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Abstract

This study examines how the long-run growth rate of per capita income is determined when population growth is negative. It uses the augmented Solow growth model as a tool for this investigation. The results reveal four distinct types of dynamics, depending on the parameter combinations. In all these dynamics, the long-run growth rate of per capita income remains positive. This finding implies that sustainable growth in per capita income is achievable, even in the context of negative population growth.

Keywords: augmented Solow model; human capital accumulation; declining population

JEL Classification: J11; O15, O41

1 Introduction

The phenomenon of population decline is becoming a global issue. Countries such as Germany and Italy have already experienced this decline, and Japan has been witnessing a continuous decrease in population since 2010. The United Nations World Population Prospects 2019 indicates that high-income economies, as classified by the

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World Bank, are projected to see a population decline post-2050, and middle-income economies are expected to follow suit after 2075. Given these circumstances, there is a growing emergence of economic growth models that take into account population decline.[1]

Christiaans (2011) develops a Solow model that incorporates increasing returns to scale due to a positive externality with capital accumulation, showing that the long-run growth rate of per capita income can remain positive, even if the population growth rate is negative. This result is possible because the effect of capital deepening becomes more powerful when the absolute value of the population decline rate is sufficiently large. Sasaki and Hoshida (2017) apply an R&D growth model, following the approach of Jones (1995), and consider negative population growth. They discover that while R&D activities may stagnate as the population decreases, the effect of capital deepening intensifies, leading to positive growth in per capita income.[2] In these models, when the rate of population decline is high, the capital stock per effective labor continues to rise, meaning capital deepening occurs. Consequently, the balanced growth path (BGP) typically seen in growth models does not exist. However, owing to decreasing returns in relation to capital in the production function, the growth rate of capital stock per effective labor decreases and converges to a positive value. Consequently, the growth rate of per capita income also converges to a positive value. This is a growth path specific to a negative population growth economy (NPGP: negative population growth path).

The aforementioned studies consider the accumulation of physical capital and the progress of endogenous technology, but do not consider the accumulation of human capital. Elgin and Tumen (2012) incorporate a Lucas (1988) style of human capital accumulation into a continuous time growth model by Barro and Sala-i-Martin (2003, ch. 9). This model endogenizes population growth and explores the relationship between the endogenously determined rate of population growth and the rate of per capita income growth. Bucci (2023) introduces a Lucas (1988) style of human capital accumulation into an R&D growth model, following the style of Romer (1990) and Jones (1995). This study investigates the relationship between the exogenously given rate of population growth and the rate of per capita income growth. Both studies

[1]Sasaki (2023) has considered negative population growth in Prettner's (2019) model, which introduces automation capital (i.e., robots and AI) into the Solow (1956) model. Sasaki shows that, under plausible population decline, sustained growth of per capita income can be achieved if households maintain a high enough saving rate.

[2]Jones (2022) presents an R&D growth model that endogenizes the population growth rate, but omits capital accumulation. He shows that when population growth is negative, sustained growth of per capita income is unattainable because R&D activities stagnate.

conclude that, under certain conditions, the long-run per capita income growth rate can be positive.

The two studies mentioned above focus their analysis on the BGP, where the primary variables in models consistently increase at a uniform constant growth rate. Consequently, along the BGP, ratios of variables such as the output-capital ratio or capital stock per effective labor remain constant.[3] In contrast, Christiaans (2011) and Sasaki and Hoshida (2017) direct their analysis toward the Negative Population Growth Path (NPGP), where the output-capital ratio converges to zero and capital stock per effective labor becomes infinite in the long run.[4]

Drawing from the above observations, we apply the augmented Solow growth model by Mankiw, Romer, and Weil (1992), which considers the accumulation of human capital. Similarly to the approaches of Christiaans (2011) and Sasaki and Hoshida (2017), we explore a growth path that is specific to an economy experiencing negative population growth. We then explain the relationship between the rate of population decline and the growth rate of per capita income.

