Making Decisions under Uncertainty: Value Chain Development

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Abstract

This presentation provides an overview of the various approaches and theories related to decision-making when faced with uncertainty. The paper's main focus is on the decision-making process itself, including how all its various components should be combined and how they should be reflected in decision rules. While the theme is not new, significant progress has been made in the past century in terms of developing decision-making techniques and measuring and managing uncertainty, largely due to the advancements in probability theory and fuzzy set theory. The goal of this paper is to develop a Value Chain for the Decision-Making process, achieved through the integration of the main components of the decision-making system under uncertainty, namely: (i) concepts of uncertainty, (ii) ways of thinking under uncertainty, (iii) creating models, and (iv) techniques of decision-making. These issues are considered in their dialectical relationship. The presentation will not delve into the specifics of each part of the system but rather aims to explain its essence and practical applicability. Both data-driven decision-making and non-quantitative approaches to making decisions are explored in the presentation.

Keywords: Uncertainty, Risk, Probability, Fuzzy sets, Metaphor, Narrative, Decision Theory, Expected Utility Theory, Prospect Theory, Possibility Theory, Real Options.

Introduction

Uncertainty is a concept that has excited the minds of thinkers, researchers, entrepreneurs, as well as ordinary people for an extended period of human civilization. In the most common understanding, uncertainty is interpreted as an alternative to certainty, complete determinism (predictability).

Since ancient Greek philosophy, the attitude to uncertainty has been radical, i.e., determinism has been excluded. It's a well-known Socrates statement: "As for me, all I know is that I don't know anything." Sometimes, attitudes toward uncertainty were expressed more artistically. According to Voltaire, "Uncertainty is an uncomfortable position. But certainty is absurd."

In ancient times, people faced the uncertainty of the future and its associated risks but could not counter them with anything rational to predict it to some extent. Therefore,

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in their projection of the future, they relied on oracles, soothsayers, shamans, and other fortune tellers.

Much later, humanity discovered the law of physics, proving that at the heart of matter, everything material in this world consists of, is uncertainty. This law was called the "uncertainty principle" and was first formulated by the outstanding German physicist Werner Heisenberg. The uncertainty principle postulates that it is impossible to determine a microparticle's position and momentum with the same accuracy. The boundaries set by this principle cannot be overcome by improving the means of measurement. At least for the time being, the uncertainty principle is considered a fundamental proposition of quantum mechanics, according to which every object in the universe behaves both as a particle and a wave.

Aside from the material interpretation of the world, it can be argued that some absolute force created the world uncertain. In other words, uncertainty is an intrinsic property of all things.

The events of recent years, and especially the last year, when russia's war against Ukraine began and continues, have made people realize that uncertainty can be threatening. The concept of radical uncertainty appears, that is, one that can destroy the general world order. It makes you wonder if this world order was perfect. Realizing that, in principle, there is nothing perfect, humanity begins to understand that this world order must make a quantum leap. This quantum leap will lead humanity to a new state that will still have the property of uncertainty but, at the same time, will radically eradicate those factors that threaten humanity's existence. True, there may be new ...

Russia's war against Ukraine was a threatening manifestation of the state of uncertainty of the existing world order for all humanity. It destroyed the previously considered solid foundations of this world order. But we need to see a positive side to this. The war has increased Ukraine's "degree of uncertainty" and its entire environment. And this made it possible to understand how fragile the world order was and, at the same time, what new opportunities were opening up for Ukraine in particular and humanity as a whole. The war gives a chance to make the next step of humane history…

1. The decision-making system

We proceed from the fact that the goal of meaningful human activity is to make a reasoned decision and its subsequent implementation. Almost everything people do is associated with making a decision and its subsequent execution. Therefore, building decision-making theories is nearly the same as describing the human activity. When making a decision, a person pursues a specific goal. In situations considered in decision theories, there is always a set of solution options. The task of decision-making is a problem of not randomly choosing one of the options. The choice in these situations is a purposeful activity. Hence, decision theory is concerned with goal-directed behaviour in the presence of options.

The decision-making process in the practice of ordinary people is often quite simplistic. The decision-maker, by his psychological characteristics, chooses one or another approach or criterion and, with its help, finds the best alternative from his point of view. There are times when this decision is the right one and benefits it. Sometimes, he makes erroneous decisions. The correctness or fallibility of a decision is tested by actual practice, known as the criterion of truth. A decision is recognized as correct if the decision-maker achieves the goal. It should be recognized that decisions are always made under uncertainty when the decision-maker needs complete information to ensure the correctness of this decision, that is, the achievement of the set goal.

Fortunately, we do not need to make a decision every day. During almost all activities, there are periods in which we have to make decisions and other periods in which we are simply executing what we have decided. Decision theory deals with the first kind of human activity.

It is also worth dwelling on who this "decision-maker" is and in what conditions the decision-making process takes place. If decisions are made at the household level, for example, to buy a new refrigerator or repair an old one, we do not need to bother much about the criteria. Such decisions are often made intuitively or by assessing the current budget.

In making more significant decisions, there needs to be essentially more than intuition and simple financial calculations. More fundamental approaches are required because the cost of a mistake is higher and can often lead to fatal consequences. And here, a more comprehensive system of criteria is required.

Modern decision-making approaches began in the middle of the 20th century as a joint effort of several academic disciplines. Decision theory, as a whole, is a discipline invariant to the subject area of application. It is equally successfully used by economists, statisticians, psychologists, political scientists and sociologists, philosophers, and entrepreneurs. A medical doctor's determination is also far from intuitive since the price of a mistake is often a human life. Everyone solves his problem during his professional activity and even private life. A political scientist is likely to learn voting rules and other aspects of collective decision-making. A psychologist will study people's behaviour when they make decisions, and a philosopher will look for ways to describe reality and draw generalizing conclusions. Most of all, the theory of decision-making methodology excites the minds of economists. Not without reason, many of them developing economic theories have significantly contributed to the decision-making technique. Some more on this later. Finding significant overlap in approaches to decision-making in different spheres of human activity, it can be concluded that decision theory benefited from various methods that researchers with different experiences applied to the same or similar problems.

It is important to emphasize that most decision-making techniques are predominantly quantitative, involving adequate mathematical models. Developing mathematical models is always the prerogative of people with analytical and mathematical mindsets. Not always does the person making the decision have this property, but the decision should be made, and others should be convinced of its correctness. What to do in this case? Often, decision-making practices exist as ways of convincing the decision-maker himself and his environment with the help of non-quantitative models. Frequently such models use narratives and metaphors used in these narratives. We will call these models semantic. The correctness of the decision made, of course, will be proved by practice. Still, based on the semantic model, the internal conviction of the decision-maker and his team is in no way inferior to the correctness of the decision-making based on a quantitative approach. Confidence in the correctness of the decision often exceeds belief based on using quantitative methods. We are searching for a proper form in the decision-making methodology, and we will do this starting with this publication.

It is especially worth noting that most decisions are not momentary. They require time to comprehend the event or phenomenon with which the person is in contact with the decision being made. Moreover, the decision-making process itself is very complicated and requires a particularized and systemic view, which we will do below.

Fig. 1.1 challenges introducing the whole system, like a pyramid of four components. In this presentation, we will briefly describe these parts and try to link them together. First, we emphasize that it is complicated to consider these components separately since they are dialectically strongly interrelated. Therefore, we will constantly turn to other parts when describing each.

As one can see from the figure, uncertainty is the foundation of the entire system. Uncertainty is recognized as an immanent state of nature and the world. And before you begin to understand how to manage your behaviour under uncertainty, you should understand how uncertainty works.

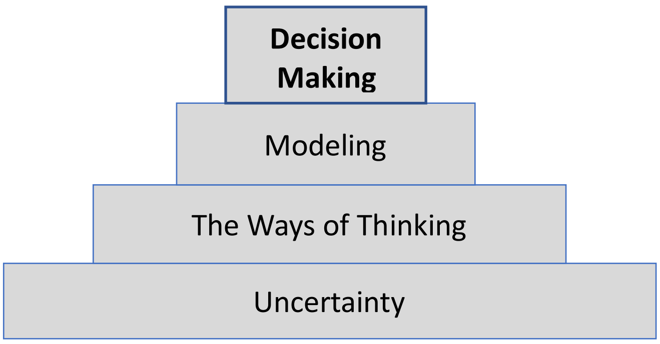

Fig. 1.1. Decision-making system

Once we realize the inevitability of uncertainty, we must accept it. Having taken it, we must develop the right attitude to uncertainty. It cannot be identified with negative manifestations, as it is typical in people's lives. It is necessary to understand that uncertainty contains new opportunities and is full of new meanings and values. If you want, you should fall in love with uncertainty, hoping that this love will be mutual. This can be done if we learn to think adequately with uncertainty. Probabilistic thinking is the most common way to think about uncertainty, which postulates the ambiguity of future events and the desire to assess their chances. This path is not the only one – there is another constructive way of thinking in the form of fuzzy sets. The decision maker must choose his way of thinking to imagine what may happen in the future, that is, to build a model.

In intelligent human activity, the model always precedes decision-making. The model formalizes the way of thinking of the decision-maker in the form of a specific image calling for action. A typical model for decision-making is mathematical; it is designed with the help of symbols. As we noted above, there are other ways to model, namely semantic models. There can exist other ways, for example, graphics. Ultimately, works of art can also be seen as a way to model a process or phenomenon, eventually awakening the desire to act. Suffice it to recall Beethoven's Ninth Symphony.

Making a decision is the final act in the general system. We are ready to make a decision. But there must be some general rules for this. By creating a model, for example, using probabilistic representations, the decision-maker can choose from a variety of alternatives the one that suits him. It is necessary to use a specific criterion for decision-making and, following it, choose the best choice. With mathematical models, the situation is quite simple. When a criterion is selected, an algorithm for calculating the best option immediately appears. And then, the responsibility for the decision is partially transferred to this algorithm. The use of non-quantitative methods leaves this responsibility to the decision-maker. This increases the chances of making a mistake.

Be that as it may, having realized the inevitability of uncertainty and learned to think adequately and create a model, it is necessary to complete the process and make a decision. We will dwell in some more details on each of these system components.

2. Uncertainty

So, uncertainty is a proper of our world and a consequence of our attempts to understand it. And it is worth attempting to understand all the intricacies of uncertainty for the decision-maker.

First, let's understand the sources of the ***origin of uncertainty***. The generally accepted point of view defines two kinds: ***aleatoric uncertainty and epistemic uncertainty***. And we consider them in some detail. But there is a solid intention to introduce one more type of uncertainty, which we will call *semantic*. So, let's devote some attention to the kinds of uncertainty regarding origin.

