Why is technical change purely labor-augmenting and skill biased in the 20th century?

Li, defu and Bental, Benjamin

School of Economics and Management, Tongji University, Department of Economics, University of Haifa

3 November 2023

Why is technical change purely labor-augmenting and skill biased in the 20th century? [*]

Defu Li[1]
School of Economics and Management, Tongji University
Benjamin Bental[2]
Department of Economics, University of Haifa.

Abstract: The history of modern economic growth indicates that technical change is not only purely labor-augmenting, but also skill biased the 20th century. Although there are papers that have separately analyzed why technical change be purely labor-augmenting or skill biased, there is no paper analyzing why it may be both labor-augmenting and skill biased. This article develops a growth model with endogenous direction and bias of technical change, in which capital accumulation process considers investment adjustment costs, and firms can undertake capital-, skill labor- and unskilled labor-augmenting technological improvements. In the steady-state equilibrium, technical change can include all of them. However, according to the results of the model, when there is no investment adjustment cost (implying capital supply with infinite elasticity), in steady state, technical change will be purely labor-augmenting. If market size effect dominated prices effect in innovation and the relative supply of skilled labor increase continuously, then technological progress will be purely labor-augmenting and skill biased, which result that the skill premium continue to rise, while the economic growth first increases and then decreases in an inverted U-shaped change, and the labor share of income first decreases and then increases in a U-shaped change.

Key words: Endogenous direction of technical change, Skilled Labor Supply, Skill Premium, Economic Growth, Labor Share, U-shaped change, Inverted U-shaped change

JEL: E13; E25; J21; J31; O15; O33; O41;

First draft, any comments would be appreciated.

---

[*] Li gratefully acknowledges support from the National Natural Science Foundation of China (NSFC:71773083; 72273096). All errors are our own.

[1] School of Economics and Management, Tongji University, 1500 Siping Road, Shanghai, P.R. China, tjldf@tongji.edu.cn.

[2] Department of Economics, University of Haifa, 199 Aba Khoushy Ave., Mount Carmel, Haifa, Israel, bbental@econ.haifa.ac.il.

1. Introduction

More than twenty years ago, Acemoglu (2002) pointed out that “there is now a large and influential literature on the determinants of the aggregate technical progress (see, among others, Romer (1990), Segerstrom, Anant and Dinopoulos (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Young (1993)). This literature does not address questions related to the ***direction*** and ***bias*** of technical change.” After two decades, although many authors have separately analyzed the ***direction*** (Acemoglu, 2003; Irmen, 2017; Irmen et al., 2017; Li and Bental, 2022; Casey and Horii, 2022) or the ***bias*** of technical change (Acemoglu,1998,2002; Kiley, 1999; Lloyd-Ellis, 1999; Galor and Maov, 2000), there is still no one analyzing both the direction and the bias in a single framework, and even the two concepts are still unclear. However, the historical facts of economic growth indicate that technological progress has generally been (even purely) labor-augmenting in long run (Kaldor, 1961), but it appears to have been unskill biased (skill replacing) during the late eighteenth and early nineteenth centuries (e.g. James and Skinner, 1985; Goldin and Katz, 1998; Mokyr, 1990, p. 137), while it has mainly been skill biased since the 20th century. For example, Figure 1 shows the labor share of income and per capita private fixed assets in the United States from 1947 to 2022. Figure 2 shows the relative supply of skilled labor and the skill premiums in the United States over the past 50 years from 1963 to 2012.

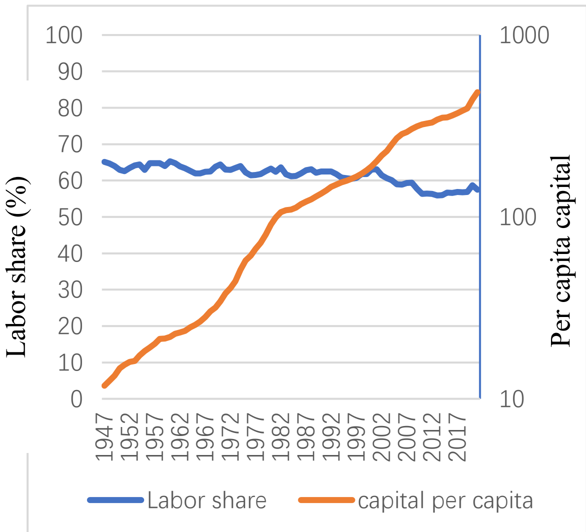

Figure 1: Capital Deepening and Labor Share Changes in the USA

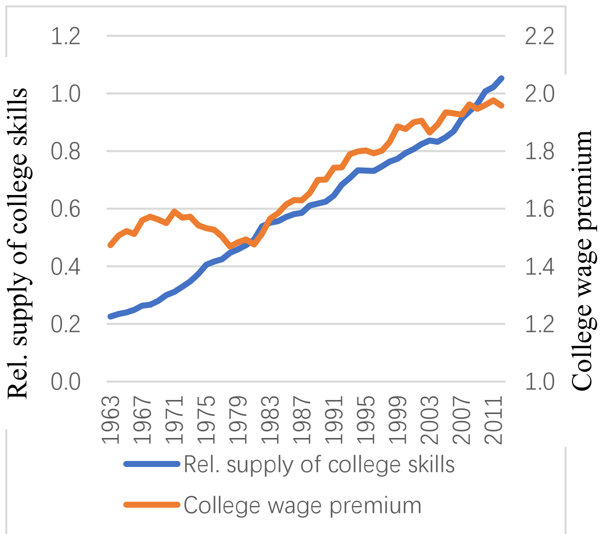

Figure 2: The relative supply of skilled labor and skill premium in the USA

Figures 1 and 2 indicate that, on the one hand, consistent with the stylized facts of modern economic growth summarized by Kaldor (1961), although per capita capital has increased steadily, the labor share has been approximately constant; On the other hand, although the relative supply of skilled labor continues to rise, the premium for skilled labor also continues to rise. Therefore, technical change is not only purely labor-augmenting, but also skill biased.[1] Why has the technical change been not only purely labor-augmenting but also skill biased in recent decades? What are the characteristics of economic growth and income distribution when technological progress is pure labor augmentation and skill biased?

To answer these questions, firstly, it requires to clearly distinguish the direction and the bias of technical change. For example, suppose that the aggregate production function is $Y = F(BK, AL)$, the direction of technical change refers to the ratio of the rates of factor-augmenting technical change, namely ($\frac{\dot{A}/A}{\dot{B}/B}$), and the bias of technical change refers to the impact of the change of relative technology on the relative marginal product of the two factors, namely $\frac{\partial(MP_K/MP_L)}{\partial(B/A)}$.[2] The determinants and applications of the two variables are different. The classic question in economic growth theory why technical change is mainly (even purely) labor-augmenting in long run requires a framework where the ***equilibrium direction*** of technical change can be studied, while the classic question in labor economics when technical change will be skill biased or skill replacing requires a framework where the ***equilibrium bias*** of technical change can be studied. Therefore, the question why technical change is both skill biased and purely labor-augmenting requires a unified framework where both the equilibrium direction and bias of technical change can be studied.

The framework we present for this purpose integrates the growth model with endogenous direction of technical change and the growth model with endogenous bias of technical change, in which capital accumulation process considers investment adjustment costs, and firms can undertake capital-, skill labor- and unskilled labor-augmenting technological improvements. In the steady-state equilibrium, technical change can include all of them. However, according to the results of the model, when there is no investment adjustment cost (implying capital accumulation with infinite elasticity), in steady state, technological progress will be purely labor-augmenting, and no capital-augmenting, that is, only skill labor- and unskilled labor-augmenting. If market size effect dominated prices effect in innovation and the relative supply of skilled labor increase continuously, then technical change will be purely labor-

[1] The labor force with education above college represents skilled labor, while high school and below represents unskilled labor. The data is calculated based on Autor (2014) data.
[2] Acemoglu (2002) pointed out the difference between the factor-augmenting and factor-biased technical change, but did not distinguish the direction and the bias of technical change.

augmenting and skill biased, which results that the skill premium increase continuously, the economic growth rate first increases and then decreases in an inverted U-shaped change, while the labor share of income first decreases and then increases in a U-shaped change. The work in this article extends existing literature on the direction and bias of technical change in the following aspects:

First, this article analyzes both the direction and the bias of technical change in an endogenous technological progress growth model, which can clearly reveal the differences of their concepts and determinants, and tries to answer the question why technical change be both skill bias and purely labor-augmenting in the 20th century. The literature on endogenous bias of technical change (Acemoglu, 1998, 2002; Kiley, 1999) only analyze the determinants of the bias but not the direction of technical change. As a result, they can explain the skill bias, but cannot explain why technological progress is purely labor-augmenting in long run. [3]On the other hand, The literature (Acemoglu, 2003; Irmen, 2017; Irmen et., 2017; Grossman et., 2017, 2021; Casey and Horii, 2022; Li and Bental, 2022) on endogenous direction of technical change only consider the determinants of direction but not the bias, and only address the question when technical change will be purely labor-augmenting, but did not explain the skill bias.