Our study reveals that, based on the parameter combinations, four types of dynamic paths emerge. In each dynamic path, the long-run growth rate of per capita income remains positive. This finding indicates that even in an economy experiencing negative population growth, sustained growth can still be achieved.

2 Model

The model aligns with the one presented by Mankiw, Romer, and Weil (1992). The production of final goods involves physical capital K, human capital H, and labor L. The production function adopts the Cobb–Douglas form, which exhibits constant returns to scale:

$$Y = K^{\alpha} H^{\beta} (AL)^{1-\alpha-\beta}, \quad 0 < \alpha < 1,\ 0 < \beta < 1,\ \alpha + \beta < 1 \tag{1}$$

$$\Longrightarrow y = k^{\alpha} h^{\beta}, \tag{2}$$

[3]Mino and Sasaki (2023) construct an endogenous growth model where the production of final goods requires exhaustible resources, in addition to physical capital and labor, and they explore the relationship between the population growth rate and the per capita income growth rate along the BGP. In the production function for final goods, they consider a positive externality caused by capital accumulation, which results in increasing returns to scale. They discover that along the BGP, positive growth of per capita income is not achieved when the population growth rate is negative.

[4]Daitoh and Sasaki (2023) conduct a detailed analysis of a canonical Ramsey–Cass–Koopmans model with negative population growth, and they find that the NPGP emerges depending on certain conditions.

where Y denotes output; A is the index of labor-augmenting technological progress; α is the output-elasticity of physical capital; and β is the output-elasticity of human capital. All parameters are larger than zero and less than unity. We define $y = Y/(AL)$, $k = K/(AL)$, and $h = H/(AL)$.

Let the population growth rate and labor-augmenting progress rate be n and g, respectively. Then, we have

$$\frac{\dot{L}}{L} = n < 0, \tag{3}$$

$$\frac{\dot{A}}{A} = g > 0. \tag{4}$$

Both growth rates are assumed to be constant. The population growth rate is negative.

Let the investment rate of physical capital and that of human capital be $s_k \in (0,1)$ and $s_h \in (0,1)$, respectively. Suppose that s_k and s_h are constant fractions of total output. Then, the dynamical equations of physical capital and human capital are as follows:

$$\dot{K} = s_k Y - \delta_k K, \tag{5}$$

$$\dot{H} = s_h Y - \delta_h H, \tag{6}$$

where $\delta_k \in (0,1)$ and $\delta_h \in (0,1)$ are the depreciation rates of physical and human capital, respectively.

Summarizing the above equations, the dynamical equations of k and h are as follows:

$$\dot{k} = s_k k^\alpha h^\beta - (n + g + \delta_k)k, \tag{7}$$

$$\dot{h} = s_h k^\alpha h^\beta - (n + g + \delta_h)h. \tag{8}$$

When $n + g + \delta_k < 0$ or $n + g + \delta_h$ holds, for $k > 0$ and $h > 0$, we have $\dot{k} > 0$ or $\dot{h} > 0$, which suggests that k or h continues to increase. In this case, the usual steady states of k and h do not exist, because $\dot{k} = 0$ or $\dot{h} = 0$ is never obtained, and we obtain the growth path specific to an NPGP.

The growth rates of k and h are given by

$$\frac{\dot{k}}{k} = s_k \frac{h^\beta}{k^{1-\alpha}} - (n + g + \delta_k), \tag{9}$$

$$\frac{\dot{h}}{h} = s_h \frac{k^\alpha}{h^{1-\beta}} - (n + g + \delta_h). \tag{10}$$

The growth rate of per capita income $g_{Y/L}$ is the sum of the growth rate of y and that of A, given by

$$g_{Y/L} = g + \alpha \left[s_k \frac{h^\beta}{k^{1-\alpha}} - (n + g + \delta_k) \right] + \beta \left[s_h \frac{k^\alpha}{h^{1-\beta}} - (n + g + \delta_h) \right]. \quad (11)$$

