Daily commuting

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Abstract

Workers generally commute on a daily basis, so we model commuting as a repeated game. The folk theorem implies that for sufficiently large discount factors, the repeated commuting game has as a Nash equilibrium any feasible strategy that is uniformly better than the minimax strategy payoff for a commuter in the one shot game, repeated over the infinite horizon. This includes the efficient equilibria. An example where the efficient payoffs strictly dominate the one shot Nash equilibrium payoffs is provided. Our conclusions pose a challenge to congestion pricing in that equilibrium selection could be at least as effective in improving welfare. We examine evidence from St. Louis to determine what equilibrium strategies are actually played in the repeated commuting game. JEL number: R41 Keywords: Repeated game; Nash equilibrium; Commuting; Folk theorem

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1 Introduction

1.1 Motivation and Related Literature

What happens to commuting behavior when a commute is repeated daily? Does behavior, namely route and departure time choice, differ dramatically from that observed in the simple context where the commuters know that they have to commute only once? One shot commuting is the major focus of the extant literature.

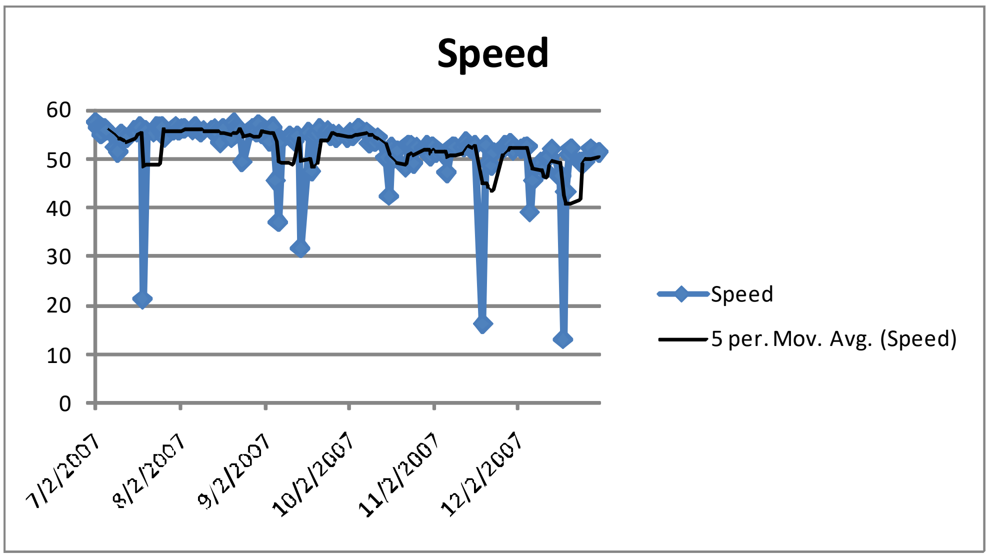

Figure 1: Evening rush hour (5-6 PM) I-64 westbound weekdays .3 miles west of Hampton Avenue

Empirical motivation for our work comes from Figure 1. The vertical axis represents speed in miles per hour, whereas the horizontal axis represents evening rush hour dates in the latter half of 2007. A major commuting highway in St. Louis was shut down on January 2, 2008. Why did rush hour traffic speed on the highway *decrease* during the last three months before closure relative to previous dates? We shall return to this in section 4 below. But first, we discuss the basic literature on the economics of commuting.

The commuting literature was initiated by Wardrop (1952). Beckmann et al (1956) introduced the canonical economic model of rush hour without time, where commuter delay is a function of the number of cars using a link. Vickrey (1963, 1969) analyzed congestion as an externality, Pigouvian taxes to correct for it, and optimal infrastructure. Arnott et al (1993) examined a commuting model with exogenous bottlenecks and Pigouvian taxes, whereas

Sandholm (2001) models road congestion as a potential game. Sandholm (2007) considers an evolutionary approach to setting optimal tolls in the case where commuters are identical (so they have the same home and work locations) using an evolutionary process to refine Nash equilibrium. Daniel et al (2009) implements the commuting model experimentally. de Palma et al (2013) provides nice surveys of most of the field. In general, this literature considers only one shot commuting, uses static models in the sense that behavior over time on a link is not modeled,[1] and employs a continuum of commuters for tractability reasons.[2] Some very interesting technical (and particularly measure-theoretic) issues with the model that has a continuum of commuters are addressed in Cerreia-Vioglio et al (2022). Our focus here is on more pragmatic issues.

Cominetti et al (2022) provide interesting theorems in the one shot, static model case on convergence of discrete commuter equilibrium to continuous commuter equilibrium. Ziemke et al (2020) examine analogous results using simulations in a dynamic model with queues.

The main difference between our work and most of the literature is that we address different questions. *That is, our primary purpose is to study the equilibria of the commuting game repeated daily rather than as a one shot game.* In fact, our framework applies to most any one shot commuting model that is used as the stage game.

The paper closest to ours is the innovative work of Scarsini and Tomala (2012). They examine a classical repeated congestion game with a finite number of commuters and a focus on monitoring and bounded rationality. They prove folk theorems in the static model using belief-free equilibrium when monitoring is imperfect and the computation capacity of commuters is limited. Our focus is on the more classical, applied context of games with a continuum of commuters and dynamics along links. We also explore Pigouvian taxes in our context. Monitoring and complexity are less important here. But there are more differences that we detail next.

We wish to emphasize two differences between the one shot static and

[1] That is, congestion cost is an exogenous function of the total measure of commuters using a link. We shall present a dynamic version of our model consistent with that used in the transportation engineering literature, for example Strub and Bayen (2006).

[2] We note here that there are inconsistencies in both terminology and notation among the related literatures, specifically transportation economics, congestion games, transportation engineering, and the mathematics of conservation laws. The lack of cross referencing should also be noted.

dynamic models that highlight the contrast between this work and the balance of the literature; we will incorporate the dynamic model in our results below. The static model has no time, as only route is a choice each day. The dynamic model has a strategic choice of both route and departure time each day, where arrival time is a consequence. We shall employ a reduced form approach to the one shot dynamic game, as this lightens the notational burden. *First, the common assumption in the literature, that congestion cost is increasing in the population using a particular strategy, will generally not be satisfied when both route and departure time are strategic choices of the commuters. Second, for the dynamic model, congestion on a route will be a function of all commuters' strategies (including departure times) rather than simply the number of people using a particular road.* These properties rule out application of the literature discussed here.

