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Endogenous Innovation Scale and Patent Policy in a Monetary Schumpeterian Growth Model*

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Abstract: This paper develops a monetary R&D-driven endogenous growth model featuring endogenous innovation scales and the price-marginal cost markup. To endogenize the step size of quality improvement, we propose a trade-off mechanism between the risk of innovation failure and the benefit of innovation success in R&D firms. Several findings emerge from the analysis. First, a rise in the nominal interest rate decreases economic growth; however, its relationship with social welfare is ambiguous. Second, either strengthening patent protection or raising the professional knowledge of R&D firms leads to an ambiguous effect on economic growth. Third, the Friedman rule of a zero nominal interest rate fails to be optimal in view of the social welfare maximum. Finally, our numerical analysis indicates that the extent of patent protection and the level of an R&D firm's professional knowledge play a crucial role in determining the optimal interest rate.

Keywords: Intellectual property rights, Economic growth, Endogenous innovation scales, Endogenous markups, Inflation

JEL Classification: O30, O40, E41, L11

1. *Introduction*

This paper explores the long-run effect of monetary policy on economic growth and social welfare in an R&D growth model with an endogenous innovation scale. To achieve this goal, we build up a Schumpeterian growth model and introduce money demand into the model via the cash-in-advance (CIA) constraint. Compared to the existing literature on monetary policy and R&D-driven growth, the salient feature of this model is that the R&D firm is motivated to choose the innovation scale freely. To be more specific, existing studies on the monetary Schumpeterian growth model, such as Chu and Cozzi, 2014 and Chu et al., 2015, generally assume that the innovation scale of the R&D firm is exogenous. This simplified assumption implies that the R&D firm cannot choose the magnitude of quality improvement in innovation to maximize its profits. However, this assumption does not fit the empirical observations, such as those in Shenhar (1993), Robertson and Gatignon (1998) and Muratori (2020), and results in the analysis being insufficiently complete to describe the R&D firm's behavior.

In this study, to reflect the empirical observations, we relax the assumption regarding the exogenous innovation scale for the R&D firm and focus on examining how the endogenous innovation scale will affect the linkages among monetary policy, economic growth, and social welfare. To rationalize the endogenous innovation scale, we emphasize that the R&D firm will choose the value of the innovation scale so as to maximize its profit. More specifically, a rise in the size of the innovation scale exerts two conflicting effects on the expected profit of the R&D firm from innovation. On the one hand, it increases the markup of monopolistic intermediate goods, and therefore causes a rise in the expected profit of the R&D firm from innovation. On the other hand, it leads the R&D firm to face a higher risk of innovation failure, and hence causes a decline in the expected profit of the R&D firm from innovation. The R&D firm thus selects the optimal innovation scale at the level where these two conflicting effects are balanced.

Apart from providing a positive analysis on how the endogenous innovation scale mechanism affects the linkage between monetary policy and economic growth, this paper also presents a normative analysis regarding how the government set its monetary policy rule from the viewpoint of welfare maximization. In his pioneering article, Friedman (1969) propounds that, to lead the economy toward an efficient circumstance, the optimal money growth targeting is set such that the nominal interest rate goes to zero. The result of a zero nominal interest rate is now well known as the Friedman rule in the literature. This paper will analyze whether the Friedman rule is valid when the innovation scale is endogenously determined by the R&D firm.

The main findings and contributions of this paper are as follows. First, in response to stronger patent protection, the R&D firm is inclined to choose a smaller innovation scale (i.e., a lower innovation challenge project). Intuitively, strengthening patent protection leads to an increase in the patent value of innovation. This implies that the R&D firm will suffer from more expected loss when innovation fails. Therefore, to reduce the risk of innovation failure, the R&D firm is motivated to perform a smaller step size of improvement in the technology. With this endogenous step size of improvement, we find that strengthening patent protection is ambiguously related to economic growth. More specifically, as documented by Li (2001), strengthening patent protection leads to a rise in the patent value, which attracts the R&D firm to hire more labor in R&D. Accordingly, it will stimulate economic growth. However, with the additional channel of an endogenous innovation scale, strengthening patent protection will lead the R&D firm to choose a smaller innovation scale, which will reduce the step size of quality improvement. This gives rise to an additional effect that will be harmful to economic growth. As a result, when the channel through which the innovation scale that is endogenously determined by the R&D firm is brought into the picture, strengthening patent protection will tend to generate an ambiguous overall effect on economic growth.

The second finding of this paper is that the R&D firm is inclined to choose a larger innovation scale when it has a higher level of professional knowledge. This finding is quite intuitive. When the R&D firm has a higher level of professional knowledge, it will face a lower risk of innovation failure. As a result, the R&D firm is motivated to choose a larger size of innovation scale to achieve profit maximization, which is similar to recent empirical research by Hsu et al. (2021).[1] Armed with this endogenous innovation scale, the R&D firm that possesses a higher level of professional knowledge is not necessarily associated with higher economic growth. Intuitively, a higher level of professional knowledge motivates the R&D firm to choose a larger innovation scale to maximize its profit. This tends to raise the risk of innovation failure, and thus is detrimental to economic growth. Nevertheless, the R&D firm that chooses a larger innovation scale is also characterized by a larger improvement in technology, which is in turn associated with a higher patent value. This would be beneficial to economic growth. Thus, with these two conflicting effects on economic growth, the relationship between the level of the R&D firm's professional knowledge and economic growth is ambiguous.

The third finding is related to the growth and welfare effects of monetary policy implemented in the form of the nominal interest rate targeting. By formulating a cash-in-advance on R&D investment, an increase in the nominal interest rate causes a rise in borrowing costs (interest payments) on R&D investment, and therefore lead R&D entrepreneurs to reduce their R&D investment, which is harmful to growth. This generates a negative effect on social welfare. However, as the fund's lender to R&D entrepreneurs, the household will earn more interest income in response to a rise in the nominal interest rate. The household will stimulate its initial consumption in response, and this generates a positive effect on the social welfare level. Accordingly, to maximize social welfare the government will choose the optimal nominal interest rate at the positive level where these two conflicting effects are balanced. This leads to the outcome that the Friedman rule of a zero nominal interest rate fails to be optimal. Moreover, in going beyond the existing studies, we highlight a key role of endogenous innovation scale on these two conflicting effects. With this, we are able to further examine how the optimal interest rate is related to the endogeneity of the innovation scale.

This paper also provides a quantitative assessment by resorting to a numerical analysis, from which two main findings emerge. First, the monetary

[1] Their research indicates that the low innovation firms (i.e., those that have weaker human capital and low knowledge stock) have the motivation to merge with high innovation firms abroad to increase their innovation knowledge, which would consequently increase their values.

authority should choose a higher optimal nominal interest rate to correct the distortions from strengthening patent protection. Second, in response to a lower level of the R&D firm's professional knowledge, the monetary authority should set a higher optimal nominal interest rate as a remedy.

2. *Related Literature*

One of the most salient features of our paper is the setting of an endogenous innovation scale, which leads to the endogenous markup of monopolistic intermediate goods. The step size of quality improvement is usually specified to be exogenous in existing studies. However, this specification regarding the scale of quality improvement does not fit realistic observations. For example, Shenhar (1993) and Robertson and Gatignon (1998) indicate that innovation plans could be classified into four types in terms of their innovation risks, which are low technological uncertainty, medium technological uncertainty, high technological uncertainty, and super high technological uncertainty. The main feature of this classification is that higher risk is associated with a higher return from innovation. Therefore, R&D firms will choose different types of innovation plan according to their capacity. In addition, by using United States Patents and Trademark Office (USPTO) data, Muratori (2020) finds that the quality of entrants' innovation increases over time during the period between 1980 and 2000. Equipped with the Shenhar (1993), Robertson and Gatignon (1998), and Muratori (2020) observation, this paper sets up an R&D-based model that is able to reflect the R&D firm's optimal decision regarding the scale of quality improvement.

There are only a few theoretical studies attempting to deal with the issue of the endogenous innovation scale in the R&D-based growth model. Among these studies, Chu and Pan (2013) introduce the profit-division rule between incumbents and new entrants in a Schumpeterian growth model and examine the effects of blocking patents (leading patent breadth) on economic growth and social welfare. Their analysis assumes that the new entrant will infringe the patent of the incumbent, and hence should transfer a share of its profit to the incumbent. In line with Chu ad Pan (2013), Lu (2022) and Lu et al. (2023) also endogenize the step size of quality improvement by resorting to the presence of blocking patents. In a recent article, Hu et al. (2021) provide another mechanism for the endogenous step size, i.e., R&D firms can increase patent value by hiring more research labor to improve the quality increment. In departing from Chu and Pan (2013), Hu et al. (2021), Lu (2022), and Lu et al. (2023), this paper instead develops the Schumpeterian growth model featuring the lagging patent breadth (i.e., patent breadth against imitation), and proposes an alternatively plausible mechanism to endogenize the step size of quality improvement. To be more specific, based on the empirical finding in Shenhar (1993) and Robertson and Gatignon (1998), this paper endogenizes the step size of innovation by way of the mechanism through which the size of the innovation scale is crucially related to the risk of innovation failure.

Our paper is also related to earlier studies that examine the patent protection-economic growth nexus in the R&D-based growth model. Some empirical studies indicate the non-monotone relationship between patent protection and economic growth. Within the literature, Thompson and Rushing (1996) find that strengthening patent protection will stimulate economic growth only for advanced countries, while it has an insignificant correlation with economic growth for developing countries. Falvey et al. (2006) find that intellectual property rights (IPR) protection is positively related to growth for low- and high-income countries, but not for middle-income countries. In addition, some theoretical studies point out that the strengthening of IPR protection may impede innovation or growth, such as in Goh and Olivier (2002), Horii and Iwaisako (2007), Iwaisako and Futagami (2013), Pan et al. (2018), and Chen (2021). However, these studies remain silent on how patent protection affects growth through the endogenous adjustment in the innovation scale. Our study aims to fill this gap and shows that the endogenous innovation scale is a plausible channel for the emergence of the inverse U-shaped relationship between patent protection and economic growth.

