The Stock of Money and Why You Should Care

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Abstract

In this paper, I will examine the problems created by incorrectly using a simple sum monetary aggregate to measure the monetary stock. Specifically, I will show that simple sum monetary aggregate confounds the current stock of money with the investment stock of money and that this confounding leads the simple sum monetary aggregate to report an artificially smooth monetary stock. This smoothing causes important information about the dynamic movements of the monetary stock to be lost. This may offer at least a partial explanation of why so many studies find that money has little economic relevance. To that end, we will conclude the paper by examining a reduced form backward looking IS equation to determine whether monetary aggregates contain information about real GDP gap. This paper differs from previous work in that it focuses on smoothing of the monetary stock data caused by the use of simple sum methodology, where the previous work focuses on the bias exhibited by simple sum monetary aggregates.

Key words: Monetary Aggregation, Money Stock, Currency Equivalent Index

JEL classification codes: C43, E49

1 Introduction

Ever since Sims (1980) demonstrated that movements in output are systematically related to changes in real interest rates and not systematically related to changes in simple sum monetary aggregates, economists have debated whether or not monetary quantity aggregates contain any economically interesting in-
formation about the macroeconomy. Numerous subsequent studies have been done showing results similar to that of Sims. One such study supporting this conclusion is that of Rudebusch and Svensson (2002), who find no support for a role for money growth in monetary policy. Thus, the consensus that has emerged is that monetary quantity aggregates have little economic significance and can be ignored.

There is a dissenting view, however. Leeper and Roush (2003), Nelson (2002) and Meltzer (2001) argue that how money is measured matters. They each show that under some sets of identifying assumptions money matters. Hafer, Haslag, and Jones (2007) answers the analysis of Rudebusch and Svensson by arguing that their finding that money growth is a statistically insignificant predictor of GDP gap when lags of GDP gap and real interest rates are included as explanatory variables is due to the use of simple sum monetary aggregates. In section 4, I will repeat Hafer et al. (2007) analysis with various measures of money. I find that measures of money that are based on monetary aggregation theory are statistically significant predictors of GDP gap even after accounting for real interest rates and lags of GDP gap, but simple sum measures of money are not significant.

Many of the studies that find that money does not matter use simple sum measurements of money. However, Barnett and others have argued for many years that the simple sum monetary aggregates are unreliable as measures of either the monetary service flow or the monetary stock. In this paper, I will examine the problems created by incorrectly using a simple sum monetary aggregate to measure the monetary stock. Specifically, I will show that the simple sum monetary aggregate confounds the current stock of money with the investment stock of money and that this confounding leads the simple sum monetary

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1See Hafer et al. (2007) for a brief listing.
aggregate to report an artificially smooth monetary stock. This smoothing causes important information about the dynamic movements of the monetary stock to be lost. This may offer at least a partial explanation of why so many studies find that money has little economic relevance. This paper differs from previous work in that it focuses on smoothing of the monetary stock data caused by the use of simple sum methodology, where the previous work focuses on the bias exhibited by simple sum monetary aggregates.

2 Notation and Preliminaries

2.1 Definition of the Current Stock of Money

To define the current stock of money (CSM), I will first consider the economic stock of money (ESM), defined by Barnett (1991) to be the present value of current and future monetary service flows. Barnett, Keating, and Kelly (2008) and Barnett, Chae, and Keating (2005) formulate ESM under uncertainty as

\[
ESM_t = E_t \left[ \sum_{s=t}^{\infty} \left( \Gamma_s \sum_{n=1}^{N} m_{ns} \psi_{ns} \right) \right],
\]

(1)

where

\[
\Gamma_s = \beta^{s-t} \frac{\partial u}{\partial C_s} / \frac{\partial u}{\partial C_t}
\]

