Another View on Growth Matters: Investment, Capital, and Solow Residual

Ikonomou, Constantinos

National and Kapodistrian University of Athens, Department of Economics, Hellenic Open University, Department of Public Administration and, University of West Attica, Department of Tourism Management

October 2023

Online at https://mpra.ub.uni-muenchen.de/119003/
MPRA Paper No. 119003, posted 04 Nov 2023 12:13 UTC
Abstract

The debate on Solow residual (SR), its extent and significance and the critic against the neoclassical growth equation (NGE) by endogenous and new growth theory (ENGT) have not managed to obliterate the contribution of NGE and of SR to the development of growth accounting and TFP measurement, to the convergence and competitiveness debates, the realization of productivity slowdown in industrialised economies and to real business cycle theory. After the proliferate critic of ENGT against NGE, other critics were left unattended, for example that by Domar on the non-incorporation of intermediate goods in growth equations. Moreover, significant amendments in the calculation of capital by OECD had unveiled its underestimation and, subsequently, TFP overestimation. The contribution of each specific type of investment to capital and subsequently to growth has also been underestimated in growth equations. This was made clearer through DeLong and Summers studies and the distinction between equipment and infrastructure investment but it was also claimed much earlier by the concept of accelerator and the various distinctions of investment types used in economics. The proclaimed “death” of NGE has harmed economic growth studies by contributing in the displacement of decreasing returns and perfect competition from their disciplinary throne. While most economists gregariously espoused such a disciplinary path, this article seeks to refresh economic thinking in growth models, by creating a new growth model that comprises labour -apart from capital- and integrates an additional, aggregate growth component that encapsulates ENGT variables and intermediate inputs. Few but significant implications for growth theory and policy are discussed.
1. **Introduction: The search for Solow residual**

In economics, the *Ockham’s razor* cuts very sharply. Whole economies could arrive at the verge of collapse if economic growth models and their policy prescriptions are neither parsimonious nor accurate; needless to mention the endless debates that may arise from such models, extending academic discussions towards infinite horizons.

The neoclassical growth equation (NGE) has left an unexplained part for decades, “Solow residual”, not attributed to changes in capital and labour inputs but to technology and its change (Solow, 1957). This part gave rise to a strong critic against the neoclassical growth model (NGM) and its assumptions that brought the abandoning of diminishing returns and perfect competition. More elaborated economic growth models and theories were created after, informing policy making in various directions.

However, Solow residual or Total Factor Productivity (TFP) or Multifactor Productivity (MFP) has opened several paths in economic thought. It has brought: i) a proliferation of growth models, their associated theories, and the rise of endogenous and new growth theory and models, almost by re-writing growth theory, ii) the development of growth accounting, the measurement and correction TFP values, iii) a debate about convergence among economies that was raised in NGE, iv) a proliferate debate about productivity slowdown in many industrialized economies and an associated debate on their competitiveness and, last but not least, v) the development of real business cycle (RBC) theory, which gave rise to new paths in macroeconomics and a critic against other macroeconomic theories.
Yet, it was realized more recently that capital accounts were underestimated (OECD, 2009; OECD, 2001a; OECD, 2001b; BEA, 2006; Jorgenson et al., 1987; Dean and Harper, 2001). Improving capital accounts ought to have some impact on diminishing the size of Solow residual, limiting the strength of the critic against NGE.

This paper, after highlighting the debate on the extent and significance Solow residual (SR) along with investigations for its explanation by endogenous and new growth theory, explains its association to the debates on convergence, productivity slowdown, competitiveness and to real business cycle theory, before reminding Domar’s critic against the NGE. Then, some of the most significant amendments in calculations of capital are emphasized that unveil that capital has been historically underestimated and subsequently TFP overestimated. Then another source of underestimation of capital to growth is highlighted, the investment type promoted, by reviewing evidence from DeLong and Summers studies, and reminding the concepts of the accelerator, the acceleration principle and past distinctions of investment types in macroeconomics. This section highlights that infrastructure investment is given a global emphasis, despite the importance of other investment types. Then, a new growth model is investigated, which, apart from capital, comprises labour and an additional aggregate growth component that encapsulates ENGT variables and intermediate inputs. This model attempts to reestablish a link among Solow’s model, ENGT factors, and intermediate inputs. Few implications for growth theory and policy are finally reached.
2. From Solow Residual to endogenous growth theory and other debates

The NGE aimed to explain growth, by offering an explanatory framework for the contribution of capital and labour to growth, using as basic assumptions decreasing returns to capital, cost minimization, competitive factor markets and constant returns to scale, much before the significant turn in economic growth modeling towards increasing returns and monopolistic competition. It accepted all assumptions of the previous Harrod-Domar model but fixed proportions on the grounds that they cause increased unemployment and inflation. As explained by Solow (2000, p. 352), the term “neoclassical” refers to the assumptions of: i) households seeking maximum satisfaction levels by buying goods and supplying their labour, given their fixed preferences and budgets, ii) firms seeking maximum profits, with their decisions for employment and investment constrained by technology, factor and production markets, and iii) markets bound to clear, by taking into account prices.

NGE has become a valuable tool to analyze growth for decades, considered as parsimonious and versatile enough. The Ramsey-Cass-Koopmans model has contributed to this end, by explicitly referring to utility and micro-foundations of consumption, while making saving rates endogenous. Prior to this, the NGE has received strong critic against its suggestion for exogeneity of savings. However, its emphasis on savings for intergenerational purposes, to reach a steady-state, was a

---

1 Solow’s growth model was written at the post-war recovery period, when most economies were investing in infrastructure development, the engines of private growth were restarting production, and the reconstruction of the European space through capital accumulation was prioritised. The post-war demographic change resulted in the significant expansion of labour size, enhanced by women’s rights recognition and the rise of female employment.
valid point; it has followed the common sense (at least at the time) that more savings reduce current consumption but bring future growth.

The contribution of NGE to economic literature has exceeded far its initial scope, as it laid down the foundations in proliferate discussions and debates. Among other contributions, it is worth referring to: a) its use as a basis to aggregate inputs, develop “growth accounting”, and accurately measure TFP, b) its use in the development of the convergence debate, c) its operation as thesis against the antithesis of endogenous and new growth theory, and d) its significance in diagnosing the productivity slowdown of industrialised economies, laying down the bases for the debate on competitiveness. Last but not least, e) it was used to develop Real Business Cycles theory, expanding the frontiers in macroeconomic analysis.

Numerous studies have found that TFP is high and that a great part of output remains unexplained by capital and labour inputs (Abramovitz, 1956; Fabricant, 1959; Kendrick, 1961; Kuznets, 1971). Extended investigations on SR brought a voluminous literature for its explanation, significance, usefulness, association to capital or labour, and other aspects, turning Solow’s model to the most investigated and criticized for its residual.

Solow (1957) had attributed this “residual” to technological change or progress, claiming that technical progress from 1909 to 1949 accounted for a substantial NGE part, greater than capital inputs. Fabricant (1954) had attributed a 90% of per capital change to technical change for the 1871-1951 period. Jones and Volrath (2013, p. 47) showed that the largest part of output produced in USA for the 1948-2010 period is
accounted by MFP\textsuperscript{2}. Aghion and Howitt (2009) provided a series of TFP and capital shares for several countries, from 1960 to 2000, which captured only the capital-deepening component for the physical capital and found an analogy 0.68:0.32 respectively (almost 2:1)\textsuperscript{3}.

Few researchers have critically raised their objections on how significant is TFP as a growth factor in comparison to capital. Denison (1967; 1969) debated with Jorgenson and Griliches (1967) about how much output can be explained by TFP and how much by capital formation (see Denison (1969) and Hulten (2001)). Baier et al. (2006) has found that TFP associated only with 14% of average growth per worker. Jorgenson et al. (2013) have found that capital changes contributed the largest share on output in the 1947-2010 period, if compared to TFP (against Solow’s 1957 suggestions), and that TFP has fallen from 1973 to 1995 but increased after. Romer (1987, p. 165) has referred to an underestimation of capital accumulation in growth accounting\textsuperscript{4}.

Felipe and McCombie (2006) suggested that growth accounting does not offer a reliable estimate for technological change, while Mankiw (1989) had simply pointed out a undeniable truth that recessions are not due to technological shocks and deficiencies in technologies alone. Since technological change in Solow’s model is exogenous and costless, the decisions of agents –firms- that realize productivity increases are not considered.

---

\footnotesize
\textsuperscript{2} Two-thirds (1.4 out of 2.6) output per worker.