In addition, our paper is related to previous theoretical studies that examine monetary policy and social welfare in the innovation-led growth model. To analyze the effects of monetary policy, we introduce money demand via cash-in-advance (CIA) constraints on R&D investment and consumption in the Schumpeterian growth model, which is in line with the following empirical findings. Hall (1992), Himmelberg and Petersen (1994), Opler et al. (1999), and Brown and Petersen (2009) show that cash flows are positively and significantly related to R&D investment in U.S. firms. Hall et al. (1999) and Brown and Petersen (2011) further point out that the sensitivity of R&D investment-cash flows is stronger than physical investment. Moreover, Bates et al. (2009) show a sharp increase in the average cash-to-assets ratio for U.S. industrial firms during 1980-2006 mainly because of increased R&D expenditures. A recent study by Brown and Petersen (2015) points out that firms with positive R&D investments tend to expend their cash reserves on buffering R&D instead of on protecting fixed investment. The empirical evidence mentioned above reveals that R&D firms will finance their R&D investment via cash holdings as a response to financial frictions. This paper therefore employs a CIA constraint on R&D, in line with Chu and Cozzi (2014), to capture the cash requirements for R&D investment.

Theoretical underpinning studies that analyze the effect of monetary policy on growth and social welfare in the R&D-based growth model have recently been developed, such as Chu and Lai (2013), Chu and Cozzi (2014), Chu et al. (2015), Zheng et al. (2021), and Huang et al. (2021). Perhaps for analytical convenience, these studies unanimously specify that the step size of innovation is constant and, as pointed out previously, the specification of an exogenous innovation scale does not fit the empirical evidence. This paper thus contributes to this strand of the literature by highlighting the importance of endogenous adjustment in the innovation scale, and then shows that the interest rate and social welfare exhibit an inverse U-shaped relationship with an endogenous innovation scale.

The remainder of this paper is organized as follows. Following the review of the related literature in Section 2, Section 3 develops a monetary Schumpeterian growth model with an endogenous innovation scale. Section 4 derives the macroeconomic equilibrium, and examines the effects of the endogenous innovation scale and monetary policy on labor allocation. Section 5 analytically examines the growth effect of the endogenous innovation scale and monetary policy. Section 6 deals with the welfare analysis and provides a numerical analysis. Finally, Section 7 concludes.

3. The model

We consider a Schumpeterian growth model in which growth is driven by innovation that improves the quality of intermediate goods (see, e.g., Grossman and Helpman, 1991). To introduce money demand, following Chu and Cozzi (2014), we impose a CIA constraint on the firm's R&D investment and consumption. The major departure from existing studies is that, in our framework, the R&D firm is allowed to choose a suitable innovation scale (innovation project) after balancing its risks and benefits. To be more specific, we introduce a variable to capture the risk of the different innovation scales. R&D firms that select a larger innovation scale (a high innovation challenge project) will bear a higher risk of innovation.[2] In what follows, we will in turn describe the economy's structure.

3.1 The household

The representative household has N_t members, and the members grow over time at the exogenous rate $n > 0$. By the law of motion, we can write $\dot{N}_t = nN_t$. The representative household derives utility from the consumption of final goods and leisure, and its lifetime utility function can be expressed as

[2] However, in the standard Schumpeterian growth model, the monopolistic intermediate firms engage in Bertrand competition in each industry. Therefore, a larger innovation scale will also result in a higher markup of intermediate goods, and thus a higher return from R&D.

follows:

$$U = \int_0^\infty e^{-\rho t} [\ln c_t + \theta \ln(1-l_t)] \, dt , \tag{1}$$

where c_t is the consumption of final goods per member of a household at time t, and l_t is the supply of labor per member of a household at time t. The parameters $\rho > 0$ and $\theta > 0$ denote the subjective time preference and leisure preference, respectively. The household maximizes lifetime utility (1) subject to the following budget constraint:

$$\dot{a}_t + \dot{m}_t = (r_t - n)a_t + w_t l_t + \tau_t - c_t - (\pi_t + n)m_t + i_t b_t , \tag{2}$$

where a_t denotes the real value of assets (in the form of equity issued by intermediate goods firms) owned by each member of the household, m_t is real money balances held by each member of the household, w_t is the real wage rate,[3] and τ_t is the lump-sum transfer. r_t is the real interest rate, π_t is the inflation rate, i_t is the nominal interest rate, and b_t is the real money balances that each member of the household lends to R&D firms to finance their R&D investment. According to the Fisher equation, the nominal interest rate can be expressed as $i_t = r_t + \pi_t$. Each member of the household holds real money balances m_t which are used partly to consume final goods and partly to lend to R&D firms. The cash-in-advance constraint takes the following form: $\xi c_t + b_t \le m_t$, where $\xi > 0$ is the fraction of consumption subject to the CIA constraint.

Each member of the household maximizes Eq. (1) subject to Eq. (2) and $\xi c_t + b_t \le m_t$, which is binding in equilibrium. The optimum conditions for consumption and labor supply are, respectively, given by:

$$\frac{1}{c_t} - \lambda_t (1 + i_t \xi) = 0 , \tag{3}$$

$$w_t (1 - l_t) = \theta c_t (1 + i_t \xi) , \tag{4}$$

where λ_t is the shadow value of the real wealth (the sum of a_t and m_t) owned by each member of the household. Moreover, the Euler equation for the dynamic optimization of consumption behavior is given by:

$$\frac{\dot{c}_t}{c_t} = r_t - \rho - n_t . \tag{5}$$

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[3] We assume that labor is perfectly mobile across sectors. This implies that all sectors provide the same real wage w_t.

3.2 Final goods

The final goods are produced by competitive firms using a unit continuum of intermediate goods industries indexed by $j \in [0,1]$, according to a standard Cobb-Douglas aggregator. The production function for the final goods is given by:

$$y_t = \exp\left(\int_0^1 \ln x_t(j)\, dj \right), \tag{6}$$

where $x_t(j)$ is the quantity of intermediate good j.

The profit-maximization problem for the final goods firm implies the following conditional demand function for intermediate good j:

$$x_t(j) = \frac{y_t}{p_{x,t}(j)}, \tag{7}$$

where $p_{x,t}(j)$ is the price of $x_t(j)$.

3.3 Intermediate goods

There is a unit continuum of intermediate goods industries indexed by $j \in [0,1]$. Each intermediate good firm is a temporary quality leader in industry j. Thus, it produces the highest-quality intermediate good and enjoys a monopoly position until the next higher-quality innovation takes place. In line with Grossman and Helpman (1991) and Chu and Cozzi (2014), we assume that labor is the only factor involved in the production of intermediate goods. The production function for each intermediate good firm is given by:

$$x_t(j) = z^{q_t(j)} L_{x,t}(j); \quad j \in [0,1], \tag{8}$$

where $L_{x,t}(j)$ denotes the labor input required to produce the intermediate good in industry j at time *t*, $z > 1$ is the step size of the quality improvement,[4] and $q_t(j)$ is the number of the quality improvements in an industry j during the time interval between 0 and t. Notice that, in departing from existing studies in which innovation comes from quality improvement, in this paper the step size of quality improvement z is an endogenous variable. It can be treated as the extent of the innovation chosen by R&D firms.

Based on Eq. (8), the marginal cost of producing an intermediate good is given by:

$$MC_t(j) = \frac{w_t}{z^{q_t(j)}}; \quad j \in [0,1]. \tag{9}$$

[4] In our model, the step size of quality improvement z is an endogenous variable. It could be treated as an innovation plan chosen by R&D firms.

Following the existing studies on quality-improving R&D, we assume that the current and former industry leaders engage in a standard Bertrand price competition. In addition, in line with Li (2001), Iwaisako and Futagami (2013), Iwaisako (2020), and Furukawa et al. (2023), we introduce a policy variable, denoted by $\beta\ (\geq 1/z)$, to capture the extent of the patent breadth. Therefore, the profit-maximizing pricing for the industry leader can be expressed as:

$$p_{x,t}(j) = \beta\, zMC_t\,(j) = \beta\, z \frac{w_t}{z^{q_t(j)}}\,;\ \ \beta \geq 1/z\,. \tag{10}$$

It should be noted that the industry sets its price equal to marginal cost when $\beta = 1/z$ is true. This case is thus associated with zero patent protection.[5]

Equipped with Eq. (8), the monopolistic profit for industry j is given by:

$$\Pi_{x,t}(j) = \left(\frac{\beta z - 1}{\beta z}\right) y_t\,. \tag{11}$$

Eq. (11) shows that a larger patent breadth β increases the amount of monopolistic profit created by innovations.

Moreover, the wage income received by workers in industry j is given by:

$$w_t L_{x,t}(j) = \frac{1}{\beta z} y_t\,. \tag{12}$$

3.4 Research and innovation

Let $v_t(j)$ denote the patent value of an industry $j \in [0,1]$ and $I_t(j)$ denote the Poisson arrival rate of innovation. Following the standard approach of R&D growth model, such as Huang and Ji (2013), Zheng et al. (2020), and Chu et al. (2021), the familiar no-arbitrage condition for $v_t(j)$ is given by:

$$r_t v_t(j) = \Pi_{x,t}(j) + \dot{v}_t(j) - I_t(j) v_t(j). \tag{13}$$

Eq. (13) indicates that the return on innovation $r_t v_t(j)$ is equal to the sum of the monopolistic profit $\Pi_{x,t}(j)$, the capital gains $\dot{v}_t(j)$, and the expected capital loss $I_t(j) v_t(j)$ stemming from creative destruction.

