Managing fundamentals versus preferences: Re-balancing portfolios and stock returns

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LATEST VERSION

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What can granular data on investor holdings tell us about stock price variation? I model the growth rate of a portfolio manager's holdings based on evolving asset fundamentals by including demand for asset-specific characteristics in a portfolio optimisation function. Alongside changes in asset characteristics, the manager re-allocates wealth according to evolving preferences. This introduces memory into the portfolio management problem, as past investments inform the choice for new allocations. Using the model, I decompose the growth rate of mutual fund holdings by the effect of i) changing stock characteristics, ii) new preferences, and iii) mean reversion in latent demand. I nest these estimated components, by aggregating holding growth rates by the fund's total net assets, into an expression for stock price growth. I find that changing preferences explain at least as much variation in stock prices as changes in fundamentals. This demonstrates the importance of studying heterogeneity in investor preferences, and their evolution, in furthering our understanding of stock market phenomena.

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1 Introduction

With data on the universe stock holdings, one could attribute a stock's price growth to the growth rate of investor-level holdings and the cross-section of investor wealth:

$$r = \boldsymbol{w}_{-1} \cdot \boldsymbol{g},$$

where r is the stock price growth rate, a cross-product of the vector of lagged market equity shares by investor $\boldsymbol{w}_{-1}$, and the vector of growth rates $\boldsymbol{g}$ in each investor's portfolio equity. Given observed wealth $\boldsymbol{w}_{-1}$, can we predict $\boldsymbol{g}$ to forecast return r?

Variation in stock returns is typically modelled from the perspective of one representative investor, who evaluates commonly observed risk factors (Sharpe 1964, Lintner 1965, Fama & French 1993, 2015). Certain avenues of research incorporate investor psychology into financial models to capture a variety of reactions to financial news (De Long et al. 1990, Hirshleifer 2015, Bordalo et al. 2020). Their predictions still focus on central tendencies in investment decisions, typically among a few groups of investor types. Taking a data-focused approach, Koijen & Yogo (2019) demonstrate the theoretical and empirical importance of disaggregated data on investor holdings in reconstructing aggregate stock price dynamics.

This paper builds on the recent efforts of Koijen & Yogo (2019) to model holding growth rates from portfolio re-allocations triggered by an evolving cross-section of asset fundamentals. The core model in this paper highlights the trade-off between i) optimising a portfolio according to fundamentals, and ii) the amount of re-balancing, which is constrained by a fixed number of transactions. The first effect is the force that data on firms' fundamental values exert on the manager's valuations. The second effect, expressed as square deviations in holding growth rates, is an adjustment cost that introduces inertia to portfolio changes.[1]

This initial framework reproduces the aggregate dynamics of well-known factor models. For example, when the portfolio manager minimises exposure to systemic risk stemming from stock returns' covariances with the market return, the framework returns the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965). Introducing more characteristics leads to a multi-factor model, such as the popular Fama-French Factor (FFF) model using book-to-market values to measure liquidation risk and firm size (Fama & French 1993).

Beyond reproducing standard models, this paper studies the inertia produced, not just by adjustment costs, but by the portfolio manager's preferences. I achieve this by incorporating the asset demand system studied by Koijen & Yogo (2019) into the portfolio management problem. Koijen & Yogo (2019) use a demand system approach to capture the heterogeneity in investor-level investment decisions. In my approach, demand matters since observed fundamental value can be surprising in a setting with noise, and perhaps unreliable. I introduce a new objective that regulates the portfolio manager's exposure to *surprises* in fundamental news is analogous to introducing diminishing returns to fundamental value at the stock level, as in Koijen & Yogo's (2019) demand system. A catch-all term for this

[1] This term sums to realised price volatility when aggregating over all investors, an interesting parallel to the model-implied variances used in Markowitz's (1952) mean-variance optimiser.

objective is *entropy production*. In addition to preserving the possible heterogeneity in managers' preference, entropy production gauges the extent to which a portfolio's exposure to fundamental value remains diversified – and robust to surprises.

The prediction of this study is that portfolio re-allocations are informed by both fundamental values, as well as past allocations. This introduces variable level and change effects of fundamental values onto the cross-section of holding growth rates for each manager. Explaining past allocations using past asset characteristics, which is the exercise in Koijen & Yogo (2019), further decomposes variation in past allocations between the share explained by observed fundamental values, versus the share attributed to latent demand. The main, novel contribution of the paper is a decomposition of holding growth rates at the investor level by three components: i) the effect of changing fundamental values, ii) the effect of changing preferences, and iii) the effect of latent asset demand.

Application The model asserts a structure on stock returns, relying on disaggregated holding growth rates by a cross-section of investors. I decompose the variation in returns explained by investor-level i) changes in value derived from fundamentals, ii) changes in preferences, iii) mean reversion in latent demand. To measure each component, I use quarterly data on mutual fund holdings from the Center for Research in Security Prices (CRSP). While the data capture only a fraction of the investment universe, they provide valuable insights into the role of varied investment mandates, classified by CRSP as, for example, growth- versus income-focused mutual funds. I use firm data from Compustat and stock price data from CRSP to construct variables for five fundamental characteristics, following the five factor model of Fama & French (2015).

I construct a proxy for stock price growth by aggregating holding growth rates by mutual funds, weighing their holdings by their reported total net assets. I find that this 'aggregated' proxy explains three quarters of variation in stock price growth, even though mutual fund assets cover roughly 18% of stock market equity. Using the repeated cross-sections for portfolio holdings at the mutual fund level, I first estimate the latent demands by adapting the asset demand specification of Koijen & Yogo (2019). In the second step, I estimate the coefficients to the dynamic asset demand model using estimated latent demands, as well as lags and first differences in the stock characteristics. I find large heterogeneity in estimated coefficients, which remains difficult to disentangle when categorising funds by their reported objectives.

The different components to holding growth rates open the doors to predicting stock returns. Changes in fundamental characteristics explain up to 15.1% of stock price growth variation through growth in mutual fund holdings, so predicting changes in firm fundamentals can help predict a similar fraction of stock price variation. Similarly, preferences account for up to 24.3% of stock price variation, which exceeds the share of variation explained by changing fundamental characteristics. Explaining changes in strategies and preferences constitutes an interesting avenue for novel research. Around half of variation in stock returns stems from unexplained variation in holding growth rates, which highlights the fact that funds likely follow a wide variety of signals, many of which may not be observed.

Related literature Different literatures lend a range of perspectives on the role of entropy production in the cross-section of portfolio holdings. Since the portfolio manager's asset demand is reflected by the market equity of firms, the model produces a factor not dissimilar from the FFF 'Small-Minus-Big' (SMB) portfolio.[2] In this fashion, this paper integrates recent advances in the study of asset demand into more traditional factor models (Koijen & Yogo 2019, Gabaix & Koijen 2021).

Koijen & Yogo (2019) use holdings data at the investor level to reconstruct aggregate stock price fluctuations as a function of *heterogeneous* demand for characteristics. The authors detail the importance of mean reversion of latent asset demand at the investor level. In a recent paper, Gabaix & Koijen (2021) demonstrate the distortions from mandates held by large mutual funds on the share of capital invested in stocks, versus bonds. Their amplifying mechanism produces significant impacts on overall valuations even when new capital flows are relatively small. In contrast to the asset demand model of Koijen & Yogo (2019), this paper proposes a dynamic asset demand model that predicts the growth rate of portfolio holdings as a function of characteristics-based demand. In particular, investors are compelled to decrease holdings in firms that reveal positive changes in fundamental value, so that their portfolio's exposure to fundamentals remains well diversified. The foundational assumption that justifies this behavior is that investors exhibit a preference for variety in their portfolio's exposure to fundamentals, so that fundamentals from one asset to the another are not perfect substitutes.

A literature in portfolio management studies the tradeoff between a portfolio's depth – its aggregate Sharpe ratio – against its breadth – the number of stocks held (Grinold 1989, Ding & Martin 2017). Chien et al. (2012) study excess stock price volatility due to investors re-balancing their portfolios infrequently. They emphasise the interaction such intermittent re-balancing has with other market actors, who need to anticipate more aggregate risk during downturns. Calvet et al. (2009) study administrative data on portfolios of Swedish households, and document the strong heterogeneity with which individual households rebalance their portfolios. This paper links portfolio re-balancing to the characteristics-based approach of Koijen & Yogo (2019), using a utility function that compels the portfolio manager to keep a diversify source of fundamental value through their exposure to different assets.

Another angle enters from the learning literature, in that the portfolio manager benefits from introducing some 'shrinkage' with which to estimate valuations from noisy data. In this fashion, the model links to the literature on reinforcement learning (Camerer & Ho 1999, Malmendier & Nagel 2011), as well as algorithmic trading (Cover 1991) This behaviour is similar to the use of *robustness* in Hansen & Sargent (2008), who propose to replace expected value in rational agents' objectives with the entropy of value. In this way,

[2] A particularly desirable property of market capitalisation as a second factor is its role as 'zero-covariance' diversification; among a sequence of totally uncorrelated asset returns, optimal allocations tend to be uniform (Samuelson 1967).

they optimise for a distribution of possible outcomes, which is desirable when the agent takes into account the likelihood their model is mis-specified. Their efforts are closely related to Sims's (2003) use of *rational inattention*, which is motivated by the limited capacity economic agents posses in processing data. Again, entropy is a natural way with which to describe this channel capacity.

The key message of this paper is that asset demand introduces entropy production into portfolio managers' optimisation programme. The result is a preference for diversification, which, on aggregate, results in a factor for price returns linked to their overall market value. This size effect is documented first by Banz (1981), and included in the attempts by Fama & French (1992, 1993) to isolate the few factors that maximally describe the cross-section of stock returns. Interestingly, Pollet & Wilson (2008) study an intermediate results, and show the improved performance achieved by funds that diversify more in response to growing larger.

Structure The paper is structured as follows. Section 2 introduces the portfolio model that links the evolution of asset demand to the growth rate of holdings. This model produces a decomposition for variation in stock price growth, which I test empirically in Section 3. Section 4 concludes.

2 Modelling a portfolio's evolution

The motivation behind the model is to study market equities, and their changes, as a function characteristics-based demand at the portfolio level. The main output is a linear model that expresses the growth rate of portfolio investments as a function of i) the characteristics valued by the portfolio manager – the asset demand function of Koijen & Yogo (2019), ii) the evolution of preferences and iii) latent demand. This evolution can be interpreted via Hansen & Sargent's (2008) use of *robustness*, so that the manager chooses allocations, not only in accordance with characteristics, but also model mispecification. In this paper, model mispecification arises from a change in the demand elasticities the manager exhibits for the relevant characteristics.

I start with a simple version to demonstrate the foundational mechanisms using portfolio management. This version is interesting on its own since it reproduces the CAPM as a special case. I then introduce entropy production to study equilibrium changes in portfolio shares as a function of changing asset demand. This behaviour manifests as a moment condition in the cross-section of stock returns corresponding to the size factor.

Notation The model includes an initial period and final period, $t-1$ and t, to describe a cross-section of M stocks indexed by j. Since the focus is on choice between risky assets, I do not include a risk-free security, although many of the results for holding growth rates can be in excess of an arbitrary risk-free rate. While the model applies for a cross-section of portfolio managers with possibly heterogeneous beliefs, I set it up from the perspective of one manager only for notational ease.

At any given time, the total portfolio equity in all M firms is $Y_t = \sum_{j=1}^{M} Y_{j,t}$, where $Y_{j,t}$ denotes the portfolio equity of firm j. The first variable of interest is the equity for firm j as a fraction of total equity $y_{j,t} \equiv Y_{j,t}/Y_t$, which is strictly positive under a short-selling constraint faced by the manager. Portfolio equities evolve over time by a holding growth rate $g_{j,t} \equiv \Delta Y_{j,t}/Y_{j,t-1}$, where Δ denotes the difference operator $\Delta x_t \equiv x_t - x_{t-1}$. These holding growth rates only reflect changes in equity, without any dividend payments.

A bar denotes the valuation-weighted average of a variable throughout the paper, $\bar{x}_t \equiv \sum_j^M y_{j,t-1} x_{j,t}$, using past valuations to stay consistent when dealing with average growth rates. Thus, the total growth rate $\bar{g}_t \equiv \sum_{j=1}^{M} y_{j,t-1} g_{j,t} = \Delta Y_t / Y_{t-1}$ is the holding growth rate averaged across j. Since the model uses several permutations of variances and covariance, I standardise their notation using

$$\sigma^2(\boldsymbol{x}_t) \equiv \sum_{j=1}^{M} y_{j,t-1} \left(x_{j,t} - \bar{x}_t \right)^2$$

for the variance of a vector $\boldsymbol{x}_t$ of variables $x_{j,t}$ weighted by portfolio equities $y_{j,t-1}$, and

$$\sigma(\boldsymbol{z}_t, \boldsymbol{x}_t) \equiv \sum_{j=1}^{M} y_{j,t-1} \left(z_{j,t} - \bar{z}_t \right) \left(x_{j,t} - \bar{x}_t \right).$$

for covariances between values $z_{j,t}$ and $x_{j,t}$ weighted by market equities $y_{j,t-1}$. As such, I define holding growth variation as the weighted sum of squared holding growth rates.

Definition 1. *Holding growth variation is*

$$\sigma^2(\boldsymbol{g}_t) = \sum_{j=1}^{M} y_{j,t-1} \left(g_{j,t} - \bar{g}_t \right)^2,$$

the value-weighted, average squared deviation in return r.