(2)

is the subjectively-discounted marginal rate of inter-temporal substitution between consumption in the current period \(t\) and the future period \(s\), \(m_{nt}\) is the quantity of monetary asset \(n\) held in period \(t\), and \(\psi_{nt}\) is the user cost holding.
monetary asset $n$ held in period $t$.$^3$ Next, define

$$\tilde{m}_{n,t+j} = \begin{cases} 
0 & \text{if } j = 0 \\
\sum_{i=1}^{j} \Delta m_{n,t+i} & \text{if } j > 0 
\end{cases}$$

and note that

$$m_{ns} = m_{nt} + \tilde{m}_{ns} \forall s \geq t. \quad (4)$$

Substituting (4) into (1) yields

$$ESM_t = E_t \left( \sum_{s=t}^{\infty} \Gamma_s \sum_{n=1}^{N} m_{nt} \psi_{ns} \right) + E_t \left( \sum_{s=t}^{\infty} \Gamma_s \sum_{n=1}^{N} \tilde{m}_{ns} \psi_{ns} \right), \quad (5)$$

where the first double summation is the current stock of money.$^4$ Thus, the CSM can be defined under risk as

$$CSM_t = E_t \left( \sum_{s=t}^{\infty} \Gamma_s \sum_{n=1}^{N} m_{nt} \psi_{ns} \right). \quad (6)$$

### 2.2 Measuring the Current Stock of Money

Given the following assumptions

1. the expectation of the stochastic discount factor in time period $t$ is $E_t(\Gamma_s) = \prod_{u=t}^{s} \left[1 + E_t(R_u)\right]^{-1}$,
2. the benchmark rate follows a martingale process, so that $E_t(R_s) = R_t$ for all $s \geq t$,
3. $\text{cov} \left( \sum_{n=1}^{N} m_{nt} \psi_{ns}, \Gamma \right) = 0$,

---

$^3$The user cost, $\psi_{nt}$, used here is the user cost under risk neutrality derived by Barnett (1995) and Barnett, Liu, and Jensen (1997). It is formulated as

$$\psi_{nt} = \frac{E_tE_t(R_t) - E_tE_t(R_t)}{1 + E_tR_t}.$$  

$^4$Also see Kelly (2008) for the same result.
4. and that $\psi_{ns}$ follows a martingale process $\forall n = 1 \ldots N$.

Kelly (2008) showed that the current stock of money reduces to the currency equivalent index (Rotemberg et al., 1995). Thus under assumptions one through four, the current stock of money can be written as

$$CSM_t = \sum_{n=1}^{N} \frac{R_t - r_{nt}}{R_t} m_{nt},$$

(7)

where $R_t$ is the rate of return on the benchmark asset in period $t$ and $r_{nt}$ is the rate of return yielded by monetary asset $n$ in period $t$.

2.3 Definition of the Investment Stock of Money

I will define the investment stock of money (ISM) in time period $t$ to be the discounted present value of the return yielded by the portfolio of monetary assets held at time period $t$. The ISM can be derived though a direct application of asset pricing theory\(^5\) to be

$$ISM_t = E_t \left[ \sum_{s=t}^{\infty} \left( \Gamma_s \sum_{n=1}^{N} m_{nt} r_{ns} \right) \right],$$

(8)

where $\Gamma_s$ is the subjectively-discoun ted marginal rate of inter-temporal substitution between consumption in the current period $t$ and the future period $s$, $m_{nt}$ is the quantity of monetary asset $n$ held in period $t$, and $r_{ns}$ is the return yielded by monetary asset $n$ held in period $t$. Applying assumptions one and two from section 2.2 yields

$$ISM_t = E_t \left[ \sum_{s=t}^{\infty} \sum_{n=1}^{N} \frac{m_{nt} r_{ns}}{(1 + R_t)^{s-t+1}} + cov \left( \sum_{n=1}^{N} m_{nt} r_{ns}, \Gamma \right) \right].$$

(9)

Finally, assume that \( \text{cov} \left( \sum_{n=1}^{N} m_{nt} r_{ns}, \Gamma \right) = 0 \) and that \( r_{ns} \) follows a martingale process \( \forall n = 1 \ldots N \), then (9) reduces to

\[
ISM_t = \sum_{n=1}^{N} \frac{r_{nt}}{R_t} m_{nt}. \tag{10}
\]

3 Measurement Error in the Simple Sum Aggregates

To begin our analysis of the measurement error exhibited in the simple sum monetary aggregates, note that combining (6) and (10) yields

\[
CSM_t + ISM_t = \sum_{n=1}^{N} R_t - \frac{r_{nt}}{R_t} m_{nt} + \sum_{n=1}^{N} \frac{r_{nt}}{R_t} m_{nt} = \sum_{n=1}^{N} m_{nt}, \tag{11}
\]

where \( \sum_{n=1}^{N} m_{nt} \) is the simple sum monetary aggregate (SSUM). Hence, the SSUM confounds together CSM and ISM. If the variations of CSM and ISM are related, then that relationship will either cause SSUM to exhibit more or less variation over time depending on whether the CSM and ISM are positively or negatively correlated.

