1.946549	
	10% level	-1.613181	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(COINTRES)
Method: Least Squares
Date: 09/23/23 Time: 19:00
Sample (adjusted): 1964 2021
Included observations: 58 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
COINTRES(-1)	-1.016598	0.132828	-7.653487	0.0000

R-squared	0.506710	Mean dependent var	0.000148
Adjusted R-squared	0.506710	S.D. dependent var	0.010086
S.E. of regression	0.007084	Akaike info criterion	-7.044871
Sum squared resid	0.002860	Schwarz criterion	-7.009346
Log likelihood	205.3013	Hannan-Quinn criter.	-7.031033
Durbin-Watson stat	1.995085		

2. Error Correction Model (ECM)

Null Hypothesis: LDCPIUSA has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic - based on SIC, maxlag=10)

		t-Statistic
Elliott-Rothenberg-Stock DF-GLS test statistic		-1.743509
Test critical values:	1% level	-2.605442
	5% level	-1.946549
	10% level	-1.613181

*MacKinnon (1996)

DF-GLS Test Equation on GLS Detrended Residuals
Dependent Variable: D(GLSRESID)
Method: Least Squares
Date: 10/27/23 Time: 14:02
Sample (adjusted): 1964 2021
Included observations: 58 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
GLSRESID(-1)	-0.129504	0.074278	-1.743509	0.0868
D(GLSRESID(-1))	0.245710	0.129593	1.896006	0.0632
D(GLSRESID(-2))	-0.330463	0.133186	-2.481213	0.0162

R-squared	0.208096	Mean dependent var	0.000579
Adjusted R-squared	0.179300	S.D. dependent var	0.016530
S.E. of regression	0.014975	Akaike info criterion	-5.514579
Sum squared resid	0.012333	Schwarz criterion	-5.408005
Log likelihood	162.9228	Hannan-Quinn criter.	-5.473066
Durbin-Watson stat	1.970499		

Null Hypothesis: LDBMUSA has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic
Elliott-Rothenberg-Stock DF-GLS test statistic		-3.370366
Test critical values:	1% level	-2.604073
	5% level	-1.946348
	10% level	-1.613293

*MacKinnon (1996)

DF-GLS Test Equation on GLS Detrended Residuals
Dependent Variable: D(GLSRESID)
Method: Least Squares
Date: 10/27/23 Time: 14:04
Sample (adjusted): 1962 2021
Included observations: 60 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
GLSRESID(-1)	-0.355725	0.105545	-3.370366	0.0013

R-squared	0.160100	Mean dependent var	0.001296
Adjusted R-squared	0.160100	S.D. dependent var	0.032593
S.E. of regression	0.029870	Akaike info criterion	-4.167367
Sum squared resid	0.052642	Schwarz criterion	-4.132462
Log likelihood	126.0210	Hannan-Quinn criter.	-4.153714
Durbin-Watson stat	2.010214		

Dependent Variable: LDCPIUSA
Method: Discrete Threshold Regression
Date: 08/27/23 Time: 19:04
Sample (adjusted): 1965 2021
Included observations: 57 after adjustments
Selection: Trimming 0.15, Max. thresholds 5, Sig. level 0.05
Threshold variable: LDCPIUSA