It may help to treat the portfolio manager as a social planner in charge of the market portfolio, in which case holding growth rates coincide with stock price returns, and holding growth rate variation is the cross-sectional variance in stock price returns. Holding growth variation, defined by Definition 1, is a measure of realised volatility I use instead of the model-implied volatilities that are commonly used to compute portfolio tracking errors. I later discuss how holding growth variation relates to the adjustment cost the manager faces when re-balancing their portfolio.

Fundamental value Firms reveal news on their fundamental value $d_{j,t}$. For now, I start with a single characteristic used to gauge fundamental value. In practice, I use characteristics from the five factor Fama-French Factor (FFF) model (Fama & French 2015). Same as with holding statistics, the average fundamental value is weighted by market equities, $\bar{d}_t = \sum_{j=1}^{M} y_{j,t-1} d_{j,t}$, and dispersion in fundamental value in the weighted cross-sectional variance in fundamental value $d_{j,t}$ is $\sigma^2(\boldsymbol{d}_t)$.

Welfare A portfolio manager derives welfare from maximising the expected fundamental value of their portfolio

$$\mathcal{W}_t = \mathbb{E}\Big[f(d_{j,t})\Big],$$

where $f(\cdot)$ is a differentiable, monotonically increasing payoff function. The foundational assumption is that the manager is required to liquidate an unknown portion of her portfolio with some probability. In that event, she derives welfare $\sum_{j=1}^{M} A_{j,t} f(d_{j,t})$ where $A_{j,t} \leq Y_{j,t}$ is the required liquidation amount, of which each unit grants $f(d_{j,t})$. For simplicity, I assume that $f(d_{j,t})$ is linear for now, so that welfare is $\mathcal{W}_t = \sum_{j=1}^{M} Y_{j,t} d_{j,t}$.

The portfolio manager faces a decision on re-allocations $\Delta Y_{j,t}$, which the change her welfare between $t-1$ and t:

$$\begin{aligned}\Delta\mathcal{W}_t &= \Delta\left(\sum_{j=1}^{M} Y_{j,t} d_{j,t}\right) \\ &= \sum_{j=1}^{M} \Delta Y_{j,t} d_{j,t} + \sum_{j=1}^{M} Y_{j,t-1} \Delta d_{j,t}. \end{aligned} \tag{1}$$

Note that the manager derives welfare from inflating holdings for all stocks uniformly by some constant $\Delta Y_{j,t} = \Delta Y_t / M$. I address this issue by fixing the change in portfolio equity to an exogenous value ΔY_t, so that improvements in welfare stem from the re-allocations of capital between assets.

Competing investors This model naturally extends to a setting where multiple investors with heterogeneous beliefs form a stock market. In particular, $Y_{i,j,t}$ would denote the portfolio holding of an investor i in stock j, who re-allocates according to their – potentially private – information $d_{i,j,t}$. Koijen & Yogo (2019) take this view by including a market clearing constraint. In order to focus the attention to the key mechanism for this paper, I only focus on the allocation decision of the manager in isolation.

2.1 Model with one factor

In this section, I gauge the change in portfolio equities $\Delta Y_{j,t}$ between two periods driven by the fundamental value revealed at the start of period t, $d_{j,t}$. In this way, the fundamental values for a cross-section of public firms trigger portfolio re-allocations.

This model yields what looks like the traditional factor structure of returns, expressed for portfolio-level holding growth rates, where the factor driving the cross-section of returns is the characteristic the manager seeks to target. If the characteristic is exposure to the market return, the result is the classic CAPM of Sharpe (1964) and Lintner (1965).

Objective: fundamental value The portfolio manager is tasked with re-balancing their portfolio allocations to incorporate information $d_{j,t}$ about their fundamental value. Given that the characteristic is constant, $d_{j,t-1} = d_{j,t}$, substituting $\Delta d_{j,t} = 0$ into Eq. 1 isolates the improvement in welfare due to better positioning:

$$\mathcal{P} = \sum_{j=1}^{M} \Delta Y_{j,t} d_{j,t}. \tag{2}$$

Quantity $\mathcal{P}$ is the projection of fundamental values $d_{j,t}$ onto portfolio re-allocations $\Delta Y_{j,t}$. This total is analogous to some 'price discovery' on behalf of the manager, who re-allocates wealth to maintain a level of exposure to the fundamental value of each stock. For simplicity, I generally refer to $\mathcal{P}$ as fundamental 'profit', because it reflects the improvement in the portfolio's average fundamental value through better positioning.

Constraint: holding growth variation The manager is assumed to faced an adjustment cost when changing portfolio allocations, which I represent as a constraint on the total squared deviations of portfolio holding growth rates. The total adjustment between the set of old an new valuations is thus the sum of squared distances:

$$\mathcal{V}^2 = \sum_{j=1}^{M} \frac{\left(\Delta Y_{j,t}\right)^2}{Y_{j,t-1}} = \sum_{j=1}^{M} Y_{j,t-1} g_{j,t}^2. \tag{3}$$

In this context, $\mathcal{V}^2$ is a measure for the non-dimensional distance between the market portfolios at times $t-1$ and t. This lends an intuitive perspective on holding growth variation as the distance between the market portfolio at time t from the portfolio at time $t-1$. The potential profit from re-allocations can be high when old allocations $Y_{j,t-1}$ are far from optimal. If the manager is compelled to re-allocate more aggressively to meet a given level of profit $\mathcal{P}$, she will raise holding growth variation as a by-product.

Constraint: liquidity A final constraint fixes the change in total portfolio equity to

$$\Delta Y_t = \sum_{j}^{M} \Delta Y_{j,t}, \tag{4}$$

labelled as the portfolio's *liquidity*. This constraint regulates the degree to which the portfolio manager is able to make new investments without funding them out of other positions. In the data, This variable corresponds to the change in total net assets of mutual fund portfolios.

Solving the programme Figure 1 demonstrates the core mechanism of the simple model. Initial valuations $Y_{j,t-1}$ and fundamentals $d_{j,t}$ are given at the start of the period. The portfolio manager has to choose new valuations $Y_{j,t}$, which generate deviations $\Delta Y_{j,t}$. Fixing liquidity ΔY_t and the sum of squared deviations, in the form of holding growth rate volatility $\mathcal{V}^2$, constrains the manager's ability to increase profit $\mathcal{P}$ by better positioning her portfolio according to fundamentals.

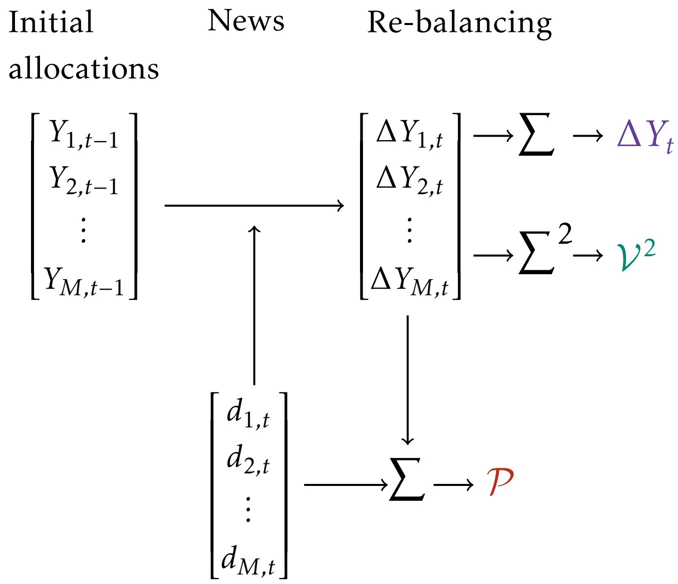


Figure 1: **Visualising the relationship between profit $\mathcal{P}$ and holding growth rate volatility $\mathcal{V}^2$**: this paper models volatility as a by-product of portfolio re-allocations $\Delta Y_{j,t}$, triggered by the revelation of fundamental values $d_{j,t}$. Returns on holding fundamentals $d_{j,t}$, $\mathcal{P}$, are in equilibrium with liquidity ΔY_t and holding growth rate volatility $\mathcal{V}^2$. Arrows indicate the chain of causality, and symbol $\sum$ indicates summation across vector elements.

The system for portfolio re-allocations $\Delta Y_{j,t}$ is in equilibrium when profit is maximised, subject to the constraint on holding growth variation and liquidity. It amounts to solving the Lagrangian

$$\mathcal{L} = \sum_{j=1}^{M} \Delta Y_{j,t} d_{j,t} - \frac{1}{2\kappa}\left(\sum_{j=1}^{M} Y_{j,t-1}\left(\frac{\Delta Y_{j,t}}{Y_{j,t-1}}\right)^2 - \mathcal{V}^2\right) - \rho\left(\sum_{j=1}^{M} \Delta Y_{j,t} - \Delta Y_t\right), \tag{5}$$

where the objective is profit $\mathcal{P}$, with constraints on volatility $\mathcal{V}^2$ and liquidity ΔY_t. Parameters κ and ρ are the relevant multipliers. I state the solution to the maximisation problem in Proposition 1.

Proposition 1 (Single factor model). *Profit $\mathcal{P}$ is maximised when portfolio equity grows at rate*

$$g_{j,t} = \rho + \beta(\boldsymbol{g}_t, \boldsymbol{d}_t) d_{j,t}, \tag{6}$$

where

$$\beta(\boldsymbol{g}_t, \boldsymbol{d}_t) = \frac{\sigma(\boldsymbol{g}_t, \boldsymbol{d}_t)}{\sigma^2(\boldsymbol{d}_t)}, \quad \sigma(\boldsymbol{g}_t, \boldsymbol{d}_t) = \sigma(\boldsymbol{g}_t)\sigma(\boldsymbol{d}_t), \quad \rho = \bar{g}_t - \beta(\boldsymbol{g}_t, \boldsymbol{d}_t)\bar{d}_t,$$

$\sigma^2(\boldsymbol{d}_t)$ is the cross-sectional variance in fundamental news, and $\sigma(\boldsymbol{g}_t, \boldsymbol{d}_t)$ is the weighted covariance of fundamentals and returns.

Proof. See Appendix A.1 □

Eq. 6 in Proposition 1 is just a linear regression of stock holding growth rates on fundamental values at the portfolio level. The subtle difference is that the average overlap between holding growth rates and fundamentals – encapsulated by covariance term $\sigma(\boldsymbol{g}_t, \boldsymbol{d}_t)$ – is related to quantity $\mathcal{P}$ the portfolio's manager actively maximises by growing their allocations at variable rate $g_{j,t}$.

Since holding growth variation $\sigma(\boldsymbol{g}_t)$ is set by $\mathcal{V}^2$, profit is ultimately constrained by the triangle inequality

$$\sigma(\boldsymbol{g}_t, \boldsymbol{d}_t) \leq \sigma(\boldsymbol{g}_t)\sigma(\boldsymbol{d}_t).$$

Therefore, the line of reasoning for the manager is that she should adjust her portfolio according to i) overall dispersion in fundamental values and ii) her tolerance to adjustment costs by incurring more variable re-allocations. She can subsequently use those aggregates to determine how her portfolio can maximise exposure to fundamentals, by constructing the slope coefficient $\beta(\boldsymbol{g}_t, \boldsymbol{d}_t)$. The intercept ρ reflects changes in total portfolio equity.

This logic follows that of finance tradition, which asserts that stock holding growth rates reflect the exposure to factor innovations that feature in the covariance of holding growth rates (Ross 1976, Fama & French 1993). In this setting, the beta accompanying a given characteristic is tied to each portfolio manager's possibly varying preferences for portfolio adjustments $\mathcal{V}^2$. This is the one source of heterogeneity on the condition that fundamentals $d_{j,t}$ are common to all managers.

In the full version of the model, I introduce portfolio equities as a second characteristic for the manager to optimise holding growth rates over, thus yielding a kind of 'Small-Minus-Big' factor. In doing so, I integrate the characteristics-based approach of Koijen & Yogo (2019) into what looks like a standard factor model. The discussion in Section 2.4 demonstrates how exposure to market risk as the measure for fundamental value yields the CAPM as a special case.

2.2 Demand as a robustness criterion

In Proposition 1, allocations adjust according to data on fundamental values. I now extend the framework to incorporate asset demand. I then discuss how asset demand acts as the portfolio manager's memory of past preferences, embedded in initial portfolio allocations, so that the manager is skeptical of adjusting to new data. This use of demand is inspired by Koijen & Yogo (2019), who model the cross-section of portfolio equities using a demand system, but then models the evolution of this demand system in the manner of Hansen & Sargent's (2008) robustness criterion.

