Information Exchange and the Limits of Arbitrage

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ABSTRACT

Evidence suggests that arbitragers exchange investment ideas. We analyze why and under what circumstances sharing occurs. Our model suggests that sharing ideas will lead to the following: more efficient asset prices, larger arbitrager profits, and correlated arbitrager returns. We predict that arbitragers will exchange ideas in markets where arbitragers are capital constrained, noise trader influence is high, and arbitrage investors are more loss averse. We also predict that arbitrage networks can lead to crowded trades, which can create systematic risk in extreme market circumstances.

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There are many indications that money managers freely share their ideas with other investment professionals and their personal networks. The mediums through which investment professionals share ideas are meeting venues, internet communities, and personal networks. Some examples of meeting venues set up expressly to allow arbitragers to share ideas are the Value Investing Congress, a venue established by Whitney Tilson, who in his own words describes the event as an opportunity to "allow top investors to meet and learn from each other and get great, actionable ideas,[1]" and the Hedge Fund Activism and Shareholder Value Summit, which allows activist hedge fund managers the opportunity to share tactics and discuss ongoing investment ideas in a closed forum. The sharing phenomenon is not limited to fundamentals based investors: investment banks and large quantitative hedge funds frequently invite other quantitative managers and academic economists to discuss new trading ideas and technologies (e.g. Lehman's Finance in Practice Conference).

Exclusive internet-based information exchanges have blossomed on the internet. For example, Sumzero.com is an invite-only internet community open to hedge fund managers. The site is specifically designed so professional investors can share investment ideas. Perhaps the most famous sharing venue utilized by professional investors is Valueinvestorsclub.com, a private community founded by Joel Greenblatt and John Petry, managers of the successful hedge fund Gotham Capital. Its founders proclaim Valueinvestorsclub.com to be an "exclusive online investment club where top investors share their best ideas."[2] The site has been heralded in many business publications as a top-notch resource for anyone who can attain membership (*Financial Times, Barrons, Business Week, Forbes,* and so forth). The investment ideas submitted on the club's site are broad, but are best described as fundamental value plays.

The membership of Valueinvestorsclub.com is highly confidential both with respect to the public and within the club itself (members all post under screen names unrelated to who they are). However, the authors have analyzed all Valueinvestorsclub.com idea submissions since the club's founding (January 1, 2000) and have reconciled the information in the recommendations with contemporaneously

[1] Found on the Value Investing Congress website: http://www.valueinvestingcongress.com.
[2] http://www.valueinvestorsclub.com/Value2/Guests/Info.aspx

filed 13-Ds, 13-Gs, and public statements to conclude that the membership of Valueinvestorsclub.com primarily consists of value hedge fund managers, activist investors, and their associates. In addition, we have spoken to multiple people in the hedge fund industry to verify that Valueinvestorsclub.com membership is exclusive and coveted by those in the industry.

Personal networks are another venue for sharing ideas. Shiller and Pound (1989) conclude that direct interpersonal communications between investors are very important in investment decisions. Indirect evidence for information sharing is documented by Hong, Kubik, and Stein (2005), who document that US fund managers living in the same city make similar portfolio choices, Feng and Seasholes (2004), who find similar behavior in the Chinese stock market for geographically close investors, and Cohen, Frazzini, and Malloy (2007), who find that mutual fund managers who went to college together have correlated portfolios.

We find further evidence that personal networks are important in the investment decision. We interviewed multiple hedge fund managers (all funds were focused value funds with assets under management ranging from $5mm to $2b[3]) and came to the conclusion, at least anecdotally, that idea exchange is rampant. One manager's comments summarized the general sentiment: "many of our best ideas come from our monthly conference calls with other value managers." We also spoke with employees at the largest hedge fund asset allocation consultant in the world[4]. Their firm listens to many of the top hedge fund managers in the world. During meetings they typically ask the fund manager for an overview of one of their best ideas. According to the firm's employees, value focused hedge fund managers (many of whom know each other) often mention the same handful of ideas. This anecdotal evidence suggests that managers are sharing information.

Why are arbitragers telling other arbitragers about their investment opportunities? According to efficient market logic (Fama (1970)), the rational arbitrager should act alone, drive the price to the fundamental level, and reap all the rewards of the arbitrage he has found. Unfortunately, arbitragers cannot do this in the real world.

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[3] The interviews were conducted with the understanding that they would remain anonymous.
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Two primary reasons for this are capital constraints and the limits to arbitrage arising from the realities in the investment management business (Shleifer and Vishny (1997)). Stein (2007) also questions why one would tell another honestly about an attractive trading opportunity when money managers care about relative performance. Stein's question is valid, however, we argue that a lack of transparency and understanding of what "relative performance" actually means, causes investors to focus simply on past returns (Shleifer and Vishny (1997)).

Dow and Gorton's (1994) analysis of arbitrager behavior suggests that arbitragers will only make investments if they believe subsequent arbitragers will buy the asset and push the value to fundamental value. One way arbitragers can help ensure other arbitragers take a position in an asset is by sharing their ideas with others. While this may be a credible motive for information exchange, we believe there are more important influences at work.

We analyze information exchange amongst arbitragers using the Shleifer and Vishny (1997) performance based arbitrage framework with an augmentation that allows for greater investor loss aversion. In our context, loss aversion means that arbitrage fund investors will withdraw large amounts of funds following poor performance because they are afraid of further declines. Using this model, we show that sharing investment ideas can make sense for arbitragers because it allows them to diversify their portfolios amongst a group of arbitrage trades, which will lower the probability they experience a large negative shock and have their funds withdrawn by loss averse investors. Our model predicts that arbitragers will exchange information in markets where noise trader influence is high and in settings where arbitrage investors are more loss averse. We also find that sharing ideas can lead to more efficient asset prices, larger arbitrager profits, and correlated arbitrager portfolios.

Sharing amongst arbitragers can also help explain the phenomenon of "crowded" trades (reference to a situation where many money managers play an identical trade at the same time). For example, a very well received idea posted on Valueinvestorsclub.com in September 2008 was the Porsche/Volkswagen negative stub trade (a full discussion and analysis of negative stub trades are described in Mitchell, Pulvino, and Stafford (2002)). In late October the Porsche/Volkswagen trade went terribly wrong and affected multiple hedge funds (Jameson and Robertson (2008)).

There is also evidence that quantitative funds crowd into trades. In August 2007, an unprecedented number of high-profile quantitative long/short equity hedge funds experienced unprecedented losses, which is thought to have been caused by one or more sizable hedge funds liquidating (Khandani and Lo (2007)). Khandani and Lo further show evidence that hedge fund returns are becoming more correlated over time. Recent evidence corroborates this result. A recent bout of hedge fund liquidations is wrecking havoc on the hedge fund industry, with the Hedge Fund Research Equity Hedge Index dropping 8.59% in September 2008[5]. Ken Griffin (Citadel Investment Group founder) speaks for the entire hedge fund industry in his September report to investors, "September was a devastating month for financial institutions and investors around the world—September was also the single worst month, by far, in the history of Citadel" (McSherry (2008)).

It is important to point out that we do not believe that all crowded trades and the recent chaos in the hedge fund markets are a result of direct information exchange between hedge fund managers. Another possibility is that arbitragers stumble across the same ideas because they are searching for investment ideas based on similar criteria (indirectly sharing). Regardless, our model predicts many of these outcomes: when arbitragers share ideas, their portfolios will become correlated, and correlated portfolios will suffer together when their underlying assets (and asset holders) falter.