Second, this paper makes a contribution to the modern literature on skill premiums and labor share of income. The "race between technological development and education" described by Tinbergen in his book on *Income distribution* (Tinbergen,1975) is the central organizing framework of the voluminous recent literature studying changes in the returns to skills (Acemoglu and Autor, 2011). However, the assumption that skill-biased technical change is exogenous is not only inconsistent with the history of technical change (James and Skinner, 1985; Goldin and Katz, 1998; Mokyr, 1990, p.137), but also cannot explain the facts that the skill premiums and skilled labor supply both increase continuously in long run (Atkinson, 2007). The endogenous technical change bias model proves that the relative supply of skilled labor itself will lead to the technological progress biased to skill labor and skill premium increase under certain conditions (Acemoglu, 1998, 2002; Kiley, 1999). However, if the increase in relative supply of skilled labor will increase the skill premium, it is likely to affect the labor share of income. Owing to without including capital and capital-augmenting technological progress, this framework cannot study the

[3] When Acemoglu (2009, ch15.6) attempts to explain the direction of technical change using the factors that determines the bias of technical change, not only encountered logical difficulties, but also lead to incorrect results Proposition 15.12 (Li,2016).

changes in the labor share of income (Hémous and Olsen, 2022). This article not only points out the impact of relative supply of skilled labor on skill premiums, but also proves that as the relative supply of skilled labor continues to rise, the labor share of income will first decrease and then increase in a U-shaped change. Numerical simulations indicate that under reasonable parameters, the increase in the relative supply of skilled labor in USA will have an undeniable impact on the labor share of income. Although Acemoglu and Restrepo (2018) and Hémous and Olsen (2022) considered the different impacts of automation on skilled and unskilled labor, and analyzed labor share and skill premium, they also did not consider the impact of relative supply of skilled labor on labor share. Grossman and Oberfield (2021) conducted a comprehensive review of the recent literature that include almost all possible affect factors of the labor share in recent years, which indicates that existing literature, whether theoretical or empirical, has not paid attention to the relative supply of skilled labor.

Finally, the framework proposes that the relative supply of skilled labor will affect the rates of technological progress and economic growth which is overlooked by existing growth models with endogenous technological progress (Romer,1990; Segerstrom, Anant and Dinopoulos, 1990; Grossman and Helpman, 1991; Aghion and Howitt,1992; Young, 1993). Moreover, this paper proves that as the relative supply of skilled labor continues to rise, the rates of technological progress and economic growth will first increase and then decrease in an inverted U-shape change. Numerical simulations indicate that the increase in the relative supply of skilled labor in USA has an undeniable impact on technological progress and economic growth.

The plan of the paper is as follows. Section 2 is the model; Section 3 is the direction and bias of technical change in steady state; Section 4 applications (labor share and economic growth); Section 5, concluding comments.

2. The model

2.1 The Environment

Following Acemoglu (2002, 2003), the economy includes four sectors: the final product, intermediate products, machines, and the research and development (R&D) of new types of machines. There are three material production factors, capital K, unskilled labor L, and skilled labor H. Scientists S specialize in research and development of new types of machines.

(1) The production functions

The production function of final product Y is as follows (1):

$$Y = \left[(1-\gamma) Y_K^{\frac{\varepsilon-1}{\varepsilon}} + \gamma \left\{ \left[\vartheta Y_L^{\frac{\sigma-1}{\sigma}} + (1-\vartheta) Y_H^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{1}$$

Which, $0 \leq \varepsilon < 1$,[4] $1 < \sigma < \infty$. Y_K is capital intensive product, Y_L is unskilled labor-intensive product, Y_H is the skilled labor-intensive product, which is produced by different machines. Their production functions are as follows:

$$\begin{cases} Y_K = \left[\int_0^M Z(j)^\beta dj \right]^{1/\beta} \\ Y_L = \left[\int_0^{N_L} X_L(i)^\beta di \right]^{1/\beta}, \qquad 0 < \beta < 1 \\ Y_H = \left[\int_0^{N_H} X_H(i)^\beta di \right]^{1/\beta} \end{cases} \tag{2}$$

Firms producing Y, Y_K, Y_L and Y_H are in the perfect competition market, and are price takers. M , N_L and N_H represent the measure of types of capital-intensive intermediates, unskilled labor-intensive intermediates, and skilled labor-intensive intermediates, respectively.

Each type machine is exclusively produced by the patent owner or authorized enterprise, thus being in a monopoly position. Machines are divided into three categories, namely capital intensive $Z(j)$ and unskilled labor intensive $X_L(i)$ and skilled labor intensive $X_H(i)$, whereby the three are produced by capital, unskilled labor and skilled labor, and their production function are given by:

$$\begin{cases} Z(j) = K(j) \\ X_L(i) = L(i) \\ X_H(i) = H(i) \end{cases} \tag{3}$$

Equations (1), (2), and (3) extend Acemoglu's (2002,2003) production department by splitting labor into unskilled and skilled categories.

(2) Factor accumulation processes

Unskilled labor L and skilled labor H are given exogenously, which is consistent with Acemoglu's (2002). But the capital accumulation function is the function provided

[4] Here, it is assumed that ε<1, mainly to ensure that the balanced growth path is the only steady state, the core conclusion of this article does not require it.

by Irmen (2013) considering the investment adjustment cost, as shown in equation (4):

$$\dot{K} = I^{\alpha} - \delta_K K, \qquad 0 < \alpha \leq 1 \tag{4}$$

$\dot{K}$ represents net investment, I represents investment, δ_K is the depreciation rate. The parameter α indicates the capital accumulation with investment adjustment cost. If $\alpha = 1$, then equation (4) degenerates into the standard neoclassical capital accumulation function which is used in Acemoglu (2003).

(3) Innovation Possibilities Frontier

Innovation is captured by a knowledge spillover model, whereby the innovation possibilities frontier for the three categories of machines are:[5]

$$\begin{cases} \dot{M} = d_M M S_M - \delta_M M \\ \dot{N}_L = d_{N_L} N_L^{\frac{1+\tau}{2}} N_H^{\frac{1-\tau}{2}} S_{N_L} - \delta_N N_L \\ \dot{N}_H = d_{N_H} N_L^{\frac{1-\tau}{2}} N_H^{\frac{1+\tau}{2}} S_{N_H} - \delta_N N_H \end{cases} \tag{5}$$

where $0 \leq \tau \leq 1$. Innovation is carried out by scientists, whose number is given exogenously.

Equation (5) assumes that the innovation of the capital-intensive machine is not affected by the stock of skilled labor-intensive machines N_H and unskilled labor-intensive machines N_L, but that the innovations of machines used in the skilled and unskilled labor-intensive sectors are influenced by each other. According to Acemoglu (2002), in equation (5), the innovation of capital-intensive machines is only influenced by the measure M, which is an innovation function with extreme state dependence which means that knowledge spillovers are limited to the same class of technologies. [6] However, the innovation function of unskilled and skilled labor-intensive machines has a normal state dependence, indicating the existence of mutual spillover effects between them. This difference in the assumption inherits the assumptions about the knowledge spillover model both in Acemoglu (2002) and Acemoglu (2003), which lead to the conclusions that can be obtained at the same time in our model but can only obtain in different Acemoglu's model.

The constraint of scientist allocation is:

$$S_M + S_{N_L} + S_{N_H} \leq S \tag{6}$$

[5] Equation (5) indicates that there is no crowding effect in innovation. If the crowding effect is considered, no matter whether the substitution elasticity of capital and labor is less than 1, there will be only a unique stable equilibrium, but the model will have no analytical expression.

[6] As defined by Acemoglu (2009, ch15, p514), state dependence refers to the phenomenon in which the path of past innovations affects the relative costs of different types of innovations.

(4) Migration of scientists

Another important difference from Acemoglu (2002, 2003) is that this framework explicitly provides migration equations for scientists among different R&D sectors based on Li and Bental (2022). Scientists are homogeneous. They can create every type machine and move freely among different sectors, but takes time. This indicates that in the short term, the number of scientists in different sectors is given, and free entry of innovation cannot guarantee that the wages of scientists in different sectors are equal at every moment. However, the wages of scientists within any sector are equalized. The wages for scientist sectors are w_M, w_{N_L} and w_{N_H}, respectively. Wage differences will induce scientists to move among sectors, according to the following processes:

$$\begin{cases} \frac{\dot{S}_{N_H}}{S_{N_H}} = \psi\left[\frac{w_{N_H}}{w_M}\right] \\ \frac{\dot{S}_{N_L}}{S_{N_L}} = \psi\left[\frac{w_{N_L}}{w_{N_H}}\right] \end{cases}, \psi[1] = 0, \psi'[.] > 0 \tag{7}$$

Equations (7) indicate that scientists will move from sectors with lower wages to sectors with higher wages. Scientists move between three innovation sectors, but only two migration functions are needed to describe them.[7] $\frac{\dot{S}_{N_H}}{S_{N_H}}$ describes the migration equation of scientists between capital-intensive machine and skilled labor-intensive machine innovation sectors, which depends on the ratio of scientists' wages in these two sectors $\frac{w_{N_H}}{w_M}$, $\frac{\dot{S}_{N_L}}{S_{N_L}}$ describes the migration equation of scientists between skilled and unskilled labor-intensive machine innovation sectors, which depends on the ratio of wages between these two sectors $\frac{w_{N_L}}{w_{N_H}}$. When scientists have enough time to move between sectors, their wage rates in different sectors will be equalized:

$$w_M = w_{N_L} = w_{N_H} \tag{8}$$

(5) Household's preference and budget constraint

The household's goal is to maximize the discounted flow of utility, given by:

$$\mathrm{U} = \int_0^\infty \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \tag{9}$$

[7] In reality, scientists may move to the sector with the highest wages from other two sectors, rather than to the sector with higher wages than their own sector, for example, if $w_M > w_{N_L} > w_{N_H}$, scientists may all move to M sector, but there is no essential difference from equation (7) for steady state equilibrium.

where $C(t)$ is consumption at time t, $\rho > 0$ is the discount rate, and $\theta > 0$ is a utility curvature coefficient of the household.

Periodic budget constraint for representative households is given by:

$$C + I \leq Y = rK + w_L L + w_H H + \mathrm{w}_{N_L} S_{N_L} + \mathrm{w}_{N_H} S_{N_H} + \mathrm{w}_M S_M + \Pi \quad (10)$$

where the LHS stands for expenditures consisting of consumption, C, and investments, I, and the RHS is income, obtained from renting out capital at the rate r, unskilled labor at the rate w_L, skilled labor at the rate w_H, scientists at the wages w_{N_L}, w_{N_H} and w_M. Π stands for total profits.