\textsuperscript{3} which turned to a 0.83:0.17 analogy for Japan and 0.86:0.14 for Greece, which are remarkably higher TFP in comparison to capital shares. They also highlighted that such results were reduced when the human-capital-deepening component was reduced but remained substantial for TFP shares.

\textsuperscript{4} Romer (1987) had claimed that “the correct weight on the rate of growth for capital in a growth accounting exercise may be closer to 1 than to 0.25”, and that positive externalities, associated with investment, interfered and raised the elasticity of output with respect to changes in capital that is more likely to be greater than the share of capital in total income (ibid, p. 166). For labour on the contrary, Romer (1987, p. 166) argued that “the exponent on labor may be substantially smaller than its share in income, possibly on the order of 0.1. or 0.2” (in the Cobb-Douglass expression), that can be explained by negative externalities associated to labour.
Other economists reminded that TFP is not a theoretical concept (Hulten, 2001), but technically created, a “measure of our ignorance” (Abramovitz, 1956). Besides, quantitative equations cannot fully rationalize and explain economic phenomena, which are subject to human action and choice\(^5\).

The search for a systematic element explaining SR has helped economists to isolate some new, hitherto unspecified growth factor(s). The very presence of SR emphasized that the productivity of all other factors (not just capital and labour) was essential to enhance growth. Unfolding the growth puzzle required solid answers to questions on how such factors are combined, by which methods and means.

Dissatisfaction from NGM and the study of SR have given rise to ENGT. Endogenous growth theory (EGT) has used more sophisticated mathematical techniques and modeling, drawing on simpler assumptions (Fine, 2000; p. 250). Growth models no longer referred to a single-sector economy. They explained growth rates as the outcome of behavior of rational maximizing agents, reflecting structural economic characteristics, such as technology or preferences (Turnovsky, 2003; p. 2). EGT aimed at endogenising growth factors, hitherto omitted or treated as exogenous, and at explaining endogenous sources of productivity increases, in various ways (Fine, 2000). Labour was thought to enhance through human capital, whose accumulation improves its quality, affecting also intermediate inputs and overall productivity (ibid, 2000).

The first endogenous growth model (EGM) took the \(Y=AK\) form, where \(A\) is a human capital proxy replacing labour. Mankiw et al. (1992) proposed an “augmented Solow

\(^5\) However, the advent of robotics and artificial intelligence requires at least a crystal-clear relationship between TFP and technology.
model” by adding human capital in Solow’s equation, confirming -through results- an equal exponent for each factor included (taking the $Y=K^{1/3}H^{1/3}L^{1/3}$ form). Lucas’s model has built on NGE, by borrowing elements from Uzawa’s EGM that emphasized human capital; it suggested that human -not physical- capital is necessary for growth, as a source of spillover effects raises technology levels. Romer (1989) emphasized the significance of human capital, education and knowledge, and assumed a production function $Y = K^{\alpha} (hL)^{1-\alpha}$, where $h$ is human capital per person.

Another wave of EGMs were product-variety models that introduced the 1977 Dixit and Stiglitz framework. Aggregate capital stock was evenly divided among $N_t$ existing varieties, and the production function took the $Y_t = N_t^{1-\alpha} K^{\alpha}$ form. Further efforts were made to introduce research and development (R&D), learning, entrepreneurship and other factors as endogenous in growth equations, all aiming at explaining the greatest part of SR. Arrow’s model that incorporates learning-by-doing, King-Robson’s model that emphasises learning-by-watching and firm innovation, Romer’s 1986 model that emphasizes -as a variant of Arrow’s model- learning by investment, and Romer’s 1990 model on endogenous technical change have all followed an endogenous growth path (Aghion and Howitt, 2009; Valdes, 1999). Productivity increases were also attributed to technological spillovers and learning-by-doing, which arise from the accumulation process, and the generation and use of new knowledge.

A special EGM are innovation-based models associating to Schumpeterian creative destruction and quality improvement (in Aghion and Howitt 1992; Aghion and Howitt 2009 etc). One such model was provided by Grossman and Helpman (1994; p.35) that
used equation $Y=K^\alpha L^\beta Z^{1-\alpha-\beta}$, where $Z$ stands for an “aggregate measure of intermediate inputs (adjusted for their quality)”.

In line with EGT, various additional empirical growth factors were tested by new growth theory. These comprised investment, financial proxies, inflation and price proxies, proxies for institutions, education, including educational quality, health, population growth and fertility, democracy, social capital, trade, geography and religion (Aghion and Durlauf, 2005; Durlauf et al., 2005; Aghion and Howitt, 2009). Despite the variety of significant growth factors tested in growth associations and the investigations for a “unified theory” (as in Galor, 2005) no single theory has offered a framework that comprises all growth factors (Capolupo, 2009; p.4). Hence, a main critic against Solow’s model on unidentified growth factors still appears in ENGT models.

The viewing of the significance of growth factors has also changed with EGT. For example many EGT models -most notably the AK- considered human capital and increased specialization of labour as the source of productivity growth, instead of labour. In EGT, productivity growth comes from within, derives from market imperfections (a central EGT component), which have an impact on growth rates rather than on output levels, as opposed to the static general equilibrium in exogenous growth theory (Fine, 2000; p. 249-250).

EGT has also offered an alternative to diminishing returns. In the neoclassical, Cobb-Douglas production function, factor inputs exhibit constant returns to scale and economic agents do not have incentives to encourage technological progress. Diminishing returns for capital set in, due to fixed labour supply and capital depreciation. On the contrary, in EGT increasing investment rates generate strong
external economies that sustain increasing returns for capital. The EGT mechanism between scale and productivity, tends to offset -if not outweigh- diminishing returns (Mare, 2004, p. 7).

EGT claims to have identified the mechanism(s) through which diminishing returns are outweighed and increasing returns placed in operation. Several models have built on Arrow’s 1962 model, who perceived learning as endogenous because capital goods contain all knowledge accumulated prior to a period of time that cannot change by future learning. Levhari (1966) and Sheshinski (1967) have emphasized that as opposed to constant individual returns of firms that make growth consistent with competitive equilibrium, it is spillover effects due to knowledge and non-rivalry of knowledge that bring spillover effects across firms. King and Robson (1993) suggested that externalities are created by a firm that uses innovation to resolve its own problem(s), and through a process of learning-by-watching by other competitors, deciding to assimilate successful innovations.

Externalities were viewed as significant for endogenous growth. Klenow and Rodriguez-Clare (2005) have claimed their significance both in the form of knowledge accumulation for firms and for workers (for human capital). They have referred to knowledge externalities, separately integrated in models (as in Romer, 1986), in models with new goods externalities (as in River-Batiz and Romer, 1991) or in models with knowledge and new goods externalities, as the most common cases.

In Romer (1986, p. 1006), externalities, increasing returns in output production and decreasing returns in new knowledge generation, formed a competitive equilibrium model of growth. Romer (1990) further suggested that ideas are non-rivalrous and that their non-rivalry, along with their excludability, brings increasing returns that
associate to imperfect competition (IC). Along with the mechanisms of increasing returns, diminishing returns are also offset by spillovers that raise productivity. Romer’s 1990 model, which combines increasing to decreasing returns, is structured across three sectors: the research sector, the intermediate goods sector and the final goods sector. Research firms make use of their available human capital and stock of knowledge to generate new knowledge. In the intermediate goods sector, firms gain monopolistic power in markets by purchasing patents and licenses that they gain from the research sector, benefiting from its spillovers. Such power generates increasing returns, up to the point when firm entry brings diminishing returns. In Romer’s 1990 model, the final goods sector combines all intermediate inputs to produce final goods for consumption purposes. The model assumes sufficiently strong research spillovers that overcome diminishing returns, excludability in the intermediate goods sector that allows monopolistic power, and transformation of intermediate goods to final. As explained by Pack (1994, p. 56) “an increasing variety or quality of machinery of intermediate inputs offsets the propensity to diminishing returns”. Finally, in Schumpeterian models, diminishing returns are offset by the forces of creative destruction.

By viewing the economy as a field of market imperfections, EGMs have espoused IC, its contribution to growth and “mechanics”. Romer’s 1986 view that ideas and knowledge production are a non-competitive, non-rival good was further expanded by Jones (2005), breaking the claims of competition elements in growth equations. Similarly, claims that human capital raises its own externalities was sufficient to consider the competitive price system as inefficient (Acemoglu, 2007; p. 493). Even if
some early EGMs (e.g. Romer, 1986; Lucas, 1988 and 1993) made use of PC, MC was introduced and explored (by Romer, 1990 and many others).