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LDCPIUSA < 0.01880259 -- 11 obs				
C	0.014309	0.007705	1.857192	0.0735
LDBMUSA	0.034344	0.050228	0.683773	0.4995
COINTRES(-1)	-0.173079	0.411458	-0.420648	0.6771
LDCPIUSA(-1)	-0.312676	0.222068	-1.408015	0.1698
LDCPIUSA(-2)	0.199677	0.291988	0.683855	0.4995
LDCPIUSA(-3)	0.208777	0.420593	0.496388	0.6234
LDCPIUSA(-4)	-0.466627	0.257329	-1.813346	0.0801
0.01880259 <= LDCPIUSA < 0.03321093 -- 21 obs				
C	0.020937	0.004510	4.642402	0.0001
LDBMUSA	-0.004781	0.040783	-0.117222	0.9075
COINTRES(-1)	0.222708	0.476137	0.467739	0.6435
LDCPIUSA(-1)	0.375926	0.251105	1.497087	0.1452
LDCPIUSA(-2)	-0.160677	0.166111	-0.967286	0.3414
LDCPIUSA(-3)	0.234133	0.104885	2.232281	0.0335
LDCPIUSA(-4)	-0.216516	0.125931	-1.719321	0.0962
0.03321093 <= LDCPIUSA < 0.05674185 -- 15 obs				
C	0.035911	0.005153	6.969271	0.0000
LDBMUSA	-0.002097	0.042875	-0.048899	0.9613
COINTRES(-1)	0.077464	0.255111	0.303647	0.7636
LDCPIUSA(-1)	0.114107	0.183997	0.620158	0.5400
LDCPIUSA(-2)	0.054145	0.216225	0.250409	0.8040
LDCPIUSA(-3)	0.168598	0.213514	0.789635	0.4362
LDCPIUSA(-4)	-0.160654	0.112735	-1.425063	0.1648
0.05674185 <= LDCPIUSA -- 10 obs				
C	-0.046899	0.025837	-1.815219	0.0798
LDBMUSA	0.471172	0.228228	2.064479	0.0480
COINTRES(-1)	-2.931964	0.396147	-7.401205	0.0000
LDCPIUSA(-1)	2.469341	0.226054	10.92366	0.0000
LDCPIUSA(-2)	-2.656812	0.292953	-9.069058	0.0000
LDCPIUSA(-3)	2.074826	0.285962	7.255607	0.0000
LDCPIUSA(-4)	-0.906994	0.181133	-5.007336	0.0000

R-squared	0.978959	Mean dependent var	0.038026
Adjusted R-squared	0.959369	S.D. dependent var	0.026141
S.E. of regression	0.005269	Akaike info criterion	-7.347122
Sum squared resid	0.000805	Schwarz criterion	-6.343518
Log likelihood	237.3930	Hannan-Quinn criter.	-6.957088
F-statistic	49.97225	Durbin-Watson stat	2.102392
Prob(F-statistic)	0.000000		

Date: 09/23/23 Time: 19:03
Sample (adjusted): 1965 2021
Q-statistic probabilities adjusted for 16 dynamic regressors

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.086	-0.086	0.4479	0.503
		2	-0.208	-0.217	3.0858	0.214
		3	-0.018	-0.062	3.1053	0.376
		4	0.195	0.150	5.5185	0.238
		5	0.003	0.027	5.5192	0.356
		6	-0.177	-0.116	7.5901	0.270
		7	-0.098	-0.125	8.2359	0.312
		8	0.157	0.059	9.9235	0.270
		9	0.078	0.062	10.350	0.323
		10	-0.142	-0.055	11.785	0.300
		11	-0.122	-0.096	12.874	0.302
		12	0.194	0.102	15.676	0.207
		13	-0.072	-0.139	16.078	0.245
		14	-0.023	0.047	16.118	0.306
		15	-0.012	0.034	16.130	0.373
		16	0.045	-0.014	16.294	0.433
		17	0.075	0.060	16.765	0.470
		18	-0.127	-0.105	18.152	0.446
		19	-0.182	-0.184	21.084	0.332
		20	0.022	-0.092	21.128	0.390
		21	0.033	-0.066	21.231	0.445
		22	-0.032	0.007	21.329	0.501
		23	-0.007	0.063	21.333	0.561
		24	-0.012	-0.078	21.348	0.618

*Probabilities may not be valid for this equation specification.