The equilibrium consists of two stages: the first stage is the *instantaneous equilibrium* (or short run equilibrium). Specifically, given K, L, H, M, N_L, N_H, S_M, S_{N_L} and S_{N_H}, and setting the final good as the numeraire, the prices r, w_L, w_H, w_M, w_{N_L} and w_{N_H} clear all markets (the product market, factor markets and scientist markets), and underly the households' intertemporal utility maximization as well as the enterprises' profit maximization.

The second stage is the *steady-state equilibrium* (or long run equilibrium), comprised of the instantaneous equilibrium with the additional condition that the scientists' wages across sectors are equalized and the dynamics of factor accumulation and technological progress is adjusted accordingly.

We start by formally analyzing the instantaneous equilibrium, then move to the steady-state equilibrium of the model.

2.2. The Instantaneous equilibrium

(1) Enterprise profit maximization and goods market equilibrium

When enterprise maximize profits, the goods market clears, the production function takes the form of a CES function (11), as summarized in Proposition 1 and the factor prices are the equations (13), Proofs in Appendix A.

Proposition 1: In the instantaneous equilibrium, the final output production function takes the form of a CES function (11), where $A_L \equiv {N_L}^{(1-\beta)/\beta}$, $A_H \equiv {N_H}^{(1-\beta)/\beta}$ *and* $B \equiv M^{(1-\beta)/\beta}$.

$$Y = \left[(1-\gamma)(BK)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma \left\{ \left[\vartheta (A_L L)^{\frac{\sigma-1}{\sigma}} + (1-\vartheta)(A_H H)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (11)$$

Letting $k \equiv \frac{BK}{A_L L} = \frac{M^{(1-\beta)/\beta}K}{{N_L}^{(1-\beta)/\beta}L}$, $h \equiv \frac{A_H H}{A_L L} = \frac{{N_H}^{(1-\beta)/\beta}H}{{N_L}^{(1-\beta)/\beta}L}$ from equation (11), the intensive production function is:

$$f(k,h) \equiv \frac{Y}{A_L L} = \left[(1-\gamma)k^{\frac{\varepsilon-1}{\varepsilon}} + \gamma\left[\vartheta + (1-\vartheta)h^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{12}$$

The factor prices for maximizing enterprise profit are as follows:

$$\begin{cases} r = \beta(1-\gamma)\left[(1-\gamma)k^{\frac{\varepsilon-1}{\varepsilon}} + \gamma v^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}}\right]^{\frac{1}{\varepsilon-1}} (k)^{\frac{-1}{\varepsilon}} M^{(1-\beta)/\beta} \\ w_L = \beta\gamma\vartheta\left[(1-\gamma)k^{\frac{\varepsilon-1}{\varepsilon}} + \gamma v^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}}\right]^{\frac{1}{\varepsilon-1}} v^{\frac{\varepsilon-\sigma}{(\sigma-1)\varepsilon}} {N_L}^{(1-\beta)/\beta} \\ w_H = \beta\gamma(1-\vartheta)\left[(1-\gamma)k^{\frac{\varepsilon-1}{\varepsilon}} + \gamma v^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}}\right]^{\frac{1}{\varepsilon-1}} v^{\frac{\varepsilon-\sigma}{(\sigma-1)\varepsilon}} h^{\frac{-1}{\sigma}} {N_H}^{(1-\beta)/\beta} \end{cases} \tag{13}$$

Where $v \equiv \vartheta + (1-\vartheta)h^{\frac{\sigma-1}{\sigma}}$.

Due to the monopoly of machines in production, like the classic Romer (1990) model, the factor prices in equations (13) are not equal to the marginal output of factors in equation (11).

(2) Scientist market equilibrium

The instantaneous equilibrium of scientist market refers to the balance between scientists supply and demand equilibrium in each sector, for given M, N_L , N_H, S_M, S_{N_L} and S_{N_H}. The demand for scientists depends on the marginal output of patent of scientists and the market value of each invention patent. Substituting the wages w_M, w_{N_L} and w_{N_H} into the migration function (7), we obtain (see Appendix B):

$$\begin{cases} \frac{\dot{S}_{N_H}}{S_{N_H}} = \psi\left[\frac{d_{N_H}}{d_M} \cdot \frac{w_H}{r} \cdot \frac{H}{K}\left(\frac{N_H}{N_L}\right)^{\frac{\tau-1}{2}}\right] \\ \frac{\dot{S}_{N_L}}{S_{N_L}} = \psi\left[\frac{d_{N_L}}{d_{N_H}} \cdot \frac{w_L}{w_H} \cdot \frac{L}{H} \cdot \left(\frac{N_H}{N_L}\right)^{1-\tau}\right] \end{cases} \tag{14}$$

Equation (14) is the result of Li and Bental (2022) in an economy with three production factors. It shows that in the instantaneous equilibrium, the forces affecting the direction of technological progress are the relative factor price $\frac{w_H}{r}$ and $\frac{w_L}{w_H}$, the relative market size $\frac{H}{K}$ and $\frac{L}{H}$, and the relative marginal productivity of scientists in the different sectors $\frac{d_{N_H}}{d_M}$ and $\frac{d_{N_L}}{d_{N_H}}$. These results not only include the price effect and market size effect emphasized by Acemoglu (2002), but also include the relative marginal productivity effect of innovation pointed out by Li and Bental (2022). This is reasonable because the wages of scientists depends not only on the market value of new patents, but also on the marginal productivity of innovation.

(3) Maximization of household utility

Households are price takers. Given the factor accumulation functions, the representative household maximizes the intertemporal utility by allocating income between consumption and the investment. The resulting Euler equations are as follows (see Appendix C):

$$\frac{\dot{C}}{C} = \frac{1}{\theta}\left\{r\alpha I^{\alpha-1} - (\alpha - 1)\frac{\dot{I}}{I} - \rho - \delta_K\right\} \tag{15}$$

The Euler equations indicate that the allocation of household income should equalize returns of the current consumption and investment in capital accumulation. Due to the existence of investment adjustment costs, one unit of investment cannot be converted into one unit of new capital, and as investment increases, the marginal efficiency of the capital converted from investment decreases. Therefore, the Euler equation is different from the usual neoclassical Euler equation, and only adjusting investment costs can be ignored, that is, $\alpha = 1$, equation (15) degenerates into the usual neoclassical Euler equation $\frac{\dot{C}}{C} = \frac{1}{\theta}(r - \rho - \delta_K)$.

(4) The dynamics of the instantaneous equilibrium

The instantaneous equilibrium delivers the optimal behavior of households and firms which underly the economy's dynamics. To obtain the equations describing those dynamics, we need to use the state variable $k(t)$. In addition we define the investment rate of capital and labor $\mathrm{s(t)} \equiv I(t)/\mathrm{Y(t)}$ as well as the following growth rates: $g(t) \equiv \dot{Y}(t)/Y(t)$, $\mathrm{g_K(t)} \equiv \dot{\mathrm{K}}(t)/\mathrm{K(t)}$.

Given the above definitions, the equations describing the dynamic evolution of the economy take the following form (see Appendix D):

$$
\begin{cases}
\dfrac{\dot{k}}{k} = \dfrac{1-\beta}{\beta}\left[d_M\left(S - S_{N_H} - S_{N_L}\right) - \delta_M\right] + g_K - \dfrac{1-\beta}{\beta}\left(d_{N_L}\left(\dfrac{N_H}{N_L}\right)^{\frac{1-\tau}{2}} S_{N_L} - \delta_N\right) \\
\dfrac{\dot{h}}{h} = \dfrac{1-\beta}{\beta}\left[\left(d_{N_H}\left(\dfrac{N_H}{N_L}\right)^{-\frac{1-\tau}{2}} S_{N_H} - d_{N_L}\left(\dfrac{N_H}{N_L}\right)^{\frac{1-\tau}{2}} S_{N_L}\right)\right] \\
\dfrac{\dot{S}_{N_H}}{S_{N_H}} = \psi\left[\dfrac{d_{N_H}}{d_M}.\dfrac{\gamma(1-\vartheta)\left[\vartheta + (1-\vartheta)h^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\varepsilon-\sigma}{(\sigma-1)\varepsilon}}(h)^{\frac{\sigma-1}{\sigma}}}{(1-\gamma)(k)^{\frac{\varepsilon-1}{\varepsilon}}}\left(\dfrac{N_H}{N_L}\right)^{\frac{\tau-1}{2}}\right] \\
\dfrac{\dot{S}_{N_L}}{S_{N_L}} = \psi\left[\dfrac{d_{N_L}}{d_{N_H}}.\dfrac{\vartheta}{(1-\vartheta)(h)^{\frac{\sigma-1}{\sigma}}}.\left(\dfrac{N_H}{N_L}\right)^{1-\tau}\right] \\
\dfrac{\dot{s}}{s} = \dfrac{(1-s)}{s}(g - g_C) \\
\dfrac{\dot{g}_K}{g_K} = \dfrac{g_K + \delta_K}{g_K}\left[\dfrac{\alpha}{s}g - \dfrac{\alpha}{s}(1-s)g_C - g_K\right]
\end{cases}
\qquad (16)
$$