Several EGMs have employed the Dixit-Stiglitz framework as a vehicle to attribute growth in the formation of MC. Growth was seen from a microeconomic perspective, as the outcome of maximisation of a utility function of a representative household consumption (at a Ramsey-Cass-Koopmans modeling framework) that permitted MC with product diversity, instead of PC (Aghion and Howitt, 2009). Love for variety and for its expansion was considered to raise utility at this model (Acemoglu, 2007; p. 597).

EGT has disassociated growth theory from price taking and accepted reduced market power for the subjects of growth, firms⁶, by espousing IC forms and especially monopolistic benefits through ideas, innovation, R&D or through product variety taking place at a monopolistic form of competition. Rather than seeking to explain how ideas will be diffused and spread in most firms, how innovation, research and other sources of monopolistic power will become available for the vast majority of entrepreneurs and enterprises, it aimed at identifying the mechanisms through which their benefits are acquired and gained by monopolistic competitors and monopolies (see Romer 1990). Monopoly’s profits contribute to growth even if earned by the market leader, since they allow benefits to accrue, at least for the generation of products used in markets (Grossman and Helpman, 1994).

⁶ In MC, like in PC, the number of firms is large. However, products are not perfect substitutes and the demand curve of the monopolistic firm has a slope, as opposed to the horizontal demand curve of the price-taker, perfectly competitive firm. The firm seeks to differentiate its products and targets customers with special preferences, even if it covers similar needs. The monopolistic competitor invests in product differentiation that brings monopolistic benefits and increases business growth, as similarly the same outcome (of increasing returns) is sought by oligopolies (for fewer firms) and monopolies (a unique seller).
This pragmatic and realistic path taken in growth modeling that no longer accepts PC and decreasing returns, has emphasized the significance of MC, monopolies and increasing returns, placing at the epicenter market power reduction, its disassociation from PC and ultimately the tolerance for the formation of market imperfections. It has thus contributed at the progressive deposition of PC from its disciplinary throne. This realistic approach has replaced the classic but ideal view of PC leads to diminishing returns. PC was scorned by many economists and deliberately accused as the most responsible culprit for diminishing returns.

Additional critc against NGE is provided by Pasinetti (2000), who referred to (i) the negligence of land, one of the three main production factors discussed by the classics and included in other attempts to build neoclassical equations (e.g. by Meade, 1962, see Pasinetti, 2000); (ii) the simultaneous treatment of capital measured in values and labour measured in physical terms. The two factors “can thus neither be placed on the same level nor be inserted symmetrically in the same function”, raising a fundamental and large “conceptual diversity” about these two production factors (Pasinetti 2000, p. 405); (iii) a reverse capital-deepening process attributed to the non-inverse monotonic relation between quantity of capital and rate of profit, irrespective of the ways employed to measure it. In the Sraffian critic against neoclassical theory, not only the profit rate is affected by the amount of capital employed (as pertained in neoclassical theory) but it affects also the amount of capital finally employed. The understanding of this two-directional causality between the profit rate and the amount of capital has contributed in turning away from neoclassical theory. Economists (for example Hahn, 1982) bypassed this problem by moving away from the initial neoclassical forms and
employing general equilibrium models that were based on inter-temporal functions of profits and utility, sought to be maximized by agents (see Pasinetti, 2000; p. 409).

If, as Pasinetti (2000) argued, omitting land in Solow’s model is misleading, then the non-incorporation of both land and labour in EGMs should also be envisaged with similar skepticism. Human capital has replaced labour, first in the AK model and then to a series of ENGT models. Influenced by Becker’s human capital theory, these models totally neglected and bypassed labour, turning against the basic perception that wages and wage costs form a basic obstacle for business growth that is considered by firms when calculating their profits, cash flows, costs, and organising their strategic planning, e.g. through calculating their net present value. Wage setting through labour demand and supply, the formation of labour market disequilibria or labour mobility across states were totally ignored. Rejecting or denying the marginal analysis of labour differs from the choice to totally exclude labour in growth models. Rather than targeting necessary adjustments that will raise employability and utilize full labour capacity in economies, endogenous and new growth models emphasized other paths, which, by omitting two out of three classical production factors suggested, bypassed the teachings of classics.

Both NGT and EGT espoused methodological individualism, the view that social phenomena can be studied as the outcome of individual behaviour of microeconomic agents (Fine, 1980; p. 5). Strategic interaction among firms, class or group dynamics is not considered in this approach, which could become a source of increasing returns. Several doubts were raised against methodological individualism, for example concerning the absence of interaction among individual agents, their separation from each other, a possible lack of rationality if such interaction finally takes place, and if
any assertion about the macro-level can be explained by the micro-level (Demulenaere, 2015). Using microfoundations may limit considering the significance of such aspects as institutions and neo-institutional foundations, such as property rights, transaction costs and even structures and hierarchies or organizational, behavioural and cultural processes and aspects, as well as historical specificities.

Despite the EGT critic against it, Solow’s model has laid down the foundations for the convergence debate, the distinction between conditional and unconditional convergence, and that among β-convergence, conditional β-convergence and σ-convergence (see Barro and Sala-i-Martin, 1992). Much before the agenda setting in growth theory for employing increasing returns and uncovering its sources, the NGE had suggested that converge will take place because most advanced states would face diminishing returns to capital and poorer countries will catch up with them. Following NGE prescription, absolute convergence occurs due to diminishing returns and the assumption of same parameters across countries (preferences and technology), without necessarily conditioning on their characteristics. Conditional β-convergence occurs if a negative relation holds between initial per capita income (GDP) and average annual growth rates (i.e. β-convergence), when controlling for other variables.

Two possible types of convergence are suggested through NGE: a) conditional convergence, i.e. for states similar in their structural characteristics (technologies, preferences, population growth rates, government policies), independently of their initial income or other conditions (see Barro, 1991; Mankiw et al., 1992; Barro and Sala-i-Martin, 1995); and club convergence, if heterogeneity among individuals takes

7 For a critic on NGT about such points see Foss (1998, p. 7).
place, provided that states have similar structural characteristics and initial conditions (see Galor, 1996, p. 4). Both types accept constant returns to scale and diminishing marginal productivity (ibid, p.5). They may occur when economies follow the model’s assumptions and prescription and there is no government policy. Club convergence may take place through AK endogenous growth models too (ibid, 1996) but conditional convergence is impossible (see Aghion and Howitt, 2009).

In general, EGMs -that employ increasing returns for an accumulating factor other than capital- offer a policy prescription that faster growth will occur in countries applying them. As a result, divergence should rise between such countries and the rest. While the convergence debate has brought a critic against the neoclassical model, it simultaneously helped to test the validity of modeling suggestions (Capolupo, 2009; p.3)\(^8\). On the other hand, the development of EGT has failed to bring a consensus on which factors affect international variation of incomes across countries. On the contrary, it appears to have widened the gap between arguments that such variations are due to factor accumulation or TFP differences (Capolupo, 2009, p.3; see also the discussion by Pack, 1994; p. 9)\(^9\).

Another source of critic against the NGE came from Domar (1961), who has focused on the aggregation of factors to explain growth. He has underlined that SR “absorbs, like a sponge, all increases in output not accounted for by the growth of explicitly

---

\(^8\) Convergence and growth differences were also investigated by new growth theory (Capolupo, 2009).

\(^9\) It is worth acknowledging that the search for significant growth factors through ENGT has substantially contributed to the development of “growth econometrics”, a field of studies that uses econometric and statistical techniques to analyse growth and convergence. Such techniques were subject to several limitations, such as model uncertainty, the overstating of the precision of inference for a finding studied by a specific model, problems in interpreting standard errors that may understate actual uncertainty in modeling parameters, country complexity and heterogeneity, the small number of countries with reliable data leading to measurement errors and parameter heterogeneity, problems with outliers, missing data, heteroskedasticity, causality of growth factors and other common econometric problems (Durlauf et al., 2005; p. 559-560; Capolupo, 2009).
recognised inputs” and that its magnitude is “completely divorced from investment and capital accumulation” (Domar, 1961, p. 712). He has also highlighted that raw materials - a significant component of the production function - are omitted in Solow’s function, “presumably in order to avoid double counting” (ibid p. 716).10 He has further reckoned that this is the reason why SR is exaggerated and that a growth equation could take the form $Y = AL^\alpha K^\beta R^\gamma$, where R stands for raw materials. Domar also referred to specific ways to handle the double-counting problem.