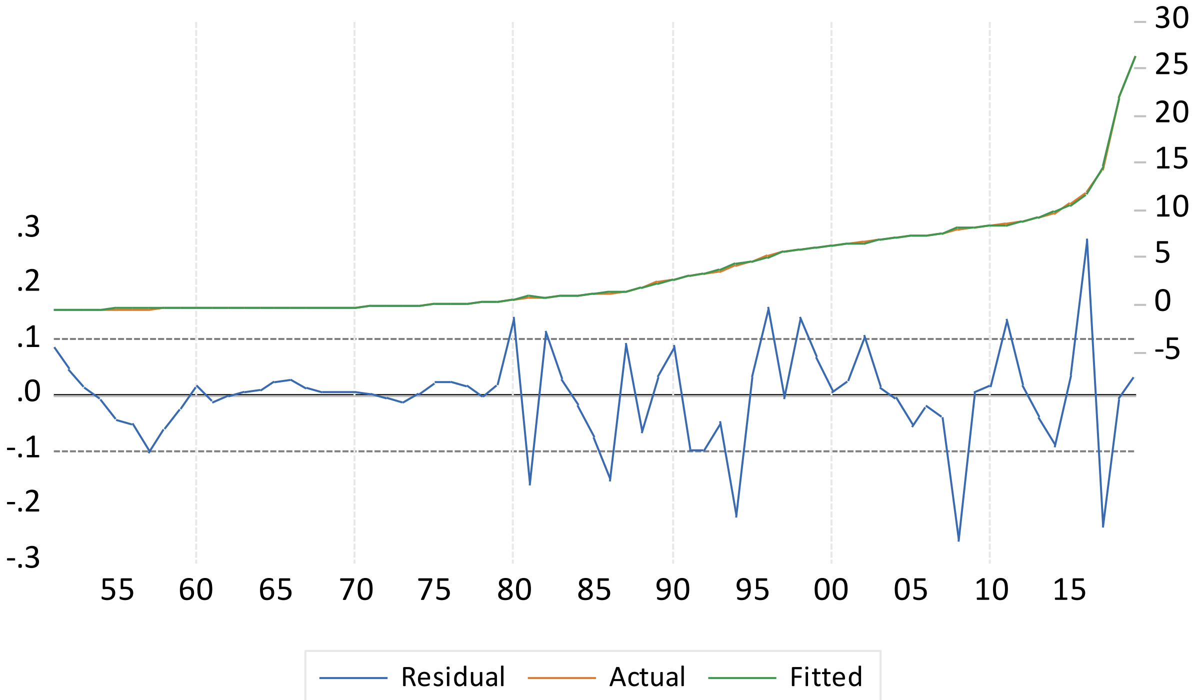
Venezuela

1. Cointegration

Dependent Variable: L_CPI
Method: Discrete Threshold Regression
Date: 09/23/23 Time: 19:29
Sample (adjusted): 1951 2019
Included observations: 69 after adjustments
Selection: Trimming 0.15, Max. thresholds 5, Sig. level 0.05
Threshold variable: LD_CPI

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LD_CPI < 0.1341597 -- 35 obs				
C	-0.612538	0.254934	-2.402727	0.0201
L_M1	0.052518	0.085170	0.616627	0.5403
@TREND	0.041412	0.012169	3.403108	0.0013
@TREND^2	-0.003429	0.000547	-6.264551	0.0000
@TREND^3	0.000100	7.37E-06	13.57647	0.0000
0.1341597 <= LD_CPI < 0.2471127 -- 12 obs				
C	-25.87220	8.024386	-3.224197	0.0023
L_M1	-0.152513	0.175111	-0.870950	0.3880
@TREND	1.492361	0.536973	2.779213	0.0077
@TREND^2	-0.023967	0.011845	-2.023422	0.0485
@TREND^3	0.000154	8.18E-05	1.884104	0.0655
0.2471127 <= LD_CPI < 0.4230561 -- 11 obs				
C	-28.56312	11.80594	-2.419386	0.0193
L_M1	0.559578	0.114942	4.868366	0.0000
@TREND	1.133250	0.746368	1.518353	0.1354
@TREND^2	-0.012723	0.015710	-0.809827	0.4220
@TREND^3	2.98E-05	0.000107	0.278340	0.7819
0.4230561 <= LD_CPI -- 11 obs				
C	-79.06160	12.26922	-6.443898	0.0000
L_M1	1.091173	0.023981	45.50094	0.0000
@TREND	4.532318	0.745861	6.076628	0.0000
@TREND^2	-0.090502	0.014786	-6.120720	0.0000
@TREND^3	0.000577	9.70E-05	5.946885	0.0000

R-squared	0.999722	Mean dependent var	3.724959
Adjusted R-squared	0.999614	S.D. dependent var	5.246121
S.E. of regression	0.103099	Akaike info criterion	-1.468832
Sum squared resid	0.520840	Schwarz criterion	-0.821264
Log likelihood	70.67469	Hannan-Quinn criter.	-1.211920
F-statistic	9264.089	Durbin-Watson stat	2.110368
Prob(F-statistic)	0.000000		



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Date: 09/23/23 Time: 19:38
Sample (adjusted): 1951 2019
Included observations: 69 after adjustments