First, we encounter uncertainty when surrounded by an unstable, ever-changing physical environment. This kind of uncertainty is called *aleatoric uncertainty and reflects the world's stochastic nature*. The second manifestation of uncertainty is the lack of knowledge or understanding of events and phenomena. Such uncertainty is called *epistemic uncertainty.* It is secondary to physical uncertainty. When people try to guess the roll of a die, the sex of a child, and the outcome of a horse race, they deal with aleatoric uncertainty. Aleatoric uncertainty might transfer into epistemic. Here is a simple example: in tossing a coin before the coin flip, we experience aleatory uncertainty; if we flip the coin and hide the result, our psychology switches to epistemic uncertainty. Usually, people experience epistemic uncertainty as more aversive. At the same time, people most often face the manifestation of epistemic uncertainty. Here are the common cases:

- Uncertainty is a measure of the sufficiency of the information; decisions must be made in conditions of limited information, and therefore there is a risk of making wrong decisions regarding the goals set.
- Uncertainty is the possibility of choosing alternatives and the variability of choices; a variety of options causes multiplicity, and it is challenging to establish criteria for optimality.
- Uncertainty creates an insufficient quality of information (reliability, completeness, value, relevance, clarity). Evaluation of information in uncertainty is associated with the reliability of information and data, their completeness and objectivity.
- Uncertainty is a natural constraint on the manageability and stability of a socio-economic system. In a socio-economic system, there is a threshold of controllability and strength caused by uncertainty.
- Uncertainty generates unpredictability in participants' behaviour in a conflict of interest. Participants in action may have different interests and goals. Do the interests and objectives of the participants coincide? There is generally no answer to this question - this creates uncertainty.
- Uncertainty is an attributive source of risk. A priori, the risk is directly dependent on uncertainty: *as uncertainty increases, so does risk*. The magnitude of the increase may vary due to the elasticity of risks concerning uncertainty.

We will dwell in more detail on the later manifestation of uncertainty. Let's look at Merriam-Webster's Collegiate Dictionary. We will find a definition according to which risk is the “possibility of loss or injury”, and to take a risk means “to expose to hazard or danger”. This is how most people understand it, i.e. risk is determined mainly with a negative connotation.

More comprehensive thinking is introduced by Eastern philosophy (see Fig. 2.1.). Chinese characters, which denote risk, very accurately characterise the features of this phenomenon.

Fig. 2.1. Eastern Philosophy of Risk

The first hieroglyph is translated as "danger", and the second is "opportunity". In the best traditions of the East, the Chinese were able to very subtly note that, on the one hand, the risk is associated with the danger of losing something or even everything. On the other hand, it provides additional opportunities if you take advantage of the situation competently.

Following the Chinese logic, in Fig. 2.1, we will present a 2x2 matrix of possible errors that generate risks. Suppose you are given two options: accept or reject a decision regarding any crucial case of your business or private life. For example, you are invited to invest a large amount of money in some project. The decision may be wrong or right. Wrong means you lose your money, and the right means you become more affluent. You can accept it or reject it. This is how the 2x2 matrix is obtained.

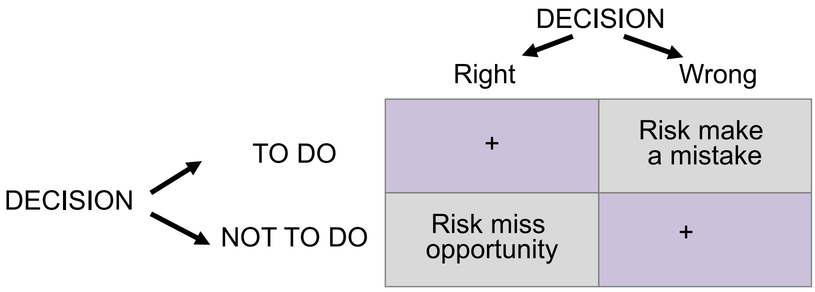

Fig. 2.2. The Error Matrix

As you can see, there are two risks: a) rejecting an option if it is correct and b) accepting an option if it is wrong. Every person in their life faces such risks. Most people are afraid to make the wrong decision, and we'll call it a first-kind risk, essentially a risk of making a mistake. In our example, it is a risk of losing money. The risk of missing out on an opportunity (second-kind risk) to become more affluent is experienced by fewer people.

We can determine their main difference if we divide all people into two categories: ordinary and ambitious people. Concerning uncertainty, ordinary people and ambitious ones differ in that the formers try to eliminate uncertainty and make the world more deterministic and predictable. They do not like risks and try to mitigate them or avoid them altogether. As for ambitious people, they behave in the exact opposite way. They love uncertainty because they are not afraid of risks and believe that the higher the degree of uncertainty, the greater the chances of discovering a new meaning of activity, creating new value. It is not for nothing that people say, "who does not take risks does not drink champagne." It's about ambitious people. They may want to increase the degree of uncertainty, creating more opportunities to find new meanings of activity and creation of new values.

Now let's come to the third kind of risk mentioned above. There is another significant source of uncertainty, which we call ***Semantic Uncertainty***. People constantly experience the negative impacts of ambiguous interpretations and understanding particular concepts and terms. The uncertainty of the interpretation of words creates uncertainty in understanding the text. Frequently this leads to misunderstanding in the transmission of meanings and, as a result, to erroneous choices.

To decompose this kind of uncertainty, let us start with Nalimov[2]. In his *Probabilistic theory of meanings***,** he assumes that consciousness is a triad consisting of meaning, texts, and language. The text reveals the meaning through the sign system of the language. The main assumptions of the approach are the following: (1) every word has a variety of meanings; (2) a text consisting of words also has a variety of meanings; (3) the variety of meanings is modelled in the form of the probability distribution.

G*eneral theoretical standpoints* are needed to study the phenomenon of Uncertainty.

We start with the approaches to understanding uncertainty advanced by **Keynes** and **Knight**. According to John Keynes[3], uncertainty is ontological (especially concerning social phenomena), and probabilistic logic is more adequate to the world of uncertainty in which we live than ordinary logic. Probability refers not to the characterisation of an event but to our assessment of the truth of a statement that a given event will occur (under certain conditions).

Wanting to find a place for the concept of uncertainty in the overall decision-making system Frank Knight[4] suggested differing risk and uncertainty. This distinction is based on an analysis of probability situations. Situations of 'risk' are ones in which it is possible to determine numerically definite probabilities (usually statistical frequencies), whereas situations of 'uncertainty' are characterized as impossible to do so. Since statistical probabilities are generally seen as a property of the external world, Knight's distinction seems to presuppose an objective interpretation of probability.

This juxtaposition of uncertainty and risk seems more semantic (i.e., relating to meaning in language and logic) than meaningful in relation to decision-making. Knight's distinction between risk and uncertainty is theoretically meaningless and practically irrelevant to the practice of making decisions.

A very interesting taxonomy came from the work of **Bradley and Drechsler** [5]. This point of view distinguishes three qualitatively different types of uncertainty - ethical, option and state space uncertainty. *Ethical uncertainty* arises if the decision maker cannot assign precise utilities to consequences. *Option uncertainty* arises when he does not know what precise consequence an act has at every state. Finally, state *space uncertainty* exists when a decision-maker is unsure how to construct an exhaustive state space. All these types of uncertainty are characterized along three dimensions - nature, object, and severity - and the relationship between them is examined.

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[2] Nalimov V.V. (1989). The spontaneity of consciousness. Moscow:, Prometheus (in Russian)
[3] Keynes, J., M. (1921) A Treatise on Probability, The Collected Writings of John Maynard Keynes, Vol. 7, London, Macmillan, [1973]
[4] Knight, F. (1921). Risk, Uncertainty and Profit, University of Chicago Press: Chicago.
[5] Bradley R., and Drechsler M., (2013) Types of uncertainty. Erkenntnis, online. pp. 1- 29. ISSN 0165-0106

This three-dimension system is useful for deeply understanding the essence of uncertainty. The first dimension (**nature**) relates the kind of uncertainty to the nature of the judgement being made. Three distinguished primary forms of uncertainty are modal, empirical and normative.

The modal uncertainty arises in connection with our possibility judgements: those concerned with what is conceivable, logically possible, feasible, and so on. For instance, in thinking about how to represent a decision problem, we need clarification as to what the world's possible states are or what consequences could follow from the choice of action. ***Empirical uncertainty*** is uncertainty about what is the case in actual circumstances. It arises in connection with the decision maker's descriptive judgements. Finally, ***normative uncertainty*** concerns what is desirable or what should be the case. It arises in connection with evaluative judgements of decision-makers.

A second dimension relates to the ***objects*** of the judgements that the decision-maker makes, the features of reality that his conclusions are directed at. Here we distinguish two fundamental classes of object – facts and counteracts – and associated forms of uncertainty. Factual uncertainty is uncertainty about the actual world, the way things are – the facts. Counterfactual uncertainty is uncertainty about how things could or would be.

Finally, the third dimension, **severity**, relates to the difficulty a decision-maker can face with different severities of uncertainty in making judgements. There exist four levels of severities (1) ignorance, when a decision has no judgement-relevant information; (2) severe uncertainty, when he only has enough information to make a partial or imprecise judgement; (3) mild uncertainty, when he has sufficient information to make a precise judgement; (4) certainty, when the value of the judgement is given or known.

The categorization of uncertainty discussed above seems complicated from the point of view of practical application. At the same time, these descriptions are very useful as they force us to look deep into the phenomenon and better understand it.

Along with the above descriptions of the phenomenon of uncertainty, this presentation proposes a more narrow-ranging classification and considers its implications. Our task is to offer the most straightforward possible idea of the decision-making system, focusing on applied aspects. First, we note that uncertainty should be considered between two extremes. The first extreme is characterized by a state of complete certainty or determinacy when the consequences of actions are known. It is not attractive from the standpoint of decision-making since there is no alternative. The opposite extreme can be characterised as absolute uncertainty (ignorance). It seems appropriate here to use the notion of "chaos" and to formalize it accordingly. We consider uncertainty to be a state of nature or our understanding of nature. And it is supposed to be done for all three origins of uncertainty, namely, aleatory, epistemic, and semantic. And we consider the risk as a primary consequence of the uncertainty. If we compare the three states of nature, we can figure out the following differences:

- In a state of chaos, we can neither model nor assess the risks and opportunities.
- Under uncertainty, we can assess risks, and there are approaches for this (which we will briefly touch upon below),
- In a state of complete order, such a task is simply meaningless.

Moreover, another distinctive feature of the state of uncertainty is the decision maker's responsibility for the results of his activities.

Chaos theory is quite seriously developed, but no one considered it from the standpoint of the extreme of "uncertainty". We can do this briefly. In everyday meaning, chaos is unpredictable or random behavior. Chaos usually carries a negative connotation associated with unwanted disorganization or confusion.

From the point of view of **metaphysics,** chaos can be represented by two options[6]. Firstly, chaos can result from the disintegration of something harmonious, beautiful, and orderly. In this case, chaos is tragic. Secondly, chaos is understood as what precedes order, harmony and beauty. It carries many possibilities and lives in anticipation of what will come out of it and what will give it meaning.