There is a continuum of R&D firms, indexed by $j \in [0,1]$, and each R&D firm employs R&D labor $L_{r,t}(j)$ to create innovations and chooses the size of the innovation scale z to innovate upon the existing products. Once an R&D firm successfully innovates in industry j, it becomes the leader in industry j and produces the new version of good j, whose quality increases in z compared with the best existing version.

The expected profit of the j-th R&D firm $\Pi_{RD,t}(j)$ is:

[5] In the seminal work by Grossman and Helpman (1991), β is set to 1 for simplification.

$$\Pi_{RD,t}(j)=I_t(j)v_t(j)-(1+i_t)w_tL_{r,t}(j)\,. \tag{14}$$

In Eq. (14), $I_t(j)v_t(j)$ is the expected revenue of the R&D firm from investing in innovation and $w_tL_{r,t}(j)$ is the wage payment of the R&D firm. Similar to Christiano et al. (2005) and Neumeyer and Perri (2005), each of the R&D firms has to pay production costs before cashing its output sales. This creates the need for working capital, and the shortage of working capital is funded by the households. As a result, following Chu and Cozzi (2014), the total amount of real money balances that the household lends to the j-th R&D firm to finance R&D investment is equal to $w_tL_{r,t}\left(j\right)$, and the cost of borrowing is $i_tw_tL_{r,t}\left(j\right)$. Thus, the total production cost of R&D is $(1+i_t)w_tL_{r,t}(j)$.

Finally, the *firm-level* arrival rate of innovation $I_t(j)$ is given by:

$$I_t(j)=\frac{\eta L_{r,t}^{\psi-1}\,L_{r,t}(j)}{\kappa N_t^{\psi}}\,, \tag{15}$$

where $L_{r,t}=\int_0^1 L_{r,t}(j)\,dj$, the parameter $\eta>0$ is an innovation productivity parameter of R&D labor, and κ denotes the complexity of innovation. To shed light on the intuition behind Eq. (15), we then rewrite it as:

$$I_t(j)=\frac{\eta}{\kappa}\;\underbrace{(\frac{L_{r,t}}{N_t})^{\psi-1}}_{\substack{\text{Allocation of resrearch}\\ \text{labor to the R\&D sector}}}\;\underbrace{(\frac{L_{r,t}(j)}{N_t})}_{\substack{\text{Allocation of resrearch}\\ \text{labor to industry } j}}\,. \tag{15a}$$

Two points related to the specification in Eq. (15a) should be addressed. First, not only the allocation of research labor to the R&D sector $(L_{r,t}/N_t)$, but also the allocation of research labor to industry j $(L_{r,t}(j)/N_t)$ is crucially related to the arrival rate of innovation of industry j. In line with existing studies, such as Jones (1995), Jones and Williams (2000), Chu and Cozzi (2014), and Chu et al. (2019), the allocation of research labor to the R&D sector is subject to the negative externality of duplication across the industries. As a result, the allocation of research labor to the R&D sector is inversely related to the arrival rate of innovation, where the parameter $\psi\in(0,1)$ reflects the extent of the duplication of innovation. This negative linkage is referred to by Jones and Williams (2000) as the *stepping on toes effect*. By contrast, the allocation of research labor to industry j is confined to the manpower allocated to the single and specific industry j, so that it does not involve cross-industry duplication. Accordingly, a rise in the allocation of research labor to industry j tends to raise the arrival rate of innovation of industry j. One point should be noted here. Our model degenerates to the Dinopoulos and Segerstrom (2010) and Chu and Cozzi (2014) model when the *stepping on the toes effect* is absent (i.e., $\psi = 1$). Armed with this specific assumption, these studies specify that the arrival rate of innovation of industry j is positively related to the allocation of research labor to industry j.

Second, the complexity of innovation leads to a negative effect on the arrival rate of innovation. The complexity of innovation is specified in the form of $\kappa = z^{\phi}$, implying that a larger innovation scale will increase the complexity of innovation, and therefore further decrease the arrival rate of innovation. This specification echoes empirical findings from Shenhar (1993) and Robertson and Gatignon (1998) mentioned in Section 2. In addition, the parameter $\phi > 0$ reflects the sensitivity of an expansion in the innovation scale to the complexity of innovation. Conceptually, the parameter ϕ can be viewed as a proxy for the level of professional knowledge that an R&D firm possesses. An R&D firm with a smaller ϕ indicates that it has a high level of professional knowledge, and hence will experience less complexity in innovation when expanding the innovation scale by one unit. Therefore, the parameter ϕ inversely measures the level of the R&D firm's professional knowledge.

To maximize the expected profit, the entrepreneur faces two decisions: choosing the size of the innovation scale z and hiring the amount of R&D labor $L_{r,t}(j)$. First, we deal with the optimal choice of the innovation scale z. Differentiating the expected profit stated in (14) with respect z yields the following result:[6]

$$\frac{\partial \Pi_{RD,t}(j)}{\partial z} = I_t(j) \underbrace{\frac{\partial v_t(j)}{\partial z}}_{+} + v_t(j) \underbrace{\frac{\partial I_t(j)}{\partial z}}_{-}. \tag{16}$$

Eq. (16) indicates that raising the innovation scale generates two conflicting effects on the expected profit of the R&D firm. On the one hand, it increases the patent value of an industry $v_t(j)$, and hence is beneficial to the expected profit of the R&D firm. On the other hand, a larger size of the innovation scale leads to the higher complexity of innovation, which further decreases the arrival rate of innovation $I_t(j)$, and hence is harmful to the expected profit of the R&D firm. Accordingly, the optimal size of the innovation scale $\tilde{z}$ is set at the value where

[6] From Eqs. (11) and (13), we can infer the result $\partial v_t(j)/\partial z = y_t / \beta(r_t + I_t(j) - \dot{v}_t/v_t) z^2 > 0$. In addition, based on Eq. (15a), we can infer the result $\partial I_t(j)/\partial z = -\phi I_t(j)/z < 0$.

these two conflicting effects are balanced:

$$\tilde{z} = \frac{1+\phi}{\beta\phi}. \tag{17}$$

Eq. (17) shows that a lower level of professional knowledge regarding innovation (a higher value of ϕ) and higher patent protection (a higher value of β) lead to a decline in the optimal size of the innovation scale. Intuitively, the R&D firm with a lower level of professional knowledge faces a higher risk of innovation failure (a lower arrival rate of innovation). Thus, it is inclined to choose a smaller size of innovation scale to avoid innovation failure. Moreover, a rise in patent protection β tends to raise the value of innovation $v_t(j)$. An R&D firm will suffer from a larger expected loss when it chooses a larger size of innovation scale and then experiences innovation failure. To reduce such an expected loss, an R&D firm is motivated to select a smaller size of innovation scale. The above discussions lead to the following proposition:

Proposition 1. *Stronger patent protection and a lower level of professional knowledge of innovation tend to reduce the optimal size of the innovation scale.*

Based on Eq. (17), to ensure that the equilibrium step size of the quality improvement is greater than 1 (i.e., $\tilde{z} > 1$), we impose the following restriction on the parameter of patent protection.

Condition PP (patent protection):

$$\beta < (1+\phi)/\phi. \tag{18}$$

We can then analyze how the complexity of innovation will react following changes in patent protection and professional knowledge of innovation after taking the R&D firm's optimal decision regarding the innovation scale into consideration.

By inserting Eq. (17) into κ, we can obtain $\tilde{\kappa}$, which is the complexity of innovation after the optimal innovation scale ($\tilde{z}$) is determined by the R&D firm:

$$\tilde{\kappa} = \tilde{z}^{\phi} = \left(\frac{1+\phi}{\beta\phi}\right)^{\phi}. \tag{19}$$

It is straightforward from Eq. (19) to infer the following result:

$$\frac{\partial\tilde{\kappa}}{\partial\beta} = \frac{\phi\tilde{\kappa}}{\tilde{z}}\frac{\partial\tilde{z}}{\partial\beta} < 0 \tag{20}$$

Based on Eq. (17), the strengthening of patent protection leads the R&D firm to choose a smaller size of innovation, and hence Eq. (20) reveals that higher patent protection is associated with a lower complexity of innovation.

We then deal with the effect of a change in professional knowledge on the complexity of innovation. Differentiating Eq. (19) with respect to ϕ yields:

$$\frac{\partial \tilde{\kappa}}{\partial \phi} = \tilde{z}^{\phi}\left(\underbrace{\ln \tilde{z}}_{+} + \frac{\phi}{\tilde{z}}\underbrace{\frac{\partial \tilde{z}}{\partial \phi}}_{-}\right) \tag{21}$$

Based on Condition PP and Eq. (17), we can infer the results: $\ln(\tilde{z}) > 0$ and $\partial\tilde{z}/\partial\phi < 0$. Accordingly, Eq. (21) shows that a decline in professional knowledge leads to two conflicting effects on the complexity of innovation. First, a fall in professional knowledge has a direct effect in terms of stimulating the complexity of innovation. Second, a fall in professional knowledge leads the R&D firm to choose a smaller size of innovation scale, and this will reduce the complexity of innovation.

Equipped with Eqs. (17) and (21), we can infer the following result concerning the signs of $\partial\tilde{\kappa}/\partial\phi$:

$$\begin{cases} \dfrac{\partial \tilde{\kappa}}{\partial \phi} > 0 \quad if \quad \dfrac{1+\phi}{\phi}\dfrac{1}{e^{1/(1+\phi)}} > \beta; \\ \dfrac{\partial \tilde{\kappa}}{\partial \phi} < 0 \quad if \quad \dfrac{1+\phi}{\phi} > \beta > \dfrac{1+\phi}{\phi}\dfrac{1}{e^{1/(1+\phi)}}. \end{cases} \tag{22}$$

Eq. (22) indicates that the extent of patent protection plays a crucial role in determining the signs of $\partial\tilde{\kappa}/\partial\phi$. More specifically, if β is relatively small, the first positive effect dominates the second negative effect in Eq. (21), thereby yielding the result $\partial\tilde{\kappa}/\partial\phi > 0$. On the contrary, if β is relatively large, the first effect falls short of the second effect, so that $\partial\tilde{\kappa}/\partial\phi < 0$ holds.