I. The Performance Based Arbitrage Model

The way we model the asset markets closely follows the framework used in Shleifer and Vishny (1990) and Shleifer and Vishny (1997). In these models the three market participants are noise traders (Delong et al., 1990), investors in arbitrage funds, and arbitragers. Noise traders participate in all markets, arbitrage funds focus on certain segments, and fund investors spread their money across many arbitrage funds. Finally, market participants are risk neutral and the market interest rate is 0.

At t=1 the fundamental value of assets, V, become known to arbitragers, but not their investors. Noise traders, who trade erratically and without regard for fundamental

[5] http:/www.hedgefundresearch.com/hfrx_reg/index.php

values, have a $1-\varphi$ probability of depressing the asset by S_2 and a φ probability of assessing the value correctly (for simplicity we only examine bearish noise traders). More formally, the price at period 2 is defined as

$$\mathrm{P}_2 = \begin{cases} V, & p(S_2 = 0) = \varphi \\ V - S_2, & p(S_2 > 0) = 1 - \varphi \end{cases} \quad s.t. \, 0 < \varphi < 1. \tag{1}$$

The efficient market price at t=1 is $P_1 = E(P_2) = \varphi(V) + (1-\varphi)(V - S_2)$. During t=1, noise traders drive down the price of an asset from the efficient market price by S_1. The aggregate noise trader demand becomes

$$QN_1 = [E(P_2) - S_1]/P_1. \tag{2}$$

Each arbitrager has funds under management of F_1, which is exogenously determined.

At t=1 arbitragers decide to allocate D_1 to their own idea and allocate $F_1 - D_1$ into cash. In this case, the arbitragers demand for a particular asset will be

$$QARB_1 = \frac{D_1}{P_1}. \tag{3}$$

Aggregate demand must equal the unit supply of an asset (Shleifer and Vishny (1997)), thus the price of an asset at t=1 is

$$P_1 = E(P_2) - S_1 + D_1. \tag{4}$$

Like Shleifer and Vishny, we assume $D_1 < S_1$ so arbitragers do not have enough resources to bring the period 1 price to fundamental value.

Shleifer and Vishny next develop a model for arbitrager funds based on a marketplace where arbitrage fund investors base their investment decisions on past performance. The authors determine an arbitrager's supply of funds at t=2, F_2, as a function (denoted by G below) of the arbitragers past returns, where D_1 is the amount of funds the arbitrager invests at t=1:

$$F_2 = F_1\left[G\left\{\left(\frac{D_1}{F_1}\right)\left(\frac{P_2}{P_1}\right) + \frac{F_1 - D_1}{F_1}\right\}\right] \; with \; G(1) = 1, G' \geq 1, and \; G'' \leq 0. \tag{5}$$

For simplicity, and because their results do not rely on the concavity of the G function, Shleifer and Vishny focus on a linear G, given by

$$G(z) = az + 1 - a, \;\; with \;\; a \geq 1, \tag{6}$$

where z is the arbitrager's gross return. Equation (5) becomes:

$$F_2 = a\left\{(D_1)\left(\frac{P_2}{P_1}\right) + (F_1 - D_1)\right\} + (1-a)F_1 \tag{7}$$

In Shleifer and Vishny's three period model an arbitrager initially has the opportunity to invest at t=1. At t=2, noise traders have either further depressed the stock price or the stock is at fair value. If noise traders have further depressed the asset's price, the arbitrager invests all they can at t=2, because at t=3 they know with certainty they will receive the fair value of the asset.

Using this basic framework, the authors find that arbitragers may rationally choose to not fully invest in an arbitrage opportunity at t=1 ($D_1 < F_1$), even though they know with certainty at t=3 they will profit, because at t=2 noise traders may make the value of their portfolio lose money. Because the arbitrager is assumed to not have the ability to signal to his investors that his poor performance is due to bad luck and not lack of skill, the arbitrager's outside investors will pull their money from the arbitrager at the exact time that expected returns are the highest. The end result is that arbitragers are sometimes limited in their ability to take advantage of an arbitrage opportunity and thus cannot drive security prices to their fundamental prices as the efficient market hypothesis would suggest (Fama (1970)).

We address other economic questions with an adaption of the Shleifer and Vishny model that includes strong loss aversion: How does the propensity to withdraw funds after poor performance affect asset markets? And, how can arbitragers sharing schemes restrain the costs created by noise traders and performance based arbitrage fund investors?

II. Loss Aversion in the Hedge Fund Industry

A. Motivation

We describe loss aversion in the asset management context as a strong relationship between fund flows and negative fund performance, or in simpler terms, if a hedge fund loses money, the manager can expect to lose a hefty portion of their funds under management. There is plenty of anecdotal evidence for strong loss aversion in the hedge fund asset market. Ever since the quantitative fund meltdown in August 2007, hedge fund managers have been forced to make public statements that their funds are stable in an effort to keep their investors from withdrawing funds. Hedge fund investors' fears of further losses have been especially strong throughout 2008. The situation in the hedge fund asset market is best captured by a recent article in the *New York Times* entitled "Hedge funds are bracing for investors to cash out" (Story (2008)). The article's main thesis is that investors are looking to withdraw their money from poor performing hedge funds because they fear further losses.

The empirical evidence on the relationship between past performance and fund flows supports the notion that investors are loss averse. Research by Ippoliot (1992) and Warther (1995) show mutual fund managers lose funds under management when they perform poorly. The characterization of the loss aversion is documented by Chevalier and Ellison (1997), Goetzmann and Pele (1997), and Sirri and Tufano (1998) who document a convex performance-flow relation.

Loss aversion is stronger in other sectors of the asset management industry. For private equity, Kaplan and Schoar (2005) find a concave relationship between fund flows and performance. In hedge funds the strength of loss aversion is debated, but evidence indicates that poor performance leads to large withdrawals (Baquero and Verbeek (2007) document a linear flow-performance relationship and Ding et al. (2008) find a concave flow-performance relation).

Specific findings from the literature support the notion that particular hedge fund investors are loss averse: long/short, emerging, and small funds (Baquero and Verbeek (2007)), and funds with various investor restrictions (subscription periods, onshore/offshore, capacity constrained styles, total redemption period, asset illiquidity, and lock-up provisions, advance notice periods, and redemption periods) (Ding et al. (2008)) are more sensitive to poor performance than the universe of hedge funds. Finally, Baquero, Horst, and Verbeek (2004) find that liquidation probabilities of hedge funds are heavily dependent on past performance.