Demand and market equity Assume that the portfolio manager can liquidate portfolio holdings, earning a payoff set according to an exponential utility function with fundamental value as variable input:

$$\mathcal{U}(d_{j,t}) = 1 - e^{-\lambda d_{j,t}},$$

where λ is the relevant elasticity. Setting the equity share of stock j equal to the marginal utility derived from exposure to fundamental value $d_{j,t}$ yields

$$y_{j,t} = ke^{-\lambda d_{j,t}}, \tag{7}$$

(a) (b)

Figure 2: **Re-pricing caused by (a) fundamentals vs. (b) demand for fundamentals**.

where k is a constant, and λ the demand elasticity with respect to fundamental d_j. Koijen & Yogo (2019) estimate this system using time-varying parameters for each fund separately, in order to investigate the heterogeneity and temporal variation in the characteristics sought out by different types of investors – households, pension funds, mututal funds, banks etc. I will refer to equity shares $y_{j,t}$ as the portfolio manager's demand for asset j. The growth in demand is

$$\log \frac{y_{j,t}}{y_{j,t-1}},$$

and reflects diverging marginal utilities in a cross-section of stocks, forced by fundamentals $d_{j,t}$. The portfolio manager is now tasked with re-allocating the market portfolio according to changes in asset demand triggered by fundamental news. Welfare improves according to

$$\Delta\mathcal{W} = \sum_{j=1}^{M} \Delta Y_{j,t} \log \frac{y_{j,t}}{y_{j,t-1}}. \tag{8}$$

The significance of replacing fundamental values in Eq. 1 with demands is to add a layer to the portfolio manager's decision to re-allocate, which I demonstrate visually in Figure 2. Depending on the specific form of demand, allocations may be invariant with respect to certain forms of the payoff function. The payoff function in Eq. 7 is one of the simpler options, in which asset demands are invariant to a shift parameter, as well as a multiplier. I discuss this in more detail after outlining the implication this has for the portfolio manager's programme.

Maximising welfare with entropy production To see the role of changing demand in the model, substitute Eq. 7 into Eq. 8:

$$\begin{aligned}
\Delta\mathcal{W} &= \sum_{j=1}^{M} \Delta Y_{j,t}\left(\log k - \lambda d_{j,t}\right) - \sum_{j=1}^{M} \Delta Y_{j,t} \log y_{j,t-1} \\
&= (\Delta Y_t)\log k - \lambda\mathcal{P} + \partial\mathcal{E}, \\
\partial\mathcal{E} &\equiv -\sum_{j=1}^{M} \Delta Y_{j,t} \log y_{j,t-1},
\end{aligned} \tag{9}$$

where I define $\partial\mathcal{E}$ as 'entropy production'. Re-allocations based on changes in demands now incorporate entropy production in addition to profit from fundamentals $\mathcal{P}$ and liquidity ΔY_t. I contrast this mechanism to the simple one studied in the preceding section in Figure

2. Welfare improvements are not just a function of better positioning to fundamentals, but also the extent to which the portfolio managers has to change her mind regarding how she derives utility from exposure to fundamentals. Profit $\mathcal{P}$ remains in the objective, and encapsulates the direct impact of fundamentals in the same fashion as in the simple version of the model.

Why entropy? In the same way that entropy governs model mispecification in Hansen & Sargent (2008), entropy production in the present model is a natural measure for the adjustment in the portfolio manager' frame of reference by re-valuing the parameters to her utility function. These parameter adjustments can change the manner in which the manager allocates new wealth to stocks.

Whether this additional step in deciding allocations increases, or decreases the amount of aggregate holding growth rate volatility will be seen to depend on aggregate outcomes as well as the specific scaling of fundamental values $d_{j,t}$. This scaling is incorporated by the demand function in Eq. 7. For instance, if fundamentals improve uniformly for all stocks, so that fundamental $d_{j,t} = a + d'_{j,t} \forall j$ relative to some benchmark value $d'_{j,t}$, there is no need to re-allocate. Demand shares $y_{j,t}$ remain constant thanks to an adjustment in parameter k, that ensures all demands sum to one. This was the case trivially in Proposition 1, which depends on fundamental dispersion, since adding a constant to a random variable does not increase its variance.

However, demands in Eq. 7 are also invariant with respect to a multiplier a, such that $d_{j,t} = a \times d'_{j,t}$, due to an adjustment in λ. This is not the case in the model without asset demand, where fundamental dispersion would change by a factor a^2. We therefore have a situation where fundamental dispersion may well have increased, and yet asset demands are the same, so no re-allocations take place.

Persistent asset demand Entropy measures inertia exhibited by asset demand in Eq. 7. Figure 3 demonstrates how persistence in asset demand counteracts the impact of fundamental news on final valuations. Despite the higher fundamental value revealed on behalf of the orange stock, the portfolio manager exhibits preference for smooth allocations – a classic motivator for utility in economics. This preference compels them to re-balance the allocations made with respect to fundamentals alone, selling off the orange stock and buying more of the green ones, so that the final portfolio is less concentrated in the orange stock. Measured as entropy production, this inertia contributes volatility to holding growth rates *on top of* dispersion in fundamental news.

There are practical reasons for persistence in asset demand, for example transaction costs, but fundamentally the model requires the manager to *change her preferences* regarding market equity as a function of incoming news on her targeted fundamental characteristic. Gabaix & Koijen (2021) motivate persistent asset demand at the fund level by noting that holdings are not perfectly responsive to news, and funds pursue fixed investment mandates.

However, it is clear that there are strong parallels to the field of robust control, and the exercise in this paper mimics the implementation of robust control methods on behalf of

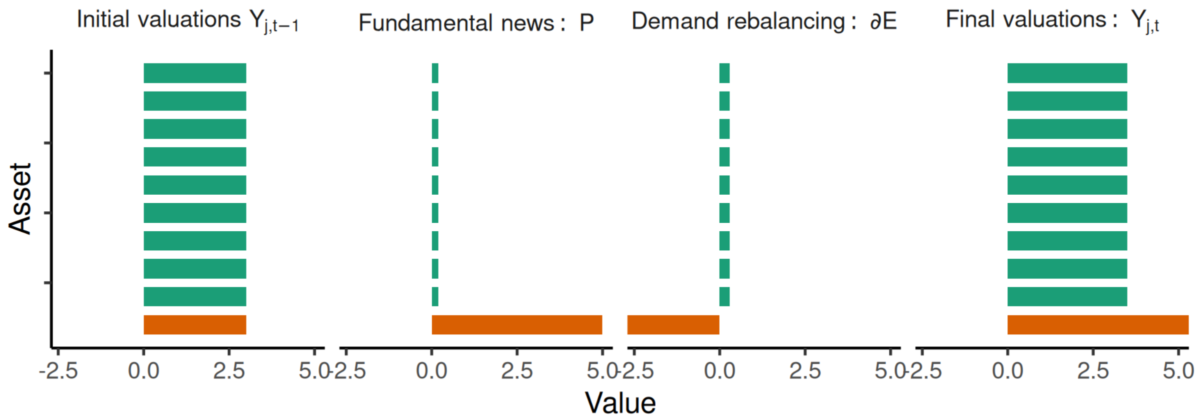


Figure 3: **Persistent asset demand contributes volatility to changes in asset prices driven by fundamental news**: in this cross-section of 10 hypothetical assets, which are uniformly values at the start of the period (leftmost plot), the orange asset receives highly positive news on fundamental valuations (center-left plot). Since the portfolio manager exhibits a preference for the demand system at the start of the period, she re-balances the portfolio by removing a fraction of funds from the orange asset, and allocating those to the green assets in the cross-section (center-right plot). The final distribution of valuations is therefore a combination of changes driven by fundamental news, and asset demand.

rational agents explored in Hansen & Sargent (2008). In their setting, it is not enough for an agent to incorporate relevant news, but they also make decisions that are least exposed to model mispecification. I provide additional motivation for entropy as a measure of diversification in Appendix C.1, and discuss the role of asset demand as an additional SMB factor in Section 2.4.

New programme Separating the manager's welfare objective into three aggregates, namely liquidity, profit and entropy production, outlines which quantities can be used for optimisation. In the programme, I set entropy production as the quantity to maximise, and set profits $\mathcal{P}$ and liquidity ΔY_t as the corresponding constraints. The implication is that the manager seeks to re-allocate based on past asset demand $-\log y_{j,t-1}$, but she cannot afford to deviate from fundamental news. This effectively mimics Bayesian updating, whereby the managers lends some weight to a prior distribution of demands $y_{j,t-1}$, then adjusts according to new data on fundamentals $d_{j,t}$.

Instead of maximising profit from fundamentals $\mathcal{P}$, the manager now seeks to maximise entropy production $\partial\mathcal{E}$ to satisfy her asset demand. Note that this amounts to introducing a second 'size' characteristic to the single-factor model in Proposition 1. Profit from fundamentals now act as a constraint, as the manager has to adjust her demand according to news on fundamentals to the extent that her elasticity of demand λ is high. I solve the problem via the Lagrangian

$$\mathcal{L} = -\sum_{j=1}^{M} \Delta Y_{j,t} \log y_{j,t-1} - \lambda\left(\sum_{j=1}^{M} \Delta Y_{j,t} d_{j,t} - \mathcal{P}\right) - \frac{1}{2\kappa}\left(\sum_{j=1}^{M} Y_{j,t-1}\left(\frac{\Delta Y_{j,t}}{Y_{j,t-1}}\right)^2 - \mathcal{V}^2\right) - \rho\left(\sum_{j=1}^{M} \Delta Y_{j,t} - \Delta Y_t\right), \tag{10}$$

where the objective is entropy production $\partial\mathcal{E}$, with constraints on profit $\mathcal{P}$, volatility $\mathcal{V}^2$ and liquidity ΔY_t. Parameters λ, κ and ρ are the relevant multipliers. Proposition 2 gives the solution to the factor model by including asset demand as a second characteristic.

Proposition 2 (Factor model with demand)**.** *Entropy production $\partial\mathcal{E}$ is maximised when*

$$g_{j,t} = \rho + \beta_d d_{j,t} - \beta_y \log y_{j,t-1}, \tag{11}$$

where

$$\begin{aligned}
\beta_d &= \frac{\sigma(\boldsymbol{d}_t, \boldsymbol{g}_t)\sigma^2(-\log \boldsymbol{y}_{t-1}) - \sigma(\boldsymbol{d}_t, -\log \boldsymbol{y}_{t-1})\sigma(\boldsymbol{g}_t, -\log \boldsymbol{y}_{t-1})}{\sigma^2(-\log \boldsymbol{y}_{t-1})\sigma^2(\boldsymbol{d}_t) - \sigma^2(\boldsymbol{d}_t, -\log \boldsymbol{y}_{t-1})}, \\
\beta_y &= \frac{\sigma(\boldsymbol{g}_t, -\log \boldsymbol{y}_{t-1})\sigma^2(\boldsymbol{d}_t) - \sigma(\boldsymbol{d}_t, -\log \boldsymbol{y}_{t-1})\sigma(\boldsymbol{d}_t, \boldsymbol{g}_t)}{\sigma^2(-\log \boldsymbol{y}_{t-1})\sigma^2(\boldsymbol{d}_t) - \sigma^2(\boldsymbol{d}_t, -\log \boldsymbol{y}_{t-1})}, \\
\rho &= \bar{g}_t - \beta_d \bar{d}_t - \beta_y \mathcal{E}_{t-1}, \\
\mathcal{E}_{t-1} &\equiv -\sum_{j=1}^{M} y_{j,t-1} \log y_{j,t-1}.
\end{aligned}$$

Proof. See Appendix A.2. □

As opposed to Proposition 1, Proposition 2 includes initial portfolio equities $-\log \boldsymbol{y}_{t-1}$ as an additional characteristic. The result effectively models holdings growth rate by a weighted linear model, with fundamentals $\boldsymbol{d}_t$ and log equity shares $-\log \boldsymbol{y}_{t-1}$ as explanatory variables. The portfolio manager can solve this problem with knowledge of the covariance between initial allocations and fundamentals values, which are both known and given exogenously.

Eq. 11 is a two-factor model of holding growth rates, where a size-like characteristic regulates the degree to which holding growth rates on large holdings are allowed to increase. The size anomaly was first investigated by Banz (1981). Under the lens of Ross's (1976) Arbitrage Pricing Theory, this may be justified by the fact that small firms face different risks than large firms. Here, no assumptions were made on risk as a function of holdings size. In fact, to the degree that riskiness is incorporated in the arbitrary value $d_{j,t}$, the covariance between fundamental value and the size of portfolio shares is also arbitrary. There exists a separate, robustness-like requirement for portfolio allocations to satisfy a smooth payoff function in Eq. 7. This is interesting in its own right, but what can be said about investment decisions at the individual level?

2.3 Dynamic asset demand

The role of robustness is for the portfolio manager to shield themselves against model mispecification. In the present context, this can be introduced in the form of time-varying demand elasticities $\beta_{d,t}$. Entropy production is a useful tool to track the degree to which changes in demand elasticities compel the portfolio manager to re-allocate.

Proposition 2 reveals the interesting structure behind holding growth rates. Namely, consider a hypothetical set of allocations that fail to trigger any re-allocation $\log y_j^* = \log y_{j,t-1} = \log y_{j,t}$. In that circumstance, holding growth rates on all stocks are equal, $g_{j,t} = \bar{g}_t$, and

$$\log y_j^* = k + \frac{\beta_{d,t}}{\beta_y} d_{j,t}, \tag{12}$$

where k is a constant, and I allow elasticity $\beta_{d,t}$ to vary with time t. Initial allocations $\log y_{j,t-1}$ can therefore be modelled as a function of variation in demands explained by characteristics, plus some latent demand $e_{j,t-1}$:

$$\log y_{j,t-1} = k + \frac{\beta_{d,t-1}}{\beta_y} d_{j,t-1} + e_{j,t-1}. \tag{13}$$

Eq. 13 is similar to the asset demand function specified by Koijen & Yogo (2019), except that they retain the non-linear form from Eq. 7. The authors subsequently use a large data set on portfolio holdings to estimate the demand elasticities – $\beta_{d,t-1}/\beta_y$ in Eq. 13 – with a repeated cross-section of investor holdings. In the current context, substituting Eq. 13 into Eq. 11, and introducing time subscripts to allow parameters to vary over time, yields an equation for the dynamic evolution of portfolio investments:

$$g_{j,t} = \tilde{\rho}_t + \Lambda_t d_{j,t-1} + \beta_{d,t} \Delta d_{j,t} - \beta_{y,t} e_{j,t-1}, \tag{14}$$

where $\tilde{\rho}_t$ is a constant, and coefficient $\Lambda_t = \beta_{y,t} \Delta\left(\beta_{d,t}/\beta_{y,t}\right)$ reflects the holding growth rate explained by a change in demand elasticities for a given level of fundamental value $d_{j,t-1}$. This equation is of particular interest when studying the growth rate of portfolio investments, and includes three key components.