The above discussions lead us to establish the following proposition:

Proposition 2. *A change in professional knowledge of innovation has an ambiguous effect on the complexity of innovation, crucially depending upon the extent of patent protection.*

Finally, we introduce the free-entry condition in the R&D sector, which implies that the following zero-expected-profit condition is satisfied:

$$v_t(j) = \frac{\tilde{z}^{\phi}(1+i_t)w_t N_t^{\psi}}{\eta L_{t,t}^{\psi-1}} \tag{23}$$

This equation is used for pinning down the allocation of the R&D labor $L_{r,t}(j)$.

3.5 Government and monetary authority

Let M_t denote the nominal money supply, P_t denote the price of the final goods, and $m_t = (M_t/P_t)/N_t$ denote real money balances per capita. Based on the definition of $m_t N_t = M_t/P_t$, the evolution of m_t can then be expressed as:

$\dot{m}_t/m_t = \dot{M}_t/M_t - n - \pi_t$, where $\pi_t \equiv \dot{P}_t/P_t$ is the inflation rate of the price of final goods. The monetary policy instrument that we consider is i_t which is exogenously chosen by the monetary authority.

The government finances its lump-sum transfer payments for each member of the household by issuing money. The balanced budget constraint faced by the government can then be expressed as: $\dot{M}_t/P_t = \tau_t N_t$. Given the definition $m_t = (M_t/P_t)/N_t$, the government's budget constraint can then be alternatively written as: $\dot{m}_t + (\pi_t + n)m_t = \tau_t$.

4. Decentralized equilibrium

The equilibrium is a time path of allocation $\{c_t, l_t, m_t, y_t, x_t(j), z, L_{x,t}, L_{r,t}\}_{t=0}^{\infty}$, a time path of prices $\{w_t, p_t(j), v_t, r_t, i_t\}_{t=0}^{\infty}$ and policies $\{i_t\}_{t=0}^{\infty}$. At each instant of time:

- households maximize lifetime utility taking $\{i_t, r_t, w_t\}$ as given;
- competitive final-goods firms produce $\{y_t\}$ and choose $\{x_t(j)\}$ to maximize profit taking $\{p_{x,t}(j)\}$ as given;
- monopolistic intermediate-goods firms produce $\{x_t(j)\}$ and choose $\{p_{x,t}(j), L_{x,t}(j)\}$ to maximize profit taking $\{w_t\}$ as given;
- R&D firms choose $\{z, L_{r,t}\}$ to maximize the expected profit taking $\{w_t, i_t, v_t\}$ as given;
- the government budget constraint is balanced such that $\dot{m}_t + (\pi_t + n)m_t = \tau_t$.
- the final goods market clears such that $y_t = c_t N_t$;
- the labor market clears such that $N_t l_t = L_{x,t} + L_{r,t}$;
- the amount of money borrowed by R&D firms is $b_t N_t = w_t L_{r,t}$.

In Appendix A, we show that the dynamic system has one positive characteristic root coupled with one jump variable. Therefore, the economy will jump immediately to a unique and stable balanced growth path. This result can be summarized in the following lemma:

Lemma 1. *The economy always jumps immediately to a unique and stable balanced growth path.*

Proof. See Appendix A. ■

4.1 Equilibrium labor allocation

In this subsection, we deal with the equilibrium labor allocation along the balanced growth path. To make the analysis tractable and clear, we assume that there is no negative duplication externality (i.e., $\psi = 1$). However, this assumption will be relaxed later in the social welfare analysis section. Under this assumption, we rewrite the market-clearing condition for labor in per capita terms as follows:

$$l_t = l_x + l_r \,, \tag{24}$$

where $l_r = L_{r,t}/N_t$ denotes the per capita R&D labor input, and $l_x = L_{x,t}/N_t$ denotes the per capita labor input required to produce intermediate goods.

Along the balanced growth path, inserting (11), (12), (15) and (23) into (13) yields:

$$(\beta\tilde{z}-1)l_x = (1+i_t)(l_r + \frac{\tilde{z}^{\phi}\rho}{\eta}) \,. \tag{25}$$

Finally, substituting Eq. (12) and $y_t = c_t N_t$ into (4), we obtain:

$$l_t = 1 - \beta\tilde{z}\theta(1+i_t\xi)l_x \,. \tag{26}$$

From Eqs. (24), (25) and (26), we can solve the equilibrium labor allocation, which is described by the following lemma:

Lemma 2. The equilibrium labor allocation is given by:

$$\tilde{l}_r = \frac{(\beta\tilde{z}-1)-(1+i_t)\dfrac{\tilde{z}^{\phi}\rho}{\eta}[1+\beta\tilde{z}\theta(1+i_t\xi)]}{(\beta\tilde{z}-1)+(1+i_t)[1+\beta\tilde{z}\theta(1+i_t\xi)]} \,, \tag{27}$$

$$\tilde{l}_x = \frac{(1+i_t)\left(1+\dfrac{\tilde{z}^{\phi}\rho}{\eta}\right)}{(\beta\tilde{z}-1)+(1+i_t)[1+\beta\tilde{z}\theta(1+i_t\xi)]} \,, \tag{28}$$

$$\tilde{l} = \frac{(\beta\tilde{z}-1)-(1+i_t)\left[\dfrac{\tilde{z}^{\phi}\rho}{\eta}\beta\tilde{z}\theta(1+i_t\xi)-1\right]}{(\beta\tilde{z}-1)+(1+i_t)[1+\beta\tilde{z}\theta(1+i_t\xi)]} \,, \tag{29}$$

where $\tilde{z} = (1+\phi)/\beta\phi$ reported in Eq. (16), and $\tilde{l}_r$, $\tilde{l}_x$ and $\tilde{l}$ are the steady-state values of l_r, l_x, and l, respectively.

4.2 Comparative statics of research labor allocation

This subsection discusses the effect of patent protection, monetary policy, and the R&D firm's professional knowledge on the allocation of research labor in the steady state. Consider first the effect of patent protection on the allocation of research labor. Differentiating (27) with respect to β yields:

$$\frac{\partial\tilde{l}_r}{\partial\beta} = \frac{\dfrac{(1+\phi)\tilde{z}^{\phi-1}}{\beta^2}(1+i_t)\left(\dfrac{\rho}{\eta}\right)\left[1+\dfrac{\theta(1+i_t\xi)(1+\phi)}{\phi}\right]}{\dfrac{1}{\phi}+(1+i_t)\left[1+\dfrac{\theta(1+i_t\xi)(1+\phi)}{\phi}\right]} > 0 \,. \tag{30}$$

Eq. (30) denotes the positive linkage between patent protection and research labor allocation. The result in Eq. (30) leads us to establish the following proposition:

Proposition 3. *Strength of patent protection is positively related to the equilibrium allocation of research labor.*

The intuition behind Proposition 3 can be explained as follows. In the steady state, strengthening patent protection has three effects on the allocation of research labor. The first effect is that strengthening patent protection (i.e., an increase in β) enables each of the intermediate firms to charge a higher markup $\beta\tilde{z}$, as exhibited in Eq. (10). This will increase the profit of intermediate firms, and in turn raise the patent value of R&D firms. Then, the R&D firms are inclined to employ more research labor.

The second effect is that, in response to strengthening patent protection, the R&D firms will choose a smaller size of innovation scale (i.e., a decline in $\tilde{z}$), which will reduce the markup $\beta\tilde{z}$. With the same reasoning (but just the opposite) as in the first effect, the R&D firms will tend to employ less research labor.

The third effect is related to the R&D firms' choice of a smaller size of innovation scale in response to stronger patent protection, mentioned in the second effect. A smaller innovation scale implies a reduction in the complexity of innovation, which will lead the R&D firms to have a higher expected profit. Thus, the R&D firms will have an incentive to hire more research labor.

Considering all three effects together, as demonstrated in Eq. (30), the two positive effects on the allocation of research labor dominate the negative effect. Thus, we can infer that strengthening patent protection will stimulate the allocation of labor to R&D.

We next examine the relationship between monetary policy and the allocation of research labor. Differentiating (27) with respect to i_t yields:

$$\frac{\partial \tilde{l}_r}{\partial i_t} = \frac{-\left\{\dfrac{\tilde{z}^{\phi}\rho}{\eta}\left[F + \dfrac{(1+i_t)(1+\phi)\theta\xi}{\phi}\right]\left[\dfrac{2}{\phi} + (1+i_t)F(1-\dfrac{\tilde{z}^{\phi}\rho}{\eta})\right]\right\}}{\left\{\dfrac{1}{\phi} + (1+i_t)F\right\}^2} < 0 . \qquad (31)$$

where $F = 1 + [\theta(1+i_t\xi)(1+\phi)/\phi]$. To make our analysis meaningful, we impose the restriction that the allocation of labor to R&D is positive (i.e., $\tilde{l}_r > 0$). With this restriction, Eq. (31) indicates that an increase in the nominal interest rate leads to a reduction in the allocation of R&D labor. Intuitively, increasing the nominal interest rate raises the working capital costs for R&D firms, and thus R&D firms will hire less research labor. The above discussion leads to the following proposition:

Proposition 4. *The equilibrium allocation of research labor is decreasing in the nominal interest rate.*

Finally, we examine how the professional knowledge of R&D firms affects the allocation of research labor. Differentiating (27) with respect to ϕ yields:

$$\frac{\partial \tilde{l}_r}{\partial \phi} = \frac{-1}{\phi^2}(1+i_t)\left(1+\frac{\rho\kappa}{\eta}\right)\left[1+\theta\left(1+i_t\xi\right)\right] - \frac{\rho(1+i_t)F}{\eta\left[\dfrac{1}{\phi}+(1+i_t)F\right]}\underset{?}{\frac{\partial \kappa}{\partial \phi}}. \tag{32}$$

Eq. (32) shows that a decline in the professional knowledge regarding the innovation of the R&D firm (a rise in ϕ) has an ambiguous effect on the allocation of labor to R&D. Intuitively, a fall in the R&D firm's professional knowledge generates two effects on the allocation of research labor. First, the R&D firm will take action to choose a smaller size of innovation scale, thereby causing the intermediate firm to charge a lower markup. This will reduce the profit of the intermediate firm, and in turn decrease the patent value of the R&D firm. Consequently, the R&D firm will employ less research labor in response. Second, a decline in the R&D firm's professional knowledge will affect the complexity of innovation $\left(\partial\tilde{\kappa}/\partial\phi\right)$. As exhibited in Proposition 2, this effect is ambiguous. If $\partial\tilde{\kappa}/\partial\phi \geq 0$ holds, a decline in professional knowledge regarding innovation will increase the complexity of innovation, and this will tend to reduce the expected profit of the R&D firm. The R&D firm will thus decrease its hired research labor. The reverse case will also apply, for if $\partial\tilde{\kappa}/\partial\phi < 0$ holds, the opposite result is true.