As for the theoretical background of chaos, it is developed quite fundamentally. One can find a good description of chaos as a scientific subject in a laconic book by Leonard Smith[7]. Considering chaos as the ultimate case of uncertainty, we realize it has a fundamentally different nature. And those patterns present in the study of uncertainty are absent when considering chaos. What are the main properties of chaos as a scientific category? Chaos, as defined in chaos theory, is not a random disorder. Chaos theory is more complex than the traditional cause-and-effect view of systems. Chaos theory helps explain the "order under disorder" of systems that cannot be defined or analyzed linearly. Chaos embodies three essential principles: 1) extreme sensitivity to initial conditions, 2) cause and effect disproportionate, and 3) nonlinearity.

We consider chaos in a general system related to uncertainty only because states of chaos and uncertainty have the same consequences that must be considered in the decision-making process. These consequences are as follows: in both a state of chaos and uncertainty, the decision-maker can either achieve his goal or not achieve it. In both cases, it is necessary to find the right way of thinking, build an adequate model and develop criteria for decision-making. In the following sections of this presentation, we will deal with this process only for a state of uncertainty.

3. The ways of thinking

Turning to the second component of the system, we immediately discover the ambiguity of the interpretation of the concept of "thinking"**.** Starting with Aristotle, the basis of the performance of the process of thinking was the principle of worldview. According to this principle, the perception of the natural world was unambiguous and

[6] Anthony Sourozhsky, (2019), Chaos, law, freedom. Conversations about meanings. "Metropolitan Anthony of Sourozh Foundation, 2019"
[7] Smith L. (2007). Chaos. A very short Introduction. Oxford University Press.

finite. The understanding of uncertainty came later. According to Descartes, one should doubt everything except one's existence. His reasoning in 1641, "Cogito ergo sum" (I think; therefore, I am), informs us that our minds exist. Unfortunately, he could not prove that our thinking fully corresponds to the state of nature and how we can fully comprehend the truth and what needs to be done to get as close as possible. And here, there is a need to rethink the actual way of thinking.

According to **John Locke[8],** sensation, remembrance, and contemplation are modes of thinking. When the mind turns its view inwards upon itself and contemplates its actions, thought is the first that occurs. In it, the mind observes a great variety of modifications and, from thence, receives distinct ideas. Probability is one of such great variety. " Probability is the appearance of the agreement upon fallible proofs… so the probability is nothing but the appearance of such an agreement or disagreement by the intervention of proofs, whose connexon is not constant and immutable, or at least is not perceived to be so, but is, or appears, for the most part, to be so, and is enough to induce the mind to judge the proposition to be true or false, rather than the contrary".

Ludwig von Mises[9] turned to the problems of uncertainty and probability, setting himself on creating a universal deductive science of human behaviour, known as praxeology, and its particular part - the theory of market behaviour, or catallactics. Mises considers uncertainty (albeit related to the limitations of our knowledge) to be a condition for freedom of choice. "The most that can be achieved relative to reality is a probability".

George Edward Moore[10] proceeded from the need (albeit ideally) to assess the probability of all possible consequences of our actions and to determine the expected value (positive or negative) to which these actions can lead. In the appeal to probability, understood in the traditional way, he saw the manifestation of rationality. "We can, in all likelihood, only claim to take into account the consequences of actions during the so-called 'near' future …

The modern view of the world around us has been successfully expressed by **Richard Feynman**[11]*:* "…one thing is, I can live with doubt, and uncertainty, and not knowing. I think it's much more interesting to live not knowing than to have answers which might be wrong. I have approximate answers and possible beliefs, and different degrees of certainty about different things. But I'm not absolutely sure of anything...".

At one time, the founder of axiomatic probability theory, Andrew Kolmogorov, said: "We have at least one severe advantage – we own probabilistic thinking[12]." Probabilistic thinking occupies an intermediate position between deductive logic and intuition. Central to probabilistic thinking is "assumption": a person assumes that an

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[8] John Locke, An Essay Concerning Human Understanding by (First published 1690), The Pennsylvania State University, 1999
[9] Mises, R. von. 1964. Mathematical Theory of Probability and Statistics. New York: Academic Press.
[10] Moore G.E. Principia Ethica, Cambridge University press, 1903.
[11] Feynman R. "Surely You're Joking, Mr Feynman!": Adventures of a Curious Character, Vintage Books, London.
[12] Nalimov *V. Spontaneity of consciousness*: Probabilistic theory of meanings and semantic architectonics of personality. Moscow.: Prometheus publisher, 1989.

event is about to happen, but he's not sure. They must assess the chances that this event will occur. Based on this assessment, he must make a specific decision, for example, to bet on a particular horse in a race.

And now, let's try to understand what probabilistic thinking is. It is generally accepted that there are three ways to think probabilistically: classical (logical), frequency and subjective.

Historically, the ***classical definition of probability*** was the first to appear. It was a consequence of the desire to predict the appearance of an event in a series of equal-possible events. The main property of such a situation was the symmetry of the outcomes. When we play dice, all the numbers on the faces of the die have the same chances of falling out as a result of tossing. And that chance was measured at 1/6. The symmetry feature is also used when two dice are rolled. It is easy to calculate that the likelihood that the dropped sum on the two dice is equal to 7 is greater than the sum of 8 since 6/36 is greater than 5/36. And you will always win, betting on 7 against 8, although you will have to play for a long time. There are many such examples, and all of them are characterized by two features: the symmetry of elementary outcomes and logic.

A prerequisite for the use of ***the frequency approach*** is carrying out repeated experiments. In each experiment, a particular event *A* may or may not appear. The probability of the occurrence of this event is estimated only after conducting a series of experiments under the same conditions. And it is equal to the ratio of the number of trials in which event *A* appeared to the total number of tests. If there are few trials, then the resulting assessment does not cause much confidence in the decision-maker. His confidence increases with the increase in the number of experiments.

The classical and frequency interpretations of probability are objective in that they are independent of the decision-maker's opinion. At the same time, the objective understanding of probability has limited application in practice since, in most practical cases, the decision-maker does not encounter situations where there is a symmetry of outcomes or a sufficient number of experiments can be conducted.

We will not be able to give the exact rapid characterization of ***subjective probabilities*** as we did above. The fact is that here we are considering the more subtle matter since when estimating probabilities, a person with all his psychological characteristics is directly present. In the most common sense, subjective probability is a way of stating our belief in the validity of a random event. A subjective probability is anyone’s opinion of the chance for an event when he/she has no sample of trials (so he cannot use relative frequency) and no theory (so he cannot use theoretical probability).

The subjective theory of probability was developed by ***Frank Ramsey[13], Bruno de Finetti[14] and Leonard Savage***[15]. Their versions of the theory are broadly similar. Let's

[13] Ramsey, F. (1931). Truth and probability. In The foundations of mathematics and other logical essays, ed. R. Braithwaite and F. Plumpton. London: K. Paul, Trench, Truber and Co..

[14] de Finetti, B. (1975): *Theory of Probability*, Vol. 2, New York: John Wiley and Sons.

[15] Savage, L. (1954): The foundations of statistics. New-York, John Wiley.

consider the subjective probabilities as betting quotients. The fundamental Ramsey-De Finetti Theorem appears: "***A set of betting quotients is coherent if and only if they satisfy the axioms of probability***". The set of axioms of probability was suggested by Andrew Kolmogorov[16] as the generalization of requirements to the probabilities regardless of their interpretation. The axioms can be summarized as follows:

1. The probability *P(A)* of any event *A* in a set of elementary events Ω, is a non-negative number in the interval [0,1].
2. The probability that at least one of the elementary events in the entire sample space will occur is equal to 1: $P(\Omega) = 1$.
   For a mutually exclusive set of events $A_1, A_2, \ldots, A_N$, the probability of an aggregate of those events is the sum of the probabilities of the individual events:
$$P(A_1 \cup A_2 \cup \ldots \cup A_N) = \sum_{i=1}^{N} P(A_i).$$

The essence of axiomatics lies in the fact that it generalizes any proposal for the construction of a technique for estimating probabilities. That is, any probabilistic technique, including subjective, must follow these axioms.

Ramsey – De Finetti Theorem gives a rigorous foundation to the subjective probability theory. The first general idea was to measure degrees of belief by betting. This was made precise by introducing betting quotients. The logical conclusion of this chain was that for betting quotients to be coherent, they must satisfy the axioms of probability and thus can be regarded as probabilities.

Savage's contribution to the subjective probability way of thinking is that he paved a bridge between subjective probabilities and making decisions, i.e. made the subjective probability theory more practical. Savage thus ties together the idea of subjective probability advocated by Ramsey and de Finetti with the concept of expected utility derived by ***von Neumann and Morgenstern***. We will come back to this issue later while discussing decision theories.

Now we stress one more critical issue of subjective interpretation of the probabilistic way of thinking. From the classical point of view (Ramsey, de Finetti), a subjective probability of any event is a matter of personal preferences. But there exists another judgment (see, for example, ***Richard Jeffreys***[17]) which argues that there is, given the totality of information that you have access to, a unique admissible probability assignment. And this judgement brings us to the Bayesian approach to modelling uncertainty. ***Tomas Bayes***[18] outlined a method known as ***Bayes' rule*** for updating probabilities in light of new information. Bayes' method does not specify how the prior

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[16] Nalimov V. V. (1989) Spontaneity of consciousness: Probabilistic theory of meanings and semantic architectonics of personality. Moscow.: «Prometheus»,
[17] Jeffrey R., (2004) Subjective Probability: The Real Thing, Cambridge University Press.
[18] Bayes T. and Price R. (1763). «An Essay towards solving a Problem in the Doctrine of Chance». Philosophical Transactions of the Royal Society of London 53: 370—418.

probabilities to be updated are determined. However, if we combine the interpretation of subjective information as a degree of belief with factual data, we can reduce the ambiguity regarding the assignment of subjective information and use it to make decisions. We will discuss the contribution of the Bayes rule in the next paragraph.

Let's look at ideas and approaches that are very close to the subjective probabilities considered, if not coincide with them completely. This will allow us to enrich the overall picture of the description of uncertainty by subjective probabilities.

We will turn again to ***John Keynes***, who defended the view that subjective probability should first obey the laws of logic, that is, be rational and do not come from the moods of the subject who expresses his degree of belief. In his fundamental work «A Treatise on Probability»[19], he argues that probabilistic logic provides a natural way to draw conclusions and make decisions. For Keynes, probability is a logical 'probability' relation between a set of evidential propositions and a conclusion. Probability is a degree of rational belief, not simply a degree of belief. As he argues: " ... in a sense important to logic, the probability is not subjective. It is not, that is to say, subject to human caprice. A proposition is not probable because we think it so. Once the facts are given which determine our knowledge, what is probable or improbable in these circumstances has been fixed objectively and is independent of our opinion. Therefore, the Theory of Probability is logical because it concerns the degree of belief it is rational to entertain in given conditions and not merely with the actual beliefs of particular individuals, which may or may not be rational."