To illustrate these two points, an example is provided. To keep it simple, we do not discuss equilibrium behavior at this juncture; that will be discussed in Section 3.5. Consider a network with one origin node and one destination node for all commuters, with one link between them. So there is no route choice. Departure times are in $[0, 1]$, and the link has length 2. Speed is given by $1 + 1/f$, where f is density (cars per mile) at a point on the link. There is measure 2 commuters. Volume is given by speed multiplied by density, in this case $1 + f$. Consider the strategy profile where departure times are uniformly distributed over $[0, 1]$. Density at the origin departure is $f = 1$ whereas volume is 2 and speed is always 2, with the last commuter arriving at time 2. To illustrate the two points from the paragraph above, shift the departure times so that departures at times $[0, \frac{1}{2}]$ have density $\frac{3}{2}$ with speed $1\frac{2}{3}$, whereas departures at times $[\frac{1}{2}, 1]$ have density $\frac{1}{2}$ with initial speed 3. The commuters departing in $[\frac{1}{2}, 1]$ will catch up with the commuters departing in $[0, \frac{1}{2}]$, and provided there is no passing, will slow down to match the speed of the cohort that departs earlier. Thus, congestion cost is actually *decreasing* in the (naïve calculation using only) number of commuters departing in $[\frac{1}{2}, 1]$, and is actually a function of the departure time strategies of all commuters, including those departing in $[0, \frac{1}{2}]$.

The folk theorem itself is covered in most textbooks for first year graduate microeconomics; see for example Mas-Colell et al (1995, Chapter 12, Appendix A). The basic idea is that if a player is seen deviating from a prescribed equilibrium strategy, then punishment strategies are executed by all other players. In the case of the classic prisoners' dilemma, this could be the grim trigger strategy, defect forever, or a tit-for-tat strategy. If players are sufficiently patient, punishment strategies will deter defection. The main difference between the standard folk theorem setting and ours is the use of a continuum of agents; this alteration is addressed in Kaneko (1982).

1.2 Outline and Results

Our results and an outline of the balance of the paper are as follows. In Section 2, we give our notation and introduce the static and dynamic models of one shot commuting. In Section 3, we study Nash equilibria of the two models, static and dynamic, when they are repeated daily. The key to the application of the folk theorem is whether the history of strategy use by each agent is observable. By applying the folk theorem, we find that the set of equilibria is much larger than in the one shot game, and includes the Pareto efficient strategies. Even when the folk theorem does not apply, the repeated game structure yields many more equilibria than the one shot game structure studied in the literature. The anti-folk theorem applies when no agent's strategy history is observable by other commuters. In that case, only Nash equilibria of the one shot game are equilibria of the daily commuting game. Evidence relevant to repeated game strategies used by commuters in St. Louis is examined in Section 4. Finally, Section 5 gives a comparison with Pigouvian taxes and our conclusions.

The main takeaway from our analysis is that equilibrium selection, perhaps through flow control, is a viable alternative to Pigouvian taxes as a method for improving commuter welfare.

2 The One Shot Commuting Game

The details and extensive analysis of the ***one shot*** *commuting game, which is the stage game for the commuting game repeated daily, can be found in Berliant (2022).* Both the static game, where only a route is chosen by commuters, and the dynamic game, where both route and departure time are chosen by commuters, are analyzed there. Here will shall be brief, so that the focus can be on the new results derived from the repeated game. Again, other models can be substituted for the stage game.

The measure space of commuters is given by $([0,1], \mathcal{C}, \mu)$ where $[0,1]$ is the set of commuter types, $\mathcal{C}$ is the collection of Lebesgue measurable subsets of $[0,1]$, and μ is a positive measure absolutely continuous with respect to

Lebesgue measure on $[0,1]$. All references to measurability are to this measure space.

There is a finite set of *nodes*, denoted by $m, n = 1, 2, ...N$, and a finite set of links between nodes. The set of all nodes is denoted by $\mathcal{N} = \{1, 2, ..., N\}$. The capacity of a direct link between nodes m and n is given by $x_{mn} \in [0, \infty]$, where $x_{nn} = \infty$; if a direct link between nodes m and n does not exist, then $x_{mn} = 0$.

We assume that commuters have an inelastic demand for one trip per day to work.

2.1 The Static Commuting Game

To complete the game-theoretic structure, the strategies and payoffs of the commuters must be specified. In the static game, there is no choice of departure time; there is only route choice. Each commuter has a fixed origin node and a fixed destination node. There is a measurable *origin map* $O : [0,1] \to \mathcal{N}$ assigning a departure node to each commuter, and a measurable *destination map* $D : [0,1] \to \mathcal{N}$ assigning a destination node to each commuter.

Let π_k be the map that projects a vector onto its coordinate k. A *route*, denoted by r, is a vector of integer length $\ell \geq 2$. The set of all routes is denoted by $\mathfrak{R}$:

$$\mathfrak{R}^\ell = \left\{ r \in \mathcal{N}^\ell \mid \text{for } i = 1, 2, ..., \ell - 1;\ x_{\pi_i(r)\pi_{i+1}(r)} > 0 \right\}$$
$$\mathfrak{R} \equiv \bigcup_{\ell=2}^{\infty} \mathfrak{R}^\ell$$

We assume that there is some route between a pair of nodes if there is a positive measure of commuters with that origin and that destination. A *commuting route structure* or *strategy profile* is a pair (l, R) where l is a *commuting length map*, namely a measurable map $l : [0,1] \to \{2, 3, ...\}$, and R is a measurable map $R : [0,1] \to \mathfrak{R}$, such that for $i = 1, 2, ..., l(c) - 1$, $x_{\pi(i)\pi(i+1)} > 0$, and almost surely for $c \in [0,1]$, $\pi_1(R(c)) = O(c)$ and $\pi_{l(c)}(R(c)) = D(c)$.

Given a commuting route structure (l, R), its *usage* $f \in \mathbb{R}_+^{N^2}$ is given by:

$$f(m,n) = \mu(\{c \in [0,1] \mid \exists k \in \{1, 2, ..., l(c) - 1\} \text{ with } \pi_k(R(c)) = m \text{ and } \pi_{k+1}(R(c)) = n\}) \quad (1)$$

$$\text{for } m, n = 1, 2, ...N.$$

If the link is congested, then the travel time increases. For example, it could increase in proportion to the usage by commuters, $f(m,n)$. More specifically, if the number of commuters using the route doubles, then travel time on the link could double. We will use other examples below.

More generally, we can allow traffic to slow down according to any well-behaved function of the number of commuters on a link and link capacity. Therefore, we specify the function $\upsilon : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_{++}$ where $\upsilon(f, x)$ is the speed of traffic with usage f on a link with capacity x. We assume that for fixed x, υ is continuous and non-increasing in f. Define the length of the link (m, n) to be $\lambda(m, n) \in \mathbb{R}_+$.

Although it is difficult to discuss travel time in a static model that is inherently atemporal, the travel time is calculated in a sort of steady state. Under this interpretation, f is the measure of commuters (repeatedly) passing through the link on their route.

For reasons of tractability and consistency with the literature on commuting, we consider only pure strategies.

The *time cost of a commuting route structure* (l, R) *for commuter* c is

$$\theta(l, R, c) = \tag{2}$$

$$-\sum_{\{(m,n)\in\mathcal{N}\times\mathcal{N} | \pi_i(R(c))=m, \pi_{i+1}(R(c))=n \text{ for some } 0\leq i\leq l(c)-1\}} \frac{\lambda(m,n)}{\upsilon\left(f(m,n)), x_{mn}\right)}$$

Thus, θ is the objective or payoff function for each commuter.