Endogenous market movements The first is the effect of changing preferences, $\Lambda_t d_{j,t-1}$. It constitutes a level effect on holding growth rates due to a change in the responsiveness of the manager's allocations to the past level of fundamentals. An example of this is a scenario where the value of one firm is unchanged, whereas those of other companies drop. In this scenario, the demand elasticity is higher, since the resource is overall more scarce. The manager sells off stock in the unchanged firm in order to keep a diversified portfolio. Conversely, if other firms experience an increase in valuations, the holding growth rate on the unchanged investment will also grow to keep up with the other allocations.

The second effect is the direct impact of improved fundamentals, $\beta_{d,t} \Delta d_{j,t}$. This term encapsulates the declining marginal utility of holding a stock that provides more fundamental value. This change effect is distinct to the level effect that depends on changing preferences, in that it regulates over-exposures into highly valuable companies by gradually penalising the growth rate within a portfolio.

The third effect is the impact of Koijen & Yogo's (2019) latent demand, $\beta_{y,t} e_{j,t-1}$. This term can be understood as a fixed effect revealed at the initial stage, in that it measures a level of demand for the asset that is not justified by fundamental value. Without any observable change in latent demand, it penalises the holding growth rate on that stock within the portfolio. The reason is the same as before: large holdings are difficult to grow in excess of smaller holdings as long as the manager prefers a diversified portfolio. The preference for diversification is embodied in their asset demand function, taken from Eq. 7.

Key predictions The first desirable outcome is to gauge the degree to which each component contributes to variation in holding growth rates, and, by extension, to holding growth rates. Besides a variance decomposition, the model makes a prediction on the persistence of latent demand, and by extension a window for return predictability. The dynamic asset demand model suggests the presence of an indirect impact from fundamentals on investment activity, through the re-balancing mechanism outlined in Eq. 14. While the direction in which preferences change may go in one direction or the other, the requirement of positive adjustment costs restricts elasticity $\beta_{y,t} > 0$. I state this as a prediction.

Prediction: portfolio holding growth rates $g_{j,t}$ are negatively correlated with past latent demand $e_{j,t-1}$, so that $\beta_{y,t} > 0$.

These predictions are consequential to our understanding of market volatility. In the spirit of Gabaix & Koijen (2021), this model introduces extra factors into the return cross-section stemming from a host of market participants re-adjusting their portfolios to meet their investment mandates, in addition to the impact of fundamental news alone. I test them in Section 3 using data on mutual fund holdings.

2.4 Discussion

I introduce the model for a portfolio holding M assets, held by a portfolio manager tasked with maximising her exposure to firms' fundamental value. The result is an equilibrium condition on holding growth variation as a function of the manager's ability to change allocations. Introducing a robustness-like effect by introducing an asset demand system produces extra factors for returns at the portfolio level, as she constantly re-balances allocations even in the absence of news for certain companies. The structural parameters that govern the problem are aggregate risk aversion, with respect to penalising holding growth variation, and the elasticity of demand with respect to fundamental value, which penalises deviation of allocations from fundamentals.

Comparison to factor models and the CAPM I begin with a simple version of the model in which the portfolio manager changes allocations, thus prices, to match data on fundamental values alone. She acts as a dictator who derives welfare from allocating resources as closely to fundamental values as possible. The only structural parameter is the degree to which the portfolio manager produces price volatility to match fundamental news, which is analogous to an aggregate risk aversion parameter.

This relationship is a factor model. Typically, $\beta_t(\boldsymbol{g}_t, \boldsymbol{d}_t)$ is measured as a time varying return to holding a stock with fundamental value $d_{j,t}$. Introducing additional factors will reduce the contribution of each fundamental to its variation left unexplained, as is the case in the full model. One important factor to consider is variation in market returns.

Proposition 1 reproduces the CAPM when the relevant fundamental value is the covariance of return $r_{j,t}$ with market return $\bar{r}_t$, $d_{j,t} = \sigma_t(r_j, \bar{r})$[3]. Setting a representative managers holding growth rate to the stock's return $g_{j,t} = r_{j,t}$, the average covariance

$$\bar{d}_t = \sum_{j=1}^{M} y_{j,t-1}\sigma_t(r_j, \bar{r}) = \sigma_t^2(m)$$

is the implicit variance of the market return (Fama & French 2004). Finally, the renowned 'market beta' is given by $\beta(r_j, \bar{r}) \equiv \sigma_t(r_j, \bar{r})/\sigma_t^2(m) = d_{j,t}/\bar{d}_t$, the proportional risk each unit invested in j contributes to the market portfolio. To see this, note that

$$\sigma(\boldsymbol{r}_t) = \sigma(\boldsymbol{d}_t) \times \frac{\bar{r}_t}{\bar{d}_t} \tag{15}$$

when substituting $r_{j,t} = (d_{j,t}/\bar{d}_t)\bar{r}_t$ into Definition 1 for holding growth variation. Substituting Eq. 15 into Eq. 6 and re-arranging yields

$$r_{j,t} = \beta(r_j, \bar{r}) \times \bar{r}_t,$$

which is the CAPM of Sharpe (1964) and Lintner (1965) once adjusting for the risk free rate and the expectation operator.[4]

In principle, the full model in Proposition 2 is just a two-factor model, and demand $\log y_{j,t-1}$ can be treated as a size characteristic. Again setting holding growth rate to the stock's return $g_{j,t} = r_{j,t}$, the model reconstruct the returns to a portfolio with weights $-\log y_{j,t-1} - \mathcal{E}_{t-1}$ in term $\sigma(\boldsymbol{r}_t, -\log \boldsymbol{y}_{t-1})$. For the representative investor, this is no more that the return on a SMB portfolio included by the FFF model, with the difference that weights are based on the logarithm of size.

Other factors Fama & French (2004, 2015) review likely fundamental characteristics that drive the cross section of asset returns. For example, firms with varying book value of assets relative to their market value are differently affected by interest rates. This is the underlying motivation behind their FFF model (Fama & French 1993, 2015). It would be interesting for future research to incorporate these additional factors in the structural model for volatility using alternative specifications for demand, for example by appealing to the term structure of equity returns (van Binsbergen & Koijen 2017).

3 Decomposing price fluctuations

This section tests the key predictions of the model in Section 2 using data on mutual fund holdings. The empirical framework can be summarised as a regression specification for the growth rate of investor holdings, in nominal terms, in addition to the demand system

[3]The covariance must vary with time, as indicated by its subscript. Otherwise, the resulting 'market beta' of asset j will mechanically vary over time.

[4]It makes sense to take a forward-looking aggregate using an expectations operator when assuming that the market beta is constant over time.

estimated by Koijen & Yogo (2019). The purpose is to provide a framework under which stock returns from price changes can be attributed to i) changing fundamentals, ii) changing preferences and iii) latent demand by mutual funds.

3.1 Data

Mutual fund data I use the CRSP mutual fund database to extract a panel of portfolio holdings for US mutual funds. These data are required by the Securities and Exchange Commission (SEC) to be reported at a quarterly frequency. In case of duplicates, I only keep the latest values reported in each quarter for a given fund's holdings and total net assets. I filter out any index and exchange traded funds. For each holding in quarter t, I impute its value $Y_{i,j,t}$ by multiplying the portfolio share by the reported total net assets of the fund. I then first-difference it with its value in the previous quarter, if it exists, and take a ratio to compute the holdings value growth rate

$$g_{i,j,t} = \Delta Y_{i,j,t}/Y_{i,j,t-1}.$$

Finally, I drop funds which report 30 or fewer holdings in a given quarter.

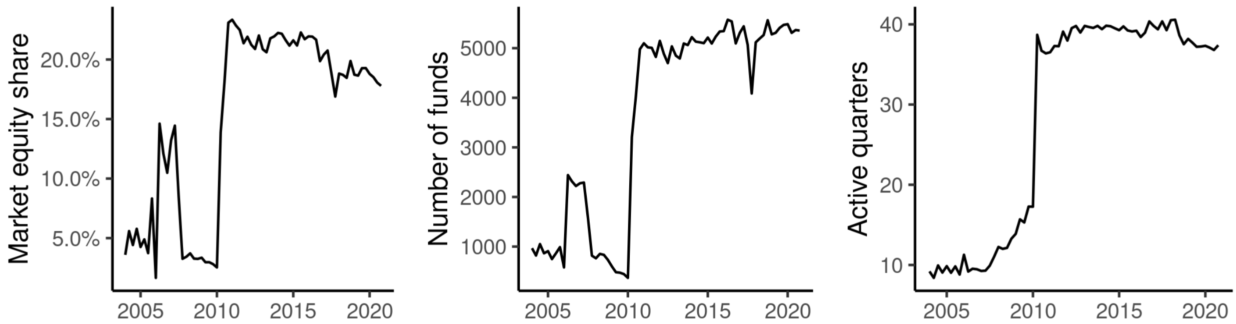


Figure 4: **Coverage of CRSP mutual fund data levels off in 2011**: this paper uses mutual fund data starting from 2011:Q1 in order to mitigate the sparse selection of funds in the early periods of data coverage.

Figure 4 illustrates the coverage of the CRSP mutual fund data, in terms of the number of portfolio observed, the total equity held as a fraction of total market equity, and the number of quarters an average fund provides data for. There is a perceptible break in 2011, even though the SEC presumably required disclosures of holdings starting in 2001. Overall, the magnitude of the data is remarkable; even though it only includes a fraction of market participants reported in Koijen & Yogo (2019), these data still account for about 18% of the US stock market. At the stock level, coverage in terms of market equity held across funds mostly varies between 10% and 40% for stocks with a market equity greater than one billion USD, as seen in the Left of Figure 5. The average fund covered by the data reports holdings for roughly 40 quarters. When removing all observations with incomplete data on portfolio holdings and their corresponding characteristics, the resulting panel includes 1.48×10^7 stock-fund-quarter observations, 8,839 mutual funds, holding 9,483 stocks among them. The number of holdings per fund-quarter, plotted on the Right of Figure 5, appears

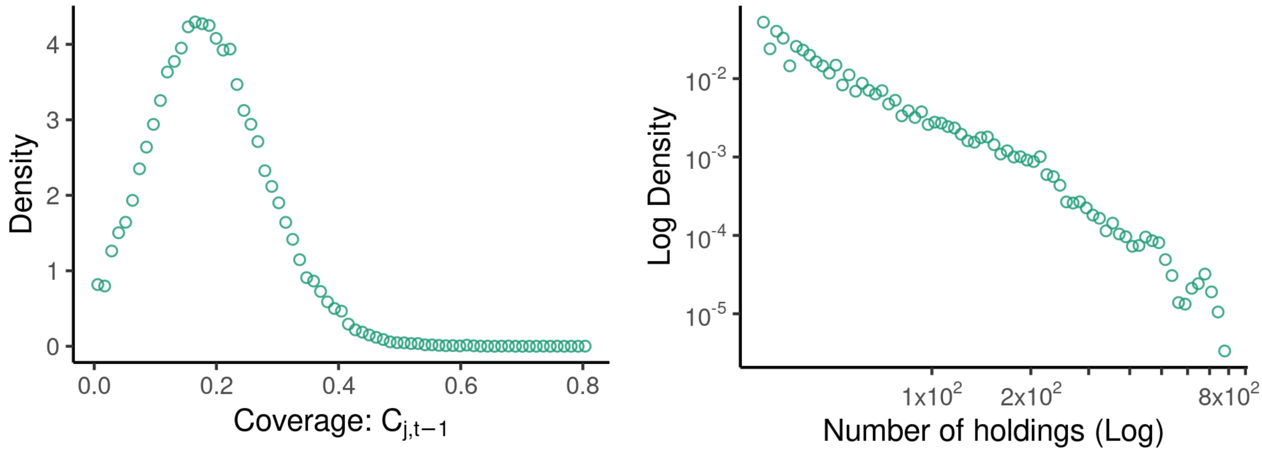


Figure 5: **(Left) Mutual funds hold around 20% of market equity in modal large companies and (Right) their number of holdings follows an approximate power-law distribution**: lag in coverage (Left), defined as the sum of observed mutual fund holdings as a share of market equity for each stock, is used to construct proxy stock returns according to Eq. 17. The number of holdings (Right) is a snapshot of the observation counts available to estimate the holding growth rates, according to Eq. 19, at the fund-quarter level of disaggregation.

to follow a power-law. This indicates that the majority of funds will hold few stocks in their portfolio, in the order of a few dozen, but some hold many hundreds of stocks in any given quarter.