3.1 Analytical Analysis

The first question to be answered is whether there is an analytical relationship between CSM and ISM. To examine this question, take the first derivative of each with respect to the return yielded by each monetary asset:

\[
\frac{\partial}{\partial r_{nt}} CSM = -\frac{m_{nt}}{R_t} \quad \forall n = 1 \ldots N, \tag{12}
\]

\[
\frac{\partial}{\partial r_{nt}} ISM = \frac{m_{nt}}{R_t} \quad \forall n = 1 \ldots N. \tag{13}
\]

Note that (12) and (13) are identical except that they have opposite signs. Therefore, it is expected that the CSM and ISM will respond to changes in
interest rates by moving in opposite directions, and since SSUM adds these two indices together, SSUM expected response is no change. Hence, most, if not all, of the dynamic response of the monetary stock to changes in interest rates is lost when SSUM is used.

3.2 Empirical Analysis

The analytical analysis in the previous section raises three empirical questions. One, how do the distributions between CSM, ISM and SSUM compare, and is there more or less variability in SSUM? Two, is there a negative relationship between CSM and ISM, and how strong is that relationship? And three, how much information, if any, about the dynamic movements of the monetary stock is contained in SSUM?

First consider question one, how do the distributions between CSM, ISM and SSUM compare? Table 1 reports descriptive statistics of the one quarter simple change in the monetary stock as measured by CSM, ISM and SSUM for the United States that was collected from the Economic Data - FRED® database maintained by the Saint Louis Federal Reserve. The sample period for each index measured at the M1, M2 and M3 levels of aggregation begins the first quarter 1960 and ends the fourth quarter 2005, and the sample period for each index measured the L level of aggregation begins the first quarter 1960 and ends the third quarter 1998. The sample period corresponds to the data that is available at the time of this paper’s publication. An interesting point to notice is that the standard deviation is much smaller in SSUM than in any of the other indices. Also, the range exhibited by one quarter change in the SSUM is much smaller. SSUM clearly exhibits less variability then either CSM or ISM.

[Table 1 about here]
Question two, is there a negative relationship between CSM and ISM? Figure 1 plots scatter plots of one quarter change in current stock of money and the present value of investment stock of money at the M1, M2, M3 and L levels of aggregation. Notice the strong negative correlation between CSM and ISM at M2 and higher levels of aggregation.

[Figure 1 about here]

But how strong is that relationship? Table 2 reports the output from regressing the standardized one quarter change in the investment stock of money on the standardized one quarter change in the current stock of money, \( \Delta CSM_t = \beta \cdot \Delta ISM_t + \varepsilon_t \). From the regression output there is strong evidence of both serial correlation and heteroskedasticity, thus robust standard errors are calculated. This analysis suggests that the negative relationship between CSM and ISM is statistically significant at the one percent confidence level with ISM able to explain an average of 90% of the variation in CSM at M2 and higher levels of aggregation. Moreover, at M2 and higher levels of aggregations the coefficient on ISM is nearly negative one.

[Table 2 about here]

Question three, how much information, if any, about the dynamic movements of the monetary stock is contained in SSUM? Table 2 also reports the output from regressing the standardized one quarter change in the the SSUM on the standardized one quarter change in the CSM, \( \Delta CSM_t = \beta \cdot \Delta SSUM_t + \varepsilon_t \). Again, robust standard errors are calculated. Note that the coefficient on SSUM is significant only at the M1 and M2 level of aggregation, and at the M2 level of aggregation only about 24% of the variability in CSM is explained by SSUM. Figure 2 plots residual, fitted and actual values for this regression.

[Figure 2 about here]

Figure 3 plots the CSM, the SSUM and the ISM at the M1, M2, M3 and L
levels of aggregation. It can be seen that while SSUM is biased at all levels of aggregation, SSUM captures the directional movements of the monetary stock at the M1 level of aggregation. But at higher levels of aggregation, SSUM clearly fails to capture the directional movements of the monetary stock.