A *Nash equilibrium of the static model* is a commuting structure (l, R) such that almost surely for $c \in [0, 1]$, there is no route r of length ℓ for commuter c such that

$$\theta(l, R, c) < -\sum_{\{(m,n)\in\mathcal{N}\times\mathcal{N} | \pi_i(r)=m, \pi_{i+1}(r)=n \text{ for some } 0\leq i\leq \ell-1\}} \frac{\lambda(m,n)}{\upsilon\left(f(m,n), x_{mn}\right)}$$

Rosenthal (1973), Sandholm (2001), and Konishi (2004) provide important results on existence and uniqueness of Nash equilibrium in this model.

A *strongly individually rational strategy profile of the stage game* is a commuting structure (l, R) such that there is $\epsilon > 0$ where for almost all $c \in [0, 1]$, there is a commuting structure (l', R') with $\theta(l', R', c) + \epsilon < \theta(l, R, c)$. This is a sufficient condition for existence of a punishment strategy in the repeated game.

Example 1: Please refer to Figure 2. Consider measure 4 commuters who must transit from home at node A to work at node M.

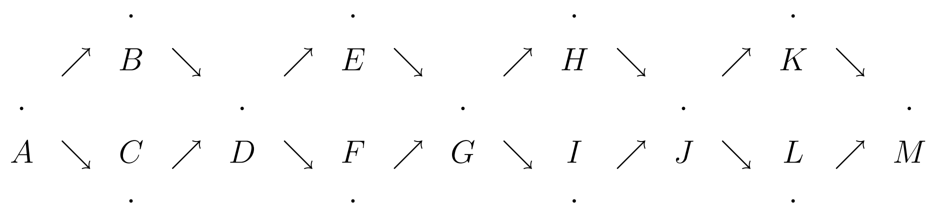

Figure 2: Nash Equilibrium that is not Pareto Optimal

Everyone must pass through nodes D, G, and J on the way from node A to node M. There are parallel routes in a series of four. The nodes B, C, E, F, H, I, K, and L appear simply to distinguish among the links and routes. For upper links ABD, DEG, GHJ, and JKM, the time a commuter spends on a link is given by f, where f is the measure of commuters using the link. For lower links ACD, DFG, GIJ, and JLM, the time a commuter spends on a link is given by 2, independent of the measure of commuters using the link. Although it is unnecessary, if the length of each link (for example ABD) is 1, speed can be computed by taking the reciprocals of the time on a link. Nash equilibrium has measure 2 using every link, for a total travel time of 8 independent of route. The identity of the users of any route is irrelevant. Turn next to a strict Pareto improvement. It will transfer measure 1 from each upper link to each lower link. In other words, measure 1 uses an upper link whereas measure 3 uses the corresponding lower link. However, the identity of the users matters. The commuters who use link ABD will only use lower links after travelling link ABD. Similarly, the commuters who use link DEG will only use lower links, including link ACD, for the remainder of their trip. In this way, use of upper links rotates among the commuters. The total travel time of each commuter is 7. This is not a Nash equilibrium, because each consumer would prefer to use upper links rather than the lower ones, for a total travel time of 4.

The classical Braess (1968) paradox provides another class of examples. That work shows that in a static model, adding new links to a network can cause Nash equilibrium travel time to increase. For our purposes, the opposite experiment works. If one begins with a network Nash equilibrium and then allows a planner to prohibit travel on some links, a Pareto improvement can be created. It is not a Nash equilibrium unless the prohibition is in place.

It is essential to mention here the important and interesting work of Milchtaich (2006) that considers the one shot static model exclusively. Our Example

1 is a modification of Milchtaich (2006, Figure 2(a)). Most important, Milchtaich (2006, Theorem 2) gives a characterization of network topologies such that every Nash equilibrium of the static one shot game is Pareto efficient.

2.2 The Dynamic Commuting Game

The dynamic model adds departure times to the static model. Departure times and routes are strategic choices of the commuters, whereas arrival times and arrival penalties are the consequences. In general, we base our model on Strub and Bayen (2006) and its successors in the various literatures. They model micro behavior on links as discontinuous differential equations.

To avoid notational overkill, we use a reduced form for the one shot dynamic model. A *commuting structure* is a triple (l, R, δ), where (l, R) is a commuting route structure and $\delta : [0, 1] \to [0, T]$ is a measurable map from the set of consumers to departure times, where $T > 0$ is fixed.

The *cost of a commuting structure* (l, R, δ) *for commuter* c is

$$\gamma(l, R, \delta, c; l(c), R(c), \delta(c)) < 0$$

The last three arguments give the route length, route and departure time actually used by commuter c a.s. This specification can include late arrival penalties, for example. Thus, γ is the objective or payoff function for each commuter.

A *Nash equilibrium of the dynamic model* is a commuting structure (l, R, δ) such that almost surely for $c \in [0, 1]$, there is no route r of length ℓ with departure time $d \in [0, T]$ for commuter c such that

$$\gamma(l, R, \delta, c; l(c), R(c), \delta(c)) < \gamma(l, R, \delta, c; \ell, r, d)$$

Berliant (2022) provides a more extensive literature review as well as examples in dynamic models where Nash equilibria are strictly Pareto dominated by strategy profiles that are not Nash equilibria. They have a flavor different from the examples in the static model, as they rely on mis-coordination of departure times (rather than route choice) in Nash equilibrium. Existence of Nash equilibrium for a specific model is proved there under some assumptions.

We make a couple of assumptions about this reduced form. First, if a commuter never arrives at work, then their payoff is $-\infty$. Second, if a positive measure (or atom) of commuters departs at time 0 along the same first link, then they never make progress along that link. This is summarized (more generally) as follows:

Assumption 1: If a commuting structure (l, R, δ) satisfies, for $(m, n) \in \mathcal{N} \times \mathcal{N}$ with $m \neq n$, $\mu(\{c \in [0,1] | \, \delta(c) = 0, \, \pi_1(R(c)) = m, \, \pi_2(R(c)) = n\}) > 0$, then $\gamma(l, R, \delta, c; l(c), R(c), \delta(c)) = -\infty$ for all $c \in [0,1]$ with $\delta(c) = 0$, $\pi_1(R(c)) = m$, and $\pi_2(R(c)) = n$.