Stock data I use data from the CRSP for common stocks traded on the three major US exchanges: the NYSE, the NYSE MKT (previously AMEX), and the NASDAQ. I build a panel of observations for market equity by firm j at the end of quarter t using prices and outstanding shares reported by CRSP, as detailed in Appendix B.1. Appendix B.1 also details the construction of other variables and additional cleaning procedures. The main variable of interest, besides stock returns and market equity, is the covariance of daily excess returns with the daily excess market return – where the risk-free rate and market returns are provided by Kenneth R. French's data library.[5] Importantly, I only include stock-quarter observations where the stock is listed both in the first and last available trading days of that quarter, in order to avoid the distortions caused by firms de-listing or joining an exchange.

Firm data I merge data from Compustat into the panel of stock returns. I use both quarterly as well as annual versions of Compustat, given that certain variables are reported at different frequencies. I detail the cleaning procedure, as well as a list of variables used, in Appendix B.2. For estimation, I use: i) the logarithm of book-to-market equity, using reported liquidation values from Compustat and the corresponding stock's market equity at the end of quarter t, ii) the profit rate, as the fraction of quarterly operating income after tax and interest payments over liquidation value, and iii) the investment rate, as the annual growth rate of the logarithm of gross property, plant and equipment plus intangible capital

[5] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

and inventories. For each stock, I lag and first difference each characteristic by reported quarter – including the market covariance – to produce the required variables to estimate Eq. 14.

3.2 Empirical framework

Stock prices from the bottom-up The link of mutual fund holdings to aggregate stock price growth was introduced in Section 1, so I make it more tangible here. The price growth on stock j in quarter t can be decomposed via

$$r_{j,t} = \sum_{i=1}^{\mathcal{N}} \frac{Y_{i,j,t-1}}{Y_{j,t-1}} g_{i,j,t}, \tag{16}$$

where $Y_{i,j,t-1}/Y_{j,t-1}$ is the past market equity share in stock j held by investor i, and $g_{i,j,t} \equiv \Delta Y_{i,j,t}/Y_{i,j,t}$ their holding growth rate in stock j. Eq. 16 assumes that the entire universe of $\mathcal{N}$ investors is observed. Notably, stock price growth excludes any dividends distributed to shareholders, an important distinction from the usual stock returns in asset pricing.

In practice, I use data on N mutual funds, who hold a market equity share

$$\hat{Y}_{j,t} = \sum_{i=1}^{N} Y_{i,j,t} \leq Y_{j,t}, \quad N \leq \mathcal{N}$$

in sum. Here, indices $1 \leq i \leq N$ refer to mutual funds only, whereas the remaining $N < i \leq \mathcal{N}$ indicate unobserved investors. This does not cover the entire investment universe, and coverage

$$C_{j,t} = \frac{\hat{Y}_{j,t}}{Y_{j,t}}$$

may vary by stock and time, a fact I document later in this section. However, I construct a proxy for stock price growth $\hat{r}_{j,t}$, by adapting Eq. 16 to include only activity observed in the mutual funds data:

$$\hat{r}_{j,t} = \sum_{i=1}^{N} \frac{Y_{i,j,t-1}}{\hat{Y}_{j,t-1}} g_{i,j,t}. \tag{17}$$

Roadmap In total, there are two stages to predicting stock price growth $r_{j,t}$. The first stage depends on the relationship between aggregate and proxied returns,

$$r_{j,t} = \delta + \psi \hat{r}_{j,t} + \varepsilon_{j,t}, \tag{18}$$

where δ is an intercept, ψ a slope coefficient, and $\varepsilon_{j,t}$ returns unexplained by mutual fund holding growth rates.

To motivate what follows, I test the relationship between stock returns and their proxy constructed from the aggregated holdings growth at the mutual fund level, according to Eq. 18. Due to the sensitivity of the results for small-cap stocks, I restrict the data to stocks with market equity exceeding one billion USD. Table 1 presents the OLS estimates to the slope $\hat{\psi}$

Table 1: **Stock price and mutual fund holdings**

	Dependent variable:	
	$r_{j,t}$	
	Unweighted	Weighted by coverage
$\hat{\psi}$	0.76 (0.004)	0.87 (0.003)
$\hat{\delta}$	0.01 (0.001)	0.02 (0.001)
Obs.	26,716	26,716
R^2	0.63	0.72
Res. Std. Error	0.11	0.03

Notes: This table provides OLS estimates for the relationship between mutual fund holding growth rates in stock j, weighed by that fund's lagged holding size, and the actual returns of that stock in quarter t. This relationship is seen in Eq. 18, where the slope coefficient is estimated to be $\hat{\psi}$, and the intercept $\hat{\delta}$. The second column weighs observations according to the lagged coverage ratio of observed mutual fund holdings as a share of the stock's total market equity, plotted in Figure 4. Standard errors are reported in brackets.

and intercept $\hat{\delta}$ in Table 1. In contrast to the first column, the second column uses a weighted OLS scheme, where the weights are equal to coverage $C_{j,t-1}$. The weighted estimates ought to be more reliable, because the relationship between the return proxy from mutual fund holdings growth should hold more sway over the stock's actual price return if funds hold a larger share of the stock's market equity. The remarkable result is not only that the slope coefficient is close to one, but the R^2 of the regression is 0.72, despite the fact that coverage is rarely above 50%, as seen in Figure 5.

The second stage involves estimating holding growth rates using Eq. 14. This is the key innovation of this paper. Holding growth rates are predicted to have a level and change relationship with respect to stock characteristics, but also depend on old latent demands $e_{i,j,t-1}$. In order to adapt Eq. 14 to the current empirical setting, I estimate weighted Ordinary Least Squares (OLS) estimates for the coefficients of

$$g_{i,j,t} = \tilde{\rho}_{i,t} + \sum_{a=1}^{A} \Lambda_{a,i,t} d_{a,j,t-1} + \sum_{a=1}^{A} \beta_{a,i,t} \Delta d_{a,j,t} - \beta_{y,i,t} \hat{e}_{i,j,t-1} + v_{i,j,t}, \tag{19}$$

which requires estimated latent demands $\hat{e}_{i,j,t}$, plus an idiosyncratic error term $v_{i,j,t}$. Lags in characteristics $d_{a,j,t-1}$ and their changes $\Delta d_{a,j,t}$ are assumed to be observed by all funds in period t.

I finalise the roadmap for this section before explaining the procedure used to estimate latent demands. Coefficient estimates in Eq. 19 can then be used to construct the aggregate components to proxied stock price growth in Eq. 17. In total, the components aggregate into stock price growth using Eq. 18, such that

$$r_{j,t} = \tilde{\delta} + \underbrace{\sum_{a=1}^{A} L_{a,t} d_{a,j,t-1}}_{\text{Preferences}} + \underbrace{\sum_{a=1}^{A} B_{a,t} \Delta d_{a,j,t}}_{\text{Fundamentals}} - \underbrace{K_t}_{\substack{\text{Latent}\\ \text{Demand}}} + \tilde{\varepsilon}_{j,t}, \tag{20}$$

where $L_{a,t}, B_{a,t}$ and K_t are the aggregated coefficients from the holdings growth rate estimation:

$$\begin{aligned} L_{a,j,t} &= \psi \sum_{i=1}^{N} \frac{Y_{i,j,t-1}}{\hat{Y}_{j,t-1}} \hat{\Lambda}_{i,a,t}, \\ B_{a,j,t} &= \psi \sum_{i=1}^{N} \frac{Y_{i,j,t-1}}{\hat{Y}_{j,t-1}} \hat{\beta}_{i,a,t}, \\ K_{j,t} &= \psi \sum_{i=1}^{N} \frac{Y_{i,j,t-1}}{\hat{Y}_{j,t-1}} \hat{\beta}_{y,i,t} \hat{e}_{i,j,t-1}, \end{aligned} \tag{21}$$

while intercept $\tilde{\delta}$ and residual $\tilde{\varepsilon}_{j,t}$ are modified to account for the intercept and residuals from the holdings growth rate equations. One note is that differences in funds' equity shares among available stocks introduce variation in aggregated coefficients $L_{a,j,t}, B_{a,j,t}$ by stock j. The notes under each term's brace in Eq. 20 indicate the possible sources of stock price growth variation, which include the effect of i) changing preferences, ii) changing fundamental characteristics, and iii) latent demand. Before delving into the data used to estimate the coefficients for Eqs. 19 and 20, I detail the estimation of latent demands and holding growth rates as a function of characteristics.

Estimating latent demands I estimate latent demands in a similar fashion to Koijen & Yogo (2019), albeit using the logarithmic version of their preferred specification. While they document that this leads to some inefficiency, and potential bias, I intend to implement their full procedure in the near future. The static asset demand specification given by Eq. 13 includes A possible characteristics:

$$\log \frac{y_{i,j,t}}{y_{i,0,t}} = k_{i,0,t} \log y_{j,t} + \sum_{a=1}^{A} \gamma_{i,a,t} d_{a,j,t} + e_{i,j,t}, \tag{22}$$

where fundamental values $d_{a,j,t}$ are assumed to be public knowledge, and $j = 0$ denotes an arbitrary holding that serves as the benchmark for the estimation procedure. An important detail is that all parameters are allowed to vary by time and fund, as per Koijen & Yogo (2019). The main idea is that each portfolio is treated as a separate cross-section, and the asset demand systems are estimated for each investor and quarter in isolation.

Log market equity share $\log y_{j,t} = \log Y_{j,t}/Y_t$ features as a variable that determines the intercept for the demand system. As a case scenario, elasticities $\gamma_{i,a,t}$ should equal zero, and intercept $k_{i,0,t}$ one, for an index fund with holdings that match market equity shares. The characteristics to asses fundamental value with are the same as in Koijen & Yogo (2019), namely the five stock characteristics from Fama & French (2015), except for size which is implicitly included in market equity shares: log book-to-market equity, investment growth rate, profit rate, and market covariance.

Hypotheses Characteristics valued by fund managers, gauged by elasticities $\beta_{a,i,t} < 1$, may significantly contribute to variation in stock price growth. This speaks to a long-standing issue regarding the excess volatility of stock returns relative to fundamentals. Here, different funds may compete more, or less, for the same assets, depending on the degree of overlap in their preferences. Eq. 20 clarifies how changing fundamentals may be amplified if aggregate demand elasticities happen to be high. This line of research is actively pursued (Gabaix & Koijen 2021).

The case for diversification, which is central to this paper, is outlined by Prediction from Section 2, which asserts a negative relationship between latent demand and holding growth rate, $\beta_{y,i,t} > 0$. Without an objective that compels the fund to increase the breadth of their portfolio, there is no reason for the level of past holdings to drive growth in corresponding holdings, beyond what is justified by changes in fundamentals. Moreover, mutual funds may find themselves compelled to divest out of positions they are unable to justify. Koijen & Yogo (2019) attribute this phenomenon to a mean-reversion in latent demands.

As discussed in Section 2, coefficient $\Lambda_{a,i,t}$ captures changes in fund i's sensitivity to characteristic $d_{a,j,t-1}$. These preference changes can be understood as a level effect, because they compel the fund to grow their holdings at variable rates despite the characteristic itself remaining constant. This paper does not offer a prediction on the sign of these coefficients, since preferences can evolve in different ways. However, it opens the door to interesting research on the evolution of investment mandates under competition (Brock & Hommes 1997, Farmer 2002). Their estimated contribution to stock price growth via Eq. 20 would be a valuable motivation to expand that line of research.

I first describe the available data on mutual fund holdings and stocks, then present estimates for Eqs. 18-22. Finally, I discuss the results in line with these three hypothetical sources of stock price variation and predictability.

3.3 How do portfolios grow?

I first estimate Eq. 22 using OLS to retrieve fitted values for latent demands $\hat{e}_{i,j,t}$. Generally, I only observe holdings held in two subsequent periods. This might raise concerns regarding a selection effect, by which a fund opens or closes a new position in a manner not captured by the model. For now, I estimate the coefficients to Eq. 19 using OLS, and lags of latent demands $\hat{e}_{i,j,t-1}$.

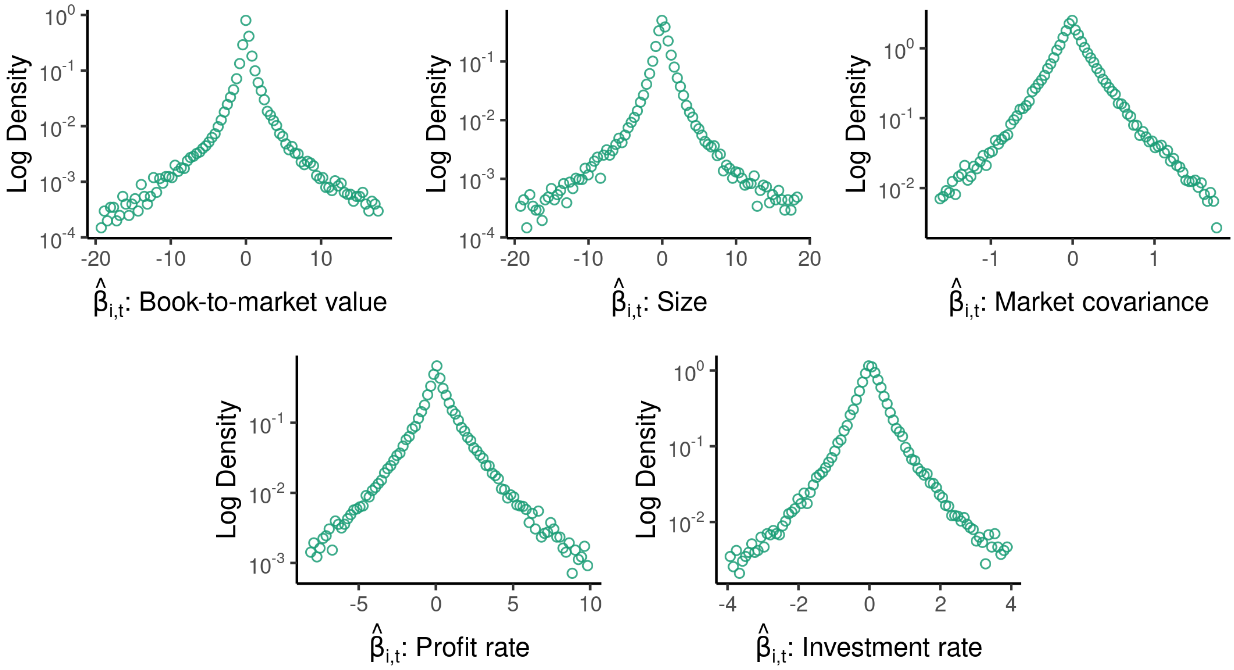


Figure 6: **Estimated demand coefficients for fundamentals $\hat{\beta}_{i,a,t}$ display strong heterogeneity**: OLS estimates of the relationship between the change in characteristics and the holding growth rate of fund i in quarter t for log book-to-market equity (Top left), the logarithm of market equity share (Top middle), market covariance (Top right), profit rate (Bottom left) and investment rate (Bottom right). 99% of observations shown.