Domar’s growth equation reminds the aforementioned equation by Grossman and Helpman (1994) that incorporated intermediate goods instead of raw materials. Raw materials are included at the intermediate goods sector, composing a significant part of it. The use of intermediate goods in growth equations as a way to reduce residuals sheds an additional light in the scanty evidence provided on the association between intermediate goods and TFP. TFP was negatively associated with an intermediate goods share by Moro (2007) and with intermediate input intensity by Baptist and Hepburn (2013). As the first index grows (TFP), the second falls. Moro (2007) has also found that the intermediate goods share accounts for a great part of TFP, in levels that start from one-fifth and reach almost two-thirds, depending on the production function selected. He referred to Jones (2007; quoted in Moro 2007) who has reminded of the linkages between firms and a multiplier effect through the production of intermediate goods. Last but not least, intermediate goods are found to comprise a large share of trade, in levels exceeding 50% of productivity in less advanced states (Ferreira and Trejos, 2009).

---

10 The distinction between the intermediate and the final goods sector was taken into account in some endogenous growth models too, e.g. in Romer (1990). Hulten (1978) proposed the incorporation of intermediate goods in measuring productivity change of sectors, confirming Domar’s aggregation procedure.
In relation to economic fluctuation, it has been observed that SR tends to follow the business cycle (Mankiw, 1989). It is large in expansion periods, turning to unusually low or even negative in recessions (Hall 1993, p. 71). The Real Business Cycle (RBC) theory was launched by Kydland and Prescott’s 1982 paper on the grounds that TFP strongly correlates to output per worker. This theory sought to investigate the precise links among the neoclassical growth model, technology and productivity shocks (see King et al., 1988).

As opposed to suggestions by RBC theory, it was claimed that technology shocks cannot explain movements of output around the trend, and the precise nature of these trends has to be identified (by Summers, as referred in Manuelli 1986). Hall (1993) has suggested that SR’s correlation to factor price movements and exogenous demand fails to confirm that TFP is produced by a shift in the production function and should rather be uncorrelated to any variable acting as a driving force for output (Hall, 1993 p. 71-72). He attributed this “invariance property” even to the possible mismeasurement of inputs and outputs, including those of capital.

Measuring TFP has played a critical role at the debate on the productivity gap of industrialised states. This took place especially after the 1973 productivity slowdown, evidenced through various researches using indexes of total or single factor -mostly labour- productivity (Link, 1983; Jorgenson et al., 2013; Dean and Harper, 2001; p.57; De Long and Summers, 1992b). While many explanations were provided for this slowdown, a debate has emerged on how important it is for states to remain productive and competitive.
It thus helped to emphasize the significance of competitiveness, a concept that became theorized, even though carrying a certain ambiguity\textsuperscript{11}. While competitiveness was associated with international trade in the past, a healthy balance of payments, price competition, the costs of domestic labour and production and non-price factors (McGeehan, 1968), it was associated more recently to factors emphasized through EGT, such as education, institutions or innovation (Schawb, 2016, p.5). The rising emphasis on competitiveness after the 1990s has led to the development of relevant institutions to promote it, such as “Councils of Competitiveness” in USA, the rising of amounts invested on it, and the use of an extended array of motives.

3. Changes in TFP and capital calculations, non-integrated in official accounts for a long period

Productivity series were re-calculated in many countries to improve their accuracy, building on new income and product accounts, developments from economics (such as the use of Tornqvist index), new data availability and enhanced needs of data users (Dean and Harper, 2001, p. 61-62; Dean and Harper, 1998). The BLS acknowledged such limitations in measuring goods and services ever since 1959 (Dear and Harper, 2001, p. 57)\textsuperscript{12}.

\textsuperscript{11} see the dialogue between Burton (1994) or Preeg (1994) and Krugman (1994); also Aigner (2006).

\textsuperscript{12} As explained by Dean and Harper (2001, p. 56), the methodology for calculating productivity indexes in USA in the much earlier BLS productivity program was based on the perception that whenever productivity improves, a worker displacement effect sets in operation, and for this reason productivity gains were weighted by associated job losses.
TFP miscalculation was further attributed to gross output TFP measures, used instead of (net) value-added (Schreyer, 2001). Value-added\(^\text{13}\) was considered a meaningful indicator to assess an industry’s contribution to an economy’s productivity growth, capturing movements of output and allowing comparability across different levels of aggregation (OECD, 2001b, p.12; OECD, 2011, p. 24-29; Schreyer, 2001, p. 40-41; Jorgenson, 1991).

Jorgenson (1991, p. 81-82) has identified various biases in productivity growth, while attributing productivity slowdown to rising energy prices after 1973. Energy shocks, their prices and more generally inflation appear to influence MFP levels (Jorgenson, 1991; Moulton, 2018; Link, 1983). Energy is necessary to produce tangible assets, especially in manufacturing, thus influencing productivity\(^\text{14}\).

Jorgenson and Grilliches (1967, p. 271-272) had referred to a series of TFP calculation errors (in aggregation, prices etc) that significantly reduced TFP contribution to output\(^\text{15}\). They proposed rental price estimates to aggregate services of assets, for different asset types (Dean and Harper, 2001, p. 62)\(^\text{16}\). Similarly, the BLS Productivity Measurement Program has adopted rental prices for aggregating different types of assets\(^\text{17}\).

\(^{13}\) i.e. gross output corrected for purchases of intermediate goods

\(^{14}\) Even the KLEMS productivity statistics project that is used for productivity measurement and contains information for capital, labour and intermediate inputs, including energy (and materials and services) has received several amendments over the last decades (see Ark and Jäger, 2017, p.10-11).

\(^{15}\) Dean and Harper believed that SR will be eliminated if productivity is more accurately measured (Hulten, 2001, p. 12). As if SR was simply a matter of productivity measurement.

\(^{16}\) Rental prices reflect an assessment of how much the owner of a particular capital good would charge to rent this capital good in a competitive market and is the sum of \(ut = (rt + \delta t - it) pt\), where \(ut\) is the rental price or user cost, \(rt\) is the required rate of return, \(\delta t\) the rate of depreciation and \(it\) the rate of asset price change, \(n\) is not high in infrastructure goods and some types of goods, causing a variation of rental prices between infrastructure and non-infrastructure goods.

\(^{17}\) For two successive periods, t-1 and t, the growth of total capital input (\(\Delta\ln K_t\)) is computed as the weighted sum over asset types \(\alpha\), of the growth rate of asset stocks (\(\Delta\ln k_{\alpha}\), where the weights are the arithmetic mean of shares in the two periods, for the implicit “rents” created by respective assets in
Omitting land or inventories was realized to cause a bias in measuring the contribution of capital to productivity (following Diewert’s work, quoted in Dean and Harper, 2001, p.62). Other sources of capital underestimation comprised inaccurately measured financial and other intangible assets (ibid, 2001). Tornqvist aggregation indexes in measuring capital were employed by BLS\(^{18}\), whose difference from hitherto applied Laspeyres-type was realized to be sizeable (OECD, 2001; p. 30). The former offers larger weights on assets that tend to depreciate faster than other indexes, because investors of short-lived assets seek to collect more rents, as they face higher depreciation costs (Dean and Harper, 2001, p. 63). Soon, hyperbolic depreciation formulas for the capital decay process were also adopted by BLS (Dean and Harper, 2001, p. 68). An enhanced capital input measure was calculated after applying more detailed asset classes, rental prices and Tornqvist aggregation techniques, which have grown by 0.8 annual percent faster, in comparison to previously applied aggregated capital stock (Dean and Harper, 2001; p. 65).

Only in USA, amendments in productivity and capital calculations comprise the 1965 revision of GNP accounts, the 1960s and 1970s expansion to include all government and business-owned fixed assets and consumer durable goods, the mid-1980s calculation of capital and TFP at a two-digit industry level by BLS, the 1976 more consistent valuation of the consumption of fixed capital, the 1985 Bureau of Economic Analysis (BEA) use of quality-adjusted price indexes for computers and peripheral equipment, the 1991 change of BEA from GNP to GDP, the 1993 System of National Accounts (SNA), the 1996 introduction of major improvements of total rents (Dean and Harper 1998, p. 12). Dean and Harper (1998) have referred to the following equation: 

\[
\Delta \ln K_T = \sum_i \Delta d \ln k_a [1/2 (k_{a_k} c_{a_i} / \sum_j k_{i,j} c_{k,j} + k_{a_{i-1}} c_{a_{j-1}} / \sum_j k_{i,j-1} c_{k,j-1})]
\]

\(^{18}\) influenced by the work of Jorgenson and Griliches (1967), as explained by Dean and Harper (2001, p. 63)
National Income and Product Account, and their 1999 and 2003 comprehensive improvements (BEA, 2006). In the 1996 NIPA, real GDP was estimated using Fischer index formula rather than a single base year (BEA, 2006; p. 2).