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
. | . |	. | . |	1	-0.063	-0.063	0.2898	0.590
. | . |	. | . |	2	-0.057	-0.062	0.5302	0.767
. | . |	. | . |	3	0.041	0.034	0.6572	0.883
.*| . |	.*| . |	4	-0.140	-0.140	2.1315	0.712
. | . |	. | . |	5	0.020	0.007	2.1610	0.826
.*| . |	.*| . |	6	-0.110	-0.130	3.0935	0.797
. | . |	. | . |	7	-0.053	-0.059	3.3177	0.854
.*| . |	.*| . |	8	-0.134	-0.188	4.7687	0.782
. | . |	. | . |	9	0.061	0.043	5.0684	0.828
.*| . |	.*| . |	10	-0.128	-0.199	6.4314	0.778
.*| . |	.*| . |	11	-0.078	-0.106	6.9445	0.804
.*| . |	**| . |	12	-0.122	-0.277	8.2205	0.768
. |*. |	. |*. |	13	0.111	0.077	9.2898	0.751
. |*. |	. | . |	14	0.132	-0.021	10.846	0.698
. | . |	. | . |	15	-0.022	-0.017	10.891	0.760
. | . |	.*| . |	16	0.070	-0.088	11.345	0.788
. | . |	. | . |	17	-0.030	-0.033	11.431	0.833
.*| . |	**| . |	18	-0.066	-0.222	11.850	0.855
. | . |	.*| . |	19	-0.053	-0.131	12.127	0.880
. |*. |	. | . |	20	0.169	0.070	14.989	0.777
.*| . |	.*| . |	21	-0.086	-0.100	15.746	0.784
. | . |	.*| . |	22	-0.003	-0.122	15.747	0.828
. |*. |	. | . |	23	0.097	-0.020	16.752	0.821
. | . |	. | . |	24	-0.006	0.022	16.757	0.859
.*| . |	**| . |	25	-0.109	-0.215	18.081	0.839
. | . |	. | . |	26	0.008	-0.032	18.089	0.872
. | . |	. | . |	27	0.043	-0.065	18.300	0.894
. | . |	.*| . |	28	-0.062	-0.095	18.762	0.905

Null Hypothesis: COINTTR has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-8.775301	0.0000
Test critical values:	1% level	-2.599413	
	5% level	-1.945669	
	10% level	-1.613677	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(COINTTR)
Method: Least Squares
Date: 09/23/23 Time: 19:46
Sample (adjusted): 1952 2019
Included observations: 68 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
COINTTR(-1)	-1.063570	0.121200	-8.775301	0.0000

R-squared	0.534725	Mean dependent var	-0.000744
Adjusted R-squared	0.534725	S.D. dependent var	0.128082
S.E. of regression	0.087366	Akaike info criterion	-2.022830
Sum squared resid	0.511396	Schwarz criterion	-1.990191
Log likelihood	69.77624	Hannan-Quinn criter.	-2.009898
Durbin-Watson stat	2.017634		

2. Error Correction Model (ECM)

Null Hypothesis: LD_CPI has a unit root
Trend Specification: Trend and intercept
Break Specification: Intercept only
Break Type: Additive outlier

Break Date: 2007
Break Selection: Minimize Dickey-Fuller t-statistic
Lag Length: 10 (Automatic - based on Schwarz information criterion, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-9.961563	< 0.01
Test critical values:	1% level	-5.347598	
	5% level	-4.859812	
	10% level	-4.607324	

*Vogelsang (1993) asymptotic one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: RESID
Method: Least Squares
Date: 10/27/23 Time: 14:47
Sample (adjusted): 1962 2019
Included observations: 58 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RESID(-1)	-4.331577	0.535215	-8.093155	0.0000
D(RESID(-1))	3.923895	0.546660	7.177943	0.0000
D(RESID(-2))	4.837210	0.587027	8.240186	0.0000
D(RESID(-3))	4.750556	0.607185	7.823900	0.0000
D(RESID(-4))	3.745941	0.739345	5.066564	0.0000
D(RESID(-5))	3.740366	0.663215	5.639751	0.0000
D(RESID(-6))	2.897696	0.748980	3.868858	0.0004
D(RESID(-7))	2.819696	0.743409	3.792929	0.0005
D(RESID(-8))	2.404812	0.712184	3.376672	0.0018
D(RESID(-9))	1.272796	0.839519	1.516102	0.1382
D(RESID(-10))	0.371183	0.915177	0.405587	0.6874
BREAKDUM	-1.481027	0.546820	-2.708435	0.0103
BREAKDUM1	-2.116766	0.466900	-4.533662	0.0001
BREAKDUM2	-1.649410	0.756960	-2.178993	0.0360
BREAKDUM3	-1.966806	0.789758	-2.490392	0.0175
BREAKDUM4	-2.783362	0.785606	-3.542950	0.0011
BREAKDUM5	-2.773057	0.818329	-3.388680	0.0017
BREAKDUM6	-3.466490	0.844471	-4.104927	0.0002
BREAKDUM7	-3.392179	0.933173	-3.635104	0.0009
BREAKDUM8	-3.610514	0.878585	-4.109465	0.0002
BREAKDUM9	-4.127061	0.886388	-4.656042	0.0000
BREAKDUM10	-3.899511	0.954873	-4.083800	0.0002