[Figure 3 about here]

4 Does the Money Measurement Matter?

In the previous section, I have demonstrated that SSUM is not only a biased measure of the monetary stock, but it also fails to capture the directional movements of the current stock of money. However, the question of whether or not monetary aggregates are of any economic interest at all still remains. If money provides no new information about the macroeconomy, then measuring money is a fruitless effort. I will address this question by estimating four reduced form backward looking IS equations that contain various measures of money. In doing so, I will determine if lags of a given money measurement are significant predictors of GDP gap even after accounting for lags of GDP gap and real interest rates.

4.1 Model Specifications

I am presently concerned with whether or not lags of a given money measurement are significant predictors of GDP gap. I am not, at this point, concerned with testing various specifications of the transmission mechanisms. Therefore, I choose to follow (Hafer et al., 2007) and estimate the following reduced form backward looking IS equation,

\[ GDPGAP_t = \beta_1 GDPGAP_{t-1} + \beta_1 GDPGAP_{t-2} + \beta_3 RFFUND_{t-1} + \beta_4 M_{t-1} + \epsilon_{n,t}, \]  

(14)
which has been a popular approach in this debate.

Each variable in (14), except $M_t$, is defined as in Rudebusch and Svensson (2002). Thus, $GDP_{GAP}^t$ is the percentage difference between chain weighted real GDP and real potential GDP published by the Congressional Budget Office and $RFFUND^t$ is the difference between a four quarter average of the effective federal funds rate and a four quarter average of inflation using the GDP chain weighted index. Four models will be estimated using various monetary aggregates. In model 1, money is omitted. In model 2, $M_t = VOLCSM2_t$ is the velocity of money using the current stock of money at the M2 level of aggregation. In model 3, $M_t = VOLSSUM2_t$ is the velocity of money using the SSUM at the M2 level of aggregation. In model 4, $M_t = \Delta \ln MSI2_t$ is the growth rate of the monetary service flow measured at the M2 level of aggregation.

4.2 Results

Table 3 reports the results of estimating (14) with various measures of money. The first thing to notice is that the coefficient on money is significant at the five percent level of confidence in models 2 and 4, but coefficient on money is not significant in model 3. This is consistent with the findings of Hafer et al. (2007) and Leeper and Roush (2003) as models 2 and 4 contain aggregation theoretic measures of money and model 3 contains the inferior simple sum aggregate. Another interesting result is in model 4, where the growth rate of the monetary service flow is included. In this model, the real effective federal funds rate is not statistically significant.

[Table 3 about here]

Each model is also tested for parameter stability. Quandt-Andrews maximum likelihood ratio test for unknown breakpoint is conducted on each model. The maximum likelihood ratio F-statistic occurs in the first quarter of 1983 for each of the four models, which is consistent with the breakpoint date found in
previous studies. However, I find that this breakpoint date is not statistically significant.

5 Conclusion

The debate over money is one that has existed for many years. A general consensus has emerged that measures of the money stock are of no economic interest, and for the purposes of macroeconomic analysis and monetary policy the money stock can be ignored. This consensus, though, has been largely based on research that has utilized SSUM. Because SSUM confounds CSM and ISM, changes in the rate of return on a monetary asset are expected to have no effect on SSUM. Thus, SSUM obscures the true relationship between interest rates and the money stock and important information about the movements of the money stock is lost. Indeed, it has been demonstrated by Barnett and others that in an economy where monetary assets yield return there is no theoretical justification for using SSUM. Thus, the foundation for concluding that money does not matter is built on data that has been mismeasured.

The results of this study indicate that proper measurement matters. When money is measured using aggregation theoretic index numbers, the quantity aggregates matter, statistically at least, even after accounting for the real interest rate and lag of the dependent variable. Moreover, the simple sum measures of money are found to not have any statistically significant explanatory power. Numerous studies, including this one, have shown that the quantity of money, if measured properly, contains a statistically significant level of information about the dynamics of the macroeconomy.