The 2009 OECD manual added to the 1993 SNA set of capital measures a whole new part on age-efficiency functions, productive stock measurement, return on capital, user cost and capital services (see Table 1). This more integrated system of national accounts (than the 1993 SNA) was based on a more detailed system of stock and flow measures of capital (OECD, 2009). It has employed capital services, i.e. “the flow of productive services from capital assets to production”, thought to be proportional of the productive stock and derived from it. The latter is calculated out of the gross stock, by the use of an age-efficiency function.

**Table 1:** Elements added in the system of Capital measures (1993 SNA) by the 2009 OECD manual

<table>
<thead>
<tr>
<th>Net capital stock</th>
<th>Age-price function</th>
<th>Net value added</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption of fixed</td>
<td>Return on capital</td>
<td>User costs</td>
</tr>
<tr>
<td>Gross stock</td>
<td>Productive stock</td>
<td>Capital services</td>
</tr>
</tbody>
</table>

**Notes:** In grey boxes all elements in the 1993 SNA system of capital measures. In white boxes all added elements in the 2009 OECD manual.  
**Source:** OECD (2009).

Capital services were not given attention in the 1993 SNA nor considered compulsory in the 2008 SNA (OECD, 2009). They were emphasized in the 2009 OECD manual, and viewed as analytical counterpart of net capital stock. Their value, necessity and
the method for their breakdown in capital service prices and volumes in income accounts were highlighted by OECD (2009, p. 26).

The OECD (2001) suggested that capital services should be calculated as a flow rather than stock variable, by converting assets to standard efficiency units, and then combining them to a volume index of capital services, using as weights the user costs of capital (OECD, 2001a; p. 11). This is a significant difference because stocks are measured at a point in time, whereas flows over an interval of time. The use of “perpetual inventory method” (PIM) to estimate gross capital stock was also suggested, based on available statistics for gross fixed capital formation, price indices for capital assets and broader information on average service lives, and how retirements are distributed around such averages (OECD, 2001a; p. 10).

Following the introduction of new classes of intangible fixed assets in the 1993 SNA\(^ {19} \), the 2009 OECD manual emphasised the more precise calculations of three types of assets, hard to measure: land, inventory and natural resources other than land. It also emphasised measuring service lives for assets, the appropriate patterns of depreciation and retirement functions.

The 2001 OECD manual suggested both an age efficiency profile of an asset, which refers to the rate it loses its productive capacity or efficiency, and its age-price profile that indicates decline of assets in market values, is associated with ageing and used for measuring fixed capital consumption (OECD, 2001; p. 34). Both asset profiles were considered to operate simultaneously. The 2009 OECD manual suggested three different age-efficiency profiles for calculating capital services: i) the hyperbolic age-

\(^ {19} \text{mineral exploration, computer software and entertainment, literary and artistic originals and research and development.}\)
efficiency profile, where productive efficiency of each asset declines at slow rates in the earlier years of its service life but then follows increasingly faster rates, giving rise to convex age-price profile (OECD, 2009; p. 35), ii) a linear decline of age-efficiency and age-price profiles at constant growth rates, as already suggested in the past (OECD, 2009; p.35), and iii) a geometric age-efficiency profile and age-price profile. This is an accelerated pattern with higher depreciation in earlier years of an asset’s life than later and is due to the decline of its efficiency. A geometric pattern in most assets is followed by the BEA since the mid-1990s (Parker and Triplet 1995; quoted in Fraumeni 1997), generally proposed at the 2009 OECD manual.

Beyond all these sources of capital underestimation, it is worth noting the possible underestimation of investment levels. For example, Parente and Prescott (2005; p. 1406) specifically highlighted that unmeasured investment reach levels between 35 and 55% of GDP, due to investments in intangible capital treated as ordinary business expenses in the NIPA of the USA.

4. Evidence on the significance of type of investment for growth

The problem of capital miscalculation and underestimation concerns both Solow model and EG and NG models. The size of unexplained part in growth equations would have been greater, had capital been estimated more accurately. TFP miscalculation is a source of underestimation for SR in particular. Another source of underestimation of the association between explanatory variables and growth at the neoclassical growth equation concerns the impact of investment on growth.
4.1 Empirics on the significance of equipment and structure investment and relevant debates

Several consecutive studies by De Long and Summers (1991; 1992a; 1992b) and De Long (1991; 1992a) (referred as DLS) have explored the association across a panel of countries between growth and the investment type. A strong association of growth with equipment and machinery investment was traced both in fast and slow-growing nations. Abel (1992, p. 206) referred to “new stylized facts of growth”, additional to kaldorian. The 1991 DLS OLS regressions revealed a high coefficient associating equipment investment with GDP per worker growth, after comparing against other investment components, such as investment in structures, and testing various alternatives (omitted variables, outliers, spatial correlation, policy regime and other). The equipment investment coefficient became even higher for high productivity nations, in the 1991 DLS (p. 458), and exceptionally high for electrical machinery, while the second higher was for non-electrical machinery. It became negative for structures and producers’ transportation equipment share.

Temple (1998) tested the credibility of several DLS findings and points. He has found that is not driven by measurement errors or a simultaneity bias. In particular for the treatment of their outliers, Temple noted that DLS results are sensitive to the method for outlier detection, proposing an alternative.

Similarly, De Long (1992a, p. 312) has regressed GDP per capita against machinery equipment and non-residential construction and found significant associations, with

---

20 Since capital is not included among independent variables, equipment investment could be acting as a proxy for capital. However, the results still emphasize the significance of the distinction between the type of investment selected and their different contribution to growth.
almost three times higher coefficients or even more for the former. In De Long (1991; 1992a) per capita national product increases by more than 0.4% average per year for each 1% of increase in machinery equipment share of national product, for six industrial nations, extending back in 1870. In their 1992 DLS study, a 0.26% annual growth occurs for each 1% increase of investment in machinery and equipment, while non-machinery plays a less significant role, since it is significantly associated to growth with a coefficient of one-quarter of the magnitude (DLS, 1992b; p. 117-118).

DLS (1992b) referred to a strong “growth-equipment nexus”, explained by the operation of an accelerator mechanism (DLS, 1991; p. 469-471), suggesting that equipment investment is driven by rightward shifts in the supply curve for equipment than rightwards in demand (ibid, 1992a). They even suggested a structural causal association running from high equipment investment to growth (DLS, 1992a; 1992b)\(^\text{21}\). They explained this causality by employing a supply-demand argument, according to which if fast growth was causing high machinery, growth would have moved the supply of machinery upwards, causing the rise of machinery prices. But since growth and machinery associate with low machinery prices, causality should run from the opposite direction (DLS, 1992b; p. 118; DLS, 1991)\(^\text{22}\). Such causality is similarly highlighted in De Long (1991; 1992a).

After breaking the aggregate investment variable into other components, DLS (1992a) have failed to provide evidence of a high correlation between growth and public investment in infrastructure or residential construction. Similarly, schooling and continent-specific proxies have not unveiled a significant association, for different samples and sub-samples (ibid, 1992a; p. 169). Non-significance of schooling is

\(^{21}\)The equipment aggregate includes electrical and nonelectrical machinery.

\(^{22}\)Jones (1994) has also found a negative association of relative machinery prices with growth.
important because DLS findings were suggested to be due to endogenous growth factors, such as R&D and its resulting knowledge that is incorporated in machinery production (Pack, 1994; p. 57).

Concerning TFP association with equipment investment, a reverse pattern from capital shares was observed (DLS, 1992a). As capital shares fell, this association turned higher and reduced as they rose, while -simultaneously- a stronger association with investment in structures was not taking place. DLS (1992a) questioned whether TFP differentials among states actually influence their growth trajectories and to what extent are due to actual resource allocation decisions and not entirely to noneconomic forces comprised in TFP. They explained strong TFP association with equipment investment by arguing that benefits from equipment investment do not depreciate rapidly, criticizing the absence of such association in NGT (DLS, 1992a; p. 187). Temple (1998, p. 17-19) has highlighted that such evidence was challenged by Koop et al. (1995, p. 35) on a weak association of TFP with equipment investment and by Maddison (1987, p. 665) on a strong association of TFP with structures investment.