B. The Performance Based Arbitrage Model with Loss Aversion

If arbitrage fund investors are loss averse, their G function will act differently when the arbitrager's gross return (z) is negative. We model this by saying G(z) takes the functional form of H(z), which is represented by

$$H(z) = \begin{cases} az + 1 - a, \ for\ z \geq 1, \ with\ a \geq 1 \\ bz + 1 - b, \ for\ z < 1, \ with\ b > a \end{cases}. \tag{8}$$

Equation (7) now becomes:

$$F_2 = \begin{cases} a\left\{(D_1)\left(\frac{P_2}{P_1}\right) + (F_1 - D_1)\right\} + (1-a)F_1, \text{ for } \left((D_1)\left(\frac{P_2}{P_1}\right) + (F_1 - D_1)\right) \geq \mathrm{F}_1 \\ b\left\{(D_1)\left(\frac{P_2}{P_1}\right) + (F_1 - D_1)\right\} + (1-b)F_1, \text{ for } \left((D_1)\left(\frac{P_2}{P_1}\right) + (F_1 - D_1)\right) < F_1 \end{cases}. \qquad (9)$$

Arbitragers maximize their wealth at t=2. Because arbitragers operate in a competitive market for investment services and marginal cost is constant, maximizing wealth at t=2 is equivalent to maximizing funds under management at t=2 (Shleifer and Vishny (1997)), or

$$\max_{D_1} E(F_2) = \varphi\left[a\left\{(D_1)\left(\frac{V}{P_1}\right) + (F_1 - D_1)\right\} + (1-a)F_1\right] + \\ (1-\varphi)\left[b\left\{(D_1)\left(\frac{V - S_2}{P_1}\right) + (F_1 - D_1)\right\} + (1-b)F_1\right] \qquad (10)$$

This equation is subject to the constraint that $E(F_2) \geq F_1$, since the arbitrager can always hold F_1 in cash and hold it until period 2, thus guaranteeing they will have F_1 funds under management at t=2. We also constrain the problem so that arbitrage is risky in the bad state of the world and there are no pure arbitrage opportunities. (See appendix for the first order condition).

Figure 1 gives perspective on the reasonableness of various *a*'s and *b's* that correspond to arbitrage fund investor's sensitivities to past performance. In this figure we calculate the funds withdrawn from or deposited to an arbitrager in three different cases: the arbitrager has investors defined by G(z) and experiences a loss, the arbitrager has loss averse investors defined by H(z) and experiences a loss, and the arbitrager experiences a gain (investors defined by G(z) and H(z) will deposit the same amount if the arbitrager does well).

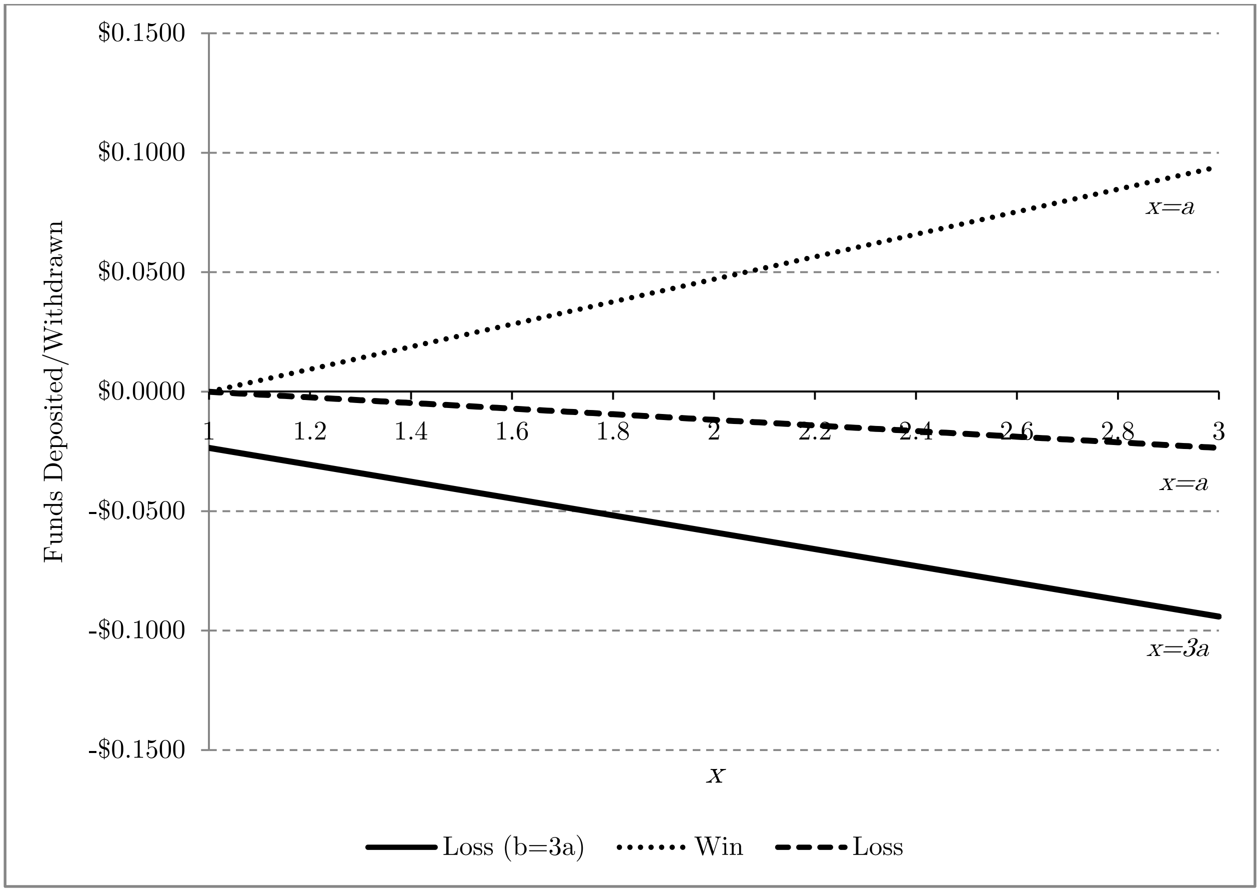

Figure 1: Fund flow and investor sensitivity to past performance. This figure plots the sensitivity arbitrage investors have to past performance (x) against investor fund flows. Initial parameters are set as $V = 1, F_1 = .1, S_1 = .3, S_2 = .4,$ and $\varphi = .7$. We also assume a simple loss aversion of b=3a. Fund flows at t=2 are measured as $\text{FundFlow}_2 = F_1\left(1 - \frac{P_2}{P_1}\right)(1 - x)$ where x is equal to b when the arbitrager has poor performance and loss averse investors, and equal to a otherwise (see appendix).

Using this set up, if arbitrage fund investors choose $a = 1$ the arbitrager will have no funds deposited if they show positive performance and have no funds withdrawn if they show negative performance; however, with loss averse investors poor performance will lead the arbitrage investors to pull .0190 from the fund, leaving the fund manager with .0715 assets under management as opposed to .0905 in the case with no loss aversion. We take no stance on the empirically observed values of a and b, but believe it is reasonable to think that real-world loss averse arbitrage fund investors set $a > 1$ and $b > a$.