When $n + g + \delta_k < 0$ or $n + g + \delta_h < 0$, we cannot use the usual phase diagram analysis, as employed in Mankiw, Romer, and Weil (1992), because we cannot obtain $\dot{k} = 0$ or $\dot{h} = 0$. Therefore, considering equations (9) and (10), we introduce the following new variables:

$$x \equiv \frac{h^\beta}{k^{1-\alpha}}, \; z \equiv \frac{k^\alpha}{h^{1-\beta}}. \quad (12)$$

The differential equations of the newly introduced variables are given by

$$\dot{x} = x[-(1-\alpha)s_k x + \beta s_h z + C_1], \quad (13)$$

$$\dot{z} = z[\alpha s_k x - (1-\beta)s_h z + C_2], \quad (14)$$

where C_1 and C_2 are defined as follows:

$$C_1 = (1-\alpha)(n + g + \delta_k) - \beta(n + g + \delta_h), \quad (15)$$

$$C_2 = (1-\beta)(n + g + \delta_h) - \alpha(n + g + \delta_k). \quad (16)$$

The parameters C_1 and C_2 can be positive or negative, and the size relationship between them is ambiguous. Substituting x and z into equation (11), we obtain

$$g_{Y/L} = g + \alpha \left[s_k x - (n + g + \delta_k) \right] + \beta \left[s_h z - (n + g + \delta_h) \right]. \quad (17)$$

To draw the phase diagram of (x, z), we find the loci of $\dot{x} = 0$ and $\dot{z} = 0$:

$$\dot{x} = 0 \Longrightarrow z = \frac{(1-\alpha)s_k}{\beta s_h} x - \frac{C_1}{\beta s_h}, \quad (18)$$

$$\dot{z} = 0 \Longrightarrow z = \frac{\alpha s_k}{(1-\beta)s_h} x + \frac{C_2}{(1-\beta)s_h}. \quad (19)$$

These are straight lines with positive slopes. The slope of $\dot{x} = 0$ is steeper than that of $\dot{z} = 0$. Both intercepts can be positive or negative.

3 Analysis

We obtain four outcomes, depending on the intercepts of the two straight lines.

3.1 Case 1

In Case 1, we define $n < 0$, but its absolute value is relatively small; hence, both $n + g + \delta_k > 0$ and $n + g + \delta_h > 0$ hold. Case 1 is the same as the case examined by Mankiw, Romer, and Weil (1992). Here, both $C_1 > 0$ and $C_2 > 0$ hold, and considering the intercepts of $\dot{x} = 0$ and $\dot{z} = 0$, we obtain Figure 1.

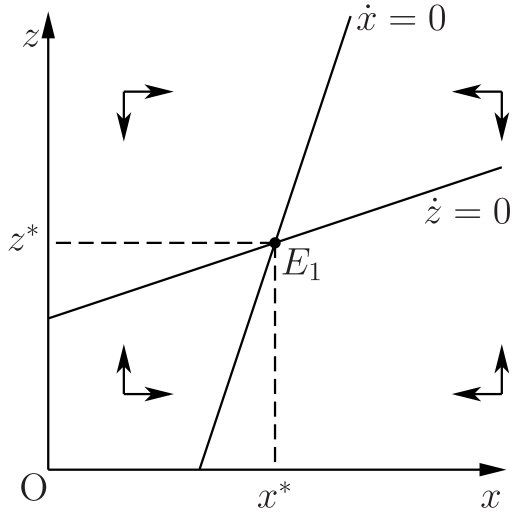

Figure 1: Phase diagram in Case 1

Both straight lines have an intersection, which gives the steady state in Case 1, E_1:

$$x^* = \frac{n + g + \delta_k}{s_k} > 0, \tag{20}$$

$$z^* = \frac{n + g + \delta_h}{s_h} > 0. \tag{21}$$

From Figure 1, the steady state is stable. The long-run growth rate of per capita income $g^*_{Y/L}$ is equal to the labor augmenting technological progress rate:

$$g^*_{Y/L} = g > 0. \tag{22}$$

3.2 Case 2

In Case 2, we define $n < 0$ and its absolute value is relatively large; hence, both $n + g + \delta_k < 0$ and $n + g + \delta_h > 0$ hold.[5] In this case, we obtain $C_1 < 0$ and $C_2 > 0$, and the phase diagram is shown in Figure 2.