Later, the imperfection of the process of assigning subjective probabilities founds its fundamental confirmation in the works of ***Kahneman and Tversky***[20]. They started with the point that beliefs concerning uncertain events are expressed numerically as odds or subjective probabilities. What determines such beliefs? How do people assess the probability of an uncertain event or the value of an uncertain quantity? In many cases, they proved that biases in judgments reveal some heuristics of thinking under uncertainty. They are (i) representativeness, (ii) availability, and (iii) adjustment and anchoring. As an example of heuristic representativeness bias, it is a well-known story about a guy maned Steve, who might be a librarian or farmer. You met Steve and found that he is a shy guy. And decided that for sure (with a probability of 90%) he is a librarian. But consider that the number of librarians is 50 times less than that of farmers. You can arrive at another judgment: the probability of the event "Steve is a librarian" is calculated to be 1/6, which is around 17%. It is an excellent example of the bias named representativeness.

If Keynes's sentences pull subjective probability in the direction of logic and rationality, then ***Karl Popper's*** theory[21] gravitates towards the psychological factors of the decision-maker. He argues that the essence of probability is **propensity**. He started with the question: What is the probability of a single event? Even when we have only

[19] Keynes, J., M. (1921) A Treatise on Probability, The Collected Writings of John Maynard Keynes, Vol. 7, London, Macmillan, [1973]
[20] Tversky A., Kahneman D. Judgment under Uncertainty and Biases, Cambridge, University Press, 1982.
[21] Popper, K.: A World of Propensities. Thoemmes Press, Bristol, 1995,

one trial, there is a propensity for a specific result because of objective experimental conditions entangled with the object of inquiry. According to Popper's theory, we live in a world of propensities. Ordinary causality is a particular type of propensity with probability 1. Here are the main **postulates of Popper's ontology:** Propensities are the ultimate ontological reality and the basis of the entanglement between subject and object. Nevertheless, he found the link with not-subjective probabilities, arguing that propensities can generate empirically observed relative frequencies and can be measured by a frequentist concept of probability.

From the point of view of the general ontology of human beings, the theory of ***Patrick Suppes***[22] deserves attention. He uses concepts of probability to deal with metaphysical and epistemological matters. He claims that it is probabilistic rather than merely logical concepts that provide a rich enough framework to justify both the ordinary ways of thinking about the world and scientific methods of investigation. Here are the main postulates of his theory. The fundamental laws of natural phenomena are probabilistic rather than deterministic in character. Causality is probabilistic, not deterministic. Consequently, no inconsistency exists between randomness in nature and the existence of valid causal laws. Our ways of thinking about rationality are intrinsically probabilistic in character. Suppes' theory can serve as a good generalization of the ways of thinking listed above since it does not contradict any of them.

There is another way to think probabilistically. It is represented by ***Dempster and Shafer Theory***[23]. It provides a mathematical basis for interval estimation of an event's probability (as a degree of belief) after combining individual parts of the initial information about this event obtained from different sources. It assumes the existence of some true probability of event *S, P(S) and* provides two extremes: "Belief $Bel(S)$ and Plausibility $Pl(S)$. In other words, by understanding the impossibility of accurately assignment of probability, the authors offer some vague representation of the estimated probability, thereby reducing the need to assign a specific value.

There exists one more approach to thinking uncertainty which we call ***radical subjectivism.*** This approach was proposed by the British economist ***Shackle***[24]. According to his theory, under uncertainty, a person sees things as he would like and then imagines the consequences of his future actions; these consequences may seem to him either extremely favourable or not. Because of these submissions, he will make a decision in the future. Shackle's contention is that granted that the very construction of probability calculus relies on a complete knowledge of the structure of the state of nature, in reality, individuals do not have such knowledge. Individual choices are made between alternatives which are subjective representations of alternative future sequels to action; choices are not between future alternatives themselves.

[22] Suppes P., (1984) Probabilistic Metaphysics. Oxford: Basil Blackwell.
[23] Dempster A (1967) Upper and Lower Probabilities Induced by a Multivalued Mapping. Annals Math. Stat. 38:325–339
[24] Shackle G. (2010). Uncertainty in Economics and Other Reflections, Cambridge Press, UK.

Here is the main analytical point based on Shackle's theory. To decision-makers who adopted the above mentioned thoughts, this point should become an indispensable analytical reference in their effort to represent decisions under genuine uncertainty. Based on this point, Shackle developed a formal theory opposed to the Bayesian approach intended to capture both the mental processes and the non-repetitive, often irreversible, nature of actual decisions. Shackle aimed to emphasise the typically imprecise domain of actual decisions by focusing on human judgements' subjective and idiosyncratic nature.

While we understand the theoretical infallibility and practical utility of the probabilistic way of thinking, we must nevertheless understand that it is not unique. There are other ways of thinking about understanding uncertainty. If we allow uncertainty to be treated as fuzziness, it becomes possible to formalize this representation with the help of so-called fuzzy sets or sets with fuzzy boundaries. This approach was proposed by the American mathematician of Azerbaijani origin, ***Lotfi Zadeh***[25]. Fuzzy logic is a calculus of compatibility[26]. Unlike probability, which is based on frequency distributions in a random population, fuzzy logic describes the characteristics of properties. According to this approach, for example, uncertainty about the reliability of your partner can be represented using his function of belonging to a set of reliable people.

Fuzziness measures how well an instance (value) conforms to a semantic concept. Fuzziness describes the degree of membership in a fuzzy set. This degree of membership can be viewed as the compatibility between an instance from the set's domain and the concept overlying the set. In the fuzzy set TALL, the height value 176 cm. has a degree not more than 0.75, meaning it is only moderately compatible with TALL. At the same time, the value of basketball player Michael Jordan height 198 cm. has this degree on the value 0,99. Of course, these estimates are subjective. And this subjectivity has the same properties of ambiguity as subjective probabilities.

The membership function is introduced to assess the degree of belonging of an object to a fuzzy set. It is fundamentally different from the idea of the characteristic property of elements, which was used earlier to construct classical sets.

If we want to compare the probabilistic way of thinking and the fuzzy one, firstly, we can figure out that, unlike probability, fuzziness does not dissipate with time. Fuzziness is an intrinsic property of an event or object. Let's consider the classical example with the chance that it will rain tomorrow. Suppose there is a 50% chance of light rain tomorrow. If we wait until tomorrow, it will either rain or not. The probabilistic uncertainty is resolved. However, the fuzzy uncertainty remains: There is still some ambiguity about whether or not the rain is light, moderate, or heavy. The interesting is the case of forecasting that tomorrow will be light rain. It seems that, in this case, we can use a fuzzy probability.

In addition to the approaches described above, there are attempts to comprehend uncertainty using representations that go beyond the probabilistic and fuzzy

[25] Zadeh L, Klir G. (1996). Fuzzy Sets, Fuzzy Logic, And Fuzzy Systems: Selected Papers, , World Scientific , USA
[26] Cox E. (1994). The Fuzzy system Handbook, AP Professionals.

representations. Uncertainty can be represented as a set of options without attributing probability to them. This way of thinking of uncertainty appeared as the simplest one from the point of view of deep penetration in the essence of uncertainty. The idea is very simple: since I am not sure that I understand uncertainty correctly, let me abstract from these complexities and simply choose from a number of equally possible alternatives. Then the choice of the preferred option will occur according to a certain criterion. It produces a set of decision-making techniques discussed in paragraph 5.

4. Modelling

Before making a decision and doing something, any reasonable person will definitely think. This is what the second component of the decision-making system tells us. But oftentimes, it is not enough to think about it. You should do this in a certain orderly way. That is where the concept of a model comes in. From the most general standpoint, a model is a formal structure, represented in semantic images, mathematical formulas, diagrams, and graphs, that helps us understand a process or phenomenon and make an effective decision.

First, let us agree that there is no perfect model. As ***George Box***[27] stated, “All models are wrong, but some are useful.” If we want to express the same idea in terms of the general system of decision-making set out here, we could say that we are always dealing with ‘second-order’ uncertainty: being uncertain about our very model of uncertainty. In other words, by creating a model of uncertainty, we increase the degree of uncertainty for decision-making because we are not sure about this model but are trying our best.

Creating a model always occurs as a conversion of inputs to outcomes. The model transforms the initial information, in whatever form it may be, into a certain finite representation, so that with its help, the decision-maker can achieve the goal. Such goals for a decision-maker can be many. Here is a list of areas of intelligent human activity for which modelling is necessary:

- Reasoning: definition of conditions and derivation of logical consequences.
- Explanation: providing (verifiable) explanations of observed phenomena.
- Development: selection of characteristics of phenomena, policies and rules.
- Communication: the transfer of knowledge and perceptions.
- Research: the study of possibilities and hypotheses.
- Forecasting: obtaining numerical and categorical forecasts of future and unknown phenomena.

In each of these cases, the decision-maker must make a choice to determine the further course of events, and the corresponding action will follow this choice. Most often, this situation will soon happen again. It means you will need to make the next

[27] George E. P. (1976). Box Journal of the American Statistical Association, Vol. 71, No. 356. pp. 791-799.

decision. Actually, this is the content of the life of a reasonable person, no matter who he is a scientist, politician, commander, entrepreneur, etc.

A model is a typical form of knowledge formalization. Models encapsulate knowledge, explain, and predict in various fields of human activity: economic models of market competition, sociological models of networks, geological models of earthquakes, ecological models, psychological models of cognition, etc.

In general, we have two ways of model creation. In the first option, we prepare for the decision based on quantitative criteria (Data-Driven Decision Making). In the second version, the model does not use quantitative categories. In each option, we must trace the model's origin and connection with the way of thinking.

From a formal standpoint, the modelling process can be represented graphically (see Fig 4.1). This is a graphical model that explains the modelling process's content, which consists of transforming the initial information into outcomes that are useful for decision-making.

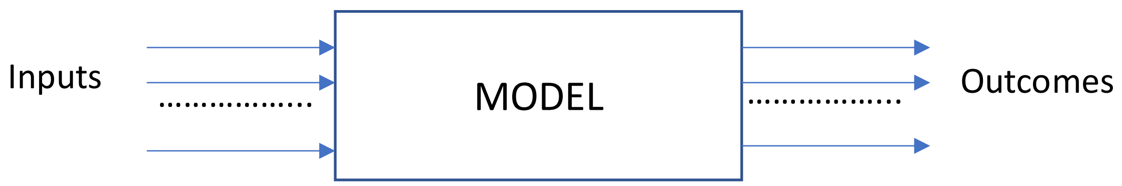

Fig. 4.1. Graphical model of modelling

One might think that this image is exclusively a transformation of quantitative data. It's not that. Texts and other non-quantitative representations can be used as input to the creation of the model, and meanings appear in the output of the model. Creating a model is more of an art than a science. This art cannot be comprehended by observing it from the outside. Full involvement of decision-makers is needed.

Firstly, we consider data-driven modelling of uncertainty. Models based on quantitative criteria are quite well formalized. Going back to the paragraph «The ways of thinking», we must recognize that there are two ways of modelling quantitative data under uncertainty: probabilistic and fuzzy. One more approach can be used to model uncertainties in a set of alternatives, each of which has no preference over the others. We consider this method of modelling uncertainty as a case of probabilistic, taking the equal probabilities for each alternative.