PROPOSITION 1: *For a given* $V, S_1, S, F_1, \varphi,$ *and* a, *there is a* b^* *such that, for* $b >$

$b^*,\ D_1 < \mathrm{F}_1, and\ for\ b < b^*, \mathrm{D}_1 = \mathrm{F}_1.$

A risk neutral arbitrager will invest in their respective arbitrage opportunity if $E(F_2) \geq F_1$. If arbitrage investors are loss averse, i.e. *b* is higher than *a*, it is plausible that the arbitrager will not invest all his capital in a risk arbitrage opportunity for fear that if the trade loses, his loss averse investors will take so many funds from his firm that he loses money in expectation. It is also possible that if arbitrage investors are not very loss averse, i.e. *b* is not much higher than *a*, that the arbitrager will invest all of his funds in his risk arbitrage opportunity. More formally, if the first order condition holds with equality, the equilibrium is given by equations (4), (9), and (A1). If the first order condition holds with inequality, equilibrium is given by $\mathrm{D}_1 = \mathrm{F}_1$ and $P_1 = V - S_1 + F_1$. We illustrate that both equilibria are possible with a numerical example. Let $V = 1,\ F_1 = .1,\ a = 1.2,\ S_1 = .3,\ S_2 = .4,$ and $\varphi = .7$. For this example, $b^* = 2.89$. If $b < 2.89$, $D_1 = F_1 = .1$ and arbitragers put all of their capital into their arbitrage opportunity. The first period price (determined by equation 4) is .68. On the other hand, if $b > b^*$, say $b = 3.6$, then $D_1 = .0904$ and $P_1 = .6704$.

Proposition 1 shows that the level of loss aversion determines how arbitragers invest. It also shows that if an arbitrager's investors are loss averse, arbitrage may be limited, as arbitragers find it will not be worth participating in an arbitrage opportunity if a loss in their fund would cause their investors to go fleeing for the exits. Because arbitrage is limited, market prices will not be efficient. For example, in the illustration above, when $b < b^*$, $P_1 = .68$, but when $b = 3.6 > b^*$, $P_1 = .6704$. In neither situation is the fundamental market price $(P_1 = E(P_2) = .5 * (1) + (1 - .5) * (1 - .4) = .8)$ realized; however, loss aversion exacerbates the key finding from SV 1997, that in a performance based world, arbitrage is limited and asset prices can drift from fundamental value.

Another question to address is how loss aversion affects arbitragers' wealth. One may hypothesize that the presence of loss averse investors will negatively affect arbitrager wealth. All else equal, having investors who are more prone to withdraw funds when performance is poor should lower expected arbitrager profits. Proposition 2 shows this to be the case.

PROPOSITION 2: *When arbitrage fund investors are loss averse and arbitrager returns are risky* ($\frac{V-S_2}{P_1} < 1$), *invested arbitragers experience lower expected wealth,* $E_{loss\ adverse}(F_2) < E(F_2)$.

The intuition for this proposition is straight forward: if there is a state of the world where an arbitrager can have a losing trade, he will lose relatively more funds in a world filled with loss averse investors (who run for the exits when the fund posts a negative return), then he will in the world where investors are not loss averse.

The implication from proposition 2 is that arbitrage funds that operate in segments with heavy noise trader influence will cater to investors who are less prone to withdrawing large amounts of money in the case of a loss, and/or they will form contracts with their investors that prevent them from withdrawing their funds in the short-term. Examples of this in the real world hedge fund industry include lock-ups and fund distribution limits, commonly referred to as "gates." We should also see that arbitragers charge higher fees when they operate in noise trader filled markets and/or have investors who are more willing to withdraw funds after poor performance.

III. Information Exchange in a Loss Averse PBA Setting

As is shown by propositions 1 and 2, arbitragers face difficulty in the presence of loss averse investors and noise trader shocks. We next analyze how information exchange amongst arbitragers can mitigate some of this difficulty.

We analyze two arbitragers operating in different segments (segments could be as broad as merger arbitrage versus statistical arbitrage or more narrowly focused, for example, fundamental value investors analyzing companies in different industries). Arbitrager A follows asset x and arbitrager B follows asset y.

Arbitragers A and B are not familiar with each other's asset because they have niche skill sets, a fixed amount of time, and limited research resources. To avert these issues, arbitrager A and arbitrager B decide to form a network. They agree that when A has a good idea he will exchange it for a good idea from B and vice versa. Because arbitrager A and B are highly sophisticated investors, they are able to understand the structure of the model determining asset prices in their respective strategies and can verify that their counterparty has an ability to identify arbitrage opportunities.

The arbitragers believe that by sharing their non-perfectly correlated arbitrage opportunities with each other they can lower the variance of their returns and appease their investors, who are not sophisticated enough to understand their investment strategies and thus use past returns as a simple heuristic on which to assess the arbitragers (Shleifer and Vishny(1997); Ippolito, (1992); and Warther (1995)).

Arbitragers also understand that loss averse investors may force them to pass on their own arbitrage opportunities (as shown in proposition 1 above) because the risk that the trade goes bust will cause them to lose a large portion of their assets under management. However, if they share ideas and thus diversify their position risk, they may find it profitable to invest in an arbitrage opportunity they would have had to pass up if they did not exchange ideas with another arbitrager. Before we can address these issues we need a baseline case from which to work. The starting point for our analysis of whether or not sharing makes sense is captured in proposition 3.

PROPOSITION 3: *In a marketplace with no loss averse investors, arbitragers are indifferent between sharing ideas and working alone.*

This is a simple notion to understand. The main benefit of sharing ideas is that arbitragers would get diversification, which smoothens returns in bad states of the world; however, if arbitrage fund investors react mildly to losses (sensitivity to performance is equal during good and bad performance), there is no benefit to sharing ideas.

A. The Information Exchange Model

At t=1 arbitragers A and B decide to split their respective funds and put D_1 to their own idea and allocate N_1 into the other's idea. In this case, the arbitragers demand for a particular asset will be $QA_1 = \frac{D_1}{P_1} + \frac{N_1}{P_1}$. Aggregate demand must equal the unit supply of an asset (Shleifer and Vishny (1997)), thus the price of an asset at t=2 is

$$P_1 = E(P_2) - S_1 + D_1 + N_1. \tag{11}$$

We assume $D_1 + N_1 < S_1$, so arbitragers do not have enough resources to bring the period 1 price to fundamental value (i.e. borrowing constraints).

At t=1, assets x and y suffer noise trader shocks of the same size, $S_{x,1} = S_{y,1}$. These shocks drive the prices of x and y below fundamental value ($E(P_2) - S_1 = P_1 < E(P_2)$), thus creating similar arbitrage opportunities for A and B.

At t=1, arbitragers A and B each have F_1 to invest and wish to maximize their wealth at t=2. The arbitragers have the option to share their ideas with one another, invest all their money in their respective ideas, or hold a portion of their portfolio in cash (C_1). If sharing ideas is a profitable decision for arbitragers A and B ($E_{share}(F_2) > E_{alone}(F_2)$), they will choose to invest N_1 in the other's arbitrage opportunity, put D_1 in their own arbitrage opportunity, and place $F_1 - N_1 - D_1$ in cash. However, because arbitragers A and B have similar arbitrage opportunities, we assume that the arbitragers agree to a simple allocation rule and set $D_1 = N_1$, or in other words, they divide their money evenly between their own idea and the other's idea. For concreteness, we also assume arbitragers will be fully invested at t=1 when arbitrage opportunities are present (this will occur when $b < b^*$ as is shown in proposition 1).

At t=2, assets x and y have a φ probability that noise trader mispricing worsens, or, $S_{x,2}, S_{y,2} = S > S_1$. There is a complementary probability of $1 - \varphi$ that $S_{x,2}, S_{y,2} = 0$, and assets x and y reach their fundamental values of V at t=2. It is assumed that noise trader influences in assets x and y are independent of one another, i.e. $\rho\left(S_{x,2}, S_{y,2}\right) = 0$ (this is a simplifying assumption and our conclusions only rely on the notion that noise traders in assets x and y trading influences are not perfectly correlated). At the end of this period arbitragers cash out of their positions and provide performance results to their investors.