DLS (1992b) have also explained the 1970s and 1980s productivity slowdown as the outcome of structural and exogenous factors and not of macroeconomic policies, which -again- they have mostly attributed to the role of equipment investment for promoting growth. They emphasized the need to raise investment “quality”, as opposed to quantity (ibid p. 112), and proposed managing public investments and identifying and introducing in policy making those investments promising extremely high social returns and substantial external benefits, not captured by the entity undertaking them (ibid, p.112).
Temple (1998) has also confirmed the strong correlation of equipment investment with growth in a large group of developing states, with returns for equipment investment exceeding by more than 50% those in structures. He considered the higher correlation of growth with equipment investment in DLS studies affected by the medium-term horizon of regressions (less than 25 years), because equipment investment brings short-term results, while structures investment requires more time. He used a Mankiw-Romer-Weil (MRW) augmented Solow’s model to accommodate DLS findings and their claim that equipment investment is more important than implied in Solow model despite their unclear mechanics (Temple, 1998; p. 58). Using different depreciation rates for structures and equipment, he concluded that their change influences the expected factor shares for equipment and structures investment (as well as for human capital). Using DLS data, he rejected Abel’s argument\(^\text{23}\) that the high coefficient of equipment investment is due to the rise of investment over time, which influences average investment ratio (Temple, 1998; p. 58).

Furthermore, Temple (1998) disaggregated capital (not only investment, as in DLS) into equipment (E) and structures (S) components and reproduced the MRW growth equation as \(Y = E^\alpha S^\gamma H^\beta (AL)^{1-\alpha-\gamma-\beta}\), with \(\alpha\) and \(\gamma\) being the exponents of equipment and structures respectively. He then transformed this equation as \(Y = K^{a+\gamma} H^\beta (AL)^{1-\alpha-\beta-\gamma}\), where \(K = E^{\alpha\gamma} S^{\alpha\gamma}\), and the growth of total factor productivity and labour force are \(\frac{\dot{A}}{A} = g\) and \(\frac{\dot{L}}{L} = n\). This equation gave the steady-state per capital income as a function of rates for equipment and structures investment, labour force, population growth (n), total factor productivity growth (g).

\(^{23}\) expressed in the Comments section of De Long and Summers 1992 study
and depreciation rate ($\delta$). He noted that since the returns of equipment investment appear to be so high, it is expected that some externalities to equipment investment will appear that should be incorporated somehow in the extension of Solow’s model selected. On the contrary, Auerbach et al. (1994), using the DLS data, have found that there are no externalities to investment and the link between investment and growth is consistent with Solow’s model.

Greenwood et al. (1997) have disaggregated the law of motion in two separate equations, one incorporating an equipment investment and one a structure investment component. Equipment investment was multiplied by the current state of technology for producing equipment, making technological change investment-specific (and not neutral), i.e. one induced by investment choices. They concluded that “investment-specific change may be a source of economic fluctuation”, explaining not only the rising of equipment-to-GNP ratio but also the fall in equipment relative prices observed (Greenwood et al., 1997; p. 359). They have also found that after removing the part attributed to investment-specific technological change (that reaches almost 60%), the level of productivity slowdown dramatically falls (ibid, p. 359). Temple (1998) has noted on this particular work that investments made in some particular capital goods require training and reorganization and not just investing on machinery and, as a result, regressions on equipment capital stocks do not capture such elements of the investment procedure and overstate the role of equipment investment.

---

24 Temple mentioned the problem of the substitution of equipment from structures (Temple 1998, p. 41).
4.2 Another source of capital and growth underestimation currently neglected in growth modeling: the type of investment and the emphasis on infrastructure

Since capital is accumulated, renewed or rendered idle through decisions about investment, the type of investment is critical for its formation, impacting on growth output. Investment is one of the most studied concepts in economic theory. Its typology though has changed over time, influencing and influenced by its use economic and macroeconomic analyses. For decades, the most valid distinction was that among business fixed investment in plant and equipment, residential construction (or housing) investment, and investment in inventories (see Ott et al., 1975).

Each of these components was studied and the rationale for investing on each one of them differentiated. Residential construction investment was observed to be quite high in times of cycles, a point necessary in their study. Business fixed investment was studied based on the microfoundations of investment function, the neoclassical theory of the firm and its choice for maximizing utility by seeking to achieve maximum net present value, the neoclassical investment model, and the marginal efficiency of capital (ibid, 1975).

The accelerator mechanism discussed in DLS is a macroeconomic concept elaborated along with the acceleration principle. The latter was common in most explanations of the trade cycle before the advent of RBC theory, and formulated by many economists, mostly for fixed business but also for inventory investment (see Eckaus, 1953, Neisser, 1954 and Ott et al., 1975). It refers to the relation among investment and/or capital stock and/or output (measured either in levels or in changes of levels of these variables), and promotes the idea that some accelerator mechanism (occasionally
called “relation”) that enhances capital and output\(^{25}\). For some economists in the past, like Kaldor, this relation depends on the availability of idle machines that have to be used and the exploration of surplus production capacity, before investment in new machines takes place to produce more consumption goods. Other critics have explained the limitations in this concept but the general idea still remains that it refers to some sort of acceleration process or mechanism is placed in operation.

Another distinction of investment currently bypassed in investment theory but extensively used in past macroeconomic analyses is that between “autonomous” and “induced investment”. The former refers to the part of investment that is autonomous of income while the latter in the part induced by changes in capital (or output) (Hamberg and Schultze, 1961; Neisser, 1954). According to Hamberg and Schultze (1961; p. 55), autonomous investments take various forms of government spending, such as infrastructure projects, social goods, residential construction and innovative investments, taken either to lower costs at prevailing levels of output or to introduce new products that displace substitute products in consumer budgets. Induced investment takes place in existing, “old” types of products, in response to pressures to expand the productive capacity, due to “high marginal costs, strain on existing staff and capital stock, bottlenecks and delays” and depends on existing income levels (ibid; p. 54).

Since investment enhances capital in growth equations, the investment type influences growth and output, irrespective of its distinction to equipment vs structure, induced vs autonomous or fixed, residential and inventory. Some investment types raise capital and income more than other, while other require more extended periods. The precise

\(^{25}\) The concept of super-multiplier was also used, to investigate the simultaneous operation of the accelerator and the multiplier, and their combined effects.
influence of investment type is a policy aspect that should not be left unattended, otherwise capital may increase in decreasing rates, may fall and its contribution to growth output may ultimately reduce. An investment type promoted that accumulates capital in the long run may leave a greater part of TFP unexplained and, subsequently, overestimate TFP in the short or medium-run.

If a certain type or balance of investment types promotes growth faster, the over-emphasis on policies on some particular type should be avoided if its impact upon the economy is negative. This is important in the light of global overemphasis on “infrastructure industry” over the last decades, its funding, financing and financialisation (O’ Brien and Pike, 2015). The emphasis is witnessed in proliferation of journals and on infrastructure investment (Preqin, 2017) and that of international institutions that had initiated data collection to analyse investment risks and returns (OECD, 2015). Institutional and other investors (mutual funds, sovereign wealth funds, life insurance companies) including large pension funds have invested billions in infrastructure, while the closing of “infrastructure gap” was assessed to require trillions of US dollars (OECD, 2007). The financialisation of infrastructure took place through publicly listed funds, private infrastructure funds, direct investment or even special pension funds (PWC and GIIA, 2017; Institute and Faculty of Actuaries, 2015; Russ et al., 2010). Specific institutions and banks such as E.I.B. or the Asian Infrastructure Investment Bank support infrastructure policies, and specific indices track the performance of infrastructure investments, such as the Dow Jones Brookfield Global Infrastructure Index or S&P Infrastructure Index.

The precise influence of infrastructure investment on private sector remains unclear. For the “Aschauer hypothesis” differences in infrastructure stocks account for
different levels of productivity, output and growth but other evidence had suggested that infrastructure investment does not generate private investment (Munnell, 1990).

5. An inclusive growth model

As emphasized above, a growth equation, apart from capital and labour, can include an aggregate component that comprises all other factors suggested in ENGT to contribute to growth, and possibly unidentified factors too, leaving ample room for intermediate goods and raw materials, by taking into account Domar’s (1961) critique and models such as that in Helpman and Grossman (1994).