3 The Repeated Commuting Game

3.1 Notation

The next step is to take the stage game detailed in the previous section and repeat it infinitely many times. Time is discrete and indexed by $t = 0, 1, 2, ...$, so the stage game is repeated a countable number of times. Strategy profiles (l, R) and induced usage f are now indexed by time, (l_t, R_t) and $f_t(m, n)$. Let $\rho \in (0, 1)$ be a discount factor common to all agents. A *strategy profile for the repeated game* is a sequence of commuting route structures $\{(l_t, R_t)\}_{t=0}^{\infty}$. Each such sequence induces a *usage sequence* $\{f_t(m, n)\}_{t=0}^{\infty}$, where at each time the usage is determined by equation (1). The *payoff* to an agent c for a sequence of strategy profiles $\{(l_t, R_t)\}_{t=0}^{\infty}$ is given by:

$$\Theta\left(\{(l_t, R_t)\}_{t=0}^{\infty}, c\right) \equiv \sum_{t=0}^{\infty} \rho^t \cdot \theta(l_t, R_t, c)$$

A *Nash equilibrium of the repeated game* is a strategy profile for the repeated game $\{(l_t, R_t)\}_{t=0}^{\infty}$ with induced usage $\{f_t(m, n)\}_{t=0}^{\infty}$ such that almost surely for $c \in [0, 1]$, there is no sequence of routes and route lengths $\{r_t, \ell_t\}_{t=0}^{\infty}$ for commuter c such that

$$\Theta\left(\{(l_t, R_t)\}_{t=0}^{\infty}, c\right) < -\sum_{t=0}^{\infty} \rho^t \cdot \left(\sum_{\{(m,n) \in \mathcal{N} \times \mathcal{N} | \pi_i(r_t) = m, \pi_{i+1}(r_t) = n \text{ for some } 0 \leq i \leq \ell_t - 1\}} \frac{\lambda(m, n)}{\upsilon\left(f_t(m, n), x_{mn}\right)} \right)$$

A *strategy profile for the repeated dynamic game* is a sequence of commuting structures $\{(l_t, R_t, \delta_t)\}_{t=0}^{\infty}$. The *payoff* to an agent c for a sequence of strategy profiles $\{(l_t, R_t, \delta_t)\}_{t=0}^{\infty}$ is given by:

$$\Gamma\left(\{(l_t, R_t, \delta_t)\}_{t=0}^{\infty}, c\right) \equiv \sum_{t=0}^{\infty} \rho^t \cdot \gamma(l_t, R_t, \delta_t, c; l_t(c), R_t(c), \delta_t(c))$$

A *Nash equilibrium of the dynamic repeated game* is a strategy profile for the dynamic repeated game $\{(l_t, R_t, \delta_t)\}_{t=0}^{\infty}$ such that almost surely for $c \in [0, 1]$, there is no sequence of routes, route lengths, and departure times $\{\ell_t, r_t, d_t\}_{t=0}^{\infty}$ for commuter c such that

$$\Gamma\left(\{(l_t, R_t, \delta_t)\}_{t=0}^{\infty}, c\right) < \sum_{t=0}^{\infty} \rho^t \cdot \gamma(l_t, R_t, \delta_t, c; \ell_t, r_t, d_t)$$

3.2 The Commuting Folk Theorem and the Commuting Anti-Folk Theorem

In repeated games with a continuum of players, the commuting game is a very nice special case. There are two important theories of equilibrium behavior, both quite famous, namely the *Folk Theorem* and the *Anti-Folk Theorem.* The conclusions of the two theorems are in a sense opposites. Loosely speaking, the first says that any individually rational, feasible payoff vectors achievable in the one shot game can be supported by a Nash equilibrium of the repeated game. Included in this set are the one shot Pareto efficient payoffs. The second theorem says that only Nash equilibria of the one shot game are equilibria at each stage of the repeated game. The critical issue in the determining which theorem applies is what players can observe about the strategies used by other players in past plays of the stage game. The formalities can get technical; see Kaneko (1982), Massó and Rosenthal (1989), and Massó (1993).

The crux of the matter is this: Fixing one particular individual, after finitely many plays of the stage game, can all players observe that individual player's past behavior? If so, then the folk theorem applies. If *no* individual's behavior can be detected by a set of players of positive measure, then the anti-folk theorem applies. Note that these two cases are not exhaustive. In the end, which theorem might apply is an empirical matter. There is some evidence that, in settings other than commuting, the folk theorem is relevant; see, for instance, Lee (1999).

3.3 The Repeated Static Commuting Game

Theorem 1 (Folk Theorem): Let (l, R) be a strongly individually rational strategy profile of the static model stage game. In the repeated static game, if for almost any commuter c, almost all commuters can observe the past strategy choices of commuter c, then for some $\rho \in (0,1)$, $\{(l,R),(l,R),...\}$ is a Nash equilibrium of the repeated game.[3]

[3]In fact, Kaneko (1982) is overly modest in the statement of this theorem. If it holds for some $\rho \in (0,1)$, it also holds for all discount factors greater than ρ but less than 1.

Proof: Follows directly from Kaneko (1982, Proposition 2.1') applied to this special case.

Theorem 2 (Anti-Folk Theorem): Assume that in the repeated static game, no commuter can observe any other commuter's past strategy choices. Then $\{(l_t, R_t)\}_{t=0}^{\infty}$ is a Nash equilibrium of the repeated game if and only if for every $t = 0, 1, 2, ...,$ (l_t, R_t) is a Nash equilibrium of the stage game.

Proof: Follows directly from Kaneko (1982, Proposition 2.3') applied to this special case.

Remarks: Please note that Kaneko (1982) provides analogous results in the case $\rho = 1$ that could be applied here. Kaneko (1982, Proposition 2.1") proves The Perfect Folk Theorem, where we can restrict even to subgame perfect Nash equilibrium and obtain similar results.

With a finite number of strategies in the stage game (routes and possibly departure times), *it is not far-fetched to think that any particular individual's strategy is observable by those who use the same departure node.* At this point, it is useful to take versions of strategies such that if a set of measure zero plays a particular strategy, then no commuter plays it. Since those commuters with the same route will be of positive measure, they can institute a punishment strategy that will be noticed by other players, who in turn can punish. Thus, the game becomes one of perfect observability. Next, let us focus on the *implications* of the two theorems for daily commuting.

We begin by assuming perfect observability and apply the folk theorem. Then there is a huge variety of equilibria. The equilibrium strategy profiles are supported by various punishment strategies that apply if the prescribed equilibrium strategy is not followed by a player. Thus, the one shot equilibrium is just one of many. Moreover, on the equilibrium path, one only observes the prescribed equilibrium strategies, not the punishments. *Thus, one expects to see the one shot equilibrium played, perhaps, but also (for example) the efficient strategies.*

Example 1 is applicable here. In that example, there is a Pareto improvement over Nash equilibrium that will not be a Nash equilibrium for the one shot game. However, it can be supported as a (subgame perfect) Nash equilibrium in the repeated game. Standard strategies that support this are the threat of Nash reversion. As we have described, the Braess paradox gives further examples of Pareto improvements over one shot Nash equilibrium that can be supported in repeated games.