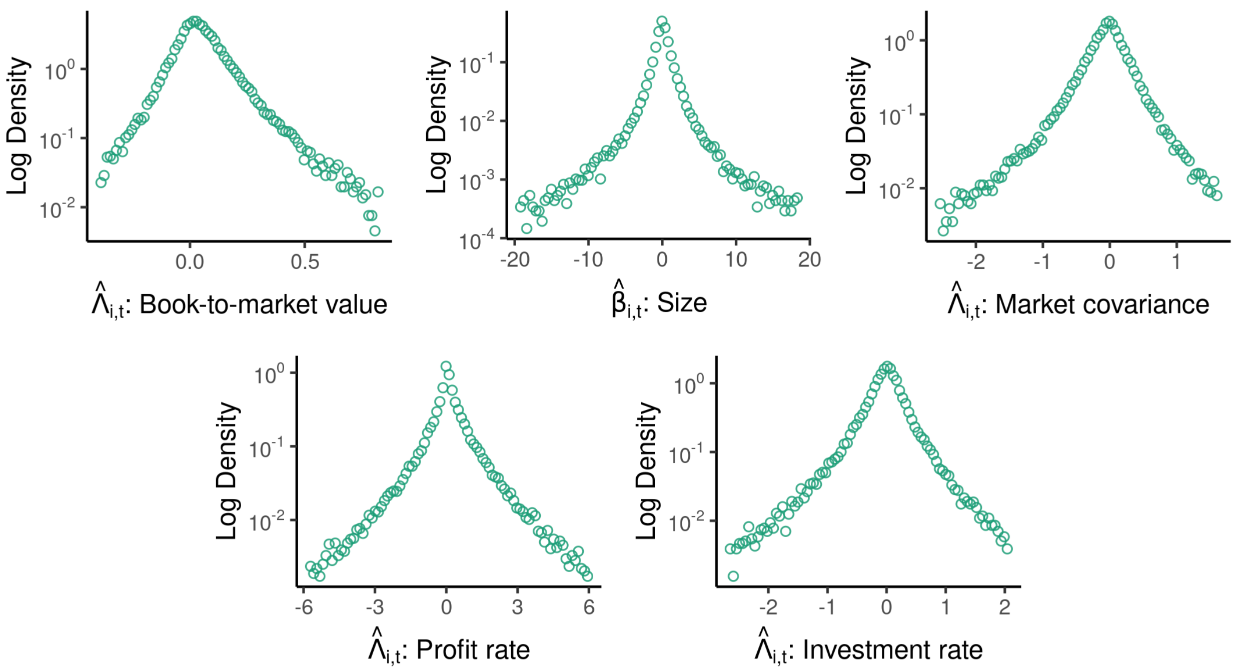


Figure 7: **Estimated demand coefficients for changes $\hat{\Lambda}_{i,t}$**: OLS estimates of the relationship between the lag in characteristics and the holding growth rate of fund i in quarter t for log book-to-market equity (Top left), the logarithm of market equity share (Top middle), market covariance (Top right), profit rate (Bottom left) and investment rate (Bottom right). 99% of observations shown.

Figure 6 plots the distributions of the coefficients estimated for changes in characteristics, which are correspond to pseudo-demand elasticities $\beta_{i,a,t}$ in Eq. 19. These plots communicate considerable heterogeneity, with estimates taking large positive and negative values. It is difficult to discern what a mean estimate might be, which makes sense given that mutual funds hold a variety of mandates and strategies. This heterogeneity also persists in Figure 7, which plots estimated values for coefficients corresponding to the level effect in the lag of a fundamental characteristic, $\Lambda_{i,a,t}$ in Eq. 19. The variable that stands out for a supposed change in preference is the coefficient for book-to-market value, which are typically much smaller than the change effects estimated in Figure 6. This may indicate that funds target book-to-market values to different degrees, yet this preference does not change much over time. Future research could shed some light on the manner in which mutual fund investments may move over the state space of preference over time.

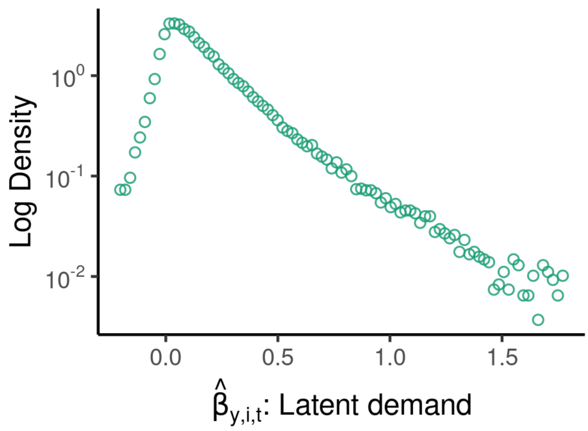


Figure 8: **Estimated adjustment costs $\hat{\kappa}_{i,t}$ are overwhelmingly positive**: adjustment cost $\beta_{y,i,t}$ reflects the average effect of a mutual fund's latent demand for an asset in the previous quarter, on its holding growth rate in the current quarter. 99% of observations shown.

Figure 8 plots the OLS estimates for the effect of lagged latent demand on the growth rate of mutual fund holdings. In the model from Section 2, these parameters correspond to the adjustment costs faced by portfolio managers, a constraint which can bind to varying degrees. The data provide strong evidence that this parameter is indeed negative, although, again, it demonstrated substantial heterogeneity among funds' portfolios. Relating it back to the model, a high value for estimate $\hat{\beta}_{y,i,t}$ suggests that a fund aggressively divests out of positions which are not justified by their fundamental values, as measured by the five FFF characteristics, and the fund's estimated preferences. In contrast, a lower value indicates that the fund tends to sit on the outsized position.

Since the data offer a total of 47,260 portfolios to estimate Eq. 19 with, I summarise the results in two ways. First, I plot distributions for all estimated coefficients, pooling them together for all quarter-fund samples. To better highlight some of the interesting heterogeneity in mutual fund investment behaviour, I summarise the key results by reporting the average coefficient for each quarter by *type* of fund, categorised by CRSP's objective codes. Specifically, I group coefficient estimates for funds categorised as i) growth funds (code

EDYG), ii) income funds (code EDYI), iii) mixed funds (code EDYB) and iv) an 'other' category of remaining funds.

Denoting each of the four fund types by subscript q, I aggregate coefficient estimates weighed by the relevant fund's reported holding value in the previous quarter, as a share of the total reported in that quarter for the same fund type,

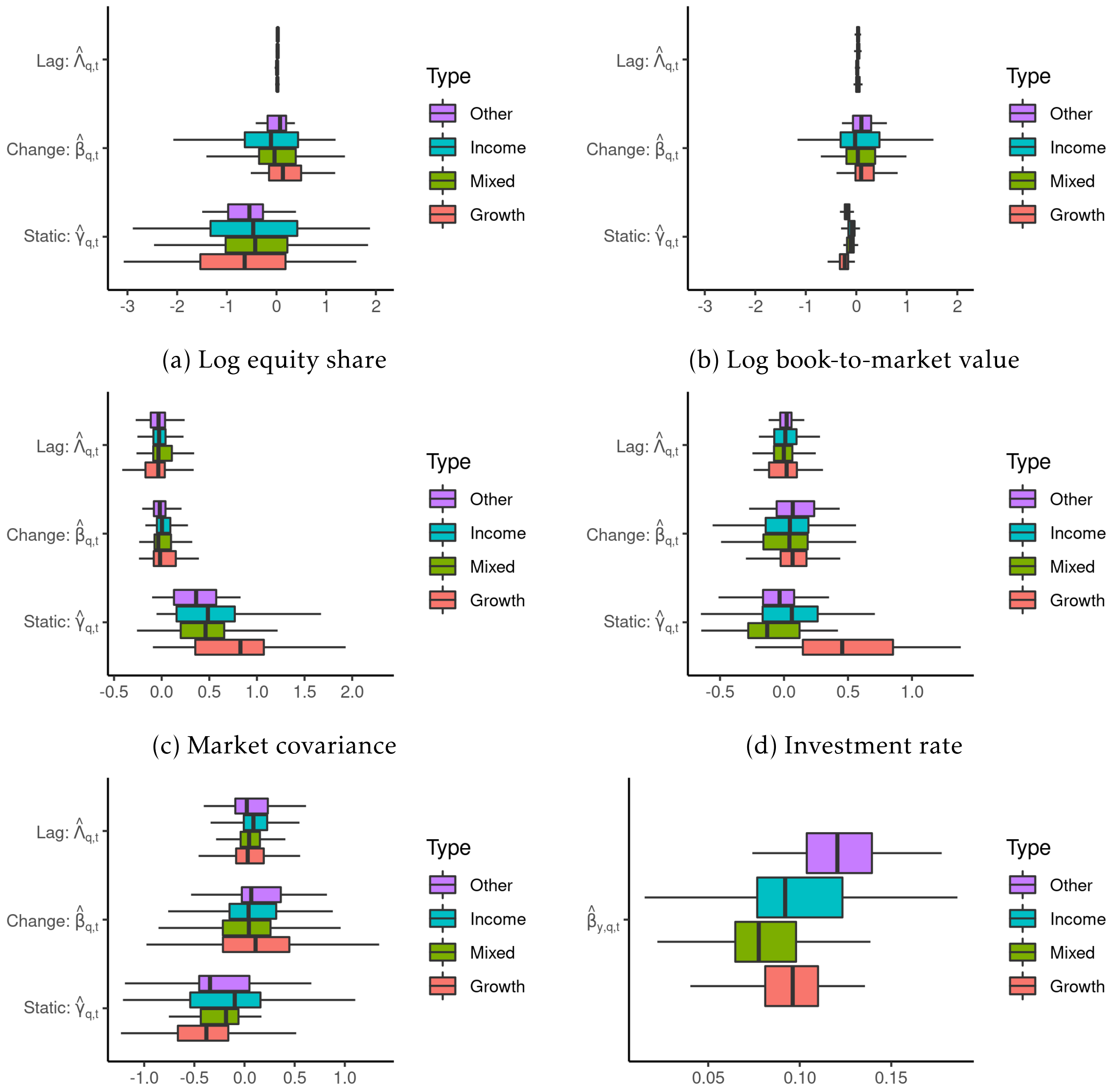
$$\hat{\Lambda}_{q,a,t} = \sum_{i=1}^{N} \frac{Y_{q,i,t-1}}{\sum_{i=1}^{N} Y_{q,i,t-1}} \hat{\Lambda}_{q,i,a,t}, \quad \hat{\beta}_{q,a,t} = \sum_{i=1}^{N} \frac{Y_{q,i,t-1}}{\sum_{i=1}^{N} Y_{q,i,t-1}} \hat{\beta}_{q,i,a,t}, \quad \hat{\gamma}_{q,a,t} = \sum_{i=1}^{N} \frac{Y_{q,i,t-1}}{\sum_{i=1}^{N} Y_{q,i,t-1}} \hat{\gamma}_{q,i,a,t}.$$

Combined, they convey the heterogeneity as well as the central tendency that describe the holding growth rates of mutual funds.

Results Figure 9 presents the coefficients aggregated for each quarter and fund type, according to weighed by funds' holding values, using standard box plots. These help discern certain patterns. The sign of the coefficients line up with fund types in some cases. For example, growth funds grow holdings more in response to growth in market shares, compared to income funds, as seen in estimates for $\hat{\beta}_{q,a,t}$ in Figure 9a. Growth funds also allocate a greater share of their holdings to stocks that move with the market in Figure 9c's estimates for $\hat{\gamma}_{q,a,t}$. Taken together, those estimates suggest that growth funds stand apart from other funds in the degree to which they follow market and stock momentum. Their holding shares also favour firms that display high investment growth rates, as seen in Figure 9d. Relative to income funds, growth funds also display a tendency to hold fewer stocks which earn a high rate of profit, as per Figure 9e. This speaks to their focus on companies seeking future profits at the expense of running short-term losses.