Total value added ($V_i$) is a function $G$ of capital ($K_i$) and labour ($L_i$) inputs and time ($T$), as in Jorgenson et al. (1987):

$$V_i = G(K_i, L_i, T) \quad \text{(i)}$$

and output is the sum of the value of added value ($V$) and intermediate inputs ($X$), as:

$$Z_i = F(X_i, V_i) \quad \text{(ii)}$$

The value of output is the sum of value of intermediate inputs and of value-added, such that:

$$q_i Z_i = p'_i X_i + p'_i V_i \quad \text{(iii)}$$

where the value-added is the sum of the value of capital and labour inputs.

$$p'_i V_i = p'_i K_i + p'_i L_i \quad \text{(iv)}$$
This identity is used to find conditions for the equilibrium point in production (see Jorgenson et al. 1987).

We can consider the maximum of total output to be a function of all quantities of added value, all intermediate inputs, all inputs of capital and labour, and time.

\[ Z = F[V(V_1, V_2, V_3, ..., V_n), X(X_1, X_2, X_3, ..., X_m), K(K_1, K_2, K_3, ..., K_p), L(L_1, L_2, L_3, ..., L_q); T] \] (v)

where \[ K = K(K_1, K_2, K_3, ..., K_p), \quad L = L(L_1, L_2, L_3, ..., L_q) \]

\[ X = X(X_1, X_2, X_3, ..., X_m) \quad \text{and} \quad V = V(V_1, V_2, V_3, ..., V_n) \]

Intermediate inputs were not taken into account in equation (iv). Similarly, none of the growth factors suggested in endogenous and new growth theory were included in equation (iv) that is used to find the equilibrium point or equation (v).

Assuming an aggregate component \( A \) that comprises all endogenous and new growth theory factors, and of the intermediate inputs \( X \), equation (v) turns to:

\[ W = H[V(V_1, V_2, V_3, ..., V_n), X(X_1, X_2, X_3, ..., X_m), K(K_1, K_2, K_3, ..., K_p), L(L_1, L_2, L_3, ..., L_q), A(A_1, A_2, A_3, ..., A_r); T] \] (vi)

where \[ A = A(A_1, A_2, A_3, ..., A_r) \]

Intermediate goods should be incorporated in the calculations for maximum value of total output. If not, the size of the aggregate component rises, because it measures every unidentified component.

We can specify each factor to have its own exponent, as below:

\[ Y = K_1^a A_1^b L_1^{1-a-b} \] (vii) \quad \text{where} \quad 0 < a, b, a + b < 1
This equation is different from the MRW constant returns production function, where each exponent accounted for one-third, for each production factors employed.

Transforming this equation gives:

\[ Y = K_t^a A_t^\beta L_t^{1-a-\beta} = K_t^a A_t^\beta \frac{L_t^{1-a}}{L_t^\beta} = K_t^a \frac{A_t^\beta}{L_t^\beta} L_t^{1-a} = K_t^a \left( \frac{A_t}{L} \right)^\beta L_t^{1-a} \]

by placing \( \left( \frac{A_t}{L} \right)^\beta = A' \), we reach \[ Y = A' K_t^a L_t^{1-a} \] (viii)

Equation (viii) is essentially a Hick’s-neutral Solow growth equation. It is open to all three possibilities of increasing, decreasing and constant returns, since the aggregate component, \( A \), expressed in labour terms, is likely to be increasing, decreasing or remain constant, depending on the levels and degree of development of factors composing this aggregate component, their interaction, the state policies followed etc.

Expressing the aggregate component in labour terms shows that its contribution to growth is subject to labour size.

Assuming that \( A \) and \( L \) are two exponential functions of time, such that \( A(t) = A(0)e^{gt} \) and \( L(t) = L(0)e^{nt} \), for time \( t \) and 0, where \( g \) and \( n \) are the growth rates for the aggregate component \( A \) and labour \( L \), respectively, then:

\[ A' = \left( \frac{A}{L} \right)_t^\beta = \left( \frac{A}{L} \right)_t^\beta = \left( \frac{A(0)e^{gt}}{L(0)e^{nt}} \right)^\beta = \left[ \frac{A(0)}{L(0)} \right]e^{(g-n)t} \beta = \left( \frac{A}{L} \right)_0 e^{(g-n)t} \beta \]

Thus, the equation (ix) becomes:

\[ Y = \left( \frac{A}{L} \right)_0 e^{(g-n)t} \beta K_t^a L_t^{1-a} \] (ix)
The aggregate component \( A = \left( \frac{A}{L} \right) e^{(g-n)t} \) affects the Cobb-Douglas function. The model’s steady state properties are those of Solow’s model, and the steady state is found at:

\[
k^* = \left[ \frac{SA}{n+d} \right]^{\frac{1}{1-a}} = \left[ \frac{s(A/L_0) e^{(g-n)t} \beta}{n+d} \right]^{\frac{1}{1-a}}
\]

and output per worker at steady state at:

\[
y^* = \left[ \frac{SA}{n+d} \right] = \left[ \frac{s(A/L_0) e^{(g-n)t} \beta}{n+d} \right]^{\frac{a}{1-a}}
\]

where \( d \) is the depreciation rate (used in the law of motion), \( n \) is the population growth rate, and \( s \) the savings rate. Both \( k^* \) and \( y^* \) depend on \((g-n)t\beta \) but \( \beta \) associates with the size of \( \alpha \). As \( \alpha \) rises, both \( 1/(1-\alpha) \) and \( a/(1-\alpha) \) rise but due to the limitation placed in the original equation (that \( 0 < a, b, a+b < 1 \)), \( \alpha \) and \( \beta \) cannot rise for ever, have upper and lower limits and are bound by the growth of each other. One should note that the exponent \((g-n)t\beta \) is likely to become negative (if \( g < n \)), thereby possibly reducing both \( k^* \) and \( y^* \). The latter is a condition often found in economies whose population expands, while the growth rate of their aggregate component falls or remains unchanged. While equations (viii) and (ix) appear to be Hicks-neutral, expressing the aggregate component in labour terms influences the final output, since it is limited by initial labour size.

The model acknowledges the significance for growth of several properties found in Solow’s model, such as savings, and EG factors (through recognizing the role of \( g \)).

As \( g \) rises, \( \beta \) is likely to rise at the expense of \( \alpha \) or at the expense of \( 1-a \) (if \( \alpha \)
remains unaltered or rises). Human capital and EG factors could bring further growth.

At the same time, it takes into account the significance of intermediate inputs, which can also be included in the aggregate component.

For the Hicks-neutral function, \( Q_t = A_t F(L_t, K_t) \), we are aware that the growth rate of output equals the growth rate of capital and of labour (weighted by their respective elasticities), plus the growth rates of the Hicksian shift parameter.

Resolving for the parameter:

\[
R_t = \frac{Q_t}{Q} - s^K_t \frac{K_t}{K_t} - s^L_t \frac{L_t}{L_t} = \frac{\dot{A}_t}{A_t} \quad \text{(vii)}
\]

where dots denote time derivatives, the ratios are the respective rates of change for capital and labour, and their respective elasticities, \( s^K_t \) and \( s^L_t \), are equal to income shares (when inputs are paid the value of their marginal products, i.e. \( \frac{\partial Q}{\partial K} = c / p \) and \( \frac{\partial Q}{\partial L} = w / p \)). This equation is a form of a Divisia index, used to calculate the residual (\( R_t \)) as the growth rate of output not explained by the income-share weighted growth rates of capital and labour inputs. We can investigate \( R_t \) for a production function after relaxing constant-returns to scale and allowing for increasing or decreasing returns (in the presence of \( A' \)).

6. Conclusions and discussion

The recent amendments of capital and productivity accounts, as reviewed, cannot be seen as corrective accounting exercises. They should have implications for the theories for capital, for growth and investment somehow. The intuition of few

\[ s^K_t = c_t L_t / [w_t L_t + c_t K_t] \]

\[ s^L_t = w_t L_t / [w_t L_t + c_t K_t] \]

\[ 26 \text{ Where } s^K_t = c_t K_t / [w_t L_t + c_t K_t] \text{ and } s^L_t = w_t L_t / [w_t L_t + c_t K_t]. \]
economists that have already discussed the underestimation of capital and investment in growth equations, as mentioned above, should be in the right direction. If capital has been underestimated for an extended period, then SR should have been overestimated. Furthermore, the strong critic against NGE should be moderated and conclusions for EGT only partially confirmed. Moreover, the productivity slowdown in industrialized economies and the theorization of competitiveness merit further consideration, since they had both sprung out of diagnosed high TFP levels during the 1980s and 1990s. Doubt is also raised for RBC theory and the work of Kydland and Prescott that it has originated from, bringing implications for several developments in macroeconomic theory.