3.4 The Repeated Dynamic Commuting Game

Next, consider informally the repeated dynamic commuting game. In this game, there is a daily choice of both commuter route and departure time. What payoffs are attainable? We shall apply a folk theorem, so the set of payoffs attainable as Nash equilibria in the repeated game is related to the payoffs attainable in the one shot game. Specifically, for large enough discount factors in the repeated game, all feasible payoffs at least ϵ greater than the maximin payoff vector for the one shot game (that is not necessarily a Nash equilibrium of the one shot game) are attainable as Nash equilibrium payoffs of the repeated game. In fact, we can show that any payoff that is feasible in the one shot game can be attained as a Nash equilibrium of the repeated game. This result is achieved by simply computing the maximin payoff of the one shot game. It will be $-\infty$. Why? Consider one individual. The worst case scenario for that individual in the one shot commuting game is that everyone else who lives at the same node "blockades" them at time zero.[4] That is, the strategy used by everyone else who lives at that commuter's home node is to depart at time 0 along the same link as the deviating commuter, whoever it is and whatever first link that may be. Then local congestion is infinite, so nobody ever reaches the destination or even moves at all, independent of what the commuter in question does (namely, what departure time and route strategy they follow).[5] Time to destination is infinite. So any feasible route and departure time strategy for the one shot game where the utility of the prescribed strategy profile in the one shot game is above $-\infty$ can be achieved in the infinitely repeated game with some discount factor (sufficiently close to 1).

Of course, if no individual's behavior is observable, then the anti-folk theorem applies to both the static and dynamic models, so the only Nash equilibria of the repeated game are the Nash equilibria of the one shot game.

Theorem 3 (Folk Theorem): Let (l, R, δ) be a strategy profile of the dynamic stage game where $\gamma(l, R, \delta, c; l(c), R(c), \delta(c)) > -\infty$ a.s. Suppose that Assumption 1 holds. In the repeated game, if for almost any commuter c, almost all commuters can observe the past strategy choices of commuter c, then for some $\rho \in (0, 1)$, $\{(l, R, \delta), (l, R, \delta), ...\}$ is a Nash equilibrium of the repeated dynamic game.

[4] Here we are assuming that positive measure lives at each departure node. If not, we can ignore the commuters who live at that node for the purpose of this argument.

[5] The implicit assumption here is that at infinite density, speed is zero.

Proof: Follows directly from Kaneko (1982, Proposition 2.1') applied to this special case.

Theorem 4 (Anti-Folk Theorem): Assume that in the repeated dynamic game, no commuter can observe any other commuter's past strategy choices. Then $\{(l_t, R_t, \delta_t)\}_{t=0}^{\infty}$ is a Nash equilibrium of the repeated dynamic game if and only if for every $t = 0, 1, 2, \ldots$, (l_t, R_t, δ_t) is a Nash equilibrium of the stage game.

Proof: Follows directly from Kaneko (1982, Proposition 2.3') applied to this special case.

3.5 Finite Commuters vs. Continuum of Commuters: The Snowball Effect

Here we consider the relevance of models with a continuum of commuters, such as the one we have used. Of course, they are only relevant in the case that their equilibria are mathematically convenient approximations to the equilibria of models with a large but finite number of commuters. See Cominetti et al (2022) for interesting convergence theorems in the one shot case. Our focus, of course, is on the repeated commuting game.

With a finite number of commuters, the anti-folk theorem becomes irrelevant, as the folk theorem applies because there is generally no issue of observability of strategies. With a continuum of commuters without observability of strategies, the anti-folk theorem applies. Due to this apparent discontinuity in the set of equilibria as the number of commuters tends to infinity, it is imperative to examine the continuity properties of the Nash equilibrium correspondence.[6]

Let us put aside the static commuting game. Given the discussion of the previous subsection, we consider two cases in the context of the repeated dynamic commuting game: when individual strategies are observable and when individual strategies are unobservable.

When individual strategies are observable, for example by commuters departing from the same node using the same route while departing at the same time, the commuting folk theorem applies to both the model with a finite number of commuters and a continuum of commuters. Thus, there is no issue of a discontinuity as the number of commuters tends to infinity.

[6] In particular, the Nash equilibrium correspondence is not upper hemi-continuous.

When individual strategies are not observable, there is the potential for such a discontinuity. The set of equilibria can contract, say in the game with no discounting, from the set of individually rational, feasible strategies to the set of one shot Nash equilibria. In the model with a continuum of commuters, when an individual commuter changes their strategy, there will be no change in what is observed by other agents, say their commuting time, so there is no basis on which to punish deviators. Thus, the anti-folk theorem applies. But now consider the model with a finite number of commuters. Even if the number of commuters is large, deviations from a prescribed along-the-equilibrium-path-strategy can be detected (for instance by commuters on the same route using the same departure time on the equilibrium path since their commuting time changes) and therefore can be punished. This explains the contraction of the equilibrium set. However, one can easily argue that *as the number of agents gets large, these individual commuter deviations become undetectable, as their effects are small* and indistinguishable from noise. For example, an analog would be to assume perfect competition in the context of a finite number of agents, where the error from this assumption is small for large economies.

If this were true, then there would be no substantial error in simply using the limit commuting game with a continuum of commuters without observability. *The big problem here is that the effect of one commuter deviations in large but finite commuting games are not small.* To see this, consider the simple example from the introduction with 2 nodes, home and work, and 1 link of length 2. Variants of this example are described more formally in Berliant (2022). Everyone commutes once a day between home and work. If density (cars per mile) at a time and place on a link is f, then speed is $1+1/f$ whereas volume is speed multiplied by density, or $1 + f$. There is measure 2 commuters. Set arrival time to 2; if consumers arrive late, the penalty is large. Consider the Nash equilibrium where departures are uniform on times $[0, 1]$, volume is 2, and density is $f = 1$. So speed is 2 and the last commuter arrives at work at time 2. If a single commuter deviates from this strategy, it is undetectable.

Next, instead of using a continuum of commuters, consider a large but finite number. In the case of a large but finite number of commuters, it's natural to think of a (fine) grid of a finite number of evenly spaced departure times in $[0, 1]$. Suppose that pure commuter strategies are uniformly distributed over departure times so that departure density is approximately 1. Speed and volume are 2. Suppose that a commuter changes their strategy from the second departure time in the grid to the first, reducing density and volume at the second departure time and increasing density and volume at the first departure time. This will slow down the first cohort. The second cohort will quickly catch up, slowing down both cohorts. The third cohort will catch up to the first two, and so forth. This "snowball effect" will not only be detectable (even if individual strategies aren't), but it also substantially changes the behavior of the entire system due to one commuter's deviation. Such a "snowball effect" is simply not possible in the commuting game with a continuum of commuters.