As discussed earlier, it is hard to comment on the lag effect of characteristics on holding growth rates – the component related to preference changes in Eq. 20. In Figure 9, estimates for $\hat{\Lambda}_{q,a,t}$ cluster around zero in all cases. This suggests that mutual fund investment patterns are relatively stable over time with respect to their sensitivity to different stock characteristics. For future research, it would be interesting to pin down exogenous variation in these coefficients to determine what might trigger funds to change their observed responsiveness to these characteristics.

Figure 9f presents strong evidence that mutual funds penalise the holding growth rates for firms in which they already hold elevated latent demand. The corresponding elasticity, which relates to multiplier $\beta_{y,i,t}$ to adjustment costs, is always positive for quarterly aggregates. These findings lend considerable support to the Prediction made in Section 2, which attributes this relationship to a preference for a diversified portfolio. To accomplish this, the mutual fund most grow their positions in small holdings faster than those of large holdings. Koijen & Yogo (2019) present a similar finding, noting that latent demands exhibit strong mean reversion in their repeated cross-sections.



(a) Log equity share

(b) Log book-to-market value

(c) Market covariance

(d) Investment rate

(e) Profit rate

(f) Latent demand

Figure 9: **OLS coefficients for the average response of holdings and their growth rates by characteristic and mutual fund type**: these figures present box plots for the aggregated coefficient estimates across mutual funds in each quarter. The averages were computed using the shares of funds' total holding values reported in the previous quarter, and are group by certain fund types q that correspond to the CRSP mutual fund objective code (labelled `CRSP_OBJ_CD`). Each characteristic, labeled by the title, is accompanied by an estimated coefficient from Eq. 19 for the effect of its lag ($\hat{\Lambda}_{q,t}$) and change ($\hat{\beta}_{q,t}$) on holding growth rates, as well as the elasticity of holding shares in response to the characteristic ($\hat{\gamma}_{q,t}$) for Eq. 22. The bottom right panel includes aggregated coefficients for latent demands ($\hat{\beta}_{y,q,t}$). Observations exceeding the 1$^{\text{st}}$ or 99$^{\text{th}}$ percentiles are not included for visualisation purposes.

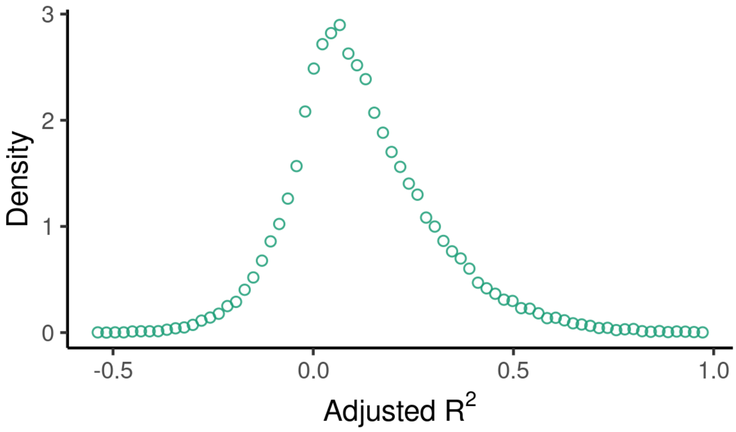


Figure 10: **Adjusted R^2 values average between 0.2 and 0.3**: this figure plots a histogram for the the adjusted R^2 of the holding growth rate model in Eq. 19, estimated separately for 49,796 quarterly mutual fund portfolios.

Summarising the uncertainty around these estimates is tricky, given the sheer amount of coefficients estimates. The range of coefficients in Figure 9 go some of the way in describing the possible realisations of these parameters between investors and over time. To get a better idea of the overall fit of the model in Eq. 19, Figure 10 plots the distribution of all adjusted R^2 values estimated for all fund-quarter sub-samples. The model explains around 0.1 to 0.2 of the variation of holding growth rates, which is good considering the rather restricted set of explanatory variables. One concern is that the number of explanatory variables may lead to over-fit models, especially for certain funds that only report a few dozen holdings.

3.4 Aggregating stock returns

I re-constructed the proxied returns from Eq. 19 using the fitted values for all coefficients. These fitted values result in a nested model for stock price growth using the proxy in Eq. 1, which I detail in Eq. 20. I re-estimated coefficients to Eq. 1, but substitute observations for holding growth rates using the components to Eq. 20. Another difference is that I do not filter out stock with market equity less than one billion USD, and instead compute the coefficients using weighted OLS, where weights correspond to the market equity share in the previous quarter, $y_{j,t-1}$.

Table 2 presents weighted OLS estimates for slope coefficients to the components for holding growth rates nested into the equation for stock price growth. To give a sense for the degree of uncertainty, the first column presents estimated components to Eq. 20 using the five FFF model, whereas the second column uses characteristics from the three FFF model, which excludes profit rate and investment rate. Removing those two characteristics significantly increases the number of observations, and while the results will not be directly comparable to those from the five FFF version, that will lend some insights into the degree of variability.

Given that proxied returns already matched stock returns closely in Table 1, it is unsurprising that the coefficients for both version of the model are close to one. The additional

Table 2: **Nesting models for mutual fund holdings to regress on stock prices**

	Dependent variable:	
	$r_{j,t}$	
	(1)	(2)
Intercept	0.85 (0.003)	0.95 (0.004)
Preferences	0.87 (0.003)	0.97 (0.004)
Fundamentals	0.86 (0.003)	0.96 (0.004)
Latent demand	0.76 (0.01)	0.86 (0.01)
Residual holdings	0.70 (0.004)	0.83 (0.003)
$\hat{\delta}$	0.02 (0.0004)	0.02 (0.001)
Obs.	41,526	118,351
Adj. R^2	0.75	0.57
Res. Std. Error	0.001	0.001

Notes: This table presents OLS estimates for the relationship between components to growth rates in mutual fund holdings in stock j and the quarterly price growth in stock j. These components, outlined in Eq. 20, include the effect of i) an intercept, by which all holdings grew by a uniform rate, ii) changing preferences, whereby funds hold larger or smaller shares in stocks that exhibit higher values for certain characteristics than other stocks, iii) changes in fundamentals, which compel the fund to increase or decrease a holding as a function of changing values for characteristics, iv) latent demand, the holding share unexplained by characteristics in the previous period, and v) a common intercept across all stocks. The characteristics used to construct these components conform to the five factors from the FFF model, namely the market equity share, the book-to-market value, the stock's return covariance with the market return, the profit rate, and the investment rate, in Column (1). Column two constructs these components using only the market equity share, book-to-market value and stock return covariance with the market, which correspond to the three factor FFF model. Standard errors are reported in brackets.

data from small cap stocks for the 5 FFF model does not seem to worsen the R^2, although this is largely attributed to the market equity weights. The coefficients are estimated to be close to one when using characteristics to the 3 FFF model, although the overall fit, as judged by an R^2 of 0.57.

Variance decomposition These nested models do not give an accurate picture with regards to stock price variation *explained* by either changes in fundamentals, preferences, or latent demand. This is because variation in fundamental characteristics, even though they map perfectly onto returns, display only a fraction of the variation in stock prices. To get a better idea for the variation in stock prices explained by mutual fund preferences, changing fundamentals or latent demand, I decompose the variance of stock price growth in Eq. 20

by each component:

$$\sigma^2(\boldsymbol{r}_t) = \underbrace{\sigma(\boldsymbol{\rho}_t, \boldsymbol{r}_t)}_{\text{Intercept}} + \underbrace{\sum_{a=1}^{A} \sigma(B_{a,t}\Delta \boldsymbol{d}_{a,t}, \boldsymbol{r}_t)}_{\text{Fundamentals}} + \underbrace{\sum_{a=1}^{A} \sigma(L_{a,t}\boldsymbol{d}_{a,t-1}, \boldsymbol{r}_t)}_{\text{Preferences}} - \underbrace{\sigma(\boldsymbol{K}_t, \boldsymbol{r}_t)}_{\text{Latent demand}} + \underbrace{\sigma(\boldsymbol{v}_t, \boldsymbol{r}_t)}_{\text{Holding residual}} + \underbrace{\sigma^2(\boldsymbol{\varepsilon}_t)}_{\text{Stock residual}},$$

weighted by the stocks' market equity shares in the previous quarter $\boldsymbol{y}_{t-1}$. The relevant components include preferences, and fundamentals and latent demand, as per Eq. 20, as well as the intercept estimated for mutual funds' holding growth rates, plus the unexplained residuals for holding growth rates and the residuals in stock price growth.

	5 Factors	3 Factors
Fundamentals		
Book-to-market	3.26	-2.82
Size	3.02	6.15
Market covariance	3.87	1.20
Profit rate	1.61	
Investment rate	3.32	
Total	15.08	4.53
Preferences		
Book-to-market	3.87	2.31
Size	3.17	0.55
Market covariance	11.80	11.47
Profit rate	2.20	
Investment rate	3.27	
Total	24.30	14.34
Latent Demand	2.40	1.99
Intercept	1.97	6.30
Residual		
Fund holdings	48.98	56.31
Stock returns	9.25	22.84
Total	58.22	79.15

Table 3: **Contributions to stock price growth variation from growth in mutual fund holdings**: This table decomposes the cross-sectional variance of quarterly stock prices growth by their covariance with different components to mutual fund holding growth rates. These components are outline in Eq. 20.

Table 3 outlines the variance decomposition of stock price growth by component, separately for the 5 FFF characterists and the 3 FFF characteristics. The individuals characteristics, on their own, contribute rather small amounts to stock price variation. One interesting feature is the large contribution stemming from mutual funds growing holdings that displayed high levels of covariance with the market return. This may speak to mutual funds' changing risk appetites over the course of the cycle. A second observation is that changes in fundamentals contribute roughly the same, if not less, price variation relative to preferences. Recall that fundamentals capture the change effect of characteristics on holdings, whereas preferences capture the level effect; re-balancing according to given fundamentals is therefore reflected in stock price variation, via the tendency of mutual funds to change holding shares in stocks to match a new set of.

Importantly, a significant amount of price variation remains unexplained. However, this is not because stock prices grow more erratically that mutual fund holdings. Rather, about one half of stock price variation is tied to growth rate in mutual fund holdings unexplained by either fundamentals or preferences for characteristics.

3.5 Discussion and limitations

These results do not provide causal evidence that mutual fund investment behaviour has price impact. Importantly, market clearing features in the crudest way possible. Mutual fund investments likely experience some interaction with other market participants. Rather, the exercise is to describe the cross-section of returns from the perspective that a vast heterogeneity in investment mandates provides different sources with which to explain stock returns, besides using fundamental characteristics alone. On that front, the framework is promising, and opens the door to further research that leans into various predictive elements of mutual fund behaviour.

Certain aspects of the empirical exercise can be improved. The selection of stocks into mutual fund portfolios likely biases the estimates for fund-level coefficients, and this extensive margin also provides more information with which to make these estimators more efficient. Another important note is that the driving motivation behind disaggregating returns is the equilibrium dynamics of portfolio shares. The application is currently missing a similar treatment for mutual fund sizes.

Finally, mutual funds may not necessarily be an important driver of daily price fluctuations, since they constitute the market's 'buy side'. Most price fluctuations would be driven by 'sell side' participants, who do not hold inventory overnight, not to mention on the quarterly frequency. The impact of mutual fund investments may therefore be more pronounced in certain liquidity-constrained situations, time scales beyond the quarterly frequency, or through the extensive margin discussed above. Unexplained stock price variation is higher when excluding profit rate and investment rate variables, however this is largely due to a selection effect for small stock that are do not report their earnings with the same regularity as larger companies.

4 Conclusion

The growth rates of mutual fund holdings are accurately modelled by a model that endogenises portfolio re-balancing in response to fundamentals, producing a level and change effect from stock fundamentals. Although a portfolio is directly more valuable after the fundamental value of one company increases, changes in preferences and an intent to remain diversified triggers re-balancing to avoid being over-exposed to a few firms. Empirically, a larger fraction of variation in stock price growth is explained by the level effect, which reflects evolving preferences aggregated across mutual funds, compared to the change effect, which reflects changes in asset characteristics. While a majority of stock price variation remains unexplained, it mostly stems from variation in holding growth rates that is not explained by fundamentals or preferences, as opposed to stock price variation. Therefore, the main empirical limitation is the search for variables which better explain the evolution of mutual fund holdings.

As demonstrated in the volume of recent research on the topic, investigating asset demand is productive in the study of stock price volatility and their co-movements. In this paper, latent asset demand compels mutual funds to re-balance aggressively in order to meet their preferred allocations according to observable characteristics. In principle, it amounts to no more than the inclusion of an additional characteristic in what is otherwise a straightforward single-factor model. Indeed, the core theoretical framework reproduces the Sharpe's (1964) and Lintner's (1965) CAPM as a special case, where demand adjusts to meet preferences on the level of exposure to market risk. However, this simple framework does not lend itself well to a scenario where changes in portfolio allocation create inertia, in the form of adjustment costs faced by the manager of the market portfolio. This inertia is closely related to uncertainty, or 'surprises' in Information Theory.

This research speaks to the great opportunities in using data on heterogeneous investors to better understand intricate behaviours that are currently missed by the aggregate models dominating finance. In particular, the manner in which investors adapt and change their strategies is shown to contribute a significant amount of stock price fluctuations. This opens the door to a host of novel questions regarding granular origins of large market events.