By adding the first and second effects mentioned above, we can infer the following two results. First, if $\partial\tilde{\kappa}/\partial\phi \geq 0$ is true, a decline in professional knowledge regarding innovation will lead the R&D firm to employ less research labor. On the contrary, if $\partial\tilde{\kappa}/\partial\phi < 0$ holds, then the relationship between the level of the R&D firm's professional knowledge and research labor allocation will be uncertain, depending upon the relative size because of the first and second effects. The above discussions can be summarized by the following proposition:

Proposition 5. *The level of the R&D firm's professional knowledge is ambiguously related to the research labor allocation in the steady state.*

5. Growth effect

This section examines how patent protection, monetary policy, and the professional knowledge of R&D firms affect economic growth. To deal with this issue, we start by deriving the steady-state equilibrium economic growth rate. Substituting the production function for each intermediate good firm reported in Eq. (8) into the production function for final goods in Eq. (6) yields:

$$y_t = L_{x,t} Z_t \,; \quad Z_t = \exp\left(\int_0^1 q_t(j)dj \ \ln(z)\right), \tag{33}$$

where Z_t is the aggregate technology. By applying the law of large numbers in the balanced growth equilibrium, the aggregate technology Z_t can be further expressed as:

$$Z_t = \exp\left(\int_0^1 q_t(j)dj \ \ln(\tilde{z})\right) = \exp\left(\int_0^t I_s \ ds \ln(\tilde{z})\right). \tag{33a}$$

Based on Eqs. (15), (33) and (33a) as well as the market-clearing condition for the final goods market, we can obtain the steady-state equilibrium economic growth rate as:

$$\tilde{g} = \frac{\dot{Z}_t}{Z_t} = \underbrace{\frac{\eta}{\tilde{\kappa}}}_{(\text{g1})} \ \underbrace{\tilde{l}_r}_{(\text{g2})} \ \underbrace{\ln(\tilde{z})}_{(\text{g3})} \ . \tag{34}$$

Eq. (34) indicates that the economic growth rate is composed of three channels which are dubbed (g1), (g2), and (g3), respectively. The first channel (g1) reveals the effect of the complexity of innovation on the economic growth rate. An increase in the complexity of innovation (a rise in $\tilde{\kappa}$) implies a reduction in the arrival rate of innovation, and thus is detrimental to economic growth. The second channel (g2) represents the effect of the research labor input on the economic growth rate. Increasing the research labor input (a rise in $\tilde{l}_r$) will increase the arrival rate of innovation, and will therefore stimulate economic growth. The third channel (g3) exhibits the effect of the size of the innovation scale on economic growth. An expansion in the size of the innovation scale (a rise in $\ln(\tilde{z})$) will improve the aggregate technology and, as a result, is beneficial to economic growth. The last two channels (g2 and g3) of Eq. (34) are already being developed in standard Schumpeterian growth models,[7] whereas the first channel (g1) is main contribution of this paper and is not put forth in the literature.

One point must be particularly emphasized here. When the innovation step size is exogenous ($\tilde{z}$ is a constant value), both channels (g1) and (g3) remain intact due to $\tilde{\kappa} = \tilde{z}^{\phi}$. Under such a situation, the relevant policies will affect the balanced growth rate only through the second channel (g2). As a result, with an

[7] See, for instance, Segerstrom (1998), Chu and Lai (2013), Minniti et al. (2013), Chu and Cozzi (2014), and Huang et al (2022).

endogenous innovation scale, two additional channels are brought into the picture in our analysis.

5.1 Patent protection and economic growth

This subsection analyzes the impact of patent protection on economic growth. Differentiating (34) with respect to β gives rise to:

$$\frac{\partial \tilde{g}}{\partial \beta} = \tilde{g}\left[\frac{-1}{\tilde{\kappa}} \underbrace{\frac{\partial \tilde{\kappa}}{\partial \beta}}_{-} + \frac{1}{\tilde{l}_r} \underbrace{\frac{\partial \tilde{l}_r}{\partial \beta}}_{+} + \frac{1}{\tilde{z}\ln(\tilde{z})} \underbrace{\frac{\partial \tilde{z}}{\partial \beta}}_{-} \right] \gtrless 0 . \tag{35}$$

Eq. (35) shows that strengthening patent protection has an ambiguous effect on economic growth. Specifically, strengthening patent protection has three effects on economic growth. First, as exhibited in Eqs. (19) and (20), strengthening patent protection tends to lower the complexity of innovation ($\partial\tilde{\kappa}/\partial\beta < 0$ with $\tilde{\kappa} = \tilde{z}^{\phi} = [(1+\phi)/\beta\phi]^{\phi}$). This in turn leads to an increase in the arrival rate of innovation, and hence will stimulate economic growth. Second, strengthening patent protection will increase the research labor input $(\partial\tilde{l}_r/\partial\beta > 0)$, as shown in Proposition 3, and will thus be beneficial to economic growth. Finally, as pointed out in Eq. (17) and Proposition 1, along the balanced growth path, strengthening patent protection results in a smaller size of innovation scale (i.e., $\partial\tilde{z}/\partial\beta < 0$). This will lower the aggregate technology, and hence is harmful to economic growth. Accordingly, with two positive effects and one negative effect, strengthening patent protection thus generates an ambiguous impact on economic growth. Compared to the previous studies, this paper provides a new plausible vehicle to explain this ambiguous relationship (see Section 2). The foregoing discussion leads to the following proposition:

Proposition 6. *The effect of patent protection on economic growth is uncertain.*

5.2 Monetary policy and economic growth

This subsection deals with the effect of monetary policy on economic growth. As mentioned above, the monetary authority implements its monetary policy by targeting the nominal interest rate i_t. Then, differentiating (34) with respect to i_t yields:

$$\frac{\partial \tilde{g}}{\partial i_t} = \eta \frac{1}{\tilde{\kappa}} \ln(\tilde{z}) \underbrace{\frac{\partial \tilde{l}_r}{\partial i_t}}_{-} < 0 . \tag{36}$$

Eq. (36) shows that an increase in the nominal interest rate will reduce the balanced economic growth rate. As shown in Proposition 4, a rise in the nominal interest rate will increase the working capital cost of R&D firms. Thus, R&D firms have an incentive to hire less research labor, which is detrimental to the economic growth rate. One point deserves special mention here. As indicated in Eq. (17), a rise in the nominal interest rate has no effect on the optimal innovation size, and therefore the growth result of monetary policy is similar to Chu and Cozzi (2014) in this endogenous innovation size model.[8] However, in Section 6 below, we will show that the channel of endogenous innovation scale is very crucial for the welfare effect of monetary policy and the validity of the Friedman rule. The result in Eq. (36) leads us to establish the following proposition:

Proposition 7. *The balanced growth rate decreases with the nominal interest rate.*

5.3 Professional knowledge of innovation and economic growth

In this subsection, we examine how the level of professional knowledge of the R&D firm affects economic growth. Differentiating (34) with respect to ϕ gives rise to:

$$\frac{\partial \tilde{g}}{\partial \phi} = \tilde{g}\left[\frac{-1}{\tilde{\kappa}}\underbrace{\frac{\partial \tilde{\kappa}}{\partial \phi}}_{?} + \frac{1}{\tilde{l}_r}\underbrace{\frac{\partial \tilde{l}_r}{\partial \phi}}_{?} + \frac{1}{\tilde{z}\ln(\tilde{z})}\underbrace{\frac{\partial \tilde{z}}{\partial \phi}}_{-}\right] \gtrless 0 . \tag{37}$$

Eq. (37) shows that the relationship between the R&D firm's professional knowledge regarding innovation and economic growth is ambiguous. As indicated in Eq. (37), a decline in the R&D firm's professional knowledge (i.e., an increase in ϕ) will affect the balanced growth rate through three channels. The first channel is the complexity of innovation. As shown in Proposition 2, a decline in the R&D firm's professional knowledge exerts an ambiguous effect on the complexity of innovation ($\partial \tilde{\kappa} / \partial \phi \lessgtr 0$). Thus, the first channel (i.e., $(-1/\tilde{\kappa})(\partial \tilde{\kappa}/\partial \phi)$) leads to an ambiguous impact on economic growth. The second channel is the research labor allocation. As exhibited in Proposition 5, a reduction in the R&D firm's professional knowledge has an ambiguous effect on research labor ($\partial \tilde{l}_r / \partial \phi \gtrless 0$) and, as a result, the second channel (i.e., $(1/\tilde{l}_r)(\partial \tilde{l}_r/\partial \phi)$) exerts an uncertain impact on economic growth. The third channel is the step size of the quality improvement. Based on Eq. (17), the R&D firm will choose the smaller step size of the innovation scale in association with a lower degree of

[8] It should be noted that even though in Eq. (34) the second channel of the growth effect $\tilde{l}_r$ is similar to Chu and Cozzi (2014), in our endogenous innovation scale model $\tilde{l}_r$ is crucially related to $\tilde{z}$ and $\tilde{\kappa}$, which are endogenous variables and have to do with the patent policy and the professional knowledge of R&D firms.

professional knowledge (i.e., $\partial\tilde{z}/\partial\phi < 0$). This will result in a smaller aggregate technology, and therefore stifle economic growth. Accordingly, putting these three channels together yields an ambiguous linkage between the R&D firm's professional knowledge of innovation and economic growth. The result leads us to establish the following proposition:

Proposition 8. *The level of the R&D firm's professional knowledge regarding innovation is ambiguously related to economic growth.*

6. Quantitative analysis of monetary policy and social welfare

In this section, we analyze the effect of monetary policy on social welfare. Given that the social welfare function derived later is rather complex, it is very difficult for us to provide a closed-form solution to solve how the welfare level is affected in response to the monetary policy. We thus need to resort to a numerical analysis. In addition, to make our numerical analysis more general, we return to our general theoretical setting and consider the effect of a negative duplication externality, $0 < \psi < 1$, on the arrival rate of innovation.