It is logical to inquire next whether this effect disappears as the number of commuters tends to infinity. The issue here, as in classical urban economics,[7] is how one takes limits as the number of commuters tends to infinity. If the number of commuters is simply increased whereas the road capacities remain constant, some densities tend to infinity and some speeds tend to zero, so the system halts. Allowing road capacity to tend to infinity seems unrealistic. The last possibility, that seems implicit in urban transportation models generally, is that one commuter in the finite model is represented by a continuum of identical commuters of positive measure (say 1) in the continuum model. In that case, deviation by a set of measure zero of commuters does not make economic sense (though it does make mathematical sense), as it does not correspond to any type of behavior in the model with a finite number of commuters. Under this interpretation, deviations can only occur for coalitions of commuters of measure 1, and we are back to the snowball effect. With this interpretation of the repeated dynamic model using a continuum of commuters, neither the snowball effect nor the folk theorem should be a surprise.

Next, we put aside the issue of economic interpretation of the model with a continuum of commuters, and focus on the mathematical relationship between that model and models with a finite number of commuters that are nearby, namely models with a large but finite number of commuters.

Consider first the model with a continuum of commuters. In the simple one route example, of course any commuter who observes a defection can punish. However, with many routes, this might not be possible. An alternative assumption is that any defection causes a snowball effect, in that a positive measure of commuters is affected. Then it is assumed that if a positive measure of commuters is affected, this is observable to all and deviators can be punished. In other words, if a commuter observes anything unexpected on the

[7] See Fujita (2020) for a survey of the relevant literature in urban economics.

way to work, they play a grim trigger strategy. Everyone sees that a positive measure is instituting this strategy (since they observe something unusual), so the grim trigger strategy is used by everyone. In terms of commuter memory, the grim trigger or tit-for-tat strategy requires only that the change in congestion caused by yesterday's strategy profile be remembered today, not that the entire history of strategies be remembered. Adding noise to the system, for example weather or accidents, involves setting a tolerance in terms of number of periods where deviation is observed before the grim trigger strategy is instituted. This requires more memory.

The problem with this idea is that there is literally no snowball effect with a continuum of commuters, but only with a large but finite (or countable) number. In fact, this is the reason there is a discontinuity of the Nash equilibrium correspondence in the limit as the number of commuters goes to infinity. A sufficient condition for a snowball effect in large but finite games close to the game with a continuum of commuters of interest is: speed is strictly decreasing in density. Under this condition, whenever a commuter deviates, there is a snowball effect of some kind; this is detected and punished by everyone, for example by using a blockade in the next period.

In summary, our conclusion is that although the snowball effect is not present in commuting games with a continuum of commuters, it is present in the large but finite games nearby. Hence, if we think that there are only a finite number of commuters in the world, *the only equilibria we should consider in the limit games with a continuum of commuters are those that are limits of equilibria in models with a finite number of commuters*. Thus, the only relevant equilibria in the limit games with a continuum of commuters are those with observability, and therefore the folk theorem applies. So it makes sense to say that the *consequence* of any individual deviation from a prescribed strategy is observable, and thus the folk theorem is applicable in the repeated dynamic game with a continuum of commuters.

Therefore, be it from observations of neighboring commuters or the snowball effect, the folk theorem in the model with a continuum of commuters seems relevant.

3.6 Limitations of the Analysis

One might argue that commuting is not a good context for repeated game punishment strategies. However, evolutionary foundations of the folk theorem studied in Vasin (1999, 2006) show how these many equilibria can be obtained as globally stable outcomes of various game-theoretic dynamical systems, including replicator dynamics and selection dynamics, justifying our interest. Beyond that, the grim trigger strategies generally used to support equilibria in the repeated game are not terribly complicated. The basic idea is that commuters play, say, a Pareto optimal strategy every period until they observe something they didn't expect on their drive to work, at which time everyone institutes a punishment strategy. It begins the next period and continues forever. On the equilibrium path, use of the punishment strategy is never actually observed. This punishment strategy will be subgame perfect (for sufficiently large discount rates), and thus credible. Stochastic elements, such as random weather or accidents, can easily be added to the model. Such elements are common in the folk theorem literature; see, for example, Fudenberg and Yamamoto (2011).

A messy alternative to our framework would employ a finite number of commuters, as in Scarsini and Tomala (2012). The drawbacks of this approach are tractability and consistency with the balance of the literature on commuting. In an important paper, Rosenthal (1973) shows under certain conditions[8] that the static, one shot commuting game with a finite number of players has a Nash equilibrium in pure strategies.[9] Nevertheless, as is well-known in the folk theorem literature, for discount factors sufficiently close to 1, *Nash equilibrium in the repeated game will generally exist even if Nash equilibrium in the one shot game does not.* So models with a finite number of commuters, although rarely employed in the literature, are a way forward.

4 Evidence

In this subsection, we examine preliminary evidence, in the context of the repeated commuting game, that can tell us whether commuters are playing a one shot Nash equilibrium in all periods, or whether other strategies, possibly more efficient, are used. The idea for the analysis is similar to that used in Lee (1999), but now in the context of commuting.

Consider a repeated game with a termination date that is finite and known to the players. In general, it is expected that only one shot Nash equilibrium will be played every period, since backward induction leads to the unravelling of other possible equilibrium strategies.

[8] In particular, congestion is strictly increasing in the number of cars using a link.

[9] In fact, to my knowledge, this is the first appearance of a potential game in the literature.

However, as described in Lee (1999, p. 123), there are various theories involving small changes in the classical repeated game model that lead to a kind of folk theorem in finitely repeated games. This is exploited by the empirical work in the field.[10] Next we proceed to try to determine which equilibrium strategy is reflected in commuting data.

If for example the players are myopic and playing one shot Nash equilibrium, then it is expected that behavior will not change as the repeated game termination date approaches. If the players are using strategies other than one shot Nash, for example they are participating in some tacit collusion as the folk theorem might predict, then one expects to see such collusive strategies played when the termination date of the game is not near, but reversion to one shot Nash equilibrium close to the termination date.

How does this work in the context of the repeated commuting game? On January 2, 2008, reconstruction was begun on I-64 (state route 40), a major east-west commuting corridor in St. Louis. A portion was completely shut down. Parts were reopened a year later, though other (adjacent) parts were shut down at that time.[11] We take this to be the termination of a repeated, daily commuting game. This closure was announced years in advance, so it was not a shock to commuters. We examine rush hour traffic speed and volume for locations that were closed in early 2008.

If commuters were playing one shot Nash equilibrium strategies, one would expect to see the same rush hour traffic speed daily until close to the closure. Near the time of the closure, traffic volume would drop off and speed would increase as commuters explored alternate routes to be used after closure.

Long before the highway shutdown, commuters could be playing a strategy other than one shot Nash, for example Pareto dominant over one shot Nash, enforced by the threat of Nash reversion. The Pareto dominant strategy profile has more commuters using alternate routes or times of departure compared to the Nash equilibrium strategy profile. By alternate we mean using a road other than I-64 or a time other than rush hour. This is similar to Example 1. Then one would expect to see high traffic speed when the closure is not imminent, followed by lower traffic speed as the closure date approaches and one shot Nash is played,[12] followed by an increase in speed near the closure

[10] If the folk theorem only applied to infinitely repeated games, those wishing to determine which strategies are played in equilibrium would be waiting a long time for all of the data.