It should be noted that ***probabilistic modelling*** historically appeared before fuzzy and received a more thorough development. As we mentioned above, there are three interpretations of probability: classical, frequency, and subjective. Subjective probability is most common when building decision-making models. There are simple models when the decision-maker assigns these probabilities and then calculates the criterion that will be the basis for decision-making by mutual analysis. In practice, the most common is the ***Bayesian approach*** [28]. According to this approach, a person first assigns prior probabilities and then combines them with observations using the Bayes rule. As a result, he arrives at posterior probabilities that serve as the basis for decision-making.

The following principles summarize the ideas of the Bayesian approach to modelling uncertainty. It's time to look at it in more detail. Three principles refer to the subject as a bearer of probabilistic beliefs:

1. The Bayesian approach follows probability axioms which are the same as those for classical and frequency probability.

2. The Bayesian decision-maker has a complete set of probabilistic beliefs. In other words, to each proposition, he/she assigns a subjective probability, *P(H).* A Bayesian decision-maker can assign a degree of belief about everything. Therefore, Bayesian decision-making is always decision-making under certainty.

3. When exposed to new information, the event with conditional probability *P(A/H)* (the probability that A occurs, given that H is true), the Bayesian decision-maker changes his beliefs under new information according to Bayes' rule.

$$P(H/A) = \frac{P(H) \cdot P(A/H)}{P(A)}.$$

This rule works equally in the case of personalistic (classical) meaning of subjective probability as well as for rationalistic one assuming unique admissible probability assignment (we discussed this difference, considering different views of subjective probabilities). The Bayesian approach postulates a subject-independent probability function. However, in both cases, the probabilities referred to are subjective in the sense of being dependent on the information available to the subject rather than on propensities or frequencies of the material world.

To illustrate the application of the Bayesian approach, consider a simple example that belongs to ***Lewis Carroll***[29], although he did not use the Bayesian rule. Let there be a ball in the urn, equally likely to be white and black. We have two hypotheses: *H_1 is a white ball, and H_2 is a black ball*. According to subjective estimates, the probabilities of these hypotheses are equal to 1/2: $P(H_1) = P(H_2) = 1/2.$ These probabilities are called prior ones.

To get new information, a white ball of the same size and weight is lowered into the urn; the balls are mixed, and one ball is taken out of the urn. It turned out to be white. The question is, have the prior probabilities of hypotheses changed? Many people will say that the chances of the hypotheses haven't changed, as we put a white ball in the urn and took out a white ball. But is that true? If you mentally repeat this experience a few more times, and each time you take out the white ball, it becomes clear that the

[28] Savchuk V., Tsokos C., (2011). Bayesian Theory and Methods with Application, Atlantis Press.
[29] Lewis Carroll, (1958). The Mathematical Recreations of Lewis Carroll: Pillow Problems and a Tangled Tale (Dover Recreational Math), Later Printing

chances that at the beginning of the experiment, there was a white ball are much higher than 0.5.

How can this problem be solved with the Bayesian approach? Very simple. We're just going to Bayes' rule. The conditional probability of the event to pull a white ball out of the urn is one since, under the condition of this hypothesis, there are two white balls in the urn, that is, $P(A/H_1) = 1$. Similarly, it is not difficult to understand that $P(A/H_2)=1/2$ since, under the condition of this hypothesis, there are two balls in the urn (one white, the other black). The total probability of event *A*, regardless of which hypothesis takes place, is:

$$P(A) = P(H_1) \cdot P(A/H_1) + P(H_2) \cdot P(A/H_2) = 1/2 \cdot 1 + 1/2 \cdot 1/2 = 3/4.$$

Now let's substitute all the found probabilities into Bayes' formula:

$$P(H_1/A) = \frac{P(H_1) \cdot P(A/H_1)}{P(A)} = \frac{1/2 \cdot 1}{3/4} = 2/3.$$

So, after receiving new information (one lowered the white ball and took out the white ball), the probability of the H_1 hypothesis (before the experiment, there was a white ball in the urn) became equal to 2/3, which is more than 1/2.

The fourth issue of the Bayesian approach closely deals with the decision-making process. It was suggested by ***Savage***[30]. It states that the rational decision-maker chooses the option with the highest expected utility. It will be the topic of the next paragraph. Now we will focus only on the model that was proposed by Savage to create a clear system of decision-making in conditions of uncertainty.

To decompose this basic uncertainty, Savage suggests a convenient representation of a decision problem by a matrix of the kind exhibited in the table of Fig. 4.2.

	States of the world		
Options	S_1	$\dots$	S_n
A^1	C_1^1	$\dots$	C_n^1
$\vdots$	$\vdots$	$\ddots$	$\vdots$
A^m	C_1^m	$\dots$	C_n^m

Fig. 4.2. Decision Matrix

Savage's model of presenting decision problems shows that in trying to decide what to do, a decision-maker is uncertain about: (i) what states and consequences there are, (ii) what actions are available, (iii) which states of the world are actual and what the consequences are of performing an action. This model was the basis for choosing the best alternative. We will discuss this issue in the next paragraph.

[30] Savage, L. (1954): The foundations of statistics. New-York, John Wiley.

Probabilistic modelling can be produced using random events, variables, and stochastic processes. Now we'll discuss approaches to building decision models using random variables, which can be discreet and continuous. Again, we are using the Fig. 4.1. model. This time, random variables will be used as inputs. The random variable is modelled using a probability distribution, particularly the probability density function (pdf). This function is designed so that the area under the pdf curve, corresponding to a specific interval, equals the probability of a random variable falling into this interval (see Fig 4.3).

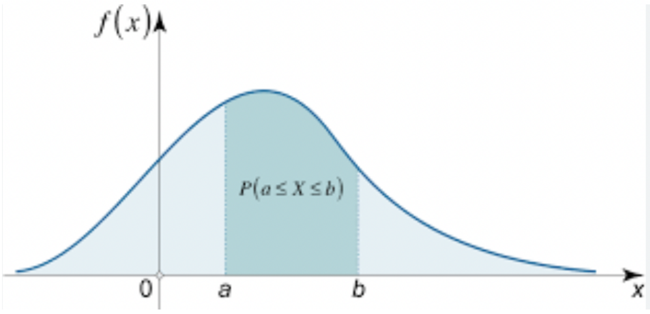

Fig 4.3. Probability density function

Now suppose that the inputs of the model $X_1, X_2, \ldots, X_n$ are random variables with given pdf $f(x_i)$. The outcomes of the model $Y_1, Y_2, \ldots, Y_m$ are random variables as well because each outcome depends on the inputs through the function:

$$Y_k = \varphi_k(X_1, X_2, \ldots X_n)\ \forall\ k = 1,2, \ldots, m.$$

The problem is to find out the pdf of each Y_k given the pdf for each $k=1,2,\ldots,m$.

In general, the 'inputs to outcomes' conversion models are very complicated, which makes it difficult to solve this problem analytically. This is where the Monte-Carlo simulation technique comes in. This technique is universal. That is, it has no limitation in terms of the content and complexity of the model. In essence, the Monte-Carlo method is an approach to modelling random variables with a given pdf utilizing the generation of the pseudo-random variables by special software.

Let's consider a small illustrative example. Imagine a situation in which we want to assess the characteristics of the uncertainty of the Operating Profit of a portfolio of two products. The profit estimation model has the following inputs: Q – a volume of products sold; p – unit price; v – variable costs per unit; F - fixed costs. The model output is $Y = \big((p_1 - v_1) + (p_2 - v_2)\big) \cdot Q - F$. All the inputs are uncertain and uniformly distributed in the interval [-10%,+10%]. We need to assess the uncertainty of Operating Profit. Fig 4.4 demonstrates the result of the modelling.

	#1	#2
Price ($)	80	120
Variable cost ($)	60	90
Volume (units)	500	300
Fixed Cost ($)	2300	
Target Profit ($)	12000	

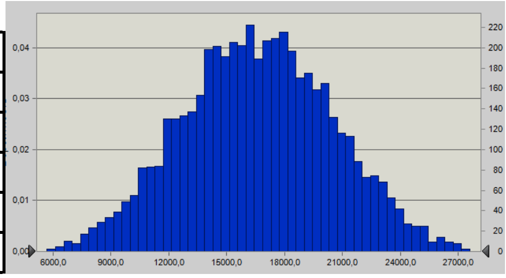

Fig 4.4. Monte Carlo simulation

So, we have simulated the uncertainty of the operating profit, obtaining its empirical pdf, a model of the uncertainty of operating profit. This gives us a source to assess the risks that the operating profit will be less than the value required by the manager.

Can the Bayesian approach be used in this design? Of course, yes, as it is also universal. Using the Bayesian approach, we can refine the prior probability distributions when we have the actual values of inputs.

Now let's switch to fuzzy modelling. The fuzzy way of thinking also has a strict mathematical formalization. The basis for creating a fuzzy model is the so-called membership function. It establishes a correspondence between the elements of the universal set *U* ($u_1, u_2, \dots u_n$) and the numerical values of their degrees of belonging to some target set *A*. Going back to the example of human height (see paragraph 3) set *U* is the set of all men and set *A* is the set of tall people. The membership function $\mu_A(u)$ takes values in the interval [0,1]. The value of the membership function $\mu_A(u)$ for an element $u \in U$ shows to what extent that element belongs to the *A*. If the degree of membership takes only two values of 0 or 1, then set *A* has unfuzzy bounders; otherwise, this set is fuzzy (the degrees of membership can take any value on the interval [0,1], for example, 0.2 or 0.8).

As we understood from the example discussed, the membership function models uncertainties in the decision-maker's perception of a linguistic variable. Fig. 4.5 demonstrates a more comprehensive illustration of such modelling. As the linguistic variable, the interest rate is used. This variable has five linguistic values: very small, small, moderate, large, and very large. Semantic intervals for each linguistic fuzzy value are getting together in the interval [0,20%]. The membership function for each fuzzy value changes in the interval [0,1], characterizing a decision-maker's judgment on how much a number corresponded to his perception of how large and small the interest rate was.

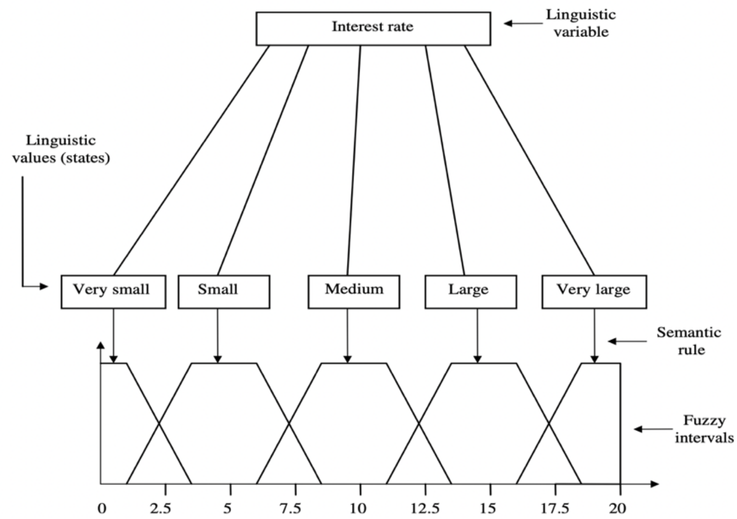

Fig.4.5. Membership function of interest rate[31]

To calculate the values of the membership function, direct methods are usually used, in which the expert expresses his opinion based on his understanding of the object under study. Indirect methods can also be used when the degree of membership is determined based on measurements of the properties of the elements. This is usually used in engineering applications. As a result, specific (non-random) numerical values of the degrees of belonging of elements to a fuzzy set are established each time.