DLS studies highlighted the importance of investment type for growth, in particular of investment in equipment in comparison to structures. The concept of accelerator and the earlier distinctions between autonomous and induced investments or among fixed business investment, residential and investment in inventories remind that each investment type has different influence on growth. A potentially inaccurate estimation of the role and contribution of investment type to capital and growth further validates the above-mentioned concerns on the extent of productivity slowdown, the actual fall in competitiveness and RBC theory claims. Productivity slowdown may derive from limited investments in particular forms of capital, especially on equipment.

Even if the possibility that the investment type differently associates to capital is excluded, and other problems at the NGE or the factor aggregation method employed are totally bypassed, the differential association of investment type to growth is likely to relate to the non-inclusion of intermediate inputs or raw materials as explanatory variables at the NGE, as suggested by Domar’s critic. A great part of equipment
investment is likely to be undertaken in the production of intermediate goods and services, with multiplied effects on outcome that allow faster economic growth. As if there is a missing component of capital in growth equations that is somehow enhanced by intermediate inputs and equipment investment. Whether intermediate inputs can be treated separately from capital (and whatever is finally contained at the variable of capital) is another point to consider in theory.

Many of these equipment investments take place due to technology, require elaborated human capital or the capacity to learn, and are related to the employment of endogenous growth factors, in some ways. This is a reason to expect endogenous growth variables to partially act as proxies for this particular missing component, intermediate inputs. The overestimated residual in growth equations neglecting intermediate inputs is found in endogenous, new growth or the NGE (only the latter residual was carefully identified to remain unexplained). Recording intermediate inputs in growth equations and their disentanglement from endogenous growth variables is a useful task but the prospect of an endogeneity problem is present if they are simultaneously employed in equations. On the other hand, the significance of ENGT variables in explaining growth has already been proved and cannot be rejected.

If the greater part of SR is attributed to the intermediate goods sector, it is important to take into account this sector for economic growth purposes, which is composed of raw materials, unfinished and semi-finished goods. Economies are not just the outcome of finished goods and materials used to calculate capital and labour. It is worth investigating growth models incorporating a perception of economies not as perfect establishments but as semi-finished, half-complete, interlinked, integrated, with leakages and traded.
Since a great part of intermediate inputs is traded, covering the needs of international demand (as in the case of the large Chinese economy), it should be expected to be subject to international fluctuations. Fluctuations in the size of intermediate goods and services and their trade may account for some significant part of an economic cycle, which is sought to be explained by RBC theory. Hence, intermediate inputs may represent a systematic component missing for the more accurate prediction of cycles. If this is the case, some valid claims and conclusions from RBC theory require reviewing, influencing -in this respect- the new neoclassical synthesis.

Another relevant conclusion is that an investment type and the investment balance influence growth and its texture more than actually considered in growth equations and should therefore better be integrated in them. This can effectively take place through empirically reassessing the extent of significance of endogenous and new growth theory factors, through incorporating intermediate inputs in growth equations and better integrating the investment type along the lines of equipment vs structure, autonomous vs induced or fixed business, residential and inventory investment.

In terms of policy recommendations, economies should emphasize specific types of investment and materials that expand their intermediate goods sector and total outcome. This will help them to limit their productivity slowdown and enhance their competitiveness. Bringing new materials in production created in laboratories could offer new possible growth avenues to sustain or enhance competitiveness. In this perspective, each economy has its own domestic size and capacity to produce that relates to its intermediate goods sector and resembles to an accordion: the more it emphasizes intermediate goods the more it expands but its stretching is also likely, subject to international fluctuations. Therefore, cautiousness in needed in the
prevailing emphasis on infrastructure investment taking place in many economies worldwide.

Some room is created for investigating new paths in growth modeling, since SR has been over-estimated, intermediate inputs were not attributed some significance, savings had been neglected in ENGT models, and growth theory deliberately espoused IC instead of PC. The aforementioned model incorporates intermediate inputs, which are associated to endogenous growth factors through an aggregate component, while more specific modelling configurations can be followed (as in the Appendix). Testing this model is needed as future research.

As far as its implications for competition are concerned, as opposed to growth models sustaining IC -and in particular MC and the employment of monopolistic power- the present model follows the tradition of models espousing PC. Globalization has unveiled new, global competitors outside national borders, making oligopolies and MC part of economic reality in most countries. Economic growth theory is neither about identifying how large-scale firms can manage to compete globally nor about the prevailing forms of IC, their sustaining and how product variety will raise growth; It is a theory about economic opportunity, freedom, and the capacity to develop in the long-run. One can criticise endogenous growth to have brought an in-depth investigation on precisely how capital exercises monopolistic power that allows its increasing returns, instead of exploring how output shall be produced and better distributed across economic agents and firms, benefiting from its spread PC and economic freedom.

If a great part of SR is due to intermediate inputs, causing higher and increasing returns, the over-emphasis placed on explanations that comprise capital but exclude
labour, its role and significance, seeking to replace it by other causes, is at best misleading for economic growth theory. Besides, one cannot ignore that the Chinese economy, for instance, has managed to compete against that of USA because of its abundance of labour, producing vast quantities of exported intermediate goods and inputs. Less sophisticated mathematically growth models that acknowledge the role of labour, such as the suggested here, may offer valuable directions in growth theory.

The policy prescription of models that, on the one hand side, emphasize the abandoning of the competition framework (that is necessary for strengthening domestic competition in states), and, on the other declare the lack of significance for labour, is expected to bring the fall of whole economic empires, not just of weaker economies that will not manage to cope with competition. One expects from economists to highlight the significance of growth models envisaging problems and the malaise in their economies and states, not just those espousing large-scale capital and its intentions. Finally, such conclusions have broader implications for regional economies too.
References


Institute and Faculty of Actuaries: Infrastructure investment: policy summary, November 2015


Appendix

To indicate the various possible configurations of a growth model that incorporates intermediate inputs assume two varieties of capital: \( K^\alpha \) that refers to finished goods and services and \( K^\beta \) that refers to intermediate inputs. The sum of capital used to produce final and intermediate goods and services is given by \( K^\alpha + K^\beta = K^{\alpha+\beta} \). Assume also that labour employed in the production of intermediate goods and services, \( L^\beta \), is different from that of finished goods and services, \( L^\alpha \).

Capital and labour were multiplied in the production function, when output for final goods and services was calculated. It is similarly expected that labour and not just capital is required in the production of intermediate inputs, and both these two have to be multiplied.

Hence, the aggregate component \( A^\beta \) of the suggested growth equation in the present text, \( Y = K^\alpha A^\beta L^{-a-\beta} \), can be written as: \( A^\beta = (BKL)^\beta = B^\beta K^\beta L^\beta \). The equation turns to the following:

\[
Y = K^\alpha A^\beta L^{-a-\beta} = K^\alpha (KBL)^\beta L^{\gamma-a-\beta} = K^\alpha K^\beta B^\alpha L^{\gamma-a-\beta} L^\beta = B^\beta K^{\alpha+\beta} L^{\gamma-a-\beta-\beta} = B^\beta K^{\alpha+\beta} L^{-a}
\]

or

\[
Y = B^\beta K^{\alpha+\beta} L^{-a}
\]

The above growth equation, compared to the initial growth equation, shows that capital is increased by the amount of intermediate inputs \( K^\beta \), and labour by the amount of labour required to produce these intermediate inputs, \( L^\beta \). The reduced in size SR, \( B^\beta \), is attributed now solely in the growth factors suggested through
endogenous and new growth theory. The size fell from $A^\beta$ to $B^\beta$ but $B^\beta$ contributes in the production of $K^{\alpha+\beta}$ as a whole and of the enhanced size of labour, $L^{1-a}$.

Overall, this model indicates that the contribution of capital and labour to growth is higher than suggested and that of the missing component lower (that suggested) and that $K^\beta$ and the exponent $\beta$ (by which capital grows) can have a significant impact in the growth of economies.

The purpose of the present note was to expose the various possible configurations of a growth model that incorporates intermediate inputs and raw materials. The exponent of capital and labour for intermediate inputs may be below the levels of $\beta$, for instance $c$ and $d$ respectively ($c, d < \beta$) such that $A^\alpha = B^\beta K^c L^d$, turning the growth equation to $Y = B^\beta K^{\alpha+c} L^{1-a-\beta+d}$. Empirical investigations are required before identifying the most appropriate model.