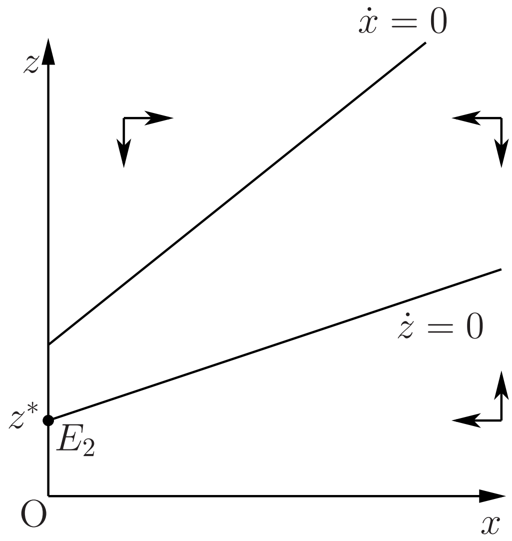

Figure 2: Phase diagram in Case 2

From Figure 2, the economy converges to the corner solution E_2, and the long-run situations are as follows:

$$x^* = 0, \tag{23}$$

$$z^* = \frac{(1-\beta)(n+g+\delta_h) - \alpha(n+g+\delta_k)}{(1-\beta)s_h} > 0. \tag{24}$$

From equation (17), the long-run growth rate of per capita output is given by

$$g^*_{Y/L} = g - \frac{\alpha}{1-\beta}\underbrace{(n+g+\delta_k)}_{-} > 0. \tag{25}$$

3.3 Case 3

In Case 3, we define $n < 0$ and its absolute value is relatively large; hence, both $n + g + \delta_k > 0$ and $n + g + \delta_h < 0$ hold. In this case, we have both $C_1 > 0$ and $C_2 < 0$,

[5]The condition $n+g+\delta_k < 0$ can be rewritten as $n < -(g+\delta_k)$, which suggests that the population declining rate is relatively large. For example, if $g = 0.01$ and $\delta_k = 0.03$, we need n smaller than -4%. Jones (2022) criticizes this as unrealistic. For additional discussion, see footnote 8 in Section 3.

and the phase diagram is shown in Figure 3.

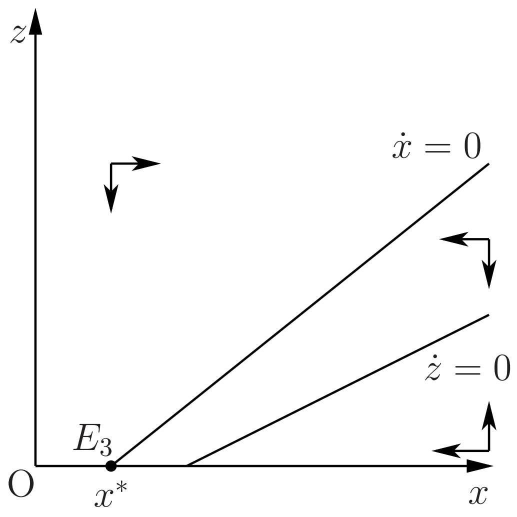

Figure 3: Phase diagram in Case 3

From Figure 3, the economy converges to the corner solution E_3, and the long-run situations are as follows:

$$x^* = \frac{(1-\alpha)(n+g+\delta_k) - \beta(n+g+\delta_h)}{(1-\alpha)s_k}, \tag{26}$$

$$z^* = 0. \tag{27}$$

From equation (17), the long-run growth rate of per capita income is given by

$$g^*_{Y/L} = g - \frac{\beta}{1-\alpha} \underbrace{(n+g+\delta_h)}_{-} > 0. \tag{28}$$