Substituting the optimal values of consumption and labor supply into Eq. (1), the social welfare function (i.e., the indirect lifetime utility of the representative household) $\tilde{U}$ is given by:

$$\tilde{U} = \frac{1}{\rho}\left[\ln(c_0) + \frac{\tilde{g}}{\rho} + \theta\ln(1-\tilde{l})\right], \tag{38}$$

where c_0 is the initial consumption. Using the market-clearing condition for final goods, (6), and (8), we have $c_0 = Z_0 l_x$, where $Z_0 = \exp(\int_0^1 q_0(j)dj\ln(z))$. In line with Dinopoulos and Segerstrom (2010), we assume that $q_0(j) = 0$, and thus we can infer the results $Z_0 = 1$ and $c_0 = l_x$. From (6), (8), (15) and the market-clearing condition for final goods, we can show derive that, when the negative duplication externality is brought into the picture, the balanced growth rate reported in Eq. (34) should be modified as follows:

$$\tilde{g} = \frac{\dot{Z}_t}{Z_t} = \frac{\eta}{\tilde{\kappa}}\tilde{l}_r^{\psi}\ln(\tilde{z}). \tag{39}$$

Finally, inserting $c_0 = l_x$, (39), and the labor market-clearing condition into Eq. (38) yields:

$$\tilde{U} = \frac{1}{\rho}\left[\ln(\tilde{l}_x) + \frac{\eta}{\rho\tilde{\kappa}}\tilde{l}_r^{\psi}\ln(\tilde{z}) + \theta\ln(1-\tilde{l})\right]. \tag{40}$$

By inserting Eqs. (27), (28), and (29) into (40), we can then examine the linkage between the social welfare level and the nominal interest rate and discuss whether the government can choose a positive nominal interest rate that maximizes the $\tilde{U}$ reported in (40). With this examination, we can infer whether the Friedman rule of a zero nominal interest rate may fail to be optimal.

We then use the numerical simulation to evaluate the validity of the Friedman rule and illustrate how the optimal interest rate is related to the relevant parameters. To perform the numerical analysis, we assign the following eight structural parameters $\{\rho, i, \xi, \eta, \beta, \phi, \theta, \psi\}$. The baseline parameters are chosen from the commonly used values in the existing literature or calibrated to match the U.S. empirical data. Following Acemoglu and Akcigit (2012), the discount rate ρ is set to 0.05. In line with Chu and Cozzi (2014), the nominal interest rate i is set to 8%, and the consumption-CIA parameter ξ is set to 0.2. To make the markup lie within the reasonable range estimated across industries (e.g., Norrbin, 1993; Basu, 1996; and Jones and Williams, 2000), the parameter for the level of professional knowledge ϕ is chosen as 3.03, which makes the markup 1.33. In addition, in line with Acemoglu and Akcigit (2012) and Chu and Cozzi (2014), the innovation scale z is set to 1.05, which allows us to pin down the parameter for patent protection, $\beta = 1.267$.

Similar to Chu et al. (2012) and Yang (2018), the empirical long-run growth rate of GDP per capita in the U.S. is 1.5%, which enables us to calibrate the R&D productivity, $\eta = 0.852$. Next, the leisure parameter θ is calibrated to be 0.177 so as to match the per capita labor supply $l = 0.3$. Finally, following Jones and Williams (2000), the parameter for the negative duplication externality ψ is set to 0.5. Table 1 reports the baseline parameter values.

Table 1. Baseline parameters

Parameter	Value	Source/Target
ρ	0.05	Acemoglu and Akcigit (2012)
i	0.08	Chu and Cozzi (2014)
ξ	0.2	Chu and Cozzi (2014)
ψ	0.5	Jones and Williams (2000)
ϕ	3.03	Monopolistic markup $=1.33$

β	1.267	Innovation scale = 1.05
η	0.852	Per capita output growth rate = 1.5%
θ	0.177	Per capita labor supply = 0.3

Figure 1 depicts the effects of patent protection and the R&D firm's professional knowledge on the optimal innovation scale. Strengthening patent protection (i.e., $\beta\uparrow$) and the lower level of the R&D firm's professional knowledge (i.e., $\phi\uparrow$) will cause the R&D firm to choose a smaller innovation scale, which is in line with Eq. (17). Strengthening patent protection raises the cost of an innovation failure. In addition, a lower level of the R&D firm's professional knowledge increases the risk of innovation failure at the same innovation scale. Accordingly, as illustrated in both the left and right panels of Figure 1, in response to a rise in either β or ϕ, the R&D firm is motivated to choose a smaller innovation scale to reduce the risk of innovation failure.

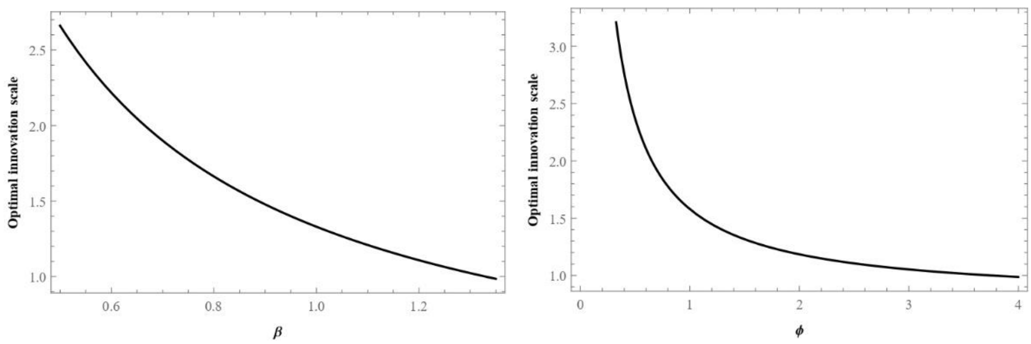

Figure 1. The innovation scale effect of patent protection and the R&D firm's professional knowledge.

Figure 2 shows how patent protection and the R&D firm's professional knowledge affect the complexity of innovation. Strengthening patent protection (i.e., $\beta\uparrow$) leads the R&D firm to choose a smaller innovation scale and thus reduces the complexity of innovation, as illustrated in the left panel of Figure 2. Moreover, as mentioned in Eq. (21), a lower level of the R&D firm's professional knowledge (i.e., $\phi\uparrow$) has two effects on the complexity of innovation. On the one hand, it will directly raise the complexity of innovation at the same innovation scale. On the other hand, the R&D firm is inclined to choose a smaller innovation scale to respond to the lower level of professional knowledge, and this will reduce the complexity of innovation. Accordingly, as exhibited in the right panel of Figure 2, the R&D firm's professional knowledge generates an inverted-U effect on the complexity of innovation depending on the relative size between these two effects.

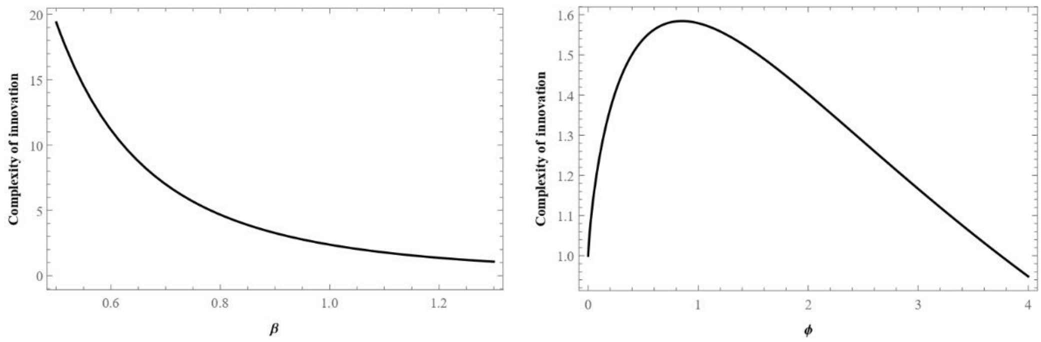

Figure 2. The complexity of innovation effect of the R&D firm's professional knowledge and patent protection.

Figure 3 displays the effects of patent protection and the R&D firm's professional knowledge on economic growth. Strengthening patent protection (i.e., $\beta\uparrow$) reduces the complexity of innovation and increases the research labor input, as shown in Eq. (20) and Eq. (30), both of which tend to stimulate economic growth. On the other hand, strengthening patent protection also decreases the innovation scale, and hence leads to a reduction in the technology improvement for the R&D firm, which is detrimental to economic growth. As a result, taking the channel of the endogenous innovation scale into consideration enables us to show that patent protection and economic growth exhibit an inverted U-shaped relationship, which is illustrated in the right panel of Figure 3.