In this world

$$E_{share}(F_{i,2}) = \varphi\varphi\left[a\left\{(F_1)\left(\frac{V}{P_1}\right)\right\} + (1-a)F_1\right] + (1-\varphi)\varphi\left[x\left\{\left(\frac{F_1V - D_1S_2}{P_1}\right)\right\} + (1-x)F_1\right] + \varphi(1-\varphi)\left[x\left\{\left(\frac{F_1V - N_1S_2}{P_1}\right)\right\} + (1-x)F_1\right] + (1-\varphi)(1-\varphi)\left[b\left\{F_1\left(\frac{V-S_2}{P_1}\right)\right\} + (1-b)F_1\right] \;\; s.t. D_1 = N_1, \tag{12}$$

where,

$$x = \begin{cases} a, if \ \dfrac{(portfolio\ value\ at\ t = 2)}{P_1} \geq 1 \\ b, if \ \dfrac{(portfolio\ value\ at\ t = 2)}{P_1} < 1 \end{cases}.$$

PROPOSITION 4: *When arbitragers act alone, for a given V, S_1, S, F_1, φ, and a, there is a b^* such that, for $b > b^*, D_1 < F_1$, and for $b < b^*, D_1 = F_1$; however, when arbitragers share ideas there is a $b' > b^*$ such that $D_1 + N_1 = F_1$.*

This proposition is best illustrated by a numerical example. Let $V = 1, F_1 = .1, a = 1.2, S_1 = .3, S_2 = .4,$ and $\varphi = .7$. For this example, $b^* = 2.89$. If $b < 2.89$, $D_1 = F_1 = .1$ and arbitragers put all of their capital into their arbitrage opportunity. The first period price (determined by equation 4) is .68. On the other hand, if $b > b^*$, say $b = 3.6$, then $D_1 = .0904$ and $P_1 = .6704$, thus implying that arbitrage hold back some of their capital. In contrast, when arbitragers share ideas their willingness to invest in arbitrage opportunities increases. For example, if $b' > b^*$, say b=3.6, then $N_1 + D_1 = F_1 = .1$ and $P_1 = .68$.

The key insight from this proposition is that sharing can encourage profit maximizing arbitragers to invest more funds into arbitrage opportunities than they would if they were not exchanging ideas. Sharing also pushes asset prices closer to fundamental value. The next proposition further explores under what circumstances arbitragers will share ideas.

PROPOSITION 5: *Capital constrained arbitragers ($F_1 < S_1 + S_2(1 - \varphi)$) operating in a marketplace with loss averse investors will always shares ideas.*

Noise traders affect the decision for arbitragers to share ideas in a couple of ways: they can either drive prices lower with high shocks (S_1 and/or S_2 are large), or they can have a low probability of assessing the t=2 market price correctly (φ is low). Arbitragers can counterbalance these influences if they have a large amount of resources relative to the noise trader mispricing (F_1 is large).

In a real world setting, proposition 5 suggests that empirically we should see arbitragers sharing ideas in markets that are more prone to noise trader influences (e.g. small stocks versus large stocks) or in markets where they have more capital constraints (equity markets versus bond markets).

PROPOSITION 6: *Arbitragers who share ideas will have correlated portfolios and crowd into trades.*

In the simple setup described, sharing arbitragers will hold the same portfolio of assets. In this basic case, the portfolios of sharing arbitragers will be perfectly correlated. In general, if arbitragers maintain a portfolio of positions and hold only a portion of their portfolio in shared assets, arbitrager portfolios will show less correlation, but it will still likely be positive. Moreover, we do not rigorously analyze the case of multiple arbitragers sharing multiple ideas; however, it is intuitive to understand that if arbitragers are sharing ideas amongst each other and investing in many of the same assets, trading ideas will become crowded with arbitrage investors all clamoring for the same arbitrage (Colla and Mele (2007) and Ozsoylev (2005) develop social network models and rigorously show that correlations and the likelihood of crowded trades increases when agents share information).

IV. Discussion

Our simple model of arbitrager behavior in the face of loss averse investors is a way to capture the notion that arbitragers may choose to share ideas in certain circumstances. This contrasts with the view that all arbitragers are secretive operations that operate with black boxes and crystal balls.

Loss aversion was the primary mechanism behind the desire to share ideas, and the circumstance we believed to be most credible after reviewing the literature, speaking with hedge fund managers, and following anecdotal evidence in the main stream financial press; however, other elements such as arbitrager risk aversion, herd behavior (Scharfstein and Stein (1990)), laziness, or internal resource deficiency could also lead to similar conclusions. We do not model these other mechanisms, but believe they will lead to the same conclusion: under certain circumstances, sharing trading ideas can make sense.

A. *Empirical Implications*

Loss averse investors can have real affects on arbitragers and asset prices. As proposition 1 shows, if loss aversion is strong enough, arbitragers will scale down their investment and market prices will stray from the efficient price. This prediction has implications for the real-world. All other things being equal, if certain asset classes attract arbitragers who tend to have loss averse investors, we would expect these assets to be more mispriced. We see evidence for this in the real world: arbitragers with more risk averse investors include long/short, emerging, and smaller hedge funds (Baquero and Verbeek (2007)). The hedge fund managers who operate these funds typically invest in smaller stocks, which are often cited as being mispriced (e.g. Banz (1981) or Fama and French (1992)).

As proposition 2 suggests, loss averse investors lower the expected profits of arbitragers. To counteract this phenomenon we should expect to see higher fees charged to certain classes of investors more prone to loss aversion and/or contracts that prevent investors from withdrawing funds in the short-term. Empirically, this behavior exists in the hedge fund market. Many funds vary their fee structure depending on the type of investor, and the contract signed. If investors agree to longer lock-up periods they are charged lower fees and shorter lock-ups are charged higher fees. A report by the Alternative Investment Advisory Specialist Group says it best, "Managers desire lock-ups in general because a stable asset base eases pressure for smoother return streams (for fear that investors may pull capital because of poor short-term results) and permits the deployment of capital over longer time horizons" (AIAS (2005)).

Propositions 4 and 5 seem to indicate that information exchange between arbitragers can be a good idea. If this is true, at the very minimum we should see mechanisms that facilitate information exchange for arbitragers in the marketplace. Indeed, this is exactly what we see. The more established invite-only venues that allow professional hedge fund managers and affiliates to discuss ideas include Valueinvestorsclub.com, Sumzero.com, Value Investing Congress, Hedge Fund Activism and Shareholder Value Summit, and the Lehman Finance in Practice Conference. Moreover, our interviews with hedge fund managers and hedge fund asset allocation consultants suggest, at least anecdotally, that information exchange is prevalent.

Other implications from proposition 4 and 5 are that asset markets with loss averse investors, such as long/short strategies, emerging hedge funds, or smaller hedge funds (Baquero and Verbeek (2007)), should see more sharing amongst managers. We should also see more sharing in markets with higher noise trader risk (e.g. small capitalization stocks, stub arbitrage (Mitchell, Pulvino, and Stafford, 2002), and pairs/twin arbitrage (Froot and Dabora, 1999)). In fact, Gray and Kern's analysis of roughly 3000 investment ideas submitted to the private internet community Valueinvestorsclub.com website from January 2000 to June 2008 corroborates the notion that sharing occurs in assets commonly thought to be subject to high levels of noise trader risk. Specifically, they find that a vast majority of the ideas submitted to Valueinvestorsclub.com are recommendations for micro-capitalization stocks (<\$500mm), or special situations such as stub and pair arbitrages, liquidations, and spin-offs in relatively illiquid markets (Gray and Kern (2008)).