Equations (16) consist of a set of differential equations, including six independent equations and six variables other than time t, which describe the dynamic behavior of economy in instantaneous general equilibrium, where $[g, g_C]$ are represented by (k, h, S_{N_H}, S_{N_L}, s, g_K) as follows:

$$
\begin{cases}
g = \dfrac{1-\beta}{\beta}\left(d_{N_L}\left(\dfrac{N_H}{N_L}\right)^{\frac{1-\tau}{2}} S_{N_L} - \delta_N\right) + \dfrac{\frac{\partial f}{\partial k}k}{f(k,h)}\dfrac{\dot{k}}{k} + \dfrac{\frac{\partial f}{\partial h}h}{f(k,h)}\dfrac{\dot{h}}{h} \\
g_C = \dfrac{\alpha\beta(g_K + \delta_K)(1-\gamma)(k)^{\frac{\varepsilon-1}{\varepsilon}}/[s\theta + (1-s)(1-\alpha)]}{(1-\gamma)k^{\frac{\varepsilon-1}{\varepsilon}} + \gamma\left[\vartheta + (1-\vartheta)h^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}}} + \dfrac{(1-\alpha)g - s(\rho + \delta_K)}{[s\theta + (1-s)(1-\alpha)]} \\
\dfrac{\frac{\partial f}{\partial k}k}{f(k,h)} = \dfrac{(1-\gamma)k^{\frac{\varepsilon-1}{\varepsilon}}}{(1-\gamma)k^{\frac{\varepsilon-1}{\varepsilon}} + \gamma\left[\vartheta + (1-\vartheta)h^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}}} \\
\dfrac{\frac{\partial f}{\partial h}h}{f(k,h)} = \dfrac{\gamma(1-\vartheta)\left[\vartheta + (1-\vartheta)h^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}-1} h^{\frac{\sigma-1}{\sigma}}}{(1-\gamma)k^{\frac{\varepsilon-1}{\varepsilon}} + \gamma\left[\vartheta + (1-\vartheta)h^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}}}
\end{cases}
\qquad (16a).
$$

2.3. The Steady State equilibrium (or the long run equilibrium)

The steady-state equilibrium: steady-state equilibrium refers to the zero solution of the dynamic system that describes the instantaneous equilibrium of the model, that is, all dynamic equations in equation (16) are equal to zero.[8]

(1) Existence and uniqueness of steady state

For the existence and uniqueness of the steady state of the specific model described in this section, we provide proposition 2 (proof see appendix E):

Proposition 2: When ε<1 and $(1-\tau)\beta\sigma-(1-\beta)(\sigma-1)>0$, an economy characterized by equations (16) possesses a unique and stable steady-state growth equilibrium. The output growth rate is described by:

$$g^{*}=\frac{1-\beta}{\beta}\frac{\left(S-\frac{\delta_M}{d_M}\right)\left(d_{N_L}d_{N_H}\frac{1-\vartheta}{\vartheta}(h^{*})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{2}}-\delta_N\left(1+\frac{1-\vartheta}{\vartheta}(h^{*})^{\frac{\sigma-1}{\sigma}}\right)}{1+\frac{1-\vartheta}{\vartheta}(h^{*})^{\frac{\sigma-1}{\sigma}}+\frac{1-\alpha}{d_M}\left(d_{N_L}d_{N_H}\frac{1-\vartheta}{\vartheta}(h^{*})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{2}}} \tag{17}$$

Where $h^{*}=\left(\frac{d_{N_H}}{d_{N_L}}.\frac{1-\vartheta}{\vartheta}\right)^{\frac{\sigma(1-\beta)}{\beta\sigma(1-\tau)-(\sigma-1)(1-\beta)}}.\left(\frac{H}{L}\right)^{\frac{\beta\sigma(1-\tau)}{\beta\sigma(1-\tau)-(\sigma-1)(1-\beta)}}$.

The solving process indicates that the model has a steady-state equilibrium which is furthermore unique.

(2) Steady state technological progress

Substitute the steady-state $S_{N_L}^{*}$ and $S_{N_H}^{*}$ into the technological innovation function (5) to obtain:

$$\begin{cases}\frac{\dot{N}_H}{N_H}=\frac{\dot{N}_L}{N_L}=\frac{\left(S-\frac{\delta_M}{d_M}\right)\left(d_{N_L}d_{N_H}.\frac{1-\vartheta}{\vartheta}(h^{*})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{2}}-\delta_N\left(1+\frac{1-\vartheta}{\vartheta}(h^{*})^{\frac{\sigma-1}{\sigma}}\right)}{1+\frac{1-\vartheta}{\vartheta}(h^{*})^{\frac{\sigma-1}{\sigma}}+\frac{1-\alpha}{d_M}\left(d_{N_L}d_{N_H}.\frac{1-\vartheta}{\vartheta}(h^{*})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{2}}}\\ \frac{\dot{M}}{M}=(1-\alpha)\frac{\left(S-\frac{\delta_M}{d_M}\right)\left(d_{N_H}d_{N_L}.\frac{1-\vartheta}{\vartheta}.(h^{*})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{2}}-\delta_N\left[1+\frac{1-\vartheta}{\vartheta}.(h^{*})^{\frac{\sigma-1}{\sigma}}\right]}{1+\frac{1-\vartheta}{\vartheta}.(h^{*})^{\frac{\sigma-1}{\sigma}}+\frac{1-\alpha}{d_M}\left(d_{N_H}d_{N_L}.\frac{1-\vartheta}{\vartheta}.(h^{*})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{2}}}\end{cases} \tag{18}$$

The results of equations (18) indicate that in the steady state, if $\alpha<1$, then the technical change includes skilled labor-, unskilled labor- and capital-augmentation.

[8] We think this is a rigorous definition. In existing literature, the usual definition requires the various quantities grow at constant (perhaps zero) rates. but the exact variables included are sometimes inconsistent in different literature. For example, Grossman et al. (2017) require the shares of factor income be constant, but Barro and Sala-i-Martin (2004) does not include this as a precondition though in the steady state the shares indeed be constant.

3. The direction and bias of technical change in steady state

Due to the important impact of the direction and bias of technical change on many economic issues, what are their determinants is a very important question. However, the direction and bias of technical change are fundamentally different, and they also address different issues. The important issue related to the direction of technical change is why technical change in modern economic growth is purely labor-augmenting, while the important issue related to the bias of technical change is when technical change is skilled labor biased. Although Acemoglu (2002) raised both questions, he only answered the issue on endogenous skill-biased technical change, but did not give the answer for why technological progress is purely labor-augmenting. On the other hand, Li and Bental (2022) develops a model for analyzing the direction of technical change but not addressing the bias of technical change. Now we try to use the results of the model in this article to answer the two issues.

1. Why was technical change purely labor-augmenting in modern economic growth?

From equation (18), if $\alpha = 1$, then the technical change in steady state is as follows,

$$\begin{cases} \frac{\dot{N}_H}{N_H} = \frac{\dot{N}_L}{N_L} = \frac{[S - \delta_M/d_M]\left(d_{N_L} d_{N_H} \cdot \frac{1-\vartheta}{\vartheta}(h^*)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{2}}}{1 + \frac{1-\vartheta}{\vartheta}(h^*)^{\frac{\sigma-1}{\sigma}}} - \delta_N \\ \frac{\dot{M}}{M} = 0 \end{cases} \tag{19}$$

Eqaution (19) shows that, as long as $S > \delta_M/d_M$, the rates of technical change of skill labor- and unskilled labor-augmenting both are greater than zero, while the rate of capital-augmenting technical change is zero. Therefore, the technical change is purely labor-augmenting. That is to say, if $\alpha = 1$, although firms can undertake both skilled labor-, unskilled labor- and capital-augmenting technological improvements, the firms would choose purely labor-augmenting (including both skilled labor-, unskilled labor-augmenting) technological improvements in the steady-state. On the contrary, if $\alpha < 1$, then the rational choice of firms would include capital-augmentation in the steady-state. Therefore, technological progress in modern economic growth is purely labor-augmenting, not due to the requirements of steady-state growth, but rather the result of the constraints of resource endowment. When $\alpha = 1$, the capital accumulation function (4) degenerates into the standard neoclassical capital accumulation function $\dot{K} = I - \delta_K K$. This is why the technical change must be purely labor-augmenting in steady state in the neoclassical growth models even with endogenous direction of technological progress (Acemoglu, 2003; Irmen,2017; Irmen et., 2017).

Why is technical change purely labor-augmenting when the capital accumulation function is $\dot{K} = I - \delta_K K$? Li and Bental (2022) provided an explanation for this. They argue that if the economy has a neoclassical production function, the direction of technological progress in steady-state depends on the relative size of factor supply elasticities as follows:

$$\begin{cases} \dfrac{\dot{B}/B}{\dot{A}_H/A_H} = \dfrac{\varepsilon_H + 1}{\varepsilon_K + 1} \\ \dfrac{\dot{B}/B}{\dot{A}_L/A_L} = \dfrac{\varepsilon_L + 1}{\varepsilon_K + 1} \\ \dfrac{\dot{A}_H/A_H}{\dot{A}_L/A_L} = \dfrac{\varepsilon_L + 1}{\varepsilon_H + 1} \end{cases} \tag{20}$$

Where ε_H, ε_L and ε_K denote the supply elasticities of skilled labor H, unskilled labor L and capital K, repectively. According to the definition of Li and Bental (2022) and the results of steady-state equilibrium of this model, the elasticities of supply of factors in steady state is obtained as follows (see appendix F):

$$\begin{cases} \varepsilon_K = \dfrac{\dot{K}/K}{\dot{r}/r} = \dfrac{\alpha}{1-\alpha} \\ \varepsilon_H = \dfrac{\dot{H}/H}{\dot{w}_H/w_H} = 0 \\ \varepsilon_L = \dfrac{\dot{L}/L}{\dot{w}_L/w_L} = 0 \end{cases} \tag{21}$$

Substituting equation (21) into equation (20) yields

$$\begin{cases} \dfrac{\dot{B}/B}{\dot{A}_H/A_H} = \dfrac{\varepsilon_H + 1}{\varepsilon_K + 1} = 1-\alpha \\ \dfrac{\dot{B}/B}{\dot{A}_L/A_L} = \dfrac{\varepsilon_L + 1}{\varepsilon_K + 1} = 1-\alpha \\ \dfrac{\dot{A}_H/A_H}{\dot{A}_L/A_L} = \dfrac{\varepsilon_L + 1}{\varepsilon_H + 1} = 1 \end{cases} \tag{22}$$