[11] The entire highway was reopened on December 7, 2009.

[12] The exact timing depends on both the model used for the folk theorem in the finitely repeated game and the discount factor.

date due to commuters exploring alternate routes.

Thus, it is detection of this counterintuitive decrease in traffic speed as the closure date approaches that can distinguish among the equilibria of the system.

Before presenting the data, it is useful to recall the fundamental identity of traffic, namely: Traffic volume is equal to speed times density. We have obtained data on volume and speed, so density can be calculated. But there are two important points to be made. First, volume is not terribly informative on its own in general, as there can be two equilibria with the same volume, one with low speed and high density, the other with low density and high speed. Second, the externality actually perceived by commuters is in speed, so we focus on that.

We have obtained data from two sensor locations, one toward the east end (closer to the downtown area) of the closure, the other at the west end.[13] Let's examine the east location first, studying evening then morning rush hour. The figures graph average traffic speed and total volume in the hour by date. We have deleted weekends, but we have not deleted holidays that fall on weekdays.

To be more precise about the data, it consists of evidence from two sensors for every hour of calendar year 2007. Eastbound (toward downtown) and westbound (away from downtown) traffic statistics are collected separately. For each date, hour, and direction of travel, each sensor gives speed, volume, and several other readings (such as vehicle class counts) that are not relevant to our analysis. What is crucial in selection of this particular data is the shutdown of a major commuting highway where the alternatives are more costly. Commuting diaries of individuals would be desirable, but we don't have these.[14] For example, schedule delay costs are important, but we can't address them with our data.

[13]The author was offered more data than the one calendar year at two sensors actually provided, but at the cost of relinquishing rights to all future work (whether related to this project or not), as well as other considerations.

[14]It is also important to note that monitoring technology is much more advanced now than it was in 2007.

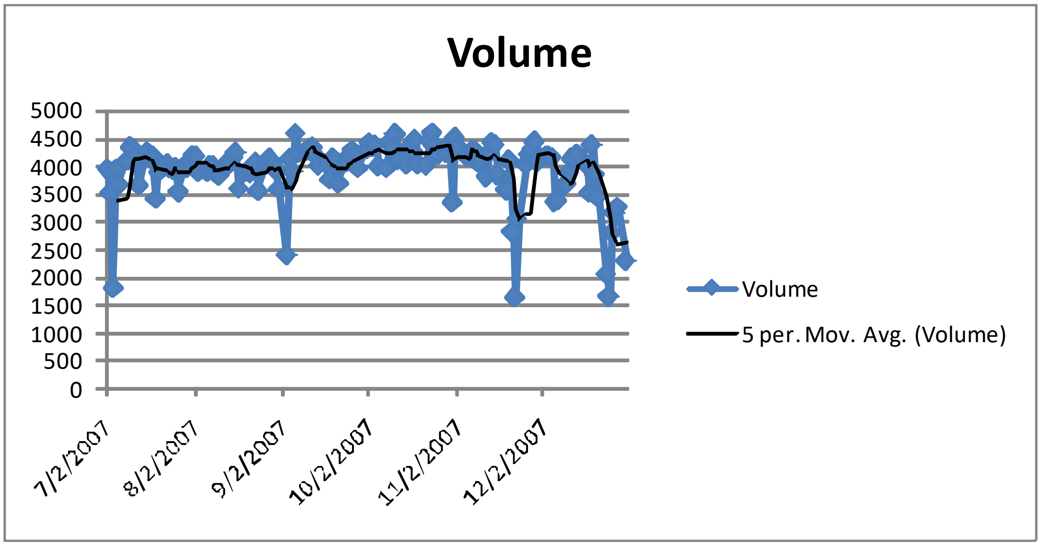

Figure 3: Evening rush hour (5-6 PM) I-64 westbound weekdays .3 miles west of Hampton Avenue

Notice that in early October, there is a decrease in speed and an attendant increase in volume, as seen in Figures 1 and 3. The outliers in the data are obviously accidents, weather issues, and holidays. For morning rush hour, as seen in Figure 4, there is a similar effect, though not as large in magnitude and with speed increasing over the Thanksgiving holiday.

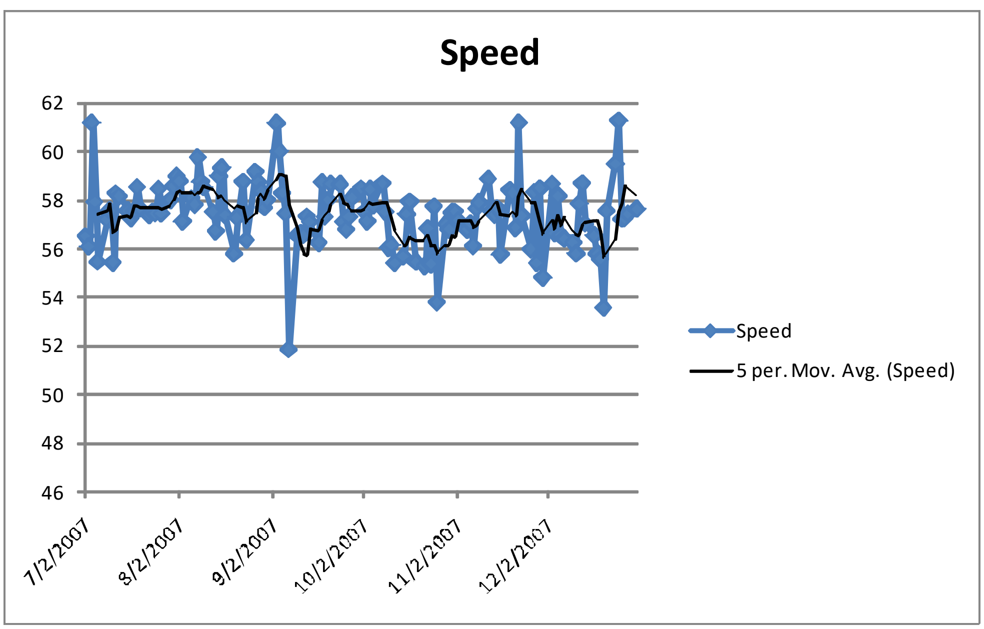

Figure 4: Morning rush hour (8-9 AM) I-64 eastbound weekdays .3 miles west of Hampton Avenue

Evening rush hour for the west sensor is displayed in Figure 5. In general, for the west sensor (more distant from the central business district), traffic moves at the speed limit. We conjecture that this is due to the fact that traffic in this area is not congested enough to cause speeds to drop below the speed limit during rush hours.