Clearly, one study cannot, by itself, end the debate over money. However, I believe that in this paper I have presented compelling evidence that the debate is not over, and that to answer the question of what role monetary aggregation
has to play in monetary policy and macroeconomic analysis, we must use the best measurements available. Measurements that are made using methodology that is coherent with the assumptions made in the models within which the data is to be used.

References


Table 1: Descriptive statistics of the one quarter simple change in the monetary stock as measured by CSM, ISM and SSUM.

<table>
<thead>
<tr>
<th></th>
<th>M1&lt;sup&gt;a&lt;/sup&gt;</th>
<th>M2&lt;sup&gt;a&lt;/sup&gt;</th>
<th>M3&lt;sup&gt;a&lt;/sup&gt;</th>
<th>L&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSM</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>18.647</td>
<td>85.197</td>
<td>107.299</td>
<td>59.152</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>32.327</td>
<td>306.115</td>
<td>510.138</td>
<td>345.894</td>
</tr>
<tr>
<td>Minimum</td>
<td>-45.831</td>
<td>-863.249</td>
<td>-1094.355</td>
<td>-1054.580</td>
</tr>
<tr>
<td>Maximum</td>
<td>165.812</td>
<td>1585.274</td>
<td>3026.776</td>
<td>1661.064</td>
</tr>
<tr>
<td>ISM</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.507</td>
<td>18.901</td>
<td>53.112</td>
<td>71.423</td>
</tr>
<tr>
<td>Median</td>
<td>4.338</td>
<td>18.310</td>
<td>29.246</td>
<td>37.791</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>20.598</td>
<td>273.123</td>
<td>476.035</td>
<td>371.350</td>
</tr>
<tr>
<td>Minimum</td>
<td>-79.197</td>
<td>-1323.618</td>
<td>-2362.069</td>
<td>-1664.187</td>
</tr>
<tr>
<td>Maximum</td>
<td>54.706</td>
<td>889.033</td>
<td>1510.374</td>
<td>1315.880</td>
</tr>
<tr>
<td>SSUM</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>20.155</td>
<td>104.098</td>
<td>160.410</td>
<td>130.575</td>
</tr>
<tr>
<td>Median</td>
<td>12.408</td>
<td>78.840</td>
<td>78.840</td>
<td>102.327</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>28.916</td>
<td>92.274</td>
<td>164.887</td>
<td>110.056</td>
</tr>
<tr>
<td>Minimum</td>
<td>-60.789</td>
<td>-49.070</td>
<td>-65.058</td>
<td>-42.520</td>
</tr>
<tr>
<td>Maximum</td>
<td>130.310</td>
<td>403.099</td>
<td>798.767</td>
<td>602.002</td>
</tr>
</tbody>
</table>

<sup>a</sup> Sample period for each index measured at the M1, M2 and M3 levels of aggregation begins the first quarter 1960 and ends the fourth quarter 2005 (183 obs.).

<sup>b</sup> Sample period for each index measured at the L level of aggregation begins the first quarter 1960 and ends the third quarter 1998 (153 obs.).
Table 2: Estimation of relationship to the current stock of money

<table>
<thead>
<tr>
<th></th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$M_3$</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISM$^b$</td>
<td>Coefficient</td>
<td>-0.475</td>
<td>-0.956</td>
<td>-0.946</td>
</tr>
<tr>
<td></td>
<td>R-squared</td>
<td>0.226</td>
<td>0.913</td>
<td>0.896</td>
</tr>
<tr>
<td></td>
<td>Standard Error</td>
<td>0.115</td>
<td>0.049</td>
<td>0.080</td>
</tr>
<tr>
<td></td>
<td>Durbin-Watson</td>
<td>0.376</td>
<td>0.311</td>
<td>0.431</td>
</tr>
<tr>
<td>SSUM$^c$</td>
<td>Coefficient</td>
<td>0.779</td>
<td>0.489</td>
<td>0.362</td>
</tr>
<tr>
<td></td>
<td>t-statistic</td>
<td>9.983***</td>
<td>3.626***</td>
<td>1.560</td>
</tr>
<tr>
<td></td>
<td>R-squared</td>
<td>0.607</td>
<td>0.239</td>
<td>0.131</td>
</tr>
<tr>
<td></td>
<td>Standard Error</td>
<td>0.079</td>
<td>0.135</td>
<td>0.232</td>
</tr>
<tr>
<td></td>
<td>Durbin-Watson</td>
<td>1.565</td>
<td>1.290</td>
<td>0.824</td>
</tr>
</tbody>
</table>

$^a$ Each of the variables are standardized to be mean zero and have a standard deviation of one. The symbols ***, **, * indicate statistical significance at the 1%, 5% and 10% level, respectively.