In addition, a lower level of the R&D firm's professional knowledge (i.e., $\phi\uparrow$) reduces the innovation scale. This in turn reduces the complexity of innovation, which is favorable to economic growth. However, a reduction in the innovation scale also decreases the step size of technology improvement and the research labor input of the R&D firm, both of which are harmful to economic growth. By taking into account all these growth effects, the right panel of Figure 3 reveals that the former positive effect falls short of the latter two negative effects, and hence a lower level of the R&D firm's professional knowledge impedes economic growth.

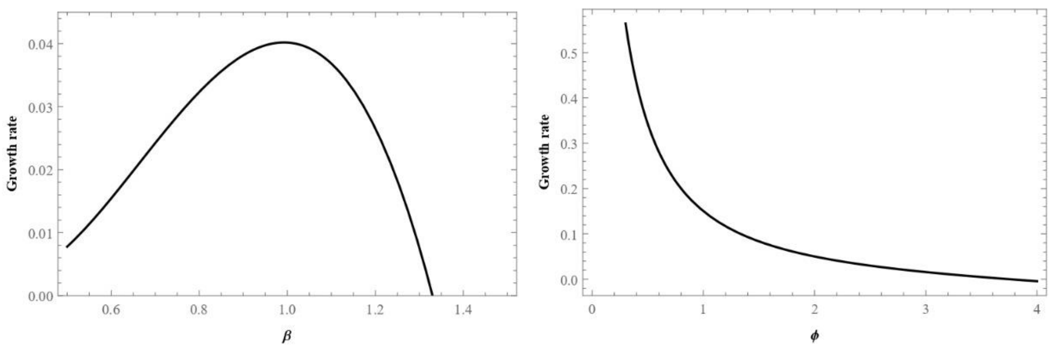

Figure 3. The growth rate effect of the R&D firms' professional knowledge and patent protection.

Figure 4 depicts the effect of monetary policy on economic growth, indicating that increasing the nominal interest rate reduces the economic growth rate. The economic intuition is reported in Subsection 5.2. Raising the nominal interest rate increases the borrowing costs (interest payments) in relation to R&D investment, and thus the R&D firm is inclined to reduce the research labor input. This tends to stifle economic growth.

Figure 5 depicts the effect of monetary policy on social welfare. Based on Eq. (40), an increase in the nominal interest rate has two conflicting effects on social welfare, and hence has a reverse U-shaped relation with social welfare. The intuition behind this result can be explained with the aid of the Segerstrom (1998) insight regarding the linkage between R&D investment and social welfare. Segerstrom (1998) points out that innovation success gives rise to two kinds of distortions.[9] The first distortion is the consumer surplus effect. The final goods sector benefits from improving the production technology once the R&D firm succeeds in innovation. However, the R&D firm does not take this external benefit into account in its profit-maximization decision. This will cause an under-investment in R&D compared to the social optimum. The second distortion is the business stealing effect. The existing intermediate firm will be driven out of business when innovation is successful. However, the loss of the monopolistic profit of the existing intermediate firms will not be considered in the R&D firm's profit-maximization decision. Thus, it will result in an over-investment in R&D compared to the social optimum. Based on our benchmark parameter values, the size of the first distortion (i.e., the under-investment in R&D) falls short of that of the second distortion (i.e., the over-investment in R&D), and hence the net effect leads the economy to be in a state of over-investment in R&D. Accordingly, as exhibited in Figure 5, to correct for this unnecessary R&D investment, the monetary authority should choose a nominal interest rate of 4.44% so as to achieve social welfare maximization. This indicates that the Friedman rule fails to be optimal in view of the social welfare maximum.

[9] Segerstrom (1998) indicates that his analytical framework has a third distortion brought about by the negative external effect on R&D investment. The reason for the presence of this third distortion is that, once innovation succeeds, future innovation becomes more difficult. However, this linkage is not considered in the R&D firm's decision, and will thus cause over-investment in R&D investment compared to the social optimum. This distortion is called the intertemporal R&D spillover effect. In departing from the Segerstrom (1998) analysis, our specification of the arrival rate of innovation $I_t(j)$ in Eq. (15) abstracts from this intertemporal R&D spillover effect.

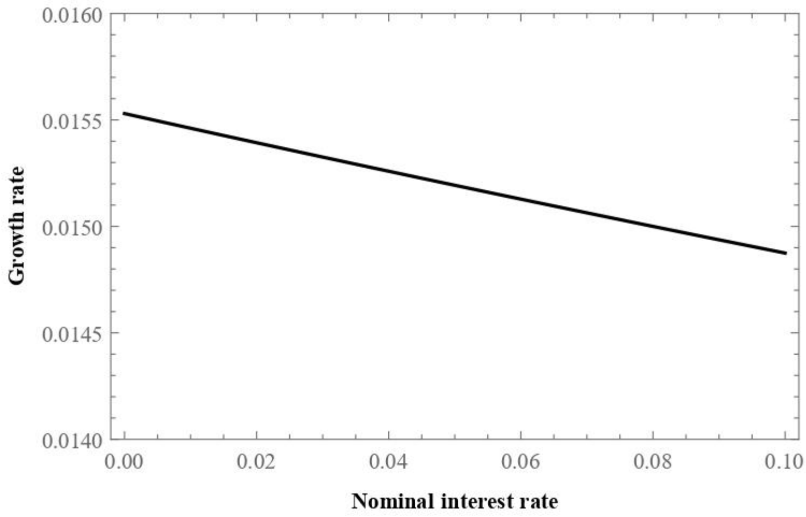

Figure 4. The effect of monetary policy on economic growth.

In what follows in this section, we will deal with how the changes in patent protection and the R&D firm's professional knowledge will affect the optimal nominal interest rate. For expository convenience, in the subsequent analysis the situation exhibited in Figure 5 (in association with the benchmark parameter values) is dubbed the benchmark case.

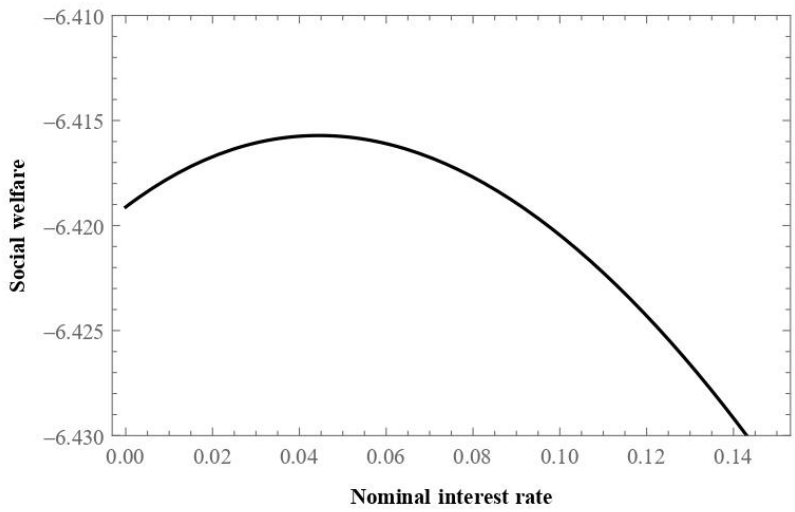

Figure 5. The effect of monetary policy on social welfare.

Figure 6 shows that, in response to a higher patent protection (i.e., $\beta\uparrow$), the monetary authority needs to choose a higher nominal interest rate so as to maximize social welfare. The intuition behind this result can also be explained by resorting to two kinds of distortions arising from R&D investment as mentioned in Figure 5 (i.e., the positive consumer surplus effect and the negative business stealing effect).

Strengthening patent protection reduces the size of the innovation scale, which will affect the size of these two kinds of distortions. On the one hand, the reduction in the innovation scale implies a smaller step size of technology improvement. It will decrease the effect of the external benefit on the final goods sector, and will thus reduce the extent of the consumer surplus effect, thereby lowering the extent of the under-investment in R&D. On the other hand, the decline in the innovation scale also leads to a higher arrival innovation rate and thus raises the expected loss in the monopolistic profit of the existing intermediate firms due to creative destruction. This causes a reduction in the monopolistic profit of the existing intermediate firms, and hence tends to reduce the extent of the business stealing effect. As a result, the extent of the under-investment in R&D is lowered in response.

Based on our benchmark parameter values, the decline in under-investment in R&D dominates that in over-investment in R&D, and hence strengthening patent protection will enlarge the extent of over-investment in R&D compared to the benchmark case in Figure 5. Accordingly, as illustrated in Figure 6, in response to stronger patent protection, the monetary authority will choose a higher optimal nominal interest rate to remedy the higher over-investment in R&D.

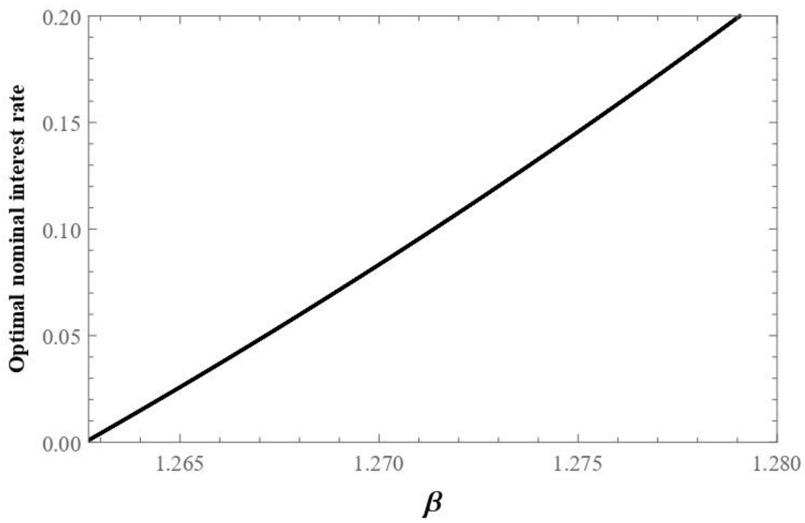

Figure 6. The effect of patent protection on the optimal nominal interest rate.