When $\alpha = 1$, the elasticities of capital is follow

$$\varepsilon_K = \frac{\dot{K}/K}{\dot{r}/r} = \frac{\alpha}{1-\alpha} = \infty \tag{23}$$

Substituting equation (23) into equation (22) yields

$$\frac{\dot{B}/B}{\dot{A}_H/A_H} = \frac{\dot{B}/B}{\dot{A}_L/A_L} = \frac{\varepsilon_H + 1}{\varepsilon_K + 1} = \frac{0+1}{\infty + 1} = 0 \tag{24}$$

Equation (24) indicates that when capital has infinite supply elasticity and both skilled and unskilled labor are inelastic, capital-augmenting technological progress must be zero (i.e, $\dot{B}/B = 0$), or the technological progress is purely labor-augmenting. Due to the same supply elasticity, technological progress is Hicks neutral for skilled and unskilled labor, (i.e., $\frac{\dot{A_H}/A_H}{\dot{A_L}/A_L} = 1$). Therefore, why technological progress in modern economic growth is purely labor-augmenting, is likely due to the process of capital accumulation, as described by the standard neoclassical accumulation function (i.e., $\dot{K} = I - \delta_K K$), which implies infinite elasticity of capital supply. In fact, when the supply of capital has infinite elasticity, technological progress cannot improve the productivity of capital, which is a result in line with economic intuition. This is just like in the Malthusian economy where labor has infinite supply elasticity and technological progress cannot improve labor productivity. This not only explains that technological progress in modern economic growth is purely labor-augmenting, but also validates the conclusion of Li and Bental (2022) in the case with three factors and three factor-augmenting technical change.

2. Endogenous skill-biased technical change

Figure 2 depicts the continuous increase in the relative supply of skilled labor and skill premiums in the United States over the past 50 years. Acemoglu (2002) provided an explanation for this phenomenon by endogenous skill-biased technical change. Our model inherits Acemoglu's (2002) assumptions about technical change and the supply of unskilled labor and skilled labor, and also encompasses all its core results.

Using $h \equiv \left(\frac{N_H}{N_L}\right)^{\frac{1-\beta}{\beta}} \frac{H}{L}$ and $h^* = \left(\frac{d_{N_H}}{d_{N_L}} . \frac{1-\vartheta}{\vartheta}\right)^{\frac{\sigma(1-\beta)}{\beta\sigma(1-\tau)-(\sigma-1)(1-\beta)}} . \left(\frac{H}{L}\right)^{\frac{\beta\sigma(1-\tau)}{\beta\sigma(1-\tau)-(\sigma-1)(1-\beta)}}$

yields

$$\left(\frac{N_H}{N_L}\right)^* = \left(\frac{d_{N_H}}{d_{N_L}} . \frac{1-\vartheta}{\vartheta}\right)^{\frac{\beta\sigma}{\beta\sigma(1-\tau)-(\sigma-1)(1-\beta)}} \left(\frac{H}{L}\right)^{\frac{\beta(\sigma-1)}{\beta\sigma(1-\tau)-(\sigma-1)(1-\beta)}} \tag{25}$$

Using the equation (13), the relative wages between skilled and unskilled labor (skill premium) can be obtained as follow

$$\frac{w_H}{w_L} = \frac{(1-\vartheta)}{\vartheta} \left(\frac{N_H}{N_L}\right)^{\frac{(\sigma-1)}{\sigma} . \frac{(1-\beta)}{\beta}} \left(\frac{H}{L}\right)^{\frac{-1}{\sigma}} \tag{26}$$

Equation (26) describes the impact of relative technology $\left(\frac{N_H}{N_L}\right)$ and relative supply of skilled labor $\left(\frac{H}{L}\right)$ on skill premiums. If the substitution elasticity between skilled and unskilled labor is greater than 1, i.e., $\sigma > 1$, the impact of technical change and the relative supply of skilled labor on skill premiums is opposite. The increase in relative technology $\left(\frac{N_H}{N_L}\right)$ leads to an increase in skill premium, while the increase in relative supply of skilled labor $\left(\frac{H}{L}\right)$ leads to a decrease in skill premium. This is the classic framework for analyzing skill premiums, that is, the "race between technical change and education". But if the two effects are independent and opposite, the skill premium should fluctuate around a certain equilibrium level. However, as shown in Figure 2, both the skill premium and the relative supply of skilled labor continue to rise in the long run. Therefore, Acemoglu (2002) proposed endogenous skill-biased technical change, as shown in equation (25), where the relative technology $\left(\frac{N_H}{N_L}\right)^*$ is a function of the relative supply of skilled labor. Substituting equation (25) into equation (26) yields

$$\left(\frac{w_H}{w_L}\right)^* = \frac{(1-\vartheta)}{\vartheta}\left(\frac{d_{N_H}}{d_{N_L}}.\frac{1-\vartheta}{\vartheta}\right)^{\frac{(1-\beta)(\sigma-1)}{\beta\sigma(1-\tau)-(\sigma-1)(1-\beta)}}\cdot\left(\frac{H}{L}\right)^{\frac{(\sigma-1)(1-\beta)-\beta(1-\tau)}{\sigma\beta(1-\tau)-(\sigma-1)(1-\beta)}} \tag{27}$$

Equation (27) describes the impact of the relative supply of skilled labor on skill premiums in the steady-state under endogenous skill-biased technical change. If $\frac{\beta(\sigma-1)}{\beta\sigma(1-\tau)-(\sigma-1)(1-\beta)} > 0$ and $\frac{(\sigma-1)(1-\beta)-\beta(1-\tau)}{\sigma\beta(1-\tau)-(\sigma-1)(1-\beta)} > 0$, that is, $\frac{1-\beta}{1-\beta(2-\tau)} > \sigma > 1$ and $\frac{2\beta-1}{\beta} < \tau < 1$,[9] then $\partial\left(\frac{N_H}{N_L}\right)^* / \partial\left(\frac{H}{L}\right) > 0$ and $\partial\left(\frac{w_H}{w_L}\right)^* / \partial\left(\frac{H}{L}\right) > 0$. That is to say, the relative supply of skilled labor increases the skill premium by promoting the skill-biased of technical change, and overcomes the inhibitory effect of the relative supply of skilled labor on the skill premium, resulting in an overall increase in skill premium. This is the core conclusion of Acemoglu (2002), and the key to this result is that market size effect dominated the prices effect in innovation.

In this section, we have used the steady-state equilibrium of the model to answer when technical change would be both purely labor-augmenting and skill-biased, and to validate the core conclusions of Acemoglu (2002) and Li and Bental (2022). The value of substitution elasticity between skilled and unskilled labor (σ), as well as the state dependence (τ) of innovation, determines the skill ***bias*** of technical change, while the relative size of factor supply elasticities determines the ***direction*** of technical change. Specifically, the parameters α that determine investment adjustment costs and implies the elasticity of capital supply determines that the technical change be must be purely

[9] The σ here is different from it in Acemoglu (2002), it corresponds to his ε.

labor-augmenting in steady state. Although Acemoglu (2002) explicitly raised both questions, he only put forward the conditions of endogenous skill-biased technical change, did not give the conditions of purely labor-augmenting technical change.

4. Labor share and economic growth

Under endogenous skill-biased technical change, the increase in the relative supply of skilled labor not only increases the skill premium, but also increases the proportion of skilled labor to total labor, which is likely to affect the labor share of income. However, since Acemoglu's (1998, 2002) framework does not include capital and capital-augmenting technological progress in analyzing skill premiums, it cannot analyze the changes in the income share of labor and capital, which is an important defect of this framework (Hémous and Olsen, 2022). The model in this article finds that the relative supply of skilled labor not only affects the labor share of income and economic growth rate, but also changes in a U-shaped and inverted U-shaped change as the relative supply of skilled labor increases, respectively.

4.1. The relative supply of skill on the labor share

Labor income includes income of skilled and unskilled labor, and the labor share in steady state is $\alpha_H \equiv \frac{w_H H + w_L L}{rK + w_H H + w_L L}$ as following (the derivation process in the Appendix G):

$$\alpha_{HL} = \frac{1 + \frac{1-\vartheta}{\vartheta}(h^*)^{\frac{\sigma-1}{\sigma}}}{1 + \frac{1-\vartheta}{\vartheta}(h^*)^{\frac{\sigma-1}{\sigma}} + \left[\frac{d_{N_L} d_{N_H}}{d_M . d_M} . \frac{1-\vartheta}{\vartheta}(h^*)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{2}}} \tag{28}$$

Owing to $h^* = \left(\frac{d_{N_H}}{d_{N_L}} . \frac{1-\vartheta}{\vartheta}\right)^{\frac{\sigma(1-\beta)}{\beta\sigma(1-\tau)-(\sigma-1)(1-\beta)}} . \left(\frac{H}{L}\right)^{\frac{\beta\sigma(1-\tau)}{\beta\sigma(1-\tau)-(\sigma-1)(1-\beta)}}$, equation (28) shows that the relative supply of skilled labor (H/L) is an important factor affecting the labor share. However, owing to (H/L) in both the numerator and denominator of α_{HL}, the change of labor share is unclear when H/L is increasing. In order to reveal the impact of the relative supply of skilled labor on labor the share, we take the derivative of the labor share α_{HL} with respect to (H/L) as following (the process see Appendix H):

$$\frac{\partial \alpha_{HL}}{\partial (H/L)} = \frac{\partial \alpha_{HL}}{\partial x} \frac{\partial x}{\partial (H/L)} = \frac{\frac{1}{2} a^{\frac{1}{2}} \left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right)}{\left[1 + x + (a.x)^{\frac{1}{2}}\right]^2} \frac{\partial x}{\partial (H/L)} \tag{29}$$

Where $x \equiv \frac{1-\vartheta}{\vartheta}(h^*)^{\frac{\sigma-1}{\sigma}}$, $a \equiv \frac{d_{N_L} d_{N_H}}{d_M . d_M}$.