A final implication of our model, captured in proposition 6, is that arbitrager information exchange causes their portfolios to be more correlated. Empirically, we see that hedge fund manager returns are correlated and that this correlation is increasing. Anecdotal examples include the Long Term Capital Management episode (Lowenstein (2000)), the August 2007 quantitative funds meltdown (Khandani and Lo (2007)), and the September/October 2008 chaos (Curran and Rogow (2008)). Comprehensive studies on the subject have been conducted by Garbaravicius and Dierick (2005) and Khandani and Lo (2007). Garbaravicius and Dierick find that individual hedge funds pairwise correlations within categories are increasing, and there is "crowding" on certain trades. Khandani and Lo confirm this result and conclude that the "hedge-fund industry has clearly become more closely connected."

Financial markets have recently highlighted the existence of crowded trades. The Porsche/Volkswagen negative stub trade is a case study (a full discussion and analysis of negative stub trades are described in Mitchell, Pulvino, and Stafford (2002)). The idea was posted in September 2008 to Valueinvestorsclub.com. The recommended trade was very well received by the Valueinvestorsclub.com community and was given one of the highest rankings in the club's history. The investment thesis was simple: Volkswagen was overvalued and Porsche's ownership of Volkswagen gave Porsche a negative equity value. The trade was to go long Porsche and short Volkswagen.

In late October, Porsche management made an announcement that it was increasing its ownership in Volkswagen. This announcement caused a massive short squeeze that sent the stock of Volkswagen soaring over 348% in two days. At one point, Porsche became the most highly valued company in the world (Zuckerman, Strasburg, and Esterl (2008)). Multiple hedge funds suffered massive losses. Laurie Pinto, a broker at North Square Capital said it best, "This [Volkswagen short squeeze] is without question the biggest single loss on a single stock in the history of hedge funds. It's a bloodbath." (Jameson and Robertson (2008)). We will never know if all the hedge fund involved in this trade randomly stumbled across the same idea or if this idea was shared amongst a group of managers; however, based on the volume of questions and general interest on Valueinvestorsclub.com following the posting of the Porsche/Volkswagen trade, it seems that many people discovered this trade via information exchange.

V. Concluding Remarks

Using the Shleifer and Vishny (1997) model as a base, we examine how loss averse arbitrage fund investors affect the marketplace. We find that loss aversion exacerbates the limits to arbitrage and decreases arbitrager's profits. We further examine what happens in the marketplace when arbitragers can share ideas with other specialized arbitragers who have unique arbitrage opportunities. Our conclusions were as follows: sharing leads to more efficient asset prices, larger arbitrager profits, systems that allow arbitragers to efficiently share ideas, and correlated arbitrager portfolios.

Sharing can have unforeseen circumstances for the marketplace. In normal times, correlated hedge fund portfolios have never been a big concern, but the recent unprecedented market events (starting September 2008) have transformed the hedge fund market. Highly levered funds are losing massive amounts of money and their investors are falling over each other to withdraw funds, which has lead to further liquidations. Unfortunately, a lot of these positions being liquidated at fire sale prices are held by other hedge funds that were sharing ideas or utilizing similar stock picking strategies. This transformation in the market has highlighted a risk of sharing and the use of leverage in the hedge fund market. Going forward, hedge fund managers will likely continue to share ideas in certain sectors, but will operate with a lower level of leverage and conduct a deeper due diligence of their sharing counterparties.

Appendix

A. First Order Condition

The first order condition is given by

$$\frac{dE(F_2)}{dD_1} = \frac{[a\varphi V - b(1-\varphi)(V-S_2)]}{P_1} - \frac{[a\varphi V D_1 - b(1-\varphi)(V-S_2)D_1]}{P_1^2} - b(1-\varphi) - a\varphi \geq 0\ ,$$

which implies

$$D_1^* \geq S_1 + S_2(1-\varphi) - V - \left[\frac{(S_1+S_2(1-\varphi)-V)(b(1-\varphi)(S_2-V)+a\varphi V}{b(1-\varphi)-a\varphi}\right]^{.5}. \qquad \text{(A1)}$$

B. Fund Flow Calculations

Explanation of fund flow calculation: We know $F_2 = a\left\{(D_1)\left(\frac{P_2}{P_1}\right) + (F_1 - D_1)\right\} + (1-x)F_1$ where *x=a* for non-loss averse arbitrage investors regardless of performance. For loss averse arbitrage investors, *x=a* when performance is positive, but *x=b* if performance is negative. Funds at t=2 can be written as $F_2 = F_1 + D_1\frac{P_2-P_1}{P_1} + D_1\frac{P_2-P_1}{P_1}(x-1)$. This equation can be decomposed into three parts: initial funds at t=1 (F_1), gains from trading ($D_1\frac{P_2-P_1}{P_1}$), and fund flows ($FundFlow_1 = D_1\frac{P_2-P_1}{P_1}(x-1)$).

C. Proofs of Proposition 2,3, and 5

Proof of Proposition 2: We want to show that $E_{loss\,adverse}(F_2) < E(F_2)$. By definition

$$
\begin{aligned}
&E_{loss\,adverse}(F_2) - E(F_2) \\
&\qquad = (1-\varphi)\left[b\left\{(D_1)\left(\frac{V-S_2}{P_1}\right) + (F_1 - D_1)\right\} + (1-b)F_1\right] \\
&\qquad - (1-\varphi)\left[a\left\{(D_1)\left(\frac{V-S_2}{P_1}\right) + (F_1 - D_1)\right\} + (1-a)F_1\right] \\
&\qquad = (1-\varphi)\left[D_1(b-a)\left(\frac{V-S_2}{P_1} - 1\right)\right]
\end{aligned}
\tag{A2}
$$

We know $(1-\varphi) > 0$, and $D_1(b-a) > 0$ by definition. If arbitrage is risky (there is a state of the world in t=2 where the arbitrager will lose money), then $\left(\frac{V-S_2}{P_1} - 1\right) < 0$, which implies equation A1 is less than 0, or $E_{loss\,adverse}(F_2) < E(F_2)$. Q.E.D.