Now let’s come to ***non-data-driven modelling***. Recently, a new notion appeared, “mental models”[32] as cognitive constructs that describe a person's understanding of the real world. A "mental model" is usually a semantic model: people understand the world by forming mental models. The general form of this hypothesis is not new: Even Immanuel Kant argued that there is no direct access to things-in-themselves. Therefore, it is necessary to build a mental model.

In the practice of building models, mainly three approaches are common:[33]

1. an approach of maximum embodiment, striving for maximum reliability,
2. the method of analogies assumes that it is possible to abstract from concrete reality and use a suitable analogue,
3. the method of an alternate reality does not intentionally represent or reflect reality.

Each approach deserves a detailed study, which will be done further. But now let's talk about what it means to make models effective, that is, convincing for decision-making. At the same time, it is necessary to consider that the decision will be made either by the person who created the model or by others to whom the model will be offered. It seems that ***narratives*** and ***metaphors*** are the most useful for constructing semantic ones. The narrative is the most suitable tool for the method of maximum embodiment

[31] Cox E. (1994), The Fuzzy System. Handbook. Academic Press inc.
[32] Hollins P. (2019) Mental Models: 30 Thinking Tools that Separate the Average From the Exceptional. Improved Decision-Making, Logical Analysis, and Problem-Solving.
[33] Page S. (2021). The Model Thinker. Basic Books, New York.

and the method of analogies. Whereas the metaphor will create the most vivid images through analogies and alternative reality methods.

A narrative refers to any narrative text whose function is precisely to model a representation of a phenomenon in verbal form. This is a specific type of exposition that has a plot, and this distinguishes it from ordinary verbal descriptions or explanations. The "refined" texts make it possible to make the model bright and as accessible as possible for perception and subsequent decision-making. The plot presented in the narrative in a general sense is a certain degree of reflection on the understanding of reality. And finally, unlike predictive models based on data, the narrative model is built to convince and transform people's mental models by presenting a certain plot.

A metaphor is usually considered a bright artistic image. Remember Gogol's vivid metaphor from Dead Souls: "... *roads sprawled* in all directions, like crayfish caught when they were poured out of *a bag*." In creating a model, metaphor has a more important and, at the same time, more pragmatic purpose. Metaphor allows us to penetrate deeper into the essence of the object of modelling, going beyond our own object. Like a paradox, we better understand the essence of what is happening, going beyond it. This works equally effectively for both the analogy method (remember crayfish) and the alternate reality method.

Metaphor is the process by which one entity or state is described in terms originally intended to describe other things. Metaphor is the change of signs that are different in meaning but used in the same semantic contexts. For this reason, metaphor is most effective in building models using the method of alternative reality.

Unlike the traditional combination of concepts, metaphor has one undeniable advantage. Metaphor always combines a concept and a vivid manifestation of emotions. If, when building a model, it is possible to find an apt metaphor, this will allow not only to build an effective model but also to form an emotional field of attraction. The latter will be crucial in the decision-making process.

A Metaphor-into-Narrative – powerful tool for semantic models. Both narrative and metaphor provide mechanisms for making sense of the world and creating a model. While metaphors elaborate and articulate particular points in a narrative, the narrative provides meaningful connections between sometimes unrelated metaphors, suggesting a symbiotic relationship between the two.

Semantic models are just one non-data-driven method of modelling. Graphic models can serve as very useful for clear perception. A visual image in the form of a diagram, a canvas, or even a painting can help a decision-maker present the overall picture of the object of decision-making and draw the correct conclusion. If we turn to business applications, we can find as a vivid example the format of the model, which has the form of a canvas on which all the essential factors of building a successful business are structurally located. The ***Business Model Canvas***[34] is a powerful strategic management tool used to document existing business models and develop new ones. It offers a visual chart with elements describing a firm's or product's value proposition, infrastructure, customers, and finances, assisting businesses in aligning their activities by illustrating potential trade-offs. All these issues are getting together and can allow the decision-maker to develop a powerful strategy.

5. Making Decisions

So, being uncertain, having learned to think adequately and create models, we are ready to make decisions. In general, there are two main approaches in decision theory: ***descriptive decision theory*** (sometimes called behavioral decision theory) and ***normative decision theory*** (sometimes called prescriptive decision theory). The first approach describes how specific people make decisions based on considerations beyond formal logic. Such descriptions may include behavioral patterns or sociological factors relevant to a particular decision. The second approach prescribes procedures for making decisions based on certain formalized logic and the application of quantitative criteria. It is assumed that a person behaves rationally and is not affected by behavioural patterns or sociological factors.

The descriptive theory is often associated with constructing a mental model, which uses metaphors and narratives in addition to the usual semantic images. And how vivid these metaphors and narratives will cause a person's emotions will depend on the degree of confidence of the person in the correctness of the decision made.

In normative decision theory, the decision is justified by using formalized criteria. The following two approaches are mainly used here. ***The first approach*** uses ***probabilistic models***. And this opens a broad palette of possibilities and practical applications in decision-making. We have already discussed probabilistic thinking and modelling, and now it's time to conclude this sequence with specific tools for decision-making based on probabilistic models. There is a special case of the first approach, which considers a situation with a set of alternatives with the same probability. It means that we eliminate probability issues while posing the making decision problem. The problem is choosing the optimal alternative from the set of possibilities for a given set of states of the world. All possible combinations of system states and options are considered, and the one that provides the maximum or minimum value of the assigned criterion is selected. In this point of view, it is possible to proceed without probabilistic modelling, and a simple set of possible options sets uncertainty. No probabilistic techniques are used for decision-making. We call this approach ***combinatorial.***

According ***to the second approach***, making decisions is based on fuzzy sets and logic. In the previous paragraph, we discussed that these approaches pursue the same goal, modelling uncertainty, but by different means.

[34] Osterwalder A., Pigneur Y. (2010). Business Model Generation: A Handbook for Visionaries, Game Changers, and Challengers. Wiley.

We will study these two approaches separately and start with the probabilistic one. As we discussed earlier, among all the interpretations of probability, subjective probabilities deserve the most attention from a practical point of view, despite the possible biases when assigned.

Regarding to applied aspects, the ***Expected Utility Theory (EUT)*** deserves the most attention. Its founder should rightly be considered ***Danial Bernoulli***[35] in 1738. His main idea came from the St. Petersburg Paradox. Bernoulli proposes a coin flip game where one flips until the coin lands tails. The payouts double for each toss that lands heads. It is easy to find that an infinite expected value as the price for the game is obtained as an expectation of a random variable. The essence of the paradox is that individuals are willing to pay a relatively small amount of money to participate in a game in which the mathematical expectation of winning is infinitely large. Since the people always set a definite, possibly relatively small upper value on the St. Petersburg Paradox, they do not price it in terms of its expected monetary value. Bernoulli argued, in effect, that individuals estimate it in terms of the Value of money outcomes, which is a sort of moral expectation, later called the Expected Utility (EU) of winning. The principal assumption of EUT is that the increment of Utility *U* is proportional to this increment of Wealth (*W*) and inversely proportional to the magnitude of the Wealth (*W*). Simply thinking, as income increases, individuals gain a correspondingly smaller increase in their satisfaction and happiness. It immediately follows from this assumption that the *Utility Function* has the form of a logarithmic function *U= Ln(W).*

Von Neumann and Morgenstern[36] made a sweeping generalization of this theory. Their great task was to lay a rational foundation for decision-making under uncertainty according to expected utility rules. Thus, EUT received its first axiomatic characterization. In particular, they state a series of axioms about the individual's preferences over indifference classes of lotteries and offer proof that an individual obeying these axioms will follow the expected utility theory. In the normative interpretation, these axioms are regarded as tenets of rational choice and should be judged by their normative appeal. In fact, if an individual does not maximize his expected utility, he violates some precise axiomatic principles, which are rationally binding in his choice. Von Neumann and Morgenstern's expected utility theory has been generally accepted as a normative rational choice model. EUT states that the decisions of a decision-maker conform to an expected utility function of the outcomes. In practice, individuals should always choose under uncertainty the alternatives that offer them the highest utility, i.e. the alternatives that offer higher earnings (wealth) or the lowest losses ever.

Years after the contribution of von Neumann and Morgenstern, Leonard Savage[37], proposed the first complete axiomatic ***Subjective Expected Utility Theory***, focusing on uncertainty. This theory is another relevant instance of the theory of choice under

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[35] Bernoulli D. (1954) «*Exposition of a New Theory on the Measurement of Risk*», Econometrica, vol.22.
[36] von Neumann and Morgenstern's (1947). "Theory of games and economic behavior". Princeton University Press.
[37] Savage L. (1954). The foundation of Statistics, Wiley and Sons.

uncertainty, while the expected utility hypothesis was originally formulated to be used with objective probabilities.

Savage introduced his new analytical framework, which was a synthesis of the ideas of Ramsey[38], de Finetti[39] and von Neumann and Morgenstern. The basic idea behind the Ramsey-de Finetti derivation is that by observing the bets people make, one can presume this reflects their personal beliefs on the race's outcome. Thus, Ramsey and de Finetti argued that subjective probabilities could be inferred from observing people's actions. de Finetti's model, in particular, was based on the notion of expected value maximizing utility.

Returning to Savage's theory, we stress that he provided necessary and sufficient conditions for the existence and joint uniqueness of utility and probability and the interpretation of individual choice under uncertainty as expected utility-maximizing behavior. In Savage's approach, the notion of probability is presented broadly, assuming the possibility of refining prior judgments with additional information. Savage subjective expected utility theory and the Bayesian rule for updating the decision-maker's information still represent the orthodoxy in making decisions under uncertainty. This treatment of decision problems reduces the decision maker's basic uncertainty concerning what to do with uncertainty regarding the true state of the world. As Karni[40] argues, it permits:

(i) the numerical expression of the decision maker's valuation of the consequences by a utility function;

(ii) the numerical expression of the decision maker's degree of belief in the likelihoods of events by a finitely additive, probability measure;

(iii) the evaluation of acts by the mathematical expectations of the utility of their consequences with respect to the subjective probabilities of the events in which these consequences materialize.