According to equation (29), when $h^{\frac{\sigma-1}{\sigma}} = \frac{\vartheta}{1-\vartheta}$, then $\frac{\partial \alpha_{HL}}{\partial (H/L)} = 0$. Let $\left(\frac{H}{L}\right)^0$ denote the value of the relative supply of skill labor when $\frac{\partial \alpha_{HL}}{\partial (H/L)} = 0$ as following

$$\left(\frac{H}{L}\right)^0 = \left(\frac{\vartheta}{1-\vartheta}\right)^{\frac{\sigma}{\sigma-1}} \left(\frac{d_{N_L}}{d_{N_H}}\right)^{\frac{(1-\beta)}{\beta(1-\tau)}} \tag{30}$$

When $\frac{\mathrm{H}}{\mathrm{L}} = \left(\frac{\mathrm{H}}{\mathrm{L}}\right)^0 = \left(\frac{\vartheta}{1-\vartheta}\right)^{\frac{\sigma}{\sigma-1}} \left(\frac{\mathrm{d}_{\mathrm{N_L}}}{\mathrm{d}_{\mathrm{N_H}}}\right)^{\frac{(1-\beta)}{\beta(1-\tau)}}$, the minimum labor share as following

$$\alpha_{HL}^{min} = \frac{2d_M}{2d_M + \left(d_{N_L} d_{N_H}\right)^{\frac{1}{2}}} \tag{31}$$

If the parameters satisfy the conditions that the increase of skilled labor relative supply leads to an increase in skill premium, i.e., $\frac{1-\beta}{1-\beta(2-\tau)} > \sigma > \frac{1-\beta\tau}{1-\beta}$ and $\frac{2\beta-1}{\beta} < \tau < 1$, then $\frac{\partial \alpha_{HL}}{\partial (H/L)} < 0$ when $\left(\frac{H}{L}\right) < \left(\frac{H}{L}\right)^0$, $\frac{\partial \alpha_{HL}}{\partial (H/L)} > 0$ when $\left(\frac{H}{L}\right) > \left(\frac{H}{L}\right)^0$. That is, as H/L increases, the labor share α_{HL} will first decrease, and arrive the minimum value $\alpha_{HL}^{min} = \frac{2}{2+\left[\frac{d_{N_L} d_{N_H}}{d_M \cdot d_M}\right]^{\frac{1}{2}}}$ at $\frac{H}{L} = \left(\frac{H}{L}\right)^0$, and then rise, showing a U-shaped change. These results are summarized by the following proposition:

Proposition 3: As long as $\frac{1-\beta}{1-\beta(2-\tau)} > \sigma > \frac{1-\beta\tau}{1-\beta}$ and $\frac{2\beta-1}{\beta} < \tau < 1$, the labor share α_{HL} in steady state decreases first and then increases as the relative supply of skilled labor $\left(\frac{H}{L}\right)$ rises from less than $\left(\frac{H}{L}\right)^0$ to more than $\left(\frac{H}{L}\right)^0$, showing a U-shaped change, the minimum labor share is $\alpha_{HL}^{min} = \frac{2d_M}{2d_M + \left[d_{N_L} d_{N_H}\right]^{\frac{1}{2}}}$.

As conditions $\frac{1-\beta}{1-\beta(2-\tau)} > \sigma > \frac{1-\beta\tau}{1-\beta}$ and $\frac{2\beta-1}{\beta} < \tau < 1$ are also the conditions for the relative supply of skilled labor to lead to an increase in relative skill level $\left(\frac{N_H}{N_L}\right)^*$ and skill premium $\frac{w_H}{w_L}$. Therefore, Proposition 3 states that as long as the increase of the relative supply of skill labor leads to increase the skill premium, it will lead the labor share showing a U-shaped change. Equation (30) gives the point of $\left(\frac{H}{L}\right)$ which labor share arrives at the minimum value.

4.2. The relative supply of skill on the economic growth

According to equation (17), the steady-state economic growth rate is also a function of the relative supply of skilled labor. Obtaining the derivative of steady-state economic growth rate over H/L as following

$$\frac{\partial g^*}{\partial(H/L)} = \frac{\partial g^*}{\partial x}\frac{\partial x}{\partial(H/L)} = \frac{1}{2}(A+\delta_N B)\frac{1-\beta}{\beta}\frac{x^{-\frac{1}{2}}-x^{\frac{1}{2}}}{\left(1+x+Bx^{\frac{1}{2}}\right)^2}\frac{\partial x}{\partial(H/L)} \quad (32)$$

where $x = \frac{1-\vartheta}{\vartheta}(h^*)^{\frac{\sigma-1}{\sigma}}$, $A \equiv \left(S-\frac{\delta_M}{d_M}\right)\left(d_{N_L}d_{N_H}\right)^{\frac{1}{2}}$, $B \equiv \frac{1-\alpha}{d_M}\left(d_{N_L}d_{N_H}\right)^{\frac{1}{2}}$.

According equation (32), when $\frac{H}{L} = \left(\frac{H}{L}\right)^0 = \left(\frac{\vartheta}{1-\vartheta}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{d_{N_L}}{d_{N_H}}\right)^{\frac{(1-\beta)}{\beta(1-\tau)}}$, $\frac{\partial g^*}{\partial(H/L)} = 0$. $\frac{\partial g^*}{\partial(H/L)} > 0$ when $\frac{H}{L} < \left(\frac{H}{L}\right)^0$, the economic growth rate increases with the increase of H/L; $\frac{\partial g^*}{\partial(H/L)} < 0$ when $\frac{H}{L} > \left(\frac{H}{L}\right)^0$, the economic growth rate decreases with the increase of H/L. The economic growth rate arrives at maximum when $\frac{H}{L} > \left(\frac{H}{L}\right)^0$. The maximum growth rate is as follow:

$$g^{max} = \frac{1-\beta}{\beta}\frac{(d_M S-\delta_M)\left(d_{N_L}d_{N_H}\right)^{\frac{1}{2}} - 2d_M\delta_N}{2d_M + (1-\alpha)\left(d_{N_L}d_{N_H}\right)^{\frac{1}{2}}} \quad (33)$$

These results are summarized by the following proposition:

Proposition 4: As long as $\frac{1-\beta}{1-\beta(2-\tau)} > \sigma > \frac{1-\beta\tau}{1-\beta}$ and $\frac{2\beta-1}{\beta} < \tau < 1$, the economic growth rate g^* in steady state increases first and then decreases as the relative supply of skilled labor $\left(\frac{H}{L}\right)$ rises from less $\left(\frac{H}{L}\right)^0$ to more than $\left(\frac{H}{L}\right)^0$, showing an inverted U-shaped change, the maximum economic growth rate is $g^{max} = \frac{1-\beta}{\beta}\frac{(d_M S-\delta_M)\left(d_{N_L}d_{N_H}\right)^{\frac{1}{2}}-2d_M\delta_N}{2d_M+(1-\alpha)\left(d_{N_L}d_{N_H}\right)^{\frac{1}{2}}}$.

Similarly, the conditions $\frac{1-\beta}{1-\beta(2-\tau)} > \sigma > \frac{1-\beta\tau}{1-\beta}$ and $\frac{2\beta-1}{\beta} < \tau < 1$ are also the conditions for the relative supply of skilled labor to lead to an increase in relative skill level $\left(\frac{N_H}{N_L}\right)^*$ and skill premium $\frac{w_H}{w_L}$, and the U-shaped change of labor share. Therefore, Proposition 4 states that as long as the increase of the relative supply of skill labor leads to increase the skill premium and the U-shaped change of labor share, it will lead the economic growth showing an inverted U-shaped change.

Given H and L, the economic growth rate is equal to the rate of labor-augmenting technical change. It is precisely because the increase of the relative supply of skilled labor leads that the labor-augmenting technical change first increases and then decreases, resulting in the first decrease and then increase in labor share of income.

4.3 Steady State and Periodic Changes of Labor Share and Economic Growth

The above analysis indicates that when the cost of investment adjustment can be ignored, i.e. $\alpha = 1$, technical change will be purely labor-augmenting, and the labor share of income and economic growth rate will remain constant. However, if the relative supply of skilled labor continues to rise, technological progress will be labor-augmenting but skill-biased, and $\left(\frac{N_H}{N_L}\right)^*$ and $\left(\frac{w_H}{w_L}\right)^*$ continue to rise, labor share and economic growth will show U-shaped and inverted U-shaped changes, respectively.

However, according to the Kaldor facts, in the Modern Economic Growth, the labor share and economic growth rate are basically stable in the long run. On the other hand, in a recent paper, Charles, Bridji, and Mcadam (2019) finds out that the share of labor income may have 30-50 year cycles change using long-term data from the UK, US, and France, and questioning the concept of balanced growth.

We do not have clear evidence, but there is a conjecture that if the labor force is periodically divided into skilled and unskilled labor, then the labor share and economic growth may periodicaly changes every decades, but basic stable in avarage in long run.

In existing literature, it usually refers to labor with college degree as skilled labor, while labor with only a high school diploma or below as unskilled labor. But a more appropriate distinction may be based on whether one has mastered the core general purpose technologies in the economy at that time. The labor mastered is skilled labor, while the labor not mastered is unskilled labor. If, with the revolution of technology, the core general purpose technology in the economy is changed, leading to the labor who previously mastered general purpose technology become unskilled labor in the face of new technologies, only through education and training can the proportion of skilled labor gradually increase last for decades. Thus, every few decades, the proportion of skilled and unskilled labor in the total labor force undergoes a cyclical change from low to high. Due to the exogeneity and randomness of major general purpose technology revolutions, such cycles may also be irregular. As a result, the labor share of income and economic growth rate also show cyclical changes, but the averages show no trend in the long run.