3.4 Case 4

In Case 4, we define $n < 0$ and its absolute value is relatively large; hence, both $n + g + \delta_k < 0$ and $n + g + \delta_h < 0$ hold. Based on the sizes of C_1 and C_2, Case 4 is divided into four sub-cases: (i) when $C_1 < 0$ and $C_2 > 0$; this sub-case is the same as Case 2; (ii) when $C_1 > 0$ and $C_2 < 0$; this sub-case is the same as Case 3; (iii) when $C_1 > 0$ and $C_2 > 0$; this sub-case is impossible as long as $\alpha + \beta < 1$; and (iv) when $C_1 < 0$ and $C_2 < 0$, we obtain the phase diagram shown in Figure 4.

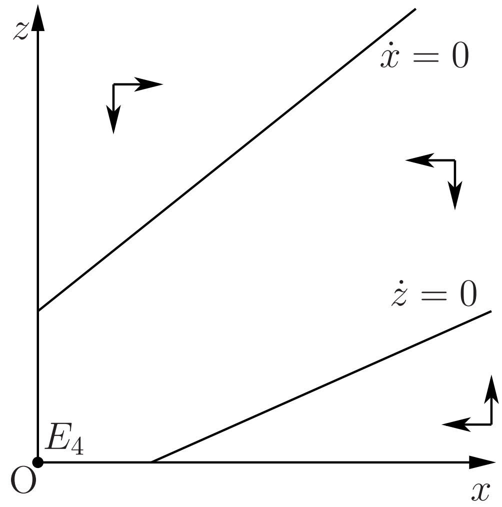

Figure 4: Phase diagram for Case 4 with $C_1 < 0$ and $C_2 < 0$

From Figure 4, we find that the economy converges to the origin, E_4, and the long-run situations are as follows:

$$x^* = z^* = 0. \tag{29}$$

From equation (17), the long-run growth rate of per capita output is given by

$$g^*_{Y/L} = g - \alpha \underbrace{(n + g + \delta_k)}_{-} - \beta \underbrace{(n + g + \delta_h)}_{-} > 0. \tag{30}$$

3.5 Numerical examples

Which case is realistic? Using data on the Japanese economy, we present a simple numerical example. We set the parameters as follows:

$$n = -0.0019,\ \alpha = 0.14,\ \beta = 0.37,\ \delta = 0.03,\ g = 0.0067. \tag{31}$$

For the population growth rate, we use the long-run economic statistics of the Annual Report on the Japanese Economy and Public Finance 2021. The annual average rate of population decline in 2010–2020 is 0.19%, and hence, we have $n = -0.0019$. The parameters α and β are taken from Mankiw, Romer, and Weil (1992). For the depreciation rates, we suppose that $\delta_k = \delta_h$, and we set $\delta = 0.03$. This value is suggested by Jones (2022). The labor augmenting technological progress rate is taken from the data of The Japan Productivity Center. We use the average growth rate of it during the period 2000–2020, $g = 0.0067$.[6] With these parameters, we have both $n+g+\delta_k > 0$ and $n+g+\delta_h > 0$, which corresponds to Case 1: $g = 0.0067 = 0.67\%$.[7]

This numerical example suggests that, given the current rate of population decline, the Japanese economy does not achieve NPGPs in Cases 2–4. Daitoh (2020) and Daitoh and Sasaki (2023) propose a solution to this problem. They recommend incorporating child-rearing costs, as outlined by Barro and Sala-i-Martin (2003, ch. 9), into growth models. This approach could potentially yield an NPGP under a realistically plausible rate of population decline.[8] Therefore, if we consider child-rearing costs, we might obtain Cases 2–4. This important topic is left to future research.

4 Conclusion

This study examines the issue of a decreasing population within the context of the augmented Solow growth model by Mankiw, Romer, and Weil (1992), which incorporates human capital accumulation. The study investigates whether the long-run growth rate of per capita income remains positive when the population growth rate is negative. The analysis reveals four potential scenarios based on the parameter sizes. In each scenario, the long-run growth rate of per capita income is positive.