Proof of Proposition 3: The expected profits for an arbitrager working alone is given by

$$
E_{alone}(F_2) = \varphi\left[a\left\{(F_1)\left(\frac{V}{P_1}\right)\right\} + (1-a)F_1\right] + (1-\varphi)\left[a\left\{(F_1)\left(\frac{V-S_2}{P_1}\right)\right\} + (1-a)F_1\right], \tag{A3}
$$

and the expected profits for arbitragers working together are expressed as

$$
\begin{aligned}
E_{share}(F_{i,2}) = \varphi\varphi&\left[a\left\{(F_1)\left(\frac{V}{P_1}\right)\right\} + (1-a)F_1\right] \\
&+ (1-\varphi)\varphi\left[a\left\{\left(\frac{F_1V - D_1S_2}{P_1}\right)\right\} + (1-a)F_1\right] \\
&+ \varphi(1-\varphi)\left[a\left\{\left(\frac{F_1V - N_1S_2}{P_1}\right)\right\} + (1-a)F_1\right] \\
&+ (1-\varphi)(1-\varphi)\left[a\left\{F_1\left(\frac{V-S_2}{P_1}\right)\right\} + (1-b)F_1\right] \\
&s.t.\, D_1 = N_1.
\end{aligned}
\tag{A4}
$$

If We substitute and let $M = a\left\{(F_1)\left(\frac{V}{P_1}\right)\right\} + (1-a)F_1$, $N = a\left\{(F_1)\left(\frac{V-S_2}{P_1}\right)\right\} + (1-a)F_1$, and $P = a\left\{\left(\frac{F_1V-N_1S_2}{P_1}\right)\right\} + (1-a)F_1$ the difference in profits between sharing and working alone becomes

$$\begin{aligned} E_{share}(F_{i,2}) - E_{alone}(F_{i,2}) &= \varphi^2 M - \varphi M + 2(1-\varphi)\varphi P + (1-\varphi)^2 N - (1-\varphi)N \\ &= (M+N)(\varphi)(\varphi-1) - 2Z(\varphi)(\varphi-1) \\ &= (M+N-2P)(\varphi)(\varphi-1). \end{aligned} \tag{A5}$$

This implies, $E_{share}(F_{i,2}) - E_{alone}(F_{i,2}) = (M+N-2P)(\varphi)(\varphi-1) = 0$ iff $(M+N-2P) = 0$, because $\underbrace{(\varphi)}_{+}\underbrace{(\varphi-1)}_{-} < 0$. Substituting again,

$$\begin{aligned} M+N-2P = a\left\{(F_1)\left(\frac{V}{P_1}\right)\right\} + (1-a)F_1 + a\left\{(F_1)\left(\frac{V-S_2}{P_1}\right)\right\} + \\ (1-a)F_1 - 2\left[a\left\{\left(\frac{F_1 V - N_1 S_2}{P_1}\right)\right\} + (1-a)F_1\right]. \end{aligned} \tag{A6}$$

We know $D_1 + N_1 = F_1$, as it is assumed $b < b^*$ (see proposition 1) and arbitragers will be fully invested. Also, because arbitragers split their portfolio evenly between their own idea and the shared idea, $D_1 = N_1$, this means $F_1 = 2D_1$, which further implies $F_1 \frac{V-S_2}{P_1} = F_1 \frac{V}{P_1} - 2N_1\left(\frac{S_2}{P_1}\right)$. Substituting this result into A3 implies $M+N-2P = 0$. Q.E.D.

Proof of Proposition 5: Capital constrained arbitragers ($F_1 < S_1 + S_2(1-\varphi)$) operating in a marketplace with loss averse investors will always shares ideas.

1. Determining investor sensitivity to gross returns

We know

$$
\begin{aligned}
E_{share}(F_{i,2}) = \varphi\varphi &\left[x\left\{(F_1)\left(\frac{V}{P_1}\right)\right\} + (1-a)F_1\right] \\
&+ (1-\varphi)\varphi\left[x\left\{\left(\frac{F_1V - D_1S_2}{P_1}\right)\right\} + (1-a)F_1\right] \\
&+ \varphi(1-\varphi)\left[x\left\{\left(\frac{F_1V - N_1S_2}{P_1}\right)\right\} + (1-a)F_1\right] \\
&+ (1-\varphi)(1-\varphi)\left[x\left\{F_1\left(\frac{V - S_2}{P_1}\right)\right\} + (1-b)F_1\right] \\
&s.t. D_1 = N_1.
\end{aligned}
\tag{A7}
$$

where,

$$
x = \begin{cases} a, if \ \dfrac{(portfolio\ value\ at\ t = 2)}{P_1} \geq F_1 \\ b, if \ \dfrac{(portfolio\ value\ at\ t = 2)}{P_1} < F_1 \end{cases}.
$$

We assume that $V - S_2 < P_1$, or $x = b$ if the portfolio value at t=2 is $F_1\left(\frac{V-S_2}{P_1}\right)$, and We assume $V > P_1$, or $x = a$ if the portfolio value at t=2 is $F_1\left(\frac{V}{P_1}\right)$. These assumptions limit arbitrage capital such that $S_2\varphi > S_1 - F_1 > S_2(\varphi - 1)$. We also know that $x = a\ if\left(\frac{F_1V-D_1S_2}{P_1}\right) \geq F_1$. We know $D_1 + N_1 = F_1$ as it is assumed $b < b^*$. Also, because arbitragers split their portfolio evenly between their own idea and the shared idea, $D_1 = N_1$, which implies $D_1 = .5F_1$, which further implies $\left(\frac{F_1V-D_1S_2}{P_1}\right) = \left(\frac{F_1(V-.5S_2)}{P_1}\right)$. Substituting the definition of $P_1 = V + S_2(\varphi - 1) - S_1 + F_1$ into the equation, we have $x = a\ if\left(\frac{F_1(V-.5S_2)}{V+S_2(\varphi-1)-S_1+F_1}\right) \geq F_1$. When we simplify this expression we have $x = a$ if $S_1 - F_1 \geq S_2(\varphi - .5)$. and $x = b$ otherwise.

2. Assessing the sharing decision when $S_1 - F_1 \geq S_2(\varphi - .5)$

We first compare the strategies of sharing and working alone when $S_1 - F_1 \geq S_2(\varphi - .5)$. In this situation, working alone profits are

$$E_{alone}(F_2) = \varphi\left[a\left\{(F_1)\left(\frac{V}{P_1}\right)\right\} + (1-a)F_1\right] + (1-\varphi)\left[b\left\{(F_1)\left(\frac{V-S_2}{P_1}\right)\right\} + (1-b)F_1\right], \qquad \text{(A8)}$$

and the expected profits for arbitragers working together are expressed as

$$\begin{aligned} E_{share}(F_{i,2}) = \varphi\varphi & \left[a\left\{(F_1)\left(\frac{V}{P_1}\right)\right\} + (1-a)F_1\right] \\ & + (1-\varphi)\varphi\left[a\left\{\left(\frac{F_1V - D_1S_2}{P_1}\right)\right\} + (1-a)F_1\right] \\ & + \varphi(1-\varphi)\left[a\left\{\left(\frac{F_1V - N_1S_2}{P_1}\right)\right\} + (1-a)F_1\right] \\ & + (1-\varphi)(1-\varphi)\left[b\left\{F_1\left(\frac{V-S_2}{P_1}\right)\right\} + (1-b)F_1\right] \\ & s.t. D_1 = N_1. \end{aligned} \qquad \text{(A9)}$$

If we substitute and let $M = a\left\{(F_1)\left(\frac{V}{P_1}\right)\right\} + (1-a)F_1$, $N = b\left\{(F_1)\left(\frac{V-S_2}{P_1}\right)\right\} + (1-b)F_1$, and $P = a\left\{\left(\frac{F_1V-N_1S_2}{P_1}\right)\right\} + (1-a)F_1$ the difference in profits between sharing and working alone becomes

$$\begin{aligned} E_{share}\left(F_{i,2}\right) - E_{alone}\left(F_{i,2}\right) &= \varphi^2M - \varphi M + 2(1-\varphi)\varphi P + (1-\varphi)^2N - (1-\varphi)N \\ &= (M+N)(\varphi)(\varphi-1) - 2Z(\varphi)(\varphi-1) \\ &= (M+N-2P)(\varphi)(\varphi-1). \end{aligned} \qquad \text{(A10)}$$

This implies, $E_{share}\left(F_{i,2}\right) - E_{alone}\left(F_{i,2}\right) = (M+N-2P)(\varphi)(\varphi-1)$, which is positive if and only if $M+N-2P<0$, because $\underbrace{(\varphi)}_{+}\underbrace{(\varphi-1)}_{-} < 0$ by assumption.