4.4 Quantitative Exercise

Due to the numerous influencing factors, it is impossible to explain all the changes in skill premiums, labor share and economic growth solely by the relative supply of skilled labor. However, in order to test whether the predicted values of the model have a certain correlation with actual data under reasonable parameter values, the following will compare the predicted values with empirical data of the United States.

(1) Parameter calibration.

In equations (17), (27) and (28), in addition to the relative supply of skilled labor (H/L), the parameters that affect the economic growth, skill premium and labor share include ϑ, σ, β, τ, d_{N_L}, d_{N_H}, d_M, etc. However, only when the parameters meet the conditions $\frac{1-\beta}{1-\beta(2-\tau)} > \sigma > \frac{1-\beta\tau}{1-\beta}$ and $\frac{2\beta-1}{\beta} < \tau < 1$, the relative supply of skilled labor increase will lead an increase in skill premium, therefore, the parameters, σ, β, τ are not entirely free to take values.

As pointed out by Acemoglu (2002), there is a relatively widespread consensus that the elasticity between skilled and unskilled workers is greater than 1, most likely, as large as 2 (Freeman, 1986; Angrist, 1995), therefore we let σ=2. Based on the evidence presented by Trajtenberg et al. (1992)'s research on patents citation, Acemoglu (2002) suggests that the parameter, τ, state dependence in innovation function may be around 0.3, so we take τ=0.3. Given σ and τ, in order to meet the conditions $\frac{1-\beta}{1-\beta(2-\tau)} > \sigma > \frac{1-\beta\tau}{1-\beta}$ and $\frac{2\beta-1}{\beta} < \tau < 1$, β must be between 0.417 and 0.588. Let β= 0.535, skill premium can be fit better. Due to the homogeneity of scientists, there may not be significant differences in the values of d_{N_L}, d_{N_H}, and d_M, and the key lies not in their absolute values, but in their relative values. Therefore, let $d_{N_L} = d_{N_H} = 0.1$. However, if $d_M = 0.1$, the predicted value of labor share is too higher than the actual value, while if $d_M = 0.09$, the predicted labor share and the actual value is closer, therefore, we take $d_M = 0.09$. ϑ=0.435. The data of skill premiums and the relative supply of skilled labor in the United States (H/L) are both come from Autor (2014), but H/L is recalculated based on his data. He reported the data H/(H+L), we used H/L which increases from 0.23 in 1963 to 1.05 in 2012.

(2) Calculation results.

According to Autor (2014), the relative supply of skilled labor in the United States increased from 0.23 in 1963 to 1.05 in 2012, the skill premium increased from 1.474 to 1.958, and the labor share decreased from 0.634 to 0.563. Under the given values of parameters previously, the corresponding relative supply of skilled labor increases from 0.23to 1.05, the skill premium increase from 1.240 to 2.026, and the labor share decreases from 0.656 to 0.629, and the economic growth after filtering change from 2.6 to 2.1, as predicted by the model showing in figure 3, 4 and 5. The predicted results of the model have some consistency with the actual data, but the accuracy is not very high, especial the economic growth. However, this is not surprising, as there are many factors that affect the skill premium and labor share and economic growth, and it cannot be fully explained by the relative supply of skilled labor alone.

Figure 3 shows the impact on premiums of H/L. The model predicted that the skill premium will continue to rise as the relative supply of skilled labor increases continuously. The significantly different in skill premium between the predicted one from the actual one is that initially increases and then decreases in the early stages. Acemoglu (2002) argues that this difference reflects the short-term effect and long-term effect of the increase in relative supply of skilled labor, while predicted premium in Figure 3 shows only the steady-state equilibrium results of the model corresponding to the relative supply of skilled labor.

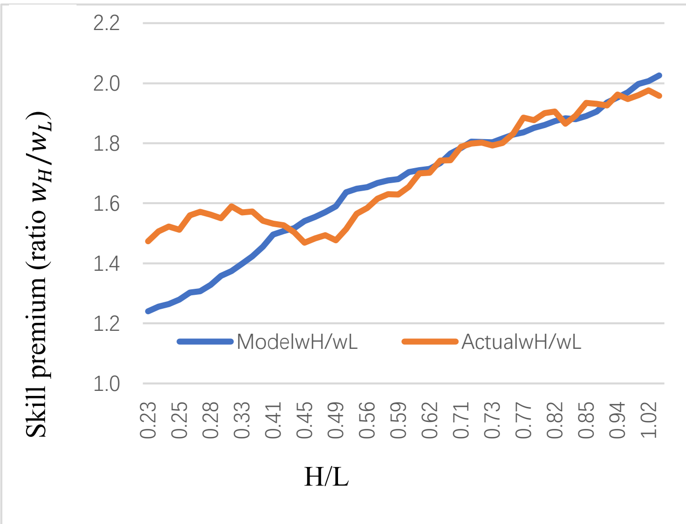

Figure 3: the impact on Skill premium of H/L

Figure 4 shows the impact on labor share of H/L. Under the given parameter values, the model predicts that the labor share is the minimum 0.613 when H/L=0.59 before H/L=0.59, the labor share decreases continuously, and after that it increase continuously as H/L increases, and return to level 0.657 around H/L=1.56, which is close to the labor share when H/L=0.23. But the actual data shows that after H/L=0.8, the labor share experienced an accelerated decline and dropped to 0.563 in 2012. Due to the fact that the actual measured H/L data only reaches a maximum of 1.05 in 2012, it is impossible to observe the specific changes in the labor share after the H/L continues to rise. Therefore, there is a significant gap between the predicted data of the model and the trend of labor share changes after H/L=0.8. However, due to many factors that affect the actual labor share, it cannot be denied that an increase in the relative supply of skilled labor will lead to an increase in the labor share from a decrease. The reason for the deviation between theoretical predictions and actual data is not only the omission of some other potentially important factors in the model, but also the measurement of the relative supply of skilled labor. It may not be accurate to represent the ratio of skilled labor to unskilled labor as the ratio of college students to non-college students. For the US economy after 2000, the technological revolution represented by the Internet had a significant impact on the economy. Therefore, skilled labor and unskilled labor perhaps should emphasize the Internet skills, rather than just the difference between college and non-college graduated. Therefore, more empirical research is needed on the impact of the relative supply of skilled labor on the labor share of income.

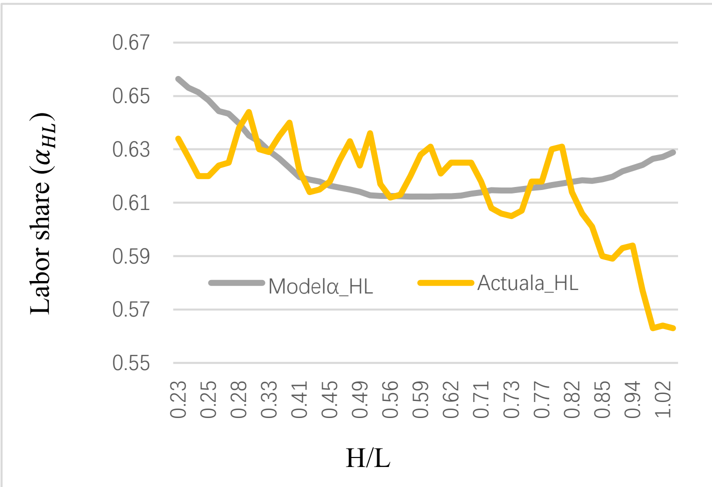

Figure 4: the impact on Labor share of H/L

Figure 5 shows the impact on economic growth of H/L. Unfortunately, the model's prediction of economic growth deviates even more from reality. From 1963 to 2012, the economic growth in the United States appeared to be a U-shaped, while the model predicted an inverted U-shaped. But this model's predictions of skill premium, labor share, and economic growth are interdependent. If the conditions that the increase of relative supply of skilled labor leads to skill premium are met, the model predicts that the labor share will inevitably be U-shaped and economic growth will inevitably be inverted U-shaped as the increase of H/L, without additional conditions. Therefore, the importance of this model lies not in whether its predictions are consistent with reality, but rather in proving that based on acceptable assumptions in existing literature, once the relative supply of skilled labor affects skill premiums, must simultaneously affect labor share and economic growth.

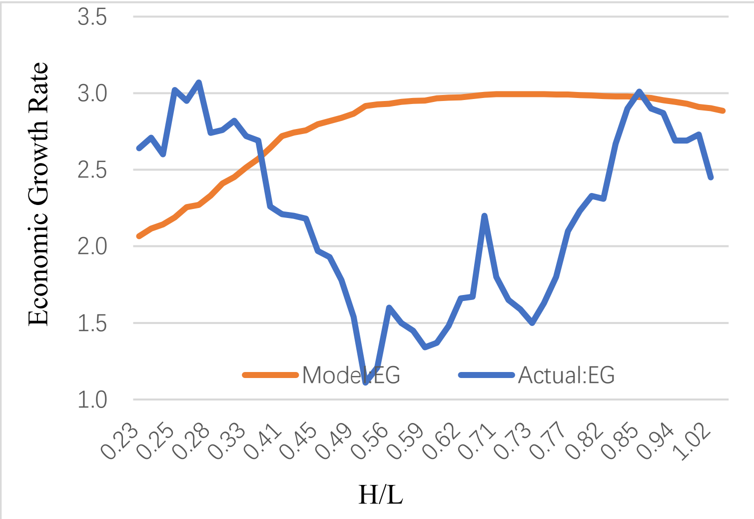

Figure 5: the impact on Economic Growth of H/L

Figure 6 shows the conjecture that the labor share and economic growth would be cycle if the labor forces are divided into skilled and unskilled labor periodically.