References

Barro, R. J. and Sala-i-Martin, X. (2003) *Economic Growth*, 2nd edition, MIT Press: Cambridge MA.

Bucci, A. (2023) "Can a Negative Population Growth Rate Sustain a Positive Economic Growth Rate in the Long Run?" *Mathematical Social Sciences* 122, pp. 17–28.

[6]The Japan Productivity Center provides data on TFP growth rate based on the Cobb–Douglas production function, with Hicks neutral technological progress. We convert it to the labor augmenting technological progress rate as $g = \dot{A}/A = 1/(1-\alpha)\,(\dot{B}/B)$, where $\dot{B}/B = 0.0058$ is the TFP growth rate with Hicks neutral technological progress rate.

[7]In Japan, during the period 2000–2020, the annual average growth rate of per capita output was about 0.5%.

[8]This is explained as follows: Let b denote the child-rearing costs directly proportional to per capita capital stock k. The dynamical equation of k is given by $\dot{k} = sf(k) - [g+\delta+(1+b)n]k$, where s denotes the saving rate of households. When the population growth rate is negative and $n < -(g+\delta)/(1+b)$, for $k > 0$, we have $\dot{k} > 0$ and, hence, we obtain the NPGP. Comparing the case with $b > 0$ and $b = 0$, we find that depending on the size of $b > 0$, we can obtain $n < -(g+\delta)/(1+b)$ under a realistically plausible size of $n < 0$.

Christiaans, T. (2011) “Semi-Endogenous Growth When Population is Decreasing,” *Economics Bulletin* 31 (3), 2667–2673.

Daitoh, I. (2020) “Rates of Population Decline in Solow and Semi-Endogenus Growth Models: Empirical Relevance and the Role of Child Rearing Cost,” *The International Economy* 23, pp. 218–234.

Daitoh, I. and Sasaki, H. (2023) “Ramsey–Cass–Koopmans Model with Declining Population,” Kyoto University, Graduate School of Economics Discussion Paper Series, No. E-23-002.

Elgin, C. and Tumen, S. (2012) “Can Sustained Economic Growth and Declining Population Coexist?” *Economic Modelling* 29, pp. 1899–1908.

Jones, C. I. (1995) “R & D-Based Models of Economic Growth,” Journal of Political Economy 103 (4), pp. 759–784.

Jones, C. I. (2022) “The End of Economic Growth? Unintended Consequences of a Declining Population,” *American Economic Review* 112, pp. 3489–3527.

Lucas Jr., R. E. (1988) “On the Mechanics of Economic Development,” *Journal of Monetary Economics* 22, pp. 3–42.

Mankiw, N. G., Romer, D., and Weil, D. N. (1992) “A Contribution to the Empirics of Economic Growth,” *The Quarterly Journal of Economics* 107 (2), pp. 407–437.

Mino, K. and Sasaki, H. (2023) “Long-Run Consequences of Population Decline in an Economy with Exhaustible Resources,” *Economic Modelling* 121, 106212.

Prettner, K. (2019) “A Note on the Implications of Automation for Economic Growth and the Labor Share,” *Macroeconomic Dynamics* 23, pp. 1294–1301.

Romer, P. M. (1990) “Endogenous Technological Change,” *Journal of Political Economy* 98 (5), pp. 71–102.

Sasaki, H. (2023) “Growth with Automation Capital and Declining Population,” *Economics Letters* 222, 110958.

Sasaki, H., and Hoshida, K. (2017) “The Effects of Negative Population Growth: An Analysis Using a Semi-endogenous R&D Growth Model,” *Macroeconomic Dynamics* 21 (7), pp. 1545–1560.

Solow, R. M. (1956) “A Contribution to the Theory of Economic Growth,” *The Quarterly Journal of Economics* 70 (1), 65–94.