Substituting again,

$$\begin{aligned} M+N-2P = a\left\{(F_1)\left(\frac{V}{P_1}\right)\right\} + (1-a)F_1 + b\left\{(F_1)\left(\frac{V-S_2}{P_1}\right)\right\} + \\ (1-b)F_1 - 2\left[a\left\{\left(\frac{F_1V-N_1S_2}{P_1}\right)\right\} + (1-a)F_1\right]. \end{aligned} \qquad \text{(A11)}$$

Equation (A11) can be simplified as $(a-b)\left[F_1 + 2\left(\frac{N_1S_2}{P_1}\right) - F_1\left(\frac{V}{P_1}\right)\right]$. We know $\underbrace{(a-b)}_{-} < 0$ by assumption; therefore, in order for $M+N-2P<0$, $\left[F_1 + 2\left(\frac{N_1S_2}{P_1}\right) - F_1\left(\frac{V}{P_1}\right)\right] > 0$. We simplify the constraint $F_1 + 2\left(\frac{N_1S_2}{P_1}\right) - F_1\left(\frac{V}{P_1}\right) > 0$ as $F_1(F_1 + S_2\varphi - S_1) > 0$, which can be expressed as $S_2\varphi > S_1 - F_1$. We know this inequality must hold because in the beginning of this problem we ensured that arbitrage is risky, $V - S_2 < P_1$, and potentially rewarding, $V > P_1$. These constraints imply $S_2\varphi > S_1 - F_1 > S_2(\varphi - 1)$, which proves that $F_1(F_1 + S_2\varphi - S_1) > 0$. With this information we can say that $E_{share}(F_{i,2}) - E_{alone}(F_{i,2}) > 0$ when $S_1 - F_1 \geq S_2(\varphi - .5)$.

3. Assessing the sharing decision when $S_1 - F_1 < S_2(\varphi - .5)$

We next compare the strategies of sharing and working alone when $S_1 - F_1 < S_2(\varphi - .5)$, but greater than $S_2(\varphi - 1)$ to ensure $V - S_2 < P_1$. In this situation working alone profits are if the portfolio value at t = 2 is

$$E_{alone}(F_2) = \varphi\left[a\left\{(F_1)\left(\frac{V}{P_1}\right)\right\} + (1-a)F_1\right] + (1-\varphi)\left[b\left\{(F_1)\left(\frac{V-S_2}{P_1}\right)\right\} + (1-b)F_1\right], \quad \text{(A12)}$$

and the expected profits for arbitragers working together are expressed as

$$\begin{aligned} E_{share}(F_{i,2}) = \varphi\varphi&\left[a\left\{(F_1)\left(\frac{V}{P_1}\right)\right\} + (1-a)F_1\right] \\ &+ (1-\varphi)\varphi\left[b\left\{\left(\frac{F_1V - D_1S_2}{P_1}\right)\right\} + (1-a)F_1\right] \\ &+ \varphi(1-\varphi)\left[b\left\{\left(\frac{F_1V - N_1S_2}{P_1}\right)\right\} + (1-a)F_1\right] \\ &+ (1-\varphi)(1-\varphi)\left[b\left\{F_1\left(\frac{V - S_2}{P_1}\right)\right\} + (1-b)F_1\right] \qquad \text{(A13)} \\ &s.t. D_1 = N_1. \end{aligned}$$

If we substitute and let $M = a\left\{(F_1)\left(\frac{V}{P_1}\right)\right\} + (1-a)F_1$, $N = b\left\{(F_1)\left(\frac{V-S_2}{P_1}\right)\right\} + (1-b)F_1$, and $P = b\left\{\left(\frac{F_1V-N_1S_2}{P_1}\right)\right\} + (1-b)F_1$ the difference in profits between sharing and working alone becomes

$$\begin{aligned} E_{share}(F_{i,2}) - E_{alone}(F_{i,2}) &= \varphi^2M - \varphi M + 2(1-\varphi)\varphi P + (1-\varphi)^2N - (1-\varphi)N \\ &= (M+N)(\varphi)(\varphi-1) - 2Z(\varphi)(\varphi-1) \\ &= (M+N-2P)(\varphi)(\varphi-1). \end{aligned} \qquad \text{(A14)}$$

This implies, $E_{share}(F_{i,2}) - E_{alone}(F_{i,2}) = (M+N-2P)(\varphi)(\varphi-1)$, which is positive if and only if $M + N - 2P < 0$, because $\underbrace{(\varphi)}_{+}\underbrace{(\varphi - 1)}_{-} < 0$ by assumption.

Substituting again,

$$M + N - 2P = a\left\{(F_1)\left(\frac{V}{P_1}\right)\right\} + (1-a)F_1 + b\left\{(F_1)\left(\frac{V - S_2}{P_1}\right)\right\} + \\ (1-b)F_1 - 2\left[b\left\{\left(\frac{F_1 V - N_1 S_2}{P_1}\right)\right\} + (1-b)F_1\right]. \tag{A15}$$

Equation (A15) can be simplified as $(a-b)\left[F_1\left(\frac{V}{P_1}\right) - F_1\right]$. We know $\underbrace{(a-b)}_{-} < 0$ by assumption, therefore, in order for $M + N - 2P < 0$, $\left[F_1\left(\frac{V}{P_1}\right) - F_1\right] > 0$. We simplify the constraint $F_1\left(\frac{V}{P_1}\right) - F_1 > 0$ as $F_1\left(\frac{V}{P_1} - 1\right) > 0$, which can be expressed as $S_1 - F_1 > S_2(\varphi - 1)$. We know this inequality must hold because in the beginning of this problem we ensured that arbitrage is risky, $V - S_2 < P_1$, and potentially rewarding, $V > P_1$. These constraints imply $S_2\varphi > S_1 - F_1 > S_2(\varphi - 1)$, which proves that $F_1\left(\frac{V}{P_1}\right) - F_1 > 0$. With this information we can say that $E_{share}(F_{i,2}) - E_{alone}(F_{i,2}) > 0$ when $S_2(\varphi - 1) < S_1 - F_1 < S_2(\varphi - .5)$.

4. Putting it all together

We have shown that sharing makes sense when $S_1 - F_1 \geq S_2(\varphi - .5)$ and when $S_2(\varphi - 1) < S_1 - F_1 < S_2(\varphi - .5)$. Combining these findings we come to the conclusion that sharing dominates the working alone strategy when $S_2(\varphi - 1) < S_1 - F_1$. When $S_2(\varphi - 1) \geq S_1 - F_1$ We violate the assumption that $V \geq P_1$, which would make arbitrage unprofitable. It can therefore be concluded that arbitragers will always share ideas when their capital is constrained to $F_1 < S_1 + S_2(1 - \varphi)$. Q.E.D.

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