Many questions about the data arise at this point. Does weather cause traffic to slow down in the fall? There are two responses. In the author's experience, most of the inclement driving weather occurs in St. Louis during the time period from mid-December to mid-February. Moreover, the sensors at the west end of the shut down, that show no decrease in speed, serve as a nice control for weather, as St. Louis is very flat and thus weather seems to be common to most of the area. Can holiday shopping can account for the increased traffic? Most stores used for shopping are now located in malls well outside the city, along with most of the area's population. Does an increasing accident rate in the fall cause the decrease in speed rather than the theory we have put forth? If this were the case, we would observe a *decrease* in volume accompanying the decrease in speed. Instead, we observe an increase in volume in the data. Could the effect we observe be due to intertemporal substitution between commuting with a car and commuting with mass transit, where commuters take advantage of the expressway when it is open? Conceptually, this would depend on the elasticity of substitution between commuting mode choices. This elasticity of substitution has been found to be quite low; see, for example, Chung (1979). Is the change in commuting speed seasonal? In theory, one could look at commuting in 2006. However, a low probability event occurred that year that disrupted commuting and corrupted data throughout the fall - the Cardinals (unexpectedly) won the World Series. The game changed when this occurred. The change might have been unexpected during the summer, but it was anticipated on a daily or weekly basis, unlike accidents for instance. For example, during the playoffs when home games were played, there was more traffic headed downtown during evening rush hour than headed away from downtown. Some commuters stayed downtown in the evening. So the pattern of origins and destinations as well as the pattern of commutes was disrupted.

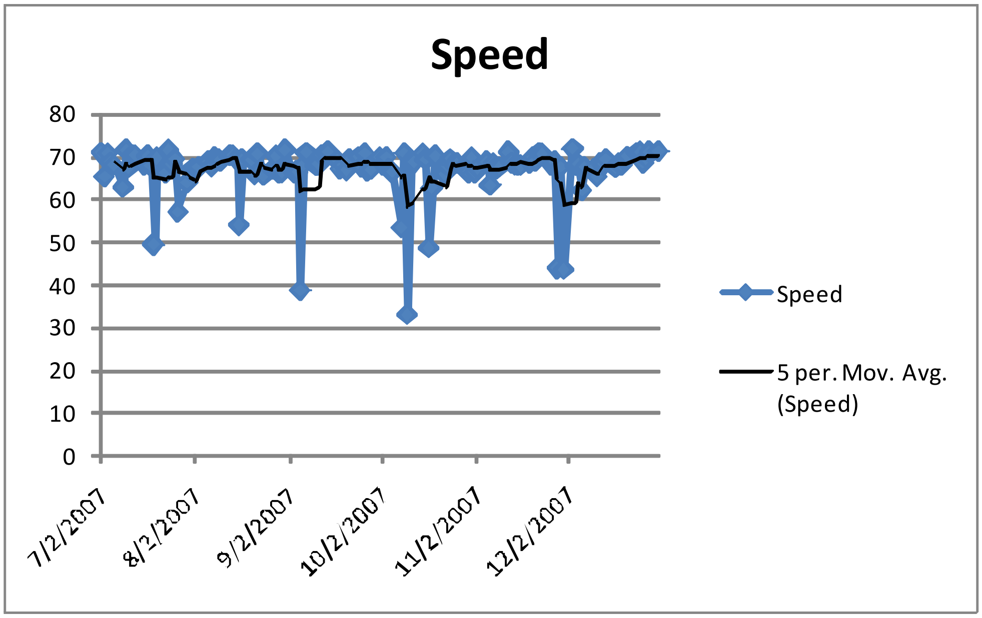

Figure 5: Evening rush hour (5-6 PM) I-64 westbound weekdays .7 miles west of Brentwood Boulevard

In summary, there is some evidence that commuters are not playing one shot Nash equilibrium. They revert to one shot Nash equilibrium strategies at around 2 1/2 to 3 months from the end of the game.

5 Conclusions

We have examined the set of Nash equilibria in the infinitely repeated versions of commuting games, using the folk theorem to obtain these large sets. Why have the additional equilibria from the repeated game, *including efficient equilibria*, been ignored by the literature? Are daily commuting equilibria inefficient due to the congestion externality? We have presented some preliminary evidence from the shutdown of an expressway in St. Louis that commuters do not always play one shot Nash equilibrium. We have also discussed the application of the anti-folk theorem to our specific game, namely conditions under which the Nash equilibria of the infinitely repeated game are only the Nash equilibria of the one shot game.

The commuting folk theorem poses a direct challenge to congestion pricing. If commuters are already playing equilibrium strategies that are efficient without tolls, congestion pricing can mess this up. If the commuters are playing efficient strategies, then the introduction of congestion pricing can jolt the system to a new repeated game equilibrium, for example one that is Pareto dominated. In order to make this idea formal, congestion pricing would have to be added to the model, likely as a penalty additive in the utility function as in Sandholm (2007). Then a folk theorem would be applied to this extended model. The equilibria of the models with and without congestion pricing could be compared. It is expected that all individually rational, feasible payoffs would be equilibria of the repeated games under a sufficiently high discount factor. The additional notation and complexity does not seem worth the trouble. However, an example is in order.

Example 1 (continued): Suppose that the commuters are commuting happily each day using Pareto optimal strategies (with total travel time of 7) supported by (say) the threat of Nash reversion if they deviate. Suddenly, one day, they experience a Pigouvian congestion tax, namely marginal damages at the optimum. Suppose that the utility function is additive in money, implying that toll revenue can be redistributed back to commuters without distortion. The Pigouvian tax is specified as follows. It is 1 for all of the upper links, such as ABD, and 0 for all the lower links, such as ACD. The Pareto optimum itself is a Nash equilibrium of the repeated game with Pigouvian taxes, but there are others. For example, consider a strategy profile where a set of measure 2 commuters uses each of the first two upper links and the last two lower links, whereas the other commuters use the the first two lower links and the last two upper links. The Pigouvian tax given above is in place. Then the total travel time of each commuter is 8 and the tax paid is 2, for a gross utility of -10. The total tax revenue is 8. If the tax revenue is rebated uniformly, that yields 2 for each commuter, so their net utility is -8. This is Pareto dominated by the efficient strategy profile without Pigouvian taxes, having total travel time 7 and no tax revenue, that is supported as a repeated game Nash equilibrium.

The bottom line is that equilibrium selection, perhaps through flow control, can be as effective as Pigouvian congestion taxes. Moreover, Pigouvian congestion taxes can disturb an equilibrium that is already efficient.

The folk theorem and anti-folk theorem can also be applied to repeated game versions of other one shot models in the literature, such as Arnott et al. (1993). For the bottleneck type of model, the punishment strategy of interest involves everyone arriving at the bottleneck simultaneously, at the earliest possible time. It would be very interesting to explore experimental complements to our theory and data; for example, see Daniel et al (2009). Which equilibrium of the repeated game is selected in the laboratory?

Future development includes examining the repeated game model with myopic commuters, specifically with discount factors above zero but below one. The set of Nash equilibria will include the equilibria from the one shot game, but not as many as in the repeated commuting game with a discount factor close to 1.

The repeated commuting model should be applied to real world commuting. For example, it can be used to perform cost benefit analysis with respect to changing road networks.

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