R-squared	0.910893	Mean dependent var	-0.026344
Adjusted R-squared	0.858914	S.D. dependent var	1.000759
S.E. of regression	0.375899	Akaike info criterion	1.162705
Sum squared resid	5.086805	Schwarz criterion	1.944252
Log likelihood	-11.71844	Hannan-Quinn criter.	1.467133
Durbin-Watson stat	0.629615		

Null Hypothesis: LD_M1 has a unit root
Trend Specification: Intercept only
Break Specification: Intercept only
Break Type: Innovational outlier

Break Date: 2016
Break Selection: Minimize Dickey-Fuller t-statistic
Lag Length: 0 (Automatic - based on Schwarz information criterion, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-9.246587	< 0.01
Test critical values:	1% level	-4.949133	
	5% level	-4.443649	
	10% level	-4.193627	

*Vogelsang (1993) asymptotic one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: LD_M1
Method: Least Squares
Date: 10/27/23 Time: 14:52
Sample (adjusted): 1952 2019
Included observations: 68 after adjustments

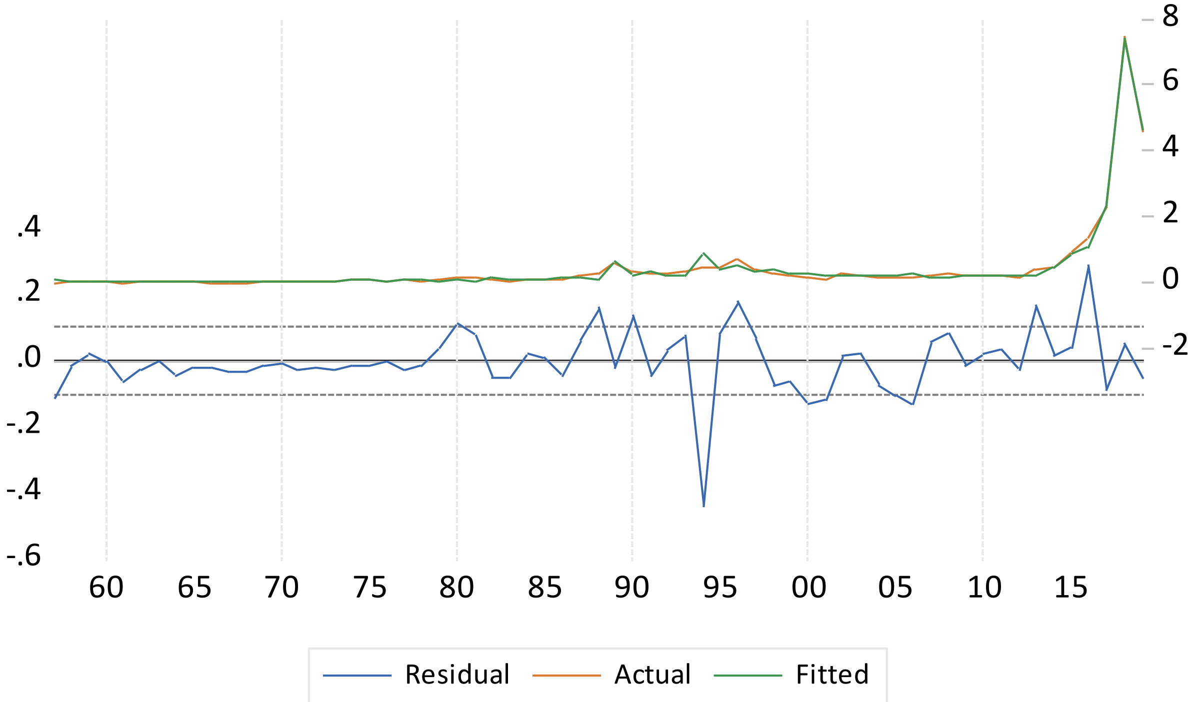
Variable	Coefficient	Std. Error	t-Statistic	Prob.
LD_M1(-1)	0.134923	0.093556	1.442162	0.1541
C	0.200124	0.054517	3.670865	0.0005
INCPTBREAK	3.648557	0.374774	9.735358	0.0000
BREAKDUM	-2.982446	0.526497	-5.664701	0.0000