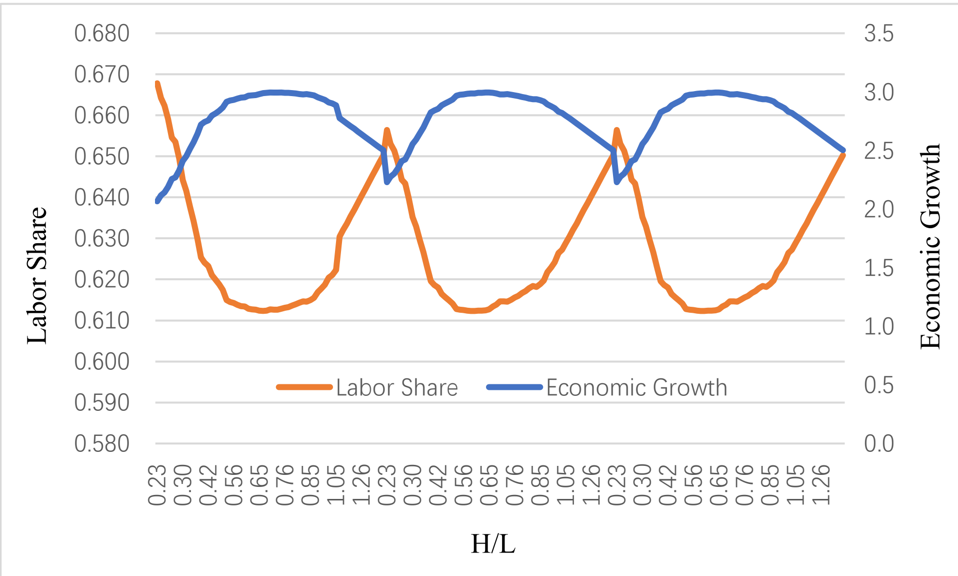

Figure 6: Periodic changes in the relative supply of skilled labor

5. Conclusion

The history of economic growth indicates that technical change is not only purely labor-augmenting, but also skill biased in the 20th century. This article develops a growth model with endogenous direction and bias of technical change, in which capital accumulation process considers investment adjustment costs, and firms can undertake capital-, skill labor- and unskilled labor-augmenting technological improvements. In the steady-state equilibrium, technical change can include all of them. However, if investment adjustment cost can be overlooked (implying capital supply with infinite elasticity), technical change will be purely labor-augmenting in steady state, that is, only skill labor- and unskilled labor-augmenting. If market size effect dominated prices effect in innovation and the relative supply of skilled labor increase continuously, then technical change will be not only purely labor-augmenting but also skill biased, which leads to the skill premium continue to rise, but the economic growth rate will first increase and then decrease in an inverted U-shaped change, while the labor share of income will first decrease and then increase in a U-shaped change. Although there is significant gap between the prediction value of the model and actual data, based on acceptable assumptions in existing literature, this article theoretically reveals that the relative supply of skilled labor may be an important factor affecting economic growth rate and labor share, which has not yet been noticed in existing literature.

In addition, although without clear empirical evidence currently, we can speculate that if skilled labor and unskilled labor are based on whether master the main general purpose technologies in the economy at that time, rather than solely on educational level, then with the technological revolution and spread, labor force may be periodically divided into skilled and unskilled labor, and the relative supply of skilled labor will also periodically increase from low ratio to high ratio. According to the results of the model, this will lead to economic growth rate and labor share being generally no trend on average over the long term, as described by the Kaldor facts, but on the other hand, they also exhibit cyclical changes of several decades in length, such as the labor share with 30-50 years cyclical changes discovered by Charles, Bridji, and Mcadam (2019) in historical data of countries such as the UK, United States and France.

References

Acemoglu, Daron. 1998. "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality." Quarterly Journal of Economics 113 (4): 1055–89.

Acemoglu, Daron. 2002. "Directed Technical Change." Review of Economic Studies 69 (4): 781–809.

Acemoglu, Daron. 2003. "Labor- and Capital-Augmenting Technical Change." Journal of European Economic Association 1 (1): 1–37.

Acemoglu, Daron. 2009. Introduction to Modern Economic Growth. Princeton, NJ: Princeton

University Press.

Acemoglu, Daron, and David Autor. 2011. “Skills, Tasks and Technologies: Implications for Employment and Earnings.” In Handbook of Labor Economics, Vol. 4B, edited by David Card and Orley Ashenfelter, 1043–1171. Amsterdam: Elsevier.

Acemoglu D, Restrepo P., 2018. “The race between man and machine: Implications of technology for growth, factor shares, and employment.” Am. Econ. Rev. 108(6):1488.542

Aghion, P. and Howitt, P., 1992, “A Model of Growth Through Creative Destruction", Econometrica, L X, 323-351.

ANGRIST, J., 1995, “The Economic Returns to Schooling in the West Bank and Gaza Strip”, American Economic Review, 85, 1065-1087.

Atkinson, Anthony B.， 2007, “The distribution of earnings in OECD countries”, International Labour Review, Vol. 146 (2007), No. 1–2.,p41-60.

Autor，David H., 2014, “Skills, education, and the rise of earnings inequality among the ‘other 99 percent’”, Science, 23 MAY 2014, New Series, Vol. 344, No. 6186 (23 MAY 2014), pp. 843-851

Barro, Robert J. and Xavier Sala-i-Martin, 2004, *Economic Growth*. Cambridge, Mass.: MIT Press.

Casey, Gregory and Ryo Horii, 2022, “A Generalized Uzawa Growth Theorem and Capital-Augmenting Technological Change,” ISER Discussion Paper 1157, Institute of Social and Economic Research, Osaka University.

Charpe, Matthieu, S. Bridji, and P. Mcadam,2019, “Labor share and growth in the long run.” Macroeconomic Dynamics:1-38.

Freeman, R., 1986, “Demand for Education”, in A. Orley and L. Richard (eds.) Handbook of Labor Economics, Chapter 6 , Vol. I (North-Holland)357-386.

Galor, O. and Maov, O., 2000, “Ability Biased Technological Transition, Wage Inequality and Economic Growth”, Quarterly Journal of Economics, 115, 469-497.

Goldin, C. and Katz, L., 1998, “The Origins of Technology-skill Complementarity”, Quarterly Journal of Economics, 113, 693-732.

Grossman, G. and Helpman, E., 1991, Innovation and Growth in the Global Economy (Cambridge, MA: MIT Press).

Grossman, Gene M., Elhanan Helpman, Ezra Oberfield, and Thomas Sampson. 2017. “Balanced Growth Despite Uzawa.” American Economic Review 107 (4): 1293–312.

Grossman, Gene M., Elhanan Helpman, Ezra Oberfield, and Thomas Sampson,2021. “Endogenous education and long-run factor shares”. American Economic Review. Insights 3(2):215.232

Grossman GM, Ezra Oberfield, 2021,“The Elusive Explanation for the Declining Labor Share”, NBER Working Paper 29165, http://www.nber.org/papers/w29165.

Hémous, David, and Morten Olsen. 2022. “The Rise of the Machines: Automation, Horizontal

Innovation and Income Inequality.” AEJ. Macroeconomics.

Irmen, Andreas, 2013, “Adjustment Costs in a Variant of Uzawa's Steady-state Growth Theorem”, *Economics Bulletin*, Vol. 33 No.4, pp. 2860-2873.

Irmen, Andreas, 2017, “Capital- and Labor-Saving Technical Change in an Aging Economy,” International Economic Review, Vol. 58(01), February 2017, pp. 261-285.

Irmen, Andreas and Amer Tabakovic, 2017, “Endogenous Capital- and Labor-Augmenting Technical Change in the Neoclassical Growth Model”, *Journal of Economic Theory* 170, pp. 346–384.

James, J. A. and Skinner, J. S., 1985, “The Resolution of the Labor-Scarcity Paradox”, Journal of Economic History, 45,513-540.

Kaldor N. 1961. Capital accumulation and economic growth. In The Theory of Capital: Proceedings of a Conference Held by the International Economic Association, ed. DC Hague, pp.177.222. London: Palgrave Macmillan.

Kiley, Michael T. 1999. “The Supply of Skilled Labor and Skill-Biased Technological Progress.” Economic Journal 109 (458): 708–24.

Li, Defu, 2016, “A Proof of the Invalidity of Proposition 15.12 in Acemoglu (2009)”, https://mpra.ub.uni-muenchen.de/75329/.

Li, Defu and Benjamin Bental,2022, “What Determines the Direction of Technological Progress?”, Tongji University Working Paper.

Lloyd-Ellis, H., 1999, “Endogenous Technological Change and Wage Inequality”, American Economic Review, 89, 47-77.

Mokyr, J., 1990, The Levers of Riches: Technological Creativity and Economic Progress, (New York: Oxford University Press).

Romer, P. M., 1990, “Endogenous Technological Change”, *Journal of Political Economy*, 98(5) (October), Part II, S71-S102.

Segerstrom, P. S., Anant, T. and Dinopoulos, E., 1990, “A Schumpeterian Model of the Product Life Cycle” American Economic Review, 80, 1077-1092.

Tinbergen, Jan, 1975. Income Difference: Recent Research. North-Holland Publishing Company, Amsterdam.

Trajtenberg, M., Henderson, R. and Jaffe, A. B., 1992, “Ivory Tower Vs. Corporate Lab: An Empirical Study of Basic Research and Appropriability” (NBER Working Paper4 146).

Young, A., 1993, “Invention and Bounded Learning by Doing”, Journal of Political Economy, 101, 443-472.