In analytical terms, Savage's approach is based on the expected utility of the set of options, i.e. weighted average value of utility for the decision-maker. In other words, if the decision-maker adheres to axioms of rationality, believing an uncertain event has possible outcomes x_i, each with a utility of $u(x_i)$, the choices of the individual can be explained by this utility function combined with the subjective belief that there is a probability of each outcome, $P(x_i)$. Therefore, the subjective expected utility is the resulting expected value of the utility:

$$E(U) = \sum_{i=1}^{N} u(x_i) \cdot P(x_i).$$

[38] Ramsey, F. (1931). Truth and probability. In The foundations of mathematics and other logical essays, ed. R. Braithwaite and F. Plumpton. London: K. Paul, Trench, Truber and Co..
[39] de Finetti, B. (1975): *Theory of Probability*, Vol. 2, New York: John Wiley and Sons.
[40] Karni, E. (2014). Axiomatic foundations of expected utility and subjective probability. In M. Machina and W. K. Viscusi (Eds.), *Handbook of the Economics of Risk and Uncertainty. Vol. 1*. Oxford: North Holland.

As we can see, the technique of using EUT is quite simple. Here is an example. Suppose there are two options, P1 and P2:

P1	Wealth	Prob.
Low	50,000	0.4
High	500,000	0.6

P2	Wealth	Prob.
Low	100,000	0.5
High	500,000	0.5

We should decide which is preferable from the point of view of EUT. We calculate the values of the utility function for each outcome as *u=Ln(Wealth)* and then compute the expectations:

E[u(P1)] = U(P1) = 0.40**u*(50,000) + 0.60**u(*500,000) = 0.40*1.6094 + 0.60*3.9120 = 2.991

E[u(P2)] = U(P2) = 0.50**u*(100,000) + 0.50**u*(500,000) = 0.50*2.3026 + 0.50*3.9210 = 3.107

Under assigned probabilities, the decision maker must choose option P2.

EUT allows determining the relations of decision makers to the risk. Regarding risk, there exist three groups of decision-makers: risk-averse, risk-neutral, and risk-seekers. Risk-averse decision-makers have a convex utility function, as shown in Fig. 5.1. They would take the expected value of a prospect with certainty rather than gamble on an uncertain outcome.

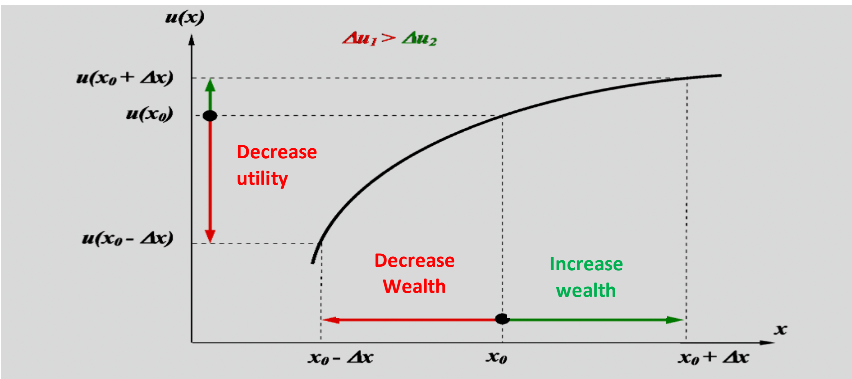

Fig 5.1. Utility function for Risk-averse decision-makers

Risk-neutral people have a linear utility function. They would be indifferent between choosing a gamble on an uncertain outcome and a prospect with certainty. Risk-seekers have a concave utility function (see Fig 5.2). They would rather gamble on the uncertain outcome than take the expected value of a prospect with certainty.

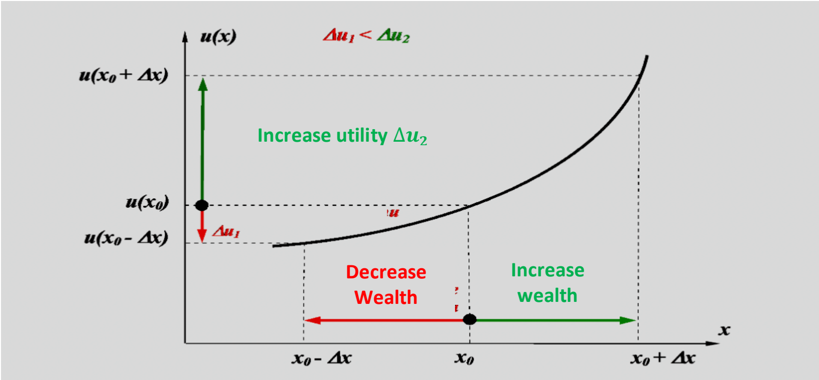

Fig 5.2. Utility function for Risk-seekers

From the point of view of the formal procedure, everything looks quite strict and understandable. But, as the saying goes, "the devil is in the details." It turns out that the result of the decision-making procedure is sensitive to the probabilities assigned by the individual. Here it is helpful to mention the well-known paradoxes of Allais[41] and Ellsberg[42]. Let's shortly discuss each of them.

We consider these paradoxes because they are examples of decision-making using subjective probabilities, which demonstrate the possibility of bias in assigning prior probabilities.

Allais was examined in the following case, presented in Fig. 5.3. There are two questions regarding the choice between the two Prospects.

Question 1	Option A		Option B	
Option 1	1 000 000	100%	-	1%
Option 2	-	0%	1 000 000	89%
Option 3	-	0%	5 000 000	10,00%

Question 2	Option C		Option D	
Option 1	-	90%	-	89%
Option 2	5 000 000	10%	1 000 000	11%

Fig 5.3. Allais' paradox tables

Allais found that most individuals prefer option ***A*** in the first pair and option ***C*** in the second. This result was perceived as paradoxical. Within the framework of the existing hypothesis, the individual who preferred the choice of ***A*** in the first pair should choose option ***D*** in the second pair, and the one who decided ***B*** should give preference to the will of ***C***. Allais mathematically explained this paradox. His main conclusion was that a rational agent prefers absolute reliability. After simple calculations, it becomes noticeable that for 1% of the risk, the expected price increases by 390 thousand francs (Allais is a Frenchman) when choosing ***B*** and ***C***, respectively. This, coupled with the coincidence of the figures 1% and 5 million, is sufficient for a paradox. Or, in other words, in the first case, we take 1% of the risk of losing 1 million, and in the second, 1% of losing 1 million. But in the first case, we increase the profit by 1.39 times for 1% of the risk, and in the second, more than 4.5 times.

[41] Allais, M. (1953). "Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'école Américaine". Econometrica.21 (4): 503–546.
[42] Ellsberg, D. (1961). Risk, ambiguity, and the Savage axioms. Quarterly Journal of Economics, 75, 643–669.

Ellsberg's paradox considers the following situation (see Fig. 5.4). You are going to draw a ball from an urn. The urn contains 30 red balls and 60 balls that are either blue or yellow, but you do not know the relative proportion of blue and yellow balls. Payoffs are based on the following matrix.

		30 Red	X Blue	60-X Yellow	
Choice 1	Option A: Bet on red	1000	0	0	People prefer
	Option B: bet on blue	0	1000	0	

		30 Red	X Blue	60-X Yellow	
Choice 2	Option C: bet on red or yellow	1000	0	1000	
	Option D: Bet on blue or yellow	0	1000	1000	People prefer

Fig 5.4. Ellsberg's paradox tables

Typical people's choices: Choice 1 prefers *Option* ***A*** (bet on red); Choice 2 *option* ***D*** (bet on blue or yellow) is preferable. Why paradox? The expected consequence says that you should prefer ***A*** to ***B*** and ***C*** to ***D*** or prefer ***B*** to ***A*** and ***D*** to ***C***. These paradoxes gave reason to think about possible biases in assigning subjective probabilities.

In turn, these doubts became the basis for conclusions obtained by Kahneman and Tversky concerning biases in judgments revealing some heuristics of thinking under uncertainty. As we mentioned, the biases are connected to representativeness, availability, adjustment and anchoring.

Kahneman and Tversky suggested the ***Prospect Theory***, which describes risk choice theoretically. The Prospect Theory differs in many ways from EUT, where decision-makers determine the value of total wealth. In contrast to some generalizations of the theory of expected utility, Kahneman and Tversky derived their theory of prospects from empirically identified and documented features of the behavior of actual respondents under uncertainty. Based on experimental studies, prospect theory makes a paradoxical conclusion: people are more likely to take on more risk to avoid losses than to receive an additional premium at high risk. Losses have a more significant effect than gains of equal size, a phenomenon known as loss aversion. According to this theory, the investor is free to hold stocks that depreciate but sell those that are rising in value. So, in the joke "a strategic investor is an unsuccessful speculator", there is some truth. "I know prices will still jump in the future; then I will sell my shares." Such reasoning is familiar to many.

As a criterion for decision-making in prospect theory, the value function (which differs from the utility function) is used, which can be negative, which means losses. At the same time, as was established by the authors, the value function has a steeper bend in the case of a loss than as shown in Fig. 5.5.

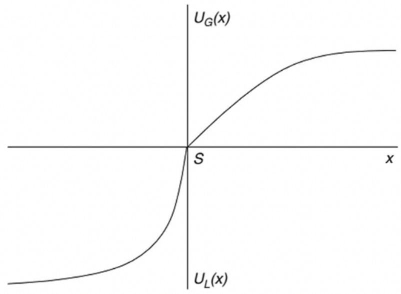

Fig 5.5. The Value in function in Prospect Theory

Prospect theory revealed another feature of decision-making under uncertainty: people inadequately perceive probabilities. Psychologically, the individual overestimates small probabilities and underestimates medium and large ones. What's more, people choose to ignore a priori probabilities in exchange for minor data and analogies. Based on the nonlinear nature of the probabilistic value function used in prospect theory, the authors explain that people's emotional perception of events creates their probabilistic interpretation.

Based on psychological research, the theory of prospects also relies on mathematical modelling methods. The model can explain behavioral reactions that deviate from the traditional theory. The pioneering role of Kahneman and Tversky lies in an unusual way for economists to construct a theory: not from a convenient formal construction to the axioms of rationality, but from the features of behavior to its formal description and then to the axioms.

Moreover, according to the prospect theory, the empirical justification has acquired the function of probabilistic values. Fundamentally, it cannot be interpreted as a probabilistic measure since it does not correspond to the axioms of probability. This is manifested in the fact that subjective estimates of probability may not be equal to objective probabilities, being less than one in total.

In real practice, the term "decision-making" implies the choice of a certain decision and its subsequent implementation. At the same time, there are often situations when the decision-maker, having made a decision, is still determining if its implementation should be started immediately. A typical example is investment decisions. These decisions involve a long implementation period, which increases the degree of uncertainty and, as a result, increases the risks of loss of invested funds. As part of the probabilistic approach to decision-making, the method of **real options** can be recommended. According to this method, assessing possible options for implementing the decision is similar to assessing the purchase or sale of option contracts in the stock market. American financier Stewart Myers[43] coined the term "real option" in 1977. He tried to apply the theory of financial options traded in the derivatives market to analyze a company's financial policy using financial leverage. More recently, this approach has been used to assess opportunities that result from strategic decisions that may arise in the future. The main difference between real and financial options is that a real

[43] Stewart Myers S. (1997), Determinants of Corporate Borrowing, Journal of Financial Economics, 5, pp. 147-175.

option is not a security. It does not circulate in the derivatives market, where it can be sold or bought. The underlying asset of a real option is the future management decisions that can be made in relation to a specific development project.