Figure 7 shows that, following a reduction in the R&D firm's professional knowledge (i.e., $\phi\uparrow$), the monetary authority will tend to choose a higher nominal interest rate so as to maximize social welfare. Similar to the analysis on strengthening patent protection in Figure 6, the intuition behind this result can be grasped by resorting to two kinds of distortion arising from the R&D investment mentioned in Figure 5 (i.e., the consumer surplus effect and the business stealing effect). On the one hand, based on Eq. (17), the R&D firm is motivated to choose the smaller step size of the innovation scale in association with a lower professional knowledge, thereby leading to a smaller step size of technology improvement. This in turn decreases the extent of the external benefit to the final goods sector, and therefore lessens the extent of under-investment resulting from the consumer surplus effect. On the other hand, a reduction in the R&D firm's professional knowledge also leads to a higher arrival innovation rate and thus stimulates the expected loss in the monopolistic profit of the existing intermediate firms due to creative destruction, thereby causing a fall in the monopolistic profit of the existing intermediate firms. This tends to lessen the extent of the business stealing effect, and therefore lowers the extent of the under-investment in R&D.

Equipped with our benchmark parameter values, the decline in under-investment in R&D exceeds that in over-investment in R&D, and hence a fall in the R&D firm's professional knowledge will enlarge the extent of the over-investment in R&D compared to the benchmark case in Figure 5. Accordingly, as exhibited in Figure 7, in response to a lower R&D firm's professional knowledge (i.e., $\phi\uparrow$), the monetary authority is inclined to choose for higher nominal interest rate to correct the higher over-investment in R&D.

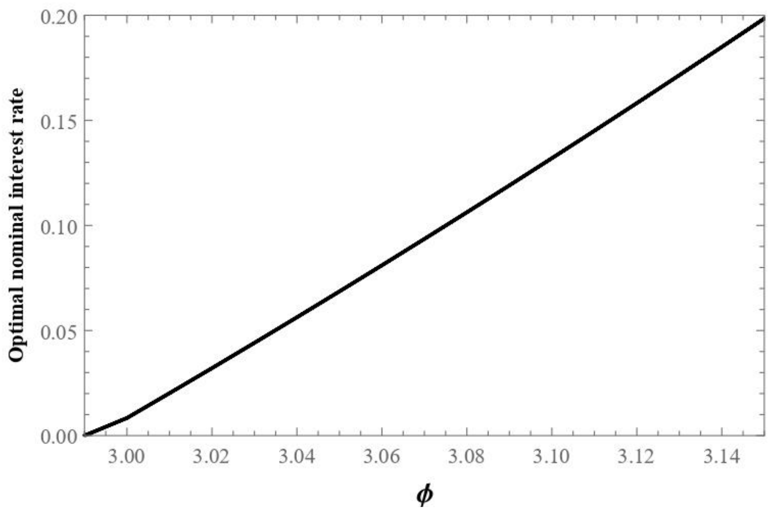

Figure 7. The effect of R&D firms' professional knowledge on the optimal nominal interest rate.

Before ending this subsection, two points deserve special mention here. First, compared with existing studies on the monetary R&D growth model (e.g., Chu and Cozzi, 2014; Chu et al., 2015; Chu et al., 2019), we not only implement a numerical analysis but also provide comprehensive economic intuition regarding how the changes in patent protection and the R&D firm's professional knowledge affect the optimal nominal interest rate. Second, compared with previous studies (Segerstrom, 1998; Li, 2003; Minniti et al., 2013) on social welfare analysis, by proposing the channel of the endogenous innovation scale, this paper finds that patent protection and the R&D firm's professional knowledge will affect not only the consumer surplus effect, but also the business stealing effect via the endogenous innovation scale. Our analysis thus provides a new insight into the social welfare implications.

7. Conclusion

This paper builds up the monetary Schumpeterian growth model which features an endogenous innovation scale. Based on this model, we examine how the endogenous innovation scale governs the effect of patent protection and monetary policy on economic growth and social welfare.

An important finding of our analysis is that when the R&D firms have a high level of professional knowledge, they are willing to choose a high innovation challenge project (i.e., a larger innovation scale) to raise the patent value of R&D. Nevertheless, the relationship between the R&D firms' professional knowledge and economic growth is ambiguous because it depends on the trade-off between the risk of innovation failure and the step size of the technology improvement. On the other hand, the R&D firms will choose a conservative innovation plan (i.e., a smaller innovation scale) to respond to the strengthening patent protection. To be specific, strengthening patent protection means a higher patent value of R&D, and thus these firms will be less willing to bear a higher risk of innovation failure. Interestingly, the strengthening patent protection may impede economic growth, which is quite different from existing studies such as Li (2001), Futagami and Iwaisako (2007), and Chu and Cozzi (2018).

Finally, we examine the effects of monetary policy on economic growth and social welfare. Increasing the nominal interest rate will impede economic growth. However, the effect of the nominal interest rate on social welfare is ambiguous. Hence, this paper employs a numerical simulation to evaluate the optimal nominal interest rate. We show that the optimal nominal interest rate is positive, which means that the Friedman rule is not optimal from the viewpoint of social welfare maximization. More specifically, if the R&D firm lacks professional knowledge or if the government strengthens patent protection, it will be more likely that the Friedman rule does not hold.

Although the model developed in this paper allows us to comprehend the interplay between the behavior of R&D firms and monetary policy, some issues are left open for future research. For instance, we could consider extending our closed-economy R&D-based growth model to one that is open (see Dinopoulos and Segerstrom, 2010, and Iwaisako and Tanaka, 2017). This extension would enable us to discuss a case where R&D firms in different countries have distinct levels of professional knowledge. In this case, it is also worth investigating how the R&D firms' behavior in the foreign country will affect the growth and welfare effects of monetary policy and the trade policy in the domestic country.

Appendix A

Proof of Lemma 1. Substituting Eq. (12) into Eq. (23) yields:

$$l_r^{1-\psi} = \frac{l_x v_t \beta \tilde{z} \eta}{y_t (1+i) \tilde{z}^{\phi}}, \tag{A1}$$

where $\tilde{z} = (1+\phi)/\beta\phi$. Equipped with Eq. (13), the economy's resource constraint $y_t = c_t N_t$, and Eq. (5), the law of motion for l_r is given by:

$$(1-\psi)\frac{\dot{l}_r}{l_r} = \frac{\dot{l}_x}{l_x} + \rho + I_t - \frac{\Pi_{x,t}}{v_t}. \tag{A2}$$

From Eqs. (11), (15), and (23), we can rewrite Eq. (A2) as follows:

$$(1-\psi)\frac{\dot{l}_r}{l_r} = \frac{\dot{l}_x}{l_x} + \rho + \frac{\eta}{\tilde{z}^{\phi}}\left(l_r^{\psi} - \frac{(\beta\tilde{z}-1)}{(1+i_t)} l_x l_r^{\psi-1} \right). \tag{A3}$$

We then derive the relationship between l_x and l_r. Combining the market-clearing condition for labor, Eq. (4), and Eq. (12) together yields:

$$l_x = \left[1+\theta(1+i_t\xi)\beta\tilde{z}\right]^{-1}(1-l_r). \tag{A4}$$

From Eq. (A4), the law of motion for l_x is given by:

$$\frac{\dot{l}_x}{l_x} = \frac{-l_r}{1-l_r}\frac{\dot{l}_r}{l_r}. \tag{A5}$$

Substituting Eqs. (14) and (A5) into Eq. (A3) yields:

$$\dot{l}_r = \left(1-\psi+\frac{l_r}{1-l_r}\right)^{-1}\left[\rho l_r + \frac{\eta(1+\Omega)l_r^{\psi+1}}{\tilde{z}^{\phi}} - \frac{\eta\Omega l_r^{\psi}}{\tilde{z}^{\phi}}\right], \tag{A6}$$

where $\Omega = \eta(\beta\tilde{z}-1)/\left\{\tilde{z}^{\phi}(1+i_t)\left[1+\theta(1+i_t\xi)\beta\tilde{z}\right]\right\}$. In the steady state, the $\dot{l}_r$ is equal to zero as follows:

$$\rho + \frac{\eta(1+\Omega)\tilde{l}_r^{\psi}}{\tilde{z}^{\phi}} - \frac{\eta\Omega\tilde{l}_r^{\psi\text{-}1}}{\tilde{z}^{\phi}} = 0. \tag{A7}$$

From Eq. (A7), we can solve the equilibrium research labor allocation $\tilde{l}_r > 0$. Then, we linearize the Eq. (A6) around the steady-state equilibrium. Differentiating the (A6) with respect to l_r yields:

$$\frac{\partial\dot{l}_r}{\partial l_r} = \left(1-\psi+\frac{\tilde{l}_r}{1-\tilde{l}_r}\right)^{-1}\left[\rho\tilde{l}_r + \frac{(1+\psi)(1+\Omega)\tilde{l}_r^{\psi}}{\tilde{z}^{\phi}} - \frac{\psi\eta\ \Omega\tilde{l}_r^{\psi-1}}{\tilde{z}^{\phi}}\right] > 0. \tag{A8}$$

Based on Eq. (A7), we obtain $\partial\dot{l}_r/\partial l_r > 0$. Since l_r is a jump variable and $\partial\dot{l}_r/\partial l_r > 0$, l_r will jump to its steady-state value and the economy will exhibit a unique and stable balanced growth path.

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