$^b$ Regression output from estimating $\Delta CSM_t = \beta \cdot \Delta ISM_t + \epsilon_t$.

$^c$ Regression output from estimating $\Delta CSM_t = \beta \cdot \Delta SSUM_t + \epsilon_t$.

$^d$ The standard errors are calculated using the Newey-West heteroskedasticity, auto-correlation consistent method.
Table 3: Various models of GDP gap (1960:3 - 2005:4)\(^a\)

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{const})</td>
<td>0.108</td>
<td>-0.589*</td>
<td>0.719</td>
<td>-0.200</td>
</tr>
<tr>
<td></td>
<td>(0.087)</td>
<td>(0.308)</td>
<td>(0.798)</td>
<td>(0.128)</td>
</tr>
<tr>
<td>(\text{GDPgap}_{t-1})</td>
<td>1.207***</td>
<td>1.177***</td>
<td>1.208***</td>
<td>1.188***</td>
</tr>
<tr>
<td></td>
<td>(0.091)</td>
<td>(0.095)</td>
<td>(0.090)</td>
<td>(0.090)</td>
</tr>
<tr>
<td>(\text{GDPgap}_{t-2})</td>
<td>-0.293***</td>
<td>-0.281***</td>
<td>-0.293***</td>
<td>-0.299***</td>
</tr>
<tr>
<td></td>
<td>(0.089)</td>
<td>(0.089)</td>
<td>(0.089)</td>
<td>(0.089)</td>
</tr>
<tr>
<td>(\text{Rfund}_{t-1})</td>
<td>-0.055*</td>
<td>-0.081**</td>
<td>-0.050*</td>
<td>-0.047</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.036)</td>
<td>(0.030)</td>
<td>(0.029)</td>
</tr>
<tr>
<td>(\text{VOLCSM}_{t-1})</td>
<td>n/a</td>
<td>0.007**</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\text{VOLSSUM}_{t-1})</td>
<td>n/a</td>
<td>n/a</td>
<td>-0.010</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
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<td></td>
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</tr>
<tr>
<td>(\Delta \ln \text{MSI}_{t-1})</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>0.050**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.020)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.898</td>
<td>0.900</td>
<td>0.898</td>
<td>0.902</td>
</tr>
<tr>
<td>(F)-Statistic</td>
<td>520.181***</td>
<td>396.511***</td>
<td>389.649***</td>
<td>405.809***</td>
</tr>
<tr>
<td></td>
<td>(0.781)</td>
<td>(0.775)</td>
<td>(0.782)</td>
<td>(0.767)</td>
</tr>
<tr>
<td>Max LR(^b)</td>
<td>3.783</td>
<td>4.42</td>
<td>7.949</td>
<td>4.204</td>
</tr>
<tr>
<td>Break Date(^c)</td>
<td>1983:1</td>
<td>1983:1</td>
<td>1983:1</td>
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\(^a\) Model 1 omits money. Model 2 and 3 include measures of the velocity of money using the CSM and SSUM, respectively. Model 4 includes the growth rate of the monetary service flow. Newey-West heteroskedasticity, auto-correlation consistent standard errors appear in parentheses. The symbols *** , ** , * indicate statistical significance at the 1%, 5% and 10% level, respectively.

\(^b\) Maximum likelihood ratio F-statistic form Quandt-Andrews test for unknown breakpoint.

\(^c\) Date of breakpoint with the maximum likelihood ratio F-statistic.
Figure 1: Scatter plot of one quarter simple change in CSM vs. ISM, (1960:01 - 2005:04).
Figure 2: Plot of residual, fitted and actual values of one quarter simple change in the CSM when SSUM is regressed on CSM, $CSM_t = \beta \cdot SSUM_t + \epsilon_t$. 
Figure 3: The CSM, ISM and SSUM, quarterly data (1960:01 - 2005:04).