R-squared	0.821225	Mean dependent var	0.419822
Adjusted R-squared	0.812844	S.D. dependent var	0.933967
S.E. of regression	0.404048	Akaike info criterion	1.082455
Sum squared resid	10.44829	Schwarz criterion	1.213014
Log likelihood	-32.80347	Hannan-Quinn criter.	1.134187
F-statistic	97.99700	Durbin-Watson stat	2.778843
Prob(F-statistic)	0.000000		

Dependent Variable: LD_CPI
Method: Discrete Threshold Regression
Date: 09/23/23 Time: 19:57
Sample (adjusted): 1957 2019
Included observations: 63 after adjustments
Selection: Trimming 0.15, Max. thresholds 5, Sig. level 0.05
Threshold variable: LD_CPI

Variable	Coefficient	Std. Error	t-Statistic	Prob.
	LD_CPI < 0.4751664 -- 54 obs			
C	0.019043	0.024201	0.786873	0.4350
LD_M1	0.154549	0.086394	1.788894	0.0796
COINTTR(-1)	-0.252943	0.186962	-1.352909	0.1821
LD_CPI(-2)	0.394842	0.173409	2.276939	0.0270
LD_CPI(-3)	0.040958	0.173781	0.235685	0.8146
LD_CPI(-6)	0.207974	0.116094	1.791423	0.0792
	0.4751664 <= LD_CPI -- 9 obs			
C	0.470400	0.121303	3.877889	0.0003
LD_M1	0.962314	0.041499	23.18874	0.0000
COINTTR(-1)	-1.283503	0.333894	-3.844042	0.0003
LD_CPI(-2)	0.390461	0.273841	1.425868	0.1600
LD_CPI(-3)	0.339048	0.570173	0.594640	0.5547
LD_CPI(-6)	-2.407992	0.609402	-3.951399	0.0002

R-squared	0.992584	Mean dependent var	0.426759
Adjusted R-squared	0.990984	S.D. dependent var	1.114892
S.E. of regression	0.105861	Akaike info criterion	-1.483727
Sum squared resid	0.571539	Schwarz criterion	-1.075511
Log likelihood	58.73741	Hannan-Quinn criter.	-1.323174
F-statistic	620.5195	Durbin-Watson stat	2.015949
Prob(F-statistic)	0.000000		



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Date: 09/23/23 Time: 20:03
Sample (adjusted): 1957 2019
Q-statistic probabilities adjusted for 6 dynamic regressors

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.021	-0.021	0.0302	0.862
		2	-0.102	-0.103	0.7344	0.693
		3	0.068	0.064	1.0534	0.788
		4	-0.107	-0.117	1.8539	0.763
		5	0.138	0.153	3.1963	0.670
		6	0.044	0.016	3.3336	0.766
		7	0.040	0.094	3.4532	0.840
		8	0.054	0.028	3.6682	0.886
		9	-0.062	-0.019	3.9580	0.914
		10	-0.127	-0.151	5.2042	0.877
		11	0.033	0.023	5.2923	0.916
		12	0.052	0.014	5.5115	0.939
		13	-0.192	-0.203	8.5158	0.808
		14	-0.184	-0.228	11.342	0.659
		15	-0.051	-0.080	11.563	0.712
		16	-0.057	-0.081	11.850	0.754
		17	-0.000	-0.034	11.850	0.809
		18	-0.004	0.004	11.851	0.855
		19	-0.119	-0.087	13.170	0.830
		20	0.080	0.123	13.780	0.841
		21	0.004	0.085	13.782	0.879
		22	-0.240	-0.199	19.520	0.613
		23	0.145	0.067	21.686	0.539
		24	-0.103	-0.179	22.790	0.532
		25	0.027	0.068	22.869	0.585
		26	0.078	-0.088	23.548	0.602
		27	-0.010	0.023	23.560	0.655
		28	0.134	-0.011	25.673	0.591

*Probabilities may not be valid for this equation specification.

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