A real option is a right, but not an obligation, to make and implement a decision in the future. The uncertainty of current conditions dictates this possibility. It is important to emphasize that the technique of real options allows you to quantify decision-makers' potential to adapt to changes. Adaptation is the most important property of systems that will enable you to reduce losses caused by uncertainty. Having the capacity to adapt can significantly reduce the negative impact of uncertainty.

The most significant property of a real option is that it should be exercised only when it is profitable. If we turn to economic activity, the peculiarity of real options is that they allow you to increase the value of development projects and, as a result, the company's value. The application of real options is especially relevant in countries with a high level of uncertainty, particularly in the conditions of modern Ukraine. The implementation of development projects in Ukraine is associated with relatively significant risks, but at the same time, with many opportunities that need to be taken into account by the classical theory of investment valuation based on the discounting of projected cash flows.

In evaluating real options, the most common is the binomial scheme, which originates from D. Bernoulli. While creating a binomial model, assumptions are used that investors are neutral about risk; there can be only two scenarios at one time. A tree model allows us to present all the many alternatives graphically and analytically to the development of events and, on their basis, make an informed decision. The calculation of the value of the option is carried out by moving from one point to another along the branches of the decision tree from left to right. At each point of transition, the subjective probabilities of following the branches of the tree are pumped up (see Fig. 5.6).

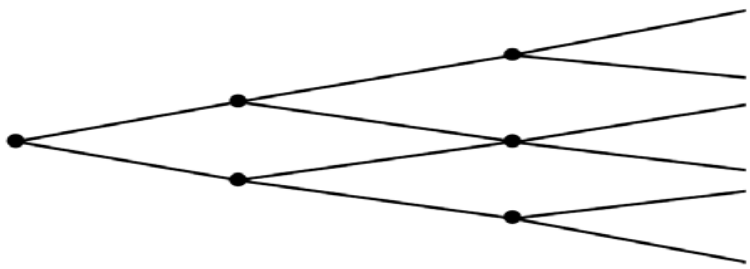

Fig 5.6. Binomial Model in Real Option Technique

More details about the technique of using real options can be found in the Practical Guide[44].

Let’s now come to the special case of the probabilistic approach, which we call combinatorial. The decision-making technique is simplest and most transparent when uncertainty is represented as a set of alternatives unrelated to probabilistic. The decision is made by a simple combinatorial search of alternatives, from which one is

[44] Copeland T. & Antikarnov V. (2001). Real options: A practitioner’s guide. New York: Texere.

chosen since it meets the criterion assigned in advance. Thus, the decision made directly depends on the criterion adopted. And there are several such criteria. The basic idea of any criterion is to replace a whole set of values with a single numerical indicator that characterizes this set from a certain point of view. Here is a list of such criteria: Wald's criterion; the "maximax" criterion; Laplace's criterion; Savage's regret criterion; Hurwitz's criterion.

Wald's criterion is the most "cautious”: the optimal alternative would be the one that provides the best outcome among all possible alternatives under the worst set of circumstances.

The "maximax" criterion is the opposite of Wald's criterion. If Wald’s reflected the view of the ultimate pessimist, then Maximax corresponds to an attitude of extreme optimism. All attention is paid only to the best outcomes.

Laplace's criterion is based on the principle of insufficient justification. Since, within the framework of the information approach in a situation of uncertainty, the probabilities of states are unknown, there is no reason to assert that they are different. Therefore, it can be assumed that they are the same. According to Laplace's criterion, the average value of outcomes is used as an estimate of the alternative.

Savage Minimax Regret Criterion is based on the following justification. Alternatives are evaluated based on the so-called "regret matrix". For an arbitrary alternative and a particular state of nature, the value of "regret" is equal to the difference between what the alternative provides and how much the maximum can be gained in a given state. From an economic point of view, the amount of "regret" can be interpreted as a lost gain compared to the maximum possible in each state of nature. The Savage criterion reflects the largest possible shortfall in winnings for a given alternative, the reason is that the less you can lose, the better.

The classical **Hurwitz’s criterion** considers only the extreme outcomes of each alternative. It can be viewed as a weighted average of the best and the worst uncertainty realizations. It allows considering the decision-maker's subjective attitude by giving these outcomes different "weights". The "optimism coefficient" λ, $0 \le \lambda \le 1$ is introduced into the criterion calculation, so if it is close to 1, the decision-maker feels optimistic and pessimistic otherwise, if λ is near zero.

We cannot conclude which criterion is more correct. The decision-maker chooses the criterion by himself. This can be considered a kind of manifestation of democracy in the decision-making theory.

It should be noted that the probabilistic approach, including the combinatorial case is presented in decision theory much more thoroughly compared to the fuzzy approach, as evidenced by the number of techniques and approaches discussed above.

Possibility theory is an uncertainty theory devoted to the handling of incomplete information. It differs from the probability by using a pair of dual set functions (possibility and necessity measures) instead of only one. Besides, it is not additive and makes sense on ordinal structures. The name “Theory of Possibility” was coined by

Zadeh[45]; ***Dubois*** and **Pradé**[46] later contributed to its development. In Zadeh's view, possibility distributions were meant to provide graded semantics to natural language statements. The meaningful interpretation of the bases of the Possibility theory differs significantly from the probabilistic ones. The possibility of an event, in contrast to probability, which estimates the frequency of its occurrence in a regular stochastic experiment, is focused on a relative assessment of the truth of this event and its preference compared with any other. That is, only the relations "more", "less", or "equals" can be interpreted meaningfully. At the same time, the possibility does not have an event-frequency interpretation (unlike probability), which connects it with the experiment. Nevertheless, the theory of possibilities allows a mathematical model of reality based on empirical facts, knowledge, hypotheses, and judgments of researchers.

The principle of minimal specificity drives possibility theory. It states that any hypothesis not known to be impossible cannot be ruled out. Human knowledge is often declarative, using statements to which belief degrees are attached. Decisions are made based on criteria connected to a membership function that reflects the decision-maker's attitude in front of uncertainty.

Possibility theory has been axiomatically justified in a decision-theoretic framework in the style of EUT, thus providing a foundation for decision-making. At the same time, possibility theory is the most straightforward framework for statistical reasoning with imprecise probabilities. As such, it has close connections with random set theory and confidence intervals and can provide a tool for uncertainty propagation with limited statistical or subjective information.

At the end of this paragraph, we devote some attention to the ***Game Theory*** as an important stage in the development of decision-making in conditions of uncertainty. The main type of uncertainty that is considered in game theory is uncertainty regarding of behavior of game participants under conditions of conflicts of interest.

Presently**,** Game Theory is a mathematical discipline that studies the resolution of conflicts between players and the optimality of their strategies. Conflict can refer to different areas of human interest: most often, it is economics, sociology, political science, cybernetics and military affairs. Conflict is any situation in which the interest of two or more participants, traditionally called players, is affected. For each player, there is a certain set of strategies that he can apply. Intersecting, the strategies of several players create a certain situation in which each player receives a certain result, called a win, positive or negative. When making decisions, it is necessary to consider not only obtaining the maximum utility for the player but also the possible steps of the enemy and their impact on the situation as a whole.

[45] Zadeh L.(1978), Fuzzy sets as a basis for a theory of possibility, Fuzzy Sets and Systems, 1: 3-28,

[46] Dubois and H. Prade, (1998) Possibility theory: Qualitative and quantitative aspects. In: D. M. Gabbay and P. Smets P., editors Handbook of Defeasible Reasoning and Uncertainty Management Systems, Vol. 1., Dordrecht: Kluwer Academic, 169-226

The game theory originates in the same work von ***Neumann and Morgenstern***[47] that we mentioned regarding the Expected Utility Theory. In different terms, von Neumann and Morgenstern analyzed the strategic behavior of players in noncooperative zero-sum games in which no pure strategy equilibrium exists. In such games, the equilibrium may require the employment of mixed strategy. By adopting the axiomatic approach to depict the decision maker's preference in relation to the set of objective risks, von Neumann and Morgenstern identified necessary and sufficient conditions for the existence of a utility function on a set of outcomes that captures the decision maker's risk attitudes and represented his/her choice as expected utility maximizing behavior.

John Nash[48] developed methods of analysis in which all participants either win or fail. These situations are called "Nash equilibrium". According to his theory, the parties should use the optimal strategy, which creates a stable equilibrium. It is beneficial for players to maintain this balance, as any change will worsen their situation. These works of Nash made a serious contribution to the development of game theory, and mathematical tools of economic modelling were revised. In particular, John Nash showed that the classic approach to the competition of Adam Smith, when everyone is for himself, is not optimal.

It should be emphasized that game theory is a very complex field of knowledge. When referring to it, a decision-maker must be careful and clearly know the boundaries of the application. Too simple interpretations are fraught with hidden danger.

6. Conclusion remarks

The main objective of this presentation was to demonstrate the diversity of approaches and methods to decision-making in conditions of uncertainty, which are not limited to data-driven systems. Not all people facing the need to make decisions under uncertainty have an analytical mindset and the ability to draw conclusions based on data. At the same time, they are ready to perceive semantic models presented by narratives, especially those that contain vivid metaphors. A technique based on narratives and metaphors is especially effective when the task of making a collective decision is faced.

And yet, the data-driven approach prevails and is represented by a considerable abundance of methods. We sought to present these methods on the broadest possible scale while simultaneously trying to penetrate the depth of specific techniques, showing their essence and primary purpose. This view was minimally saturated with mathematical calculations and models. One of the objectives of this presentation was to demonstrate the practical application of the considered methods to assess their practical usefulness.

[47] von Neumann and Morgenstern (1947). “Theory of games and economic behavior”. Princeton University Press.
[48] Nash J. (1951). Non-Cooperative Games,” Annals of Mathematics.

To better understand the decision-making process under uncertainty, the structure of this process was proposed and discussed in detail. It represents a dialectical unity of four components: (i) uncertainty itself, including the metaphysical view, (ii) the ways of thinking uncertainty, (iii) the modelling of uncertainty, (iv) the final approaches to decision-making. Each of these components is discussed separately, but the relationships between the components are addressed each time.

An overview of the considered methods gave a basis to conclude that the most advanced and methodically formalized are methods based on probabilistic models. These models extensively use subjective probabilities. At the same time, assigning subjective probabilities might cause biases from the standpoint of the rational approach. These biases, in turn, may lead to an erroneous decision. The way out of this predicament is the Bayesian approach, which suggests using subjective prior probabilities in decision-making and enriches them with factual data that appear in the decision-making process. The Bayesian approach is well-developed and formalized and demonstrates effective practical application.