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A Proofs

A.1 Proof to Proposition 1

Solving the first order conditions of Eq. 5 with respect to re-allocations $\Delta Y_{j,t}$ yield an expression for the excess holding growth rate for each stock j, as a percentage:

$$\kappa\left(g_{j,t} - \bar{g}_t\right) = d_j - \bar{d}_t, \tag{23}$$

where values a bar denotes a value-weighted mean, $\bar{x}_t \equiv \sum_{j=1}^{M} y_{j,t-1} x_{j,t}$. Squaring both sides, and multiplying by $y_{j,t-1}$ then summing across j yields:

$$\kappa = \frac{\sigma(\boldsymbol{d}_t)}{\sigma(\boldsymbol{g}_t)}. \tag{24}$$

Alternatively, multiplying both sides of Eq. 23 by $\Delta Y_{j,t}$ yields:

$$\kappa = \frac{\sigma(\boldsymbol{d}_t, \boldsymbol{g}_t)}{\sigma^2(\boldsymbol{g}_t)}, \tag{25}$$

which is the coefficient of a weighted regression of fundamentals on holding growth rates. Setting the right-hand sides of Eq. 24 and Eq. 25 and re-arranging yields the solution.

A.2 Proof to Proposition 2

For tractability, I re-write Eq. 10 as:

$$\mathcal{L} = \sum_{j=1}^{M} Y_{j,t-1}\left(\frac{\Delta Y_{j,t}}{Y_{j,t-1}}\right)^2 - 2\kappa\left(-\sum_{j=1}^{M}\Delta Y_{j,t}\log y_{j,t-1} - \partial\mathcal{E}\right) - 2\lambda\left(\sum_{j=1}^{M}\Delta Y_{j,t}d_{j,t} - \mathcal{P}\right) - 2\rho\left(\sum_{j=1}^{M}\Delta Y_{j,t} - \Delta Y_t\right), \tag{26}$$

which yields the same solution at the maximum.

The first order conditions are

$$\frac{\Delta Y_{j,t}}{Y_{j,t-1}} \equiv g_{j,t} = \rho - \kappa\log y_{j,t-1} + \lambda d_{j,t}, \tag{27}$$

and

$$\mathcal{P} = \sum_{j=1}^{M}\Delta Y_{j,t}d_j, \quad \partial\mathcal{E} = -\sum_{j=1}^{M} Y_{j,t-1}\log y_{j,t-1} \quad \Delta Y_t = \sum_{j=1}^{M}\Delta Y_{j,t}.$$

Multiplying Eq. 27 by $Y_{j,t-1}$, and summing across j solves for

$$\rho = \bar{g}_t + \kappa\sum_{j=1}^{M} y_{j,t-1}\log y_{j,t-1} - \lambda\bar{d}_t,$$

using the fact that $y_{j,t-1} = Y_{j,t}/Y_{t-1}$ and $\bar{g}_t = \Delta Y_t/Y_{t-1}$. Setting $\mathcal{E}_{t-1} \equiv -\sum_{j=1}^{M} y_{j,t-1}\log y_{j,t-1}$ yields an expression for the excess holding growth rate for each stock j, as a percentage:

$$g_{j,t} - \bar{g}_t = \kappa\left(-\log y_{j,t-1} - \mathcal{E}_{t-1}\right) - \lambda\left(d_{j,t} - \bar{d}_t\right). \tag{28}$$

The joint result for holding growth rate moments are derived from Eq. 28. Multiplying both sides by $Y_{j,t-1}d_j$ and dividing by Y_{t-1} yields

$$\frac{\mathcal{P}}{Y_{t-1}} - \bar{g}_t\bar{d} = \kappa\left(-\sum_{j=1}^{M} y_{j,t-1}d_j\log y_{j,t-1} - \mathcal{E}_{t-1}\bar{d}\right) + \lambda\sum_{j=1}^{M} y_{j,t-1}d_{j,t}\left(d_{j,t} - \bar{d}_t\right),$$
$$\Rightarrow \sigma(\boldsymbol{d}_t, \boldsymbol{g}_t) = \kappa\sigma(\boldsymbol{d}_t, -\mathbf{log}\,\boldsymbol{y}_{t-1}) + \lambda\sigma^2(\boldsymbol{d}_t),$$

where $\sigma(\boldsymbol{x}_t, \boldsymbol{z}_t) = \sum_j y_{j,t-1}(x_{j,t} - \bar{x}_t)(zj,t - \bar{z}_t)$ denotes the value-weighted covariance between x_t and z_t. Multiplying both sides of Eq. 28 by $\Delta Y_{j,t}$ and dividing by Y_{t-1} yields

$$\frac{\mathcal{V}^2}{Y_{t-1}} - \bar{g}_t^2 = \kappa\left(-\sum_{j=1}^{M} y_{j,t-1}g_{j,t}\log y_{j,t-1} - \mathcal{E}_{t-1}\bar{g}_t\right) + \lambda\sum_{j=1}^{M} y_{j,t-1}g_{j,t}\left(d_j - \bar{d}\right),$$
$$\Rightarrow \sigma^2(\boldsymbol{g}_t) = \kappa\sigma(\boldsymbol{g}_t, -\mathbf{log}\,\boldsymbol{y}_{t-1}) + \lambda\sigma(\boldsymbol{g}_t, \boldsymbol{d}_t).$$

Multiplying both sides of Eq. 28 by $-\log y_{j,t-1}$ and dividing by Y_{t-1} yields

$$\frac{\partial\mathcal{E}}{Y_{t-1}} - \mathcal{E}_{t-1}\bar{g}_t = \kappa\left(\sum_{j=1}^{M} y_{j,t-1}\left(-\log y_{j,t-1} - \mathcal{E}_{t-1}\right)^2\right) + \lambda\sum_{j=1}^{M} y_{j,t-1}g_{j,t}(-\log y_{j,t-1})\left(d_j - \bar{d}\right),$$
$$\Rightarrow \sigma(\boldsymbol{g}_t, -\mathbf{log}\,\boldsymbol{y}_{t-1}) = \kappa\sigma^2(-\mathbf{log}\,\boldsymbol{y}_{t-1}) + \lambda\sigma(\boldsymbol{d}_t, -\mathbf{log}\,\boldsymbol{y}_{t-1}).$$

Solving by substitution yields

$$\lambda = \frac{\sigma(\boldsymbol{d}_t, \boldsymbol{g}_t)\sigma^2(-\log \boldsymbol{y}_{t-1}) - \sigma(\boldsymbol{d}_t, -\log \boldsymbol{y}_{t-1})\sigma(\boldsymbol{g}_t, -\log \boldsymbol{y}_{t-1})}{\sigma^2(-\log \boldsymbol{y}_{t-1})\sigma^2(\boldsymbol{d}_t) - \sigma^2(\boldsymbol{d}_t, -\log \boldsymbol{y}_{t-1})},$$
$$\kappa = \frac{\sigma(\boldsymbol{g}_t, -\log \boldsymbol{y}_{t-1})\sigma^2(\boldsymbol{d}_t) - \sigma(\boldsymbol{d}_t, -\log \boldsymbol{y}_{t-1})\sigma(\boldsymbol{d}_t, \boldsymbol{g}_t)}{\sigma^2(-\log \boldsymbol{y}_{t-1})\sigma^2(\boldsymbol{d}_t) - \sigma^2(\boldsymbol{d}_t, -\log \boldsymbol{y}_{t-1})}.$$

These are the coefficients to a weighted linear model with two dependent variables, where the variables are lagged equity share $-\log y_{j,t}$ and fundamentals $d_{j,t}$.

B Data

B.1 CRSP stock variables

This paper uses the following variables from CRSP, with a set of accompanying filters:

- `DATE`: trading date. This corresponds to the calendar date indexed by subscript t in which an observation is made.

  Filter: I use observations spanning December 15$^{\text{th}}$, 1973 to December 31$^{\text{st}}$, 2020. This is because coverage for firms listed on the NASDAQ begins December 14$^{\text{th}}$, 1973.

- `PERMNO`: firm identification number. This identifier is issued by CRSP, and for the purposes of this paper constitutes firm identifiers denoted by subscript j.

  Filter: When two observations are available on the same trading day for one firm identifier, I keep the first line of observation and discard the rest to mitigate any duplicates.

- `SHRCD`: the share code.

  Filter: This paper only includes common stock, which are labelled as 10 and 11.

- `EXCHCD`: exchange code.

  Filter: This paper only includes data from the three US exchanges, namely the NYSE (1), the NYSE MKT (2) and the NASDAQ (3).

- `PRC`: last available price. This value is used for the share price $P_{j,t}$ to compute daily market capitalisation of firm j.

  Filter: I include observations led by a dash, which indicates an imputed value using the average bid-ask spread.

- `SHROUT`: shares outstanding. This value is used for the number of shares Q_j to compute the market equity of firm j. This variable can vary due to stock splits, but this split will also be reflected in the unadjusted price $P_{j,t}$ so that the market capitalisation is unaffected.

- `RET`: returns. This variable includes dividends. I remove values that are non-numeric, which refer to different instances of missing observations.

B.2 Compustat variables

This paper uses the following variables from Compustat, with a set of accompanying filters:

- `DATADATE`: reporting date. This variable reports the month and year corresponding to company report.

  Filter: I use observations reported on December 31st. These correspond to data on company operations from the preceding fiscal year.

- `FYEAR`: fiscal year. I use this variable to assign financial variables to the corresponding calendar year.

- `GVKEY`: firm identification number. This identifier is issued by Compustat.

- `INDFMT`: industry format. This identifier records the reporting standard used for the corresponding financial variables, either as `FS` or `INDL`.

  Filter: When two observations for financial variables are available for the same year and company, I complete observations if any are missing under industry format `INDL` using observations reported under industry format `FS`. I subsequently remove duplicated entries by company-year, prioritizing observations under the `INDL` format.

- `OIBDP`: operating income before depreciation.

- `TXT`: total income taxes.

- `XINT`: interest and related expenses.

- `PPEGT`: property, plant and equipment gross of tax.

- `INTAN`: intangible assets.

- `INVT`: inventories.

  Filter: I drop observations for which no value is reported for operating income, tax and interest payments.

- `CEQL`: liquidation value of company assets. I use this variable to construct the book value of firms.

C Additional results

C.1 Entropy as a measure of diversification

The measurement of entropy has found universal applications in any situation that involves a frequency versus quantity trade-off: originally in thermodynamics, then biology and information theory, and even chemistry (Frank 2018). The scientific productivity in identifying fluctuations of aggregates sourced from quantity, versus frequency, motivates Jaynes's (1978) Maximum Entropy Principle (MEP). Put bluntly, maximising entropy is desirable *out* *of principle*. In economics, entropy has been studied as a measure of statistical uncertainty in financial markets, and as a theoretical under-pinning to welfare-improving trade in other economic disciplines (Smith & Foley 2008).

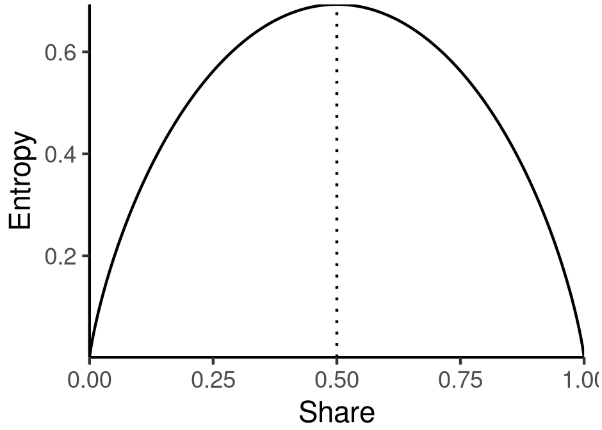

Figure 11: **Plotting entropy for two industries**: Total entropy is maximal at the dotted line, where the share of each industry is equal at 1/2.

The level of entropy $\mathcal{E}$ is often used as a measure for dispersion, taking larger values as underlying quantities are more uniform. The number of possible arrangements for those transactions is given by the multinomial coefficient:

$$\binom{Y!}{Y_1!, Y_2!, \ldots, Y_N!} = \frac{Y!}{Y_1!Y_2!\ldots Y_N!} = \frac{Y!}{(Yy_1)!(Yy_2)!\ldots(Yy_N)!},$$

where $y_i = Y_i/Y$. This is simplified, under large N, using the Stirling approximation for factorials:

$$\log\left[\frac{Y!}{(Yy_1)!(Yy_2)!\ldots(Yy_N)!}\right] \approx Y\sum_{i=1}^{N} y_i \log\left(\frac{1}{y_i}\right), \tag{29}$$

which corresponds to Shannon's (1948) entropy. To better interpret this term, assume that, on a given day, \$10 of fruit are traded on a market consisting of two types, apples and coconuts. If spending is uniform, the number of possible allocations is

$$\frac{10!}{5!5!} = 252.$$

On the other extreme, if all but one Euro is spent on one fruit alone, the number of possible arrangements is

$$\frac{10!}{9!1!} = 10.$$

This is why entropy is often described as a measure of uncertainty, or *observational variety*: from the consumer's perspective, there are many more ways to spend an endowment equally among options, rather than prioritising some ahead of others.[6] An additional Dollar

[6]Two technical remarks: i) observational zeros do not appear ($0 \log 0 = 0$), and ii) each Dollar is treated equally.

has many possible ways of being spent in a uniform economy, but fewer in a concentrated economy.